Effect of Phase Shift in Shape Changing Collision of Solitons in Coupled Nonlinear Schrödinger Equations

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Abstract

Soliton interactions in systems modelled by coupled nonlinear Schrödinger (CNLS) equations and encountered in phenomena such as wave propagation in optical fibers and photorefractive media possess unusual features : shape changing intensity redistributions, amplitude dependent phase shifts and relative separation distances. We demonstrate these properties in the case of integrable 2-CNLS equations. As a simple example, we consider the stationary two-soliton solution which is equivalent to the so-called partially coherent soliton (PCS) solution discussed much in the recent literature.

I. INTRODUCTION

The study on the formation of optical solitons and their intriguing interaction properties is becoming one of the frontier areas of research in nonlinear dynamics due to their potential technological applications[1,2]. Indeed optical solitons are becoming desirable candidates in long distance optical communication systems, in optical devices and in optical computers. In a mathematical sense these solitons appear basically as solutions of integrable coupled nonlinear Schrödinger (CNLS) type equations. For example, the intense electromagnetic wave propagation in a birefringent fiber is governed by the following set of 2-CNLS equations for the envelopes q_1 and q_2, which is in general nonintegrable,

$$\begin{aligned} iq_{1z} + q_{1tt} + 2\mu(|q_1|^2 + B|q_2|^2)q_1 = 0, \\ iq_{2z} + q_{2tt} + 2\mu(|q_2|^2 + B|q_1|^2)q_2 = 0, \end{aligned} \tag{1}$$

where z and t represent the normalized distance along the fiber and the retarded time respectively, μ represents the strength of nonlinearity and $B = \frac{2+2sin^2\theta}{2+cos^2\theta}$ is the cross phase modulation coupling parameter (θ: ellipticity angle). However this system becomes integrable for $B = 1$. The resulting set of equations

$$\begin{aligned} iq_{1z} + q_{1tt} + 2\mu(|q_1|^2 + |q_2|^2)q_1 &= 0, \\ iq_{2z} + q_{2tt} + 2\mu(|q_1|^2 + |q_2|^2)q_2 &= 0, \end{aligned} \tag{2}$$

is the celebrated Manakov equation[3]. In a recent work Radhakrishnan, Lakshmanan and Hietarinta [4] have revealed the fact that the soliton solutions of the integrable 2-CNLS (Manakov) equations undergo a fascinating shape-changing collision, resulting in a redistribution of intensity between the two solitons in the two modes, which is not observed in the scalar nonlinear Schrödinger (NLS) equation which exhibits only pure elastic collision without any redistribution of intensities of solitons. Consequently, Jakubowski, Steiglitz and Squier[5] have pointed out the possibility of using this phenomenon in constructing logic gates and in a very recent work[6] Steiglitz constructed such gates including the universal NAND gate, thereby showing the theoretical possibility of constructing all optical computers without interconnecting discrete components in a homogeneous bulk nonlinear optical medium. Also Yang[7] has studied the effect of additional perturbations on these solitons using perturbation theory. Further such integrable CNLS equations arise in the context of spatial solitons as well which are receiving renewed attention for their formation at very low optical powers in photorefractive medium[8]. The present authors have extended the results of 2-CNLS system to 3- and N-CNLS equations [9].

All the above investigations mostly concentrate on the effect of changes in the amplitude (polarization) and the consequent effect on the energy redistribution between the modes of the solitons. So far not much attention has been paid to the role of phases during optical soliton interaction. In this report, we point out the significance of amplitude dependent phase shift / relative seperation distance involved in the interaction process (sec.2) responsible for the shape change of solitons during collision along with the changes in the amplitudes (polarization) of the modes in the Manakov system. It may be noted that such amplitude dependent phase shifts do not occur in the case of scalar NLS equation. As a simple example, we consider (sec.3) the role of phase shifts / relative separation distances for stationary 2-soliton case and point out that this solution is nothing but the so-called stationary partially coherent soliton (PCS) in the recent literature[10].

II. SOLITON INTERACTION IN 2-CNLS SYSTEM

To start with let us consider briefly the nature of one- and two-soliton solutions [4,11]. Multisoliton solutions of CNLS equations and their interactions will be considered elsewhere[12].

A. One-soliton solution

The one-soliton solution to eq.(2) can be given in terms of three arbitrary complex parameters $\alpha_1^{(1)}$,$\alpha_1^{(2)}$ and k_1 as[4,11]

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \end{pmatrix} \frac{e^{\eta_1}}{1+e^{\eta_1+\eta_1^*+R}}, \tag{3}$$

$$= \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \frac{k_{1R}e^{i\eta_{1I}}}{\cosh\left(\eta_{1R}+\frac{R}{2}\right)}, \tag{4}$$

where $\eta_i = k_i(t+ik_iz)$, $i=1$, where $k_i = k_{iR}+ik_{iI}$, k_{iR} and k_{iI} represent the real and imaginary parts of k_i. Here $\sqrt{\mu}(A_1,A_2) = \frac{(\alpha_1^{(1)},\alpha_1^{(2)})}{\sqrt{|\alpha_1^{(1)}|^2+|\alpha_1^{(2)}|^2}}$ represents the unit polarization vector, $k_{1R}A_j$, j=1,2 gives the amplitude of the jth mode and $2k_{1I}$ is the soliton velocity.

B. Two-soliton solution

Introducing six complex parameters $\alpha_1^{(1)}$, $\alpha_2^{(1)}$, $\alpha_1^{(2)}$, $\alpha_2^{(2)}$, k_1 and k_2, the two-soliton solution can be given as [4,11]

$$\begin{aligned} q_1 &= \frac{\alpha_1^{(1)}e^{\eta_1} + \alpha_2^{(1)}e^{\eta_2} + e^{\eta_1+\eta_1^*+\eta_2+\delta_1} + e^{\eta_1+\eta_2+\eta_2^*+\delta_2}}{D}, \\ q_2 &= \frac{\alpha_1^{(2)}e^{\eta_1} + \alpha_2^{(2)}e^{\eta_2} + e^{\eta_1+\eta_1^*+\eta_2+\delta_1'} + e^{\eta_1+\eta_2+\eta_2^*+\delta_2'}}{D}, \end{aligned}$$

where

$$\begin{aligned} D &= 1 + e^{\eta_1+\eta_1^*+R_1} + e^{\eta_1+\eta_2^*+\delta_0} + e^{\eta_1^*+\eta_2+\delta_0^*} + e^{\eta_2+\eta_2^*+R_2} \\ &\quad + e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_3}. \end{aligned} \tag{5}$$

Here

$$\begin{aligned} \eta_i &= k_i(t + ik_iz), \quad e^{\delta_0} = \frac{\kappa_{12}}{k_1+k_2^*}, \quad e^{R_j} = \frac{\kappa_{jj}}{k_j+k_j^*}, \\ e^{\delta_1} &= \frac{k_1-k_2}{(k_1+k_1^*)(k_1^*+k_2)}(\alpha_1^{(1)}\kappa_{21} - \alpha_2^{(1)}\kappa_{11}), \\ e^{\delta_2} &= \frac{k_2-k_1}{(k_2+k_2^*)(k_1+k_2^*)}(\alpha_2^{(1)}\kappa_{12} - \alpha_1^{(1)}\kappa_{22}), \\ e^{\delta_1'} &= \frac{k_1-k_2}{(k_1+k_1^*)(k_1^*+k_2)}(\alpha_1^{(2)}\kappa_{21} - \alpha_2^{(2)}\kappa_{11}), \\ e^{\delta_2'} &= \frac{k_2-k_1}{(k_2+k_2^*)(k_1+k_2^*)}(\alpha_2^{(2)}\kappa_{12} - \alpha_1^{(2)}\kappa_{22}), \\ e^{R_3} &= \frac{|k_1-k_2|^2}{(k_1+k_1^*)(k_2+k_2^*)|k_1+k_2^*|^2}(\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21}) \\ \text{and } \kappa_{ij} &= \frac{\mu(\alpha_i^{(1)}\alpha_j^{(1)*} + \alpha_i^{(2)}\alpha_j^{(2)*})}{k_i+k_j^*}, \ i,j = 1,2. \end{aligned}$$

The above two-soliton solution represents the interaction of two coupled one solitons. The scenario behind this interaction is that there is an intensity redistribution among the two modes of the two solitons along with an amplitude dependent phase shift and relative separation distance[4,9,11]. In order to understand the nature of the collisions we can consider the following cases for $k_{1I}>k_{2I}$:
(a) $k_{1R} > 0,\ k_{2R} > 0$ (b) $k_{1R} > 0,\ k_{2R} < 0$ (c) $k_{1R} < 0, k_{2R} > 0$ (d) $k_{1R} < 0, k_{2R} < 0$. Similarly, one can consider four cases for $k_{1I} < k_{2I}$. In all these cases an asymptotic analysis ($z \to \pm\infty$) reveals the following structures.

1) Limit$z \rightarrow -\infty$

(a) *Soliton*1 :

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \rightarrow \begin{pmatrix} A_1^{1-} \\ A_2^{1-} \end{pmatrix} k_{1R} e^{i\eta_{1I}} \text{sech} \left(\eta_{1R} + \hat{\phi}^{1-}\right), \tag{6}$$

(b) *Soliton*2:

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \rightarrow \begin{pmatrix} A_1^{2-} \\ A_2^{2-} \end{pmatrix} k_{2R} e^{i\eta_{2I}} \text{sech} \left(\eta_{2R} + \hat{\phi}^{2-}\right). \tag{7}$$

2) Limit$z \rightarrow +\infty$

(a) *Soliton*1 :

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \rightarrow \begin{pmatrix} A_1^{1+} \\ A_2^{1+} \end{pmatrix} k_{1R} e^{i\eta_{1I}} \text{sech} \left(\eta_{1R} + \hat{\phi}^{1+}\right), \tag{8}$$

(b) *Soliton*2:

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \rightarrow \begin{pmatrix} A_1^{2+} \\ A_2^{2+} \end{pmatrix} k_{2R} e^{i\eta_{2I}} \text{sech} \left(\eta_{2R} + \hat{\phi}^{2+}\right). \tag{9}$$

Here the various quantities corresponding to the four cases for $k_{1I}>k_{2I}$ are given below.

Case(a)$k_{1R} > 0$,$k_{2R} > 0$:

$$\begin{pmatrix} A_1^{1-} \\ A_2^{1-} \end{pmatrix} = \begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \end{pmatrix} e^{-R_1/2}, \tag{10}$$

$$\begin{pmatrix} A_1^{1+} \\ A_2^{1+} \end{pmatrix} = \begin{pmatrix} e^{\delta_2} \\ e^{\delta_2'} \end{pmatrix} e^{-(R_2+R_3)/2}, \tag{11}$$

$$\begin{pmatrix} A_1^{2-} \\ A_2^{2-} \end{pmatrix} = \begin{pmatrix} e^{\delta_1} \\ e^{\delta_1'} \end{pmatrix} e^{-(R_1+R_3)/2}, \tag{12}$$

$$\begin{pmatrix} A_1^{2+} \\ A_2^{2+} \end{pmatrix} = \begin{pmatrix} \alpha_2^{(1)} \\ \alpha_2^{(2)} \end{pmatrix} e^{-R_2/2}, \tag{13}$$

$\hat{\phi}^{1-} = \frac{R_1}{2}$, $\hat{\phi}^{1+} = \frac{R_3-R_2}{2}$,$\hat{\phi}^{2-} = \frac{R_3-R_1}{2}$ and $\hat{\phi}^{2+} = \frac{R_2}{2}$. Case(b)$k_{1R} > 0$,$k_{2R} < 0$:

$$\begin{pmatrix} A_1^{j-} \\ A_2^{j-} \end{pmatrix} = \begin{pmatrix} e^{\delta_l} \\ e^{\delta_l'} \end{pmatrix} e^{-(R_l+R_3)/2}, \tag{14}$$

$$\begin{pmatrix} A_1^{j+} \\ A_2^{j+} \end{pmatrix} = \begin{pmatrix} \alpha_j^{(1)} \\ \alpha_j^{(2)} \end{pmatrix} e^{-R_j/2}, \tag{15}$$

$\hat{\phi}^{j-} = \frac{R_3 - R_l}{2}$ and $\hat{\phi}^{j+} = \frac{R_j}{2}$, $j = 1, 2$.

Case(c) $k_{1R} < 0$,$k_{2R} > 0$:

$$\begin{pmatrix} A_1^{j-} \\ A_2^{j-} \end{pmatrix} = \begin{pmatrix} \alpha_j^{(1)} \\ \alpha_j^{(2)} \end{pmatrix} e^{-R_j/2}, \tag{16}$$

$$\begin{pmatrix} A_1^{j+} \\ A_2^{j+} \end{pmatrix} = \begin{pmatrix} e^{\delta_l} \\ e^{\delta_l'} \end{pmatrix} e^{-(R_l+R_3)/2}, \tag{17}$$

$\hat{\phi}^{j-} = \frac{R_j}{2}$ and $\hat{\phi}^{j+} = \frac{R_3 - R_l}{2}$.

In both the cases (b) and (c) $l = j + (-1)^{j+1}$, $j = 1, 2$.

Case(d) $k_{1R} < 0$,$k_{2R} < 0$:

$$\begin{pmatrix} A_1^{1-} \\ A_2^{1-} \end{pmatrix} = \begin{pmatrix} e^{\delta_2} \\ e^{\delta_2'} \end{pmatrix} e^{-(R_2+R_3)/2}, \tag{18}$$

$$\begin{pmatrix} A_1^{1+} \\ A_2^{1+} \end{pmatrix} = \begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \end{pmatrix} e^{-R_1/2}, \tag{19}$$

$$\begin{pmatrix} A_1^{2-} \\ A_2^{2-} \end{pmatrix} = \begin{pmatrix} \alpha_2^{(1)} \\ \alpha_2^{(2)} \end{pmatrix} e^{-R_2/2}, \tag{20}$$

$$\begin{pmatrix} A_1^{2+} \\ A_2^{2+} \end{pmatrix} = \begin{pmatrix} e^{\delta_1} \\ e^{\delta_1'} \end{pmatrix} e^{-(R_1+R_3)/2}, \tag{21}$$

$\hat{\phi}^{1-} = \frac{R_3 - R_2}{2}$,$\hat{\phi}^{1+} = \frac{R_1}{2}$,$\hat{\phi}^{2-} = \frac{R_2}{2}$ and $\hat{\phi}^{2+} = \frac{R_3 - R_1}{2}$.

In equations(6-21) the superscripts denote the solitons and the subscripts denote the modes. It is clear from the above expressions that in all the cases there exists a redistribution of intensity among the solitons. However it should be noticed that though there is an intensity redistribution among the solitons in two modes, the total intensity of individual soliton is conserved during collision process, that is, $|A_1^{j-}|^2 + |A_2^{j-}|^2 = |A_1^{j+}|^2 + |A_2^{j+}|^2 = \frac{1}{\mu}$, $j = 1, 2$, which is of course a consequence of the integrability of the Manakov model. For example, the amplitude change in the two modes of soliton 1 after interaction can be expressed by the following transformation,

$$\begin{aligned} A_1^{1+} &= \Gamma C_{11} A_1^{1-} + \Gamma C_{12} A_2^{1-}, \\ A_2^{1+} &= \Gamma C_{21} A_1^{1-} + \Gamma C_{22} A_2^{1-}. \end{aligned} \tag{22}$$

Here $\Gamma = \Gamma(A_1^{1-}, A_2^{1-}, A_1^{2-}, A_2^{2-}) \equiv \left(\frac{a_2}{a_2^*}\right) [1/((\alpha_1^{(1)}\alpha_2^{(1)*} + \alpha_1^{(2)}\alpha_2^{(2)*})(\alpha_2^{(1)}\alpha_2^{(1)*} + \alpha_2^{(2)}\alpha_2^{(2)*}))]$ $\left[\frac{1}{|\kappa_{12}|^2} - \frac{1}{\kappa_{11}\kappa_{22}}\right]^{-1/2}$, in which $a_2 = (k_2 + k_1^*)[(k_1 - k_2)(\alpha_1^{(1)}\alpha_2^{(1)*} + \alpha_1^{(2)}\alpha_2^{(2)*})]^{1/2}$. The forms of C_{ij}'s, $i, j = 1, 2$ read as $C_{11} = \alpha_2^{(1)}\alpha_2^{(1)*}(k_1 - k_2) + \alpha_2^{(2)}\alpha_2^{(2)*}(k_1 + k_2^*)$, $C_{12} = -\alpha_2^{(1)}\alpha_2^{(2)*}(k_2 + k_2^*)$, $C_{21} = -\alpha_2^{(2)}\alpha_2^{(1)*}(k_2 + k_2^*)$, $C_{22} = \alpha_2^{(1)}\alpha_2^{(1)*}(k_1 + k_2^*) + \alpha_2^{(2)}\alpha_2^{(2)*}(k_1 - k_2)$. Note that C_{ij}'s are independent of $\alpha_1^{(j)}$'s and so of A_1^{1-} and A_2^{1-}. Similar relations for the soliton 2 hold good for A_1^{2+} and A_2^{2+} also. Then the ratios of the A_i^j's, $i, j = 1, 2$, can be connected through linear fractional transformations(LFTs). For example, for soliton 1,

$$\rho_1^+ = \frac{A_1^{1+}}{A_2^{1+}} = \frac{C_{11}\rho_1^- + C_{12}}{C_{21}\rho_1^- + C_{22}}, \tag{23}$$

where $\rho_1 = \frac{A_1^{1-}}{A_2^{1-}}$, ensuring that for every transformation there exists an inverse transformation. This idea has been profitably used in constructing logic gates[5,6]. In fact the LFT (23) is identical to the LFT given by equation(9) in ref.[5] under the change of notation $\rho_1^- \to \rho_1$, $\rho_1^+ \to \rho_R$ with C_{ij}'s identified as the expressions given therein.

Further, it is observed that the absolute value of the phase shift of the two solitons in all the above four cases is same and is given by

$$\begin{aligned} |\Phi| &= \frac{|R_3 - R_1 - R_2|}{2}, \\ &= \frac{1}{2}\log\left[\frac{|k_1 - k_2|^2(|\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21}|)}{|k_1 + k_2^*|^2\kappa_{11}\kappa_{22}}\right], \\ &= \frac{1}{2}\log\left[\frac{|k_1 - k_2|^2}{|k_1 + k_2^*|^4}\right] \\ &+ \frac{1}{2}\log\left[\frac{\bar{\kappa}_{11}\bar{\kappa}_{22}|k_1 + k_2^*|^2 - |\bar{\kappa}_{12}|^2(k_1 + k_1^*)(k_2 + k_2^*)}{\bar{\kappa}_{11}\bar{\kappa}_{22}}\right], \end{aligned} \tag{24}$$

where $\bar{\kappa}_{ij} = \mu(\alpha_i^{(1)}\alpha_j^{(1)*} + \alpha_i^{(2)}\alpha_j^{(2)*})$, $i, j = 1, 2$. and the absolute value of the change in relative separation distance $t_{12}^{\pm}$ (position of S_2 (at $z \to \pm\infty$) - position of S_1 (at $z \to \pm\infty$)) is given by

$$|\Delta t_{12}| = |t_{12}^- - t_{12}^+| = \left|\frac{(k_{1R} + k_{2R})}{2k_{1R}k_{2R}}\right| |\Phi|. \tag{25}$$

It is interesting to note that in the collision process the phase shift is not only dependent on the k_j's, $j = 1, 2$ but also on the complex parameters $\alpha_i^{(j)}$'s, $i, j = 1, 2$, and so on A_i^j's. These two properties, that is, dependence of the change in the intensity profiles of the solitons in the two modes and of the phase shift of them during collision on the parameters $\alpha_i^{(j)}$'s make the collision properties novel, not seen in general other standard $(1+1)$ dimensional soliton systems.

Now looking at the dependence of the phase shift on $\alpha_i^{(j)}$'s, we can consider two special cases.

Case(a): $\underline{\alpha_1^{(1)} : \alpha_2^{(1)} = \alpha_1^{(2)} : \alpha_2^{(2)}}$.

In this case(corresponding to parallel modes) the collision correspond to pure elastic collision $(|A_i^{j+}| = |A_i^{j-}|,\ i, j = 1, 2)$ and the phase shift is given by

$$|\Phi| = \left|\log\left[\frac{|k_1 - k_2|^2}{|k_1 + k_2^*|^2}\right]\right|. \tag{26}$$

Case(b): $\underline{\alpha_1^{(1)} : \alpha_2^{(1)} = \infty,\ \alpha_1^{(2)} : \alpha_2^{(2)} = 0}$.

This case corresponds to two orthogonal modes. Here the phase shift is given by

$$|\Phi| = \left|\log\left[\frac{|k_1 - k_2|}{|k_1 + k_2^*|}\right]\right|. \tag{27}$$

These two examples show that the phase shifts and hence the relative separation distances (see eq.(25)) vary as the complex parameters $\alpha_i^{(j)}$'s change and this variation will be reflected in the shape of the profiles of the two interacting solitons.

III. STATIONARY SOLITONS AND RELATIVE SEPARATION DISTANCES

In order to realize the effect of phase shift on the shape of the solitons we consider the simple case of stationary limit of the two-soliton solution (5). Let us consider the situation in which the velocities are zero, that is $k_{jI} = 0$, $j = 1, 2$. Further we choose $\alpha_2^{(1)} = \alpha_1^{(2)} = 0$, $\alpha_1^{(1)} = e^{\eta_{10}}$, $\alpha_2^{(2)} = -e^{\eta_{20}}$ and $k_{nI} = 0$, where $\eta_{i0}, i = 1, 2$ are real parameters (corresponding to orthogonal modes). In this limit, the two-soliton solution(5) reduces to

$$q_1 = 2k_{1R}\sqrt{\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}}\cosh(k_{2R}\bar{t_2})e^{ik_{1R}^2 z}/D_1, \tag{28}$$

$$q_2 = 2k_{2R}\sqrt{\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}}\sinh(k_{1R}\bar{t_1})e^{ik_{2R}^2 z}/D_1 \tag{29}$$

$$D_1 = \sqrt{\mu}\cosh(k_{1R}\bar{t_1}+k_{2R}\bar{t_2}) + \sqrt{\mu}\left(\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}\right) \cosh(k_{1R}\bar{t_1}-k_{2R}\bar{t_2}), \tag{30}$$

$$\bar{t_1} = t - t_1 = t + \frac{\eta_{10}}{k_{1R}} + \frac{1}{2k_{1R}}\log\left[\frac{\mu(k_{1R}-k_{2R})}{4k_{1R}^2(k_{1R}+k_{2R})}\right], \tag{31}$$

$$\bar{t_2} = t - t_2 = t + \frac{\eta_{20}}{k_{2R}} + \frac{1}{2k_{2R}}\log\left[\frac{\mu(k_{1R}-k_{2R})}{4k_{2R}^2(k_{1R}+k_{2R})}\right], \tag{32}$$

so that each soliton is in one particular mode.

It is also of interest to note that the stationary limit of the two-soliton solution obtained above is also the so-called partially coherent stationary soliton studied in the literature intensively in recent times [8,10] in connection with the existence of spatial solitons in photorefractive materials. In fact eqs.(28-32) are nothing but the stationary 2-PCS solution obtained in eqs.(13-15) in ref.[10]. Now we can identify the relative separation distance between the solitons as

$$t_{12} = t_2 - t_1 = \frac{\eta_{10}}{k_{1R}} - \frac{\eta_{20}}{k_{2R}} + \frac{1}{2k_{1R}}\log\left[\frac{\mu(k_{1R}-k_{2R})}{4k_{1R}^2(k_{1R}+k_{2R})}\right] - \frac{1}{2k_{2R}}\log\left[\frac{\mu(k_{1R}-k_{2R})}{4k_{2R}^2(k_{1R}+k_{2R})}\right]. \tag{33}$$

It can be very easily seen that the shape of the stationary soliton depends very much on the relative separation distance t_{12}. To illustrate this in fig.(1) we have plotted (i)symmetric case $t_{12}=0$ and (ii) asymmetric case $t_{12}\neq 0$.

One can proceed to consider the propagation of the above stationary soliton solution and check their shape changing properties under collision. Choosing the parameters $\alpha_1^{(2)}$ and $\alpha_2^{(1)}$ as functions of velocities (k_{jI}) such that they vanish when $k_{jI}=0$, $j=1,2$, the nature of soliton collisions is shown in figs.2 for the parameters $\alpha_1^{(1)}=1.0$, $\alpha_1^{(2)}=1.0$, $\alpha_2^{(1)}=\frac{22+80i}{89}$, $\alpha_2^{(2)}=-2.0$, $k_1=1.0+i$ and $k_2=2.0-i$.

IV. CONCLUSION

The soliton interactions in CNLS equations possess very rich structure. In this paper, we have discussed the two-soliton interaction properties of 2-CNLS equations with special emphasis on the nature of phase-shift encountered by solitons under collision and its dependence on the amplitudes of the modes. We have also pointed out that the much discussed stationary PCS solitons correspond to stationary limit of appropriate soliton solutions with 2-PCS as an example here. Because of the complex nature of soliton interaction, multisoliton solution in multicomponent systems possess highly nontrivial structures. These properties will be presented seperately [12]. Such studies are expected to have very important application in optical communications, optical devices and optical computing.

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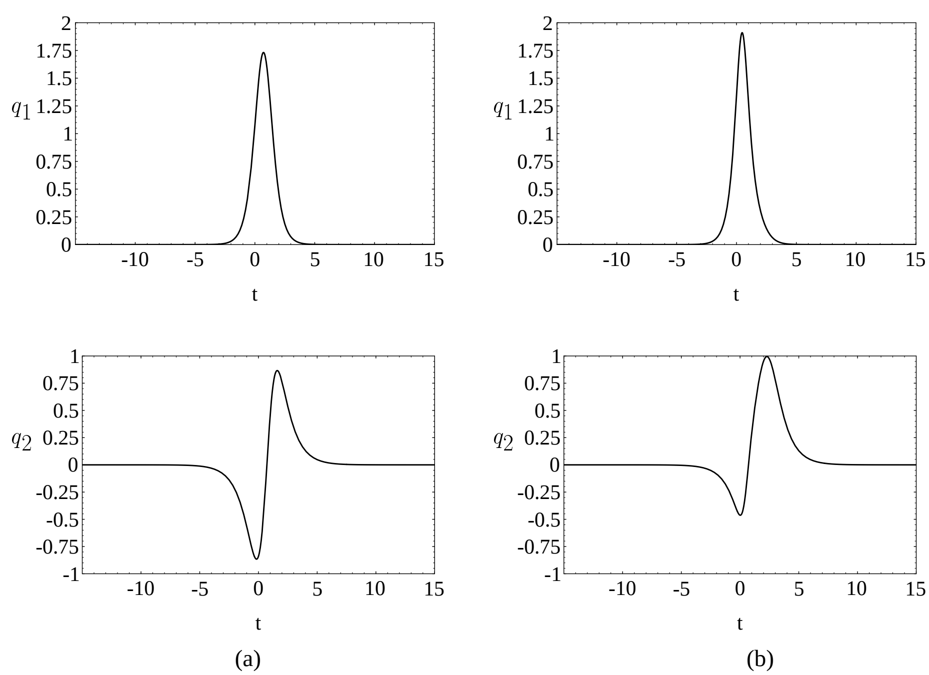

FIG. 1: Typical stationary form of the 2-soliton solution (PCS) for the 2-CNLS system for z=0, see eqs.(28-32) : (a) symmetric case ($t_{12} = 0$), (b) asymmetric case ($t_{12} = 1.0$).

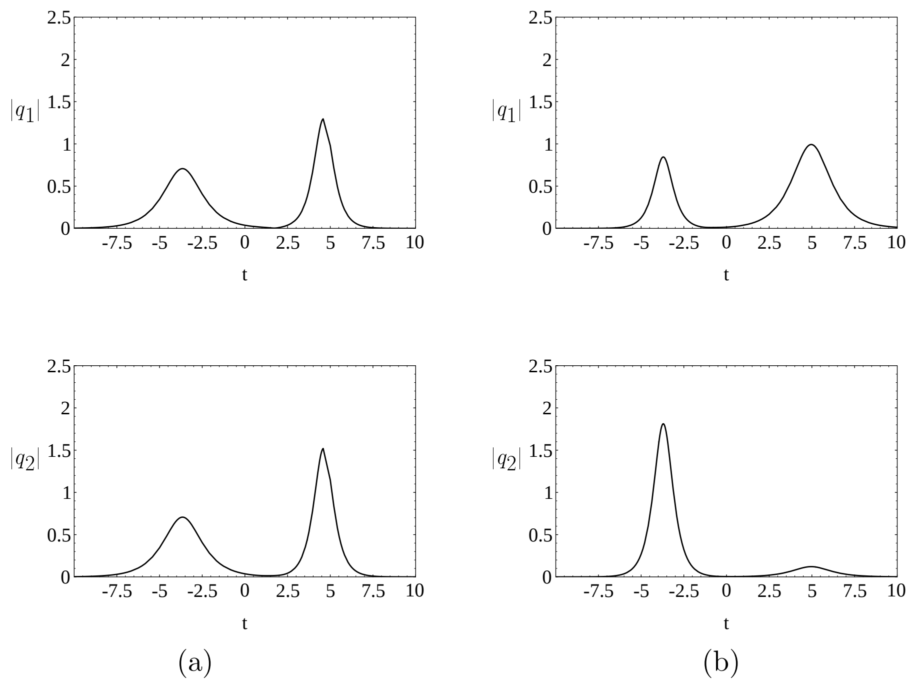

FIG. 2: Asymptotic forms of two-soliton solution (whose stationary form is similar to fig.1) of the integrable 2-CNLS equations (a) at z=-2 and (b) at z=2.

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