IP/BBSR/92-38

May, 92

SYMMETRIES OF STRING EFFECTIVE ACTION AND SPACE-TIME GEOMETRY

S. Pratik Khastgir⋆

Institute of Physics, Bhubaneswar-751005, INDIA

and

Jnanadeva Maharana†

California Institute of Technology, Pasadena, CA 91125

Abstract

Two dimensional charged black hole solution is obtained by implementing an $O(2,2)$ transformation on the three dimensional black string solution. Two different monopole backgrounds in five dimensions are related through an $O(2,2)$ transformation. It has been shown in these examples that the particular $O(2,2)$ transformation corresponds to duality transformation.

⋆ e-mail: pratik%iopb@shakti.ernet.in

† Permanent address:Institute of Physics, Bhubaneswar-751005, INDIA

e-mail: maharana@theory3.caltech.edu until June 30.

I. Introduction

The string theory offers the prospect of unifying all the four fundamental forces of nature. In the first quantized framework, one considers the evolution of the string in the background of its massless modes. This approach has proved to be quite powerful in undrstanding various aspects of the string theory. The requirement of conformal invarience of string theory imposes stringient constraints on the configurations of the background fields. In other words, if we require the beta-functions associated with these backgrounds to vanish, then we arrive at differential equations known as the equations motion for the background fields. It can be argued that so long as we are interested in the low energy effects of the string theory, it suffices to take into account the effects of the massless modes of the string. We can construct the tree level string effective action involving only the massless excitations in such a way that the equations of motion derived from the effective action exactly correspond to the requirements of the vanishing of the beta-functions [1] as discussed above.

Recently, there has been an extensive study of the properties of the string effective action in order to explore the consequences of string theory in cosmology [2] as well as to investigate properties of black holes [3]. Other interesting types of space-time structures like strings and branes [4,5] have also emerged in this investigation. The monopole and dyon [6,7] solutions were also obtained in this context. Furthermore many of these solutions can be shown to correspond to exact conformal field theory [5,8,9]. In this context, a rich symmetry structure of the string effective action has been unravelled. Indeed, these symmetry properties have played an important role in discovering new vacuum configurations of the string theory (in general they correspond to inequivalent string vacua). These vacuum configurations are identified as different spacetime geometries.

We may recall that target space duality is a very important symmetry property of the string theory. A familiar example is the R duality [10]: when we consider a compactified string on a circle of radius R, the spectrum of this string remains invariant under the transformation $R \rightarrow \lambda_s^2/R$ ($\lambda_s^2 = 2\alpha'\hbar$). The consequences of duality are far reaching than that meets the eye and have been explored in several directions. It has led to introduction of a minimum compactification scale. The idea of duality has been employed to restrict the form of scalar (or super) potentials and to study nonperturbative supersymmetry breaking. Furthermore, the consequences of duality in cosmology [11] have turned out to be quite interesting and imoprtant.

Another important symmetry property of the string efective action has been discovered by Meissner and Veneziano [12] when the massless backgrounds are only allowed to depend on time. It was shown that that the effective action is invaraint under global $O(d,d)$ transformations where d is the spatial dimension and spacetime dimension, $D = d+1$. Subsequently, it was shown that one can generate new cosmological solutions [13] from a given solution. In particular, it was demonstrated that it was possible to generate spacetime with nontrivial geometries by implementing $O(d,d)$ transformations on an initial background with trivial (flat geometries [13]). Furthermore, a large class of black hole solutions have been generated through suitable choice of $O(d,d)$ transformations [14].

The purpose of this article is to explore further the consequences of the $O(d,d)$ transformations in string theory. We demonstrate that given a background space-time geometry which satisfies vanishing beta-function constraints, it is possible to generate a new class of background configurations through $O(d,d)$ transformations. It is shown that a two dimensional charged black hole solution [9] can be obtained by implementing a suitable $O(d,d)$ transformation on the three dimensional black string solution of Horne-Horowitz [5]. Furthermore, we explicitly demostrate that starting from a generalized Kaluza-Klein monopole solution obtained by Sorkin and Gross and Perry [6] and studied by Banks et al. [15], in the context of string theory, another solution of Ref. [15] can be generated by an appropiate $O(d,d)$ transformation. Moreover, we are able to provide an interpretation of these transformations from the duality point of view discussed in Refs. [16,17].

The paper is organized as follows: In section II we recall some of the important results of Meissner and Veneziano in the context of $O(d,d)$ symmetries of string effective action and discuss how to generate new solution. Then we describe briefly isometries of target space backgruond fields and their relation to duality transformations. Section III deals with the specific string theoretic models we are interested in and the configurations of the background fields. We introduce explicit $O(d,d)$ transformations and obtain the new background field configurations. The sigma model interpretation of the newly generated backgrounds is discussed. Next, we elucidate on the relations of our work with the results of Ref [16,17]. The summary and conclusions are given in Sec IV.

II. The symmetries of string effective action

In this section we shall try to summarize some of the important results obtained by Veneziano, Meissner, and Sen [12,14]. We shall also discuss the duality transformations briefly [16,17] at the end.

We consider a closed bosonic string in the background of its massless modes such as graviton, dilaton and antisymmetric tensor fields. The equations of motion of the tree level string effective action given below corresponds to the requirement of the vanishing of the beta-functions associated with these background fields.

$$S = -\int d^D x \sqrt{-detG} e^{-\phi} \left[R + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - V\right] \tag{1}$$

where V is the cosmological term proportional to $D-26$ (proportional to $D-10$ for superstring), ϕ is the dilaton field, $G_{\mu\nu}$ is the D-dimensional metric and $H_{\mu\nu\rho}$ is the field strength for the antisymmetric tensor field $B_{\mu\nu}$:

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + cyclic. \tag{2}$$

If the backgrounds G, B, and ϕ are functions of only one coordinate (say time t), then the metric G and the antisymmetric tensor B can be written in the following form implementing the general coordinate transformations and the Abelian gauge transformations on

G and B respectively.

$$G = \begin{pmatrix} -1 & 0 \\ 0 & \mathcal{G}(t) \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{B}(t) \end{pmatrix}, \tag{3}$$

where $\mathcal{G}(t)$ and $\mathcal{B}(t)$ are $d \times d$ matrices with $d = D - 1$. Then the reduced action obtained from eqn. (1) can be written in a manifestly $O(d,d)$ invariant form obtained by Meissner and Veneziano [12].

$$S = \int dte^{-\Phi}[V + \dot{\Phi}^2 + \frac{1}{8}Tr(\dot{M}\eta\dot{M}\eta)], \tag{4}$$

with

$$\Phi = \phi - ln\sqrt{det\mathcal{G}}, \tag{5}$$

$$M \equiv \begin{pmatrix} \mathcal{G}^{-1} & -\mathcal{G}^{-1}\mathcal{B} \\ \mathcal{B}\mathcal{G}^{-1} & \mathcal{G} - \mathcal{B}\mathcal{G}^{-1}\mathcal{B} \end{pmatrix}, \tag{6}$$

and

$$\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{7}$$

In last expression I stands for d-dimensional unit matrix.

The action (4) is manifestly invariant under the global $O(d,d)$ group [12] acting as,

$$\Phi \to \Phi, \qquad M \to \Omega M \Omega^T. \tag{8}$$

Here Ω is an $O(d,d)$ matrix satisfying

$$\Omega\eta\Omega^T = \eta. \tag{9}$$

This $O(d,d)$ transformations relate different string vacua (geometries) which are not equivalent in general. Recently, Sen [14] has shown in the frame work of string field theory that the space of solutions has $O(d) \times O(d)$ symmetry, which is a subgroup of $O(d,d)$. Moreover, it is argued that the diagonal subgroup $O(d)$ of $O(d) \times O(d)$ generates only spatial rotations. Thus, the coset is equivalent to $(O(d)\times O(d))/O(d)$ and has dimensionality

$d(d-1)/2$ in contrast to the time independent case where the dimensionality is d^2. It was shown [14] that $O(d) \times O(d)$ symmetry persists to all orders in string tesion $\alpha^{'}$. Recently, several interesting solutions (both cosmological [13] and black hole type [14]) were generated from known backgrounds exploiting this symmetry.

Next we briefly describe the isometry and duality [16,17]. Starting with graviton ($g_{\mu\nu}$), antisymmetric field tensor ($B_{\mu\nu}$), and dilaton (ϕ) background, which has a translational symmetry (isometry) in x, one can generate a dual background given by [16],

$$\tilde{g}_{xx} = \frac{1}{g_{xx}}, \qquad \tilde{g}_{ax} = \frac{B_{ax}}{g_{xx}}, \qquad \tilde{g}_{ab} = g_{ab} - \frac{g_{ax}g_{xb} + B_{ax}B_{xb}}{g_{xx}}$$

$$\tilde{B}_{ax} = \frac{g_{ax}}{g_{xx}}, \qquad \tilde{B}_{ab} = B_{ab} - \frac{g_{ax}B_{xb} + B_{ax}g_{xb}}{g_{xx}}, \qquad \tilde{\phi} = \phi - \ln g_{xx}, \tag{10}$$

where a and b run over all directions except x. The original background and its dual satisfy the same equations of motion. This duality of low energy field equations exists whether or not x is compact. Recently it has been shown [17] that if x is compact, the original solution and dual are both low-energy approximation to the same conformal field theory. This completes our recapitulation.

III. Applications of $O(d,d)$ transformation

In this section we present two explicit examples where $O(d,d)$ transformations are implemented and dual solutions are generated. The first example is in three dimensions and the other is in five dimensions. The similarities of these two examples will be distinct when we describe the examples.

Our starting point is a classical solution which was first obtained by Horne and Horowitz [5] as an exact conformal field theory. This result was obtained by gauging a one dimensional subgroup of $G = SL(2,R) \times R$. In this consruction, which is a generalisation of Witten's [8] construction, a free boson x is added to the theory. The action is,

$$S = \frac{1}{\pi}\int_{\Sigma} d^2\sigma \Big[\frac{k'\partial_+ r\partial_- r}{8r^2(1-\frac{M}{r})(1-\frac{Q^2}{Mr})} - (1-\frac{M}{r})\partial_+ t\partial_- t$$

$$+(1-\frac{Q^2}{Mr})\partial_+ x\partial_- x+\frac{Q}{M}(1-\frac{M}{r})(\partial_+ x\partial_- t-\partial_- x\partial_+ t)\Big] \tag{11}$$

The space-time metric has the form,

$$ds^2=-(1-\frac{M}{r})dt^2+(1-\frac{Q^2}{Mr})dx^2+(1-\frac{M}{r})^{-1}(1-\frac{Q^2}{Mr})^{-1}\frac{k'dr^2}{8r^2} \tag{12}$$

whereas the antisymmetric tensor field and the dilaton are given by,

$$B_{tx}=\frac{Q}{M}(1-\frac{M}{r}),\qquad \phi=-\ln r-\frac{1}{2}\ln\frac{k'}{2}, \tag{13}$$

Q and M are the axionic charge and mass per unit length of the black string. The k' is the WZW level. The central charge of the theory is $\frac{3k'}{k'-2}$. The equations of motion of the low energy string effective action,

$$S=\int d^3x\sqrt{detG}e^{-\phi}\big[R+G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{12}H^2+\frac{8}{k'}\big]. \tag{14}$$

are the conditions for vanishing of β-functions. Here $\frac{8}{k'}$ plays the role of cosmological constant.

We observe that the backgrounds are independent of two coordinates x and t. Thus one can perform an $O(2,2)$ transformation to generate a new solution and we take $\Omega\equiv O(2,2)$ (which is also an element of $O(2)\times O(2)$) as [14],

$$\Omega=\frac{1}{2}\begin{pmatrix} S+R & R-S \\ R-S & S+R \end{pmatrix}. \tag{15}$$

Note that $O(2)\times O(2)$ is a subgroup of $O(2,2)$ and S and R are $O(2)$ matrices given by,

$$S=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\qquad R=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{16}$$

We first identify the $x-t$ block of the metric and antisymmetric tensor,

$$\mathcal{G}=\begin{pmatrix} -(1-\frac{M}{r}) & 0 \\ 0 & (1-\frac{Q^2}{Mr}) \end{pmatrix},\qquad \mathcal{B}=\begin{pmatrix} 0 & \frac{Q}{M}(1-\frac{M}{r}) \\ -\frac{Q}{M}(1-\frac{M}{r}) & 0 \end{pmatrix}. \tag{17}$$

Now performing the $O(2,2)$ transformation on this block of metric and antisymmetric tensor, we obtain new $M' = \Omega^T M \Omega$, M defined earlier by eqn. (6). The new backgrounds $\mathcal{G}'$ and $\mathcal{B}'$ as,

$$\mathcal{G}' = \begin{pmatrix} -(1-\frac{M}{r}) + \frac{Q^2}{M^2}(1-\frac{M}{r})^2(1-\frac{Q^2}{Mr})^{-1} & \frac{Q}{M}(1-\frac{M}{r})(1-\frac{Q^2}{Mr})^{-1} \\ \frac{Q}{M}(1-\frac{M}{r})(1-\frac{Q^2}{Mr})^{-1} & (1-\frac{Q^2}{Mr})^{-1} \end{pmatrix}, \qquad \mathcal{B}' = 0. \quad (18)$$

The metric in three dimensions takes the form,

$$ds^2 = \big[-(1-\frac{M}{r}) + \frac{Q^2}{M^2}(1-\frac{M}{r})^2(1-\frac{Q^2}{Mr})^{-1}\big]dt^2 + \frac{2Q}{M}(1-\frac{M}{r})(1-\frac{Q^2}{Mr})^{-1}dtdx$$

$$+(1-\frac{Q^2}{Mr})^{-1}dx^2 + (1-\frac{M}{r})^{-1}(1-\frac{Q^2}{Mr})^{-1}\frac{k'dr^2}{8r^2}, \quad (19)$$

and the new dilaton is given by (recall $\Phi = \phi - \ln\sqrt{det\mathcal{G}}$, expression (8), remains invariant under this transformation),

$$\phi' = \phi - \frac{1}{2}\ln\frac{det\mathcal{G}}{det\mathcal{G}'} = -\ln r(1-\frac{Q^2}{Mr}) - \frac{1}{2}\ln\frac{k'}{2}. \quad (20)$$

If we make the coordinate transformation

$$r \rightarrow \frac{Q^2}{M} + M(1-\frac{Q^2}{M^2})\cosh^2 r, \quad (21)$$

with

$$\frac{Q}{M} = e, \qquad \frac{1}{2}(1-\frac{Q^2}{M^2}) = \frac{k'}{4}, \quad (22)$$

the metric elements will reduce to,

$$g_{xx} = \frac{r}{r-\frac{Q^2}{M}} \rightarrow \frac{\frac{Q^2}{M} + M(1-\frac{Q^2}{M^2})\cosh^2 r}{M(1-\frac{Q^2}{M^2})\cosh^2 r} = 1 + \frac{2e^2}{k'}\frac{1}{\cosh^2 r} \quad (23a)$$

$$g_{xt} = \frac{Q}{M}\frac{(r-M)}{(r-\frac{Q^2}{M})} \rightarrow \frac{Q}{M}\frac{M(1-\frac{Q^2}{M^2})(\cosh^2 r - 1)}{M(1-\frac{Q^2}{M^2})\cosh^2 r} = e\tanh^2 r \quad (23b)$$

$$g_{rr} \to \frac{k'}{2}, \quad etc.. \tag{23c}$$

The metric in three dimensions and the dilaton read,

$$ds^2 = \frac{k'}{2}dr^2 - \frac{k'}{2}\tanh^2 r dt^2 + 2e\tanh^2 r dx dt + (1 + \frac{2e^2}{k'}\frac{1}{\cosh^2 r})dx^2 \tag{24a}$$

and

$$\phi = -\ln\cosh^2 r + const. \tag{24b}$$

If we take x as a compact direction this is exactly the 2-dimensional charged black hole solution obtained originally by Ishibashi, Lie and Steif [9] by gauging a subgroup $U(1)$ of $SU(2) \times U_i(1)$, where i denotes the internal direction. The 2-dimensional metric is,

$$ds^2 = 2kdr^2 - 2k\tanh^2 r dt^2 \tag{25a}$$

and the gauge field,

$$A_r = 0, \quad A_t = \tanh^2 r. \tag{25b}$$

There are two scalar backgrounds such as dilaton and the “Higgs” field given by,

$$\phi = -\ln\cosh^2 r + const., \qquad \psi = (1 + \frac{e^2}{2k}\frac{1}{\cosh^2 r}). \tag{25c}$$

We mention in passing that the parameter k', appearing in the WZW action given by eqn. (11) is related to the parameter of Ishibashi *etal*. as $k' = 4k$. As a consequence the cosmological constant is $\frac{8}{k'}$ in eqn. (14) whereas it is $\frac{2}{k}$ in the action of Ref. [9]. The relation between the three dimensional black string solution and the charged black hole solution can be envisaged from the point of view of duality transformations. Since the backgrounds are independent of the coordinate x, there is x translation symmetry. Thus one can apply the duality transformations on (12,13) and get (19,20). The equivalence between momentum and axionic charge was discussed in Ref. [18] by using the arguments of duality transformation. In (21) if one does not assume x to be compact one can think the g_{xt} term in eqn. (24a) as momentum along x direction. The $O(2,2)$ transformation we have used in this case is equivalent to duality transformation (10). Although we can arrive at solution (25) from solution (12) through an $O(d,d)$ or duality trnsformation but the underlying conformal field theories in two cases are very different because in eqn. (12) x is non-compact whereas in eqn. (24) the x direction was taken to be compact to arrive at solution (25) (Ref. [17]).The space-time geometry and other properties of these solutions are extensively discussed in Refs. [5,9].

Now we consider the construction of monopole background in the string effective action discussed by Banks *etal*. [15]. The string effective action is a generalisation of the Kaluza-Klein theory considered by Sorkin, Gross and Perry [6]. The string effective action could arise from the following scenario for a closed string. We can have a configuration as envisaged by Gaspirini, Maharana and Veneziano [13], that 21 space dimensions of the bosonic string are flat and of the remaining five dimensions, one corresponding to x^{22}, is compactified with radius R_0. The five dimension theory (before compactification) is endowed with graviton, antisymmetric tensor field and a dilaton satisfying the equations of motion required to satisfy conformal invarance. When the coordinate x^{22} is compactified the massless spectrum consists of a spacetime graviton ($g_{\mu\nu}$), antisymmetric field ($b_{\mu\nu}$), and two gauge fields A_μ and B_μ coming from five dimensional metric and antisymmetric tensor field. Moreover, there are two scalar fields Φ and $g_{55} = R$. The equations of motion for the background fields are derived from the action,

$$S = \int d^5x\sqrt{detG}e^{-\phi}\left[R^{(5)} + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}H^2\right], \tag{26}$$

where $R^{(5)}$ is the five dimensional scalar curvature. The indices should run over five dimensions. The specific background fields which are cosistent with the vanishing β-function conditions are given by

$$ds^2 = -dt^2 + R^2(dx^5 + A_\phi d\phi)^2 + \frac{1}{R^2}(dr^2 + r^2 d\Omega^2), \tag{27}$$

where $A_\phi = R_0 \sin^2(\frac{\theta}{2})$ and $R^2 = (1 + \frac{R_0}{2r})^{-1}$. The dilaton $\Phi = \Phi_0 = const.$, and the antisymmetric tensor field $B = 0$. The direction x^5 is compact. So A_ϕ here is like a gauge field. This solution was obtained by solving $R_{MN} = 0$, where M and N run over all the five coordinates, so in five dimension it is a Ricci flat solution. One can verify that this field configuration satisfies the equations of motion following from the effective action (26). The above metric is independent of three coordinates: t, x, and ϕ. So one has the $O(3,3)$ symmetry. $O(3,3)$ rotation can be performed on $t - \phi - x^5$ block and new solution can be generated. Again we use the same Ω (defined earlier in (15)). The $O(2) \times O(2)$ is a subgroup of $O(3,3)$. We want to perform this transformation on the $\phi - x^5$ block of metric. The $\phi - x^5$ block of the metric (27) reads,

$$\mathcal{G} = \begin{pmatrix} r^2 \sin^2\theta R^{-2} + A_\phi^2 R^2 & A_\phi R^2 \\ A_\phi R^2 & R^2 \end{pmatrix}, \qquad \mathcal{B} = 0. \tag{28}$$

After the action of Ω on this the new generated metric and antisymetric field tensor are

$$\mathcal{G}' = \begin{pmatrix} r^2 \sin^2\theta R^{-2} & 0 \\ 0 & R^{-2} \end{pmatrix}, \qquad \mathcal{B}' = \begin{pmatrix} 0 & A_\phi \\ -A_\phi & 0 \end{pmatrix}, \tag{29a}$$

and new dilaton is,

$$\Phi = \Phi_0 + \ln(\frac{1}{R^2}). \tag{29b}$$

So the full metric and torsion field can be written as,

$$ds^2 = -dt^2 + \frac{1}{R^2} {dx^5}^2 + \frac{1}{R^2}(dr^2 + r^2 d\Omega^2), \qquad B_{\phi 5} = A_\phi \tag{30}$$

with $R^2 = (1 + \frac{R_0}{2r})^{-1}$. This is exactly the dual solution obtained by Banks *etal.* [15] using the symmetry properties of the effective action. It is interesting to note that this solution has non trivial antisymmetric field tensor component as well as non trivial dilaton, whereas in the original solution both this fields were trivial. This solution is similar to the original solution of first example. We observe that in both the cases the particular $\Omega(O(2) \times O(2))$ interchanges gauge fields and antisymmetric field tensor. Again solution

(30) can be obtained from the Sorkin, Gross and Perry solution (27) using the duality transformations (10). For this one uses the translation symmetry of x^5 and assumes $x \equiv x^5$ in (10). The dual solution is also a monopole solution and this time it is magnetic. Its properties etc are discussed in [15].

IV. Summary and Conclusions

The $O(d,d)$ transformations transform a given string background geometry to another (in general inequivalent) geometry. Here we have presented two examples where appropiate $O(d,d)$ transformation corresponds to duality transformation eqn. (10). The $O(2,2)$ transformations employed by us interchange the gauge field and the antisymmetric tensor field in both the cases. In the first example we show that the axionic charge (in a given background configuration) can be transformed in another background with electric charge in a lower dimension.We are also able to relate two different backgrounds (12) and (25), which were obtained very differently, via $O(2,2)$ (equivalently duality) transformation.

Acknowledgements: One of us (JM) would like to acknowledge useful discussions with John Schwarz and he would like to thank the High Energy Physics group, especially John Schwarz, for warm hospitality at Caltech.

References:

[1] C. G. Callan, D. Friedan, E. J. Martinec and M. J. Perry, *Nucl. Phys.* **B262** (1985) 593; A. Sen, *Phys. Rev. Lett.* **55** (1985) 1846.

[2] A. A. Tseylin and C. Vafa, *Nucl. Phys.* **B372** (1992) 443; A. A. Tseytlin Cambridge Univ. preprint *DAMTP*-37-1991.

[3] G. W. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741; C. G. Callan, R. C. Myers and M. J. Perry, *Nucl. Phys.* **B311** (1988/89) 673; H. J. Vega and N. Sanchez, *Nucl. Phys.* **B309** (1988) 552; *Nucl. Phys.* **B309** (1988) 557; G. Mandal, A. M. Sengupta and S. Wadia, *Mod. Phys. Lett.* **A6** (1991) 1685; D. Garfinkle, G. T. Horowitz and A. Strominger, *Phys. Rev.* **D43** (1991) 3140; S. P. de Alwis and J. Lykken, *Phys. Lett.* **269B** (1991) 264; A. Tseytlin, Johns Hopkins preprint *JHU-TIPAC*-91009 (1991); A. Giveon, LBL preprint *LBL*-30671 (1991).

[4] S. B. Giddings and A. Strominger, *Phys. Rev. Lett.* **67** (1991) 2930; G. T. Horowitz and A. Strominger, *Nucl. Phys.* **B360** (1991) 197; M. J. Duff and J. X. Lu, *CTP-TAMU*-81/90 (1990); A. Sen, *Phys. Lett.* **274B** (1991) 34.

[5] J. H. Horne and G. T. Horowitz, *Nucl. Phys.* **B368** (1992) 444.

[6] R. Sorkin, *Phys. Rev. Lett.* **51** (1983) 87; D. J. Gross and M. J. Perry, *Nucl. Phys.* **B226** (1983) 29.

[7] J. A. Harvey and J. Liu, *Phys. Lett.* **268B** (1991) 40; A. Shapere, S. Trivedi and F. Wilczek, Institute of Advanced Study, Princeton preprint *IASSNS-HEP*-91/33 (1991).

[8] E. Witten, *Phys. Rev.* **D44** (1991) 314; R. Dijgraaf, E. Verlinde and H. Verlinde, Institute for Advanced Study, Princeton preprint *IASSNS-HEP*-91/22 (1991); E. B. Kiritsis, *Mod. Phys. Lett.* **A6** (1991) 2871; I. Bars, University of South California preprint, *USC*-91/*HEP*-B3 (1991); *USC*-91/*HEP*-B4 (1991); E. Martinec and S. Shatasvili, *Nucl. Phys.* **B368** (1992) 338; M. Bershadsky and D. Kutasov, *PUPT*-1261, *HUTP*-91/A024; P. Horava, Chicago preprint *EFI*-91-57 (1991); S. Nojiri, *FERMILAB-PUB*-91/230-*T*, KEK preprint 91-99, *KEK-TH*-388.

[9] N. Ishibashi, M. Li and A. R. Steif, *Phys. Rev. Lett.* **67** (1991) 3336.

[10] K. Kikkawa and M. Yamasaki, *Phys. Lett.* **149B** (1984) 357; N. Sakai and I. Senda, *Prog. Theor. Phys.* **75** (1986) 692; V. Nair, A. Shapere, A. Stominger and F. Wilczek, *Nucl. Phys.* **B287** (1987) 402; A. Giveon, E. Rabinovici and G. Veneziano, *Nucl. Phys.* **B322** (1989) 167; For a recent review see "Space-time duality in string theory", J. H. Schwarz in Elementary Particles and the Universe, Essays in Honour of Murray Gell-Mann, ed. J. H. Schwarz, Cambridge University Press 1991 and references therein, and Caltech Preprint, *CALT*-68-1740, May 1991.

[11] A. A. Tseytlin, "Space-time duality, dilaton and string cosmology", Proc. of the First International A. D. Sakharov Conference on Physics, Moscow 27-30 May 1991, ed. L. V. Keldysh et al. Nova Science Publ., Commack, N. Y., 1991.

[12] K. A. Meissner and G. Veneziano, *Phys. Lett.* **267B** (1991) 33; G. Veneziano, *Phys. Lett.* **265B** (1991) 287. K. A. Meissner and G. Veneziano, *Mod. Phys. Lett.* **A6** (1991) 3397.

[13] M. Gasperini, J. Maharana and G. Veneziano, *Phys. Lett.* **272B** (1991) 167; M. Gasperini and G.Veneziano, *Phys. Lett.* **277B** (1991) 256.

[14] A. Sen, *Phys. Lett.* **271B** (1991) 295; A. Sen, *Phys. Lett.* **274B** (1991) 34; S. F. Hassan and A. Sen, Tata preprint, *TIFR/TH/*91-40, (1991) (to appear in *Nucl. Phys. B*); S. P. Khastgir and A. Kumar, *Mod. Phys. Lett.* **A6** (1991) 3365; S. K. Kar, S. P. Khastgir and A. Kumar, Institute of Physics, Bhubaneswar preprint, *IP/BBSR/*91-51 (1991) (to appear in *Mod. Phys. Lett. A*); A. Sen, Tata preprint, *TIFR/TH/*92-20, (1992).

[15] T. Banks, M. Dine, H. Dijkstra and W. Fischler, *Phys. Lett.* **212B** (1988) 45.

[16] T. Busher, *Phys. Lett.* **194B** (1987) 59; *Phys. Lett.* **201B** (1988) 466; *Phys. Lett.* **159B** (1985) 127.

[17] M. Rocek and E. Verlinde, *Nucl. Phys.* **B373** (1992) 630.

[18] J. H. Horne, G. T. Horowitz and A. R. Steif, *Phys. Rev. Lett.* **68** (1991) 568.