ON ELEMENTARY HEAVY PARTICLES WITH ANY INTEGRAL CHARGE

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It is well known that all quantised field theories lead to divergent results in higher approximations. In the quantised theory of the charged meson field and its interaction with the heavy particles these divergences are even greater than they are in the quantum theory of the electron. This difference is due to two causes. First, the meson field has an interaction with heavy particles which, in addition to the nuclear charge g_1 which corresponds to the electric charge e, contains an explicit dipole moment g_2. Secondly, the meson field may itself be electrically charged, so that electric charge may leave a heavy particle and transfer itself to the meson field or *vice versa*. As a result of recent work the position has become much clearer with regard to these divergences, and it is possible to distinguish the different causes leading to divergent results and to discuss each of them separately.

In the quantum theory of the electron the second order perturbation of the energy due to the interaction of the electron with the electromagnetic field is infinite, and this has usually been interpreted as the quantum mechanical equivalent of the classically infinite energy of a point charge. I believe that this interpretation must now be rejected, since the work of Dirac (1938) and Pryce (1938) has shown that a complete classical theory for *point* charge can be built up entirely free from all difficulties about infinite self-energies.

The difficulties in the classical theory arise by first letting the charge occupy a certain finite volume and then considering a point charge as the limit of this distribution when the volume is made to tend to zero, the amount of charge inside the volume remaining constant. The work done against electrostatic forces in compressing a charge which was originally distributed over a finite volume into a point is naturally infinite. The usual infinite energy of the field surrounding a point charge is but another result of looking at a point charge in this way as the limit of a continuous distribution. This becomes clear at once if we consider the way in which the concept of field energy arises. By the use of Maxwell's equations, the potential energy of any *non-singular* distribution of charge can be transformed into an integral over the whole of space occupied by the field, and can hence be regarded as the energy of the field. *But this transformation cannot be performed if point charges are present from the beginning.*

There is, however, no reason why a point charge should be regarded as the limit of a finite distribution. One may simply regard the point charges in Nature as given. The only idea of energy which now has a physical meaning is the mutual energy of the point charges. Pryce (1938) has shown that it is possible in a relativistically invariant way to regard this potential energy as the energy of the field. The tensor giving the energy of the field at every point is now naturally different from the usual tensor.

Another approach to the same problem is that of Dirac, and leads to the same result. When a point charge is present, one does not concern oneself with the total energy of the field, but with the way this energy changes. More precisely, one considers the rate at which energy flows out of a small tube surrounding the world line of the point charge. It is then found that those terms which become infinite when the radius of this tube is made to tend to zero are all perfect differentials, *i.e.*, that this singular flow of energy out of the tube can be regarded as the change of some quantity which depends only on the state of motion of the point charge. This quantity returns to its original value when the point charge returns to its original state of motion. Thus, the singular part of the flow of energy out of the tube is reversible, and does not represent radiation in any real sense. Only for this reason can it be subtracted, as has been done by Dirac. Indeed, it can be shown that the new definition of field energy as given by Pryce leads at once to the classical equation of Dirac, and conversely, the equation of Dirac leads to Pryce's definition of field energy.

The great advantage of the classical equation of Dirac is that it may be regarded as *exact*, since it takes full account of the reaction of the emitted radiation on the motion of the point charge.

This radiation reaction had already been derived by Lorentz by considering a finite model of the electron. In this theory, as is well known, the term representing the electrostatic energy of the charge could be identified with the mass of the electron. The 'radius' of the electron was, in fact, *defined* as that radius for which its electrostatic energy became equal to the observed mass. Besides the above two terms there were others in Lorentz's theory which do not appear in the equation of Dirac since the electron is treated as a point. The difficulties in the way of the Lorentz model of the electron are well known. The idea of a rigid body is contrary to the theory of relativity. To fix the shape of the body in a relativistically invariant way, one would have to introduce another field besides the electromagnetic field. Indeed, there is no reason in Lorentz's theory of the electron why the charge should continue to remain in its initial configuration at all. There are therefore great advantages in regarding the electron as a mathematical point charge. It is far simpler than any other rigorous solution of the problem. Moreover, what reasons there may once have been for regarding the mass of the electron as electromagnetic in origin have now completely disappeared since the discovery of the meson a particle of the same charge but nearly a hundred and seventy times the mass of the electron.*

We may then sum up the position with regard to divergences in the quantum theory of the electron in the following way. The terms which should express the effects of the reaction of radiation on the motion of the electron are absent in the quantum theory. The effect of these terms would be to diminish the motion of the electron, as they do in the classical theory, whenever the frequency becomes very high. The well-known divergences in the quantum theory are then to be attributed *not* to the fact that we are dealing with a point charge, but *to the neglect in this theory of the effects of radiation reaction.* The reason for going into this point in such detail will appear presently.

Coming now to the meson field and its interaction with heavy particles, we can consider each of the differences from the quantum theory of the electron separately. Let us consider first a neutral meson field. It has already been proved (Bhabha, 1939) that the finite rest mass of the meson introduces no fundamental differences from Maxwell theory, either in the quantum or the classical treatment. It was shown there that we can at any stage let the meson mass μ tend to zero and pass over mathematically to the Maxwell theory. The only differences introduced by the finite mass of the meson are for energies smaller than or comparable with μc^2 while for higher energies the formulæ approach ever more closely those of radiation theory.

* This is supported by the calculation of the second order contribution to the self-energy of the electron by Weisskopf on the basis of Dirac's positron theory. For an electron at rest, this self energy diverges and may be written

$$E \sim 2 \frac{e^2}{\hbar c} mc^2 \log \frac{k}{mc}.$$

In view of the arguments given in the text this divergence would disappear if radiation damping had been taken into account. We might put $k \sim 137\, mc$, being the limit at which radiation damping becomes important for a point charge, or even $k \sim \sqrt{3 \times 137}\, mc$ (Bhabha, 1940 *a*, *b*) being the limit at which radiation damping becomes important for a spinning point dipole. In either case, the self energy is small compared with mc^2.

The g_1 interaction, which we shall call the " mesic charge " is exactly equivalent to the electric charge e in the Maxwell theory. What we have said above about the divergences in electron theory then applies in every particular to this term.

The term corresponding to the g_2 interaction, which we shall describe as a " mesic dipole ", is absent from electron theory, although there is mathematically no reason why it should not appear there. In the quantised theory of neutral mesons this term is known to lead to greater divergences than the g_1 term, but an exact classical treatment has shown that there are really no divergence difficulties with this term either. It will be proved in a paper with Corben that the singularities in the field energy of a point dipole can be dealt with exactly as in the case of a point charge. As the classical theory shows quite clearly (Bhabha, 1940 *a*, *b*) the greater divergences in the quantum theory for this term are entirely due to the *greater importance of radiation reaction for this term than for the* g_1 *term.* This radiation reaction can be taken account of classically, but has so far been neglected in the quantum theory. Moreover, a comparison with the classical theory shows that due to the *empirical magnitude* of the constant g_2, this radiation reaction becomes important at comparatively low energies, namely, $\hbar\omega \gtrsim \hbar\sqrt{3\hbar/2\, g_2^2} \sim 3\,\mu c^2$. The rapid increase of various cross-sections above this frequency, and the apparent appearance of multiple processes due to the g_2 term are then to be regarded as entirely spurious, for a proper consideration of radiation reaction would do away with them. *They have also nothing to do with the largeness of* g_2. Indeed, it is interesting to note in this connection that in the *exact* classical theory *the* g_1 *and* g_2 *terms give almost the identical cross-sections for the scattering of neutral mesons of very high frequency* ω *by heavy particles.* Due to the g_1 term this cross-section is $6\,\pi/\omega^2$ (Bhabha, 1939), and due to the g_2 term it is $4\,\pi/\omega^2$ (Bhabha, 1940 *a*, *b*). For the purposes of this note, then, we shall put $g_2 = 0$ whenever we have to deal with very high energies, for the exact classical theory shows that the effect of this term will at most be of the same order of magnitude as that of the g_1 term.

The above considerations show that at least for neutral mesons a proper consideration of radiation reaction would not only remove all divergences, but cut down the cross-sections of various processes at high energies. It is also clear that at least for neutral mesons no Heisenberg explosions and multiple processes will appear.

We are now in a position to discuss the last and perhaps the most important difference from radiation theory, namely that the meson field itself may carry electric charge. Here one must distinguish two causes. All those difficulties and divergences which appear in the quantum theory of neutral mesons (but not in the classical theory) will appear for charged mesons also. The solution of these in the quantum theory must be the same as for neutral mesons. But besides these, the theory of charged mesons leads to divergences and other difficulties such as unduly large scattering cross-sections which do not appear for neutral mesons. The purpose of this paper is to try and remove these.

Now, there are very great difficulties in the way of a classical theory of a charged meson field. The electric charge density of the meson field is (Bhabha, 1938)

$$\sigma_0 = ie \sum_{k=1}^{3} (\overline{U}_k G_k - U_k \overline{G}_k)$$

where U_k are the quantised vector potentials of the meson field, $\overline{U}_k$ the conjugate complex quantities, and $\overline{G}_k$ and G_k the momenta conjugate to U_k and $\overline{U}_k$ respectively. In a classical theory all the above quantities would commute. To prevent the above expression for the electric charge density of the field emitted by a heavy particle from vanishing identically, the "mesic" charge and current of the heavy particle and its dipole moment would have to be complex quantities. It is difficult to see what physical meaning could be attached to a complex quantity in a classical theory.

I think it is only to be regarded as satisfactory that a classical theory of a charged meson field does not seem possible, for in a classical theory the jump of a heavy particle from the proton to the neutron state would have to be replaced by a continuous process in which the electric charge gradually left the heavy particle and spread into space in the form of a charged meson field. Now if we regarded the electric charge, while attached to the heavy particle as concentrated in a point, then its field energy would *in reality* be infinite, for it would be infinitely higher than the state in which the same charge was spread over a finite volume in the form of the meson field. As can be seen at once from the previous discussion, the subtracting of the infinite field energy of a point charge is only possible if we regard the point charge as given and existing unchanged, for then this infinite field energy plays no part in any physical phenomenon. This would no longer be the case in a theory such as we are discussing now, and we should have to look at the electric charge when attached to the heavy particle as spread over a finite volume. The difficulties in the way of treating such a picture of a charged particle have already been discussed.

The above difficulty, however, does not exist for a quantised meson field. For now, although when a proton turns into a neutron the electric charge becomes distributed over a finite volume as a charged meson field, this distribution is only statistical, and we may still regard the electric charge as always concentrated in a point. This is obvious if we consider the measurement of the electric charge carried by the meson field in any finite volume. As is well known, the result of such a measurement is that the charge is a positive or negative integral multiple of e, and in the present example it would be just e or 0 if the volume in which the measurement is made were sufficiently large. Thus in a quantised theory in which an electrically charged meson field could be radiated by the heavy particles there does not appear to be this difficulty in regarding the electric charge as always concentrated in a point.

The above considerations at least make it plausible that a solution of those specific difficulties which occur in the theory of charged but not neutral mesons and their interaction with heavy particles, is only to be found on a quantum mechanical basis. The most serious of these difficulties is that the cross-section for scattering of charged longitudinal mesons by heavy particles is of the order (Heitler, 1938 ; Bhabha, 1938)

$$4\pi \left(\frac{g_1^2}{\mu c^2}\right)^2 \frac{p^4}{E^2 \mu^2}.$$

Here p is the momentum of the meson, and E its energy. It continues to increase quadratically with the energy. The corresponding cross-section for neutral mesons is zero if the meson be longitudinal, and of the order (Bhabha,

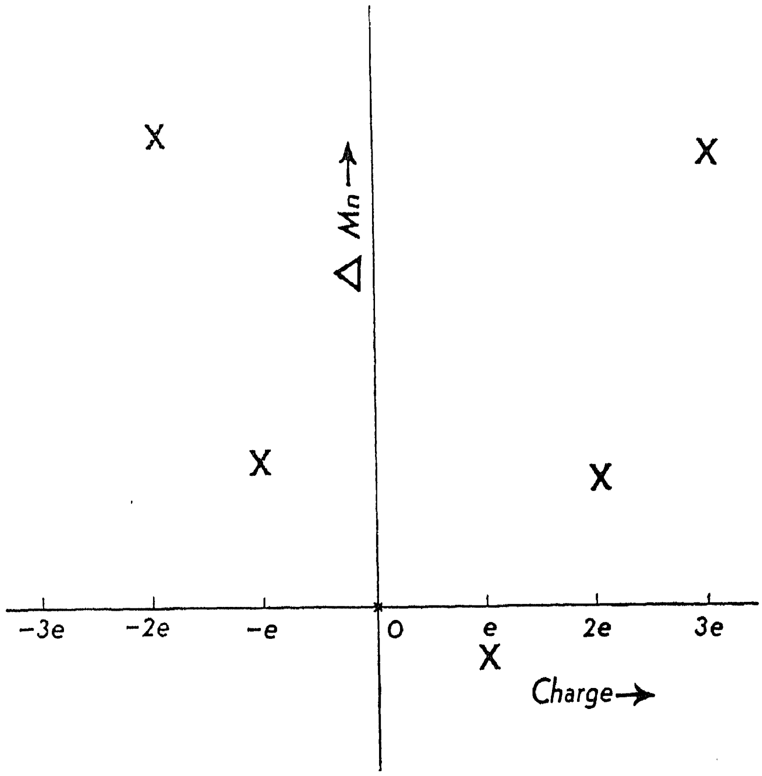
$$\frac{8\pi}{3} \left(\frac{g_1^2}{Mc^2}\right)^2$$

1939) if the meson be transverse. Even in the quantum theory, this cross-section does not increase with the energy, but for very large $h\omega \gtrsim Mc^2$ when quantum effects set in, it actually decreases in analogy with the Klein-Nishina formula, as shown by Wilson and Booth. The reason for this difference has been discussed at length in a previous paper (Bhabha, 1939). As mentioned there, the scattering is a second order process and there are only half as many intermediate states for a charged as for an uncharged meson. For example, for the scattering of a positive meson by a neutron, the process can only take place in one way. The meson is first absorbed by the neutron which turns into a proton. It is then re-emitted. For the scattering of a neutral meson, there are two ways. The neutron could either absorb the original meson first and then emit the scattered meson, or alternately, it could emit the scattered meson first and then absorb the original one. To avoid this difference I suggested that we might suppose the heavy particles to exist in all states of positive and negative charge, of which only the two with the lowest rest mass, namely, the proton and the neutron occur with any frequency in nature. This assumption not only reduces the scattering, but it puts the theory of charged mesons on the same footing as the quantum theory of neutral mesons.

This is the central idea of this paper, and we may formulate it thus. *A heavy particle can exist in any state with an electric charge ne where n is a positive or negative integer, or zero, the rest mass in the corresponding state being* M_n. Denoting by ΔM_n the difference between this mass and the mass of the neutron,

$$M_n = M_0 + \Delta M_n,$$

we must assume in order to account for the fact that so far only the proton or neutron have been observed to occur in Nature that $\Delta M_n c^2$ is positive and larger than about 15 million e-volts for all values of n except 1. We might suppose that ΔM_n increases rapidly with increasing n forming a curve something like a parabola with its cavity turned upwards. The figure schematically represents ΔM_n as a function of n.



This idea can at once be incorporated into the theory of charged mesons, even in the relativistic case, by a slight modification of the present formalism. The operators τ_{PN} and τ_{NP} which change a neutron into a proton and *vice versa*, have now to be replaced by τ_+ and τ_-, the former leading to an increase in the electric charge of the heavy particle by one unit and the latter to its decrease by one unit. Heitler (1940) has recently used this idea to recalculate the cross-section for the scattering of charged mesons. The main result of this calculation is that ΔM_2 and ΔM_{-1} have to be taken to be approximately 35 mc^2 or 17 million *e*-volts. This is high enough to prevent the proton of charge $2e$ or $-e$ from occurring as a constituent of nuclei.

It is clear that the proton of charge $-e$ is not the " anti-proton " since its mass is very appreciably different from the mass of the proton. Since the existence of an anti-proton of exactly the same mass as a proton is a consequence of the Dirac equation the acceptance of the above idea would probably necessitate rejecting the Dirac equation for a relativistic theory of heavy particles, for it is improbable that a heavy particle could have two states both of charge $-e$ with masses differing by about 17 M.e-V.

Now if these postulated states merely remained as hypothetical intermediate states, they would be uninteresting. However there are a number of processes by which a proton of charge $2e$ or $-e$ could be produced in the free state. The circumstances under which we should expect these particles to be produced will form the investigations of this paper. It will appear that although the chance of their being seen is small, so that they may have escaped observation so far, it is not so small as to make their observation very difficult if a proper search is made to this end. This position is satisfactory, for it allows one to decide by reference to experiment if the idea postulated above is correct or not. The purpose of this note is to draw attention to this possibility so that a proper search may be made for these particles.

Disintegration of Protons of Higher Charge

It is clear that since the rest mass of a heavy particle in a state of charge other than 0 or e is much greater than in the proton or neutron state, all such states will be unstable as a result of β-disintegrations.

The calculation of the life time can be carried out sufficiently well on the original Fermi theory (1934). We give only the result here as the calculation is straightforward. Since ΔM is of the order $35\,m$, the recoil momentum P of the initially stationary heavy particle is small compared to Mc, M being the approximately equal mass of the proton or neutron. The heavy particle then only takes up momentum but negligible kinetic energy. We find for the rate of disintegration in which the β-particle emerges with momentum between p and $p + dp$ the expression

$$\frac{64\,\pi^4\,g^2}{h^7}\;(\Delta M - E)^2\,p^2\,dp.$$

This is but a special case of Fermi's expression for the β-decay, where we have neglected the influence of the charge of the heavy particle on the wave function of the β-particle. This is justified in our case as the heavy particle charge is of the order e. Here g is Fermi's constant, and has the value

$$g \sim 4 \cdot 10^{-50} \text{ erg cm.}^3$$

Integrating this rate of disintegration over all momenta of the emerging β-particle, we find that the life term τ of the heavy particle is given by

$$\frac{1}{\tau} = \frac{64\,\pi^4\,\delta^2\,m^5\,c^4}{h^7}\left[-\frac{\eta_0}{4} - \frac{\eta_0^3}{12} + \frac{\eta_0^5}{30} + \frac{\sqrt{1+\eta_0^2}}{4}\log\{\eta_0 + \sqrt{1+\eta_0^2}\}\right],$$

where $mc\eta_0$ is the maximum momentum of the β-particle given by

$$\eta_0 = \sqrt{\left(\frac{\Delta M}{m}\right)^2 - 1} \approx \frac{\Delta M}{m}.$$

The expression outside the square brackets is of the order $9 \cdot 2 \times 10^{-6}$ sec.$^{-1}$ For large ΔM the above expression can be written

$$\frac{1}{\tau} \approx 3 \cdot 08 \times 10^{-7}\left(\frac{\Delta M}{m}\right)^5.$$

Putting $\Delta M/m \sim 35$ we find

$$\tau \sim \tfrac{1}{6} \text{ sec.}$$

This is quite long enough for the heavy particle of higher charge to be observed in a Wilson chamber if it emerged as a free particle.

Collision of Heavy Particles

We now consider the cross-section for the following processes which may lead to the creation of protons of charge $2e$ or $-e$ if sufficient energy is available.

$$\begin{aligned} P + P &\rightarrow P_2 + N \\ N + N &\rightarrow P_{-1} + P \end{aligned} \qquad (1)$$

P denotes a proton, N a neutron, and P_2 and P_{-1} the proton of charge $2e$ and $-e$ respectively. We shall restrict ourselves to the case where the heavy particles may be treated non-relativistically.

This process, like all those involving two heavy particles, can take place in two ways. First, it may take place by the intermediate emission or absorption of a charged meson. The interaction for this has been given by several authors.* We will take it in the form given by $\mathcal{J}_0$ equation (58 *a*) of my paper (1938). The only alteration to this interaction in the present calculation is to replace the operators τ_{PN} and τ_{NP} by τ_+ and τ_- respectively.

* Kemmer (1938); Fröhlich, Heitler and Kemmer (1938); Bhabha (1938) called A in this paper; Yukawa, Sakata and Takitani (1938).

Secondly, there may also be a direct interaction between two heavy particles in the form of δ-functions. These must be so chosen as to exactly cancel the δ-functions that appear in the interaction between two heavy particles through the meson field. As shown in A, this can be done exactly in the relativistic case. With the above expression for $\mathcal{J}_0$ this δ-function interaction in the non-relativistic approximation for the heavy particles to which we restrict ourselves here may be written*

$$D = 4\pi\ (\tau_+^{(1)}\tau_-^{(2)} + \tau_-^{(1)}\tau_+^{(2)}\ [g_1^2 + g_2'^2\ (\sigma^{(1)}, \sigma^{(2)})] \left(\frac{\hbar}{\mu c}\right)^2 \delta\ (\mathbf{X}_2 - \mathbf{X}_1) \qquad (2)$$

where $\sigma^{(1)}$, $\sigma^{(2)}$ are the three Pauli matrices for the two heavy particles 1 and 2, and $\mathbf{X}_1$, $\mathbf{X}_2$ their co-ordinates. The meson mass is denoted by μ, and $g_2' = g_2\ \mu c/\hbar$.

Let one of the particles be initially at rest, and the other move with momentum p_0 and kinetic energy E_0. Let this latter particle be scattered through an angle θ and let its final momentum be p_f, so that $(\mathbf{p}_f, \mathbf{p}_0) = \cos\theta$. We denote by M the approximately equal masses of the proton and neutron, and just write ΔM in place ΔM_2 and ΔM_{-1}. The total final kinetic energy is then

$$E_{tot} = \frac{p_f^2}{2(M+\Delta M)} + \frac{(\mathbf{p}_0 - \mathbf{p}_f)^2}{2M} \approx \frac{p_f^2}{2M} + \frac{(\mathbf{p}_0 - \mathbf{p}_f)^2}{2M}. \qquad (3)$$

Conservation of energy demands that

$$\frac{p_0^2}{2M} = \frac{p_f^2}{2M} + \frac{(\mathbf{p}_0 - \mathbf{p}_f)^2}{2M} + \Delta M c^2. \qquad (4)$$

from which it follows that

$$p_f = \tfrac{1}{2}\, p_0 \cos\theta \pm \tfrac{1}{2}\, \sqrt{p_0^2 \cos^2\theta - 4M\,\Delta M c^2} \qquad (5)$$

Thus for every scattering angle θ there are *two* values of the momentum possible. The maximum scattering angle θ is given by

$$\cos^2\theta_{\max} = \frac{4M\,\Delta\,Mc^2}{p_0^2} = \frac{2\,\Delta Mc^2}{E_0}. \qquad (6)$$

Thus this process will not take place unless

$$E_0 \geqslant 2\,\Delta M c^2.$$

This is important, for it means that if $\Delta Mc^2 \sim 17$ M.e-V., one of the heavy particles must have a kinetic energy of some 35 M.e-V. before the process becomes possible.

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* The constants g_1 and g_2' have the dimensions of a charge, and differ from those of A by a factor $1/\sqrt{4\pi}\,\hbar c$.

The differential cross-section for the scattering process in which the heavy particle is scattered into a solid angle $d\Omega$ in the direction θ is as usual

$$dQ = \frac{4\pi^2}{h} \,|\,N\,|^2 \frac{p_f^{\,2}\, Mc}{h^3\, p_0} \left(\frac{dp_f}{dE_{tot}}\right) d\Omega, \tag{7}$$

where the matrix element N is given by

$$N = (f\,|\,D\,|\,i) + \sum_m \frac{(f\,|\,\mathcal{J}_0\,|\,m)\,(m\,|\,\mathcal{J}_0\,|\,i)}{E_i - E_m}. \tag{8}$$

Here i, m and f denote the initial, intermediate and final states of the system of heavy particles plus the meson field and E_i and E_m the total energies of the whole system in these respective states. The matrix elements $(f\,|\mathcal{J}_0|\,m)$, $(m\,|\,\mathcal{J}_0\,|\,i)$ are exactly the same as before. The only difference between this calculation and the previous one of the neutron proton scattering is in the denominators $E_i - E_m$. There are now two intermediate states m.

$$P^{(1)} + P^{(2)} \rightarrow \begin{Bmatrix} P_2^{(1)} + Y^- + P^{(2)} \\ P^{(1)} \;\;+ Y^+ + N^{(2)} \end{Bmatrix} \rightarrow P_2^{(1)} + N^{(2)}. \tag{9}$$

For the top intermediate state we have

$$E_i - E_m \approx -E - \Delta Mc^2, \tag{10 a}$$

where E is the energy $c\sqrt{\mu^2c^2 + p^2}$ for the meson Y^- in the intermediate state. Its momentum is given

$$\mathbf{p} = \mathbf{p}_0 - \mathbf{p}_f. \tag{11}$$

For the other intermediate state

$$E_i - E_m \approx -E. \tag{10 b}$$

Using the expression (58 *a*) of A for $\mathcal{J}_0$ and the expression (2) above we easily find as in A that N can be written as the matrix element $(f\,|\,W\,|\,i)$ of a function W, where W is given by

$$W = \sum_p \left(\frac{\hbar}{\mu c}\right)^2 \left([g_1^{\,2} + g_2^{\prime 2}\,(\sigma^{(1)}, \sigma^{(2)})] - \frac{c^2 p_2}{E^2}\,[g_1^{\,2} + g_2^{\prime 2}\,(\sigma_{1p}^{(1)}\sigma_{1p}^{(2)} + \sigma_{2p}^{(1)}\,\sigma_{2p}^{(2)})] \right.$$

$$\left. \times \left\{\frac{E + \Delta Mc^2/2}{E + \Delta Mc^2}\right\}\right) e^{\frac{i}{\hbar}(\mathbf{p},\, \mathbf{X}_2 - \mathbf{X}_1)} \tag{12}$$

The summation is over all momentum states p of the meson. In the notation of the previous paper $\sigma^{(1)}$, $\sigma^{(2)}$ are the Pauli spin matrices of the two heavy particles, and σ_{1p} and σ_{2p} are the components of these matrices along the two transverse polarisation directions of the meson in its intermediate state of momentum p. The only difference from previous calculations is the appearance of the factor in curly brackets in (12) which was previously unity. We may write for this factor

$$\frac{E + \frac{1}{2} \Delta Mc^2}{E + \Delta Mc^2} = 1 - \frac{\Delta Mc^2}{2(E + \Delta Mc^2)} \approx 1 - \frac{\Delta Mc^2}{2E}, \tag{13}$$

since $E > \mu c^2 \gg \Delta Mc^2$. For the actual cross-section (7) we only require $(f|W|i)$ in which case due to the exponential at the end of the expression (12), the summation over p drops out, and only that momentum p satisfying (11) gives a non-vanishing contribution.

To get the final cross-section for the process (1) we must remember that both the colliding particles are identical, so that exchange will have to be taken into account. This can be done easily if we remember that the particle with the initial momentum $\mathbf{p}_0$ may either go into the state of momentum $\mathbf{p}_f$ the other particle recoiling with momentum $\mathbf{p}_0 - \mathbf{p}_f$ or alternately, the colliding particle may itself go into the state of momentum $\mathbf{p}_0 - \mathbf{p}_f$ in which case its change of momentum is $\mathbf{p}_0 - (\mathbf{p}_0 - \mathbf{p}_f) = \mathbf{p}_f$. Thus to the matrix element $(f|W|i) \equiv (\mathbf{p}_f|W|\mathbf{p}_0)$ we have to add $(\mathbf{p}_0 - \mathbf{p}_f|W|\mathbf{p}_0)$, *i.e.* the same expression with $\mathbf{p}_f$ in place of $\mathbf{p}$ and $E_f \equiv c\sqrt{\mu^2c^2 + p_f^2}$ in place of E.

As stated, for a given scattering angle θ, there are two definite values of p_f. Conversely, we can at once express $\cos\theta$ in terms of p_f. In the present instance this is more convenient. From (4) or (5)

$$p_0 \cos\theta = p_f + \frac{M \Delta Mc^2}{p_f}, \tag{13}$$

so that

$$d\cos\theta_0 = \left(1 - \frac{M \Delta Mc^2}{p_f^2}\right)\frac{dp_f}{p_0}. \tag{14}$$

The momentum given by (11) can also be expressed in terms of p_f by using (4)

$$p = \sqrt{p_0^2 - p_f^2 - 2M \Delta Mc^2}. \tag{15}$$

Using (3) we find that

$$\frac{\partial E_{tot}}{\partial p_f} = \frac{1}{M}\sqrt{p_0^2\cos^2\theta - 4M \Delta Mc^2} = \frac{1}{M}\left(p_f - \frac{M \Delta Mc^2}{p_f}\right). \tag{16}$$

Averaging over the two possible spin directions of the heavy particles and using the above relations, we find after some calculation that (7) reduces to

$$d\theta = 8\pi\left(\frac{\hbar}{\mu c}\right)^2\left(\frac{M}{\mu}\right)^2\left\{\left(\frac{g_1^2}{\hbar c}\right)^2 R^2 - 2\left(\frac{g_1 g_1'}{\hbar c}\right)^2 R + 2\left(\frac{g_2'^2}{\hbar c}\right)^2 (R^2+1)\right\}\frac{p_f\,dp_f}{p_0^2}.$$

where

$$R = \frac{\mu^2 c^4}{E^2} + \frac{c^2 p^2}{E^3} \frac{\Delta M c^2}{2} + \frac{\mu^2 c^4}{E_f^2} + \frac{c^2 p_f^2}{E_f^3} \frac{\Delta M}{2}. \tag{18}$$

The last two terms of R are due to the effects of exchange. The terms containing ΔM in R can be neglected at once, for they only become comparable with the other terms when

$$\frac{1}{2} \frac{p^2}{\mu^2 c^2} \frac{\Delta M c^2}{E} \gtrsim 1,$$

i.e.,

$$p \gtrsim 2\left(\frac{\mu}{\Delta M}\right) \mu c \sim 10 \mu c.$$

For such high momenta the heavy particles would have to be treated relativistically and the above cross-section would in any case not be correct. For very high momenta the effect of relativity is again to diminish the effect of the difference in the mass ΔM as a more accurate consideration of the resonance denominators (10 *a*) and (10 *b*) shows. The only effect of ΔM is in determining the maximum and minimum values of p_f. From (5) we find

$$\begin{aligned} (p_f)_{max} &= \tfrac{1}{2} p_0 + \tfrac{1}{2} \sqrt{p_0^2 - 4M \Delta M c^2} \\ (p_f)_{min} &= \tfrac{1}{2} p_0 - \tfrac{1}{2} \sqrt{p_0^2 - 4M \Delta M c^2} \end{aligned} \tag{19}$$

Integrating (17) we find that the total cross-section can be written

$$Q = 8\pi \left(\frac{\hbar}{\mu c}\right)^2 \left[\left\{\left(\frac{g_1^2}{\hbar c}\right)^2 + 2\left(\frac{g_2'^2}{\hbar c}\right)^2\right\} \frac{\mu^2 c^4}{E_0} \left\{\frac{\sqrt{E_0 (E_0 - 2 \Delta M c^2)}}{E'^2 - E_0 (E_0 - \Delta M c^2)} + \frac{1}{2E'} \log \frac{E' + \sqrt{E_0 (E_0 - 2 \Delta M c^2)}}{E' - \sqrt{E_0 (E_0 - 2 \Delta M c^2)}}\right\} - \left(\frac{g_1 g_2'}{\hbar c}\right)^2 \frac{M}{\mu} \frac{\mu c^2}{E_0} \log \frac{E' + \sqrt{E_0 (E_0 - 2 \Delta M c^2)}}{E' - \sqrt{E_0 (E_0 - 2 \Delta M c^2)}} + \left(\frac{g_2'^2}{\hbar c}\right)^2 \left(\frac{M}{\mu}\right)^2 \sqrt{1 - \frac{2 \Delta M c^2}{E_0}}\right] \tag{20}$$

where

$$E' = \frac{\mu^2 c^2}{M} + E_0 - \Delta M c^2. \tag{21}$$

We notice that this expression after reaching a maximum decreases for increasing E_0. It goes over into the usual cross-section for neutron proton scattering if ΔM is put equal to zero. Taking $g_1^2/\hbar c \sim 1/25$ and putting $g_2 = 0$, the cross-section due to the g_1 term for $E_0 \sim 50$ M.e-V. becomes

$$Q^{(1)} \sim 2 \cdot 3 \times 10^{26} \text{ cm.}^2 \tag{22}$$

The contribution due to the g_2 term alone is larger, mainly due to the factor $(M/m)^2$. Putting $g_2'^2/\hbar c \sim \frac{1}{16}$ and $E_0 = 50$ M.e-V. we get

$$Q^{(2)} \sim 3\cdot 2 \times 10^{-25}\ \text{cm.}^2 \tag{23}$$

The cross-sections for the reverse of the processes (1) in which a proton of charge $2\,e$ or $-\,e$ is converted into an ordinary heavy particle by collision with a neutron or proton respectively are given by (17), (20) and (21) if we merely change the sign of ΔM. The reverse processes in which the kinetic energy of the whole system increases are naturally possible for all kinetic energies. The cross-sections are of the same order of magnitude. For $E_0 \sim 50$ M.e-V. and $g_2'^2/\hbar c \sim \frac{1}{16}$ we get

$$Q_r \sim 8 \times 10^{-25}\ \text{cm.}^2 \tag{24}$$

We shall require the cross-section for this process in particular when E_0 is very small. Remembering that $E_0 = Mv^2/2$, v being the initial velocity of the colliding particle, (20) with the sign of ΔM reversed becomes

$$Q_r(0) \approx 16\,\pi \left(\frac{\hbar}{\mu c}\right)^2 \sqrt{\frac{\Delta M}{M}} \left[\left(\frac{g_2'^2}{\hbar c}\right)^2 \left(\frac{M}{\mu}\right)^2 - 2\left(\frac{g_1\,g_2'}{\hbar c}\right)^2 \frac{Mc^2}{E''} + \left\{\left(\frac{g_1^2}{\hbar c}\right)^2 + 2\left(\frac{g_2'^2}{\hbar c}\right)^2\right\} \frac{2\mu^2 c^4}{E''^2}\right] \frac{c}{v}, \tag{25}$$

with

$$E'' = \frac{\mu^2 c^2}{M} + \Delta\ Mc^2, \tag{26}$$

which holds when $E_0 \ll \Delta Mc^2$. The only dependence on the energy is expressed by v, the rest being constant. With the same values for the constants as above, we find

$$Q_r(0) = \frac{c}{v}\, 2\cdot 15 \times 10^{-25}\ \text{cm.}^2 \tag{27}$$

It is also of interest to know the interaction of a proton of charge $2e$ with a proton. This is of course again an exchange interaction

$$P_2^{(1)} + P^{(2)} \rightarrow P^{(1)} + P_2^{(2)}, \tag{28}$$

Again, the only difference between (28) and the process (1) lies in the different energy denominators. It is easily seen that we now have in place (10 *a*) and (10 *b*) respectively

$$\begin{aligned} E_i - E_{mI} &\approx -E + \Delta Mc^2\,, \\ E_i - E_{mII} &\sim -E - \Delta Mc^2, \end{aligned} \tag{29}$$

so that the only change is in the factor in curly brackets in (12) which now becomes

$$\frac{E^2}{E^2 - (\Delta Mc^2)^2} \approx 1 + \frac{(\Delta Mc^2)^2}{E^2}. \tag{30}$$

The interaction is got by inserting (29) in the curly bracket in (12) and carrying out the summation over all momenta. This summation may as usual be replaced by an integral. We find that

$$W = W_1 + W_2, \tag{31}$$

where, writing $\chi = \mu c/\hbar$,

$$W_1 = (\tau_+^{(1)}\, \tau_-^{(2)} + \tau_-^{(1)}\, \tau_+^{(2)})\, [g_1^2 + g_2'^2\, (\sigma^{(1)}, \sigma^{(2)}) - \frac{g_2'^2}{\chi^2}\, (\sigma^{(1)}, \text{grad})\, (\sigma^{(2)}, \text{grad})]\, \frac{e^{-\chi|X_2 - X_1|}}{|X_2 - X_1|}, \tag{32}$$

is the same as the neutron proton interaction, and

$$W_2 = -4\pi\, (\tau_+^{(1)}\, \tau_-^{(2)} + \tau_-^{(1)}\, \tau_+^{(2)})\, (\Delta\, Mc^2)^2 \iiint \frac{d\mathbf{p}}{h^3} \Big[\, g_1^2 + g_2'^2\, (\sigma^{(1)}, \sigma^{(2)}) - \frac{g_2'^2 \hbar^2}{p^2}\, (\sigma^{(1)}, \text{grad})\, (\sigma^{(2)}, \text{grad})\Big]\, \frac{c^2 p^2}{E^4}\, e^{\frac{i}{\hbar}(\mathbf{p},\, X_2 - X_1)} \tag{33}$$

After some easy calculation we find that the second part becomes

$$W_2 = -\, (\tau_+^{(1)}\, \tau_-^{(2)} + \tau_-^{(1)}\, \tau_+^{(2)})\, \left(\frac{\Delta M}{\mu}\right)^2 \Big[\Big\{g_1^2 + g_2'^2\, (\sigma^{(1)}, \sigma^{(2)})\Big\}\, \frac{e^{-\chi r}}{r} + \Big\{g_1^2 + g_2'^2\, (\sigma^{(1)}, \sigma^{(2)}) - \frac{g_2'^2}{\chi^2}\, (\sigma^{(1)}, \text{grad})\, (\sigma^{(2)}, \text{grad})\Big\}\, \frac{\chi}{2}\, e^{-\chi r}\Big]. \tag{34}$$

This additional interaction is smaller than the usual interaction by a factor $(\Delta M/\mu)^2$ and hence is unimportant in most cases. A term like (34) exists also for the neutron proton interaction but ΔM being of the order m in this case is entirely negligible.

The above calculations have been made on the unsymmetrical meson theory where only charged mesons are assumed to exist but no neutral mesons. If the calculations had been done on the assumptions which lead to the charge independence hypothesis, then the scattering cross-sections (20) would have been smaller, since the values to be assumed for $g_1^2/\hbar c$ and $g_2'^2/\hbar c$ are smaller. According to Kemmer (1938 *b*) we must assume $g_1^2/\hbar c \sim 1/35$, $g_2'^2/\hbar c \sim 1/14$ which reduces (22) roughly by a factor two, (23) remaining the same.

Besides this, the introduction of neutral mesons naturally leads to an interaction of the same magnitude as (32) between a proton of charge $2e$ and a neutron. The term (34) is now absent. The same applies, of course, to the interaction of a proton of charge $-e$ with a proton or neutron.

Creation of Protons of Charge 2e and − e by Mesons and Protons

We now consider the process in which a positive meson is absorbed by a proton with the emission of a quantum of radiation, the proton changing into a proton of charge $2e$ and also the corresponding process for a negative meson.

$$Y^- + N \rightarrow P_- + h\nu \qquad (35)$$
$$Y^+ + P \rightarrow P_2 + h\nu.$$

The difference in mass between a proton of charge $2e$ and a normal proton will naturally not effect this process appreciably, except in so far as it concerns the conservation of energy and momentum. The matrix element will be the same as for the process

$$Y^+ + N \rightarrow P + h\nu. \qquad (36)$$

The processes (35) and (36) can take place either as a second order processes, involving the separate interactions of the meson with the heavy particles and light quanta respectively, or it can take place as a first order process through an interaction term which involves both the mesic and the electromagnetic interactions at once.* This term is given explicitly by (58 b) of A. In calculating (35) we meet with one difficulty. The process involves the absorption of a meson from the free state, and a large contribution will come from the g_2 interaction. Moreover, as we have mentioned above the quantum theory will give an entirely wrong energy dependence for this term for high energies due to the neglect of radiation reaction. Hence we can only make a fair estimate of the cross-section by considering the g_1 term alone, for which in the non-relativistic region for the heavy particles with which we are concerned, the effects of radiation reaction are negligible.

We can now take over the result of Kobayasi and Okayama (1939) for the process (36).† Denoting by $\mathbf{p}$ the momentum of the meson, by E its energy and by $\mathbf{k}$ the momentum of the light quantum, the differential cross-section for the process (35) in which the light quantum is emitted in the direction is, according to Kobayasi and Okayama

$$dQ = \pi g_1^2 e^2 \left\{\frac{\mu^2 c^4}{E} \cos^2\theta + E\,(1 + \sin^2\theta)\right\} \frac{Ek}{E_1^4 p}\, d\cos\theta \qquad (37)$$

* The process (36) was calculated by Heitler (1938), but by an error the term which gives a first order contribution to the process was omitted. Since the effect of this term is largely to cancel the effect of the second order process, unfortunately no reliance can be put in his result.

† According to Belinfante (1939) an error of sign has occurred in the calculations of these authors. Belinfante however does not give the correct result. I have not rechecked the cross-section (37) as given by Kobayasi and Okayama.

where

$$E_1^2 = c^2p^2 + c^2k^2 - 2\,c^2pk\cos\theta + \mu^2c^4. \tag{38}$$

Here k is connected with E by

$$E = ck + \Delta Mc^2. \tag{39}$$

Integrating (37) over all angles, we get for the total cross-section

$$Q = \pi\left(\frac{g_1^2}{\hbar c}\right)\left(\frac{e^2}{\hbar c}\right)\left(\frac{\hbar}{\mu c}\right)^2\Bigg[\frac{2\,(2E^2 - p^2)\,k\mu^2}{p\left\{\left(\frac{E}{c}+k\right)^2(\Delta M)^2 + 4\mu^2c^2k^2\right\}}$$

$$+\frac{\{(E)^2 + c^2k^2\}\,\mu^2c^2}{4c^2p^2k^2}\log\frac{E^2 + c^2k^2 + 2c^2pk}{E^2 + c^2k^2 - 2c^2pk} - \frac{\mu^2c^2}{pk}\Bigg]. \tag{40}$$

For $p \gg \mu c$ the expression in square brackets of (40) tends to the constant value $\frac{1}{2}$ so that the cross-section does not increase indefinitely with increasing energy. Taking $g_1^2/\hbar c \sim \frac{1}{35}$ this gives for $E = 1\cdot 4\,\mu c^2$

$$Q \sim 5 \times 10^{-29}\ \text{cm.}^2 \tag{41}$$

Two other processes leading to the creation of protons of charge $2e$ and $-e$ are

$$\begin{aligned} h\nu + P &\rightarrow P_2 + Y^- \\ h\nu + N &\rightarrow P_{-1} + Y^+. \end{aligned} \tag{42}$$

The cross-sections for these would only differ from (40) by a factor p^2/k^2. They are, of course, of the same order of magnitude as (41).

Discussion

For brevity in the present discussion it will be convenient to speak of a " heavy " proton whenever our remarks are meant to apply both to a proton of charge $2\,e$ and a proton of charge $-e$. The above calculations show that by far the most probable processes leading to the creation of heavy protons are those involving the collision of two heavy particles as represented by (1). The cross-section (23) shows that an energetic proton would have to transverse on the average about 5 grs./cm.^{2} of hydrogen in order to produce a proton of charge $2\,e$. The only necessary condition is that the proton at every point of its path should have a minimum energy of about 35 M.e-V. This minimum energy is conditioned by the need to conserve energy and momentum in the collision. For collision with a proton in a nucleus the minimum energy could be much less. On the other hand, the effective cross-section per nuclear proton will be smaller for two reasons. First, the proton of charge $2\,e$ will only escape from the nucleus if it is produced near the surface and hit in the outward direction. Otherwise it will get reconverted into an ordinary proton by collision with another particle in the same nucleus. Secondly, its double charge will make the chance of its penetrating the Coulomb barrier of the nucleus much smaller than for a proton. As a rough estimate an energetic proton would have to traverse some 25 cms. of water in order to produce a proton of charge $2e$ as a free particle. Correspondingly, a neutron of high energy would have to traverse about the same distance in water to produce a proton of charge $-e$.

Now protons or neutrons of this energy are very rare at sea level. They do, however, occur frequently in nuclear explosions produced by cosmic rays, as photographed in a Wilson chamber or recorded in the emulsions of photographic plates. Experiments with photographic plates have shown that the radiation responsible for these explosions increases rapidly with height. Experiments at high altitudes either with a Wilson chamber or photographic plates would be most likely to reveal the existence of a proton of charge $2e$ or $-e$. We shall return to this point later.

Formula (41) shows that the chance of a heavy proton being produced by a meson or a photon is extremely small. Moreover, the cross-section (41) is so small that it would not in any way affect the observed penetrating power of charged mesons. In fact, the above calculations and discussion show that the assumption of allowing heavy particles to exist in states of all integral charge so reduces all cross-sections for charged mesons as to bring the theory into harmony with the observed penetrating power of mesons. One also sees that the cross-sections do not continue to increase with increasing energy, as was originally the case for charged mesons even with the g_1 interaction alone. The calculations of Heitler (1940) on this idea have shown that it also decreases the divergence in the expression for the anomalous magnetic moment of the proton. Of course, the g_2 interaction will still lead to cross-sections which increase with the energy but reasons have been given in the discussion of the introduction for attributing this increase to neglect of the effects of radiation reaction.

It is also possible for a meson to be absorbed by a nuclear particle without the emission of a quantum of radiation. The extra momentum is now taken up by the other nuclear particles. The cross-section for the absorption of a positive or negative meson by the conversion in a nucleus of a neutron into a proton or *vice versa* has been calculated by Sakata and Tanikawa (1939) and is of the order $4 \cdot 5 \times 10^{-28}\, c/v$ cm.^{2} v being the velocity of the meson. This is some ten times greater than (41) or even more, depending on the velocity. The corresponding process for the creation of heavy protons would be of the same order. It would become quite effective for very slow mesons.

The ionisation produced by a particle only depends on its charge and velocity, and this combined with its mass then determines the range of the particle. The rate of energy loss of a heavy particle moving with non-relativistic velocities due to ionisation may conveniently be written in the form

$$-\frac{\partial \epsilon}{\partial x} = a \log \epsilon, \tag{43}$$

where

$$a = \frac{32\, \pi \rho\, e^4 z^2 m}{\bar{E}^2 M}, \tag{44}$$

and ϵ is connected with the energy E by

$$\epsilon = \left(4 \frac{m E}{M \bar{E}}\right)^2. \tag{45}$$

Here ρ represents the number of electrons per cubic centimetre of the substance traversed, m the mass of the electron, M that of the ionising particle, ze its charge and E its kinetic energy. $\bar{E}$ is the mean ionisation potential of the substance traversed and following Bloch is to be put equal to 13·5 Z e-V., Z being the atomic number of the substance. For air we may take $\bar{E}$ = 94·5 e-V*. The formula (43) can be integrated at once and gives the range l by

$$al = \int_{\epsilon_0}^{\epsilon} \frac{d\,\epsilon'}{\log \epsilon'} - \overline{Ei}\,(\log_e \epsilon_0), \tag{46}$$

$\overline{E_i}$ $(\log_e \epsilon)$ is the well-known logarithmic integral and has been tabulated (see, for example, Jahnke-Edme Table of Functions). ϵ_0 is some suitably chosen lower limit. We can determine it by comparison with the ranges calculated by Bethe for low energy particles. In this paper ϵ_0 has been so chosen that for E = 10^6 e-V. for protons and α-particles the ranges given by (46) should agree with the corresponding ranges given by Bethe. Using this formula the ranges of protons, protons of charge $2e$ and α-particles have been calculated and are given in the table below:

Ranges of particles in centimetres air

Energy in e-V.	10^6	10^7	10^8	10^9
Protons of charge e	1·3	128	$8{\cdot}79 \times 10^3$	
Protons of charge $2e$	0·3	32	$2{\cdot}2 \times 10^3$	
α-Particles	0·1	11·04	681	$4{\cdot}86 \times 10^4$

* The ranges calculated in this paper are different from those of Bethe (1933) because of his having taken $\bar{E}$ = 37 e-V. The Bloch formula fits experiments better.

The mass of a proton of charge $2e$ being nearly equal to that of an ordinary proton, (44) shows that its range will be exactly a quarter that of a proton of the same energy. For the same *velocity*, the track of a proton of charge $2e$ will look exactly like that of an α-particle, or for the same *energy*, the proton of charge $2e$ will show somewhat more than half the ionisation of an α-particle.

In many experiments it is of interest to know the average ionisation along the whole track of the particle as a function of the range. This average ionisation may simply be defined as E/l. For $\log_e \epsilon \gg 1$, (46) may be written

$$al \backsimeq \frac{\epsilon}{\log_e \epsilon},$$

which leads very roughly to

$$\epsilon \backsimeq al \log_e al.$$

Using (44) and (45) we get

$$\frac{E}{l} \sim z \sqrt{\frac{M}{m}} \sqrt{\frac{2\pi \rho e^4 \log al}{l}}.$$

Thus, for the same range, the average ionisation of a proton of charge $2e$ is roughly twice that of an ordinary proton, and that of an α-particle twice that of a proton of charge $2e$. Using (46), a more accurate calculation can be performed numerically. This has been done and it is found that for large l greater than about 10 cm. air, the average ionisation of a proton of charge $2e$ is $2 \cdot 2$ times that of an ordinary proton of the same range, while that of an α-particle is $1 \cdot 8$ times that of a proton of charge $2e$. Even for shorter ranges the above figures are not far out.

Experiments have been made by Schopper and Schopper (1939), in which the average distance between two developed grains along the track of particles in photographic emulsions has been plotted as a function of the range of the particle. In this way it is possible to distinguish at least two types of particles, one group being identified as protons and the other as α-particles. The identification of the second group as due to α-particles is by no means certain, especially for the longer ranges. One can only say that it is due to a particle with a greater charge than the proton. In fact their farthest point for the second group is at a range of $2000\,\mu$, while the next farthest is at a range of $100\,\mu$. While the latter point might almost certainly be attributed to α-particles as it is very near the point for α-particle from known sources, it is not clear to me that the farthest point is also to be so interpreted. On a diagram like theirs, the curve for a proton of charge $2e$ would lie between that for a proton and that for an α-particle. Unfortunately the statistical errors are large, and the average distance between two grains in a track in a photographic emulsion is not very sensitive to variation in the ionisation. It is, therefore, not possible from their experiments to draw a definite conclusion one way or the other on the existence or non-existence of a proton of charge $2e$. *If* one is correct in attributing the farther point at a range of $2000\,\mu$ to α-particles, then their experiments might be taken to indicate that a proton of charge $2e$ does not appear as frequently as an ordinary proton. This is in any case to be expected theoretically from what we have said above about the emergence of a proton of charge $2e$ from heavy nuclei. More extended experiments with plates at high altitudes especially if surrounded by hydrogen containing substances could settle this point. The Wilson chamber technique is of course more sensitive in revealing differences in ionisation of particles of the same range due to differences of charge and mass.

Summary

The previous paper having shown that all divergences and large cross-sections for neutral mesons being due entirely to neglect of radiation reaction, an attempt is made in this paper to remove those difficulties in the theory of *charged* mesons which do not occur in the theory of neutral mesons by following up an idea put forward tentatively by the present author some time ago on the ground that it would diminish the excessive scattering of charged mesons. It is assumed that the heavy elementary particles can exist in states of all integral charge, positive, negative or zero, the different states having different rest masses, of which the states with charge 0 and e (neutron and proton) must be assumed to have the lowest rest masses, while the proton states of charge $-e$ and $2e$ are assumed to have the next lowest. The cross-sections for the creation and annihilation of protons of charge $2e$ and $-e$ by several processes are calculated. The collision of a fast proton with another stationary proton is the most effective process for creating protons of charge $2e$, the cross-section being of the order 10^{-27} cm.^{2} The colliding proton must have a kinetic energy of at least 35 M.e-V. Neutrons of the same energy would produce protons of charge $-e$ on colliding with neutrons. The cross-sections for the production of protons of charge $2e$ and $-e$ by mesons or photons are of the order 10^{-27} cm.^{2} The life time for spontaneous decay of these particles is of the order of $\frac{1}{6}$ seconds, while the life time in air for reconversion into ordinary protons or neutrons by collision with a nucleus is of the order Z 10^{-7} secs. for low velocities. These particles have an interaction with the proton or

neutron which is the same as the proton-neutron interaction with small additional terms. The energy-range relationship is calculated. The mean ionisation along a tract of a proton of charge $2e$ is nearly twice that of a proton or half that of an α-particle of the same range. If the theory is correct these particles are expected to occur in the nuclear explosions produced by cosmic rays, though less frequently than ordinary protons. Study of Wilson chamber photographs and photographs of nuclear explosions in the emulsions of photographic plates especially at high altitudes might be expected to reveal or disprove the existence of these particles.

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