TIFR-TH/97-56, MIT-CTP-2694, MRI-PHY/P971030, hep-th/9711003

Boosts, Schwarzschild Black Holes and Absorption cross-sections in M theory

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D dimensional neutral black strings wrapped on a circle are related to $(D-1)$ dimensional charged black holes by boosts. We show that the boost has to be performed in the covering space and the boosted coordinate has to be compactified on a circle with a Lorentz contracted radius. Using this fact we show that the transition between Schwarzschild black holes to black p-branes observed recently in M theory is the well-known black hole- black string transition viewed in a boosted frame. In a similar way the correspondence point where an excited string state goes over to a neutral black hole is mapped exactly to the correspondence point for black p-branes. In terms of the p brane quantities the equation of state for an excited string state becomes identical to that of a $3+1$ dimensional massless gas for all p. Finally, we show how boosts can be used to relate Hawking radiation rates. Using the known microscopic derivation of absorption by extremal 3-branes and near-extremal 5D holes with three large charges we provide a microscopic derivation of absorption of 0-branes by seven and five dimensional Schwarzschild black holes in a certain regime.

October, 1997

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1. Introduction

Over the past year there has been a growing feeling that it is useful to examine M theory in a boosted frame, to be better able to formulate its structure and consequences. In a recent set of papers this notion was applied to attempt an understanding of Schwarzschild black holes in various dimensions [1] [2] [3].

The idea is to use the fact that neutral black holes become RR-charged holes in a frame boosted along x^{11}. On the other hand, it has been conjectured in [4] and shown in [5] that the hamiltonian of discrete light cone quantization of M-theory is the finite N version of M(atrix) theory [6]. Thus DLCQ M(atrix) theory should give a microscopic understanding of the thermodynamics of neutral black holes in various dimensions. In [1] this is done for M theory compactified on T^3 for which the DLCQ M(atrix) theory is the well understood $N = 4$ super-Yang-Mills theory in $3 + 1$ dimensions. In [2] compactification on other T^p have been considered and the black hole results have been shown to be consistent with known strong coupling results in M(atrix) theory for $p = 4$.

It is important to understand the precise benefits of using the boosted frame. When do we get something new, and when do we simply get something already known but seen in a Lorentz transformed frame? In this paper we ask the following questions :

[A] Suppose spacetime has a spatial compact direction. If we take a Schwarzschild black hole, and boost it in this compact direction, then from the viewpoint of the noncompact space it acquires Kaluza-Klein charge. Can we therefore relate some properties of charged black holes to properties of neutral black holes?

[B] Which of the arguments in [1] and [2] depend on properties of M(atrix) theory and which of them follow from known facts in general relativity and the kinematics of boosts ?

[C] Can we derive some interesting results about black holes which were not otherwise calculable, by using the idea of boosts?

We have found the following answers to the questions above.

[A]

(i) While the answer to the first question appears to be obviously 'yes', we need to be careful about the fact that boosting in a compact direction is not a Lorentz symmetry. We need to discuss what boost transformation is required to establish a precise connection between neutral and charged black holes. We start with a "black string" in $(d + 1)$ dimensions along $x^{d+1} = z$ with no momentum along it. x^{d+1} lies on a circle of radius R. Some of the other transverse directions may also be compact, lying on a T^p with each side of length L. Then this is a neutral black hole in $(d-p)$ dimensions. We then perform a boost in the *covering space* by an angle α. Finally we compactify the new z coordinate z' on a circle of radius R'. As a result we get a solution which after standard Kaluza Klein reduction becomes an electrically charged $(d-p)$ black hole. If $d=10$ and the theory is M-theory, the charge is a 0-brane charge of the ten dimensional string theory. In that case one can further perform T-dualities to get a black Dp-brane solution [7].

(ii) The new point which we observe is that the resulting charged black hole has the same entropy as the original neutral black hole when $R' = R/\cosh\alpha$, i.e. the new radius is taken to be precisely the Lorentz contracted radius. This identifies the precise nature of the "boost transformation" relevant to us.

[B]

(i) We show that a crucial ingredient of [1] and [2]- viz. that the entropies of the boosted $(11-p)$ dimensional Schwarzschild black hole is equal to the entropy of a black p-brane - is precisely the boosted version of the black hole-black string transition [8] when the black hole radius ρ_0 becomes greater than the radius of the transverse compact direction R. As such this fact is largely independent of any property of the 11 dimensional M theory or M(atrix) theory; this is a result in any theory which contains General Relativity in $(d+1)$ dimensions. In M theory the charge is a 0-brane charge of the string theory [9]. We can then T-dualize the charged hole with p compact transverse dimensions to a D-p brane and use the microscopic understanding of the resulting black D-p brane.

[C]

(i) We consider the correspondence principle formulated in [8] which says that string states at weak coupling transit to black holes at a critical coupling, where both string and black hole descriptions have the same entropy. We show that the correspondence point for neutral black holes may be related to the correspondence point for black p-brane (and hence to a large class of black holes with a single large charge) by "boosts" described above.

(ii) For Schwarzschild black holes the weak coupling regime is described by highly excited states of a single string. We rewrite the corresponding equation of state in terms of the boosted variables and find that for all values of p the equation of state is that of a gas in $3+1$ dimensions. This equation of state then gives a uniform description of the weak coupling regime of black p-branes. Its implications are not very clear at the moment.

(iii) We show that the above boosts may be used to relate the absorption cross-sections of particles of different charges. In particular we check that this procedure correctly predicts the emission rate of charged scalars from the $4+1$ dimensional near extremal black hole with three large charges from the emission rate of neutral particles. This correspondence between rates is independent of the way it is calculated (i.e. semiclassical or microscopic) and simply a result of the kinematics of boosts.

(iv) We show that the relation between absorption/emission rates may be now used to provide a microscopic description of absorption by a seven dimensional Schwarzschild black hole - at least in a certain limit. This is because by boosts and T-duality we can go from this black hole to a black 3-brane. In the extremal limit of the latter it is known that the absorption of massless neutral scalars may be reproduced exactly by a microscopic calculation in the $3+1$ dimensional Yang-Mills theory [10]. By the kinematics of boosts this is related to absorption of certain charged particles by the seven dimensional Schwarzschild black hole, in a suitable limit.

(v) In a similar way we argue that one can have a microscopic understanding of absorption or emission of certain particles in five dimensional Schwarzschild black holes in string theory. We show how such black holes are related to the well known five dimensional black hole with three large charges by a chain of boosts and T-dualities. Since absorption/emission cross-sections in the latter has a microscopic derivation [11] we have a microscopic derivation for certain absorption/emission process in the former. In this case one can deal with near-extremal rather than exactly extremal holes so that we can make a statement about Hawking radiation.

2. Boosted Schwarzschild black holes as charged black holes

In this section we identify the precise nature of boost transformations which relate black strings in $(d-p+1)$ dimensions, or equivalently Schwarzschild black holes in $(d-p)$ dimensions, to charged black holes in $(d-p)$ dimensions.

Consider pure Einstein gravity in $d+1$ space-time dimensions with a Planck length l_{pl}. We will consider compactifications of this theory to $(d-p)$ dimensions with the compact space being a torus $T^p \times S^1$. The torus T^p has volume L^p while the radius of the circle S^1 is R. In this theory there is a Schwarzschild black string solution

$$ds^2_{d+1} = -(1-(\frac{r_0}{r})^n)dt^2 + \frac{dr^2}{(1-(\frac{r_0}{r})^n)} + r^2 d\Omega_{n+1} + dz^2 + \sum_{i=1}^{p}(dx^i)^2 \tag{2.1}$$

where

$$n = d-3-p \tag{2.2}$$

and we have labelled the direction along S^1 by z. By standard Kaluza-Klein reduction this is a black hole in $(d-p)$ dimensions. The mass and entropy of this black hole are given by

$$\begin{aligned} M &= \frac{(n+1)\Omega_{n+1}L^p R}{8l_{pl}^{d-1}}\ r_0^n \\ S_{bs} &= \frac{\Omega_{n+1}L^p \pi R}{2l_{pl}^{d-1}}\ r_0^{n+1} \end{aligned} \tag{2.3}$$

where Ω_m denotes the volume of a unit m-sphere and the $(d+1)$ dimensional Planck length l_{pl} is defined in terms of the $(d+1)$ dimensional Newton constant $G_{d+1} = l_{pl}^{d-1}$

We now go to the covering space (i.e. a noncompact z) and boost along the z direction by a boost angle α. Thus the coordinates in the boosted frame (z', t') are

$$\begin{aligned} z' &= z\cosh\alpha + t\sinh\alpha \\ t' &= t\cosh\alpha + z\sinh\alpha \end{aligned} \tag{2.4}$$

The metric now becomes

$$\begin{aligned} ds^2_{d+1} =& -(1-\frac{r_0^n}{r^n}\cosh^2\alpha)(dt')^2 + (1+\frac{r_0^n}{r^n}\sinh^2\alpha)(dz')^2 \\ &+ \frac{r_0^n}{r^n}\sinh(2\alpha)\ dz'\ dt' \\ &+ \frac{dr^2}{(1-(\frac{r_0}{r})^n)} + r^2 d\Omega_{n+1} + \sum_{i=1}^{p}(dx^i)^2 \end{aligned} \tag{2.5}$$

Finally we compactify the boosted coordinate on a radius R'. The momentum P in the z direction is then quantized and given in terms of an integer N by

$$P = \frac{N}{R'} = M(\frac{R'}{R})\cosh\alpha\sinh\alpha \tag{2.6}$$

with M given by (2.3).

By standard Kaluza-Klein procedure the solution represents a charged black hole in $(d-p)$ dimensions [12]. The Einstein metric of this black hole is given by

$$
\begin{aligned}
ds_d^2 = [f(r)]^{(\frac{1}{d-2})}\{-[f(r)]^{-1}(1-\frac{r_0^n}{r^n})(dt')^2 \\
+\frac{dr^2}{(1-(\frac{r_0}{r})^n)}+r^2 d\Omega_{n+1}+\sum_{i=1}^{p}(dx^i)^2\}
\end{aligned}
\tag{2.7}
$$

where

$$
f(r) = 1+\frac{r_0^n \sinh^2\alpha}{r^n} \tag{2.8}
$$

The dilaton ϕ and the gauge field A_0 are given by

$$
\begin{aligned}
e^{2\phi} &= [f(r)]^{(\frac{d-1}{d-4})} \\
A_0(r) &= \frac{r_0^n \sinh\alpha\cosh\alpha}{r^n + r_0^n \sinh^2\alpha}
\end{aligned}
\tag{2.9}
$$

Note that the procedure which we have used is not a symmetry of the theory. If we boost along a compact direction z, the new coordinate z' is not compact with any radius. Such a procedure has been implictly used in earlier work on classical solutions [13].

The Bekenstein entropy of this solution may be read off from (2.7)

$$
S_{ch} = \frac{\Omega_{n+1}L^p\pi R'}{2l_{pl}^{d-1}}\; r_0^{n+1}\cosh\alpha \tag{2.10}
$$

while the ADM mass is given by

$$
M_{ADM} = M(\frac{R'}{R})[1+\frac{n}{n+1}\sinh^2\alpha] \tag{2.11}
$$

with M given by (2.3).

The entropy, which should have an interpretation in terms of the total number of states, should not change under boosts. ¿From (2.10) and (2.3) it is clear that $S_{bs} = S_{ch}$ if R' is the Lorentz contracted radius

$$
R' = \frac{R}{\cosh\alpha} \tag{2.12}
$$

Note, however, that when viewed as black holes in $(d-p)$ dimensions, the area of the $(d-p-2)$ dimensional horizon is *not* boost invariant. The entropy is invariant since the $(d-p)$ dimensional Newton's constant defined by

$$
G_{d-p} = \frac{G_{d+1}}{L^p R} \tag{2.13}
$$

increases due to the contraction of R to exactly compensate for the increase of the area of $(d-p-2)$ dimensional horizon.

The momentum in (2.6) transforms correctly as may be seen by using (2.12) to obtain

$$P = M \sinh\alpha \tag{2.14}$$

as expected. The ADM mass (2.11) can be seen to follow as a simple consequence of the Lorentz transformation of an appropriate $(d+1)$-dimensional energy momentum tensor $T_{\mu\nu}$ which generates the metric (2.1) asymptotically. We briefly outline the construction [14].

Setting $g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$, the appropriate energy momentum tensor $T_{\mu\nu}$ is given by

$$R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)} = 8\pi G T_{\mu\nu} \tag{2.15}$$

where $(\partial_\lambda \equiv \frac{\partial}{\partial x^\lambda})$

$$R^{(1)}_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\partial_\nu h^\lambda_\lambda + \partial_\lambda\partial^\lambda h_{\mu\nu} - \partial_\mu\partial_\lambda h^\lambda_\nu - \partial_\nu\partial_\lambda h^\lambda_\mu\right) \tag{2.16}$$

is the part of the Ricci tensor linear in $h_{\mu\nu}$ and the indices are to be raised or lowered using $\eta_{\mu\nu}$ [15]. Choosing the harmonic gauge where

$$\partial_\lambda h^\lambda_\mu = \frac{1}{2}\partial_\mu h\ , \qquad h \equiv \eta^{\mu\nu}h_{\mu\nu}\ , \tag{2.17}$$

the energy momentum tensor $T_{\mu\nu}$ is given by

$$\partial_\lambda\partial^\lambda(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h) = 16\pi G T_{\mu\nu}\ . \tag{2.18}$$

The metric (2.1) written in isotropic coordinates becomes

$$ds^2_{d+1} = -\left(\frac{4\rho^n - r_0^n}{4\rho^n + r_0^n}\right)^2 dt^2 + \left(1 + \frac{r_0^n}{4\rho^n}\right)^{\frac{4}{n}}(d\rho^2 + \rho^2 d\Omega^2_{n+1}) + dz^2 + \sum_{i=1}^{p}(dx^i)^2 \tag{2.19}$$

where ρ is related to r by

$$2\rho^n = r^n - \frac{r_0^n}{2} + \sqrt{r^{2n} - r_0^n r^n}\ . \tag{2.20}$$

Then to the leading order in $\frac{r_0^n}{\rho^n}$ we get $(a,\ b = 1,\ 2,\ \cdots p\ ; \quad i,\ j = p{+}1,\ p{+}2,\ \cdots p{+}n{+}2)$

$$h_{00} = \frac{r_0^n}{\rho^n}\ , \quad h_{ij} = \frac{r_0^n}{n\rho^n}\delta_{ij}\ , \quad h_{zz} = 0\ , \quad h_{ab} = 0\ , \tag{2.21}$$

satisfying the harmonic gauge condition (2.17). Note that $\rho^2 = \sum_{p+1}^{p+n+2} (x^i)^2$.

Defining the mass M through the relation $T_{00} = M\delta^{(n+2)}(x^i)$ gives the expression for M as in (2.3). Furthermore, it follows that

$$T_{ij} = 0 \; ; \quad T_{zz} = -\frac{1}{n+1} T_{00} \; ; \quad T_{ab} = -\frac{1}{n+1} T_{00}\delta_{ab} \; . \tag{2.22}$$

Now boost along the z direction by a boost angle α. Note that the Newton's constant G increases to $G' = G\cosh\alpha$ due to the Lorentz contraction of R as can be seen from equations (2.12) and (2.13). Therefore, it follows from (2.15) that the combination $GT_{\mu\nu}$ must transform like a tensor. We thus get for the 00-component

$$G'T'_{00} = (1 + \frac{n}{n+1} \sinh^2\alpha) \; GT_{00} \; . \tag{2.23}$$

Defining the mass M' in the boosted frame through the relation $T'_{00} = M'\delta^{(n+2)}(x^i)$ gives

$$M' = M_{ADM} \tag{2.24}$$

where M_{ADM} is given in (2.11).

All the results discussed so far are results in $(d+1)$ dimensional General Relativity and has nothing to do with string theory. If our starting point is 11-dimensional supergravity the dimensionally reduced theory is Type IIA string theory. In that case we can perform T-duality transformations to convert the charged black hole solution into a Dirichelt p-brane solution. The identification of the entropy, energy and momentum of course continue to hold.

3. The black hole-black string transition

In this section we demonstrate the relationship between the neutral black hole - string transition [8] to the boosted Schwarzschild black hole - black p-brane transition observed in [1] and [2].

The object of interest in [1] and [2] is the $(d+1-p)$ dimensional Schwarzschild black hole, still keeping z to be compact. Consider such an object in its rest frame with horizon radius ρ_0. The solution is an array with a spacing $2\pi R$ along the covering space of the circle along z [16]. The mass and entropy of the black hole are

$$\begin{aligned} \tilde{M} &\sim \frac{L^p}{l_{pl}^{d-1}} \; \rho_0^{n+1} \\ S_{bh} &\sim \frac{L^p}{l_{pl}^{d-1}} \; \rho_0^{n+2} \end{aligned} \tag{3.1}$$

Consider now the $(d+1-p)$ dimensional Schwarzschild hole which has the same mass as that of the Schwarzschild string. From (2.3) this means

$$(\frac{r_0}{\rho_0})^n \sim (\frac{\rho_0}{R}) \tag{3.2}$$

The ratio of the entropies is

$$(\frac{S_{bs}}{S_{bh}}) \sim (\frac{\rho_0}{R})^{\frac{1}{n}} \tag{3.3}$$

Thus for $\rho_0 < R$ upto a numerical factor of order one, the black hole array is entropically favored while for $\rho_0 > R$ the black string is entropically favored [8]. The common entropy at the black hole- black string transition point $\rho_0 \sim R$ is

$$S' \sim (RM) \tag{3.4}$$

This transition is related to the instability of a black string [17]. Some other properties of this transition have been studied in [18].

Now boost the system in the sense described above. The black string becomes a $(d-p)$ dimensional charged black hole. There will be a similar transition from the boosted Schwarzschild black hole in $(d+1-p)$ dimensions and a charged black hole in $(d-p)$ dimensions. Since the entropies do not change under boosts the transition takes place once again when $\rho_0 \sim R$. However, expressed in terms of the radius R' after the boost this point is

$$\rho_0 \sim R' \cosh\alpha \tag{3.5}$$

In [1] and [2] it was assumed that the longitudinal size of the $(d+1-p)$ dimensional Schwarzschild black hole contracts due to the boost and the transition happens when this contracted radius is of the same order as the radius of the circle. This gives precisely the relation (3.5). The common entropy at the transition is

$$S' \sim (N \coth\alpha) \tag{3.6}$$

When the boost parameter is large one has a near-extremal charged black hole. In this case $\coth\alpha \sim 1$ and $S' \sim N$.

It is in general not easy to find the precise radius of compactification where the entropy of the black hole will equal the entropy of the black string carrying the same energy. The black hole in the compact space can be thought of as an array in the covering space, and the effective energy of each hole in the array is influenced by the gravity of the other members of the array; thus the metric of a single hole is not directly applicable to the array.

But by what was said above, we can find this critical radius of compactification for black holes carrying momentum charge, if we know it for neutral holes. Thus start with the neutral hole of mass M in $d+1$ spacetime dimensions. Let the compact circle have radius R_c which is the point where the hole is unstable to formation of a black string stretching in the compact direction. On dimensional grounds

$$R_c = \mu[G_N M]^{1/(d-2)}$$

where μ is a dimensionless constant of order unity, and G_N is the gravitational constant in the $d+1$ dimensional spacetime.

In the covering space we see an array of black holes at spacing R_c. Now boost along the direction of the array, with boost angle α. We now have an array of holes, each with mass $M \cosh\alpha$ and with momentm charge $P = M \sinh\alpha$. The array is of course still at the point of instability, but the separation between holes now appears as $R_c/\cosh\alpha$.

Thus we conclude that the critical radius for a hole with mass M and charge P will be

$$R_c = \mu[G_N M]^{1/(d-2)}[1 - \frac{P^2}{M^2}]^{\frac{(d-1)}{2(d-2)}}$$

Once again the results of this section are results in $(d+1)$ dimensional General Relativity or any theory which contains it.

4. The correspondence principle and boosts

In this section we will restrict ourselves to 11-dimensional M-theory and the ten dimensional string theory which follows from it and examine the correspondence principle of [8] in the light of boosts in the 11th direction.

The string coupling g_s and the string length l_s are related to the 11 dimensional Planck length l_{pl} and the size of the 11th dimension R by the well known relations

$$l_s = (\frac{l_{pl}^3}{R})^{\frac{1}{2}} \qquad g_s = (\frac{R}{l_{pl}})^{\frac{3}{2}} \tag{4.1}$$

In [8] a correspondence principle was formulated which relates a weak coupling description in terms of a string state with a strong coupling description in terms of a classical solution. For neutral black holes the weak coupling description is in terms of a highly excited state of a single string with an entropy

$$S_1 \sim m l_s \tag{4.2}$$

while at strong coupling the black hole entropy is

$$S_2 \sim m r_0 \tag{4.3}$$

where m and r_0 are the mass and the radius of the black hole. Thus the correspondence point is

$$r_0 \sim l_s \tag{4.4}$$

At this point the curvature at the horizon is of the string scale.

For black D- P branes the stringy description is in terms of a Yang-Mills gas in $p+1$ dimensions for large α and in terms of a single string state for small α. The black brane description is given precisely by the T-dual of the metric (2.7). The correspondence point once again occurs when the string metric curvature reaches the string scale. This happens when [8]

$$r_0 \sim \frac{\tilde{l}_s}{(\cosh\alpha)^{\frac{1}{2}}} \tag{4.5}$$

where $\tilde{l}_s$ is the relevant string length.

However we saw that the D p-brane can be obtained by boosting a neutral black string along x^{11}, i.e. a $(10-p)$ dimensional Schwarzschild black hole and then performing T dualities. Thus the relevant string length $\tilde{l}_s$ in (4.5) has to be identified with the string length in the boosted frame. Because of the boost the radius of x^{11} changes from R to R', given by (2.12). As a result the ten dimensional string theory obtained after the boost has a different string length l'_s and string coupling g'_s which may be read off from (4.1)

$$\begin{aligned} l'_s &= (\frac{l_{pl}^3}{R'})^{1/2} = l_s(\cosh\alpha)^{\frac{1}{2}} \\ g'_s &= (\frac{R'}{l_{pl}})^{\frac{3}{2}} = g_s(\cosh\alpha)^{-\frac{3}{2}} \end{aligned} \tag{4.6}$$

Indeed, identifying $\tilde{l}_s$ in (4.5) with l'_s in (4.6), we see that the correspondence point of neutral black holes written in terms of the string scale in the boosted frame is precisely the correspondence point for D p-branes given by (4.5).

To write the relations in terms of the string coupling and the volume relevant to the p-brane description we perform a T-duality along the T^p directions. The new string coupling g and the volume of the brane becomes

$$
\begin{aligned}
g &= g_s'(\frac{l_s'}{L})^p \\
V &= \Sigma^p = (\frac{l_s'^2}{L})^p
\end{aligned}
\tag{4.7}
$$

For near-extremal D p-branes the stringy description is a Yang-Mills gas. The energy of the gas ΔE has to be identified with the excess energy of the black hole above extremality, which, from (2.11) is given by

$$
\Delta E = M(\frac{R'}{R}) \tag{4.8}
$$

The ratio of the entropy of the p-brane S_p to that of the YM gas S_g is

$$
\frac{S_p}{S_g} \sim [(gN)(\frac{\Delta E}{N^2 V})^{\frac{4-n}{8-n}}]^{\frac{4-n}{2n}} \tag{4.9}
$$

which can be easily derived from the formulae given in [8].

It may be easily checked using (4.7), (4.8)and (4.6) that the formulae for the energy and entropy in Section 2. are the standard formuale for D p-branes. In terms of the string coupling the correspondence point for the brane-gas transition is at $g = g_c$ where

$$
g_c N = (\frac{\Delta E}{N^2 V})^{\frac{4-n}{n-8}} \tag{4.10}
$$

However, (4.9) shows that the gas phase has more entropy for $g < g_c$ only when $n < 4$ ($p > 3$). For $n = 4$ ($p = 3$) one has $S_g \sim S_p$ for all couplings, while $n > 4$ ($p < 3$) one has $S_p > S_g$ at weak couplings. This pathological behavior for $p < 3$ is probably related to the fact that for small values of p there are strong infra-red divergences in the worldbrane theory which makes the gas picture unrealiable even at weak coupling.

On the other hand for $(10-p)$ dimensional neutral black holes the system would be in the string phase at weak couplings for all values of p. Since the black p-branes and the neutral black holes are related by a boost, it is of some interest to see what the equation of state for a string state becomes when expressed in terms of the boosted variables.

When the boost α is very large (so that the boosted black hole is near-extremal), it follows from (2.14), (2.12) and (2.6) that

$$\begin{aligned} M &\sim \frac{N}{R} \sim \frac{N}{g_s l_s} \\ \Delta E &\sim M e^{-\alpha} \end{aligned} \tag{4.11}$$

Using (4.11), (4.6), (4.7) and (4.8), the entropy of excited string state of mass M in rest frame (4.2) may be expressed as

$$S_1 \sim M l_s \sim [N^2 (\Delta E)^3 (\frac{l_s'^{3-p} \Sigma^p}{gN})]^{1/4} \tag{4.12}$$

Remarkably this is precisely the equation of state for a massless gas in 3 space dimensions with N^2 degrees of freedom and a volume

$$V = \frac{l_s'^{3-p} \Sigma^p}{gN} \tag{4.13}$$

and the result is true for all p. In the above we have expressed everything in terms of the product (gN) since this is the effective open string coupling in the p-brane.

It may be easily checked that this expression for the microscopic entropy agrees with the p-brane entropy at the correspondence point and of course for all values of p the entropy S_1 is greater than S_p at weak couplings.

We have reached an intriguing conclusion : an excited massive string state appears like a three dimensional gas in a boosted frame. In other words if we take such a massive string state and add some 0-brane excitations to it by a boost we get a three dimensional gas. The reason for this is not known to us.

It has been problematic to reproduce the correct absorption properties of black holes in the context of correspondence principle [19] [20] - at least when the naive guess about the degrees of freedom in the stringy phase (which gives the entropy correctly) is used. For black holes with two large charges, the degrees of freedom has been correctly identified in [18] and then the absorption properties follow correctly. For black holes with a single large charge this is not known yet. The above equation of state may be useful in this regard.

5. Emission and absorption using boosts

In this section we show how to use boost transformations to relate emission or absorption cross-sections for different processes.

5.1. General formalism

First we describe the general strategy. Consider a theory in $\tilde{D}$ space-time dimensions with q compact directions, one of which we will label z and consider a black hole in $D = \tilde{D} - q$ dimensions which carries a Kaluza-Klein charge coming from momentum along z. Such a black hole may be obtained by boosting (in the sense used in this paper) a neutral hole along z by some boost parameter α. The coordinate radius of the black hole is r_0.

Consider emission of a particle which is massless in the full $\tilde{D}$ dimensions with some momentum $\vec{k}$ along the $(D-1)$ transverse noncompact spatial directions and some momentum e along z, but no momentum along the other compact directions. From the D dimensional perspective this is a charged particle of charge e. The energy of the particle ω is then

$$\omega^2 = k^2 + e^2 \tag{5.1}$$

Let the absorption cross-section of the particle be denoted by $\sigma = \sigma(\omega, e; r_0, \alpha)$. If the temperature of the black hole is $T = T(r_0, \alpha)$, and $A_0 = A_0(r_0, \alpha)$ is the electrostatic potential at the horizon $r = r_0$, then the distribution function for such particles is given by

$$\rho(\omega, e; r_0, \alpha) = [e^{\frac{\omega - eA_0}{T}} - 1]^{-1} \tag{5.2}$$

The rate of emission of particles with energy between $(\omega, \omega + d\omega)$ and charge between $(e, e + de)$ is given by

$$\Gamma(\omega, e; r_0, \alpha) = \sigma \;\; \rho \;\; (\frac{d^{D-1}k}{(2\pi)^{D-1}}) \; (Rde) \tag{5.3}$$

Now boost the system further with some angle β. The boost parameter of the new black hole obtained this way is then $(\alpha + \beta)$. The particle of energy and charge (ω, e) now becomes a particle of energy and charge (ω', e'), where

$$\begin{aligned} \omega' &= \omega \cosh\beta + e \sinh\beta \\ e' &= e \cosh\beta + \omega \sinh\beta \end{aligned} \tag{5.4}$$

Let us denote the rate of emission of these particles in the boosted frame $\Gamma'(\omega', e'; r_0, \alpha+\beta)$. We want to obtain Γ' from a knowledge of Γ.

In fact, Γ and Γ' are related by time dilation factor

$$\Gamma(\omega, e; r_0, \alpha) = \Gamma'(\omega', e'; r_0, \alpha + \beta) \; \frac{\cosh(\alpha + \beta)}{\cosh(\alpha)} \tag{5.5}$$

To obtain Γ' we thus express Γ in terms of the primed variables. We make a change of variables from $(|k|, e) \to (\omega, e)$ so that

$$|k|d|k|de = \omega d\omega de \tag{5.6}$$

and write the phase space factor as follows

$$d^{D-1}kde = d\Omega_{D-2}\ |k|^{D-3}\omega d\omega de = d\Omega_{D-2}\ |k|^{D-3}\omega d\omega' de' \tag{5.7}$$

since $d\omega de = d\omega' de'$ under boosts and the transverse momentum magnitude $|k|$ is unchanged. Furthermore the new radius of the z direction after the boost, R' is related to R by

$$R = R'\ \frac{\cosh(\alpha+\beta)}{\cosh(\alpha)} \tag{5.8}$$

The potential at the horizon of the black hole before the boost is

$$A_0 = \tanh\alpha \tag{5.9}$$

whereas the temperature is

$$T = \frac{T_0}{\cosh\alpha} \tag{5.10}$$

The potential and the temperature of the boosted black hole are then

$$\begin{aligned} A_0' &= \tanh(\alpha+\beta) \\ T' &= \frac{T_0}{\cosh(\alpha+\beta)} \end{aligned} \tag{5.11}$$

It then follows from (5.4) and (5.9) and (5.11) that

$$(\omega - eA_0) = (\omega' - e'A_0')\frac{\cosh(\alpha+\beta)}{\cosh(\alpha)} \tag{5.12}$$

so that

$$\rho(\omega, e; r_0, \alpha) = \rho(\omega', e'; r_0, \alpha+\beta) \equiv \rho' \tag{5.13}$$

Thus we get, using (5.6),

$$\Gamma(\omega, e; r_0, \alpha) = \frac{\sigma(\omega, e; r_0, \alpha)}{[e^{\frac{\omega'-e'A_0'}{T'}} - 1]}\ \frac{\omega}{\omega'}\ \frac{d\Omega_{D-2}\ |k|^{D-3}\omega' d\omega' R' de'}{(2\pi)^{D-1}}\ \frac{\cosh(\alpha+\beta)}{\cosh(\alpha)} \tag{5.14}$$

Thus we get, using (5.5) and (5.7),

$$\Gamma'(\omega', e'; r_0, \alpha+\beta) = \sigma\ (\frac{\omega}{\omega'})\ \rho'\ \ (\frac{d^{D-1}k'}{(2\pi)^{D-1}})\ (R'de') \tag{5.15}$$

Comparing with (5.3) we therefore have

$$\sigma'(\omega', e'; r_0, \alpha+\beta) = \sigma(\omega, e; r_0, \alpha)\ \frac{\omega}{\omega'} \tag{5.16}$$

which gives the absorption cross-section σ' for particles of energy ω' and charge e' by the boosted black hole.

5.2. Application to five-dimensional black holes with three charges

A good check of our formalism is offered by emission of scalars from the near-extremal five dimensional black holes with three charges Q_1, Q_5, N where N is a Kaluza Klein charge and Q_1 and Q_5 are D1-brane and D5-brane charges. The 1-brane is wrapped on a circle of radius R and the 5-brane is wrapped on a T^4 times this circle. The volume of T^4 is V.

The boost parameter is then associated with the charge N which is the quantized momentum along x^5. The general answer for the low energy absorption cross-section for charged particles [21] is known to be

$$\sigma(\omega, e; r_0, \alpha) = A_H(r_0, \alpha) \ (\frac{\omega - eA_0}{\omega}) \tag{5.17}$$

Here e is the charge of the particle which is the momentum along x^5 and A_H is the area of the three dimensional horizon given by

$$A_H(r_0, \alpha) = A_H(r_0, 0) \ \cosh\alpha \tag{5.18}$$

Note that this horizon area is *not* invariant under boosts along x^5. However the entropy is invariant under boosts. This is because

$$S = \frac{A_H}{4G_5} \tag{5.19}$$

where G_5 is the five-dimensional Newton constant which is related to the ten dimensional Newton constant by

$$G_5 \sim \frac{G_{10}}{VR} \tag{5.20}$$

Thus the Lorentz contraction of the radius R results in an increase of G_5 which exactly compensates for the expansion of A_H implied by (5.18). In other words the area of the *four* dimensional horizon (which is the product of the *three* dimensional horizon with the circle along x^5) is indeed boost invariant.

Now boost further along x^5 with a parameter β. It immediately follows, using (5.16). (5.17) and (5.12) that

$$\sigma'(\omega', e'; r_0, \alpha + \beta) = A_H(r_0, \ \alpha + \beta) \ (\frac{\omega' - e'A_0'}{\omega'}) \tag{5.21}$$

which is exactly what one would expect.

In particular we can choose $e = 0$ and obtain charged emission from neutral emission without doing a separate calculation. Similarly if we know how to calculate only neutral emission from a neutral black hole ($\alpha = 0$) we could have used this formalism to calculate the emission of charged particles from a charged black hole.

The important point to realize in all this is that the relation between cross-sections derived above does not depend on *how* the cross-sections are calculated in the first place - semiclassical or microscopic. This is entirely given by the boost properties.

5.3. Application to seven dimensional Schwarzschild black holes

The above formalism may be used to obtain microscopic derivation of absorption cross-section by a seven dimensional Schwarzschild hole in string theory.

Consider a seven dimensional neutral black hole, or a black string in eight dimensions. The corresponding 11 dimensional metric is given by (2.1) with $n = 4$. This hole may Hawking radiate particles which are massless in 11 dimensions but carry some momentum in the 11th direction, i.e. 0 branes of the corresponding string theory. The classical absorption cross-section may be calculated in principle. However, *a priori* there is no obvious microscopic derivation of the absorption cross-section.

Now boost the system along x^{11} by some angle α. The black string now acquires a momentum. In the string theory language this has become a 0-brane black hole with three compact direction. The particles which are absorbed now have a different 0-brane charge determined by boost properties. After T-duality we have therefore a 3-brane absorbing 3 branes.

We may further choose the boost parameter such that the x^{11} momentum of the absorbed particles vanishes, so that after T-duality we have a 3-brane absorbing massless neutral particles.

When such particles are minimally coupled scalars, and the 3-brane is exactly extremal, the classical absorption cross-section can be exactly reproduced by a microscopic calculation in the $3+1$ dimensional Yang-Mills theory living on the brane [10]. Therefore, at least in a certain limit, we can use the boost properties of absorption cross-sections to have a microscopic derivation of absorption properties of certain "0-branes" from the seven dimensional neutral black hole.

The limit is of course peculiar, since the extremal 3-brane can be obtained from (2.1) by infinite boosting ($\alpha \to \infty$) and then taking the limit of $r_0 \to 0$ keeping

$$\lambda^4 = r_0^4 \sinh^2 \alpha \tag{5.22}$$

fixed.

To see the precise limits consider the S-wave equation of motion of a minimally coupled scalar of energy ω and charge e in the background of the original neutral solution

$$\frac{1}{r^5}\partial_r(r^5 g(r)\partial_r \chi) + \frac{\omega^2 - e^2 g(r)}{g(r)}\chi = 0 \tag{5.23}$$

where

$$g(r) = 1 - \frac{r_0^4}{r^4} \tag{5.24}$$

This scalar field has an energy ω' and charge e' after the boost. If we choose $e' = 0$ one therefore has

$$\omega = \omega' \cosh\alpha \qquad\qquad e = -\omega' \sinh\alpha \tag{5.25}$$

and the equation (5.23) becomes

$$\frac{1}{r^5}\partial_r(r^5 g(r)\partial_r\chi) + \frac{\omega'^2}{g(r)}[1 + \frac{\lambda^4}{r^4}]\chi = 0 \tag{5.26}$$

where λ is defined in (5.22).

For $(\omega' r_0) << 1$ this is precisely the equation which is solved in [10] to obtain the classical cross-section for absorption of neutral scalars by extremal 3-branes. In terms of ω, e and m this regime of parameters is

$$\omega r_0 << (1 - \frac{e^2}{\omega^2})^{-1/2} \tag{5.27}$$

This guarantees that the *classical* cross-section of our original problem is known once the classical cross-section for the three brane is known. The latter has been obtained as an expansion in the product $\omega'\lambda$. Written in terms of the original parameters this product is

$$\omega'\lambda = (\omega r_0)[(\frac{e}{\omega})^2(1 - (\frac{e}{\omega})^2)]^{\frac{1}{4}} \tag{5.28}$$

which means that $\omega'\lambda$ can be kept small and finite with $e \sim \omega$.

The fact that the cross-section for the 3-brane can be reproduced by a microscopic calculation shows that the cross-section for the neutral black hole has a microscopic derivation in the regime of parameters given by (5.27) and $e \sim \omega$.

5.4. Application to five dimensional Schwarzschild black holes

The above microscopic derivation of absorption by seven dimensional Schwarzschild black holes holds only in the limit where the 3-brane to which it is related by boosts and T-duality is *exactly* extremal. We now outline a much cleaner example where one can once again obtain an exact derivation of absorption of certain 0-branes by a five dimensional Schwarzschild black hole in ten dimensions - or a black string in eleven dimensions with five of the transverse directions compact. This is related to the well known five dimensional black hole with a 1-brane charge Q_1, a five brane charge Q_5 and some momentum along the 1-brane direction by the following series of steps

(i) Start with the 11-dimensional metric corresponding to black string along x^{11} and the directions $(x^1 \cdots x^5)$ compact.

(ii) Perform a boost along x^{11} by a parameter γ. The ten dimensional description of this is a 0-brane with five compact directions with a charge $Q_5 \sim r_0^2 \sinh(2\gamma)$

(iii) Perform a T-duality along the directions $(x^1 \cdots x^4)$. The 0 brane now becomes a 4-brane with the same charge Q_5

(iv) Consider the 11-dimensional object whose ten dimensional description is given by (iii). Perform a boost along x^{11} with some parameter α. The resulting ten dimensional object is a 4-brane with some additional 0-brane charge $Q_1 \sim r_0^2 \sinh(2\alpha)$, and a compact transverse direction x^5.

(v) Now perform a T-duality along x^5. The 4-brane becomes a 5-brane with the same charge Q_5 and the 0-brane becomes a 1-brane with the charge Q_1.

(vi) Finally perform a boost along the compact direction x^5. One now has the well known system of 1-branes, 5-branes and momentum.

Following the above chain of boosts and dualities it is easy to check that the metric obtained starting from the standard metric (2.1) is the metric of 1-brane - 5-brane system obtained in [22] and [23]. The 11-dimensional metric may be viewed as intersecting M-branes [24]. The above steps have been discussed earlier in [25] [5]. Thus the absorption of certain 0-brane charged objects by the five dimensional Schwarzschild black hole is related to absorption of neutral objects by the five dimensional black hole with three charges.

The low energy absorption cross-section for the 5D black hole with three charges is, however, *exactly* reproduced by a microscopic calculation not just in the extremal limit, but in the near-extremal situation as well [11]. In other words there is a microscopic derivation of the rate of Hawking radiation for this case. This implies that we have a microscopic derivation of Hawking radiation of related particles from a five dimensional Schwarzschild as well.

Note that the T-dualities and the boosts in the above chain do not commute. This makes the identification of the particles emitted from the Schwarzschild black hole which are related to neutral particles in the 5D black hole with three large charges rather involved. A more detailed discussion of emission amplitudes will appear in a future manuscript.

[5] We thank A. Tseytlin for pointing out these papers to us

Note added

While this paper was being typed we saw [26] which has some overlap with Section 2. of our work. Another related paper is [27]

Acknowledgements

We would like to thank A. Dabholkar and P. Joshi for discussions. S.K.R. would like to thank the Theoretical Physics Group of Tata Institute of Fundamental Research for hospitality. S.D.M was partially supported by DOE cooperative agreement No. DE-FC02-94ER40818.

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