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The Sign Convention for Quadrature Parkinson Arrows in Geomagnetic Induction Studies

F. E. M. LILLEY

Research School of Earth Sciences, Australian National University, Canberra, A.C.T. 2600, Australia

B. R. ARORA

Indian Institute of Geomagnetism, Colaba, Bombay 400-005, India

Time series analysis, which is basic to modern geophysical data processing, involves a choice between working with a time dependence of $e^{+i\omega t}$ or $e^{-i\omega t}$. In published work the choice made is sometimes not explicitly stated, leaving ambiguity in the interpretation of complex quantities with quadrature parts. Parkinson arrows are used in geomagnetic induction studies to summarize anomalous vertical magnetic fluctuations at different observing stations and to indicate regions of high electrical conductivity. Such arrows are now regularly computed as real and quadrature pairs. The general convention is often adopted of 'reversing' a calculated real arrow so that it will point toward a conductivity increase, but for quadrature arrows the practice between various published papers has generally not been so consistent. The present paper demonstrates that consistent practice for reversing or not reversing quadrature Parkinson arrows is possible when the initial convention for time dependence is taken into account. A reversal practice is determined for interpretation in terms of a simple channeling model. A related matter is the definition of phase. Phase values are also generally ambiguous unless the time dependence used ($e^{-i\omega t}$ or $e^{+i\omega t}$) is stated.

INTRODUCTION

The spectral analysis of a time series of geophysical data involves the following steps: (1) the specification that time dependence shall be according to either $e^{-i\omega t}$ or $e^{+i\omega t}$, where t denotes time, ω denotes angular frequency, and i denotes $-1^{1/2}$, (2) the determination of a time-independent function which is complex with real (or in phase) and quadrature (or out of phase) parts, and (3) the understanding that the actual time series is given by the real part of its complex spectral representation.

Thus the component $f_\omega(t)$ at frequency ω of a function $f(t)$ may be written as

$$f_\omega(t) = \chi e^{-i\omega t}$$

where $\chi = \chi_r + i\chi_q$, so that

$$\begin{aligned} f_\omega(t) &= (\chi_r + i\chi_q)(\cos \omega t - i \sin \omega t) \\ &= (\chi_r \cos \omega t + \chi_q \sin \omega t) + i(\chi_q \cos \omega t - \chi_r \sin \omega t) \\ &= \chi_r \cos \omega t + \chi_q \sin \omega t \end{aligned} \tag{1}$$

taking only the real part of the right-hand side.

Alternatively, $f_\omega(t)$ may be written as

$$f_\omega(t) = \psi e^{+i\omega t}$$

where $\psi = \psi_r + i\psi_q$, so that

$$\begin{aligned} f_\omega(t) &= (\psi_r + i\psi_q)(\cos \omega t + i \sin \omega t) \\ &= (\psi_r \cos \omega t - \psi_q \sin \omega t) + i(\psi_q \cos \omega t + \psi_r \sin \omega t) \\ &= \psi_r \cos \omega t - \psi_q \sin \omega t \end{aligned} \tag{2}$$

upon taking only the real part of the right-hand side. Because (1) and (2) hold over all t,

$$\chi_r = \psi_r \tag{3}$$

$$\chi_q = -\psi_q \tag{4}$$

so that the sign of a quadrature coefficient changes if the time dependence specified changes between $e^{-i\omega t}$ and $e^{+i\omega t}$. This dependence in the sign of a quadrature coefficient upon the time dependence initially specified may have far-reaching effects in the interpretation of quadrature coefficients themselves and of any other parameters derived from them.

THE BASIC EQUATION

A Parkinson arrow as used in geomagnetic induction studies is based on an empirical fit of observed magnetic fluctuation data to an equation such as

$$Z = AX + BY \tag{5}$$

where X, Y, and Z are the components of the magnetic fluctuation field in the usual observatory coordinates and A and B are constants, depending ideally only upon the local electrical conductivity structure of the earth. All the quantities in (5) are frequency-dependent and complex, in that they may have real and quadrature components.

The general question of induction arrow representation has been recently reviewed by *Gregori and Lanzerotti* [1980] (see also *Jones* [1981]). These authors discuss also (5) above, pointing out its early statement by *Rikitake and Yokoyama* [1953], *Schmucker* [1964], and *Everett and Hyndman* [1967]. Denoting the real and quadrature components of X by X_r and X_q, so that

$$X = X_r + iX_q$$

and similarly for Y, Z, A, and B, then (5) may be expanded into its real and quadrature parts as

$$Z_r = A_rX_r - A_qX_q + B_rY_r - B_qY_q \tag{6}$$

$$Z_q = A_rX_q + A_qX_r + B_rY_q + B_qY_r \tag{7}$$

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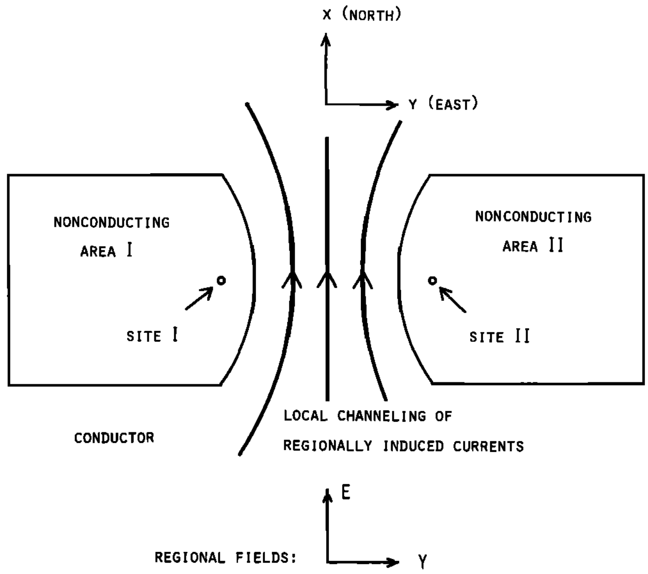

Fig. 1. Plan view of simple channeling model.

Traditionally, in-phase response arrows are formed with a component A_r north and B_r east and are then reversed to conform with *Parkinson*'s [1962] convention. The question is: Should quadrature response arrows, formed with A_q north and B_q east, be reversed or not?

It can be seen by inspection of (6) and (7) that changing the signs of all X_q, Y_q, and Z_q data values will change the signs of the A_q and B_q coefficients which these data values generate. Thus the signs of the A_q and B_q coefficients determined by any ensemble of X, Y, and Z data observations, and the direction of a quadrature Parkinson arrow thus formed with a component A_q north and B_q east, will depend, as shown in the introduction, on the specification of time dependence initially made in the necessary time series analysis.

Therefore whether a quadrature Parkinson arrow should or should not be plotted reversed in direction will depend upon the convention taken for time dependence in the time series analysis process.

A Note on the Physics Involved

It is now appropriate to consider the physics involved in electromagnetic induction in the earth. There are many diagrams in the literature (most based on simple models of very high electrical conductivity) which demonstrate that an in-phase arrow formed with a component A_r north and B_r east must be reversed in direction to point toward a good electrical conductor or the high electrical conductivity side of a conductivity contrast (see, for example, *Gregori and Lanzerotti* [1980, Figure 1]). Such models are qualitatively simple to visualize, because the vertical fluctuation component Z involved is entirely in phase with the horizontal fluctuation component (X, Y) with which it is associated, and so both components may be pictured as part of the same continuous magnetic flux line.

The quadrature case is more difficult to visualize because of the quarter-cycle (i.e., $\pi/2$) phase difference between an anomalous vertical-component fluctuation and its associated quadrature horizontal-component fluctuation, which means that both components cannot be represented by the same continuous magnetic flux line. In considering a simple induction model for quadrature arrow considerations, therefore, use will be made of a basic theoretical result (derived, for example, by *Cagniard* [1953, equations 1 and 5]) concerning electromagnetic induction at the surface of a uniform half space. The result is that the phase of the surface magnetic field is retarded by an angle of $\pi/4$ with respect to that of the telluric field, where the electric field E is taken positive in the horizontal x direction, the magnetic field Y positive in the horizontal y direction, and x, y, and z form a right-handed system with z positive vertically downward. This phase relationship, to form the basis of a simple model to be discussed in the next section, is shown in Figure 2 below.

Cases more complicated than the case of a uniform half space are often solved individually; for example, *Schmucker* [1970, p. 23] quotes the result that superficial eddy currents have a marked phase lead relative to the horizontal magnetic fields inducing them and gives an example [*Schmucker*, 1970, p. 78, Figure 35*c*] where the anomalous vertical field fluctuations have a phase lead of 70° relative to the inducing regional horizontal variations.

Arrows for a Simple Channeling Model

Imagine now a situation where near-surface electric currents are channeled and concentrated as shown in Figure 1. On a regional basis the currents obey the result that the telluric voltages lead by $\pi/4$ the magnetic field variations which induce them. In Figure 1 an electric current flowing north associated with and in phase with positive E causes in area I an upward Z field, which is negative by definition. In area II the same electric current causes a downward or positive Z field. The relative phases of the signals are as shown in Figure 2, where the E signal with a phase lead of

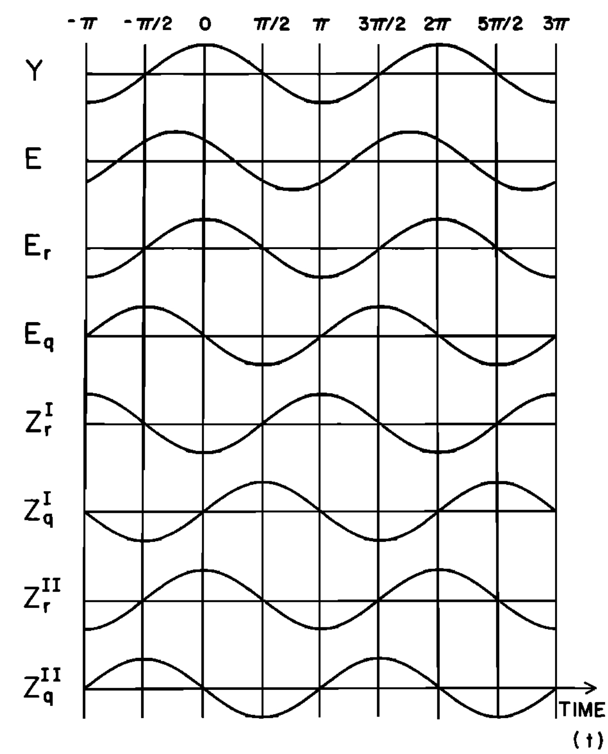

Fig. 2. Relative phases of the waveforms for the channeling model of Figure 1. Amplitude scales are arbitrary. Z^{I} and Z^{II} denote the vertical fluctuation signals for areas I and II, respectively, and the subscripts r and q denote real and quadrature components.

$\pi/4$ is decomposed into a real (or in phase) component E_r and a quadrature component E_q.

Inspection of Figure 2 indicates the signs which would be determined for the real and quadrature parts of the coefficients A and B of (5) for observing sites in areas I and II of Figure 1. For simplicity, assume the model is such that $A = 0$ at both sites, so that (6) and (7) simplify to

$$Z_r = B_r Y_r \tag{8}$$

$$Z_q = B_q Y_r \tag{9}$$

as the phase zero is defined to make Y_q zero. The two possible time dependences, $e^{-i\omega t}$ and $e^{+i\omega t}$, must be considered separately.

First consider a site in area I.

Case 1: Time Dependence $e^{-i\omega t}$

Using the notation given in the introduction, for area I the signal Z_r^{I} will give a negative value for χ_r (as Z_r^{I} is of the form $-\cos \omega t$ rather than $+\cos \omega t$). Thus the B_r value (which by equation (8) links Z_r and Y_r) will for area I be negative, and an arrow formed with component B_r to the east will point westward. Thus such an arrow should be reversed to point to the line of current channeling, consistent with the tradition for in-phase Parkinson arrows, as expected.

The Z_q^{I} waveform for area I (being of form $+\sin \omega t$) will give a positive value for χ_q. Thus the B_q value for area I will be positive, and an arrow formed with component B_q to the east will point eastward: toward the line of channeling, unreversed.

Case 2: Time Dependence $e^{+i\omega t}$

With this time dependence the Z_r^{I} value for ψ_r is still negative, and the Z_q^{I} waveform is now negative also. It thus follows that both B_r and B_q arrows should be reversed to point toward the line of channeling.

Area II

Area II may be considered similarly, and thus the summary of results in Table 1 compiled.

Table 1 thus shows that an in-phase arrow, formed with components A_r north and B_r east, should be reversed to point toward the line of channeling of the model of Figure 1, independent of which time dependence is used. A quadrature arrow, however, with components A_q north and B_q east, will point toward the line of channeling unreversed for a time dependence of $e^{-i\omega t}$ but must be reversed to point toward the line of channeling for a time dependence of $e^{+i\omega t}$.

Induction models more complicated than the case in Figure 1 may need to be solved individually for rules concerning them to be determined regarding the reversal of quadrature arrows. However, the case in point demonstrates that care with time dependence is always necessary.

THE ASSOCIATED DEFINITION OF PHASE

Given spectral representations of data as specified in the introduction, it is customary to define a phase angle as being the arc tangent of the quotient of a quadrature coefficient divided by a real coefficient.

From equation (1), for time dependence $e^{-i\omega t}$, a phase angle θ may thus be defined as

$$\theta = \arctan (\chi_q/\chi_r) \tag{10}$$

which, taking the respective signs of χ_q and χ_r into account, defines θ over a range of 2π; though if the respective signs of χ_q and χ_r are ignored, θ is defined only over a range of π. From equation (2), for time dependence $e^{+i\omega t}$, a phase angle ϕ may be defined as

$$\phi = \arctan (\psi_q/\psi_r) \tag{11}$$

where again taking the respective signs of ψ_q and ψ_r into account, ϕ is defined over a range of 2π; otherwise over π. Corresponding to (10) and (11), the two phase angles θ and ϕ are not equal, but for the same function $f_\omega(t)$ they have a relationship which from (3) and (4) can be seen to be

$$\theta + \phi = 2n\pi$$

when the respective signs of χ_q, χ_r, ψ_q, and ψ_r are taken into account to determine the quadrant of an arc tangent, and

TABLE 1. Summary of Results for Areas I and II in Figure 1, Concerning Rules for Reversing (or Not Reversing) Real and Quadrature Parkinson Arrows

	Area				
	I and II	I	I	II	II
Signal	Y	Z_r^{I}	Z_q^{I}	Z_r^{II}	Z_q^{II}
Waveform in Figure 2	$\cos \omega t$	$-\cos \omega t$	$\sin \omega t$	$\cos \omega t$	$-\sin \omega t$
For $e^{-i\omega t}$ analysis					
Sign of χ component	χ_r positive	χ_r negative	χ_q positive	χ_r positive	χ_q negative
Component and direction of computed B		B_r west	B_q east	B_r east	B_q west
Direction of conductor		east	east	west	west
Action for arrow to point towards conductor		reverse	do not reverse*	reverse	do not reverse*
For $e^{+i\omega t}$ analysis					
Sign of ψ component	ψ_r positive	ψ_r negative	ψ_q negative	ψ_r positive	ψ_q positive
Component and direction of computed B		B_r west	B_q west	B_r east	B_q east
Direction of conductor		east	east	west	west
Action for arrow to point towards conductor		reverse	reverse*	reverse	reverse*

*Cases where 'reversing' or 'not reversing' depends upon time dependence used.

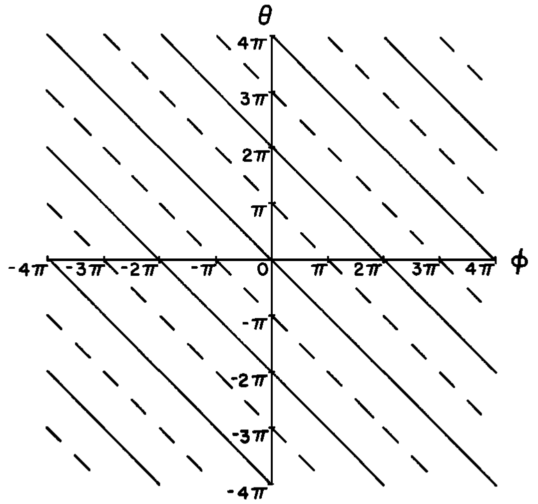

Fig. 3. Relationships between the two phase angles θ and ϕ defined by equations (10) and (11). The solid lines are for $\theta + \phi = 2n\pi$, and the solid and dashed lines are for $\theta + \phi = n\pi$, where $n = 0, \pm1, \pm2, \pm3, \cdots$.

$$\theta + \phi = n\pi$$

when only the resultant signs of χ_q/χ_r and ψ_q/ψ_r are taken into account (where $n = 0, \pm1, \pm2, \pm3, \cdots$). These relationships are shown in Figure 3.

Because (10) and (11) and Figure 3 demonstrate that quoted phase values are generally ambiguous unless the basic time dependence which underlies them is specified, it is relevant to examine two examples of how time series analysis according to either $e^{-i\omega t}$ or $e^{+i\omega t}$ may be carried out for a time series $f(t)$ which has been recorded from time t_1 to time $t_1 + T$.

Example 1: Implied Time Dependence of $e^{-i\omega t}$

The signal is expanded as a Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{T} + b_n \sin \frac{n\pi t}{T} \right) \tag{12}$$

where a_n and b_n, the Fourier cosine and sine coefficients, respectively, are given by

$$a_n = \frac{1}{T} \int_{t_1}^{t_1+T} f(t) \cos \frac{n\pi t}{T}\, dt \qquad n = 0, 1, 2, 3, \cdots$$

$$b_n = \frac{1}{T} \int_{t_1}^{t_1+T} f(t) \sin \frac{n\pi t}{T}\, dt \qquad n = 1, 2, 3, \cdots$$

Consistent with the custom given above, the phase θ of the signal at frequency $n\pi/T$ is defined by

$$\theta = \arctan (b_n/a_n)$$

Expanding the expression $\cos (\omega t - \theta)$ as $\cos \omega t \cos \theta + \sin \omega t \sin \theta$ and comparing it with (12) above shows that angle θ thus specifies the 'phase lag' of a sinusoidal signal

$$f(t) = \cos (\omega t - \theta)$$

as shown in Figure 4. Note that making such a phase lag θ more positive shifts the signal to a later real time.

Example 2: Implied Time Dependence of $e^{+i\omega t}$

An equally common method of spectral analysis is to compute the Fourier transform of the signal $f(t)$ according to some definition such as

$$g(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt \tag{13}$$

(where because of the truncation of the basic signal the integral will in fact be taken from $t = t_1$ to $t = t_1 + T$) and then to define the phase ϕ at frequency ω as being given by

$$\phi = \arctan [g_q(\omega)/g_r(\omega)]$$

where $g_r(\omega)$ and $g_q(\omega)$ are the real and quadrature components of $g(\omega)$, that is,

$$g(\omega) = g_r(\omega) + i g_q(\omega)$$

The transform definition (13) implies the inverse transform

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega) e^{i\omega t}\, d\omega \tag{14}$$

for which, at frequency ω, the real part of $g(\omega)e^{i\omega t}$ is $g_r(\omega) \cos \omega t - g_q(\omega) \sin \omega t$, of form $\cos (\omega t + \phi)$ for ϕ as defined above. The phase angle ϕ is thus a 'phase lead' as shown in Figure 4, and making ϕ more positive shifts a waveform to an earlier real time.

Discussion

Because an expansion in terms of Fourier coefficients (as in example 1) is suitable for a periodic signal, whereas expression in terms of a Fourier transform (as in example 2) is suitable only for an aperiodic signal, in principle, the two methods should be mutually exclusive. In practice, however, commonly either one or the other method may be used because geophysical signals are recorded for finite lengths of time, and the significance of any truncation that a signal may have suffered is often a matter of individual physical interpretation.

As both methods may thus be applied to the same data, it is therefore appropriate to note with *Everett and Hyndman* [1967], *Schmucker* [1970], and *Lilley* [1975] that for the Fourier transform as defined in (13),

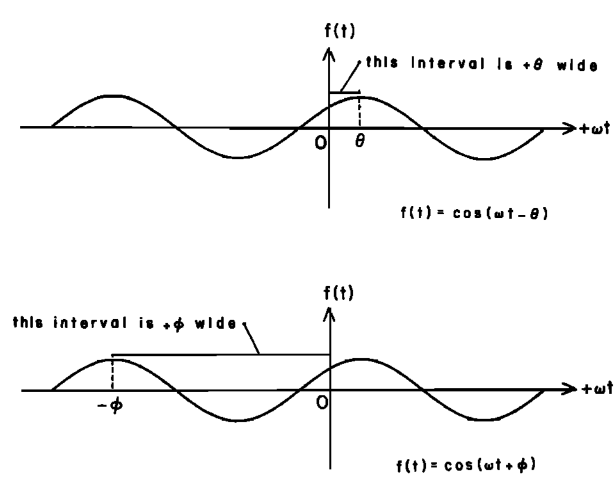

Fig. 4. Demonstration of the difference between 'lag' (θ) and 'lead' (ϕ) phases defined by equations (10) and (11).

$$g_r(\omega) = \int_{t_1}^{t_1+T} f(t) \cos \omega t \, dt \quad (15)$$

$$g_q(\omega) = -\int_{t_1}^{t_1+T} f(t) \sin \omega t \, dt \quad (16)$$

where the negative sign (which arises from the negative exponent of $e^{-i\omega t}$ in (13)) may cause confusion if it is not guarded for, as it causes the quadrature part of the full Fourier transform to be the negative of a traditional Fourier sine transform.

Examples of Plotting Conventions in Some Earlier Work

Schmucker [1964, 1970] and *Everett and Hyndman* [1967] in definitive papers on the computation of transfer functions (A and B) took Fourier transforms with the form of that in (13) above, thus implying a time dependence in the data of $e^{+i\omega t}$. Then, as discussed above, phase values computed as arc tangents of quadrature parts of transforms divided by real parts are phase leads.

Real arrows formed by plotting A_r north and B_r east were reversed by *Schmucker* [1970] to conform to *Parkinson*'s [1962] convention and so to point toward good conductors. *Schmucker*'s [1970] quadrature arrows, formed by plotting A_q north and B_q east, were not reversed and were considered to point away from good conductors (consistent with Table 1), so that substantial quadrature arrows opposed to real arrows were taken to suggest that near-surface conductivity anomalies were involved.

Cochrane and Hyndman [1970, 1974] and *Hyndman and Cochrane* [1971] formed real arrows from the negative of the real transfer function components, and quadrature arrows from the positive of the quadrature transfer function components. The 1971 and 1974 papers by these authors demonstrate both conventions for quadrature arrows, as may be seen by comparing the arrows for the common sites of the two papers.

Gough et al. [1973], in presenting quadrature arrows for the South African region, follow the convention of *Schmucker* [1970]; for consistency between real and quadrature arrows, *Gough et al.* [1974] and *Alabi et al.* [1975] reverse quadrature arrows.

Examples of Significant Applications of Quadrature Arrows

A rapid spatial variation of real arrows coupled with pronounced quadrature arrows was interpreted by *Cochrane and Hyndman* [1970] as suggesting superficial sedimentary structures to the east of their station at Grand Forks in western Canada. Similarly, *Schmucker* [1970] interpreted substantial quadrature arrows along the western and eastern slopes of the Sierra Nevada in the southwest United States to indicate concentrations of shallow currents in the San Joaquin Valley to the west of the Sierra Nevada and also along the eastern slopes of the mountains.

In addition to establishing the nature or type of a conductor as being either reactive or resistive, the separation of induction effects into real and quadrature parts can also prove advantageous in identifying distinct conductors which might exist near each other. The orientation and frequency dependence of real and quadrature arrows enabled *Gough et al.* [1973] to isolate the effect associated with closely located conductors in southern Africa: the Karroo sedimentary basin and the deep conductivity body under the Cape fold belt. *Gothe et al.* [1977] showed that while it might be possible to calculate theoretically the arrows resulting from the combined effects of two conductors, the inverse task of decomposing observed arrows into parts resulting from the conductors separately is not possible, even were the strikes of the two conductors known. The examples quoted above illustrate that separation of combined conductors may be possible, however, by examination of real and quadrature Parkinson arrows, for those cases where one conductor has a predominantly real or in-phase response and the other conductor a predominantly quadrature response.

A further application of quadrature arrows was that of *Alabi et al.* [1975] in analyzing data from an array study in the North American central plains. Northern recording stations were close to the auroral electrojet, and source field bias was considered to dominate real arrows computed, directing them away from the flow paths of ionospheric current. Computed quadrature arrows, however, were much less affected by source field bias and (reversed in this particular case) were used to show the presence and position of a major electrical conductor running through the array area.

A balancing cautionary note should be added regarding the possible vulnerability of transfer function estimates to the effects of source field bias, even at mid-latitudes. Recent discussions of this problem, and precautions for guarding against it, are given by *Anderson et al.* [1978], *Beamish* [1979], and *Gough and de Beer* [1980].

Conclusions

Consistent with reversing an in-phase Parkinson arrow so that it will point to a good electrical conductor, results for quadrature Parkinson arrows should carry a clear statement of the time dependence upon which they are based.

For a time dependence of $e^{-i\omega t}$ (as implied, for example, in a simple Fourier series expansion) a quadrature Parkinson arrow unreversed points toward a simple near-surface channeling like that of Figure 1.

For a time dependence of $e^{+i\omega t}$ (as implied, for example, in a simple Fourier transform like equation (12)) a quadrature Parkinson arrow should be reversed to point toward a simple near-surface channeling like that of Figure 1.

Similarly, phase values quoted may be ambiguous unless the time dependence upon which they are based is stated. Generally, phases associated with $e^{-i\omega t}$ will be lags, and phases associated with $e^{+i\omega t}$ will be leads.

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