View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Publications of the IAS Fellows

Boundary element method based computation of stress intensity factor by modified crack closure integral

S. K. Maiti, N. K. Mukhopadhyay, A. Kakodkar

Abstract The local smoothing scheme in conjunction with the modified crack closure integral technique has been adopted in the boundary element method to improve the accuracy of computed stress intensity factors. Simple relations have been derived for the case of linear, quadratic and quarter point elements around the crack tip. Case studies are presented to demonstrate improvement in the accuracy. While the displacement method gives a difference with the standard handbook solution up to 26%, the suggested method helps to reduce it to within 2%.

List of symbols

G_I , G_{II}	strain energy release rate in mode I, mode II
K	stress intensity factor
l	crack tip element length
P	total load
p_1, p_2	tractions
r_1, r_2	internal and external radii
s_{avg}, s_j	x-component of traction
t, t_1, t_2, t_{avg}, t_j	y-component of traction
u	x-component of displacement
v	y-component of displacement
W	crack closure work
x, y	cartesian coordinates
Y	SIF correction factor
ξ	natural coordinate

1 Introduction

The boundary element method (BEM) provides an attractive alternative to finite element method (FEM) for the extraction of stress intensity factors (SIFs). The issue has received a considerable attention, e.g. Cruse and Buren (1971), Cruse (1972, 1978), Cruse and Wilson (1978), Tan and Fenner (1978, 1979), Nadiri et al. (1982), Bakr and Fenner (1985), Perucchio and Ingraffea (1985) and Gerstle (1988). The SIFs can be evaluated in principle using several methods, e.g. displacement and stress method (Watwood 1969, Chan et al. 1970), weight function technique, J-integral method (Rice 1968), stiffness derivative procedure (Parks 1974), crack closure integral (Irwin 1958), etc., for 2-D crack configurations. Out of these the first two has been mostly adopted in the BEM.

In the BEM displacements and tractions are used as independent variables. This offers scope for imposing different rates of variation for the two entities. While using the analog of the wellknown quarter point finite element (Barsoum 1976) in the BEM, the assumption of a variation of displacement in proportion to square root of distance from the crack tip does guarantee a square root strain singularity. This cannot however automatically ensure a square-root traction singularity. Blandford et al. (1981) and Martinez et al. (1984) have shown that more accurate SIFs are obtained employing the traction singularity elements than strain singularity elements. They have employed the displacement based extrapolation method for the extraction of the SIFs.

In the FEM, the adoption of the concept of modified crack closure integral (CCI) have contributed to the improvement of accuracy of SIFs over the computation based on the displacement method. This has been demonstrated by several investigators, e.g. Rybicki and Kanninen (1977), Krishnamurthy et al. (1985), Ramamurthy et al. (1986) and Sethuraman and Maiti (1988). The method can also help obtaining solution for problems with mixed mode loading. This technique was first adopted in the FEM by Rybicki and Kanninen (1977) to evaluate the SIF. They explained the method considering a linear variation of the displacement field around the crack tip. Consequently, the element ensures a constant strain field. They termed the method as modified crack closure integral technique. Later it has been shown by Krishnamurthy et al. (1985), Ramamurthy et al. (1986) and Sethuraman and Maiti (1988) that crack line displacement and stresses can be locally smoothed using the computed nodal data. These smoothed field can then be employed to compute the crack closure work. This procedure gives the energy release rate, and hence the SIFs, which are more accurate than the results based on displacement method or the modified CCI. The similar possibility has not yet been exploited in the BEM. This has provided the motivation for the present study. The formulation is presented and the performance is illustrated for the case of linear, quadratic and quarter point elements used around the crack tips.

Communicated by S. N. Atluri, 14 August 1996

S. K. Maiti
Mechanical Engineering Department,
Indian Institute of Technology, Bombay,
Mumbai 400076, India

N. K. Mukhopadhyay, A. Kakodkar
Reactor Design & Development Group,
Bhabha Atomic Research Centre,
Mumbai 400085, India

Correspondence to: S. K. Maiti

2 Mathematical formulation

2.1 Linear element

For a typical BEM discretisation (Fig. 1a), with crack tip at node j and the two adjacent nodes j − 1 and j + 1, the displacement variation over OA can be written in the form

$$v = a_0 + a_1(1 - x/l) \tag{1}$$

where a_0 and a_1 are arbitrary constants, l is the length of the crack tip element and x is the coordinate. It is straightforward to show that $a_0 = 0$ and $a_1 = v_{j-1}$. A linear variation of the traction along OB can be represented in the same way as the displacement. That is

$$t = 0.5(t_j + t_{j+1}) - 0.5(t_j - t_{j+1})\xi \tag{2}$$

where ξ a is natural coordinate. The total load P on the face OB

$$P = \int_0^l t\ dx = 0.5(t_j + t_{j+1})l \tag{3}$$

In the smoothing scheme the traction variation is assumed consistent with the displacement variation. For a linear element, the traction variation is assumed constant over each element span. The normal traction p_1 over OB is so chosen that the total load $p_1 l = P$. This gives

$$p_1 = 0.5(t_j + t_{j+1}) \tag{4}$$

In the boundary element formulation the work done corresponding to the span AO of the boundary is not zero, though this segment is free of any normal traction. This is because traction at the node j is nonzero. The constant traction p_2 along the face AO is assumed to be equal to the traction at the node j for the crack closure work calculations, i.e. $p_2 = t_j$. The total crack closure work W is given by

$$W = \frac{1}{2}\int_0^l (v\,p_1 + v\,p_2)\ dx \tag{5}$$

Finally this leads to

$$G_I = \frac{v_{j-1}}{2}[0.75\ t_j + 0.25\ t_{j+1}] \tag{6}$$

A similar expression can be derived for mode II, which will involve x components of tractions and displacements. The SIF can then be obtained using the standard relations between G and K.

If the traction p_1 over OB is arbitrarily assumed to be a constant and is equal to t_j, a different expression for G is obtained.

$$G_I = v_{j-1}t_j/2 \tag{7}$$

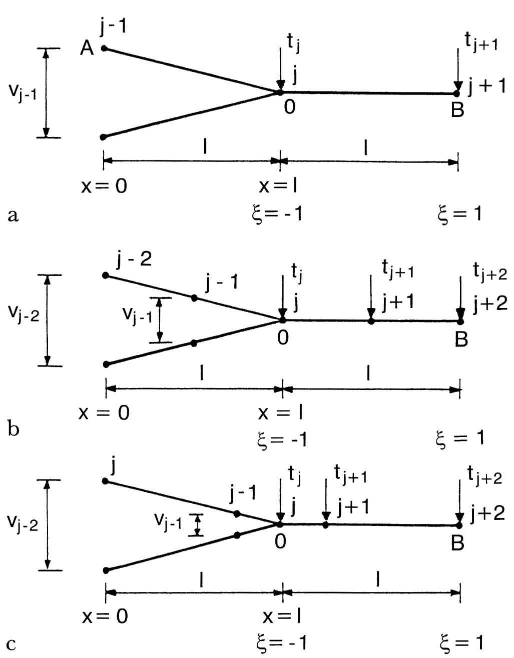

Fig. 1a–c. (a) Linear element. (b) Quadratic element. (c) Quarter point element

2.2 Quadratic element

In the case of quadratic elements around the crack tip (Fig. 1b) the displacement variation over OB is given by

$$v = v_{j-1} - 0.5\ v_{j-2}\,\xi + (0.5\ v_{j-2} - v_{j-1})\,\xi^2 \tag{8}$$

Similarly the traction variation, which is also quadratic, has the form

$$t = t_{j+1} + 0.5(t_{j+2} - t_j)\ \xi + [0.5\ (t_{j+2} + t_j) - t_{j+1}]\ \xi^2 \tag{9}$$

The total normal load arising out of the traction on the face OB

$$P = \int_0^l t\ dx = (t_j + 4t_{j+1} + t_{j+2})l/6 \tag{10}$$

The traction variation for smoothing is assumed to be linear. The variation of traction p_1 along AO is given by

$$p_1 = \frac{1}{2}(1 + \xi)\ t_j \tag{11}$$

The variation of traction p_2 along OB is assumed to be

$$p_2 = \frac{1}{2}(1 - \xi)\ t_1 + \frac{1}{2}(1 + \xi)\ t_2 \tag{12}$$

where t_1 and t_2 are the assumed tractions at the nodes j and j+2. t_1 and t_2 are assumed so that the total load is the same as P. t_1 and t_2 can be taken in the following form

$$t_1 = (t_j + 2\ t_{j+1})/3 \tag{13}$$

$$t_2 = (2\ t_{j+1} + t_{j+2})/3 \tag{14}$$

The total crack closure work

$$W = \frac{1}{2}\int_0^l (v\,p_1 + v\,p_2)\ dx = [2\ t_j v_{j-1} + t_1(2\ v_{j-1} + v_{j-2}) + 2\ t_2\ v_{j-1}]/12 \tag{15}$$

The strain energy release rate

$$G_I = \frac{1}{6}[t_j v_{j-1} + t_1(v_{j-1} + 0.5\ v_{j-2}) + t_2\ v_{j-1}] \tag{16}$$

A similar expression for G_{II} can be obtained involving x-component of tractions and displacements.

2.3
Quarter point element

In the case of quarter point elements (Fig. 1c) the displacement is assumed to vary as $\sqrt{x}$ along OA. That is

$$v = 2\,(v_{j-2} - 2\,v_{j-1})\ (1 - x/l) + (4\ v_{j-1} - v_{j-2})\sqrt{(1 - x/l)} \tag{17}$$

The traction too has a similar variation and can be represented in the form

$$t = t_j\{-0.5\,\xi(1-\xi)\} + t_{j+1}(1-\xi^2) + t_{j+2}\{0.5\,\xi(1+\xi)\} \tag{18}$$

where $1 + \xi = 2\sqrt{(x/l)}$.
The total load over OB

$$P = \int_0^l t\ dx = (2\ t_{j+1} + t_{j+2})\ l/3 \tag{19}$$

This equation shows that the total load is independent of the traction at the crack tip node j. Therefore, total load for the span AO is zero, because t_{j-1} and t_{j-2} are both zero. Hence the traction p_1 along AO can be neglected. In order to ensure a singularity variation of traction along the span OB and the same total load P, the traction p_2 along OB is taken as follows.

$$p_2 = \frac{t_{avg}}{2\sqrt{(x/l)}} \tag{20}$$

$$\text{where } t_{avg} = (2\ t_{j+1} + t_{j+2})/3 \tag{21}$$

The total crack closure work

$$W = \frac{1}{2}\int_0^l v\,p_2\ dx = 0.5\ t_{avg}[v_{j-2}\,(4/3 - \pi/4) + v_{j-1}(\pi - 8/3)] \tag{22}$$

The strain energy release rate

$$G_I = 0.5\ t_{avg}[v_{j-2}(4/3 - \pi/4) + v_{j-1}(\pi - 8/3)] \tag{23}$$

A similar expression can be derived for G_{II}, the mode II energy release rate.

$$G_{II} = 0.5\ s_{avg}[u_{j-2}(4/3 - \pi/4) + u_{j-1}(\pi - 8/3)] \tag{24}$$

where $s_{avg} = (2\ s_{j+1} + s_{j+2})/3$, s_{j+1} and s_{j+2} are the x-components of nodal tractions.

3
Case studies

Numerical studies have been carried out on mode I considering problems of centre crack, single edge crack and circular ring with radial cracks (Fig. 2). The a/w ratio is considered in the range 0.2 to 0.8 for the centre crack problem (Fig. 2a) and 0.2 to 0.7 for the edge crack problem (Fig. 2b). A similar parameter $a/(r_2 - r_1)$, is considered in the range of 0.2 to 0.8 for the circular ring with radial inner edge crack under external uniform tension (Fig. 2c) and 0.1 to 0.6 for the circular ring with radial outer edge crack under internal pressure (Fig. 2d). The material is assumed to be isotropic. In the case of quadratic and quarter point elements the crack tip element size is 0.01a. For the case of linear element the size is 0.005a. The subsequent element sizes, away from the crack tip, are 0.02a, 0.04a, 0.08a, 0.15a, etc. In the case of centre and edge cracks, there are approximately 22 elements and 44 nodes in the discretisation when quadratic and quarter point elements are used around the crack tip. When the linear element is used, the number of elements and nodes are 48. These extra nodes come up because two closely spaced nodes are used at the four corners. In the case of circular ring the number of elements and nodes are 26 and 52 when quadratic and quarter point elements are used. The SIF has been computed through both the displacement and the proposed CCI methods. In the displacement technique the SIF is evaluated considering separately the displacement of the first and second corner nodes from the crack tip. The results in the form of SIF correction factor Y in the case of centre crack, edge crack and radial inner edge crack are compared with the handbook solutions (Rooke and Cartwright, 1976 and Murakami 1987) in Tables 1 to 10 and the data for the case of radial outer edge crack are presented in Table 11.

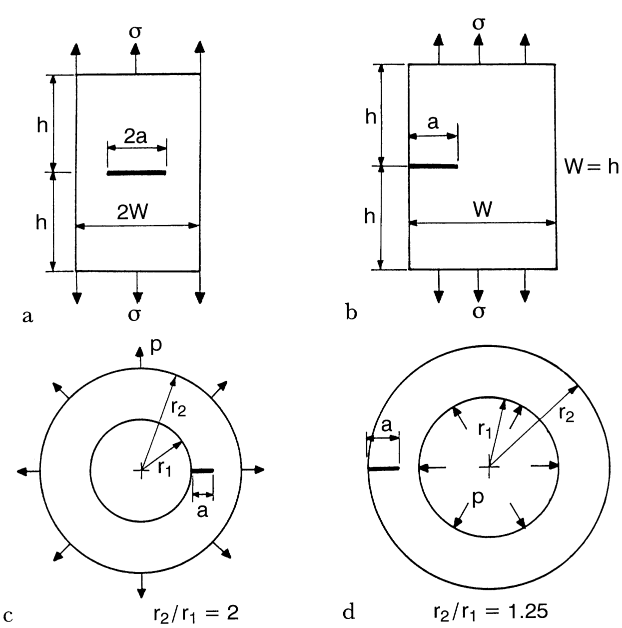

Fig. 2a–d. Geometries considered for analysis. (**a**) Centre crack. (**b**) Edge crack. (**c**) Circular ring with radial inner edge crack. (**d**) Circular ring with radial outer edge crack

The effect of the crack tip element size on the accuracy of the results has been studied for both the centre crack and edge crack problems considering a/w = 0.5. The results are shown in Figs. 3 and 4.

4
Discussion

The accuracy of the displacement method is dependent on where the displacements are compared. The accuracy improves substantially in the case of linear elements considering the second corner node rather than the first corner node for a comparison. Thereby the maximum error reduces from 27% to about 6.5%. The CCI method helps to keep this difference to within 11.39% for the centre crack problem. In the case of both quadratic and quarter point elements, the comparison of displacements at the second corner node is preferable. However in these cases, the CCI method gives the best accuracy. The error is less than about 1.5%.

In the case of edge crack in the rectangular panel, again comparison of displacement at the second corner node is preferable. The error is around 30% when linear element is employed and SIF is compared at the first corner node. This error reduces to within 14% for quarter point element and less than 4.2% for quadratic element. The error is within 6% for all the three elements when the SIF is

Table 1. Comparison of SIF correction factor Y for centre crack for linear element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (6)	%Error
		Y	% Error	Y	% Error		
0.2	1.0254	0.7611	−25.778	0.9696	−5.443	0.9228	−10.002
0.3	1.0594	0.7831	−26.081	0.9973	−5.862	0.9495	−10.372
0.4	1.1118	0.8205	−26.197	1.0454	−5.972	0.9949	−10.510
0.5	1.1891	0.8760	−26.331	1.1157	−6.174	1.0622	−10.670
0.6	1.3043	0.9988	−23.422	1.2189	−6.547	1.1856	− 9.103
0.7	1.4842	1.0859	−26.835	1.4201	−4.317	1.3191	−11.125
0.8	1.7989	1.3121	−27.059	1.7136	−4.745	1.5940	−11.388

Table 2. Comparison of SIF correction factor Y for centre crack for quadratic element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (16)	% Error
		Y	% Error	Y	% Error		
0.2	1.0254	0.9743	−4.984	0.9858	−3.864	1.0226	−0.270
0.3	1.0594	1.0079	−4.866	1.0265	−3.110	1.0572	−0.204
0.4	1.1118	1.0567	−4.952	1.0695	−3.808	1.1094	−0.217
0.5	1.1891	1.1293	−5.025	1.1425	−3.918	1.1857	−0.289
0.6	1.3043	1.2332	−5.449	1.2471	−4.385	1.2944	−0.760
0.7	1.4842	1.4020	−5.535	1.4268	−3.868	1.4713	−0.871
0.8	1.7989	1.6938	−5.845	1.7222	−4.265	1.7778	−1.173

Table 3. Comparison of SIF correction factor Y for centre crack for quarter point element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (23)	% Error
		Y	% Error	Y	% Error		
0.2	1.0254	1.1384	11.024	1.0343	0.871	1.0157	−0.944
0.3	1.0594	1.1771	11.124	1.0508	−0.816	1.0516	−0.735
0.4	1.1118	1.2352	11.096	1.1220	0.913	1.1023	−0.857
0.5	1.1891	1.3203	11.031	1.1997	0.889	1.1784	−0.898
0.6	1.3043	1.4437	10.686	1.3117	0.571	1.2886	−1.201
0.7	1.4842	1.6398	10.484	1.4626	−1.454	1.4655	−1.263
0.8	1.7989	1.9830	10.236	1.7609	−2.112	1.7727	−1.458

Table 4. Comparison for SIF correction factor Y for edge crack for linear element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (6)	% Error
		Y	% Error	Y	% Error		
0.2	1.3736	1.0293	−25.066	1.3159	−4.203	1.2471	− 9.207
0.3	1.6629	1.2452	−25.117	1.5951	−4.078	1.5081	− 9.309
0.4	2.1066	1.5763	−25.176	2.0237	−3.933	1.9072	− 9.463
0.5	2.8297	2.0886	−26.191	2.6962	−4.718	2.5257	−10.743
0.6	4.0299	2.9119	−27.743	3.7953	−5.823	3.5141	−12.675
0.7	6.3610	4.4314	−30.335	5.9779	−6.022	5.3575	−15.776

Table 5. Comparison for SIF correction factor Y for edge crack for quadratic element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (16)	% Error
		Y	% Error	Y	% Error		
0.2	1.3736	1.3160	−4.195	1.3360	−2.738	1.3793	0.418
0.3	1.6629	1.6008	−3.737	1.6272	−2.149	1.6767	0.828
0.4	2.1066	2.0380	−3.257	2.0762	−1.441	2.1315	1.180
0.5	2.8297	2.7284	−3.579	2.7907	−1.379	2.8475	0.627
0.6	4.0299	3.8950	−3.348	4.0091	−0.516	4.0518	0.543
0.7	6.3610	6.1061	−4.007	6.4362	1.182	6.3594	−0.026

Table 6. Comparison for SIF correction factor Y for edge crack for quarter point element

a/w	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (23)	% Error
		Y	% Error	Y	% Error		
0.2	1.3736	1.5410	12.186	1.4042	2.230	1.3726	−0.075
0.3	1.6629	1.8755	12.756	1.7117	2.936	1.6695	0.396
0.4	2.1066	2.3903	13.466	2.1870	3.819	2.1245	0.848
0.5	2.8297	3.2052	13.270	2.9441	4.041	2.8433	0.482
0.6	4.0299	4.5911	13.927	4.2438	5.309	4.0582	0.703
0.7	6.3610	7.1817	12.901	6.6361	4.325	6.3669	0.093

Table 7. Comparison of SIF correction factor Y for circular ring with radial inner edge crack under external tension for linear element

$a/(r_2-r_1)$	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (6)	% Error
		Y	% Error	Y	% Error		
0.2	2.7760	2.0871	−24.817	2.6256	−5.418	2.5273	− 8.958
0.3	2.8672	2.1237	−25.931	2.6711	−6.841	2.5719	−10.301
0.4	2.9887	2.1948	−26.565	2.7601	−7.646	2.6581	−11.602
0.5	3.1360	2.2679	−27.683	2.8547	−8.970	2.7457	−12.445
0.6	3.3152	2.3919	−27.851	3.0123	−9.137	2.8954	−12.662
0.7	3.5541	2.5502	−28.247	3.2152	−9.535	3.0859	−13.174
0.8	3.9125	2.8202	−27.918	3.5608	−8.989	3.4118	−12.796

Table 8. Comparison of SIF correction factor Y for circular ring with radial inner edge crack under external tension for quadratic element

$a/(r_2-r_1)$	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (16)	% Error
		Y	%Error	Y	% Error		
0.2	2.7760	2.6380	−4.972	2.6624	−4.094	2.7620	−0.503
0.3	2.8672	2.7193	−5.160	2.7441	−4.293	2.8476	−0.682
0.4	2.9887	2.8386	−5.022	2.8648	−4.146	2.9726	−0.539
0.5	3.1360	2.9663	−5.413	2.9955	−4.479	3.1041	−1.017
0.6	3.3152	3.1395	−5.301	3.1721	−4.315	3.2837	−0.950
0.7	3.5541	3.3513	−5.705	3.3889	−4.648	3.5019	−1.468
0.8	3.9125	3.6982	−5.478	3.7434	−4.322	3.8604	−1.331

Table 9. Comparison of SIF correction factor Y for circular ring with radial inner edge crack under external tension for quarter point element

$a/(r_2-r_1)$	SIF Correction Factor Y						
	Reference Solution	Computed by					
		Displacement Method				CCI Method	
		1st Corner Node		2nd Corner Node		Y Eqn. (23)	% Error
		Y	% Error	Y	% Error		
0.2	2.7760	3.0793	10.927	2.8635	3.153	2.7394	−1.301
0.3	2.8672	3.1768	10.800	2.9537	3.018	2.8274	−1.388
0.4	2.9887	3.3191	11.054	3.0862	3.261	2.9540	−1.160
0.5	3.1360	3.4685	10.603	3.2273	2.913	3.0849	−1.629
0.6	3.3152	3.6739	10.818	3.4201	3.163	3.2657	−1.493
0.7	3.5541	3.9247	10.428	3.6567	2.888	3.4856	−1.929
0.8	3.9125	4.3384	10.888	4.0462	3.416	3.8489	−1.625

Table 10. Comparison of SIF correction factor Y based on a different scheme of CCI calculation for centre crack, edge crack and circular ring with radial inner edge crack for linear element

a/w	SIF Correction Factor Y Computed by Eqn. (7)					
	Centre crack		Edge crack		Circular ring	
	Y	% Error	Y	% Error	Y	% Error
0.2	1.1016	−1.348	1.3673	−0.460	2.7713	−0.170
0.3	1.0408	−1.753	1.6535	−0.563	2.8200	−1.647
0.4	1.0906	−1.906	2.0914	−0.719	2.9145	−2.484
0.5	1.1644	−2.081	2.7703	−2.099	3.0108	−3.993
0.6	1.2721	−2.470	3.8597	−4.224	3.1751	−4.227
0.7	1.4440	−2.711	5.8736	−7.662	3.3842	−4.7781
0.8	1.7449	−3.000	–	–	3.7421	−4.355

compared at the second corner node. The CCI method helps to contain the error within 16% in the case of linear element. The error reduces drastically when the quadratic or quarter point elements are employed around the crack tip. The maximum error is 1.18% for the range of a/w = 0.2 to 0.7 for both the quadratic and quarter point elements. Similar trend is observed in the case of circular rings. The error is found to be less than 2% for the range of $a/(r_2 - r_1) = 0.2$ to 0.8 when quadratic or quarter point elements are employed in the proposed method.

The error in Y through Eqn. (6) is in the range of 10–16% in the case of centre crack, edge crack and circular ring with inner edge crack. However, if Eqn. (7) is employed instead (Table 10), the error reduces to within 3% in the case of centre crack, 7.66% for the edge crack and 4.8% for the circular ring. This means that an assumption of a constant traction $p_1 = t_j$ over the edge OB produces better results than the assumption of a value $p_1 = (t_j + t_{j+1})/2$. It is not possible to provide any justification for this observation at this stage.

The results for the case of circular ring with radial outer edge crack (Table 11) shows that, for a/w, for example, equal to 0.3, the difference with Rooke and Cartwright (1976) is less than 10%, 1.62% and 1% based on linear

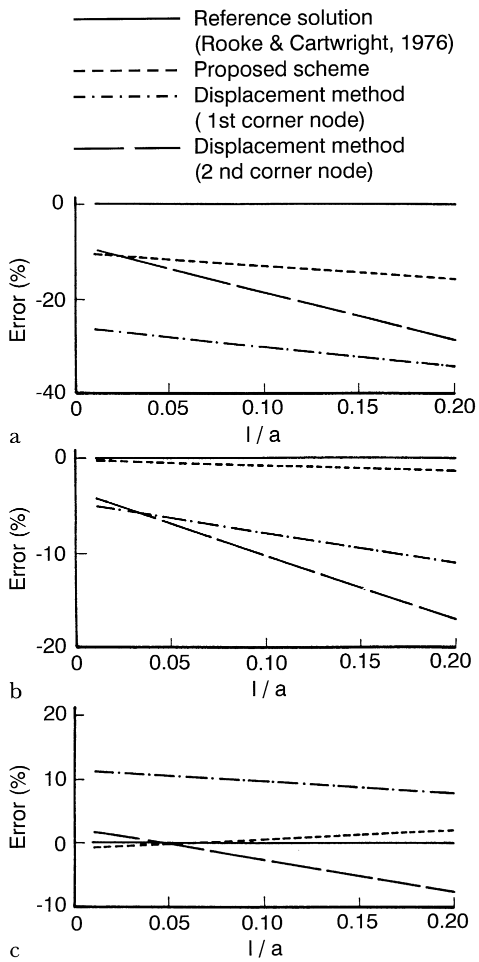

Fig. 3a–c. Effect of crack tip element size on error in the case of centre crack. (**a**) Linear element. (**b**) Quadratic element. (**c**) Quarter point element

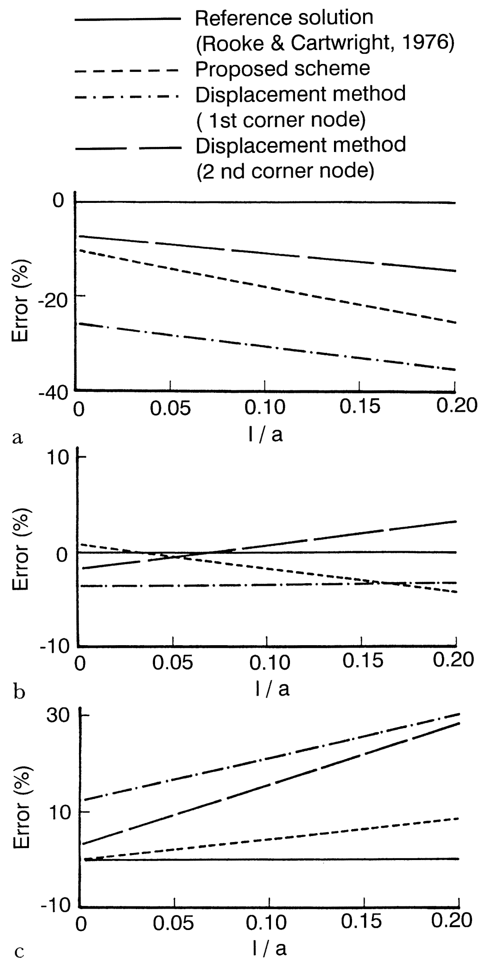

Fig. 4a–c. Effect of crack tip element size on error in the case of edge crack. (**a**) Linear element. (**b**) Quadratic element. (**c**) Quarter point element

element, quadratic element and quarter point element respectively with adoption of the CCI.

The effect of crack tip element size on the accuracy of SIF shows (Figs. 3 and 4) that the displacement method is very sensitive to the crack tip element size. The accuracy reduces as the element size increases. On the other hand the proposed method is less sensitive to the mesh refinement. In the case of the linear elements the difference with the reference solution increases steadily with the element size. The minimum difference observed is about 10% for a crack tip element size of 0.01a for both the cases. In the case of quadratic elements, for a crack tip element size up to 0.2a, the difference is around 1% for the centre crack and it is less than 4.5% for the edge crack. In the case of the quarter point elements there is very good accuracy for element size up to 0.12a in the case of centre crack problem and 0.06a in the case of edge crack.

5 Conclusion

Thus it is clear that the crack closure integral and local smoothing can be used together to improve the accuracy

Table 11. SIF correction factor Y for circular ring with radial outer edge crack using CCI method

$a/(r_2-r_1)$	SIF Correction Factor Y Computed By		
	Linear element	Quadratic element	Quarter point element
	Y	Y	Y
0.1	1.085	1.1649	1.1619
0.2	1.2012	1.3088	1.3008
0.3	1.3860	1.5650	1.5551
0.4	1.6557	1.8296	1.8213
0.5	2.0271	2.2416	2.2337
0.6	2.4653	2.7271	2.7174

of boundary element based computation of SIFs. While locally smoothing the traction and displacement variation along the crack line, it is beneficial to keep the order of displacement variation the same as in the BEM but the order of traction variation one order less than the displacement. By adopting the CCI along with local smoothing the error can be reduced to less than 1% in the case of mode I problems. In general the crack tip element size less than about 0.06a is recommended.

References

Bakr, A. A.; Fenner, R. T. (1985): Axisymmetric fracture mechanics analysis by the boundary element method. Int. J. Pressure Vessels and Piping. 18: 55–75

Barsoum, R. S. (1976): On the use of isoparametric finite elements in linear fracture mechanics. Int. J. Numer. Methods Eng. 10: 25–37

Blandford, G. E.; Ingraffea, A. R.; Liggett, J. A. (1981): Two-dimensional stress intensity factor computations using the boundary element method. Int. J. Numer. Methods Eng. 17: 387–404

Chan, S. K.; Tuba, I. S.; Wilson, W. K. (1970): On the finite element method in linear fracture mechanics. Engng Fracture Mechanics. 2: 1–17

Cruse, T. A.; Buren, W. Van. (1971): Three-dimensional elastic stress analysis of a fracture specimen with an edge crack. Int. J. Fracture Mechanics. 7: 1–16

Cruse, T. A. (1972): Numerical evaluation of elastic stress intensity factors by the boundary integral equation method. In: Swedlow, J. L. (ed): The Surface Crack: Physical Problems and Computational Solutions ASME. pp 153–170, New York

Cruse, T. A. (1978): Two-dimensional BIE fracture mechanics analysis. Appl. Math. Modelling. 2: 287–293

Cruse, T. A.; Wilson, R. B. (1978): Advanced applications of boundary-integral equation methods. Nucl. Engr. Design. 46: 223–234

Gerstle, W. H.; Ingraffea, A. R.; Perucchio, R. (1988): Three-dimensional fatigue crack propagation analysis using the boundary element method, Int. J. Fatigue. 10: 189–192

Irwin, G. R. (1958): In: Flugge, S. (ed): Fracture I, Hanbuch der physik VI, pp 558–590, Springer, Berlin

Krishnamurthy, T.; Ramamurthy, T. S.; Vijayakumar, K.; Duttaguru, B. (1985): Modified crack closure integral method for higher order finite elements. In: Kant, T. (ed): Proc. Int. Conf. Finite Elements in Computational Mechanics. pp 891–900, Bombay

Martinez, J.; Dominguez, J. (1984): On the use of quarter-point boundary elements for stress intensity factor computations. Int. J. Numer. Methods Eng 20: 1941–1950

Murakami, Y. (ed). (1987): Stress intensity factors handbook. Pergamon Press

Nadiri, F.; Tan, C. L.; Fenner, R. T. (1982): Three-dimensional analysis of surface cracks in pressurised thick walled cylinders. Int. J. Pressure Vessels and Piping, 10: 159–167

Parks, D. M. (1974): A stiffness-derivative finite element technique for determination of crack tip stress intensity factors. Int. J. Fracture. 10: 487–502

Perucchio, R.; Ingraffea, A. R. (1985): An integrated boundary element analysis system with interactive computer graphics for three-dimensional linear elastic fracture mechanics. Computers and Structures. 20: 157–171

Ramamurthy, T. S.; Krishnamurthy, T.; Narayan, K. Badri.; Vijayakumar, K; Duttaguru, B. (1986): Modified crack closure integral methods with quarter point elements. Mech. Res. Commun, 13: 179–186

Rice, J. R. (1968): A path independent integral and the approximate analysis of strain concentration by notches and cracks. Trans. ASME J. Appl. Mech. 35: 379–386

Rooke, D. P.; Cartwright, D. J. (1976): Compendium of stress intensity factors, HMSO, London

Rybicki, E. F.; Kanninen, M. F. (1977): A finite element calculation of stress intensity factors by a modified crack closure integral. Engng Fracture Mech. 9: 931–938

Sethuraman, R.; Maiti, S. K. (1988): Finite element based computation of strain energy release rate by modified crack closure integral. Engng. Fracture Mechanics. 30: 227–231

Tan, C. L.; Fenner, R. T. (1978): Three-dimensional stress analysis by the boundary integral equation method. J. Strain Analysis. 13: 231–237

Tan, C. L.; Fenner, R. T. (1979): Elastic fracture mechanics analysis by the boundary integral equation method. Proc. Roy. Soc. Lond. A369: 243–260

Watwood, V. B. (1969): The finite element method for prediction of crack behaviour. Nuclear Engng. and Des. 11: 323–332