A SEARCH FOR A CONSISTENT MODEL FOR THE ELECTROMAGNETIC SPECTRUM OF THE CRAB NEBULA*

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Abstract. An attempt is made to search for a consistent model to explain the electromagnetic spectrum of the Crab nebula (Tau A). It is assumed that there is a continuous injection of electrons at the centre of the nebula with an energy spectrum $E^{-1.54}$ as evidenced by radio data. This spectrum must steepen to a slope larger than 2 at some energy E_i in order to ensure that the energy input into electrons remains finite. The spectrum must also steepen beyond an energy E_c depending on the magnetic field because of synchrotron energy losses. Two types of models are considered: Class I, in which the whole nebula is characterised by a uniform magnetic field, and Class II, in which besides the general field H_0, small filamentary regions of strong field H_s are postulated.

In models of Class I, the best fit to the observed data is obtained when $E_i > E_c$ and $H_0 \simeq 5 \times 10^{-4}$ gauss. However, this predicts a decrease in X-ray source size beyond $\sim$ 40 KeV. There are two possibilities of Class II model depending on the residence time of electrons in strong field regions being small or large. The former case explains the flattening in the optical spectrum.

Experiments to distinguish between the various models are indicated.

1. Introduction

There have been many attempts to understand the electromagnetic spectrum of the Crab Nebula (Shklovski, 1968; Apparao, 1967; Tucker, 1967; Burbidge and Hoyle, 1969), which has been studied over a wide range of frequencies, ranging in the radio region from $\sim 10^7$ Hz to the hard X-ray region $\sim 10^{20}$ Hz; at higher frequencies, up to $\sim 10^{29}$ Hz, useful upper limits have been placed on the flux. The observations are summarised in Figure 1. In this paper we discuss a number of consistent models for interpreting the observed spectrum, and indicate future measurements which might distinguish between them.

A recent study of the optical spectra of the moving wisps in the Crab nebula (Scargle, 1969) indicates a continuous activity near the centre of the nebula and a possible renewal of the relativistic electrons. Also a pulsar, NP0532, has been identified at this site and is the probable cause of this activity. This pulsar has a period of $\sim$30 ms and a characteristic slowing down time of $\sim$2000 years. It has been suggested (Gold, 1968) that pulsars would be important sources of cosmic rays and that the above rate of slowing down could be due to emission of relativistic particles at a rate 10^{37-38} ergs/sec. Therefore, in the following discussions we shall assume that there is a continuous injection of high-energy electrons at the centre of Crab.

The radio spectrum, from 10 to 1000 MHz, has an index of $\sim$0.27, indicating a slope of $\sim$1.54 for the electron spectrum. The injection spectrum of electrons should

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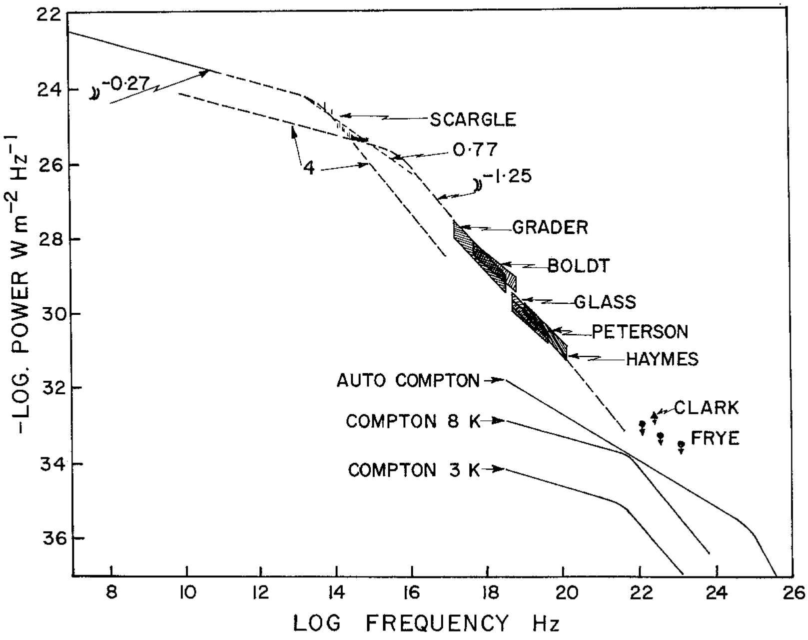

Fig. 1. The available spectral data on the Crab in the frequency interval 10^7–10^{26} Hz are summarised. The optical data, corrected for interstellar absorption ($A_v = 2$; Crab distance = 1.8 Kpc), are taken from Scargle (1969). The X-ray and the γ-ray data are from Clark *et al.* (1968); Frye and Wang (1969); Grader *et al.* (1966) and (1969); Boldt *et al.* (1969); Glass (1969); Peterson *et al.* (1966) and Haymes *et al.* (1968). The γ-ray spectra arising from Compton scattering of the synchrotron quanta (auto-Compton) and the 3 K and sub-millimeter quanta (Houck and Harwit, 1969) are also plotted for a magnetic field of 5×10^{-4} gauss. An assumption of substantially smaller field would contradict the γ-ray upper limits (see text, model 3). The dashed curves marked (4) represent the contributions to the spectrum from the general and strong field regions as per model 4.

also have the same slope, since with a field of $\sim 5 \times 10^{-4}$ gauss, the spectrum of electrons at low energies (~ 1 GeV) remains unaltered during the lifetime (~ 900 yrs) of Crab. However, in order that the total energy of the injected electrons be finite, the injection spectrum must steepen at some high energy E_i to a logarithmic slope greater than 2.

Relativistic electrons suffer energy losses mainly because of synchrotron radiation and betatron deceleration in the expanding envelope. With continuous injection (Kardashev, 1962) the electrons at present would have the same spectral slope as that of the injected spectrum, up to an energy $E_c = 4/bT$, T being the age of the nebula (~ 900 yrs) and $b \simeq 2e^4 \langle H^2 \rangle / 3m^4c^7 = 1.95 \times 10^{-9} \langle H^2 \rangle / mc^2$. Beyond E_c the spectral slope increases by unity.

The resulting spectrum may again be injected into localised regions of stronger magnetic field. Then, if b_s is the characteristic energy loss parameter in this field and τ the containment time therein, the spectrum will experience a further steepening by one in the index at energies beyond $1/b_s\tau$.

2. The Models

We will consider two broad classes of models viz. class I, in which the magnetic field is reasonably uniform over the whole nebular volume and the continuous injection occurs at a point (close to the pulsar) near its centre; class II, in which there is a general magnetic field H_0 over the volume, but in addition, there are filamentary regions throughout the volume where the value of the magnetic field H_s is much higher. Clearly either of these model types is an idealised representation of the actual situation.

2.1. MODELS OF CLASS I

Three possibilities exist, which provide different levels of fit to the experimental data. These models are basically distinguished by the criteria that the steepening (at E_i) in the injection spectrum occurs at the same energy, earlier or later than the steepening (at E_c) due to synchrotron energy loss.

(1) $E_i \simeq E_c$: In this case, the slopes of the optical and X-ray spectra will be the same. It may be barely possible to reconcile the experimental data with a single power law spectrum $\nu^{-1.1}$. Then we have to choose E_c to correspond to a frequency $\sim 5 \times 10^{13}$ Hz (see Figure 1) and thus obtain $H_0 \simeq 5 \times 10^{-4}$ gauss. The injection spectrum beyond E_i will have to have a slope of 2.2, which, because of synchrotron-steepening becomes 3.2, giving a slope of $(3.2-1)/2=1.1$ for the synchrotron radiation spectrum.

(2) $E_i > E_c$: In this case we have to put E_c at the same place as in the last case, giving a break at $\sim 5 \times 10^{13}$ Hz and a magnetic field $H_0 = 5 \times 10^{-4}$ gauss. The optical spectrum would then have an index $(1.54+1-1)/2 = 0.77$, which is not inconsistent with experiment. If E_i now corresponds to a frequency $\sim 1.6 \times 10^{16}$ Hz, the appropriate slope of the X-ray spectrum can then be explained by a suitable choice of the spectral slope of the injected electrons beyond E_i.

(3) $E_i < E_c$: The injection spectrum could steepen to a slope of ~ 2.5, e.g., at E_i corresponding to $\sim 5 \times 10^{13}$ Hz, giving an optical index of 0.75. Synchrotron-steepening at E_c corresponding to 1.6×10^{16} Hz would then lead to an X-ray spectrum with an index equal to 1.25. The required magnetic field in this case would be lower than in the previous cases by a factor $(1.6 \times 10^{16}/5 \times 10^{13})^{1/3} \simeq 7$. Thus $H_0 \simeq 7 \times 10^{-5}$ gauss.

2.2. GENERAL COMMENTS ON MODELS OF CLASS I

Model 3 requires a very low value of the magnetic field. As a result (see Table I) the energy density of the electrons exceeds by a large factor, the energy density in the magnetic field. Also, the required intensity of electrons is so high that auto-Compton process (scattering of synchrotron photons by high energy electrons) would predict high γ-ray fluxes which would contradict the experimental limits (Clark *et al.*, 1968; Frye and Wang, 1969). Therefore model 3 is not acceptable.

In models 1 and 2, with a field of 5×10^{-4} gauss the half-life of electrons radiating at energies ~ 40 KeV is $\sim \frac{1}{2}$ year, leading to a source diameter of 1–2 light years, which is in agreement with the measured size. However, the expected size would decrease with increasing X-ray energy. This would be an important point to check by experiment.

TABLE I

Model	Optical index	X-ray index	$\nu(E_c)$ (Hz)	$\nu(E_i)$ (Hz)	H_0 (gauss)	H_s (gauss)	$\int EN_e(E)\,\mathrm{d}E$ (erg/cm^3)	$\frac{1}{8\pi}\int H^2\,\mathrm{d}V$ (erg/cc)
1	1.1	1.1	5×10^{13}	5×10^{13}	5×10^{-4}	–	1.6×10^{-9}	10^{-8}
2	0.77	1.25	5×10^{13}	1.6×10^{16}	5×10^{-4}	–	1.9×10^{-9}	10^{-8}
3	0.77	1.25	1.6×10^{16}	5×10^{13}	7×10^{-5}	–	4.5×10^{-7}	2×10^{-10}
4	1.1	1.25	5×10^{13}	5×10^{13}	2.5×10^{-4}	2.5×10^{-2} in 7×10^{-4} of the volume	4.5×10^{-9}	1.8×10^{-8}
5	0.77	1.27	5×10^{13}	cuts off at 10^{17}	1.7×10^{-4}	1.7×10^{-1} in 2.5×10^{-5} of the volume	8.3×10^{-9}	2.9×10^{-8}

Model 1 invokes a sort of accident that $E_i \simeq E_c$ and, in any case, gives a rather poor representation of the X-ray data. Therefore we feel that Model 2 is the best candidate among the class I models.

2.3. MODELS OF CLASS II

Two situations are possible depending on whether the residence time in the localised strong field regions is small or large. We would discuss these two cases in models 4 and 5 respectively.

(4) τ Small: In this case we take $E_i \simeq E_c$ and the injection slopes for electron spectrum as 1.54 and 2.5 on the two sides of E_i. The spectrum beyond the synchrotron-break ($\sim 5 \times 10^{13}$ Hz) would then have an index of $(2.5+1-1)/2 = 1.25$. This spectrum does not match the X-ray intensity, and for this we need the 'optical' electrons to traverse a magnetic field which is ~ 100 times stronger than the general field occupying $\sim 7 \times 10^{-4}$ of the nebular volume. In order that E_c corresponds to a frequency of $\sim 5 \times 10^{13}$ Hz in the general field, we need $H_0 \simeq 2.5 \times 10^{-4}$ gauss and $H_s \simeq 2.5 \times 10^{-2}$ gauss. The spectral contributions from the general and strong field regions are shown in Figure 1. The total spectrum reproduces the observed flattening of the optical spectrum beyond 3×10^{14} Hz.

In order that the spectrum of X-ray emitting electrons does not steepen in the strong field region at least up to energies corresponding to $\sim 10^{19}$ Hz, the upper limit on the residence time of electrons in these regions should be 5×10^4 sec. This implies a diameter of 0.1″ of arc for the condensations and there is some evidence for such fine filamentary structures in the nebula.

(5) τ Large: If the size of high field regions is about 1″ of arc, corresponding to the measured sizes of some of the major filaments and wisps in the nebula, then the residence time of electrons in the strong field regions would exceed 10^6 sec. Here, we should have $E_i \gg E_c$, which is taken to correspond to $\sim 5 \times 10^{13}$ Hz, so that the spectrum of the optical photons generated in the general field has a slope ~ 0.77. The electron spectrum corresponding to this, having a slope 2.54, relaxes to a slope of 3.54 in the regions of high magnetic field where the X-ray photons are, therefore, generated with a slope 1.27. In order that the electrons in the general field, having a spectral slope of 2.54 at high energies do not contribute to the X-rays, the injection has to be terminated at energies corresponding to $\lesssim 10^{17}$ Hz. To reproduce the observed breaks and intensities we need $H_0 \simeq 2.5 \times 10^{-4}$ gauss and $H_s \simeq 2.5 \times 10^{-1}$ gauss, the latter field occupying $\sim 2.5 \times 10^{-5}$ of the nebular volume.

3. Discussion

The properties and parameters characterising various models are listed in Table I.

It is clear that if cosmic-ray energy density is not to exceed substantially the magnetic energy density, acceleration of electrons should be preferred over the acceleration of protons; it is to be remembered that protons, not being subject to synchrotron energy losses, add much more to the energy inventory for the same energy at injection.

For reasons already discussed, the preferred models are numbers 2, 4 and 5. Model 2 predicts an energy-dependence of size beyond ~40 KeV. Model 4 predicts correctly the flattening of the spectrum around and beyond 3×10^{14} Hz and would also imply a steepening of X-ray spectrum beyond 10^{19} Hz. Experiments bearing on these characteristic features are suggested.*

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* Burbidge and Hoyle (1969) have argued for the existence of highly condensed objects in Crab, about 100 in number, having fields of 10^3–10^5 gauss in their envelopes. We fear that such a requirement (of high field) arose because of a possible numerical error in estimating the energy of electrons which radiate in the optical region in a field of 5×10^{-4} gauss. This energy should be in the range 150–500 GeV rather than ~ 10 GeV as they have used.