Having fixed y, we can find from (2) integers $n_1, \ldots, n_5$ (not all zero) such that

(4) $$y < c_1 n_1^2 + \ldots + c_5 n_5^2 \leqslant y + \epsilon.$$

From (3) and (4),

$$\left| \sum_{s=1}^{r} c_s n_s^2 \right| \leqslant \epsilon,$$

where the n's are not all zero; this proves the theorem.

A THEOREM IN ARITHMETIC

S. CHOWLA*.

HYPOTHESIS. *Let $\theta_1, \ldots, \theta_5$ be positive numbers and such that at least one of the ratios θ_s/θ_1 ($s = 2, 3, 4, 5$) is irrational. Let $[y]$ denote the greatest integer contained in y.*

THEOREM. *Every $n \geqslant n_0(\theta_1, \ldots, \theta_5)$ satisfies*

$$n = [\theta_1 n_1^2] + \ldots + [\theta_5 n_5^2] + c,$$

where c may be 0, 1, 2, 3, or 4, and the n's are integers.

Remarks. Two points about this theorem are:

(i) It is not a consequence of Schnirelmann's recent generalization† of Waring's problem.

(ii) It is not capable, as proved here, of generalization to higher powers.

Proof. It follows from (1) of the preceding paper that the number of solutions of

$$x < \theta_1 n_1^2 + \ldots + \theta_5 n_5^2 \leqslant x + \tfrac{1}{2}$$

is asymptotically $Bx^{\frac{3}{2}}$ for all $x \geqslant x_0(\theta_1, \ldots, \theta_5)$, where $B > 0$. Hence

$$[\theta_1 n_1^2] + \ldots + [\theta_5 n_5^2]$$

is equal to one of x, $x-1$, $x-2$, $x-3$, $x-4$, where x is a sufficiently large integer. This proves the theorem.

* Received 27 January, 1934; read 15 March, 1934.

† "Über additive Eigenschaften von Zahlen", *Math. Annalen*, 107 (1933), 649–691 (682, § 3).