Entropy production, energy loss and currents in adiabatically rocked thermal ratchets

Raishma Krishnan* and A. M. Jayannavar†

Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, India

Abstract: We study the nature of currents, input energy and entropy production in different types of adiabatically rocked ratchets using the method of stochastic energetics. The currents exhibit a peak as a function of noise strength. We show that there is no underlying resonance or synchronisation phenomena in the dynamics of the particle with these current peaks. This follows from the analysis of energy loss in the medium. We also show that the maxima seen in current as well as the total entropy production are not directly correlated.

PACS numbers: 05.40.-a; 05.70.lw
Keywords: Ratchets, entropy production, noise, energy loss.

I. INTRODUCTION

The subject of noise induced transport has attracted much theoretical as well as experimental interest for the past few years [1, 2, 3, 4]. The motivation for such a study stems from the challenge to develop models to explain the reliable unidirectional transport observed in biological systems amidst a very noisy environment in the absence of bias. Systems that combine the asymmetry and nonequilibrium fluctuations to generate systematic motion in the absence of a macroscopic bias are termed as ratchets or Brownian motors. In thermal equilibrium the principle of detailed balance prohibits any net particle current in the system. Hence a net current in the absence of any bias can appear only as a consequence of the interaction of the particle with its noisy nonequilibrium environment. Thus it is possible to extract energy from the random fluctuations and put it into use. These ratchet systems are information engines analogous to the Maxwell's demon which extract work out of bath at the expense of an overall increase in entropy (or entropy production) [5, 6]. There are several ways in which one can incorporate nonequilibrium effects arising out of the irreversible interaction of the system with its external surroundings. This has led to various types of ratchets, namely, flashing ratchets, rocking ratchets, frictional ratchets, time asymmetric ratchets, etc [2]. Extensive studies have been carried out on the nature of current and their possible reversals as a function of various physical parameters. These studies are found to be useful in identifying proper models for biological motors and also to develop machines at the molecular scales including nanoparticle separation devices [4].

The subject of the energetics of Brownian motors or ratchets has developed into an entire subfield of its own right [7, 8]. A general framework has been developed wherein, the compatibility between the Langevin and the Fokker-Planck formalisms used for various types of ratchets or motor models and the laws of thermodynamics have been proved [8]. Using this framework one can readily calculate various physical quantities like the efficiency of energy transduction, energy dissipation (hysterisis loss), entropy (entropy production), input energy, change in internal energy, work etc., in systems far from linear response regime into the relam of nonequilibrium domain. Some recent studies have also tried to reveal the relations between two completely unrelated phenomena, namely, stochastic resonance (SR) and Brownian ratchets in a formal way through the consideration of Fokker-Planck equations [9, 10, 11, 12, 13]. It has been argued that the rate of flow of particles in a Brownian ratchet is analogous to the rate of flow of information in the case of stochastic resonance. Qian et al. have investigated a simple flashing ratchet model for ratchet effect as well as SR and have pointed out that the consis-

*Electronic address: raishma@iopb.res.in
†Electronic address: jayan@iopb.res.in

tency between these two phenomena are just due to the existence of circular flux in nonequilibrium state [13].

In our present work we have analysed the nature of input energy (energy loss) and the total entropy production in a class of adiabatically rocked ratchets as a function of temperature of the bath (or noise strength). These systems exhibit peak in the noise induced current (in the absence of any net bias) as a function of temperature. The question now arises as to whether this peak is related to the underlying resonance due to the synchronization of the position of the particle with the external drive induced by the noise. Our analysis of input energy E_{in}, rules out the presence of any resonance features in the dynamics of the position of the particle in these systems in the adiabatic regime.

The presence of net currents in the ratchets increases the amount of known information about the system than otherwise. This extra bit of information comes from the negentropy or the physical information supplied by the external nonequilibrium bath. The amount of information transferred by the nonequilibrium bath is quantified in terms of algorithmic complexity. It has been argued that the algorithmic complexity or Kolmogorov information entropy is maximum when the current is maximum [14]. Since the currents are generated at the expense of entropy we naively expect the maxmima in current to be related to the maxmima in the overall entropy production as a function of noise strength. However, we show that the maxima in current and the entropy production do not correlate with each other.

II. THE MODEL:

We study the motion of an overdamped Brownian particle in a potential $V(q)$ subjected to a space dependent medium with friction coefficient $\gamma(q)$ and an external periodic force field $F(t)$ at temperature T. The motion is described by the Langevin equation [15]

$$\dot{q} = -\frac{V'(q) - F(t)}{\gamma(q)} - \frac{k_B T \gamma'(q)}{2[\gamma(q)]^2} + \sqrt{\frac{k_B T}{\gamma(q)}}\xi(t) \qquad (1)$$

where $\xi(t)$ is a randomly fluctuating Gaussian white noise with zero mean and correlation: $< \xi(t)\xi(t') > = 2\,\delta(t-t')$. We take the potential $V(q)$ to be periodic in space and is given by $V(q) = -sin(q) - (\mu/4)\,sin(2q)$ with the asymmetry parameter μ taking values between -1 and 1. Also, we take the friction coefficient $\gamma(q)$ to be periodic: $\gamma(q) = \gamma_0(1 - \lambda sin(q+\phi))$, where ϕ is the phase difference with respect to $V(q)$ and the coefficient λ takes values between 0 and 1. The equation of motion is equivalently given by the Fokker-Planck equation [16]

$$\begin{aligned} \frac{\partial P(q,t)}{\partial t} &= \frac{\partial}{\partial q}\frac{1}{\gamma(q)}\Big[k_B T\frac{\partial P(q,t)}{\partial q} \\ &+ [V'(q) - F(t)]P(q,t)\Big] \end{aligned} \qquad (2)$$

This equation can be solved for the probability current j when $F(t) = F_0 = constant$, and is given by

$$j = \frac{1 - exp\,[\frac{-2\pi F_0}{k_B T}]}{\int_0^{2\pi} dy I_-(y)} \qquad (3)$$

where $I_-(y)$ is given by

$$\begin{aligned} I_-(y) &= exp\left[\frac{-V(y) + F_0 y}{k_B T}\right] \\ &\int_y^{y+2\pi} dx\ \ \gamma(x) exp\left[\frac{V(x) - F_0 x}{k_B T}\right] \end{aligned} \qquad (4)$$

In the case of inhomogeneous ratchets (space dependent frictional case, $\lambda \neq 0$) [15, 17] with the spatial asymmetry parameter $\mu = 0$ it may be noted that $j(F_0)$ may not be equal to $-j(-F_0)$ for $\phi \neq 0, \pi$. This fact leads to the rectification of current in the presence of an applied ac field $F(t)$. In these inhomogeneous systems directed currents can be obtained even in a spatially periodic symmetric potential. The inversion symmetry in these systems being broken dynamically by the space dependent frictional coefficient which is periodic in space having a phase lag of ϕ with the potential profile. In the second case where $\lambda = 0$ (purely homogeneous case) the net currents are generated due to the spatial asymmetry of the potential ($\mu \neq 0$). We assume that $F(t)$ changes slowly enough (adiabatic regime), i.e., its frequency is smaller than any other frequency related to the relaxation rate in the problem such that the system is in a steady state at each instant of time. For a field $F(t)$ of a square wave of amplitude F_0, an average current over the period of oscillation is given by, $< j > = \frac{1}{2}[j(F_0) + j(-F_0)]$ [15, 18]. In the quasi static limit following the method of stochastic energetics it can be shown [18] that the input energy E_{in} (per unit time) is given by $E_{in} = \frac{1}{2}F_0[j(F_0) - j(-F_0)]$.

We also consider another type of ratchet, namely, the time asymmetric ratchets [19, 20, 21] where the driving force has zero mean, $< F(t) >= 0$, but is asymmetric in time i.e.,

$$F(t) = \frac{1+\epsilon}{1-\epsilon} F_0, \ (n\tau \le t < n\tau + \frac{1}{2}\tau(1-\epsilon)), \quad (5)$$
$$= -F_0, \ (n\tau + \frac{1}{2}\tau(1-\epsilon) < t \le (n+1)\tau).$$

The time averaged current in this case is given by

$$< j >= \frac{1}{2}(j_1 + j_2) \quad (6)$$

with

$$j_1 = (1-\epsilon) j(\frac{1+\epsilon}{1-\epsilon} F_0) \quad (7)$$
$$j_2 = (1+\epsilon) j(-F_0)$$

The input energy E_{in} per unit time for this time asymmetric ratchet is given by $E_{in} = \frac{1}{2} F_0 (\frac{1+\epsilon}{1-\epsilon} j_1 - j_2)$ [21]. For the case where $\mu = 0$, $\lambda = 0$ and $\epsilon \neq 0$ the currents are generated in the absence of broken spatial symmetry, but in the presence of a temporal asymmetric driving with zero mean. This type of temporal asymmetry is particularly common in biological systems [19].

III. RESULTS AND DISCUSSIONS

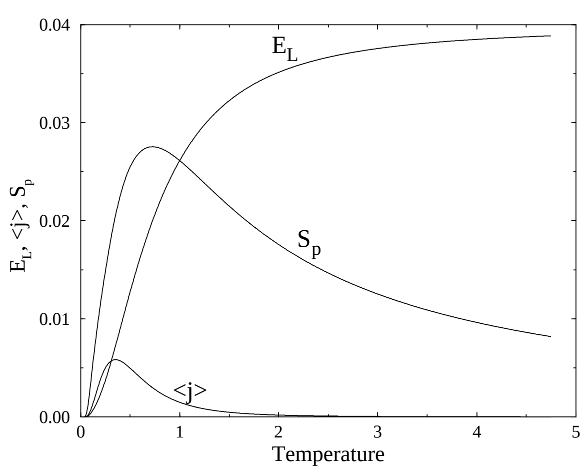

FIG. 1: $E_L, < j >$ and S_p vs temperature for $\mu = 1.0$, $\lambda = 0.0$, $\epsilon = 0$ with fixed $F_0 = 0.5$ and $\phi = 0.3\pi$.

In the following we analyse all these special classes of adiabatic ratchets mentioned above and also their combinations. We study the average current $< j >$, the total entropy production S_p, and the energy loss E_L as a function of temperature T (noise strength) for seven different cases of adiabatically rocked ratchet systems described below. All these quantities are averaged over the period of external drive and are in appropriate dimensionless units. It has been argued that for the case of a driven double well system input energy is a reliable quantity for the identification of SR taking into account the detailed comparison between various measures of SR [22]. Further analysis based on input energy has shown the SR to be a bonafide resonance [23] in that one obtaines peak in the input energy as a function of noise strength as well as the frequency of the

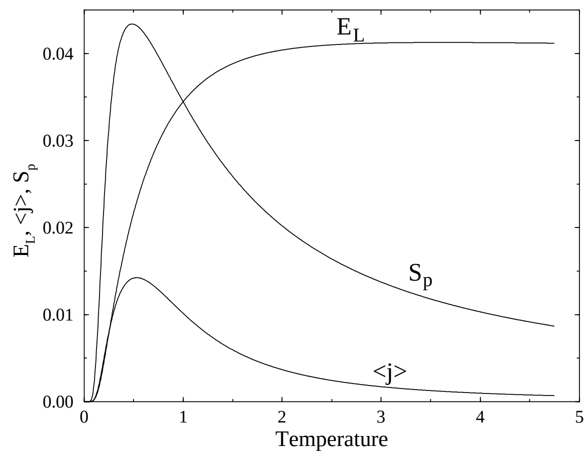

FIG. 2: $E_L, < j >$ and S_p vs temperature for $\mu = 0$, $\lambda = 0.9$, $\epsilon = 0$ with fixed $F_0 = 0.5$ and $\phi = 1.3\pi$.

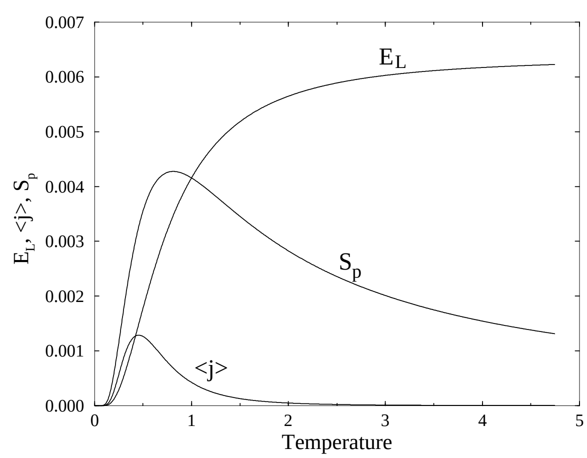

FIG. 3: $E_L, < j >$ and S_p vs temperature for $\mu = 0$, $\lambda = 0$, $\epsilon = 0.6$ with fixed $F_0 = 0.1$ and $\phi = 0.3\pi$.

external drive [24]. Thus the peak in the input energy represents the matching condition of the escape rate out of the potential well (or synchonization in the dynamics of the particle) and the external driving frequency. It should be noted that as the system on the average does not perform any useful work the input energy in the steady state equals the energy loss (hysterisis loss) in the medium. Hysterisis loss being a good measure to identify SR is already known in the literature [25]. It is quite natural that when the system dynamics exhibits a resonance feature (or a peak) by tuning certain physical parameters, then at resonance the input energy

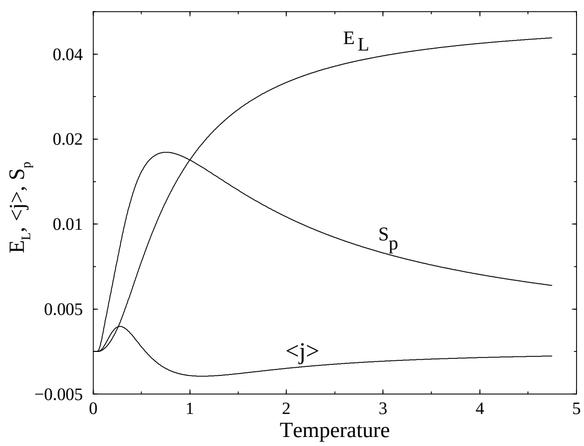

FIG. 4: $E_L, < j >$ and S_p vs temperature for $\mu = 1.0\,,\, \lambda = 0.9\,,\, \epsilon = 0$ with fixed $F_0 = 0.5$ and $\phi = 0.3\pi$

.

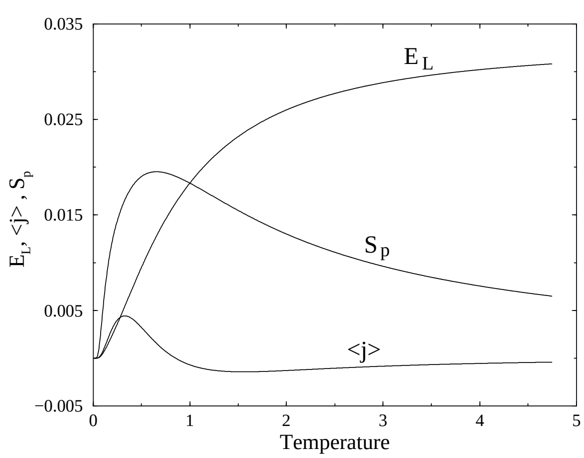

FIG. 5: $E_L, < j >$ and S_p vs temperature for $\mu = 0\,,\, \lambda = 0.9\,,\, \epsilon = 0.4$ with fixed $F_0 = 0.3$ and $\phi = 0.3\pi$.

extracted from the source (and the concomittant energy loss in the medium) is expected to be high. Thus the study of input energy or energy loss is expected to reveal the resonances if any in the dynamics of the particle as a function of various physical parameters.

In our present work the particle performs a motion in a periodic potential in the presence of an adiabatic drive. As there is no load applied to the system, the system does not perform any useful work or stores energy and

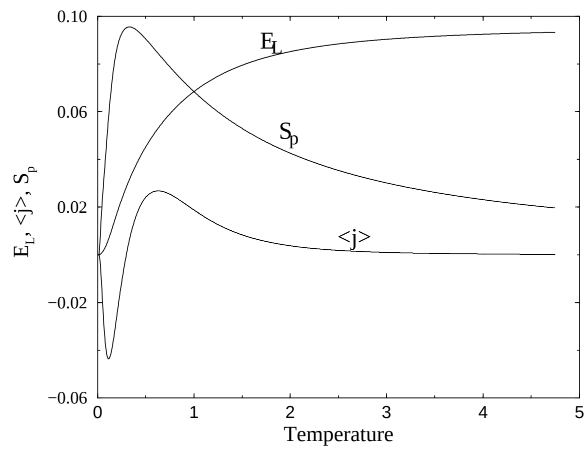

FIG. 6: $E_L, < j >$ and S_p vs temperature for $\mu = -1.0\,,\, \lambda = 0\,,\, \epsilon = 0.25$ with fixed $F_0 = 0.6$ and $\phi = 0.3\pi$. The current is scaled by a factor of 10 to make it comparable with E_L and S_p.

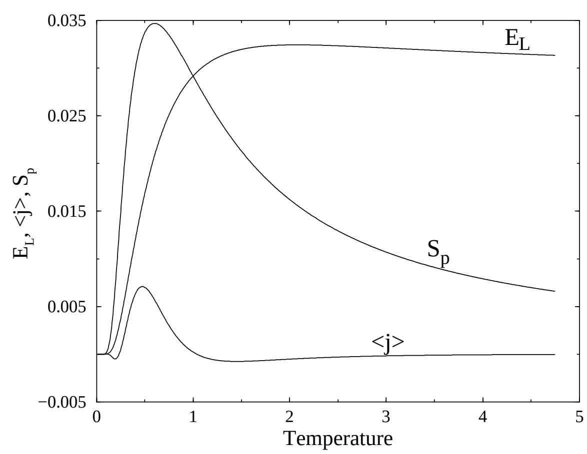

FIG. 7: $E_L, < j >$ and S_p vs temperature for $\mu = -1.0\,,\, \lambda = 0.9\,,\, \epsilon = 0.34$ with fixed $F_0 = 0.3$ and $\phi = 1.005\pi$. The current is scaled by a factor of 10 to make it comparable with E_L and S_p.

as a result all the input energy in the steady state will be dissipated away. Hence the energy loss in the medium is given by $E_L = E_{in}$. E_L inturn is equal to the heat dQ transferred to the bath and thus entropy production $S_p = dQ/T = E_L/T$ [8]. Thus the total increase in the entropy (or the entropy production) of the bath (universe) integrated over the period of the external drive is given by $S_p = \frac{dQ}{T} = \frac{E_{in}}{T} = \frac{E_L}{T}$ [8]. As discussed in the introduction, currents in the ratchet systems are generated at the expense of entropy and thus we expect a correlation between the magnitude of current and the total entropy production.

In figures 1 to 7 we have plotted energy loss E_L (equal to input energy), average current $< j >$ and entropy production S_p as a function of temperature T for various types of adiabatically driven ratchet systems. All the physical parameters are in dimensionless units and their values are mentioned in the figure caption. All these figures are representative of the different classes of ratchet systems chosen to make our analysis clear.

Figures 1,2 and 3 correspond to the spatially asymmetric case $\mu \neq 0, \lambda = 0, \epsilon = 0$; inhomogeneous case $\lambda \neq 0, \mu = 0, \epsilon = 0$ and temporal asymmetric case $\epsilon \neq 0, \lambda = 0, \mu = 0$ respectively. In each case currents are generated via different types of mechanisms arising out of spatial asymmetry in the potential or frictional inhomogenity or temporal asymmetry of the external force. Here nonlinearity of the system, external drive and asymmetry conspire to generate unidirectional currents in the absence of any bias. In all these figures (1 to 3) current exhibits a single peak as a function of temperature. The input energy being a monotonically increasing function of T rules out any resonance in the dynamics of the particle as a function of noise strength as discussed earlier. Entropy production exhibits a single peak as a function of noise strength. Moreover, the peak in the average current, $< j >$ and total entropy production S_p does not occur at the same T. This clearly indicates that maxima in the entropy production does not take place at the same value where current is maximum. To make this point explicit in figures 4,5 and 6 we have plotted $E_{in}, < j >$ and S_p as a function of T. They correspond to a combination of spatial asymmetry and system inhomogenity $\mu \neq 0, \lambda \neq 0, \epsilon = 0$; system inhomogenity and temporal inhomogenity $\lambda \neq 0, \epsilon \neq 0, \mu = 0$ and temporal asymmetry and spatial asymmetry $\epsilon \neq 0, \mu \neq 0, \lambda = 0$ respectively. All these figures from 4 to 6 exhibit the phenomena of single current reversal or the absolute magnitude of current exhibits two peaks. However, the entropy production exhibits only a single maxima as a function of noise strength. This rules out clearly any correlation between current maxima and the maxima in the entropy production. The behaviour of input energy rules out any resonance phenomena in these systems as well.

In fig. 7 we plotted $< j >, E_{in}$ and S_p for a system which incorporates frictional inhomogenity, temporal asymmetry as well as spatial asymmetry. The system with all these combinations exhibit rich variety in the nature of current as a function of various physical parameters and moreover in some parameter region large efficiency of energy transduction is observed [26]. For parameters given in fig. 7 the current exhibits two reversals as a function of T. Our observation of double current reversal in the adiabatic regime is in itself a novel phenomena. The absolute magnitude of current exhibits three peaks as against to the single peak structure in entropy production. This again reinforces the fact that maxima in the entropy production and current are totally unrelated. The behaviour of input energy again rules out the resonance dynamics in the system.

In conclusion, by considering different cases of adiabatically rocked ratchets we have shown that the resonance like feature observed in the nature of current as a function of temperature or the noise strength is not related to the intrinsic resonance in the dynamics of the particle and that the total entropy production does not extremize at the same parameter value at which the current exhibits a maximum. Our present results are valid only for the case of an adiabatically rocked thermal ratchet. This does not rule out the resonance in the dynamics of the particle in the nonadiabatic regimes as well as in other ratchet systems like flashing ratchets considered earlier [9, 10, 11, 12]. The fact that the noise strength at which the current maxima and the maxima in entropy production occurs do not coincide may be related to the quality of the current. Noise induced currents are always associated with a dispersion or diffusion. When the diffusion is large then the quality of transport degrades and the coherence in the unidirectional motion is lost. The coherent transport (optimal transport) refers to the case of large mean velocity at fairly small diffusion. It can be quantified [27, 28, 29, 30] by a dimensionless Pećlet number. For a given magnitude of current transport may be coherent or incoherent. Thus analysis of the relation between current and the entropy production requires not only the magnitude of current but also the quality of transport. It is also shown that the algorithmic complexity or the Kolmogorov information entropy of the thermal ratchet motion exhibits maxima at the same value at which current is maximum [14]. It will be of interest to see multiple maxima in algorithmic complexity as a function of system parameter in a multiple current reversal regime. These studies are expected to reveal a deep connection between efficiency, quality of transport, entropy and information. Works along these different directions are in progress.

[1] F. Jülicher, A. Adjari and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).

[2] P. Reimann, Phys. Rep. **361**, 57 (2002) and references therein.

[3] A. M. Jayannavar, cond-mat 0107079.

[4] Special issue on "Ratchets and Brownian motors: basics, experiments and applications" ed: H. Linke, Appl. Phys. (special issue) **A75(2)** 2002.

[5] M. Millonas, Phys. Rev. Lett. **74**, 10 (1995).

[6] A. M. Jayannavar, Phys. Rev. **E 53**, 2957 (1996).

[7] K. Sekimoto, J. Phys. Soc. Jpn. **66**, 6335 (1997).

[8] J. M. R. Parrondo and B. J. De Cisneros, Appl. Phys. **A75**, 179 (2002).

[9] A. Allison and D. Abbott, Fluct. Noise Lett. **1**, L239 (2001).

[10] M. Bier, Stochastic Dynamics **386**, 81-87, Springer, Berlin (1997).

[11] V. Berdichevsky and M. Gitterman, Phys. A **249** 88 (1998).

[12] R. D. Astumian and F. Moss, Chaos, **8** (3), 533 (1998).

[13] Qian Min, Wang Yan and Ahang Xue-Juan, Chin. Phys. Lett. **20**, 810 (2003).

[14] J. R. Sanchez, F. Family and C. M. Arizmendi, Phys. Lett. **A249**, 281 (1998).

[15] A. M. Jayannavar and M. C. Mahato, Pramana-J. Phys. **45**, 369 (1995); M. C. Mahato, T. P. Pareek and A. M. Jayannavar, Int. J. Mod. Phys. B **10**, 3857 (1996), D. Dan, M. C. Mahato and A. M. Jayannavar, Phys. Lett. A **258**, 217 (1999); Int. J. Mod. Phys. B **14** 1585 (2000); Phys. Rev. **E60**, 6421 (1999); Phys. Rev. **E63**,56307 (2001).

[16] H. Riskin, The Fokker-Planck Equation (Springer Verlag, Berlin, 1984).

[17] M. Buttiker, Z. Phys. B **68**,161 (1987).

[18] H. Kamegawa, T. Hondou and F. Takagi, Phys. Rev. Lett. **80**, 5251 (1998); F. Takagi and T. Hondou, Phys. Rev. **E60**, 4954 (1999); D. Dan and A. M. Jayannavar, Phys. Rev. **E66**, 41106 (2002).

[19] D. R. Chialvo, M. M. Millonas, Phys. Lett. A **209**, 26 (1995).

[20] Mangal C. Mahato and A. M. Jayannavar, Phys. Lett. A **209**, 21 (1995).

[21] Bao-Quan Ai, X. J. Wang, G. T. Liu, H. Z. Xie, D. H. Wen, W. Chen and L. G. Liu, physics 0306163.

[22] T. Iwai, Physica A **300**, 350 (2001); T. Iwai, J. Phys. Soc. Jpn. **70**, 353 (2001).

[23] L. Gammaitoni, F. Marchesoni and S. Santucci, Phys. Rev. Lett. **74**, 1052 (1995); L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).

[24] Debasis Dan and A. M. Jayannavar, cond-mat 0303417.

[25] Mangal C. Mahato and A. M. Jayannavar, Physica A **248** 138 (1998); Phys. Rev. **E 55**, 6266 (1997); Mod. Phys. Lett. B **11**, 815 (1997).

[26] R. Krishnan and A. M. Jayannavar, unpublished.

[27] B. Linder and L Schimansky-Geier, Phys. Rev. Lett. **89**, 230602 (2002).

[28] D. Dan and A. M. Jayannavar, Phys. Rev. **E66**, 41106 (2002); Raishma Krishnan, D. Dan and A. M. Jayannavar, cond-mat 0309617.

[29] J. A. Freund and L. Schimansky-Geier, Phys. Rev. **E60**, 1304 (1999); T. Harms and R. Lipowsky, Phys. Rev. Lett. **79**, 2895 (1997).

[30] P. Reimann, C. Van den Broeck, H. Linke, P. Hänggi, J. M. Rubi and A. Perez-Madrid, Phys. Rev. Lett. **87**,10602 (2001); B. Linder and L. Schimansky-Geier, Fluct. Noise Lett. **1**, R25 (2001).