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Brane Waves, Yang-Mills theories and Causality

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We provide evidence for the validity of AdS/CFT correspondence in the Coulomb branch by comparing the Yang-Mills effective action with the potential between waves on two separated test 3-branes in the presence of a large number of other 3-branes. For constant gauge fields excited on the branes, this requires that the supergravity potential in a $AdS_5 \times S^5$ background is the *same* as that in flat space, despite the fact that both propagators and couplings of some relevant supergravity modes are different. We show that this is indeed true, due to a subtle cancellation. With time-dependent gauge fields on the test branes, the potential is sensitive to retardation effects of causal propagation in the bulk. We argue that this is reflected in higher derivative (acceleration) terms in the Yang-Mills effective action. We show that for two 3-branes separated in flat space the structure of lowest order acceleration terms is in agreement with supergravity expectations.

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1. Introduction.

The duality between large-N strongly coupled $\mathcal{N} = 4$ Yang-Mills theory in $3 + 1$ dimensions and supergravity in a $AdS_5 \times S^5$ background [1] [2] has served as an appropriate setting for discussing holographic nature of theories of gravity. Traditionally this duality is conjectured for the Higgs branch of the Yang-Mills theory, corresponding to a large number of coincident three branes, though the possibility that this could be valid in the Coulomb branch - corresponding to branes separated from each other - was suggested already in [1]. More recently, it has been argued by Douglas and Taylor [3] that the duality indeed holds for the Coulomb branch with brane positions identified with Higgs vacuum expectation values.

If the correspondence is valid for the Coulomb branch, one would be able to get new insight into the description of *local* physics in the bulk in terms of Yang-Mills theory and thus eventually understand black hole complementarity. Attempts to understand motion of brane probes have been made in [4]. Some evidence for AdS/CFT correspondence in the Coulomb branch has appeared in [5] and a different point of view is discussed in [6]

Consider for example a Higgs vev in the $\mathcal{N} = 4$ theory of the form

$$\begin{pmatrix} z_1^i & 0 & 0 \\ 0 & z_2^i & 0 \\ 0 & 0 & \mathbf{0}_{(N-2)\times(N-2)} \end{pmatrix} \tag{1.1}$$

The proposal is to identify z_1^i and z_2^i with the transverse positions of a pair of three branes in the presence of $(N - 2)$ other branes - with all the branes parallel to each other.

Now consider exciting this pair by turning on gauge fields F_1 and F_2 respectively. At strong 't Hooft coupling, the low energy effective action for these fields should then give the interaction energy between the branes. Non-renormalization theorems [7] may be then used to calculate this energy by performing a one-loop computation for special brane waves like those made of constant gauge fields. The general one-loop answer for the $O(F^4)$ term is given by [8] [9] [3]

$$\begin{aligned} \int \prod_i d^4p_i [F^\mu_\nu(p_1)F^\nu_\rho(p_2)F^\rho_\kappa(p_3)F^\kappa_\mu(p_4) - \frac{1}{4}F^\mu_\nu(p_1)F^\nu_\mu(p_2)F^\rho_\kappa(p_3)F^\kappa_\rho(p_4)] \\ \delta^4(\sum_{i=1}^4 p_i)\ [G(p_1,p_2,p_3,p_4) + \text{permutations}] \end{aligned} \tag{1.2}$$

with $F = F_1 - F_2$ and

$$G(p_i) = \int d^4k[(\rho^2 + k^2)(\rho^2 + (k - p_1)^2)(\rho^2 + (k - p_1 - p_2)^2)(\rho^2 + (k + p_4)^2)]^{-1} \quad (1.3)$$

and

$$\rho^2 = \sum_{i=1}^{6}(z_1^i - z_2^i)^2 \quad (1.4)$$

We can expand $G(p_i)$ around $p_i = 0$ and obtain the position space effective action in a derivative expansion. The first term is

$$\frac{1}{\rho^4}\int d^4y\ [F^\mu_\nu F^\nu_\rho F^\rho_\kappa F^\kappa_\mu - \frac{1}{4}F^\mu_\nu F^\nu_\mu F^\rho_\kappa F^\kappa_\rho] \quad (1.5)$$

This one loop contribution is exact for $N = 2$ [7]. It is also exact for $SU(N)$ in the Coulomb branch where all the higgs have nonzero expectation values in which case the expression (1.2) should include a sum over all $U(1)$ factors [10] . In [3] it has been argued that the nonrenormalization theorems should still hold for our case, where $SU(N) \to SU(N-2) \times [U(1)]^2$, by considering (1.1) as a limit from the Coulomb branch - though there is no proof as yet. This situation could be therefore compared to a supergravity calculation. In this limit, (1.2) leads to an effective potential which has the following terms

$$\frac{1}{\rho^4}[O_1^\phi O_2^\phi + O_1^\chi O_2^\chi + 2T_1^{\mu\nu} T_{2\ \mu\nu} + 2O_1^{\mu\nu} O_{2\ \mu\nu}|_4] \quad (1.6)$$

where for each $i = 1, 2$

$$\begin{aligned}
O_i^\phi &= \frac{1}{4}(F_i)^{\mu\nu}(F_i)_{\mu\nu} \\
O_i^\chi &= \frac{1}{4}(F_i)^{\mu\nu}(\tilde{F}_i)_{\mu\nu} \\
T_i^{\mu\nu} &= \frac{1}{2}[(F_i)^\mu_\alpha (F_i)^{\alpha\nu} - \frac{1}{2}\eta^{\mu\nu}(F_i)^{\alpha\beta}(F_i)_{\alpha\beta}] \\
O_i^{\mu\nu} &= \frac{1}{2}[(F_i)^{\mu\nu} + (F_i)^{\mu\alpha}(F_i)_{\alpha\beta}(F_i)^{\beta\nu} - \frac{1}{4}(F_i)^{\mu\nu}(F_i)^{\alpha\beta}(F_i)_{\alpha\beta}]
\end{aligned} \quad (1.7)$$

In (1.6) the subscript $|_4$ in the last term means that we retain terms containing four factors of the gauge field in the product.

Let us first consider the case where the gauge fields on the branes are constants. Then (1.5) is the only contribution to the effective action upto $O(F^4)$.

On the supergravity side, the force between the pair is due to the exchange of supergravity modes. With only the gauge field excited these modes are the dilaton, axion, longitudinally polarized graviton and the longitudinally polarized 2-form fields. When $N = 2$ this is propagation in flat space. In this case it is straightforward to understand the terms in (1.6). The overall factor $1/\rho^4$ comes from the static massless propagator in the six transverse dimensions, which appears because the supergravity modes which couple to constant gauge fields on the brane have zero momentum along the brane directions. The first term is due to the exchange of a dilaton, which couples to $\mathrm{Tr}F^2$ on each of the branes [11]. The second term comes from axion exchanges which couples to $\mathrm{Tr}(F\tilde{F})$. The third term comes from the exchange of a longitudinally polarized graviton which couples to the energy momentum tensor $T_{\mu\nu}$ on the worldvolumes [11]. The last term comes from the exchange of a 2-form field, whose couplings have been obtained in [12]. (Here the 2-form couples to $F_{\mu\nu}$ on one of the branes and to a cubic in the fields on the other one). Moreover, the relative coefficients between the various operators in (1.6) are exactly what is expected from the couplings and propagators. The fact that the supergravity answer agrees with the gauge theory truncation of the open string theory on the brane is well known in related contexts [13].

For large N and in the scaling limit, however, the pair of branes are situated in the $AdS \times S^5$ produced by the $N - 2$ other branes and one has to use the couplings and propagators in this space-time. It is puzzling how the same Yang-Mills answer in (1.6) could be reproduced by supergravity in a nontrivial space-time. In particular, the flat space propagator $1/\rho^4$ depends on the coordinate distance between the branes and not on their individual locations - a feature which is not *a priori* expected in $AdS_5 \times S^5$.

Remarkably, as was shown in [3], the $AdS_5 \times S^5$ propagator for fields which obey the massless Klein-Gordon equation is identical to the flat space propagator when restricted to the zero brane momentum sector. Examples of such fields are the dilaton and the longitudinally polarized graviton. Moreover, as is clear from the analysis of [12], the couplings of these fields to the individual branes are the same as that in flat space. Thus the first three terms in (1.6) indeed follow from dilaton and graviton exchanges in $AdS_5 \times S^5$.

In [3], it was claimed that the last term of (1.6) can be also explained by 2-form exchange in $AdS_5 \times S^5$. However, because of the presence of a nonzero 5-form field strength in the $AdS_5 \times S^5$ background , the NS-NS and the R-R 2-forms mix with each other through a Chern-Simons term in IIB supergravity [14], leading to two independent branches and these branches behave as *massive* fields. Various other supergravity modes mix with each other in a similar fashion. This phenomenon is crucial in a supression of the classical s-wave absorption cross-section of the 2-form field by three branes [15]. Consequently, as will be shown below, this leads to rather different propagators which reflect the mixings and also depend on individual brane locations. Moreover, as shown in [12], the coupling of the 2-form fields to the brane are *different* in $AdS_5 \times S^5$ and flat space. It would be rather miraculous if inspite of such differences, the supergravity calculation is able to reproduce (1.6).

In this paper we show that this miracle indeed happens. The difference in couplings and the propagators conspire to reproduce the exact form of the two-form mediated potential expected from Yang-Mills theory. We conjecture that this mechanism is quite general and would be manifest in the interaction between other brane waves which involve exchange of other supergravity modes displaying a similar mixing. Our results provide strong evidence for the validity of Maldacena conjecture in the Coulomb branch.

Finally we address the question of causality in the bulk and its manifestation in the Yang-Mills effective action. In supergravity, the interaction between test branes occurs through *retarded* potentials arising from causal propagation of supergravity modes. For constant gauge fields on the branes, retardation effects are invisible and static propagators in transverse space are relevant. However, for nonconstant waves, causality manifests itself by producing an interaction energy which is bi-local on the brane [2]. From the point of view of the AdS/CFT correspondence, it may appear puzzling how the boundary Yang-Mills theory "knows" about causality in the bulk. In particular when the two test branes are separated only in the radial direction, the two locations map into the *same* point on the boundary and causality in the boundary theory does not impose any restriction. In fact, the Yang-Mills effective action is usually written as a sum of *local* terms.

We will argue that bulk causality is reflected in the Yang-Mills theory in terms involving derivative of the fields. Supergravity then predicts a specific structure of these terms. We show this explicitly for the lowest order acceleration terms involving gauge fields in the case of two test branes in flat space by comparing the result with the effective action of $SU(2)$ Yang-Mills theory. Fortunately this term is not renormalized, thus a comparison with supergravity is allowed. We expect that this will continue to hold in $AdS_5 \times S^5$, which we will discuss in a future publication [16]. In general such considerations may lead to a supergravity understanding of the "acceleration" terms in the Yang-Mills effective action.

[2] This point has been emphasized to me by S. Mathur

2. Propagators at zero brane momentum

Consider the following metric in $AdS_5 \times S^5$

$$ds^2 = (\frac{r}{R})^2[dy \cdot dy] + (\frac{R}{r})^2[dr^2 + r^2 \sum_{i=1}^{5} f_i(\theta_i)(d\theta_i)^2] \tag{2.1}$$

We will use the following conventions. The ten dimensional coordinates will be denoted by $y^a, a = 0, \cdots 9$. Out of these we continue to denote the brane worldvolume directions by $y^\mu, \mu = 0, \cdots 3$. The remaining six transverse coordinates $y^5 \cdots y^9$ will be relabelled as $z^i, i = 1, \cdots 6$. $r = \sqrt{\sum_{i=1}^{6}(z^i)^2}$ is the radial coordinate in the transverse space and θ_i are angles on the S^5. (r, θ_i) are related to the cartesian coordinates z^i in the transverse space by the standard transformations and the metric coefficients $f_i(\theta_i)$ are determined from these transformations.

In the following we will set $R = 1$ without loss of generality, and restore them using dimensional analysis when required.

The action for a minimally coupled massless scalar in this background may be easily seen to be

$$S_\phi = \frac{1}{2}\int dt\ d^3x\ d^6z[\frac{1}{r^4}(\partial_{x^\mu}\phi)^2 + (\partial_{z^i}\phi)^2] \tag{2.2}$$

Thus when the fields do not depend on the brane worldvolume coordinates, the action is in fact identical to that of a massless scalar field in flat space. This means that the propagator with zero worldvolume momentum is the flat space propagator and given by

$$G_\phi(z_1, z_2) = \frac{1}{4\pi^3|z_1 - z_2|^4} \tag{2.3}$$

The dilaton and the longitudinally polarized graviton [3] behave like massless minimally coupled scalars from the point of view of the six dimensional transverse space and therefore has propagators given by (2.3). This is the result used in [3].

The longitudinally polarized 2-form field is also a scalar from the point of view of the transverse space, but it is not a minimally coupled scalar. More significantly, there are two such 2-form fields, the NS-NS field which we denote by b_{ab} and the R-R field which we will denote by a_{ab}. These two fields are coupled with each other through the background

[3] Longitudinal polarization means that the tensor indices of the fields are along the 3-brane worldvolume.

five form field strength. It is conveninent to combine these two fields into a single complex field

$$B_{ab} = b_{ab} + i a_{ab} \tag{2.4}$$

the relevant part of the supergravity action [4] is given by

$$S_B = \frac{1}{12} \int d^{10}x \ \sqrt{g}[H^*_{abc} H^{abc} + i F^{abcde}(H_{abc} B_{de} - H^*_{abc} B_{de} - (c.c.))] \tag{2.5}$$

where

$$H_{abc} = \partial_a B_{bc} + \partial_b B_{ca} + \partial_c B_{ab} \tag{2.6}$$

The last term is a Chern-Simons term which couples the two types of fields. This leads to the well known equations of motion

$$\frac{1}{\sqrt{g}} \partial_c(\sqrt{g} H^{cab}) = -\frac{2i}{3} F^{abcde} H_{cde} \tag{2.7}$$

In the $AdS_5 \times S^5$ background the five form field has a value

$$F^{12r03} = \frac{1}{r^3} \tag{2.8}$$

the other nonzero components being determined by antisymmetry and self-duality in the usual fashion.

We are interested in the longitudinal components of the 2-form field, so that in B_{ab} the indices (a, b) take values $a, b = 0, \cdots 3$. The equations (2.6) and (2.7) then show that a given component $B_{\mu\nu}$ mixes only with its dual $\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}$. It is therefore convenient to define three pairs of complex fields (ϕ_1^A, ϕ_2^A), $A = 1 \cdots 3$ denoting the electric and magnetic parts of $B_{\mu\nu}$

$$\phi_1^A = \frac{1}{2} \epsilon^{ABC} B_{BC} \qquad\qquad \phi_2^A = B_{0A} \tag{2.9}$$

We fix a gauge in which the fields are independent of the coordinates $y^\mu, \mu = 0, \cdots 3$. We also introduce a coordinate

$$x = \log r \tag{2.10}$$

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[4] In this paper we do not consider fluctuations of the five form field strength, so that usual problems of writing an action for a self dual five form gauge field are not relevant.

The action (2.5) for fields which depend only on the transverse coordinates z^i in the background given by (2.1) and (2.8) then becomes

$$
\begin{aligned}
S_B = \frac{1}{2}\int dx\ [d\Omega_5] \sum_{A=1}^{3} [\partial_x \phi_1^{A\ *} \partial_x \phi_1^A + \sum_{i=1}^{5} \frac{1}{f_i} \partial_i \phi_1^{A\ *} \partial_i \phi_1^A \\
- \partial_x \phi_2^{A\ *} \partial_x \phi_2^A - \sum_{i=1}^{5} \frac{1}{f_i} \partial_i \phi_2^{A\ *} \partial_i \phi_2^A \\
+ 4i(\phi_1^{A\ *} \partial_x \phi_2^A + \phi_2^{A\ *} \partial_x \phi_1^A)]
\end{aligned}
\tag{2.11}
$$

Here the measure on S^5 is given by

$$
d\Omega_5 = (\prod_{i=1}^{5} d\theta_i)\ h(\theta_i) \qquad h(\theta_i) = (f_i)^{\frac{1}{2}} \tag{2.12}
$$

The negative signs in the kinetic terms of ϕ_2^A come from lowering a timelike index.

Clearly the action (2.11) is *not* the same as that in flat space, unlike the minimally coupled scalar discussed above. This, together with the mixing between ϕ_1^A and ϕ_2^A makes the propagator nontrivial. Furthermore the different pairs ϕ_n^A are independent of each other and may be treated separately.

The propagators for these fields may be obtained by performing a standard mode decomposition to diagonalize the action. The details are given in Appendix B. The final result for the propagator is, after restoring factors of R

$$
N^{AB}(\vec{z}_1, \vec{z}_2) = \frac{\delta^{AB}}{8\pi^3 R^4 |\vec{z}_1 - \vec{z}_2|^4} \begin{pmatrix} (r_1^4 + r_2^4) & -i(r_1^4 - r_2^4) \\ -i(r_1^4 - r_2^4) & -(r_1^4 + r_2^4) \end{pmatrix} \tag{2.13}
$$

The propagator may be, of course, expressed in terms of geodesic distances. However that will not be necessary for our present purposes.

It may be easily checked that the propagators for the 2-form fields in *flat* space is

$$
N_{\text{flat}}^{AB} = \frac{\delta^{AB}}{4\pi^3 |\vec{z}_1 - \vec{z}_2|^4} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.14}
$$

in sharp contrast with (2.13). The relative factor of 2 in the overall normalizations in (2.13) and (2.14) will be crucial in what follows.

Finally let us consider current couplings in the supergravity theory with currents $J(x, \theta_i)$ which depend only on the transverse directions

$$
\frac{1}{4}\int dx d\Omega_5 \sum_{A,n} [(J^A)^*_n(x,\theta_i)\phi_n^A(x,\theta_i) + J_n^A(x,\theta_i)(\phi^A)^*_n(x,\theta_i)] \tag{2.15}
$$

Integrating out the fields one gets the current-current coupling

$$\frac{1}{4}\int dx d\Omega_5 \int dx' d\Omega'_5 [(J^A)^*_n(x,\theta) N^{AB}_{nm}(x,\theta;x',\theta') J^B_m(x',\theta')] \tag{2.16}$$

Note that reality requires

$$(N^{AB})^*_{mn}(x,\theta;x',\theta') = N^{BA}_{nm}(x',\theta';x,\theta) \tag{2.17}$$

which is satisfied by our propagator (2.13).

3. Couplings in the Dirac-Born-Infeld-Wess-Zumino action

The couplings of the relevant supergravity modes to a single brane in $AdS_5 \times S^5$ may be obtained from the Dirac-Born-Infeld-Wess-Zumino (DBI-WZ) action, and have been studied in [12]. The action for a D3-brane in a general background of dilaton, graviton and rank-2 fields is given by [17]

$$S = -\int d^4\xi \sqrt{-det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + \int (\hat{C}_{(4)} + \mathcal{F} \wedge \hat{A} + \hat{C}_{(0)} \mathcal{F} \wedge \mathcal{F}) \tag{3.1}$$

[18] The two terms above correspond to the DBI action and the WZ term respectively. $G_{\mu\nu}$ refers to the induced world-volume metric, obtained as the pull-back of the spacetime metric. Similarly,

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu} - \hat{B}_{\mu\nu} \tag{3.2}$$

where $F_{\mu\nu}$ stands for the gauge field on the D3-brane and $\hat{B}_{\mu\nu}$ is the pullback of the NS-NS two form potential. In the W-Z term $\hat{C}_{(4)}$, $\hat{A}$ and $\hat{C}_{(0)}$ refer to the pullback of the R-R four form, two form and zero form fields respectively. The DBI-WZ action may be viewed as the effective action of the Yang-Mills theory with $SU(N) \rightarrow SU(N-1) \times U(1)$, with derivatives on gauge fields ignored. The diagonal Higgs which breaks the symmetry interpreted as the position of a 3-brane probe. Conformal transformations of the Higgs fields in the Yang-Mills description are "metamorphosed" into those of the transverse coordinates in $AdS_5 \times S^5$ due to modifications of Ward identities in the gauge fixed theory [19].

We fix a static gauge, setting the four worldvolume parameters to be equal to the coordinates y^μ in the metric and also fix the kappa symmetry following [18] by setting half of the fermionic fields in the brane action to zero. The couplings of the various supergravity modes may be then obtained by performing an expansion around background values (given by the $AdS_5 \times S^5$ solution and the five form background field strength) and then expanding the determinant to the required order.

We will consider the case when only the bosonic gauge fields are excited on the brane. Then the operator on the worldvolume which couples to the dilaton obtained by the above procedure is

$$\mathcal{O}_\phi = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{3.3}$$

where indices of worldvolume fields are raised and lowered using the flat metric. The same operator couples to a brane in flat space [11]. Similarly the operator coupling to the longitudinal components of the metric is also of the same form as in flat space

$$(\mathcal{O}_g)^{\mu\nu} = \frac{1}{2}[F^{\mu\alpha} F^{\nu}_{\alpha} - \frac{1}{4}\eta_{\mu\nu}(F^{\mu\nu}F_{\nu\mu})] \tag{3.4}$$

In contrast the operator coupling to the antisymmetric tensor fields $a_{\mu\nu}$ and $b_{\mu\nu}$ differ in important detail from their form in flat space. The operator for the NS-NS form comes from the DBI term and is given by [12]

$$(\mathcal{O}_b)^{\nu\mu} = -\frac{1}{2}[F^{\nu\mu} + \frac{1}{r^4} G^{\nu\mu}] \tag{3.5}$$

where

$$G^{\nu\mu} = [F^{\nu}_{\rho} F^{\rho}_{\kappa} F^{\kappa\mu} - \frac{1}{4}(F^{\rho\kappa}F_{\rho\kappa})F^{\nu\mu}] \tag{3.6}$$

while that for the R-R field comes from the WZ term

$$(\mathcal{O}_a)^{\nu\mu} = \frac{1}{4}\epsilon^{\nu\mu\rho\kappa} F_{\rho\kappa} \tag{3.7}$$

In (3.5) r is the location of the brane in question.

If the brane was located in flat space one would simply have

$$(\mathcal{O}_b)^{\nu\mu}|_{\text{flat}} = -\frac{1}{2}[F^{\nu\mu} + G^{\nu\mu}] \tag{3.8}$$

The factor of $1/r^4$ in front of $G^{\nu\mu}$ is now absent.

In [12] it was shown that the operator (3.5), modified by the prescription of [20], for the nonabelian analog, represents the 2-form field in the dual description in terms of a Yang-Mills theory. Note that for this to hold the supergravity modes to which they couple have to be *on shell*. This played a crucial role in cancellation of dimension four fermionic operators, which would have jeopardazied the AdS/CFT connection.

For our purposes such an interpretation is not necessary - we will simply consider these operators for what they stand : coupling of individual branes to supergravity modes. By the same token we remain *off-shell*.

The presence of the factor of $1/r^4$ (which is actually $(R/r)^4$ once the R is restored) in front of the dimension six term is related to the relationship between the infrared cutoff in AdS space and the ultraviolet cutoff in the dual gauge theory - a fact that is crucial for holography [1],[21],[4]. When the dual theory is considered to live on the boundary at large r this term may be thought of providing the ultraviolet cutoff necessary to write down a higher dimension operator in the gauge theory. The presence of this factor of $1/r^4$ in (3.5) will turn out to be crucial in what follows.

Note the asymmetry between the NS-NS and R-R fields in the couplings. The 3-brane is of course self-dual. In the dual formulation, the NS-NS fields are interchanged with the R-R fields and the field strength is replaced by its dual as well [22],[18].

Because of the presence of the $\epsilon_{\mu\nu\rho\kappa}$ in (3.7), it is natural to rewrite the coupling

$$\mathcal{L}_I = b_{\mu\nu}(\mathcal{O}_b)^{\nu\mu} + a_{\mu\nu}(\mathcal{O}_a)^{\nu\mu} \tag{3.9}$$

in terms of the fields ϕ_n^A introduced in the previous sections.

$$\mathcal{L}_I = \frac{1}{2}\sum_{A,n}[\phi_n^{A\ *}\mathcal{P}_n^A + c.c.] \tag{3.10}$$

where we have

$$\begin{aligned} \mathcal{P}_1^A &= \frac{1}{2}[\frac{1}{2}\epsilon^{ABC}(F_{BC} + \frac{1}{r^4}G_{BC}) + iF^{0A}] \\ \mathcal{P}_2^A &= \frac{1}{2}[(F^{0A} + \frac{1}{r^4}G^{0A}) + \frac{i}{2}\epsilon^{ABC}F_{BC}] \end{aligned} \tag{3.11}$$

In the full ten dimensional theory, the interaction of the 2-form field with a pair of branes located at $\vec{z} = \vec{z}_1$ and $\vec{z} = \vec{z}_2$ may be then written as

$$\int d^4y \int d^6z \sum_{A,n}[\phi_n^{A\ *}(y,z)\tilde{J}_n^A(y,z) + c.c.] \tag{3.12}$$

where

$$\tilde{J}_n^A(y,z) = [\delta^6(z-z_1) + \delta^6(z-z_2)]\mathcal{P}_n^A(y,z) \tag{3.13}$$

We are interested in the situation where the brane waves are constant along the brane, so that the operators $\mathcal{P}$ are independent of y. In that case the y integration in (3.12) projects out the zero brane momentum part of the fields ϕ_n^A and one is left with an expression of the form (2.15). Since the measure in (2.15) is $dx d\Omega_5$ while that in (3.12) it is $dr\ d\Omega_5\ r^5$ one has

$$J_n^A(x,\theta) = r^6\tilde{J}_n^A(x,\theta) \tag{3.14}$$

4. Interaction Energy of constant field brane waves

We can now use the formulae in Section 2. to derive the interaction energy between brane waves due to 2-form exchange. This is the connected piece in (2.16), where we substitute (3.14) and (3.13). Note the additional factor of r^6 present in (3.14) can be absorbed by changing the measure in (2.16) to yield

$$\int d^6z \int d^6z'[\delta^6(z-z_1)+\delta^6(z-z_2)][\delta^6(z'-z_1)+\delta^6(z'-z_2)]\mathcal{P}_m^{A\ *}(z)N_{mn}^{AB}(z,z')\mathcal{P}_n^B(z') \quad (4.1)$$

where N_{mn}^{AB} is the zero momentum propagator which has been calculated above. The interaction energy is given by the connected piece

$$E = \mathcal{P}_m^{A\ *}(z_1)N_{mn}^{AB}(z_1,z_2)\mathcal{P}_n^B(z_2) + \mathcal{P}_m^{A\ *}(z_2)N_{mn}^{AB}(z_2,z_1)\mathcal{P}_n^B(z_1) \quad (4.2)$$

Evaluating (4.2) using (3.11) and (2.13) is straightforward. The final result is

$$E = \frac{1}{4\pi^3\rho^4}[(F_1)^\mu_\nu(F_2)^\nu_\rho(F_2)^\rho_\kappa(F_2)^\kappa_\mu - \frac{1}{4}(F_1)^\mu_\nu(F_2)^\nu_\mu(F_2)^\rho_\kappa(F_2)^\kappa_\rho + (1\to 2)] \quad (4.3)$$

Using (3.8) and (2.14) it is easily seen that we get an *identical* result for just two three branes located in flat space. The relative factor of two in the overall normalizations of the flat space and AdS propagators is crucial for this agreement.

Two sets of important cancellations happened for each term over the indices (A,B)

1. Terms quadratic in F's, like $(F_1)^\mu_\nu(F_2)^\nu_\mu$, which could have been present because of terms in $\mathcal{P}_n^A$ linear in F, cancelled. If this did not happen, there would be no correspondence with Yang-Mills. These would be loop corrections to the kinetic energy terms, which cannot be present in this $N=4$ theory.
2. Both the propagator and the couplings depend on the individual brane locations $\vec{z}_1$ and $\vec{z}_2$. However these translation-noninvariant terms conspire to cancel each other leaving with an answer which depends only on $|\vec{z}_1 - \vec{z}_2|$.

The structure in (4.3) is in *precise* agreement with the result of Yang-Mills theory given in the last line (1.6).

Since the couplings and zero momentum propagators for the dilaton and the graviton are identical in $AdS_5 \times S^5$ and flat space we would trivially reproduce the first two lines of (1.6).

5. Other brane waves

Even for the simple brane waves considered above, i.e. constant gauge fields, the agreement of Yang-Mills effective action in Coulomb branch and the interaction between branes in supergravity through single mode exchange depends on the non-trivial cancellation demonstrated above. It is certainly worth understanding this mechanism by studying other kinds of brane waves, e.g. excitations of fermions or Higgs fields on the worldvolume.

Of particular interest are fermionic operators. These would couple to the gravitons via their contribution to the energy momentum tensor and to the two-form field via operators which have been derived in [12]. It may be easily verified, using the nature of the 2-form propagators derived above, that the fermionic operators do not have a net contribution from 2-form exchange *both* in $AdS_5 \times S^5$ as well as in flat space. This again is due to cancellations, but now the contributions from the diagonal and the off-diagonal parts of the propagators cancel separately [5]. The Yang-Mills contributions may be read off from the results of [8],[7]and [23].

When other brane waves are excited, various other supergravity modes will contribute to the exchange and *a priori* their propagators would not be the same as in flat space. For example, with the Higgs field excited, there is a coupling with the trace of the S^5 metric which mixes with the rank-4 gauge field polarized along S^5 [14]. It would be interesting to see whether similar cancellations hold in this case as well.

6. Time dependent brane waves and causality in the bulk

So far we have restricted our attention to interactions mediated by supergravity modes with zero brane momentum. This restriction hides an important piece of physics in the bulk : causality. The point is that the force between any two objects is mediated by *retarded* propagators reflecting causal propagation and not by instantaneous action. This does not have an obvious meaning in the Yang-Mills description. The base space-time of Yang-Mills theory is identified with the directions y in the bulk, but there is no analog of the radial distance r. Consider for example two points which are separated in the AdS_5 space along the radial direction. A physical signal takes a finite time to travel between these points. However in the Yang-Mills description these points are in fact the *same*

[5] The same mechanism is responsible for the *on shell* cancellation of dimension four fermionic operators required for AdS/CFT correspondence to hold [12].

point in the boundary space. It seems rather mysterious as to how the Yang-Mills theory encodes this finite time lag.

In the following we will argue that the supergravity prediction for force between branes due to causal propagation of massless modes leads to a precise prediction for the structure of higher derivative operators in the effective action in the Yang-Mills theory.

Consider a general coupling to the test branes of the form given by (3.12) and (3.13). For an arbitrary supergravity field Φ_M this is given by

$$\sum_M \int d^4y \int d^6z [\Phi^*_M(y,z)\tilde{J}_M(y,z) + c.c.] \tag{6.1}$$

where

$$\tilde{J}_M(y,z) = [\delta^6(z-z_1) + \delta^6(z-z_2)]\mathcal{P}_M(y,z) \tag{6.2}$$

where indices M label various supergravity fields. The currents $\mathcal{P}_M$ are made out of fields on the brane. To illustrate the point, we will consider currents $\mathcal{P}_M$ which depend only on time. This, in fact, highlights the issue since the coupling of the fields in (3.12) are to currents on the two branes which are at the same *spatial* position on the brane. Then the interaction energy is given by the expression

$$E = \sum_{MN} \int dt \int dt'\ \mathcal{P}^*_M(t,z_1)\ \Delta^R_{MN}(t-t';z_1,z_2)\ \mathcal{P}_N(t',z_2) \tag{6.3}$$

where Δ^R_{MN} denotes the retarded propagator, and we have assumed time translation invariance. In the special case considered in the previous sections, i.e. with time independent $\mathcal{P}_M$, the time integrals pass through the currents and convert the retarded propagator into a static propagator in transverse space. For general time dependence, this does not happen and one is left with a bilocal expression for the interaction energy, given above.

The Yang-Mills effective action, however, is given as a sum of various terms which are integrals of local densities on the brane worldvolume. In our example this involves a single integral over time since the fields are assumed to be constant in space.

Our proposal for comparing the supergravity and Yang-Mills expressions is to expand the currents in (6.3) around the average time. Introducing

$$t_0 = \frac{1}{2}(t+t') \qquad \delta t = t - t' \tag{6.4}$$

we find from this Taylor expansion

$$
\begin{aligned}
E = \sum_{MN} [\int dt_0 \mathcal{P}^*_M(t_0, z_1) N_{MN}(z_1, z_2) \mathcal{P}_N(t_0, z_2) \\
+ \frac{1}{2} \int dt_0 (\partial_0 \mathcal{P}^*_M(t_0))(\partial_0 \mathcal{P}_N(t_0)) \ \int [d(\delta t)] \ (\delta t)^2 \ \Delta^R_{MN}(\delta t; z_1, z_2) + \cdots]
\end{aligned}
\tag{6.5}
$$

where the dots denote higher order terms in δt. The first term involves the static propagator N_{MN}

$$
N_{MN}(z_1, z_2) = \int dt \Delta^R_{MN}(t; z_1, z_2) \tag{6.6}
$$

which we considered in the previous sections. However the currents $\mathcal{P}_M(t)$ are general functions of time. In the general case they may be considered as general functions of the brane worldvolume coordinates. This explains how supergravity generates F^4 terms in the effective action, as in (1.5), even when the fields are not constant.

Since the currents $\mathcal{P}_N$ are composite operators involving gauge fields, the successive terms in (6.5) should corrrespond to terms in the Yang-Mills effective action which are higher order in a time derivative expansion. Moreover, as we will see shortly, the integral over δt converts the expansion in terms of the time lag into an expansion in terms of the magntitude of the transverse distance. The latter is, however, the magnitude of the Higgs expectation value and hence the scale below which the low energy effective action is valid. This is a direct manifestation of the IR-UV correspondence. This has played a role in earlier discussions of bulk causality [24].

Causality in the bulk therefore provides a specific structure for these higher derivative terms for the strongly coupled Yang-Mills theory, strong coupling being required for the validity of the supergravity approximation of IIB string theory. To check this proposal we need to find such operators which are protected by non-renormalization theorems, so that we can perform a weak coupling calculation in the gauge theory.

In fact the simplest test involves currents which are linear in the gauge fields, which couples to the 2-form field in the bulk. In terms of the notation introduced above we then have

$$
\begin{aligned}
\mathcal{P}^A_1(t, z) &= \frac{1}{2} \epsilon^{ABC} F_{BC}(t) + i F^{0A}(t) \\
\mathcal{P}^A_2(t, z) &= F^{0A}(t) + i \frac{1}{2} \epsilon^{ABC} F_{BC}(t)
\end{aligned}
\tag{6.7}
$$

The first term in (6.5) then involves two powers of the gauge field and no derivatives - these cancel as shown in section 4. The second term in (6.5) is of the form $(\partial F)^2$, which is of "weight" four and hence protected by nonrenormalization theorems of [7]. We will show soon that there is a net contribution to these derivative terms.

6.1. Branes in flat space and $SU(2)$

The considerations of causality are equally relevant to the situation with two separated three branes, with no other branes present. In supergravity, these are then located in flat space and we can evaluate the expressions easily. The Yang-Mills description is then in terms of a $SU(2)$ gauge theory.

As before we deal with the 2-form fields with polarizations along the brane worldvolume. The fields are assumed to depend on time and the transverse directions. We will work in a gauge $\partial^a B_{ab} = 0$. The action for these modes in a flat background can be easily worked out to be

$$S = \frac{1}{2} \sum_A \int dt \int d^6 z [-|\partial_t \phi_1^A|^2 + |\partial_z \phi_1^A|^2 - |\partial_z \phi_2^A|^2] \tag{6.8}$$

where the fields ϕ_n^A have been defined in (2.9). Note that the action does not involve time derivatives of the "electric" components ϕ_2^A, so that the propagator for this is essentially the static propagator in transverse space. The contributions of ϕ_1^A and ϕ_2^A to the first term in (6.5) cancel, essentially due to the negative sign of the kinetic term for ϕ_2^A - as has been shown in the previous sections. In the second term, only ϕ_1^A contributes and one is thus left with a term

$$\sum_A \int dt_0 (\partial_t \mathcal{P}_1^A(t_0))^* (\partial_t \mathcal{P}_1^B(t_0)) \int d\delta t \ (\delta t)^2 \ \Delta_{11}^{R,AB}(\delta t; z_1, z_2) \tag{6.9}$$

where the retarded propagator for ϕ_1^A can be read off from the action (6.8)

$$\Delta_{11}^{R,AB}(\delta t, z_1, z_2) = \delta^{AB} \int \frac{dp_0}{2\pi} \int \frac{d^6 p}{(2\pi)^6} \frac{e^{-ip_0 \delta t + i\vec{p}\cdot(\vec{z}_1 - \vec{z}_2)}}{(p_0 + i\epsilon)^2 - (\vec{p})^2} \tag{6.10}$$

The integral in (6.9) may be easily seen to be

$$\int d\delta t \ (\delta t)^2 \ \Delta_{11}^{R,AB}(\delta t; z_1, z_2) \sim \frac{\delta^{AB}}{\rho^2} \tag{6.11}$$

where ρ is the transverse distance defined in (1.4). Inserting the expressions for $\mathcal{P}^A$ we get a contribution for the interaction energy of the form

$$E \sim \frac{1}{\rho^2} \int dt_0 \ (\partial_t F_{1\ \mu\nu})(\partial_t F_2^{\mu\nu}) \tag{6.12}$$

This is precisely a term of weight four in the effective action of $SU(2)$ gauge theory, as may be seen in [23]. This term is not renormalized since it is of weight four [7]. Note that this term has two less powers of ρ in the denominator compared to the F^4 terms which appear in the zero momentum potential. This reflects the IR-UV connection : an expansion in the time interval gets translated into an expansion in the transverse distance, which is the magnitude of the Higgs and hence a scale in the effective Yang-Mills theory. Also note that this is the *only* term of weight four other than the F^4 terms when fields other than the gauge field are set to zero.

Terms with higher weight will come from the higher terms of the Taylor series, from the F^3 terms in the 2-form coupling, and from exchange of other supergravity modes. It would be interesting to see whether the corresponding operators also obey nonrenormalization theorems.

When the brane waves depend on the spatial coordinates on the branes we expect the time derivatives in (6.12) to be converted into space-time derivatives on the brane.

Finally, we note that the calculation described above does not really probe the *retarded* nature of the propagator - an advanced propagator would lead to the same result. Strictly speaking we have been investigating consequences of the finite speed of light rather than causality. However, we expect that at higher orders the difference of retarded and advanced propagators will play a role.

6.2. Branes in $AdS_5 \times S^5$

The calculation outined above is a test for our proposal in the simplest possible setting. A test of the proposal in the context of the AdS/CFT correspondence requires an analysis which involves propagators of fields in the $AdS_5 \times S^5$ background with nonzero brane momentum. These propagators have been obtained in full generality [25]. We expect that the signature of causal propagation in the $AdS_5 \times S^5$ background in terms of the higher derivative operators in the Yang-Mills effective action would hold in this case as well. For this to work the cancellation which made the constant field interaction energy in $AdS_5 \times S^5$ equal to the flat space result should continue to work for non-constant fields. Our results in this direction will appear in a future publication [16].

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8. Appendix A : The scalar propagator in flat 6d space

Consider flat six dimensional (euclidean) space with the metric

$$dr^2 + r^2 \sum_{i=1}^{5} f_i(\theta_i)(d\theta_i)^2 \tag{8.1}$$

The action of a complex massless scalar field is

$$S_\phi = \frac{1}{2} \int dr\ d\Omega_5\ r^5\ \phi^* [\frac{1}{r^5}\partial_r(r^5 \partial_r \phi) + \sum_{i=1}^{5} \frac{1}{h\ r^2} \partial_i (h f_i^{-1} \partial_i \phi)] \tag{8.2}$$

The relevant mode decomposition for this action is

$$\phi(r, \theta_i) = \int \frac{d\beta}{2\pi} \sum_{k\vec{m}} \frac{1}{r^2} r^{i\beta}\ Z_{k,\vec{m}}(\theta_i)\ \phi_{\beta,k,\vec{m}} \tag{8.3}$$

where the S^5 (scalar) spherical harmonics $Z_{k,\vec{m}}(\theta_i)$ satisfy [14], [26]

$$\frac{1}{h}\partial_i(\frac{1}{f_i} h \partial_i) Z_{k,\vec{m}}(\theta_i) = -k(k+4)\ Z_{km}(\theta_i) \tag{8.4}$$

with integer k and are chosen to be orthornormal with the measure $[d\Omega_5]$. The action then becomes

$$S_\phi = \int \frac{d\beta}{2\pi} \sum_{k,\vec{m}} [\beta^2 + (k+2)^2] \phi^*_{\beta,k,\vec{m}} \phi_{\beta,k,\vec{m}} \tag{8.5}$$

Thus the propagator is

$$G(\vec{z}_1, \vec{z}_2) = \int \frac{d\beta}{2\pi} \sum_{k,\vec{m}} \frac{1}{(r_1 r_2)^2} (\frac{r_1}{r_2})^{i\beta} \frac{1}{\beta^2 + (k+2)^2} Z^*_{k,\vec{m}}(\theta_1) Z_{k,\vec{m}}(\theta_2) \tag{8.6}$$

Integrating over β for $r_1 > r_2$ now gives

$$G(\vec{z}_1, \vec{z}_2) = \frac{\pi}{r_1^4} \sum_{k,\vec{m}} \frac{1}{2(k+2)} (\frac{r_2}{r_1})^k\ Z^*_{k,\vec{m}}(\theta_1) Z_{k,\vec{m}}(\theta_2) \tag{8.7}$$

However we know that the position space propagator in six dimensions is

$$G(\vec{z}_1, \vec{z}_2) = \frac{1}{4\pi^3 |\vec{z}_1 - \vec{z}_2|^4} \tag{8.8}$$

Comparing (8.8) and (8.7) we get

$$\frac{1}{2\pi^3 |\vec{z}_1 - \vec{z}_2|^4} = \frac{1}{r_1^4} \sum_{km} \frac{1}{k+2}\ (\frac{r_2}{r_1})^k\ Z^*_{km}(\theta_1)\ Z_{km}(\theta_2) \qquad (r_1 > r_2) \tag{8.9}$$

This equation can be also proved by using explcit properties of the S^5 spherical harmonics.

9. Appendix B : Propagator for 2-forms in $AdS_5 \times S^5$

The mode decomposition which diagonalizes the action (2.11) is

$$\phi_n^A(x,\theta_i) = \int_{-\infty}^{\infty} \frac{d\beta}{2\pi} \sum_{k,\vec{m}} e^{-i\beta x} Z_{k,\vec{m}}(\theta_i)\ \phi^A_{n,(\beta,k,\vec{m})} \qquad n = 1,2 \tag{9.1}$$

With this mode decomposition and a partial integration the action S_B may be diagonalized to yield

$$S_B = \frac{1}{2}\sum_A \int \frac{d\beta}{2\pi} \sum_{k\vec{m}} (\ \phi_1^{A\ *} \quad \phi_2^{A\ *}\ ) \begin{pmatrix} (\beta^2 + k(k+4)) & -4\beta \\ -4\beta & -(\beta^2 + k(k+4)) \end{pmatrix} \begin{pmatrix} \phi_1^A \\ \phi_2^A \end{pmatrix} \tag{9.2}$$

In (9.2) ϕ^A stands for $\phi^A_{(\beta,k,\vec{m})}$.

The eigenvalues of the kinetic energy matrix may be easily seen to be

$$\lambda_\pm = \pm\sqrt{(\beta^2 + k^2)(\beta^2 + (k+4)^2)} \tag{9.3}$$

This clearly shows the two branches of this field found in [14]. These have masses k and $(k+4)$ respectively.

The propagator may be now found by inverting the matrix. The result is

$$N^{AB}(\beta k) = \frac{\delta^{AB}}{(\beta^2 + k^2)(\beta^2 + (k+4)^2)} \begin{pmatrix} \beta^2 + k(k+4) & -4\beta \\ -4\beta & -(\beta^2 + k(k+4)) \end{pmatrix} \tag{9.4}$$

The nature of the propagator may be made more transparent by rewriting the matrix elements of N as

$$\begin{aligned} N_{11}^{AB}(\beta,k) = -N_{22}^{AB}(\beta,k) &= \frac{\delta^{AB}}{2k+4}[\frac{k}{\beta^2 + k^2} + \frac{k+4}{\beta^2 + (k+4)^2}] \\ N_{12}^{AB}(\beta,k) = N_{21}^{AB}(\beta,k) &= -\frac{\delta^{AB}}{2k+4}[\frac{\beta}{\beta^2 + k^2} - \frac{\beta}{\beta^2 + (k+4)^2}] \end{aligned} \tag{9.5}$$

The position space propagators may be now easily calculated

$$N_{mn}^{AB}(\vec{z}_1,\vec{z}_2) = \delta^{AB} \int \frac{d\beta}{2\pi} \sum_{km} e^{i\beta(x_1 - x_2)}\ Z^*_{km}(\theta_1)\ Z_{km}(\theta_2)\ N^{AB}_{mn}(\beta,k) \tag{9.6}$$

The integral over β may be now performed to get, for $x_1 > x_2$

$$\begin{aligned} N_{11}^{AB}(\vec{z}_1,\vec{z}_2) &= \delta^{AB} \sum_{km} \frac{1}{4(k+2)}[(\frac{r_2}{r_1})^k + (\frac{r_2}{r_1})^{k+4}]\ Z^*_{km}(\theta_1)\ Z_{km}(\theta_2) \\ N_{12}^{AB}(\vec{z}_1,\vec{z}_2) &= -i\delta^{AB}\pi \sum_{km} \frac{1}{4(k+2)}[(\frac{r_2}{r_1})^k - (\frac{r_2}{r_1})^{k+4}]\ Z^*_{km}(\theta_1)\ Z_{km}(\theta_2) \end{aligned} \tag{9.7}$$

where we have used (2.10). Using the relation (8.9) in Appendix A, we get the final answer for the propagator matrix

$$N^{AB}(\vec{z}_1,\vec{z}_2) = \frac{\delta^{AB}}{8\pi^3|\vec{z}_1 - \vec{z}_2|^4} \begin{pmatrix} (r_1^4 + r_2^4) & -i(r_1^4 - r_2^4) \\ -i(r_1^4 - r_2^4) & -(r_1^4 + r_2^4) \end{pmatrix} \tag{9.8}$$

Restoring powers of R yields (2.13).

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