Nonintegrability of (2+1)-dimensional continuum isotropic Heisenberg spin system: Painlevé analysis

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Abstract

While many integrable spin systems are known to exist in (1+1) and (2+1) dimensions, the integrability property of the physically important (2+1) dimensional isotropic Heisenberg ferromagnetic spin system in the continuum limit has not been investigated in the literature. In this paper, we show through a careful singularity structure analysis of the underlying nonlinear evolution equation that the system admits logarithmic type singular manifolds and so is of non-Painlevé type and is expected to be nonintegrable.

Key words: Painlevé property; Integrability
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The nonlinear dynamics underlying magnetic spin systems is a fascinating topic of study and it is of considerable interest especially from the points of view of soliton theory and condensed matter physics. The underlying evolution equations are highly nonlinear and they give rise to many integrable cases both in (1+1) and (2+1) dimensions.

The standing example of an integrable spin system in (1+1) dimensions is the isotropic Heisenberg ferromagnetic spin (IHFS) chain [1]-[3] in its continuum

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limit. The underlying spin evolution equation is

$$\mathbf{S}_t = \mathbf{S} \wedge \mathbf{S}_{xx}, \tag{1}$$

where $\mathbf{S} = (S_1, S_2, S_3), \quad \mathbf{S}^2 = 1$. It is equivalent geometrically [2] and through gauge transformation [3] to the ubiquitous soliton possessing nonlinear Schrödinger equation [2]. Also the corresponding spin evolution equation itself is associated with a Lax pair and the inverse scattering transform analysis can be carried out for the system directly [4].

Besides the isotropic spin system, there exists a number of other spin systems in (1+1) dimensions which possess Lax pairs, gauge equivalent counterparts and complete integrability property. These include the addition of anisotropy and magnetic field to the isotropic case leading to the spin evolution equation [5],

$$\mathbf{S}_t = \mathbf{S} \wedge \mathbf{S}_{xx} - 2A(\mathbf{S}.\mathbf{n})\mathbf{n} + \mu \mathbf{B}, \tag{2}$$

where $\vec{n} = (0, 0, 1)$, $\vec{B} = (0, 0, B)$, A is the strength of anisotropy and B is the strength of the magnetic field along the z-direction.

One more interesting integrable spin evolution equation is the bianisotropic equation studied by Sklyanin [6],

$$\mathbf{S}_t = \mathbf{S} \wedge \mathbf{S}_{xx} + \mathbf{S} \wedge \mathbf{J}\mathbf{S}, \tag{3}$$

where $\mathbf{J} = diag(J_1, J_2, J_3)$ is the anisotropic matrix. The above type of spin equations are also special cases of the Landau-Lifshitz(L-L) equation deduced from phenomenological arguments [7]. Besides the aforementioned systems, various higher order and inhomogeneous integrable extensions also exist. For example, the spin evolution equation

$$\mathbf{S}_t = (\nu_2 + \mu_2 x)\mathbf{S} \wedge \mathbf{S}_{xx} + \mu_2 \mathbf{S} \wedge \mathbf{S}_x - (\nu_1 + \mu_1 x)\mathbf{S}_x - \gamma[\mathbf{S}_{xx} + \frac{3}{2}(\mathbf{S}_x)^2.\mathbf{S}]_x, \tag{4}$$

is integrable [8]. Also an SO(3) invariant deformed Heisenberg spin system has been shown to be integrable [9]

$$\mathbf{S}_t = \mathbf{S} \wedge \mathbf{S}_{xx} + \alpha \mathbf{S}_x (\mathbf{S}_x)^2, \tag{5}$$

and it is equivalent to the integrable derivative NLS equation [10]

$$iq_t + q_{xx} + 2|q|^2 q - 2i\alpha(|q|^2 q)_x = 0. \tag{6}$$

Many other integrable generalizations have also been obtained by Myrzakulov and coworkers [11]-[14]. All the above equations admit Lax pairs and satisfy Painlevé property.

Naturally, the question arises as to what is the situation in (2+1) dimensions. The well known integrable generalization of Eq. (1) in (2+1) dimensions are the Ishimori equation [15],

$$\mathbf{S}_t = \mathbf{S} \wedge (\mathbf{S}_{xx} + \sigma^2 \mathbf{S}_{yy}) + \phi_y \mathbf{S}_x + \phi_x \mathbf{S}_y, \tag{7a}$$

$$\phi_{xx} - \sigma^2 \phi_{yy} = -2\sigma^2 \mathbf{S}.\mathbf{S}_x \wedge \mathbf{S}_y, \tag{7b}$$

where $\mathbf{S} = (S_1, S_2, S_3), \quad \mathbf{S}^2 = 1$ and $\phi(x, y, t)$ is a scalar field and $\sigma^2 = \pm 1$, and the Myrzakulov M-I equation [13]

$$\mathbf{S}_t = \{\mathbf{S} \wedge (\mathbf{S}_y + u\mathbf{S}\}_x, \tag{8a}$$

$$u_x = -\mathbf{S}.\mathbf{S}_x \wedge \mathbf{S}_y, \tag{8b}$$

where $u(x, y, t)$ is a scalar field.

Again these equations possess Lax pairs and admit the Painlevé property. However, till today the integrability nature of the physically interesting (2+1) dimensional direct generalization of (1), namely

$$\mathbf{S}_t = \mathbf{S} \wedge (\mathbf{S}_{xx} + \mathbf{S}_{yy}), \tag{9}$$

where $\mathbf{S} = (S_1, S_2, S_3), \quad \mathbf{S}^2 = 1$, has not been studied, though the special case of Eq.(9) with circular symmetry

$$\mathbf{S}_t = \mathbf{S} \wedge (\mathbf{S}_{rr} + \frac{1}{r}\mathbf{S}_r), \tag{10}$$

where $r = \sqrt{x^2 + y^2}$, is known to be integrable [16].

In this paper, we wish to investigate the singularity structure property of the isotropic Heisenberg spin equation (9) in (2+1) dimensions and prove that it is of non-Painlevé type and so is expected to be non-integrable, even though the special cases (1) and (10) are of Painlevé type and so integrable. The Painlevé analysis of the Heisenberg spin type equations is rather tricky as was shown for the case of (1+1) dimensional system with anisotropy and transverse magnetic field [17], where the "Taylor" type expansion can lead to logarithmic singular manifolds leading to non-integrability.

In order to investigate the Painlevé singularity structure underlying Eq. (9), we first rewrite it in terms of the complex stereographic field variable $\omega(x, y, t)$ through the transformation

$$S^+ = S_1 + iS_2 = \frac{2\omega}{1 + |\omega|^2}, S_3 = \frac{1 - |\omega|^2}{1 + |\omega|^2}. \tag{11}$$

In terms of this variable, the equation of motion for the (2+1) dimensional Heisenberg spin system can be written as

$$(1+\omega\omega^*)[i\omega_t+\omega_{xx}+\omega_{yy}]-2\omega^*(\omega_x^2+\omega_y^2)=0, \tag{12}$$

and its complex conjugate. Representing $\omega \rightarrow F$ and $\omega^* \rightarrow G$, Eq. (12) and its complex conjugate equation can be written as

$$(1+FG)(iF_t+F_{xx}+F_{yy})-2G(F_x^2+F_y^2)=0, \tag{13a}$$
$$(1+FG)(-iG_t+G_{xx}+G_{yy})-2F(G_x^2+G_y^2)=0. \tag{13b}$$

We carry out a Painlevé analysis of Eqs.(13) by seeking a generalized Laurent expansion for each dependent variable in the form,

$$F=F_0\phi^p+\sum_j F_j\phi^{p+j}, \quad F_0\neq 0, \tag{14a}$$
$$G=G_0\phi^q+\sum_j G_j\phi^{q+j}, \quad G_0\neq 0, \tag{14b}$$

in the neighbourhood of the noncharacteristic singular manifold $\phi(x,y,t)=0$, $\phi_t,\phi_x,\phi_y\neq 0$. The results are as follows.

1.Leading Order Behaviour

Looking at the dominant terms, we distinguish the following possibilities corresponding to (i) $p\leq 0$, $q\leq 0$, (ii) $p\leq 0$, $q\geq 0$, (iii) $p\geq 0$, $q\leq 0$.

Case(i): $p\leq 0,\ q\leq 0$:

Upon using the leading order solution $F=F_0\phi^p$, $G=G_0\phi^q$, substituting it in Eq.(13), and balancing the most dominant terms, we obtain

$$F_0^2G_0[p(p-1)-2p^2](\phi_x^2+\phi_y^2)\phi^{2p+q-2}=0, \tag{15a}$$
$$F_0G_0^2[q(q-1)-2q^2](\phi_x^2+\phi_y^2)\phi^{2q+p-2}=0. \tag{15b}$$

From the above, we have the following three possibilities of leading order behaviour:

Branch (i)	$p=-1, q=-1,$	F_0, G_0: arbitrary
Branch (ii)	$p=-1, q=0,$	F_0, G_0: arbitrary
Branch (iii)	$p=0, q=-1,$	F_0, G_0: arbitrary

In addition, there is a possibility that $p = 0$, $q = 0$, which requires a more detailed analysis, see below.

Case(ii): $p \leq 0$, $q \geq 0$:

$$\Big(F_0(p-1)\phi^{p-2} - F_0^2 G_0(p+1)\phi^{2p+q-2}\Big)(\phi_x^2 + \phi_y^2) = 0, \tag{16a}$$
$$\Big(G_0(q-1)\phi^{q-2} - F_0 G_0^2(q+1)\phi^{p+2q-2}\Big)(\phi_x^2 + \phi_y^2) = 0. \tag{16b}$$

From Eqs. (16a,16b) we obtain $p+q = 0$ and $F_0G_0 = \frac{p-1}{p+1}$ from Eq.(16a), and $F_0G_0 = \frac{q-1}{q+1}$ from Eq.(16b), respectively. We also obtain the same result for the case $p \geq 0, q \leq 0$. This suggests that $p = q = 0$ is the only possibility here. Looking at this case more carefully, by using Eq. (14) in (13), we obtain the following.

Branch (iv): $p = 0$, $q = 0$

$$\begin{aligned}(1+F_0G_0)[i(F_{0t}+F_1\phi_t) + F_{0xx} + 2F_{1x}\phi_x + F_1\phi_{xx} + 2F_2\phi_x^2 + F_{0yy}\\ +2F_{1y}\phi_y + F_1\phi_{yy} + 2F_2\phi_y^2] - 2G_0[F_{0x}^2 + F_{0y}^2 + F_1^2(\phi_x^2+\phi_y^2)\\ +2F_1(F_{0x}\phi_x + F_{0y}\phi_y)] = 0\end{aligned} \tag{17a}$$
$$\begin{aligned}(1+F_0G_0)[-i(G_{0t}+G_1\phi_t) + G_{0xx} + 2G_{1x}\phi_x + G_1\phi_{xx} + 2G_2\phi_x^2 + G_{0yy}\\ +2G_{1y}\phi_y + G_1\phi_{yy} + 2G_2\phi_y^2] - 2F_0[G_{0x}^2 + G_{0y}^2 + G_1^2(\phi_x^2+\phi_y^2)\\ +2G_1(G_{0x}\phi_x + G_{0y}\phi_y)] = 0\end{aligned} \tag{17b}$$

We consider two separate cases of the manifold (i) $F_0G_0 \neq -1$ (ii) $F_0G_0 = -1$. In the former case, from eqn. (17a) and (17b), the coefficient functions F_2 and G_2 can be expressed in terms of F_0, G_0, F_1 and G_1 leaving the later functions arbitrary. For the case $(1+F_0G_0) = 0$, we assume for simplicity the Kruskal's reduced manifold $\phi(x,y,t) = x + \psi(y,t) = 0$. Using this in (17), we find two sets of solutions.

Case (1):

$$F_1 = \frac{iF_{0y}}{(1-i\psi_y)}, \tag{18a}$$
$$G_1 = \frac{iG_{0y}}{(1-i\psi_y)}. \tag{18b}$$

Case (2):

$$F_1 = \frac{iF_{0y}}{(1 - i\psi_y)}, \tag{19a}$$
$$G_1 = \frac{-iG_{0y}}{(1 + i\psi_y)}. \tag{19b}$$

2. Resonances

To find the resonances, that is the powers of the Laurent series (14) at which arbitrary functions enter, for branches (i), (ii) and (iii) we expand

$$F = F_0\phi^p + ... + \alpha\phi^{p+r}, \tag{20a}$$
$$G = G_0\phi^q + ... + \beta\phi^{q+r}, \tag{20b}$$

(α, β not both zero) and substitute in the equations (13) containing the dominant terms alone to fix the values of r. Detailed calculation leads to the following results.

Branch(i) $p = -1$, $q = -1$: $r = -1, -1, 0, 0$

Branch(ii) $p = -1$, $q = 0$: $r = -1, 0, 0, 1$

Branch(iii) $p = 0$, $q = -1$: $r = -1, 0, 0, 1$

For the case of branch (iv), $p = 0$, $q = 0$, we proceed with the expansion

$$F = F_0 + F_1\phi + ... + F_r\phi^r, \tag{21a}$$
$$G = G_0 + G_1\phi + ... + G_r\phi^r, \tag{21b}$$

and substitute them into the equations (13) and collect the coefficients of ϕ^{r-2} and ϕ^{r-1} (after making use of eqs.(17).

(a) Coefficients of ϕ^{r-2}:

When $(1 + F_0G_0) \neq 0$, we have the condition

$$(1 + F_0G_0)(1 + \psi_y^2)r(r - 1)F_r = 0, \tag{22a}$$
$$(1 + F_0G_0)(1 + \psi_y^2)r(r - 1)G_r = 0. \tag{22b}$$

It follows that the resonance values are $r = 0, 0, 1, 1$. For the case $(1+F_0G_0) = 0$, the conditions become identities.

(b) Coefficients of ϕ^{r-1}:

When $(1+F_0G_0) \neq 0$, the resulting condition is in confirmity with the resonance values $r = 0, 0, 1, 1$ noted above. When $(1+F_0G_0) = 0$, we have

$$r[(F_0G_1 + F_1G_0)(1+\psi_y^2)(r-1) - 4(G_0F_1(1+\psi_y^2) + G_0F_{0y}\psi_y)]F_r = 0, \tag{23a}$$
$$r[(F_0G_1 + F_1G_0)(1+\psi_y^2)(r-1) - 4(F_0G_1(1+\psi_y^2) + F_0G_{0y}\psi_y)]G_r = 0. \tag{23b}$$

These equations reduce to the following forms for the cases 1 and 2, respectively.

Case(1):

$$4irG_0F_{0y} = 0, \tag{24a}$$
$$4irF_0G_{0y} = 0. \tag{24b}$$

For this case, the resonance values are $0, 0$.

Case(2):

In this case, we have

$$r[F_{0y}G_0(r-5) - F_0G_{0y}(r-1)] = 0, \tag{25a}$$
$$r[G_{0y}F_0(r-5) - G_0F_{0y}(r-1)] = 0. \tag{25b}$$

Since $(1+F_0G_0) = 0$, $F_{0y}G_0 + F_0G_{0y} = 0$ and consequently from Eqs. (25), we find the resonance values to be $r = 0, 0, 3, 3$.

3. Analysis of the Laurent expansion for arbitrary functions

In the case of the branches (i), (ii) and (iii) we have verified that the resonance conditions are indeed satisfied in the sense that apart from the arbitrariness of the singular manifold, required number of arbitrary functions occur at $r = 0$ and $r = 1$ in the Laurent series and also that no logarithmic singularity can occur in the leading order for the branch (i). We now carry out the calculations for the analysis of the Taylor like expansion corresponding to the branch (iv) (again in terms of the Kruskal's reduced manifold $x + \psi(y,t) = 0$) by writing

$$F(x,y,t) = F_0(y,t) + F_1(y,t)\phi + F_2(y,t)\phi^2 + F_3(y,t)\phi^3 + ..., \tag{26a}$$
$$G(x,y,t) = G_0(y,t) + G_1(y,t)\phi + G_2(y,t)\phi^2 + G_3(y,t)\phi^3 + ... \tag{26b}$$

Substituting the above into Eq.(13), and collecting the coefficients of different powers of ϕ we obtain the following results.

Zeroth order in ϕ:

a) For the manifold $F_0G_0 \neq -1$, the Taylor like series (26) can be easily shown not to admit any movable singular manifold, where four arbitrary functions can enter into the series (while the manifold ϕ can be absorbed into F_1 or G_1). This is in confirmity with the resonance values $r = 0, 0, 1, 1$ pointed out after Eq.(22).

b) For the manifold $F_0G_0 = -1$, one can obtain two sets of the expression for F_1 and G_1 which are the same as cases 1 and 2 given by Eqs. (18) and (19), respectively. We will consider each of the cases separately.

Case(1)-Eqs.(18):
(a) First order in ϕ: With $(1 + F_0G_0) = 0$, we have

$$\begin{aligned}&(F_0G_1 + F_1G_0)[i(F_{0t} + F_1\psi_t) + 2F_2 + (F_{0yy} + 2F_{1y}\psi_y + F_1\psi_{yy} \\ &+2F_2\psi_y^2)] - 4G_0[F_{0y}F_{1y} + 2F_{0y}F_2\psi_y + F_1(2F_2(1 + \psi_y^2) + F_{1y}\psi_y)]] \\ &-2G_1[F_1^2(1 + \psi_y^2) + F_{0y}(F_{0y} + 2F_1\psi_y)] = 0, \quad (27a)\\ &(F_0G_1 + F_1G_0)[-i(G_{0t} + G_1\psi_t) + 2G_2 + (G_{0yy} + 2G_{1y}\psi_y + G_1\psi_{yy} \\ &+2G_2\psi_y^2)] - 4F_0[G_{0y}G_{1y} + 2G_{0y}G_2\psi_y + G_1(2G_2(1 + \psi_y^2) + G_{1y}\psi_y)]] \\ &-2F_1[G_1^2(1 + \psi_y^2) + G_{0y}(G_{0y} + 2G_1\psi_y)] = 0. \quad (27b)\end{aligned}$$

Using the results of the previous order for F_1 and G_1, we obtain

$$F_2 = \frac{-F_{0yy}(1 - i\psi_y) - iF_{0y}\psi_{yy}}{2(1 - i\psi_y)^3}, \tag{28a}$$

$$G_2 = \frac{-G_{0yy}(1 - i\psi_y) - iG_{0y}\psi_{yy}}{2(1 - i\psi_y)^3}. \tag{28b}$$

(b) Second order in ϕ:

Here we obtain F_3 and G_3 as

$$
\begin{aligned}
F_3 = \frac{i}{12G_0F_{0y}}\Big[&2G_0[4F_2^2+(F_{1y}^2+4F_2^2\psi_y^2+4F_{1y}F_2\psi_y)+(2F_{0y}F_{2y}\\
&+F_1F_{2y}\psi_y)]+2G_1[4F_1F_2+2(F_{0y}F_{1y}+2F_{0y}F_2\psi_y+F_1F_{1y}\psi_y\\
&+2F_1F_2\psi_y^2)]-(F_0G_2+F_1G_1+F_2G_0)[i(F_{0t}+F_1\psi_t)+2F_2\\
&+(F_{0yy}+2F_{1y}\psi_y+F_1\psi_{yy}+2F_2\psi_y^2)]\Big],
\end{aligned} \tag{29a}
$$

$$
\begin{aligned}
G_3 = \frac{i}{12G_{0y}F_0}\Big[&2F_0[4G_2^2+(G_{1y}^2+4G_2^2\psi_y^2+4G_{1y}G_2\psi_y)+(2G_{0y}G_{2y}\\
&+G_1G_{2y}\psi_y)]+2F_1[4G_1G_2+2(G_{0y}G_{1y}+2G_{0y}G_2\psi_y+G_1G_{1y}\psi_y\\
&+2G_1G_2\psi_y^2)]-(F_0G_2+F_1G_1+F_2G_0)[-i(G_{0t}+G_1\psi_t)+2G_2\\
&+(G_{0yy}+2G_{1y}\psi_y+G_1\psi_{yy}+2G_2\psi_y^2)]\Big].
\end{aligned} \tag{29b}
$$

In a similar way, one can compute (F_4, G_4), (F_5, G_5), etc. No indeterminate coefficients appear in the series (at least upto the order deduced) and thus no possibility for singularity arises. We also note that either F_0 or G_0 and $\psi(y,t)$ are the only arbitrary functions in the Taylor like series (26) in confirmity with the resonance values $r = 0, 0$.

Case(2)-Eqs.(19):
(a) First order in ϕ: From Eq. (13) we obtain

$$
\begin{aligned}
F_2 = \frac{i}{4F_{0y}G_0}\Bigg(&\frac{-i(F_{0y}G_0-F_0G_{0y})}{1+\psi_y^2}\Big[i(F_{0t}+i\frac{F_{0y}}{1-i\psi_y}\psi_t)+F_{0yy}\\
&+\frac{2iF_{0yy}}{1-i\psi_y}\psi_y-\frac{2F_{0y}}{(1-i\psi_y)^2}\psi_y\psi_{yy}+\frac{iF_{0y}}{1-i\psi_y}\psi_{yy}\Big]\\
&+4G_0\Big[\frac{F_{0y}}{1-i\psi_y}(\frac{iF_{0yy}}{1-i\psi_y}-\frac{F_{0y}}{(1-i\psi_y)^2}\psi_{yy})\Big]\Bigg),
\end{aligned} \tag{30a}
$$

$$
\begin{aligned}
G_2 = \frac{-i}{4F_0G_{0y}}\Bigg(&\frac{-i(F_{0y}G_0-F_0G_{0y})}{1+\psi_y^2}\Big[-i(G_{0t}-i\frac{G_{0y}}{1+i\psi_y}\psi_t)+G_{0yy}\\
&-\frac{2iG_{0yy}}{1+i\psi_y}\psi_y-\frac{2G_{0y}}{(1+i\psi_y)^2}\psi_y\psi_{yy}-\frac{iG_{0y}}{1-i\psi_y}\psi_{yy}\Big]\\
&+4F_0\Big[\frac{G_{0y}}{1+i\psi_y}(\frac{-iG_{0yy}}{1+i\psi_y}-\frac{G_{0y}}{(1+i\psi_y)^2}\psi_{yy})\Big]\Bigg).
\end{aligned} \tag{30b}
$$

(b) Second order in ϕ:

$$
\begin{aligned}
2G_0[4F_2^2 + (F_{1y}^2 + 4F_2^2\psi_y^2 + 4F_{1y}F_2\psi_y) + (2F_{0y}F_{2y} + F_1F_{2y}\psi_y)] \\
+2G_1[4F_1F_2 + 2(F_{0y}F_{1y} + 2F_{0y}F_2\psi_y + F_1F_{1y}\psi_y + 2F_1F_2\psi_y^2)] \\
-(F_0G_2 + F_1G_1 + F_2G_0)[i(F_{0t} + F_1\psi_t) + 2F_2 + (F_{0yy} + 2F_{1y}\psi_y \\
+F_1\psi_{yy} + 2F_2\psi_y^2)] - (F_0G_1 + F_1G_0)[i(F_{1t} + 2F_2\psi_t) \\
+(F_{1yy} + 4F_{2y}\psi_y + 2F_2\psi_{yy})] = 0, \quad \text{(31a)} \\
2F_0[4G_2^2 + (G_{1y}^2 + 4G_2^2\psi_y^2 + 4G_{1y}G_2\psi_y) + (2G_{0y}G_{2y} + G_1G_{2y}\psi_y)] \\
+2F_1[4G_1G_2 + 2(G_{0y}G_{1y} + 2G_{0y}G_2\psi_y + G_1G_{1y}\psi_y + 2G_1G_2\psi_y^2)] \\
-(F_0G_2 + F_1G_1 + F_2G_0)[-i(G_{0t} + G_1\psi_t) + 2G_2 + (G_{0yy} + 2G_{1y}\psi_y \\
+G_1\psi_{yy} + 2G_2\psi_y^2)] - (F_0G_1 + F_1G_0)[-i(G_{1t} + 2G_2\psi_t) \\
+(G_{1yy} + 4G_{2y}\psi_y + 2G_2\psi_{yy})] = 0. \quad \text{(31b)}
\end{aligned}
$$

It may be noted that in this order both F_3 and G_3 are absent indicating that they are arbitrary functions corresponding to the resonance values $r = 3, 3$. Note that from Eqs. (30) and the relation $(1 + F_0G_0) = 0$, two of the three functions F_0, G_0 and ψ are arbitrary corresponding to the values $r = 0, 0$. However, simplifying the above set of equations (31) by using the expressions obtained for the coefficients F_1, F_2, G_1, G_2 in terms of F_0, G_0 and ψ (vide Eqs. (18), (19), (30)), we find that the equations (31) reduce to two nontrivial conditions which are incompatible, unless the y-dependence is dropped (corresponding (1+1) dimensional system (1)) or one carries out the analysis with the radial variable $r = \sqrt{x^2 + y^2}$ (vide Eq.(10)). As a consequence logarithmic singularity appears in the series expansion (26). Consequently the (2+1) dimensional continuum isotropic Heisenberg spin system (12) and so (9) does not satisfy the Painlevé property [18] and is expected to be nonintegrable.

One can also carry out the analysis with the general manifold $\phi(x, y, t)$ instead of the Kruskal's reduced manifold and one can check that the same conclusion results in here also.

To conclude, in this letter we have shown that the physically important (2+1) dimensional isotropic Heisenberg continuum spin system (9) does not admit the Painlevé property and so it belongs to the class of non-integrable nonlinear evolution equations. It will be of considerable interest to investigate the underlying spatiotemporal structures of such a nonlinear evolution equation in detail.

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