Persistent currents in coupled mesoscopic rings.

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Abstract

We have analysed the nature of persistent currents in open coupled mesoscopic rings. Our system is comprised of two ideal loops connected to an electron reservoir. We have obtained analytical expressions for the persistent current densities in two rings in the presence of a magnetic field. We show that the known even-odd parity effects in isolated single loops have to be generalised for the case of coupled rings. We also show that when the two rings have unequal circumferences, it is possible to observe opposite currents (diamagnetic or paramagnetic) in the two rings for a given Fermi level.

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I. Introduction

It was predicted by B$\ddot{u}$ttiker et. al. [1] that an equilibrium persistent current flows in an ideal one dimensional mesoscopic ring threaded by a magnetic flux ϕ. Persistent current flows in a ring as a response to magnetic field which destroys the time reversal symmetry and is periodic in magnetic flux , with a period ϕ_0 , ϕ_0 being the elementary flux quanta (ϕ_0=hc/e). At zero temperature the amplitude of persistent current is given by ev_f/L , where v_f is the Fermi velocity and L is the circumference of the ring. For spinless electrons persistent current can be either diamagnetic or paramagnetic depending upon whether the total number of electrons present is odd or even, respectively [2,3]. This behaviour of persistent current is also known as parity effect. The existence of persistent currents in mesoscopic rings has subsequently been confirmed by several experiments [4-6].

Persistent currents occur in both open and isolated closed systems [7-13]. A simple open system [7] is a metallic ring connected to an electron reservoir, characterized by chemical potential μ_1. Several new effects related to persistent currents can arise in open systems which have no analog in closed or isolated systems. Recently we have also shown [13-15] that large circulating currents can arise in open mesoscopic rings in the presence of a transport current ,in the absence of magnetic field. This is purely a quantum effect and is related to the property of current magnification in the loop.

So far theoretical treatments of persistent currents deal with single rings(open and closed systems) threaded by magnetic flux. Studies in a closed ring have been extended to include finite temperature effects, multichannel rings, disorder, spin-orbit coupling and electron-electron effects [2,3,16-14]. In our present work we study persistent currents in coupled mesoscopic rings. Specifically, we consider two normal one dimensional(single channel) coupled rings con-

contact with the left and the right ring at junctions J_1 and J_2, respectively. This lead, in turn, is connected to an external electron reservoir characterized by a chemical potential μ, via an another ideal lead making connection at point X. The circumference of the left and the right rings are l_1 and l_2 respectively. The distances J_1X ,J_2X are l_3, l_4, respectively. The electron reservoir acts as a source as well as a sink of electrons, and by definition their is no phase relationship between the absorbed and the emitted electrons. Electrons emitted by the reservoir propagate along the lead and are partially reflected by the junction point X and are partially transmitted along the loop via repeated scatterings at junction points X, J_1, and J_2. Electrons in the loops will eventually reach the reservoir after some time delay. This gives rise to finite life time broadening for the electron states of the coupled rings. Scattering processes in the coupled rings are elastic. Only the reservoir acts as an inelastic scatterer. There is complete spatial separation between the sources of elastic and inelastic scattering. Our present analysis concerns non-interacting spinless electrons. In the presence of an external uniform magnetic field B, The magnetic flux through the left and the right rings are given by $\alpha_1 = B{l_1}^2/4\pi$ and $\alpha_2 = B{l_2}^2/4\pi$, respectively; α_1 and α_2 are not independent quantities as the magnetic flux in both the rings arise from the same applied uniform magnetic field B. We have obtained analytical expressions for the persistent current densities in both the rings in the presence of a magnetic field. We show that persistent currents in the two rings are very sensitive to the geometric features (such as lengths l_1, l_2, l_3 and l_4) of the system. Even though we have obtained an analytical expression for the general case, we restrict ourselves to a case where $l_3 = l_4$. When the rings are of the same size the magnitude and sign of the persistent currents are same in both the rings (due to symmetry). We observe that if the distance between the rings $l = (l_3 + l_4)$ is much larger than the circumference of the two identical rings the known considered rings of unequal circumferences. In such a situation at a given Fermi level it is possible to observe diamagnetic current in one of the rings and simultaneously paramagnetic current in the other ring. In sec.II we give a brief account of theoretical treatment and sec.III is devoted to results and conclusions.

.II Theoretical treatment

In this section we derive expressions for persistent current in the left and the right rings for the general case when $l_3 \neq l_4$. For this we consider a noninteracting electron system. Our system is considered as a free electron network, i.e., the quantum potential V throughout the network is assumed to be identically zero. The scattering arises solely due to junctions (or geometric scattering) at J_1, J_2 and X. For scattering at the junctions we do not assume any specific form of scattering matrix , instead the junction scattering matrix follows from principles of quantum mechanics. We use the Griffiths boundary condition (current conservation) and the single valuedness of wave function at the junctions [21]. Following exactly the same procedure as the earlier ones [11-15] one can readily derive analytical expressions for the persistent current densities (dJ/dk) (i.e. persistent current density in the small wave vector interval k and k+dk) in the left (dJ_L/dk) and right (dJ_R/dk) loops [7] and are given by

$$dJ_L/dk = -(ek\hbar/2\pi m)256Sin[\alpha]Sin[kl_1]\left\{Sin[k(l_2-l_4)] -3Sin[k(l_2+l_4)] - 2Sin[\beta - kl_4] + 2Sin[\beta + kl_4]\right\}^2/({\Omega_1}^2+{\Omega_2}^2), \quad (1)$$

$$dJ_R/dk = -(ek\hbar/2\pi m)256Sin[\beta]Sin[kl_2]\left\{Sin[k(l_1-l_3)] -3Sin[k(l_1+l_3)] - 2Sin[\alpha - kl_3] + 2Sin[\alpha + kl_3]\right\}^2/({\Omega_1}^2+{\Omega_2}^2). \quad (2)$$

$$
\begin{aligned}
\Omega_1 = 2\{&-3Cos(k(l_1-l_2-l_3-l_4)) - Cos(k(l_1+l_2-l_3-l_4)) + 9Cos(k(l_1-l_2+l_3-l_4)) \\
&+3Cos(k(l_1+l_2+l_3-l_4)) + Cos(k(l_1-l_2-l_3+l_4)) + 3Cos(k(l_1+l_2-l_3+l_4)) \\
&-3Cos(k(l_1-l_2+l_3+l_4)) - 9Cos(k(l_1+l_2+l_3+l_4)) - \\
&4Cos(\alpha-\beta-kl_3-kl_4) - 4Cos(\alpha+\beta-kl_3-kl_4) + 6Cos(\beta-kl_1-kl_3-kl_4) \\
&+2Cos(\beta+kl_1-kl_3-kl_4) + 6Cos(\alpha-kl_2-kl_3-kl_4) + 2Cos(\alpha+kl_2-kl_3-kl_4) \\
&+4Cos(\alpha-\beta+kl_3-kl_4) + 4Cos(\alpha+\beta+kl_3-kl_4) - 2Cos(\beta-kl_1+kl_3-kl_4) \\
&-6Cos(\beta+kl_1+kl_3-kl_4) - 6Cos(\alpha-kl_2+kl_3-kl_4) - 2Cos(\alpha+kl_2+kl_3-kl_4) \\
&+4Cos(\alpha-\beta-kl_3+kl_4) + 4Cos(\alpha+\beta-kl_3+kl_4) - 6Cos(\beta-kl_1-kl_3+kl_4) \\
&-2Cos(\beta+kl_1-kl_3+kl_4) - 2Cos(\alpha-kl_2-kl_3+kl_4) - 6Cos(\alpha+kl_2-kl_3+kl_4) \\
&-4Cos(\alpha-\beta+kl_3+kl_4) - 4Cos(\alpha+\beta+kl_3+kl_4) + 2Cos(\beta-kl_1+kl_3+kl_4) \\
&+6Cos(\beta+kl_1+kl_3+kl_4) + 2Cos(\alpha-kl_2+kl_3+kl_4) + 6Cos(\alpha+kl_2+kl_3+kl_4)\}, \quad (3)
\end{aligned}
$$

$$
\begin{aligned}
\Omega_2 = 4\{&-3Sin(k(l_1-l_2-l_3-l_4)) - Sin(k(l_1+l_2-l_3-l_4)) + \\
&3Sin(k(l_1-l_2+l_3+l_4)) + 9Sin(k(l_1+l_2+l_3+l_4)) - \\
&4Sin(\alpha-\beta-kl_3-kl_4) - 4Sin(\alpha+\beta-kl_3-kl_4) + 6Sin(\beta-kl_1-kl_3-kl_4) + \\
&2Sin(\beta+kl_1-kl_3-kl_4) + 6Sin(\alpha-kl_2-kl_3-kl_4) + 2Sin(\alpha+kl_2-kl_3-kl_4) + \\
&4Sin(\alpha-\beta+kl_3+kl_4) + 4Sin(\alpha+\beta+kl_3+kl_4) - 2Sin(\beta-kl_1+kl_3+kl_4) - \\
&6Sin(\beta+kl_1+kl_3+kl_4) - 2Sin(\alpha-kl_2+kl_3+kl_4) - 6Sin(\alpha+kl_2+kl_3+kl_4)\}, \quad (4)
\end{aligned}
$$

where $\alpha = 2\pi\alpha_1/\phi_0$ and $\beta = 2\pi\alpha_2/\phi_0$,$\phi_0 = hc/e$ the elementary flux quanta. The wavevector

E=$\hbar^2 k^2/2m$. Since we are considering the case wherein the magnetic field B is due to the same source and consequently the flux (α and β are written in a dimensionless form) piercing through the two loops are related by the following relation (i.e., α and β are dependent variables).

$$\alpha = (\left({l_2}^2/{l_1}^2\right) * \beta\right) . \tag{5}$$

For the above case from equation (1)-(5) one can readily verify that persistent current densities are antisymmetric in B or the persistent currents in two loops change sign on the reversal of magnetic field (B $\rightarrow$ $-$B). Henceforth we rescale the current densities in the dimensionless form and denote $dj_L = (dJ_L/dk)(2m\pi/\hbar ek)$ and $dj_R = (dJ_R/dk)(2m\pi/\hbar ek)$. We have also rescaled all the lengths with respect to the length l_1 of the left hand loop. The wave vector is written in the dimensionless form as kl_1.

.III Results and Discussions

We would like to point out that our expression for the persistent current densities obtained in equation (1) and (2) are quite general and valid even for the case, where, the flux enclosed by two rings α_1 and α_2 are independent variables. This case corresponds to a situation in which the enclosed magnetic flux in the left and right rings may arise respectively from two independent magnetic field sources. However, in our present detailed analysis we have not considered this case. If the two rings are identical ($l_1 = l_2$) we notice that the magnitude of the persistent current densities in the left and the right rings are unequal. This follows from the fact that there is a asymmetry in the system. This asymmetry arises because of the junction scattering point X, which is not placed at a symmetrical position with respect to the position of the two rings ($l_3 \neq l_4$). Henceforth we restrict our discussion further to the case $l_3 = l_4$ (symmetrical situation). For this special case, when $l_1 = l_2$, the magnitude and the direction of the persistent current are same in both the rings.

In fig.2 we have plotted persistent current density dj_L as a function of dimensionless wave vector kl_1 for a fixed value of l_2/l_1=1, l_3/l_1=l_4/l_1=0.5 , and α=0.5. Since in this particular case the system is symmetric about the junction X , we expect that current in the left or the right ring will be same. As one varies kl_1 the persistent currents oscillate between diamagnetic and paramagnetic behaviour. In our problem the coupled rings are connected to a reservoir, which, in turn, leads to finite life time broadening of the electron states in the system and as a consequence the persistent current shows a broadened feature as a function of kl_1 compared to an isolated system. The amplitude extrema in persistent current occur approximately at the values of kl_1=$2\pi(n + \alpha_1/\phi_0)$, where n=0,±1,±2,...etc., which correspond to the allowed states in a single isolated loop of length l_1. The observed small deviation from values of kl_1 for isolated ring follows from the fact that there is a coupling between the rings and additional scatterings at J_1, J_2 and X.

In fig.(3) we have plotted persistent current as a function of α for a fixed value of $kl_1 = 6.0$. Other parameters being the same as used for fig.(2). We notice that results obtained in fig.2 and fig.3. are qualitatively same as one observes in a single loop of length l_1 connected to an electron reservoir [7]. It is also to be noted that the simple periodicity observed in fig.(2) and fig.(3) is due to the fact that all lengths (l_1, l_2, l_3, and l_4) are simple rational multiples of each other, otherwise we would have obtained a complicated structure in the behaviour of persistent current as a function of kl_1 as well as α.

From now on we discuss the case when the length of the connecting lead ($l_3 + l_4$) is much larger than the circumference l_1 of the loops. We have taken both the loops to be of equal circumference. We show that in this simple case the even-odd parity effect known for ratio $(l_3 + l_4)/l_1$. In the absence of magnetic field, an isolated single loop has eigenstates corresponding to wave vector k $=2\pi n/l_1$, whereas isolated connecting wire of length l_3+l_4 has eigen states with k $= n\pi/(l_3 + l_4)$ (n=0,$\pm$1,$\pm$2,...etc.). Therefore for the length $(l_3 + l_4) > l_1$ energy levels in the isolated lead are closely spaced than the energy levels in the loop. These closely spaced energy levels, leak into the loops (hybridized with the states within the loop) in a connected ring system and consequently additional quasi bound states arise which have energies lying between the states of the isolated ring. Naturally the energies of these new states of the coupled system will be shifted from either of those of the separate lead and the ring due to the coupling(perturbation). In the presence of magnetic field such an additional state contributes to the persistent current diamagnetically or paramagnetically depending on whether it is near respectively to a diamagnetic or paramagnetic state of the isolated loop (in the presence of a magnetic field). These states basically owe their existence to the resonant states in the isolated lead and their contribution to the magnitude of persistent current is small compared to the contribution of persistent current from the states near the resonant states of the isolated loops. Thus a situation can arise a system of coupled loops (with $(l_3 + l_4) >> l_1$) such that first N_1 states are diamagnetic and the next N_1 states will be paramagnetic ($2N_1$ is the number of resonant states, of the lead, lying between the two successive levels of the isolated ring) and process repeats as we go to higher states. In a single isolated loop, for spinless electrons, it is well known[2,3] that current in a loop is diamagnetic or paramgnetic depending on whether the number of particles is odd or even, respectively(even-odd parity effect). Now for coupled mesoscopic rings this simple even-odd parity effect gets altered and instead we have first N_1 levels contributing to a diamagnetic current but the next N_1 levels contribute a paramagnetic current, where N_1 depends on the ratio $(l_3 + l_4)/l_1$. This is true identical loops). For this case the underlying concepts will become a little complicated as we have to discuss parity effects in the left and right loops separately as they carry different currents for any given state, which will be discussed below. In fig.(4) we have plotted the persistent current dj as a function of kl_1, for the case when $(l_3 + l_4)/l_1$=2,l_2/l_1=1 and for a fixed value of $\alpha = 1.2$. For this situation we have two additional states of the connecting lead (lying between eigen states of the isolated loops), which leak into the loops. We clearly observe from fig.(4) that as we vary kl_1 we get the first two peaks which are diamagnetic and the later two peaks are paramagnetic and the sequence repeats.

In our problem we basically solve a scattering problem wherein electrons are injected in the system from the reservoir which get reflected back to the reservoir. From a scattering matrix structure one can get the information about quasi bound states. This can be achieved by looking at the poles in a complex kl_1 plane of the complex reflection amplitude. The real part of the poles (R) gives the wave vector values of the resonant states, whereas the imaginary part gives the information about the lifetime of these states. In fig.(5) we have plotted the real part R of these complex poles as a function of α. All the parameters used here are the same as in fig.(4). We clearly observe that additional states (in the present case 2) appears within the intervals of kl_1 values of isolated loops. Moreover, one can readily notice that the first two resonant states carry diamagnetic current (as their slopes with respect to the magnetic flux are positive [2,3]) and the next two carry paramagnetic currents and so on. As expected on the general grounds values of R are periodic in flux α_1 with a period ϕ_0. In fig.(6) we have plotted dj versus kl_1 for the case $(l_3 + l_4)/l_1 = 10.0$ and for a fixed value of $\alpha = 1.2$ while $l_2/l_1 = 1.0$. It is clear from this figure that the first six states carry diamagnetic current, next six states carry paramagnetic current and so on. The fig.(4) and fig.(6) clearly indicate that rings and moreover the emergence of new parity effect as discussed above is sensitive to the length ratio $(l_3 + l_4)/l_1$.

We further consider the case for which the loops are not identical, in that their circumferences are different. In such a situation one has to discuss the persistent currents in the right and the left loops separately. Consider a situation where $l_2 > l_1$. Naturally resonant states in the right loop are more closely spaced than those in the left loop. There will be mixing between these states due to the coupling. However, it is possible that as one varies wave vector kl_1 persistent current in the right loop oscillates between diamagnetic and paramagnetic behaviour much more rapidly than the persistent current in the left hand loop, i.e., in a given interval of kl_1, persistent current does not change sign for the case of left hand loop whereas in the same interval persistent current in the right hand loop changes sign several times. We can have a situation where for a given state (kl_1) current in the left and right loops have either the same sign or different (i.e., current in left loop are diamagnetic where as current in the right loop is paramagnetic). This is illustrated in fig.(7), where .... lines and —— lines indicate persistent current in the right(dj_R) and the left (dj_L) loop, respectively. For the above case we have taken $l_2/l_1 = 4$ and $(l_3 + l_4)/l_1 = 1$. In fig.(8) and fig.(9) we have plotted persistent currents as a function of α for a fixed value of $kl_1 = 2.2$ for the right and the left loop respectively. The other parameters are same as those used in fig.(7). From fig.(8) and fig.(9) we notice that dj_L and dj_R are periodic in α_1 with a period ϕ_0. We would like to mention that this is so because for our case we have considered a commensurate ratio $l_2/l_1 = 4$. In general if we choose the ratio to be incommensurate (or irrational) dj_L or dj_R will have much larger value of the period with respect to α. It should be kept in mind that as one varies α_1 (the flux through the left ring) by ϕ_0, the flux through the right ring (α_2)

to 2π the persistent current density in the left loop changes sign once while the persistent current density in the right loop changes the sign 16 times.

It is well known that for a simple case of isolated single loop (or a single hole in the sample) the persistent current carried by the nth state of energy E_n is given by $I_n = -(1/c)\partial\epsilon_n/\partial\phi$, where ϕ is the flux piercing through the loop (or hole). In the present case of multiply connected nonidentical rings one cannot infer the values of persistent current in the individual rings from the above definition. To calculate persistent current in the presence of magnetic field in each loop of the system of coupled rings one has to calculate quantum mechanical wave function in each ring explicitly and from that one can calculate the currents.

In our analysis throughout we have discussed the persistent current densities dj in the small wave vector interval k and k+dk. However, experimentally it is the total persistent current generated by all the conducting electrons in the system that can be observed. This can be calculated by integrating the persistent current densities up to the Fermi wave vector k_f using eqns.(1) and (2). In conclusion , we have studied the nature of persistent currents in open mesoscopic coupled ring system in presence of magnetic field. Throughout we have considered simple commensurate ratios of l_1/l_2 and $(l_3 + l_4)/l_1$. For coupled identical rings one observes different parity effects. The parity effect depends on the ratio $(l_3 + l_4)/l_1$ of the connecting lead length to the circumference of the rings. In the case of non identical loops, for a given state, it is possible that persistent current in one loop is diamagnetic whereas in the other it may be paramagnetic or diamagnetic. Moreover all these effects are very sensitive to the length ratio involved in the system as the problem is inherently quantum mechanical in nature, where interference effects dominate.

References

[1] M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365(1983).

[2] H. F. Cheung, Y. Geffen, E. K. Riedel and W. H. Shih, Phys. Rev. B **37**, 6050(1988).

[3] H. F. Cheung and E. K. Riedel, Phys. Rev. B **40**, 9498(1989).

[4] L. P. Levy, G. Dolan, J. Dunsmuir and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1900).

[5] V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).

[6] D. Mailly, C. Chapelier and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).

[7] M. Büttiker, Phys. Rev. B **32**, 1846(1985).

[8] M. Büttiker in SQUIDS'85-*Superconducting quantum interference devices and their applications* (de Gruyter, Berlin, 1985),p. 529.

[9] P. A. Mello, Phys. Rev. B **47**, 16358(1993).

[10] D. Takai and K. Ohta, Phys. Rev. B **48**, 14318(1993).

[11] P. Singha Deo and A. M. Jayannavar, Mod. Phys. Lett. B **7**, 1045(1993).

[12] P. Singha Deo and A. M. Jayannavar, Phys. Rev. B **49**, 13685(1994).

[13] A. M. Jayannavar, P. Singha Deo and T. P. Pareek, in Proceedings of International Workshop on "Novel Physics in Low-Dimensional Electron Systems," Madras, India, Physica B **212**,216(1995).

[15] T. P. Pareek, P. Singha Deo and A. M. Jayannavar, Phys. Rev. B (1995) in print.

[16] Quantum Coherence in Mesoscopic Systems, edited by B. Kramer, Vol 254 of Nato Advanced Study Institute Series B: Physics (Plenum, New York, 1991).

[17] G. Montambaux, H. Bouchiat, D. Sigeti, and R. Freisner Phys. Rev. B **42**, 7647(1990).

[18] O. Entin-Wohlman, Y. Geffen, Y. Meier and Y. Oreg Phys. Rev. B **45**, 11890(1992).

[19] P. Kopeitz, Int. J. Mod. Phys. B **8**, 2593(1994) and references therein.

[20] P. Singha Deo, Phys. Rev. B **51**, 5441(1995).

[21] P. Singha Deo and A. M. Jayannavar, Phys. Rev. B **50** 11629(1994).

Figure captions

Fig. 1. Two metal loops connected to an electron reservoir with chemical potential μ_1.

Fig. 2. Plot of circulating current versus kl_1 for a fixed value of α=0.2. For this case $l_2/l_1 = 1.0$, l_3/l_1=l_4/l_1=0.5.

Fig. 3. Plot of circulating current versus α for a fixed value of $kl_1 = 6.0$. For this case $l_2/l_1 = 1.0$, l_3/l_1=l_4/l_1=0.5.

Fig. 4. Plot of circulating current versus kl_1 for a fixed value of α=1.2. For this case $l_2/l_1 = 1.0$, l_3/l_1=l_4/l_1=1.0.

Fig. 5. The plot of real part R of the complex poles in the kl_1 plane of the reflection amplitude as a function of α for $l_2/l_1 = 1.0$, l_3/l_1=l_4/l_1=1.0.

Fig. 6. Plot of circulating current versus kl_1 for a fixed value of α=1.2. For this case $l_2/l_1 = 1.0$, l_3/l_1=l_4/l_1=5.0.

Fig. 7. The persistent current as a function of kl_1 in the left loop (solid line) and the right loop (dashed lines) for a fixed value of $\alpha = 1.2$. For this case $l_2/l_1 = 4$ and $l_3/l_1 = l_4/l_1$=0.5.

Fig. 8. Plot of persistent current in the left loop as a function of α for a fixed value of $kl_1 = 2.2$. For this case $l_2/l_1 = 4$ and $l_3/l_1 = l_4/l_1$=0.5.

Fig. 9. Plot of persistent current in the right loop as a function of α for a fixed value of $kl_1 = 2.2$. For this case $l_2/l_1 = 4$ and $l_3/l_1 = l_4/l_1$=0.5.