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Zeno blocking of interplanar tunneling by intraplane inelastic scattering in layered superconductors: A generalized spin-boson analysis

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Following an earlier proposal that the observed temperature dependence of the normal-state c-axis resistivity of oxide superconductors can be understood as arising from the inhibition of electron transport along the c axis due to in-plane incoherent inelastic scatterings suffered by the tagged electron, we consider a specific form for the interaction Hamiltonian. In this, the tagged electron is coupled to bosonic baths at adjacent planes (the baths at any two planes being uncorrelated) and is coupled also to the intraplane momentum-flip degree of freedom via the bath degrees of freedom. Thus our model Hamiltonian incorporates the earlier proposed picture that each in-plane inelastic scattering event is like a measurement of which plane the electron is in, and this, as in the quantum Zeno effect, leads to the suppression of interplane tunneling. In the present scenario it is the baths which bring about a coupling between the intraplane and interplane degrees of freedom. For simplicity we confine ourselves to dynamics in two adjacent planes and allow for two states only, as far as momentum flips due to scattering are concerned. In the case when the intraplane dynamics is absent, our model reduces effectively to the usual spin-boson model. We solve for the reduced tunneling dynamics of the electron using a non-Markovian master equation approach. Our numerical results on the survival probability of the electron in the initial plane show that the intraplane momentum flips lead to further inhibition of the interplane tunneling over and above the inhibition effected by pure spin-boson dynamics.

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I. INTRODUCTION

The cuprate superconductors continue to capture the imagination of theorists, leading to a plethora of proposals for describing different physical properties. One of these properties is the unusual suppression of the c-axis resistivity compared to the ab-plane resistivity, in the normal state, which cannot be simply explained by the anisotropy of the underlying crystal structure. It is also clear that any proposed mechanism for suppression of the c-axis resistivity has to concomitantly account for loss-free pair tunneling, which leads to superconductivity. A model to elucidate and encompass the above-mentioned phenomena was earlier presented by one of us in collaboration with others.[1] The basic premise of this analysis is that the strong intralayer electron-electron scattering blocks the single-electron interlayer tunneling but not the tunneling of (the time-reversed) electron pairs. This proposal is much in the spirit of and complementary to the work of Chakravarty *et al.*[2] and that of Kumar,[3] all based on the idea of confinement by "orthogonality catastrophe."[4]

The above-mentioned idea was incorporated by Kumar and Jayannavar[1] (KJ) in terms of a Hamiltonian

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}', \tag{1.1}$$

where $\mathcal{H}'$, though not specified, was assumed to *not* commute with $\mathcal{H}_0$ and taken to describe in-plane scattering processes. The term $\mathcal{H}_0$, on the other hand, takes into account the single-electron energies in two adjacent cuprate layers (designated by α and β) and tunneling between them:

$$\mathcal{H}_0=\sum_{k,\sigma}\epsilon_k\alpha^\dagger_{k,\sigma}\alpha_{k,\sigma}+\sum_{k,\sigma}\epsilon_k\beta^\dagger_{k,\sigma}\beta_{k,\sigma}-\frac{\hbar\delta}{2}\sum_{k,\sigma}(\beta^\dagger_{k,\sigma}\alpha_{k,\sigma}+\mathrm{H.c.}). \tag{1.2}$$

In Eq. (1.2), the summations are over the wave vector k and spin σ, although the latter can be suppressed as there is no spin-flip scattering. In the KJ model the role of $\mathcal{H}'$ was imagined such as to cause repeated interruptions in the time evolution of the electron state, due to successive in-plane scattering events. Thus an initial state of the electron lying in the α plane with wave vector k_0 and spin σ and denoted by $|\alpha,k_0,\sigma\rangle$ would transform into $|\alpha,k_1,\sigma\rangle$, then into $|\alpha,k_2,\sigma\rangle$, and so on. Invoking then the "watched pot effect" of Simonius,[5] KJ had argued that the survival probability of the electron in the α layer, which is related to the transport coefficient across the c axis, is suppressed. In this paper we provide an explicit treatment of this analysis, with the aid of a model for $\mathcal{H}'$.

Before we specify $\mathcal{H}$ it is useful to rewrite $\mathcal{H}_0$ in a simplified notation by introducing pseudospin operators (for spin 1/2) which describe the two-level system of adjacent cuprate layers. Thus,

$$\mathcal{H}=\sum_k\epsilon_k|k\rangle\langle k|\otimes(|\alpha\rangle\langle\alpha|+|\beta\rangle\langle\beta|)-\frac{\hbar\delta}{2}\sum_k|k\rangle\langle k|\otimes(|\alpha\rangle\langle\beta|+|\beta\rangle\langle\alpha|). \tag{1.3}$$

On using the closure property of the states, $\mathcal{H}_0$ can be further reduced to

$$\mathcal{H}_0 = \frac{\hat{p}^2}{2m} - \frac{\hbar\delta}{2}(T^+ + T^-), \tag{1.4}$$

where the first term represents the free-electron Hamiltonian in terms of the momentum operator $\hat{p}$, and $T^{\pm}$ are pseudospin operators which connect the α and β layers. Similarly $\frac{1}{2} \pm T^z$ project a given state on to the $|\alpha\rangle$ and $|\beta\rangle$ states, respectively:

$$T^+ = |\alpha\rangle\langle\beta|, \quad T^- = |\beta\rangle\langle\alpha|,$$

$$T^z = \frac{1}{2}(|\alpha\rangle\langle\alpha| - |\beta\rangle\langle\beta|). \tag{1.5}$$

We are now set to write down $\mathcal{H}'$ in accordance with our stated objective; i.e., $\mathcal{H}'$ should contain terms that cause momentum flips as well as couple to a heat bath[6] that incorporates quantum dissipative processes of inelastic scattering. Thus following Caldeira and Leggett,[8] we may write

$$\mathcal{H}' = \left[\left(\frac{1}{2} + T^z\right)\sum_q g_q(a_q + a_q^\dagger) + \left(\frac{1}{2} - T^z\right)\sum_q G_q(b_q + b_q^\dagger)\right]\hat{x} + \sum_q \hbar(\omega_q a_q^\dagger a_q + \Omega_q b_q^\dagger b_q), \tag{1.6}$$

where $\hat{x}$ is the position operator conjugate to $\hat{p}$. In order to understand the structure of the Hamiltonian $\mathcal{H}$, it is useful to project Eq. (1.6) onto the α or the β plane. Thus

$$\langle\alpha|\mathcal{H}|\alpha\rangle = \frac{\hat{p}^2}{2m} + \hat{x}\sum_q g_q(a_q + a_q^\dagger) + \sum_q \hbar\omega_q a_q^\dagger a_q, \tag{1.7}$$

where we have dropped the term proportional to ω_q as it is inconsequential. Similarly,

$$\langle\beta|\mathcal{H}|\beta\rangle = \frac{\hat{p}^2}{2m} + \hat{x}\sum_q G_q(b_q + b_q^\dagger) + \sum_q \hbar\Omega_q b_q^\dagger b_q, \tag{1.8}$$

where again we have omitted the term proportional to ω_q. Taken separately, either Eq. (1.7) or (1.8) describes the "quantum Brownian motion" of a free electron in which the dissipative friction arises from linear coupling to a quantum heat bath comprised of bosonic excitations.[9,10] Viewed differently, as $\hat{x}$ causes transitions among the free-particle states, Eqs. (1.7) and (1.8) account for inelastic scattering processes in α and β planes, respectively. On the other hand, the off-diagonal element of $\mathcal{H}$ is given by

$$\langle\alpha|\mathcal{H}|\beta\rangle = \langle\beta|\mathcal{H}|\alpha\rangle = -\frac{\hbar\delta}{2}, \tag{1.9}$$

where δ is simply the tunneling frequency for coherent propagation across the c axis.

Combining Eqs. (1.4) and (1.6), the full Hamiltonian can be written as

$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \left[\left(\frac{1}{2} + T^z\right)\sum_q g_q(a_q + a_q^\dagger) + \left(\frac{1}{2} - T^z\right)\sum_q G_q(b_q + b_q^\dagger)\right]\hat{x} - \frac{\hbar\delta}{2}(T^+ + T^-) + \sum_q \hbar(\omega_q a_q^\dagger a_q + \Omega_q b_q^\dagger b_q). \tag{1.10}$$

Thus, $\mathcal{H}$ operates in the product Hilbert space of (i) the discrete two-level system of adjacent cuprate layers, (ii) continuous phase space of a free quantum particle, and (iii) independent sets of quantum oscillators (or bosonic excitations) belonging to the α or the β plane. As $(\frac{1}{2} \pm T^z)$ are projection operators associated with the two adjacent cuprate layers we may view $\mathcal{H}$ as describing a free electron coupled to an environment of quantum oscillators in which the coupling itself depends on which layer the electron is in. Additionally, the term proportional to δ accounts for coherent tunneling of the electron across the layers.

A comment is now in order as to what has motivated us to use the phrase "zeno blocking" in the title of the paper. If $\mathcal{H}'$ is absent and $\mathcal{H}_0$ is the only operative part of the Hamiltonian, then the electron happens to reside in a superposed state of layers α and β, in each of which it moves like a free particle [Eq. (1.4)]. When $\mathcal{H}'$ is switched on, the heat bath comes into play and causes inelastic scattering of the electron through momentum flips. The strength of the scattering process which can be measured in terms of a scattering cross section, say, depends on either $|g_q|^2$ or $|G_q|^2$, depending on which layer the electron belongs to, in view of the presence of the projection operators $(\frac{1}{2} \pm T^z)$ in the Hamiltonian [Eq. (1.6)]. Thus, each in-plane inelastic scattering event[11] is like a quantum measurement of which plane the electron is in, if we view the heat bath as a measuring apparatus.[12] Therefore as in the quantum zeno effect, it is expected that a succession of such scattering events would lead to a suppression of interplane tunneling.

One other noteworthy point is that if the intraplane dynamics is absent, i.e., if the kinetic energy of the electron goes to zero, then the position operator $\hat{x}$ can be replaced by a constant. Because the bath operators $a_q(a_q^\dagger)$ and $b_q(b_q^\dagger)$ belonging to two distinct layers are taken to be independent, the Hamiltonian in Eq. (1.10) reduces to the usual spin-boson Hamiltonian

$$\mathcal{H}_{SB} = -\frac{\hbar\delta}{2}(T^+ + T^-) + T^z\sum_q \hbar g_q(a_q + a_q^\dagger) + \sum_q \hbar\omega_q a_q^\dagger a_q. \tag{1.11}$$

As has been extensively discussed in the review article by Leggett *et al.*,[4] Eq. (1.11) can describe the dissipative dynamics of a mechanical particle moving in a symmetric double-well potential. The minima of the two wells correspond to $T^z = \pm\frac{1}{2}$ states. It has been argued by several authors that when the damping, occasioned by the heat bath coupling, exceeds a certain critical value, the system undergoes a spontaneous symmetry breaking transition at $T=0$. When that happens, tunneling gets suppressed and the particle is localized in one of the two wells. While this phenomenon can also be viewed as a quantum Zeno effect of some sort and is subsumed[14] by the the more general Hamiltonian in Eq. (1.10), the effect discussed in the preceding paragraph is a more subtle one, with richer consequences, as discussed below.

Coming back to the full Hamiltonian in Eq. (1.11), we would like to compute the survival probability in, e.g., the α layer, defined as

$$P_\alpha(t) = \frac{1}{2} + \langle T^z(t)\rangle, \tag{1.12}$$

with the initial condition

$$\langle T^z(t=0)\rangle = \frac{1}{2}. \tag{1.13}$$

Thus, $P_\alpha(t)$ measures the stay-put probability of the electron in the α layer given that it was localized in the α layer at $t=0$. The "leakage" in $P_\alpha(t)$ would clearly then be a measure of the transport across the *c*-axis.

The calculation of $P_\alpha(t)$ based on the complete Hamiltonian of Eq. (1.10) is a rather formidable one. We instead study a simpler Hamiltonian in this paper, in order to set up the required theoretical machinery and check the relevant trends in the result. For this we assume that there are only *two momentum states* of the electron between which the momentum flips occur. Thus the continuous phase space of the electron, described in terms of the position operator $\hat{x}$ and the momentum operator $\hat{p}$, is drastically reduced to a truncated Hilbert space of pseudospin operators (for spin 1/2) S^z and $S^{\pm}$. The simplified Hamiltonian can then be expressed as

$$\begin{aligned}\mathcal{H} = &-\frac{\hbar\Delta}{2}(S^+ + S^-) + 2S^z\left[\left(\frac{1}{2}+T^z\right)\sum_q \hbar g_q(a_q + a_q^\dagger)\right.\\ &\left.+\left(\frac{1}{2}-T^z\right)\sum_q \hbar G_q(b_q+b_q^\dagger)\right] - \frac{\hbar\delta}{2}(T^+ + T^-)\\ &+\sum_q \hbar(\omega_q a_q^\dagger a_q + \Omega_q b_q^\dagger b_q).\end{aligned} \tag{1.14}$$

Note that now the system-bath oscillator coupling constants g_q and G_q have dimensions of frequency. It is pertinent to mention that a model very similar to that in Eq. (1.14) but in a very different context of hopping of a particle along a chain of sites, each coupled independently to a bath, had been treated earlier, within a functional integral formalism.[16]

Even though the Hamiltonian in Eq. (1.14) is a much simplified version of Eq. (1.10), its analysis can be quite complicated. This is the subject of this paper, the breakup of which is as follows. In Sec. II we review and compare the relative merits and demerits of the various existing treatments of the spin-boson Hamiltonian (1.11). This analysis helps us to motivate a similar treatment for the more general case of Eq. (1.14) which is presented in Sec. III. In Sec. IV we discuss the numerical results for the survival probability (1.12) and present certain conclusions. An analysis of the full Hamiltonian in Eq. (1.10) is deferred for future work.

II. SPIN-BOSON HAMILTONIAN: DILUTE BOUNCE GAS APPROXIMATION AND BEYOND

The first step in the analysis of the spin-boson Hamiltonian (1.11) is to subject it to a unitary transformation, well known in polaron physics. This transformation is defined by the operator

$$U = \exp\left[-\sum_q \left(\frac{g_q}{\omega_q}\right)(a_q - a_q^\dagger)T^z\right]. \tag{2.1}$$

This changes the Hamiltonian $\mathcal{H}_{SB}$ to $\tilde{\mathcal{H}}_{SB}$ where

$$\tilde{\mathcal{H}}_{SB} = U\mathcal{H}_{SB}U^{-1} = -\frac{\hbar\delta}{2}(T^+A_- + T^-A_+) + \sum_q \hbar\omega_q a_q^\dagger a_q, \tag{2.2}$$

with

$$A_\pm = \exp\left[\pm\sum_q \left(\frac{g_q}{\omega_q}\right)(a_q - a_q^\dagger)\right]. \tag{2.3}$$

Note that we have ignored the counterterm that would occur in Eq. (2.2) as a result of transformation, Eq. (2.1), since it does not affect the dynamics.

The point about the structure of $\tilde{\mathcal{H}}_{SB}$ is that in any theory in which the first term in Eq. (2.2) is treated as a perturbation, the coupling (parametrized by g_q) is essentially considered to all orders. However, a shortcoming of such a treatment would be that the tunneling frequency δ would have to be taken to be small. This explains why this analysis goes under the name of the dilute bounce gas approximation (DBGA) or the noninteracting blip approximation in a path integral formulation.[4] We discuss below the DBGA using a master equation technique (well known in the quantum optics literature[17]), following Aslangul *et al.*[18] who showed the equivalence of their results with those derived from the path integral method. Incidentally, both these approaches, viz., the path integral and master equation, have also been shown to be equivalent to a resolvent operator technique, which is formulated in the Laplace transform domain instead of in the time domain.[19]

In general, the Hamiltonian in a system-plus-bath decomposition can be expressed as

$$\tilde{\mathcal{H}}_{SB} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{H}_I, \tag{2.4}$$

where in the present case, of course,

$$\mathcal{H}_S = 0, \tag{2.5}$$

while

$$\mathcal{H}_B=\sum_q \hbar\omega_q a_q^\dagger a_q. \tag{2.6}$$

The interaction term $\mathcal{H}_I$ has a general structure

$$\mathcal{H}_I=\hbar\sum_j S_j B_j, \tag{2.7}$$

where in the present case the summation index j runs from 1 to 2 and

$$S_1=T^+,\quad S_2=T^-,\quad B_1=-\frac{\delta}{2}A_-,\quad B_2=-\frac{\delta}{2}A_+. \tag{2.8}$$

We assume that at $t=0$ the system and bath are decoupled so that the total density operator has the factorized form

$$\rho(0)=\rho_S(0)\otimes\rho_B. \tag{2.9}$$

Further, the bath is taken to be in thermal equilibrium, i.e.,

$$\rho_B=\frac{\exp(-\beta\mathcal{H}_B)}{Z_B},\quad \beta=\frac{1}{KT}, \tag{2.10}$$

Z_B being the partition function of the bath. Thus, ρ_B commutes with $\mathcal{H}_B$.

Defining a reduced system density operator ρ_S as

$$\rho_S=\mathrm{Tr}_B\rho \tag{2.11}$$

and using standard techniques to eliminate the bath degrees of freedom under the Born approximation, the equation of motion for ρ_S can be written as[17]

$$\begin{aligned}\dot{\rho}_S=&-i[\mathcal{H}_S,\rho_S]-i\sum_j\langle B_j\rangle[S_j,\rho_S]\\&-\int_0^t d\tau\sum_{jk}\{\langle\langle B_j(t-\tau)B_k(0)\rangle\rangle\\&\times[S_j,e^{-i\mathcal{H}_S(t-\tau)}S_k\rho_S(\tau)e^{i\mathcal{H}_S(t-\tau)}]\\&-\langle\langle B_k(0)B_j(t-\tau)\rangle\rangle[S_j,e^{-i\mathcal{H}_S(t-\tau)}\rho_S(\tau)S_k e^{i\mathcal{H}_S(t-\tau)}]\}.\end{aligned} \tag{2.12}$$

In the above we have defined

$$\langle\langle XY\rangle\rangle=\langle XY\rangle-\langle X\rangle\langle Y\rangle, \tag{2.13}$$

where the angular brackets denote bath averages in the ensemble defined by ρ_B in Eq. (2.10). Using earlier results[20]

$$\begin{aligned}\langle A_\pm\rangle=0,\quad &\langle A_\pm(t)A_\pm(0)\rangle=0,\\ \langle A_+(0)A_-(t)\rangle&=\langle A_-(0)A_+(t)\rangle\\&=\Phi(t)\\&=\exp\left\{-\sum_q\frac{4g_q^2}{\omega_q^2}\left[\coth\left(\frac{1}{2}\beta\omega_q\right)\right.\right.\\&\qquad\left.\left.\times(1-\cos\omega_q t)-i\sin\ \omega_q t\right]\right\},\\ \langle A_+(t)A_-(0)\rangle&=\langle A_-(t)A_+(0)\rangle=\Phi(-t),\end{aligned} \tag{2.14}$$

the equation of motion for $\langle T^z(t)\rangle$ can be written as

$$\langle\dot{T}^z(t)\rangle=-\frac{\delta^2}{4}\int_0^t d\tau[\Phi(-\tau)+\Phi(\tau)]\langle T^z(t-\tau)\rangle. \tag{2.15}$$

Aslangul *et al.* [18] have shown that the solution of Eq. (2.15) agrees with the one arrived at from the path integral approach within the DBGA.

As stated earlier and as is manifestly clear from Eq. (2.15), the DBGA, though valid in the strong-coupling regime, is actually of second order in the tunneling frequency δ. Because of this, the DBGA has an inherent defect that correct thermal equilibrium results are not recovered from time-dependent solutions, in the appropriate asymptotic limit.[4] Weiss and Wollensak[21] have shown how to alleviate this problem, within the path integral approach, by considering interacting blips. Alternatively, this issue has been addressed in the resolvent operator method by Qureshi and Dattagupta[22] by adding and substracting the "free"-tunneling term in Eq. (2.2). Thus $\tilde{\mathcal{H}}_{SB}$ is rewritten as

$$\begin{aligned}\tilde{\mathcal{H}}_{SB}=&-\frac{\hbar\delta}{2}(T^++T^-)-\frac{\hbar\delta}{2}[T^+(A_--1)+T^-(A_+-1)]\\&+\sum_q\hbar\omega_q a_q^\dagger a_q.\end{aligned} \tag{2.16}$$

The idea behind the above decomposition is that in any perturbative treatment of the second term in Eq. (2.16), the first term (i.e., the free-tunneling term) is dealt with exactly. Therefore *now* in accordance with the separation indicated in Eq. (2.4), $\mathcal{H}_S$ is *not* zero but given by

$$\mathcal{H}_S=-\frac{\hbar\delta}{2}(T^++T^-). \tag{2.17}$$

The master equation (2.12) in this case leads to the following (closed) equations of motion:

$$\begin{aligned}\langle\dot{T}^z(t)\rangle=&-\int_0^t d\tau\{K_{11}(\tau)\langle T^z(t-\tau)\rangle\\&+K_{13}(\tau)\langle T^+(t-\tau)-T^-(t-\tau)\rangle\},\\ \langle\dot{T}^+(t)-\dot{T}^-(t)\rangle=&-\int_0^t d\tau\{4K_{13}(\tau)\langle T^z(t-\tau)\rangle\\&+K_{33}(\tau)\langle T^+(t-\tau)-T^-(t-\tau)\rangle\},\end{aligned} \tag{2.18}$$

where

$$K_{11}(t)=\frac{\delta^2}{4}(1+\cos\delta t)[\Phi(-t)+\Phi(t)],$$

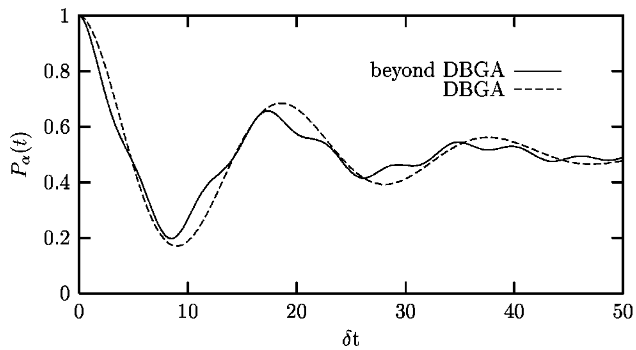

FIG. 1. Survival probability as a function of normalized time in the spin-boson model. Solid line (dashed line) corresponds to beyond the DBGA (DBGA). The values of the various parameters are $\beta\hbar\delta=50$, $D=1000$, and $K=0.1$.

$$K_{13}(t)=i\frac{\delta^2}{8}\sin\delta t[\Phi(-t)+\Phi(t)],$$

$$K_{33}(t)=\frac{\delta^2}{4}\cos\delta t[\Phi(-t)+\Phi(t)]. \tag{2.19}$$

Note, however, that terms corresponding to the systematic part of the evolution in Eqs. (2.19) get canceled off and only the structure of the kernel matrix elements in Eqs. (2.19) gets altered due to the decompositon in Eq. (2.16). Note further that the DBGA equation [Eq. (2.15)] is recovered from Eq. (2.18) on putting $\cos\delta t=1$ and $\sin\delta t=0$ in Eq. (2.19), implying that the DBGA is valid over time scales much shorter than the inverse tunneling frequency.

Equations (2.18) can be solved using Laplace transform techniques. We consider the spectrum of bath oscillators to be Ohmic; i.e., we replace the expression for $\Phi(t)$ in Eq. (2.14) by

$$\Phi(t)=\exp\left\{-2K\int_0^\infty d\omega\frac{e^{-\omega/D}}{\omega}\left[\coth\left(\frac{\beta\hbar\omega}{2}\right)(1-\cos\omega t)-i\sin\omega t\right]\right\}, \tag{2.20}$$

where K is a dimensionless constant that parametrizes damping (strength of coupling to the bath) and D is a cutoff frequency. In the limit of $\beta\hbar D\gg1$ and $Dt\gg1$, the Laplace transform of $\Phi(t)$ has the expression[20]

$$\hat{\Phi}(z)=\exp(i\pi K)\left(\frac{2\pi}{\hbar\beta D}\right)^{2K-1}\frac{\Gamma(1-2K)\Gamma(K+z\hbar\beta/2\pi)}{\Gamma(1-K+z\hbar\beta/2\pi)}, \tag{2.21}$$

where $\Gamma(z)$ is the gamma function of argument z. The survival probability, Eq. (1.12), can then be computed by numerically inverting the expression for its Laplace transform.

It is instructive to compare the results for the survival probability $P_\alpha(t)$ (1.12) in the DBGA and beyond the DBGA in the sense of Eq. (2.15) and Eq. (2.18), respectively. These are presented in Fig. 1. It can be seen from the figure that there are many more coherent oscillations in the survival probability as a function of time in the result beyond the DBGA than in the DBGA result.

III. EFFECT OF MOMENTUM FLIPS ON THE SURVIVAL PROBABILITY

We turn our attention in this section to the main focus of the present study, viz., the c-axis transport in layered superconductors. Before we discuss the calculation it is useful to recall the relevant phenomenology in order to put matters in perspective. Experiments in $YBa_2Cu_3O_{7-x}$ suggest[23] that both the in-plane (ab-plane) resistivity and the c-axis resistivity, in the normal state, have an identical temperature dependence over a significantly wide range of temperatures:

$$\rho=A/T+BT. \tag{3.1}$$

What is of particular interest, in addition, is the prefactor B of the linear term which shows an order-of-magnitude variance between the ab-plane and c-axis values:

$$B_{ab}=1.4\times10^{-6},\quad B_c=3\times10^{-5}. \tag{3.2}$$

Our central proposal is that in-plane inelastic scattering events determine the off-plane transport via incoherent tunneling processes. Now, in the incoherent regime one would expect for the stay-put probability an exponential relaxation

$$P_\alpha(t)\approx\exp(-\tilde{\delta}^2\tau t), \tag{3.3}$$

where $\tilde{\delta}$ is a '''renormalized'' tunneling rate and τ is the in-plane scattering time. Thus the rate of transmittance across the adjacent planes is given by

$$\lambda=\tilde{\delta}^2\tau. \tag{3.4}$$

If d is the interplane separation, one can define a ''mobility''

$$\mu=d\lambda. \tag{3.5}$$

Thus the c-axis resistivity

$$\rho_c\propto\mu^{-1}(=1/d\tilde{\delta}^2\tau). \tag{3.6}$$

On the other hand, the in-plane resistivity ρ_{ab} can be expected to have the Drude form

$$\rho_{ab}=m^*_{ab}/ne^2\tau, \tag{3.7}$$

where m^*_{ab} is the effective mass of electrons in the ab plane, n the number density, and e the electron's charge. Therefore, the important point to note is that both ρ_c and ρ_{ab} are governed by the temperature dependence of τ.

The next issue to address is what is the relevance of the spin-boson model as far as the inelastic scattering processes in general and the temperature dependence of τ in particular are concerned. Here we may refer to the detailed work of Chang and Chakravarty[24] wherein it has been shown that the electron-hole excitations above the Fermi surface are indeed described by a spectral density of bosonic excitations which have the Ohmic form. Further within the DBGA it is well known that [cf. Eq. (2.21)]

$$\tau^{-1} \propto T^{2K-1}. \tag{3.8}$$

Therefore, the phenomenology contained in Eq. (3.1) would suggest that $K=1$, which further implies that one is in the strong-coupling regime, thus justifying the calculational scheme outlined in Sec. II

We return now to the issue at hand concerning the role of momentum flips due to in-plane scattering, accounted for in terms of the pseudospin operators $S^{\pm}$, as in Eq. (1.14). In analogy with the spin-boson case we introduce the unitary transformation

$$U=\exp\left\{-\sum_q \left[\frac{g_q}{\omega_q}(a_q-a_q^{\dagger})\left(\frac{1}{2}+T^z\right)\right.\right.$$
$$\left.\left.+\frac{G_q}{\Omega_q}(b_q-b_q^{\dagger})\left(\frac{1}{2}-T^z\right)\right]2S^z\right\}. \tag{3.9}$$

The Hamiltonian in Eq. (1.14) becomes

$$\tilde{\mathcal{H}}=-\frac{\hbar\delta}{4}(T^+ + T^-)(A_+ + A_-)$$
$$+\frac{\hbar\delta}{2}(T^+ - T^-)S^z(A_+ - A_-)-\frac{\hbar\Delta}{4}S^+(B_-^{(1)}+B_-^{(2)})$$
$$-\frac{\hbar\Delta}{4}S^-(B_+^{(1)}+B_+^{(2)})-\frac{\hbar\Delta}{2}S^+T^z(B_-^{(1)}-B_-^{(2)})$$
$$-\frac{\hbar\Delta}{2}S^-T^z(B_+^{(1)}-B_+^{(2)})+\sum_q \hbar(\omega_q a_q^{\dagger}a_q+\Omega_q b_q^{\dagger}b_q), \tag{3.10}$$

where

$$A_{\pm}=\exp\left\{\pm\sum_q \left[\frac{g_q}{\omega_q}(a_q-a_q^{\dagger})+\frac{G_q}{\Omega_q}(b_q-b_q^{\dagger})\right]\right\}, \tag{3.11}$$

$$B_{\pm}^{(1)}=\exp\left\{\pm\sum_q \frac{2g_q}{\omega_q}(a_q-a_q^{\dagger})\right\},$$

$$B_{\pm}^{(2)}=\exp\left\{\pm\sum_q \frac{2G_q}{\Omega_q}(b_q-b_q^{\dagger})\right\}. \tag{3.12}$$

Again we ignore counterterms which would occur in Eq. (3.10) as a result of the transformation, Eq. (3.9), since these do not affect the dynamics, assuming $g_k^2/\omega_k = G_k^2/\Omega_k$. This is a valid assumption since we consider any pair of cuprate layers to be an unbiased two-state system.

As before we rewrite $\tilde{\mathcal{H}}$ by pulling out the free-tunneling terms, thus yielding

$$\tilde{\mathcal{H}}=\mathcal{H}_S+\mathcal{H}_B+\mathcal{H}_I, \tag{3.13}$$

where

$$\mathcal{H}_S=-\frac{\hbar\Delta}{2}(S^+ + S^-)-\frac{\hbar\delta}{2}(T^+ + T^-), \tag{3.14}$$

$$\mathcal{H}_B=\sum_q \hbar(\omega_q a_q^{\dagger}a_q+\Omega_q b_q^{\dagger}b_q), \tag{3.15}$$

$$\mathcal{H}_I=-\frac{\hbar\delta}{4}(T^+ + T^-)(\overline{A}_+ + \overline{A}_-)$$
$$+\frac{\hbar\delta}{2}(T^+ - T^-)S^z(\overline{A}_+ - \overline{A}_-)-\frac{\hbar\Delta}{4}S^+(\overline{B}_-^{(1)}+\overline{B}_-^{(2)})$$
$$-\frac{\hbar\Delta}{4}S^-(\overline{B}_+^{(1)}+\overline{B}_+^{(2)})-\frac{\hbar\Delta}{2}S^+T^z(\overline{B}_-^{(1)}-\overline{B}_-^{(2)})$$
$$-\frac{\hbar\Delta}{2}S^-T^z(\overline{B}_+^{(1)}-\overline{B}_+^{(2)}). \tag{3.16}$$

In the above an overbar is used to denote, for instance,

$$\overline{X}\equiv X-1. \tag{3.17}$$

The interaction part of the Hamiltonian can be expressed in the compact form

$$\mathcal{H}_I=\hbar\sum_{j=1}^{6} S_j B_j, \tag{3.18}$$

where

$$S_1=T^+ + T^-,\quad S_2=(T^+ - T^-)S^z,\quad S_3=S^+,$$
$$S_4=S^-,\quad S_5=S^+T^z,\quad S_6=S^-T^z,$$
$$B_1=-\frac{\delta}{4}(\overline{A}_+ + \overline{A}_-),\quad B_2=\frac{\delta}{2}(\overline{A}_+ - \overline{A}_-),$$
$$B_3=-\frac{\Delta}{4}(\overline{B}_-^{(1)}+\overline{B}_-^{(2)}),$$
$$B_4=-\frac{\Delta}{4}(\overline{B}_+^{(1)}+\overline{B}_+^{(2)}),\quad B_5=-\frac{\Delta}{2}(\overline{B}_-^{(1)}-\overline{B}_-^{(2)}),$$
$$B_6=-\frac{\Delta}{2}(\overline{B}_+^{(1)}-\overline{B}_+^{(2)}). \tag{3.19}$$

Using the Born master equation (2.12) we can now write down a closed set of equations of motion for the average values of the pseudospin operators representing the momentum flip and the tunneling degrees of freedom. For this purpose it proves convenient to define the following complete set of operators, viz.,

$$X_0=1,\quad X_1=T^z,\quad X_2=T^+ + T^-,\quad X_3=T^+ - T^-,$$
$$X_4=S^z,\quad X_5=S^+,\quad X_6=S^-,$$
$$X_7=T^zS^z,\quad X_8=(T^+ + T^-)S^z,\quad X_9=(T^+ - T^-)S^z,$$
$$X_{10}=T^zS^+,\quad X_{11}=(T^+ + T^-)S^+,\quad X_{12}=(T^+ - T^-)S^+,$$
$$X_{13}=T^zS^-,\quad X_{14}=(T^+ + T^-)S^-,\quad X_{15}=(T^+ - T^-)S^-. \tag{3.20}$$

The equations of motion for the averages of the above operators can be expressed in the compact form

$$\langle \dot{X}_i \rangle = -\int_0^t d\tau \left[\sum_{n=1}^{15} K_{in}(\tau) X_n(t-\tau) + K_{i0}(\tau) \right], \tag{3.21}$$

where the matrix elements of the kernel in the integro-differential equation are given by

$$K_{in}(t) = \sum_{j,k=1}^{6} [\alpha_{jk}^{in}(t) B_{jk}^{+}(t) + \beta_{jk}^{in}(t) B_{jk}^{-}(t)], \tag{3.22}$$

$$B_{jk}^{\pm} = \frac{1}{2}(\langle B_j(t) B_k(0)\rangle \pm \langle B_k(0) B_j(t)\rangle), \tag{3.23}$$

$$\alpha_{jk}^{in}(t) = \sum_{l,m=0}^{15} \kappa_{ij}^{l} r_{lm}(t) \kappa_{mk}^{n},$$

$$\beta_{jk}^{in}(t) = \sum_{l,m=0}^{15} \kappa_{ij}^{l} r_{lm}(t) \lambda_{mk}^{n}. \tag{3.24}$$

In the above expressions $B^{\pm}$ represent the symmetrized (antisymmetrized) correlation functions of the bath operators. The matrix $r(t)$ determines the time evolution of X_i, $i = 1,2, \ldots ,15$, under the free Hamiltonian $\mathcal{H}_S$ [Eq. (3.14)],

$$e^{i\mathcal{H}_S t} X_i e^{-i\mathcal{H}_S t} = \sum_j r_{ij}(t) X_j, \tag{3.25}$$

while the κ and λ terms are the structure constants defined through the commutator and anticommutator algebra:

$$[X_i, S_j] = \sum_k \kappa_{ij}^{k} X_k, \quad \{X_i, S_j\} = \sum_k \lambda_{ij}^{k} X_k. \tag{3.26}$$

A careful examination shows that the only nonzero bath correlations are

$$B_{11}^{\pm}(t) = \frac{\delta^2}{16}[\Phi_1(-t) \pm \Phi_1(t)],$$

$$B_{22}^{\pm}(t) = -\frac{\delta^2}{4}[\Phi_1(-t) \pm \Phi_1(t)],$$

$$B_{34}^{\pm}(t) = B_{43}^{\pm}(t) = \frac{\Delta^2}{16}[\Phi(-t) \pm \Phi(t)],$$

$$B_{56}^{\pm}(t) = B_{65}^{\pm}(t) = 4 B_{34}^{\pm}(t). \tag{3.27}$$

Again we consider an Ohmic spectrum of bath oscillators. Note that $\Phi(t)$ is given by the same expression as in Eq. (2.20) and $\Phi_1(t)$ is given by the same expression as that of $\Phi(t)$ but with K relaced by $K/2$. We calculate the survival probability [Eq. (1.12)] by solving Eqs. (3.21) following the method outlined at the end of Sec. II.

One is tempted to ask if the structure of Eqs. (3.21) simplifies under the DBGA. Note that the DBGA equations are recovered by putting $r_{lm}(t) = \delta_{lm}$. It so happens that in our problem, the structure constants defined in Eq. (3.26) are such that the kernel matrix K becomes diagonal under the DBGA. Further, it turns out that $\langle T^z(t)\rangle$ obeys the same equation as in the spin-boson case under the DBGA, namely, Eq. (2.15), except for the coupling constant K in the spin-boson problem being replaced by $K/2$. Hence, under the DBGA, the tunneling particle does not sense the presence of the momentum-flip degree of freedom at all, pointing to a physical limitation of the DBGA in this context. Consequently, in order to evaluate the effect of momentum-flips on the tunneling dynamics, one is *forced* to go beyond the DBGA.

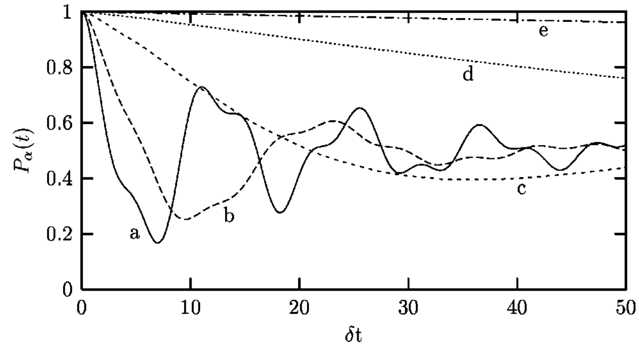

FIG. 2. Survival probability as a function of normalized time in the generalized spin-boson model. Values of various parameters are $\Delta = 0$, $\beta\hbar\delta = 50$, and $D = 1000$. Different curves correspond to different values of K: (a) $K = 0.1$, (b) $K = 0.25$, (c) $K = 0.5$, (d) $K = 0.75$, and (e) $K = 1$.

IV. DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

In Fig. 2 we have plotted the survival probability of the electron in the initial α plane in the case when the in-plane inelastic scattering is absent (i.e., $\Delta = 0$). In this case, as we have noted in Sec. III, the tunneling dynamics is governed by an effective spin-boson Hamiltonian with a modified coupling constant. Figure 2 shows that as the coupling to the bath becomes stronger (K becomes larger), the survival probability evolves in time more and more slowly on average. This behavior is well known in the literature from studies of spin-boson dynamics. What is new in our work is evident in Fig. 3, in which we have plotted the same quantity when $\Delta \neq 0$ (i.e., inelastic scattering is present). Figure 3

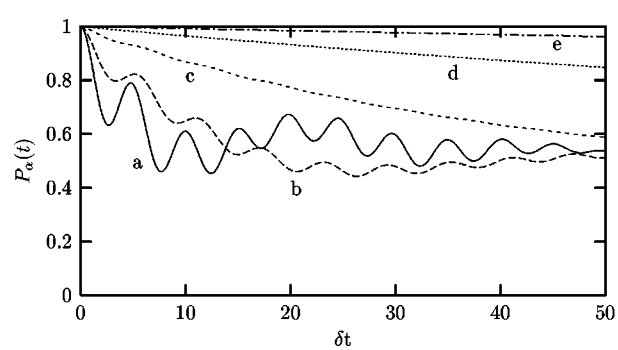

FIG. 3. Same as in Fig. 2, except now $\Delta = \delta$.

shows that, in comparison with Fig. 2, there is a further slowing down of the time evolution. Thus, it turns out that the in-plane inelastic scattering events lead to further inhibition of the tunneling of the tagged electron, across the c axis, over and above what occurs due to the spin-boson dynamics.

Figures 3(c), 3(d), and 3(e) can be fitted to exponentials [cf. Eq. (3.3)] and the c-axis resistivity ρ_c can be extracted from the respective exponents [cf. Eq. (3.6)]. As already stated, the inclusion of $\Delta \neq 0$ terms has led to further inhibition of the transmittance across layers, over and above what is permitted within the spin-boson dynamics. It is fair to state, however, that in-plane scattering processes $(\Delta \neq 0)$ have been treated rather simplistically in that only two momentum states have been allowed. But even through this oversimplified picture we have been able to capture the essential physics of the ''Zeno blocking'' of interplane tunneling. This is not surprising because it is known, in classical stochastic theory, that momentum-reversing ''collisions'' lead to Brownian motion, described by classical Langevin equations. Since the Ohmic dissipation model is known to yield *quantum* Brownian motion,[8,10] our simplified model, described by Eq. (1.14), works reasonably well. It would of course be important to extend the present analysis to a full phase-space treatment of the electron's momentum. We shall return to this matter elsewhere.

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[11] A difference to note between the present model and the one of Kumar and Jayannavar (Ref. 1) is that in the latter the in-plane scattering events were assumed to occur only in the α plane, whereas in the present model we allow for such events in both the planes.

[12] It is interesting to note that our model is similiar in spirit to that of Garg *et al.* (Ref. 13), where, as the authors argue, the bath via the momentum degree of freedom ''observes'' the site state and thus inhibits the tunneling of the particle. However our model differs from that of Garg *et al.* in important ways—whereas in the latter the tunneling pseudospin operator is directly coupled to the momentum degree of freedom, in our model such a coupling is mediated by the bath.

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