# Suppression and Enhancement of Soliton Switching During Interaction in Periodically Twisted Birefringent Fiber 

R. Radhakrishnan and M. Lakshmanan<br>Centre for Nonlinear Dynamics, Department of Physics, Bharathidasan University, Tiruchirapalli - 620 024, Tamil Nadu, India

(December 14, 1998)
Soliton interaction in periodically twisted birefringent optical fibers has been analysed analytically with refernce to soliton switching. For this purpose we construct the exact general two-soliton solution of the associated coupled system and investigate its asymptotic behaviour. Using the results of our analytical approach we point out that the interaction can be used as a switch to suppress or to enhance soliton switching dynamics, if one injects multisoliton as an input pulse in the periodically twisted birefringent fiber.

PACS Numbers 42.81Dp, 42.65Tg, 03.40kf
It is a well-known fact that single-core fiber supports two distinct modes of propagation as a result of birefringence effect, which can be introduced through twisting the fiber during the preform stage of formation or through stress induced birefringence mechanism. For many years the propagation of solitons in stress induced birefringent nonlinear Kerr media with reference to optical fibers is an intensive research subject for theoretical as well as experimental investigations [1]. The topic of propagation in twisted birefringent optical fiber is also gaining considerable interest theoretically and experimentally in recent times as most of the nonlinear directional couplers are based on such fibers. One may mention the example of the rocking rotator [1] which can be used as a switching device at high-power [2] and as a filter at low-power [3].

Soliton propagation in the periodically twisted birefringent fiber is usually described by using the coupled nonlinear Schrödinger (CNLS) family of equations [1,2]. However such CNLS equations are in general not completely integrable. Interestingly if we assume the
value of ellipticity angle to be $35^{\circ}$ then the dynamics of soliton interaction in a periodically twisted birefringent fiber coupler can be described by coupled wave equations of the form [4]

$$
\begin{align*}
& i q_{1 z}+q_{1 t t}+\rho q_{1}+\varkappa q_{2}+2 \mu\left(\left|q_{1}\right|^{2}+\left|q_{2}\right|^{2}\right) q_{1}=0 \\
& i q_{2 z}+q_{2 t t}-\rho q_{2}+\varkappa q_{1}+2 \mu\left(\left|q_{1}\right|^{2}+\left|q_{2}\right|^{2}\right) q_{2}=0 \tag{1}
\end{align*}
$$

where $q_{1}(z, t)$ and $q_{2}(z, t)$ are slowly varying envelopes of two orthogonally polarized modes, z and t are respectively the normalized distance and time and $\varkappa$ and $\rho$ are the normalized linear coupling constants caused by the periodic twist of the birefringence axes and the phase-velocity mismatch from resonance respectively. If the linear coupling constants are absent (that is, $\varkappa=\rho=0$ ), then one can easily recognize the system(1) to be the celebrated integrable Manakov model [5]. The resulting Manakov equation is receving renewed attention recently as it describes the effects of averaged random birefringence on an orthogonally polarized pulse in a real fiber [6]. When the birefringene axes of fiber are periodically twisted during the drawing process there is a periodic intensity exchange between the orthogonally polarized modes [1-4] and it can be modelled by Eq.(1) if the value of ellipticity angle is $35^{\circ}$ [4]. The linear coupling length where the maximum power is transfered from one mode to the other is $\pi /\left(2 \sqrt{\varkappa^{2}+\rho^{2}}\right)$. At the resonance wavelength, the linear parameter $\rho=0$ and the linear coupling length increases to $\pi /(2 \varkappa)$. The schematic of the experimental apparatus used to observe switching characters in the periodically twisted birefringent fiber is given in Ref. [2]. Particularly the dependence of switching characters on the input power, operating wavelength, twist magnitude and twist period are described for example in Refs. [2] and [4].

Considerable attention has been paid in the literature [ $1,5,7,8$ ] to study soliton collision in the birefringent fiber. Particularly by using the system (1) in the absence of linear coupling terms Manakov [5] pointed out that during soliton collision their velocities and amplitudes (intensities) do not change but the associated unit polarization vectors do change provided they are neither parallel nor orthogonal (see also the paper of Silmon-Clyde and Elgin [8] for a discussion in terms of Stokes vectors). Further Menyuk [7] has observed at the value
of the ellipticity angle $35^{\circ}$, where the Manakov equation holds good, that a soliton of one polarization when interacting with a switching pulse of the other polarization does not develop a shadow and also does not change shape. However very recently we have proved [9] by constructing the most general two-soliton solution of the Manakov model that it has the property that soliton in birefringent fiber can in general change its shape after interaction due to change in intensity distribution among the modes enenthough the total energy is conserved. In this letter we investigate the implication of this property of the solitons when the additional effects due to the periodic rotation of birefringence axes are included by constructing the exact two-soliton solution of the system (1). In particular we point out that interaction can be used as a switch to suppress or to induce soliton switching, if we inject multisoliton as an input pulse in the periodically twisted birefringent fiber.

The coupled system (1) reduces to the celebrated integrable Manakov model [5],

$$
\begin{align*}
& i q_{1 M z}+q_{1 M t t}+2 \mu\left(\left|q_{1 M}\right|^{2}+\left|q_{2 M}\right|^{2}\right) q_{1 M}=0, \\
& i q_{2 M z}+q_{2 M t t}+2 \mu\left(\left|q_{1 M}\right|^{2}+\left|q_{2 M}\right|^{2}\right) q_{2 M}=0, \tag{2}
\end{align*}
$$

(the subscript M refers to the Manakov model) under the transformation [4],

$$
\begin{align*}
& q_{1}=\cos (\theta / 2) e^{i \Gamma z} q_{1 M}-\sin (\theta / 2) e^{-i \Gamma z} q_{2 M} \\
& q_{2}=\sin (\theta / 2) e^{i \Gamma z} q_{1 M}+\cos (\theta / 2) e^{-i \Gamma z} q_{2 M} \tag{3}
\end{align*}
$$

where $\Gamma=\left(\rho^{2}+\varkappa^{2}\right)^{1 / 2}$ and $\theta=\tan ^{-1}(\varkappa / \rho)$. Bélanger and Paré [10] and also briefly in [1] have shown that the system (1) without linear self-coupling ( $\rho=0$ ) has simple solitary wave solutions exhibiting energy exchange between the modes. By using the one-soliton solution of the Manakov model (2) in (3), Potasek [4] has pointed out the possibility of periodic intensity exchange between the orthogonally polarised modes $q_{1}$ and $q_{2}$ in the coupled system (1), when both the linear coupling constants $\varkappa$ and $\rho$ are present. An interesting question arises here when one considers multi-soliton solutions for the Manakov model that admits both elastic and inelastic (shape changing) type of collisions depending upon the initial conditions or arbitrary parameters as shown in Ref. [9]. Then how does the switching and
energy sharing properties get modified for the multi-soliton solutions of the system (1)? We show in this letter that indeed novel features in the intensity sharing and different switching properties do arise when the most general two-soliton solution is considered for (1).

For our analysis we make use of the general two-soliton solution of the Manakov system (2), reported in [9], and obtain the corresponding two-soliton solution of (1) through (3). It has the form:

$$
\begin{aligned}
q_{1}= & \left\{\left[\cos (\theta / 2) e^{i \Gamma z} \alpha_{1}-\sin (\theta / 2) e^{-i \Gamma z} \beta_{1}\right] e^{\eta_{1}}+\left[\cos (\theta / 2) e^{i \Gamma z} \alpha_{2}-\sin (\theta / 2)\right.\right. \\
& \left.e^{-i \Gamma z} \beta_{2}\right] e^{\eta_{2}}+\left[\cos (\theta / 2) e^{i \Gamma z+\delta_{1}}-\sin (\theta / 2) e^{-i \Gamma z+\delta_{1}^{\prime}}\right] e^{\eta_{1}+\eta_{1}^{*}+\eta_{2}}+ \\
& {\left.\left[\cos (\theta / 2) e^{i \Gamma z+\delta_{2}}-\sin (\theta / 2) e^{-i \Gamma z+\delta_{2}^{\prime}}\right] e^{\eta_{1}+\eta_{2}+\eta_{2}^{*}}\right\} / D_{n}, }
\end{aligned}
$$

$q_{2}=$ as $\mathrm{q}_{1}$ above with the replacements $\cos (\theta / 2) \rightarrow \sin (\theta / 2)$ and

$$
\begin{equation*}
\sin (\theta / 2) \rightarrow-\cos (\theta / 2) \tag{4}
\end{equation*}
$$

(the symbol * denotes complex conjugation). Here $D_{n}=1+\exp \left(\eta_{1}+\eta_{1}^{*}+R_{1}\right)+\exp \left(\eta_{1}+\right.$ $\left.\eta_{2}^{*}+\delta_{0}\right)+\exp \left(\eta_{1}^{*}+\eta_{2}+\delta_{0}^{*}\right)+\exp \left(\eta_{2}+\eta_{2}^{*}+R_{2}\right)+\exp \left(\eta_{1}+\eta_{1}^{*}+\eta_{2}+\eta_{2}^{*}+R_{3}\right)$ and $\eta_{j}=$ $k_{j}\left(t+i k_{j} z\right), \mathrm{j}=1,2$. The parameters $\exp \left(\delta_{1}\right)=\left(k_{1}-k_{2}\right)\left(\alpha_{1} \kappa_{21}-\alpha_{2} \kappa_{11}\right) /\left(k_{1}+k_{1}^{*}\right)\left(k_{1}^{*}+\right.$ $\left.k_{2}\right), \exp \left(\delta_{2}\right)=\left(k_{2}-k_{1}\right)\left(\alpha_{2} \kappa_{12}-\alpha_{1} \kappa_{22}\right) /\left(k_{2}+k_{2}^{*}\right)\left(k_{1}+k_{2}^{*}\right), \exp \left(\delta_{1}^{\prime}\right)=\left(k_{1}-k_{2}\right)\left(\beta_{1} \kappa_{21}-\right.$ $\left.\beta_{2} \kappa_{11}\right) /\left(k_{1}+k_{1}^{*}\right)\left(k_{1}^{*}+k_{2}\right), \exp \left(\delta_{2}^{\prime}\right)=\left(k_{2}-k_{1}\right)\left(\beta_{2} \kappa_{12}-\beta_{1} \kappa_{22}\right) /\left(k_{2}+k_{2}^{*}\right)\left(k_{1}+k_{2}^{*}\right), \exp \left(\delta_{0}\right)=$ $\kappa_{12} /\left(k_{1}+k_{2}^{*}\right), \exp \left(R_{1}\right)=\kappa_{11} /\left(k_{1}+k_{1}^{*}\right), \exp \left(R_{2}\right)=\kappa_{22} /\left(k_{2}+k_{2}^{*}\right), \exp \left(R_{3}\right)=\left|k_{1}-k_{2}\right|^{2}$ $\left(\kappa_{11} \kappa_{22}-\kappa_{12} \kappa_{21}\right) /\left(k_{1}+k_{1}^{*}\right)\left(k_{2}+k_{2}^{*}\right)\left|k_{1}+k_{2}^{*}\right|^{2}$ and $\kappa_{i j}=\mu\left(\alpha_{i} \alpha_{j}^{*}+\beta_{i} \beta_{j}^{*}\right)\left(k_{i}+k_{j}^{*}\right)^{-1}, i, j=$ 1,2 . The six arbitrary complex parameters $\alpha_{1}, \alpha_{2}, \beta_{1}, \beta_{2}, k_{1}$ and $k_{2}$ determine the amplitude, velocity and phase of the asymptotic soliton forms of (4). Now in order to bring out the nature of the solitons of (1) and their interactions including exchange of energy we carryout an asymptotic analysis of the solution (4). To be specific we choose the arbitrary complex parameters $k_{i}, i=1,2$, as $k_{1 I}>k_{2_{I}}, k_{1 R}>0$ and $k_{2 R}>0$ (here subscripts I and R refer to the imaginary and real parts).
I. Limit $\mathrm{z} \rightarrow-\infty$ : As $\mathrm{z} \rightarrow-\infty$ we can identify two independent solitons denoted by soliton 1 and soliton 2 with the above choices of $k_{1}$ and $k_{2}$. Soliton 1 will be centered around $\eta_{1 R}=k_{1 R}\left(t-2 k_{1 I} z\right) \simeq 0$ (when $\left.\eta_{2 R} \rightarrow-\infty\right)$ and soliton 2 will be centered around
$\eta_{2 R}=k_{2 R}\left(t-2 k_{2 I} z\right) \simeq 0\left(\right.$ when $\left.\eta_{1 R} \rightarrow \infty\right)$.
(a) Soliton $1\left(\eta_{1 R} \simeq 0, \eta_{2 R} \rightarrow-\infty\right)$ :

$$
\begin{align*}
& q_{1} \cong\left[\cos (\theta / 2) e^{i \Gamma z} A_{1 M}^{1-}-\sin (\theta / 2) e^{-i \Gamma z} A_{2 M}^{1-}\right] q^{1-}, \\
& q_{2} \cong\left[\sin (\theta / 2) e^{i \Gamma z} A_{1 M}^{1-}+\cos (\theta / 2) e^{-i \Gamma z} A_{2 M}^{1-}\right] q^{1-}, \tag{5}
\end{align*}
$$

where $q^{1-}=k_{1 R} \exp \left(i \eta_{1 I}\right) \sec h\left(\eta_{1 R}+R_{1} / 2\right), \eta_{1 I}=k_{1 I} t+\left(k_{1 R}^{2}-k_{1 I}^{2}\right) z,\left(A_{1 M}^{1-}, A_{2 M}^{1-}\right)=[\mu($ $\left.\left.\alpha_{1} \alpha_{1}^{*}+\beta_{1} \beta_{1}^{*}\right)\right]^{-1 / 2}\left(\alpha_{1}, \beta_{1}\right)$ and $\left|A_{1 M}^{1-}\right|^{2}+\left|A_{2 M}^{1-}\right|^{2}=1 / \mu$. Here $\left(A_{1 M}^{1-}, A_{2 M}^{1-}\right)$ refers polarization unit vector of Manakov one-soliton solution, which can be obtained from (5) by substituting $\theta=\Gamma=0$, superscripts 1 - denote soliton 1 at the limit $\mathrm{z} \rightarrow-\infty$ and subscripts 1 and 2 refer to the modes $q_{1}$ and $q_{2}$. Eq.(5) exhibits the same form of one-soliton solution of (1) reported by Potasek in the Ref. [4]. If we parametrize [8] the unit polarization vector as $\left(A_{1 M}^{1-}\right.$, $\left.A_{2 M}^{1-}\right)=\left(\cos \left(\theta_{p}^{1-}\right) \exp \left(i \alpha_{p 1}^{1-}\right), \sin \left(\theta_{p}^{1-}\right) \exp \left(i \alpha_{p 2}^{1-}\right)\right)$ then we can identify $\theta_{p}^{1-}$ as the polarization angle and the phases $\alpha_{p 1}^{1-} \neq \alpha_{p 2}^{1-}$ corresponds to the state of elliptical polarization.
(b) Soliton $2\left(\eta_{2 R} \simeq 0, \eta_{1 R} \rightarrow \infty\right)$ :

$$
\begin{align*}
& q_{1} \cong\left[\cos (\theta / 2) e^{i \Gamma z} A_{1 M}^{2-}-\sin (\theta / 2) e^{-i \Gamma z} A_{2 M}^{2-}\right] q^{2-} \\
& q_{2} \cong\left[\sin (\theta / 2) e^{i \Gamma z} A_{1 M}^{2-}+\cos (\theta / 2) e^{-i \Gamma z} A_{2 M}^{2-}\right] q^{2-} \tag{6}
\end{align*}
$$

where $q^{2-}=k_{2 R} \exp \left(i \eta_{2 I}\right) \sec h\left(\eta_{2 R}+\left(R_{3}-R_{1}\right) / 2\right), \eta_{2 I}=k_{2 I} t+\left(k_{2 R}^{2}-k_{2 I}^{2}\right) z,\left(A_{1 M}^{2-}, A_{2 M}^{2-}\right)=$ $\left(\mathrm{a}_{1} / \mathrm{a}_{1}^{*}\right) \mathrm{c}\left[\mu\left(\alpha_{2} \alpha_{2}^{*}+\beta_{2} \beta_{2}^{*}\right)\right]^{-1 / 2}\left[\left(\alpha_{1}, \beta_{1}\right) \kappa_{11}^{-1}-\left(\alpha_{2}, \beta_{2}\right) \kappa_{21}^{-1}\right],\left|A_{1 M}^{2-}\right|^{2}+\left|A_{2 M}^{2-}\right|^{2}=1 / \mu$, in which $\mathrm{a}_{1}=\left(k_{1}+k_{2}^{*}\right)\left[\left(k_{1-} k_{2}\right)\left(\alpha_{1}^{*} \alpha_{2}+\beta_{1}^{*} \beta_{2}\right)\right]^{1 / 2}$ and $\mathrm{c}=\left[1 /\left|\kappa_{12}\right|^{2}-1 / \kappa_{11} \kappa_{22}\right]^{1 / 2}$. It is interesting to note from (5) and (6) that the form of $A_{1 M}^{2-}$ and $A_{2 M}^{2-}$ in (6) differs from the values $A_{1 M}^{1-}$ and $A_{2 M}^{1-}$ in (5) and the former contains more number of parameters, eventhough (6) is an exact one-soliton solution of the system (1) just like the solution (5). Further in the special case $\alpha_{1}: \alpha_{2}=\beta_{1}: \beta_{2}$, the form of (6) reduces to the form of (5) with parameters specifying soliton 2. So Eq.(6) may be considered as the most general one-soliton solution form of (1).
II. Limit $\mathrm{z} \rightarrow \infty$ : We now analyse the form of the solitons after interactions as $\mathrm{z} \rightarrow \infty$ :
(a) Soliton $1\left(\eta_{1 R} \simeq 0, \eta_{2 R} \rightarrow \infty\right)$ :

$$
q_{1} \cong\left[\cos (\theta / 2) e^{i \Gamma z} A_{1 M}^{1+}-\sin (\theta / 2) e^{-i \Gamma z} A_{2 M}^{1+}\right] q^{1+},
$$

$$
\begin{equation*}
q_{2} \cong\left[\sin (\theta / 2) e^{i \Gamma z} A_{1 M}^{1+}+\cos (\theta / 2) e^{-i \Gamma z} A_{2 M}^{1+}\right] q^{1+}, \tag{7}
\end{equation*}
$$

where $q^{1+}=k_{1_{R}} \exp \left(i \eta_{1 I}\right) \sec h\left(\eta_{1 R}+\left(R_{3}-R_{2}\right) / 2\right),\left(A_{1 M}^{1+}, A_{2 M}^{1+}\right)=\left(\mathrm{a}_{2} / \mathrm{a}_{2}^{*}\right) \mathrm{c}\left[\mu\left(\alpha_{1} \alpha_{1}^{*}+\right.\right.$ $\left.\left.\beta_{1} \beta_{1}^{*}\right)\right]^{-1 / 2}\left[\left(\alpha_{1}, \beta_{1}\right) \kappa_{12}^{-1}-\left(\alpha_{2}, \beta_{2}\right) \kappa_{22}^{-1}\right],\left|A_{1 M}^{1+}\right|^{2}+\left|A_{2 M}^{1+}\right|^{2}=1 / \mu$, in which $\mathrm{a}_{2}=\left(k_{2}+\right.$ $\left.k_{1}^{*}\right)\left[\left(k_{1-} k_{2}\right)\left(\alpha_{1} \alpha_{2}^{*}+\beta_{1} \beta_{2}^{*}\right)\right]^{1 / 2}$. Note that $\mathrm{A}_{1 M}^{1+} \neq \mathrm{A}_{1 M}^{1-}$ and $\mathrm{A}_{2 M}^{1+} \neq A_{2 M}^{1-}$, except when $\alpha_{1}$ : $\alpha_{2}=\beta_{1}: \beta_{2}$, corresponding to pure elastic collision in the Manakov model [9].
(b) Soliton $2\left(\eta_{2 R} \simeq 0, \eta_{1 R} \rightarrow-\infty\right)$ :

$$
\begin{align*}
& q_{1} \cong\left[\cos (\theta / 2) e^{i \Gamma z} A_{1 M}^{2+}-\sin (\theta / 2) e^{-i \Gamma z} A_{2 M}^{2+}\right] q^{2+} \\
& q_{2} \cong\left[\sin (\theta / 2) e^{i \Gamma z} A_{1 M}^{2+}+\cos (\theta / 2) e^{-i \Gamma z} A_{2 M}^{2+}\right] q^{2+} \tag{8}
\end{align*}
$$

where $q^{2+}=k_{2 R} \exp \left(i \eta_{2 I}\right) \sec h\left(\eta_{2 R}+R_{2} / 2\right),\left(A_{1 M}^{2+} A_{2 M}^{2+}\right)=\left[\mu\left(\alpha_{2} \alpha_{2}^{*}+\beta_{2} \beta_{2}^{*}\right)\right]^{-1 / 2}\left(\alpha_{2}, \beta_{2}\right)$ and $\left|A_{1 M}^{2+}\right|^{2}+\left|A_{2 M}^{2+}\right|^{2}=1 / \mu$. Here also $\mathrm{A}_{1 M}^{2+} \neq \mathrm{A}_{1 M}^{2-}$ and $\mathrm{A}_{2 M}^{2+} \neq A_{2 M}^{2-}$, unless $\alpha_{1}: \alpha_{2}=\beta_{1}: \beta_{2}$.

Now we recognise from (5-8) that not only the phase-factors but also the overall shapes of the solitons get modified (due to the intensity redistribution among the solitons) after undergoing interaction when the two coupled one-solitons (5) and (6) move from $\mathrm{z} \rightarrow-\infty$ to $z \rightarrow \infty$ as shown in Eq.(7) and Eq.(8). To facilitate the understanding of the above behaviour with reference to the optical soliton switching between the orthogonally polarised modes, it is convenient to obtain the oscillating parts of the intensities associated with the asymptotic forms (5-8) as

$$
\begin{align*}
\left|\frac{q_{l}(z, t)}{q^{n \mp}(z, t)}\right|^{2}= & \left|A_{l M}^{n \mp}\right|^{2} \cos ^{2}(\theta / 2)+\left|A_{j M}^{n \mp}\right|^{2} \sin ^{2}(\theta / 2)+(-1)^{l}\left|A_{l M}^{n \mp} \| A_{j M}^{n \mp}\right| \\
& \sin (\theta) \cos \left(2 \Gamma z+\phi^{n \mp}\right), 1, \mathrm{j}=1,2(\mathrm{l} \neq \mathrm{j}), \mathrm{z} \rightarrow \mp \infty \tag{9}
\end{align*}
$$

where $\phi^{n \mp}=\tan ^{-1}\left(\frac{A_{1 M I}^{n \mp}}{A_{1 M R}^{n \mp}}\right)-\tan ^{-1}\left(\frac{A_{2 M I}^{n \mp}}{A_{2 M R}^{n \mp}}\right)$. The presence or absence of the last term involving the factor $\cos \left(2 \Gamma z+\phi^{n \mp}\right)$ plays a crucial role in the switching behaviour of solitons as demonstrated below.

As we have mentioned before the values of $\mathrm{A}_{j M}^{n-}(\mathrm{j}, \mathrm{n}=1,2)$ change to new values $\mathrm{A}_{j M}^{n+}$ due to the collision between two co-propagating solitons, namely soliton 1 and soliton 2, without violating the condition $\left|A_{1 M}^{n \mp}\right|^{2}+\left|A_{2 M}^{n \mp}\right|^{2}=1 / \mu$. The amount of change
$\left(\mathrm{A}_{j M}^{n+}-\mathrm{A}_{j M}^{n-}\right)$ can be estimated by assigning suitable values to the arbitrary parameters $\mathrm{k}_{1}, \mathrm{k}_{2}, \alpha_{1}, \alpha_{2}, \beta_{1}$, and $\beta_{2}$, appearing in the expressions for $\mathrm{A}_{j M}^{n \mp}(\mathrm{j}, \mathrm{n}=1,2)$. Further from (9) it is obvious that depending upon the values of $\left|\mathrm{A}_{j M}^{n \mp}\right|$, the nature of switching dynamics supported by the system (1) also changes. The change in $A_{j M}^{n \mp}$ can be parmeterized as $\left(\cos \left(\theta_{p}^{n \mp}\right) \exp \left(i \alpha_{p 1}^{n \mp}\right), \sin \left(\theta_{p}^{n \mp}\right) \exp \left(i \alpha_{p 2}^{n \mp}\right)\right)$, where as long as the phases $\alpha_{p 1}^{n \mp} \neq \alpha_{p 2}^{n \mp}$ the state of polarization is preserved during interaction. Therefore in general without affecting the state of polarization, the switching dynamics can be changed just by changing $A_{j M}^{n \mp}$ with the help of the polarization angle. In the following we briefly discuss the different changes which can occur in the intensity exchange between $q_{1}$ and $q_{2}$ modes with respect to soliton 1 and soliton 2 due to the above mentioned collision by considering $\left|\frac{q_{l}}{q^{n}-}\right|^{2}$ and $\left|\frac{q_{l}}{q^{n+}}\right|^{2}$ defined in (9) for each $\mathrm{l}=1,2$ and $\mathrm{n}=1,2$ values.

Case 1: All the $\left|A_{j M}^{n \mp}\right|$ 's $(\mathrm{j}, \mathrm{n}=1,2)$ are nonzero. In this case due to the presence of $\cos \left(2 \Gamma z+\phi^{n \mp}\right)$ term on the right hand side of (9), there is a periodic intensity switching which is always present in both the solitons and in both the components before as well as after the interaction. Of course the conservation relations $\left|A_{1 M}^{n-}\right|^{2}+\left|A_{2 M}^{n-}\right|^{2}=\left|A_{1 M}^{n+}\right|^{2}$ $+\left|A_{2 M}^{n+}\right|^{2}=1 / \mu$ for the total intensity are always valid. However the switching dynamics appearing before and after interaction is not similar in form, due to the condition $\mathrm{A}_{j M}^{n+}$ $\neq \mathrm{A}_{j M}^{n-}, \mathrm{j}, \mathrm{n}=1,2$, except when $\alpha_{1}: \alpha_{2}=\beta_{1}: \beta_{2}$ as mentioned before, giving rise to a partial suppression or enhancement of the periodically varying intensities.

Case 2: Any one of the $\left|A_{j M}^{n \mp}\right|$ 's is zero and others are nonzero. For example if $\left|A_{1 M}^{1+}\right| \sim$ 0 , corresponding to the condition $\alpha_{2}\left(k_{2+} k_{2}^{*}\right)\left(\alpha_{1} \alpha_{2}^{*}+\beta_{1} \beta_{2}^{*}\right)=\alpha_{1}\left(\left|\alpha_{2}\right|^{2}+\left|\beta_{2}\right|^{2}\right)\left(k_{1}+k_{2}^{*}\right)$, then the switching in the intensity of the soliton 1 gets fully suppressed in both the modes $q_{1}$ and $q_{2}$, while it persists for the other soliton. This is illustrated in Fig. 1a for the choosen parameters values, namely $\mathrm{k}_{1}=1+i, k_{2}=2-i, \alpha_{1}=\beta_{1}=\beta_{2}=1, \alpha_{2}=(39+i 80) / 89(\simeq$ $\left.\exp \left(i 64^{\circ}\right)\right), \rho=0.25$ and $\varkappa=0.5$, for which $\left|A_{1 M}^{1-}\right| \sim 0.7,\left|A_{1 M}^{2-}\right| \sim 0.5,\left|A_{2 M}^{1-}\right| \sim 0.7,\left|A_{2 M}^{2-}\right| \sim$ 0.86, $\left|A_{1 M}^{1+}\right| \sim 0.06,\left|A_{1 M}^{2+}\right| \sim 0.7,\left|A_{2 M}^{1+}\right| \sim 0.99$ and $\left|A_{2 M}^{2+}\right| \sim 0.7$, satisfying the condtion that $\left|A_{1 M}^{1+}\right| \sim 0$. Similar phenomenon can be seen if $\left|A_{2 M}^{1+}\right| \sim 0$ instead of $\left|A_{1 M}^{1+}\right| \sim 0$. But
if we choose $\left|A_{j M}^{2+}\right| \sim 0(\mathrm{j}=1$ or $\mathrm{j}=2)$ the periodic intensity exchange with respect to soliton 2 will be suppressed, while it persists in soliton 1 . On the other hand, if $\left|A_{j M}^{n-}\right| \sim 0(\mathrm{j}=1$ or $j=2 ; n=1$ or $n=2$ ), then there is no switching in the intensity of soliton $n$ before interaction, but the switching appears after interaction in that soliton and so there is an inducement of switching due to the inteaction. Thus the interaction itself acts as a switch to suppress or to enhance the switching dynamics.

Case 3: Any two of the $\left|A_{j M}^{n \mp}\right|$ 's are zero and others are nonzero without violating the conservation conditions. For concreteness, let $\left|A_{1 M}^{n+}\right| \sim 0(n=1,2)$, which implies the condition $\alpha_{1} \sim \alpha_{2} \sim 0$. It implies that $\left|A_{1 M}^{n-}\right|(\mathrm{n}=1,2)$ should also simultaneously vanish. Consequently there is no switching between the modes $q_{1}$ and $q_{2}$ either before or after interaction and there will be only inelastic (shape changing) scattering as discussed for the Manakov model in Ref. [9]. Similar observations can also be made if one makes any two of the $\left|A_{j M}^{n \mp}\right|$ 's to vanish. However, one can identify the interesting possibility of switching existing in soliton 1 only before interaction, which gets interchanged with soliton 2 after interaction, by allowing one of the two $\left|A_{j M}^{2-}\right|$ 's and another one of $\left|A_{j M}^{1+}\right|$ 's simultaneously to take the value zero, corresponding to the condition $\left[\left|k_{1}+k_{2}^{*}\right|^{2}\left(\left|\alpha_{1}\right|^{2}+\left|\beta_{1}\right|^{2}\right)\left(\left|\alpha_{2}\right|^{2}\right.\right.$ $\left.\left.+\left|\beta_{2}\right|^{2}\right)\right]=\left[\left(k_{1}+k_{1}^{*}\right)\left(k_{2}+k_{2}^{*}\right)\left|\alpha_{1} \alpha_{2}^{*}+\beta_{1} \beta_{2}^{*}\right|^{2}\right]$. Figure 1b, for the choosen parameters namely $k_{1}=1+i 0.1, k_{2}=1-i 0.1, \alpha_{1}=0.86+i 0.5, \alpha_{2}=0.5+i 0.86, \beta_{1}=0.7+i 0.72$, $\beta_{2}=0.44+i 0.9$, and $\rho=\varkappa=0.25$, for which $\left|A_{1 M}^{1-}\right| \sim 0.7,\left|A_{1 M}^{2-}\right| \sim 0.05,\left|A_{2 M}^{1-}\right| \sim 0.7, \mid$ $A_{2 M}^{2-}\left|\sim 0.99,\left|A_{1 M}^{1+}\right| \sim 0.04,\left|A_{1 M}^{2+}\right| \sim 0.7,\left|A_{2 M}^{1+}\right| \sim 0.99\right.$ and $| A_{2 M}^{2+} \mid \sim 0.7$ satisfying the above condtion, shows that the interaction induces the periodic intensity exchange between the two modes of soliton 2 while it suppresses the switching dynamics in soliton 1.

To conclude, by studying the interaction between two co-propagating solitons in the periodically twisted birefringent fiber with reference to soliton switching, we have observed several possible ways to use interaction as a switch to suppress or to induce the switching dynamics. The basic underlying mechanism of such possibilities is the inelastic (shape changing) nature of the soliton interaction which arises essentially due to changes in the
polarization angle and so in the overall amplitude of the solitons. Since Mollenauer et al [8] have demonstrated polarization scattering by soliton-soliton collision, it shoud also possible to experimentally study the phenomena described in this paper by using specially fabricated optical fibers. These possibilities should have important ramifications in nonlinear switching devices like the rocking rotator.

Acknowledgments : The authors are grateful to Dr. Jarmo Hietarinta for useful discussions. This work has been supported by the Deparment of Science and Technology, Government of India.
[1] G.P. Agrawal, Nonlinear Fiber Optics -Second Edition (Academic Press, New York 1995)
[2] S. Wabnitz, S. Trillo, E. M. Wright, and G. I. Stegeman, J. Opt. Soc. Am. B 8, 602(1991); S. Trillo, S. Wabnitz, W. C. Banyai, N. Finlayson, C. T. Seaton, G. I. Stegeman, and R. H. Stolen, IEEE J. Quantum Electron. QE25, 104 (1989); A. B. Aceves and S. Wabnitz, Opt. Lett. 17, 25 (1992)
[3] R. H. Stolen, A. Ashkin, W. Pleibel, and J. J. Dziedzic, Opt. Lett. 9, 300 (1984); P. Ferro, S. Trillo, and S. Wabnitz, Opt. Lett. 19, 263 (1994)
[4] M.J. Potasek, J. Opt. Soc. Am. B 10, 941(1993)
[5] S. V. Manakov, Zh. Eksp. Teor. Fiz 65, 505 (1973) [Sov.Phys. JETP 38, 248(1974)]
[6] P. K. A. Wai, C. R. Menyuk, and H. H. Chen, Opt. Lett. 16, 1231 (1991); C. De Angelis, S. Wabnitz, and M. Haelterman, Electron. Lett. 29, 1568 (1993); C. De Angelis and S. Wabnitz, Opt. Commun. 125, 186 (1996); P. K. A. Wai and C. R. Menyuk, J. Light Wave Technol. 14, 148 (1996); M. Karlsson, D. J. Kaup and B. A. Malomed, Phys. Rev. E 54, 5802(1996)
[7] C. R. Menyuk, IEEE J. Quantum Electron. QE25, 2674 (1989); C. R. Menyuk and P. K. Wai in Optical Solitons - Theory and Experiment, edited by J. R. Taylor (Cambridge University

Press, New York, 1992) pp. 314-377
[8] L. F. Mollenauer, J. P. Gordon, and F. Heismann, Opt.Lett.20, 2060(1995); Silmon-Clyde and Elgin Opt. Lett. 23, 180(1998)
[9] R. Radhakrishnan, M. Lakshmanan and J. Hietarinta, Phys. Rev. E 56, 2213 (1997)
[10] P. A. Bélanger and C. Paré, Phys. Rev. A 41, 5254(1990)

## Part I

## Figure Caption

Fig.1: Typical evolution of the intensity profiles $\left|q_{1}\right|^{2}$ and $\left|q_{2}\right|^{2}$ of the two-soliton solution (4) (a) showing the suppression in switching between the two modes of s1 soliton and (b) showing the suppression in switching of s1 soliton and enhancement in s2 soliton (while undergoing a large phase shift) for the parameter values given in the text.





