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Estimation of hypocentral parameters of local earthquakes when crustal layers have constant *P*-velocities and dipping interfaces

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Abstract. The paper describes an algorithm for estimating the hypocentral coordinates and origin time of local earthquakes when the wave speed model to be employed is a layered one with dipping interfaces. A constrained least-squared error problem has been solved using the penalty function approach, in conjunction with the sequential unconstrained optimization technique of Fiacco and McCormick. Joint confidence intervals for the computed parameters are estimated using the approach of Bard for nonlinear problems. These results show that when a hypocentre lies outside the array of recording stations and head waves from a dipping interface are involved, then its inclination must be taken into account for dip angles exceeding 5°.

Keywords. Hypocentral parameters; dipping interfaces; sequential unconstrained optimization; penalty function approach; earthquakes.

1. Introduction

Seismicity studies provide an illuminating perspective for seismic risk assessment in specific regions of interest. These studies, in turn, require accurate and systematic mapping of hypocentral parameters of local earthquakes in space and time. Determination of the four hypocentral parameters i.e. epicentral latitude and longitude, depth and time of occurrence are made from the arrival times of seismic *P* (and *S*) waves at a number of stations. Since arrival time observations may contain some errors, a minimum of 5 station data pertaining to each earthquake are required to obtain, in some prescribed sense, an optimized estimate of its hypocentral parameters, provided of course that the distribution of *P*-wave speeds in the region is known. When this latter information is not available, one proceeds by making some arbitrary assumptions. Alternatively, the *P*-wave distribution is considered as a set of unknowns to be determined along with the hypocentral parameters. But the number of station records required for this analysis has to be adequate enough to render the inverse problem an overdetermined one.

A number of algorithms have been developed for estimating hypocentral parameters, in a horizontally layered earth, from *P*-arrival times at 5 or more local stations i.e. for ray paths short enough to justify the neglect of the earth's curvature; (Flinn 1960; Norquidst 1962; Bolt 1960; Lee and Lahr 1975; Hermann 1979). These broadly fall into two groups depending upon whether the *P*-wave speed distribution is assumed or posed as unknowns to be determined by inverting *P*-arrival times at a larger number of recording stations.

The algorithm presented in this paper enables one to determine hypocentral parameters in a layered earth even when the various interfaces have different inclinations in three-dimensional space. This would be particularly useful in application to active fold belts of recent orogenies where lithostratigraphic boundaries dip variously in different regions. The algorithm is indeed quite versatile and capable of simulating a wide variety of P-wave speed distributions over small regions of the earth. It can easily deal with a horizontally layered earth simply by setting all dip angles to zero. It also allows one to seek solutions where the crustal structure is more representative of a framework of constant speed blocks, by simulating their vertical faces with an appropriately striking boundary that has a dip of 90°. The algorithm works very well for a small number of relatively larger blocks. However, it begins to pose some difficulties, largely of a book keeping nature, for a larger number of smaller blocks.

The motivation for developing this algorithm arose from an anticipated need of having to analyse P-wave arrival times recorded in the structurally complex region of Garhwal-Kumaon Himalaya (Gaur *et al* 1985), where an array of portable seismographs had been deployed to investigate the space-time regime of local earthquakes. It was of course realized at the time of initiating this exercise that the stage when P-wave speed distributions will be actually simulated for hypocentral locations, on the level of detail actually inferred from outcropping boundaries in the Himalaya, was still far off. However, it was felt that if crustal layers were indeed found to have appreciable inclination in the Himalayan region, then they would have to be modelled with care.

Estimation of hypocentral parameters from arrival times of seismic waves, constitutes a geophysical inverse problem which requires that the solution to the corresponding forward problem be already available. The direct problem here is to obtain a formalism whereby P-wave arrival times from a given hypocentre may be calculated at a given recording station, for a given P-wave speed distribution in the region. This, in turn, requires that the ray path between the known hypocentre and the recording stations should be traceable. The ray tracing problem has been discussed by Julian and Gubbins (1977), Pereyra *et al* (1980) and Chander (1977). Here we adopt Chander's ray tracing algorithm for our formalism. Recently, Kanasewich and Chen (1985) also assessed Chander's approach and adopted it for crustal investigations in parts of Canada.

2. The algorithm

Optimized estimates of hypocentral parameters are obtained using the least-squared error criterion. Let (X_H, Y_H, Z_H) be the three spatial coordinates of the hypocentre in a convenient local coordinate system and T_H the time of earthquake occurrence; then we optimize the function $E(X_H, Y_H, Z_H, T_H)$ with respect to its arguments.

$$E(X_H, Y_H, Z_H, T_H) = \sum_{i=1}^{n} [T_{i0} - T_{ic}(X_H, Y_H, Z_H, T_H)]^2 . \tag{1}$$

Here T_{i0} is the observed arrival time at the ith station with known coordinates (X_i, Y_i, Z_i) and T_{ic} is the corresponding calculated time

$$T_{ic} = T_H + t_{ic}(X_H, Y_H, Z_H), \tag{2}$$

where t_{ic} is the P-travel time between the assumed hypocentre and the ith recording station. As indicated above, we assumed that the wave speed distribution between the hypocentre and all the stations can be simulated by constant wave speed layers with plane interfaces which may dip. The algorithm of Chander (1977) is used to trace rays between the hypocentres and the stations and thus to calculate t_{ic}'s and T_{ic}'s.

After Geiger (1912), the optimization of E in equation (1) has been traditionally conducted by first linearizing the t_{ic}'s (defined in (2)) with respect to X_H, Y_H, Z_H and estimating the hypocentral parameters iteratively (Flinn 1960; Bolt 1960; Norquidst 1962; Lee and Lahr 1975; Hermann 1979). If at each iteration the expected changes in hypocentral parameter estimates are represented by a column vector $\Delta\mathbf{X}$, then

$$\Delta\mathbf{X} = (A^T A)^{-1} A^T \mathbf{b}, \tag{3}$$

where the matrix A is composed of derivatives of T_{ic}'s with respect to hypocentral parameter estimates and $\mathbf{b}$ is the column vector involving T_{i0}'s. Both matrix A and column vector $\mathbf{b}$ are evaluated at current estimates of hypocentral parameters.

The common experience is that the matrix $A^T A$ is often near singular and the column vector $\Delta\mathbf{X}$, instead of tending to zero, grows without limit with each iteration. This arises from a number of situations notably (a) poor initial estimates of hypocentral parameters (b) majority of the recording stations being on one side of the hypocentre (c) excessive random errors in T_{i0}'s and (d) extremely unrealistic seismic P wave speed model. Thurber (1985) advocated retention of nonlinearity of t_{ic}'s and solution of conditional equations using Newton's method. Aki and Lee 1976; Hermann 1979; Hawley *et al* 1981; Koch 1985; Kanasewich and Chen 1985 followed the method of Levenberg (1944) and added a damping term λI to $A^T A$ to overcome this problem, λ being a constant and I the identity matrix. Here we adopt a constrained optimization procedure.

The search for optimized values of hypocentral parameters is constrained as follows

$$X_{H\min} < X_H < X_{H\max},$$
$$Y_{H\min} < Y_H < Y_{H\max},$$
$$Z_{H\min} < Z_H < Z_{H\max},$$
$$T_{H\min} < T_H < T_{H\max}.$$

The values of $X_{H\min}$, $X_{H\max}$, $Y_{H\min}$, $Y_{H\max}$, $Z_{H\min}$, $Z_{H\max}$, $T_{H\min}$ and $T_{H\max}$ are prescribed.

We use the penalty function approach to incorporate the constraints and pose an unconstrained optimization problem. We define

$$E^*(X_H, Y_H, Z_H, T_H) = E(X_H, Y_H, Z_H, T_H) +$$

$$\lambda \left[\frac{W_{X\max}}{X_{H\max} - X_H} + \frac{W_{X\min}}{X_H - X_{H\min}} + \frac{W_{Y\max}}{Y_{H\max} - Y_H} + \frac{W_{Y\min}}{Y_H - Y_{H\min}} \right.$$

$$\left. + \frac{W_{Z\max}}{Z_{H\max} - Z_H} + \frac{W_{Z\min}}{Z_H - Z_{H\min}} + \frac{W_{T\max}}{T_{H\max} - T_H} + \frac{W_{T\min}}{T_H - T_{H\min}} \right],$$

where λ is a weight function and $W_{X\max}$, $W_{X\min}$, $W_{Y\max}$, $W_{Y\min}$, $W_{Z\max}$, $W_{Z\min}$, $W_{T\max}$ and $W_{T\min}$ are constants to describe the relative importance of the different constraints. These constants too are prescribed.

Using the SUMT (sequential unconstrained optimization technique) of Fiacco and McCormick (1968) we solve a sequence of problems in which E^* is optimized with respect to its arguments. Always a smaller value of λ is used. The values of hypocentral parameters obtained in the preceding iteration are used as the starting values in the new iteration. In the limit, as λ tends to zero, the values of the hypocentral parameters optimizing E^* also optimize E of (1).

During each iteration, the conditional equations

$$\partial E^*/\partial X_H = 0, \ \partial E^*/\partial Y_H = 0, \ \partial E^*/\partial Z_H = 0, \ \partial E^*/\partial T_H = 0 \qquad (4)$$

are solved using the Newton-Raphson technique. Alternative procedures may be used for the purpose, e.g., the Powell's algorithm (Kanasewich and Chen 1985).

Following Flinn (1965) joint confidence intervals around the estimated hypocentral parameters are also estimated. However, while Flinn adopted Geiger's method of linerization, we follow Bard (1974) whose procedure permits retention of the nonlinearity.

3. Tests of the algorithm

The algorithm has been tested extensively by (i) using the synthetic data generated by us (table 1a and 1b); (ii) using our algorithm on the synthetic data set provided by Lee and Lahr (1975) in their manual on HYPO 71 and also by comparing the results of our algorithm and HYPO 71 for other synthetic data sets generated by us. The results of some of these tests are reported below.

(i) A plane with equation $-0{\cdot}02X - 0{\cdot}02Y + Z - 5{\cdot}0 = 0$ separating a region of P-wave speed 3 km/sec which overlies a region of P-wave speed 5 km/sec is taken

Table 1a. Coordinates of recorder positions and corresponding P-wave arrival time data.

Recorder coordinates	Arrival time data
$S_1(3{\cdot}404, 3{\cdot}404, 0{\cdot}0)$	$0^h\ 0^m\ 7^s{\cdot}142$ (P)
$S_2(68{\cdot}079, 2{\cdot}861, -200{\cdot}0)$	$0^h\ 0^m\ 77^s{\cdot}108$ (P)
$S_3(-35{\cdot}545, 55{\cdot}969, -100)$	$0^h\ 0^m\ 46^s{\cdot}446$ (P)
$S_4(1{\cdot}066, 5{\cdot}782, 0{\cdot}0)$	$0^h\ 0^m\ 7^s{\cdot}366$ (P)
$S_5(-4{\cdot}473, 5{\cdot}614, -5{\cdot}0)$	$0^h\ 0^m\ 12^s{\cdot}289$ (S)

as the seismic velocity model. The hypocentre is assumed to be in the higher velocity region. Table 1a gives details of the five assumed recorder coordinates and the arrival time data at these recording stations. Table 1b gives the starting and final values of the hypocentral parameters of the program well as the (true) assumed hypocentral parameters.

The poor agreement in Z-coordinate is because all the stations are on one side of the hypocentre.

(ii) Table 2 displays the results of our program and those of HYPO 71 using P-wave arrival time data and wave speed model provided by Lee and Lahr (1975).

On 28 December 1979 an earthquake occurred within the recording array operated by our group in the vicinity of the Main Central Thrust in the Garhwal Himalaya. Hypocentral parameters of this earthquake were also estimated by NOAA using regional permanent stations. Our results are compared with those of NOAA for this earthquake (table 3).

Table 1b. Starting, final and assumed value of hypocentral parameters.

Starting value of hypocentral parameters	Final value of hypocentral parameters	Assumed value of hypocentral parameters
$X = 0{\cdot}5$	$X = 1{\cdot}07$	$X = 1{\cdot}00$
$Y = 0{\cdot}5$	$Y = 0{\cdot}99$	$Y = 1{\cdot}00$
$Z = 4{\cdot}0$	$Z = 3{\cdot}92$	$Z = 6{\cdot}00$
$t = 0^h 0^m 4^s{\cdot}0$	$t = 0^h 0^m 4^s{\cdot}97$	$t = 0^h 0^m 5^s{\cdot}00$

Table 2. Comparison of hypocentral parameters estimated by HYPO 71 and our algorithm.

Hypocentral parameters	HYPO 71	Our algorithm
Latitude	38°29′·53N	38°29′·05N
Longitude	122°42′·08W	122°41′·94W
Depth	3·85 km	1·95 km
Origin Time	$12^h 6^m 44^s{\cdot}56$	$12^h 6^m 44^s{\cdot}23$

Table 3. Comparison of hypocentral parameters estimated by NOAA and our algorithm.

Hypocentral parameters	NOAA	Our algorithm
Latitude	30°·628N	30°·822N
Longitude	78°·445E	78°·521E
Depth	33 km	15·531 km
Origin Time	$1^h 59^m 18^s{\cdot}8$ GMT	$1^h 59^m 18^s{\cdot}406$ GMT

The epicentre is thus found to shift 15·9 km to the NE using local data. Also a definite estimate of the focal depth (15·531 km) has now been obtained using local network data in contrast to the routine value of 33 km as assigned by NOAA.

We are thus led to conclude that our algorithm and the computer program based on it are working satisfactorily.

4. Results

The algorithm has been used to analyse the actual data obtained in the Garhwal Kumaon Himalaya (Gaur *et al* 1985). It may be mentioned that for the vast majority of local earthquakes whose data were reported in that study, a uniform half-space wave speed model was adapted and the ray tracing facility of our algorithm was not needed. However, as estimates of focal depths for a number of earthquakes turned out to be in excess of 70 km, the data were reanalysed with a layered wave speed model and making use of the ray tracing facility of the algorithm. It has also been used to analyse synthetic data with a view to investigate several questions of theoretical nature pertaining to hypocentral parameter estimation. One of these questions is to decide the angle of interface dip at which it is desirable to abandon the use of flat-layered P-wave speed simulation of the earth and adopt a dipping interface simulation. In its simplest form the problem can be posed and examined using synthetic data for a one-layer over a half-space P-wave speed model in which the top surface of the layer is horizontal and the bottom interface dipping.

The stations (S_1, S_2, S_3, S_4, S_5) in figure 1 represent in a schematic fashion one of the arrays used by Gaur *et al* (1985) to record microearthquakes in the Garhwal Kumaon Himalaya. The dashed MSL surface implies that the stations are situated above it at varying heights up to 1·5 km. The maximum array dimension is 45 km. According to the geotectonic model of Seeber and Armbruster (1981) for this section of the Himalaya, a wedge of metasedimentary rocks is resting on Indian shield material. We assume that the contact between the two is planar and dips in the NE direction. The P-wave speed in the sedimentary wedge is 5·2 km/sec after Chander *et al* (1986) while that in shield material is taken to be 6·2 km/sec after Ni and Barazangi (1983). Using synthetic data, we estimate the order of error incurred in hypocentral location if the interface is assumed to be horizontal while it actually has dips of 5°, 10° and 15°.

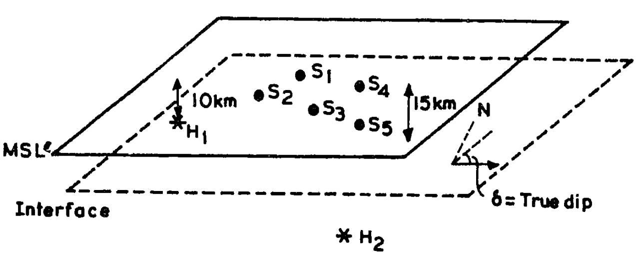

Figure 1. Schematic diagram showing recorder positions in the seismic array and the hypocentres H_1 and H_2. MSL is mean sea level. Interface dips as shown by true dip vector.

The data displayed in tables 4 and 5 refer to two situations, one in which the epicentre is situated outside the array and the other in which it is inside the array. In this former case, H_1 (figure 1) is taken sufficiently far away so that the first arriving P waves at every station are head waves.

5. Discussion of results

We start by commenting on the errors in hypocentral parameter estimates when the correct dip model is used (see columns 2 in tables 4 and 5). They are ascribed firstly to round off errors in prescribing observed (actually synthetic in this hypothetical situation) arrival times. Secondly, although the rays leave the hypocentre in the same direction, downward in case of H_1 and upward in case of H_2, the distance travelled along rays is much greater in the former case and so are the hypocentral shifts i.e. the respective distances between the estimated and known exact hypocentres.

In the no dip model calculations (see column 3 in tables 4 and 5) while both the above types of errors are present, the errors arising from the neglect of dip worsen with increase in dip of the interface in the actual model.

We conclude from this experiment that the effect of dip is much more pronounced when the epicentre is outside the array and head waves are involved. In this case, if the aim is to relate specific earthquakes to specific tectonic features

Table 4. Hypocentral parameter estimates when earthquake occurs outside the seismic array.

Earthquake occurs at H_1 in figure 1
Correct values X_H = 200·0 km, Y_H = −130·0 km, Z_H = 10 km, T_H = $0^h 0^m 0^s \cdot 0$

Apparent dip of interface		Parameter estimates: With corect dip model	With zero dip model
15°	X_H	199·7	191·6
	Y_H	−128·6	−137·4
	Z_H	6·7	9·6
	T_H	$0^h 0^m 0 \cdot 4^s$	$0^h 0^m 0^s \cdot 1$
	Hyp shift	3·6 km	11·2 km
10°	X_H	199·6	193·5
	Y_H	−128·7	−135·0
	Z_H	6·7	9·6
	T_H	$0^h 0^m 0^s \cdot 4$	$0^h 0^m (-0^s \cdot 3)$
	Hyp shift	3·4 km	8·2 km
5°	X_H	199·4	196·2
	Y_H	−128·8	−131·7
	Z_H	6·6	9·6
	T_H	$0^h 0^m 0^s \cdot 4$	$0^h 0^m (-0^s \cdot 1)$
	Hyp shift	3·6 km	4·2 km

Table 5. Hypocentral parameter estimates when earthquake occurs within the seismic array.

Earthquake occurs at H_2 in figure 1
Correct values $X_H = 120{\cdot}0$:, $Y_H = -60{\cdot}0$, $Z_H = 30{\cdot}0$, $T_H = 0^h 0^m 0^s$

Apparent dip of interface		Parameter estimates	
		With correct dip model	With zero dip model
15°	X_H	120·0	118·8
	Y_H	−60·0	−60·6
	Z_H	30·0	31·0
	T_H	$0^h 0^m 0^s{\cdot}5$	$0^h 0^m 0^s{\cdot}0$
	Hyp shift	0·0 km	1·7 km
10°	X_H	120·0	119·2
	Y_H	−60·0	−60·4
	Z_H	30·0	29·8
	T_H	$0^h 0^m 0^s{\cdot}5$	$0^h 0^m 0^s{\cdot}4$
	Hyp shift	0·0 km	0·9 km
5°	X_H	120·0	119·6
	Y_H	−60·0	−60·2
	Z_H	30·0	31·0
	T_H	$0^h 0^m 0^s{\cdot}5$	$0^h 0^m 0^s{\cdot}1$
	Hyp shift	0·0 km	1·1 km

(such as fault surfaces in 3D and fault traces in outcrops) then even 5° interface dip may be too significant to be ignored.

6. Discussion of the hypocentral parameters estimation algorithm

At this writing, hypocentral parameters of more than 500 earthquakes occurring in the Garhwal Kumaon Himalaya have been estimated using this algorithm. Between 150 and 200 synthetic data sets have been analyzed. We find that the constraints $Z_{H\min}$, $Z_{H\max}$ and $T_{H\max}$ are the most critical. A flag is put up during computations if a constraint is violated. The corresponding parameter is brought into the feasible region and the computations resume. But repeated violation of a constraint warn the operator that the prescription of that constraint needs scrutiny. When a converged constrained solution is obtained, a final run is carried out without any constraint ($\lambda = 0$ in equation (2)).

A somewhat unsatisfactory feature of the computer program as in current use is when for an assumed hypocentral location, a station is situated close to the crossover distance for direct and head waves. Then the ray path has to be changed from direct to critically refracted or vice versa and a fresh computation has to be started. But this is in no way an insurmountable programing problem. Otherwise we find that we have a well-tested and satisfactorily working algorithm and computer program.

7. Conclusion

A number of algorithms have been reported already in the literature for estimation of hypocentral parameters of local earthquakes using *P*-arrival time data. But in this domain a particular niche had remained vacant. It pertained to the need for an algorithm capable of handling of models with constant wave speed layers separated by dipping interfaces. That niche is now filled.

Within the overall framework of a least-squared error procedure whose tendency for instability is curbed through the use of constraints incorporated into the objective function via penality functions, estimation of hypocentral parameters in quite general wave speed models is achieved using a variational ray tracing procedure. Joint confidence intervals around the estimated hypocentral parameters are also computed.

Use of such an algorithm is indicated when the interface dips exceed 5°.

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