Distribution of Wigner delay time from single channel disordered systems

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Abstract

We consider the scattering of an electron from a semi-infinite one-dimensional random medium. The random medium is characterized by force, $-\partial V/\partial L$ being the basic random variable. We obtain an analytical expression for the stationary delay time (τ) distribution $P_s(\tau)$ within a random phase approximation. Our result agrees with earlier analytical expressions, where the random potential is taken to be of different kind, indicating universality of the delay time distribution, i.e., delay time distribution is independent of the nature of disorder.

Keywords: A. disordered systems, D. electronic transport, D. quantum localization

In recent years universal parametric correlations of phase shifts and delay times in mesoscopic systems is being studied intensively [1]. The time delay in a scattering event (or duration of a collision event) is an interesting aspect in itself in the general theory of quantum scattering. Wigner was the first to establish the relation between the time delay and the energy derivative of scattering phase shift [2]. Distribution of delay times in quantum chaotic regime have been shown to be universal as it depends only on the symmetry property of the Hamiltonian or scattering matrix [1,3]. The delay time statistics is intimately connected with the issue of dynamic admittance of microstructres (or mesoscopic systems) [3,4], for example quantum capacitance and its fluctuation [5]. The wave packet incident on the surface a sample is not backscattered (or reflected) immediately. There will be some time delay before it is reflected. This leads to a non-cancellation of the instantaneous currents at the surface involving the incident and the reflected wave. This in-turn is expected to lead to a low temperature $1/f$-type noise for the fluctuating surface currents in the random systems [6–8]. The study of change of density of states due to scatterer is also directly related to the phase derivative of scattering phase shift with respect to the energy, i.e., to the delay time.

The distribution of delay time and its correlations in higher dimensions, where system exhibits the Anderson localization has not been addressed so far. The first study of the stationary distribution $P_s(\tau)$ of delay time τ for a disordered semi-infinite one-dimensional chain was carried out in reference [7]. Here authors used the invariant imbedding approach. The underlying random potential $V(x)$ is treated as a Gaussian white noise with zero mean. Using the random phase approximation (RPA) analytical expression for the $P_s(\tau)$ was obtained, which exhibits $1/\tau^2$ dependence for the tail of the delay distribution. Further developments for $P_s(\tau)$ using supersymmetric potentials lead to same distribution function $P_s(\tau)$ within RPA. This has lead to a conjecture that within RPA, $P_s(\tau)$ is independent of nature of disorder and, hence, is universal [9]. Our recent numerical study has clearly indicated that long time delay distribution is universal beyond RPA [10]. In our present work we calculate the distribution of delay time where we take $-\partial V/\partial x$ as the basic random variable with delta correlated Gaussian distribution and we obtain analytical expression for

$P_s(\tau)$ in RPA. The stationary distribution has the same functional form obtained earlier with different random potential indicating the universal nature of $P_s(\tau)$.

The model Hamiltonian for the 1-D disordered system is

$$H = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \tag{1}$$

where $V(x)$ for $0 < x < L$ is the random potential. The disordered sample extends from $x = 0$ to $X = L$, the two ends being connected to perfect leads. Consider an electron of wave number k incident at $x = L$ from right. It is partially reflected with the complex reflection coefficient $R(L)$ and partially transmitted. The transmission and reflection coefficients are emergent quantities of direct physical interest for the conductance problem. The method of invariant imbedding was proposed originally by S. Chandrashekhar in the context of radiative transfer through stellar atmosphere [11]. His method consists of viewing the given sample of length L as imbedded in a larger sample of length $L + \Delta L$ and then setting up an equation for the resulting change in the S-matrix ΔS as $\Delta L \to 0$. In order to look for the complex reflection coefficient, we transform Eqn. 1 to the invariant imbedding equation for the complex reflection amplitude $R(L) = |R(L)|Exp(i\theta(L))$. The evolution equation for $R(L)$ is now given by [12]

$$\frac{\partial R(L)}{\partial L} = f_1(L) + 2if_0(L)R(L) - f_1(L)R^2(L), \tag{2}$$

with

$$f_1(L) = \frac{2}{k(L)}\frac{\partial k}{\partial L},$$

$$f_0(L) = k(L)$$

and

$$k^2 = \frac{2m}{\hbar^2}\left(E - V(L)\right).$$

The above equation Eqn. 2 was first studied in Ref. [13] to evaluate the resistance and its fluctuation in a disordered quantum wire. The invariant imbedding method has been generalized to N-channel case and in an equivalent random phase approximation has lead to DMPK (Dorokhov-Mello-Pereyra-Kumar) equation [14], using which coherent transport properties have been analyzed extensively in mesoscopic systems.

In the present problem we consider random potential $V(L)$ to be bounded having a small amplitude. However, $-\partial V/\partial L$ can be unbounded and we treat $\xi(L) = -\partial V/\partial L$ as our basic random variable. The energy of incident electron is assumed to be large, i.e., much larger than the magnitude of the upper bound on the potential. In that case we have

$$f_1(L) = \frac{-1}{E}\frac{\partial V(L)}{\partial L} \quad \text{and} \quad \mathrm{f_0(L)} = \sqrt{\frac{2\mathrm{m}}{\hbar^2}}\sqrt{\mathrm{E}} \tag{3}$$

We take $\xi(L)$ to be Gaussian delta correlated random number with zero mean and

$$\langle \xi(L)\xi(L')\rangle = 2\alpha\ \delta(L-L'). \tag{4}$$

Here, the $< .... >$ denotes the ensemble average with respect to the realizations of the stochastic variable ξ and α denotes the strength of the disorder. The equation for the phase (θ) is readily obtained from Eqn. 2 as

$$\frac{\partial \theta}{\partial L} = 2\sqrt{\frac{2m}{\hbar^2}}\sqrt{E} - 2\frac{\xi(L)}{E} sin(\theta) \tag{5}$$

where we have set $|R| = 1$ since we will be interested in the limit $L \to \infty$ (semi-infinite medium), i.e., total back-reflection with probability one. The delay time is given by $\tau = \hbar\partial\theta/\partial E$. Differentiation of Eqn. 5 with respect to E leads to the following equation for the evolution of τ:

$$\frac{\partial \tau}{\partial L} = \frac{\sqrt{2m}}{\sqrt{E}} + \frac{2\hbar}{E^2}\xi(L) sin(\theta) - \frac{2}{E}\xi(L) cos(\theta)\tau \tag{6}$$

From Eqns. 5 and 6 one can obtain readily obtain the equation governing the evolution of the joint probability distribution $W(\tau, \theta; L)$ for θ and τ by using the Van Kampen lemma [15] and Novikov's theorem [16–18]. In our case, however, we are interested only in the marginal probability distribution $P(\tau; L) = \int_0^{2\pi} W(\tau, \theta; L) d\theta$ of delay time τ. The delay time being the derivative of phase we expect it to fluctuate much more rapidly as compared to the phase itself. We therefore make the decoupling approximation, as done in earlier literature by [7,8], treating θ and τ as statistically independent variables in the large length (L) limit. As mentioned earlier, we are interested in the case of high energy particles ($E \gg V$) and in this limit the distribution of θ becomes uniform [19–21], i.e., $P(\theta) = 1/2\pi$. This is generally referred to as random phase approximation (RPA). Within the above mentioned approximations after a straight forward algebra the evolution equation for $P(\tau; L)$ becomes

$$\frac{\partial P}{\partial L} = \hbar \frac{\partial}{\partial \tau} \left\{ \frac{2\alpha\hbar}{E^4} \frac{\partial P}{\partial \tau} - \frac{\sqrt{2m}}{\sqrt{E}} P + \frac{4\alpha}{\hbar E^2} \tau P + \frac{2\alpha}{\hbar E^2} \tau^2 \frac{\partial P}{\partial \tau} \right\} \tag{7}$$

The stationary distribution $P_s(\tau)$ for τ in the limit $L \to \infty$ can be obtained by setting $\partial P/\partial L = 0$. We get the following expression for normalized $P_s(\tau)$

$$P_s(\tau) = \frac{\lambda e^{\lambda tan^{-1} \tau}}{(e^{\lambda\pi/2} - 1)(1 + \tau^2)} \tag{8}$$

In the above expression we have redefined τ in a dimensionless form $\tau \equiv \tau E/\hbar$ and $\lambda = \sqrt{2mE}\ E^2/(2\alpha\hbar)$. The most probable value of τ occurs at $\tau = \lambda/2$. As $\tau \to \infty$, $P_s(\tau) \to 1/\tau^2$, i.e., the distribution has a long time tail which goes as $1/\tau^2$. This leads to the logarithmic divergence of the average value of τ indicating that the origin of such a tail is due, presumably, to the Azbel resonances [22] which make Landauer's four probe conductance infinite even for a finite sample. In case of these resonant realizations,the time spent by the particle inside the medium is large as it travels a large distance before getting reflected. It is now well established that coherent interference effects, due to elastic scattering by the serial static disorder lead to localization of eigenstates for arbitrary weak disorder. The localization length l of these eigenstates is a self averaging quantity [23] and in a one-dimensional system it is directly proportional to elastic mean free path. The most probable value τ_{max} of τ is proportional to a time taken by a particle to traverse a distance of the order of localization length l, $\tau_{max} \propto 2l/(\hbar k/m)$, where k is the incident energy. From this one can readily obtain the behavior of localization length on the material parameters, namely, $l \propto E^2/\alpha$.

Our above analytical expression has the same functional form as obtained earlier where potential itself is treated as a Gaussian random variable using a different invariant imbedding equation. From this we conclude that two different models of random variable lead to the same universal distribution of the delay time. Thus reinforcing the conjecture of universal behavior of the delay time distribution, independent of nature of disorder within RPA.

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