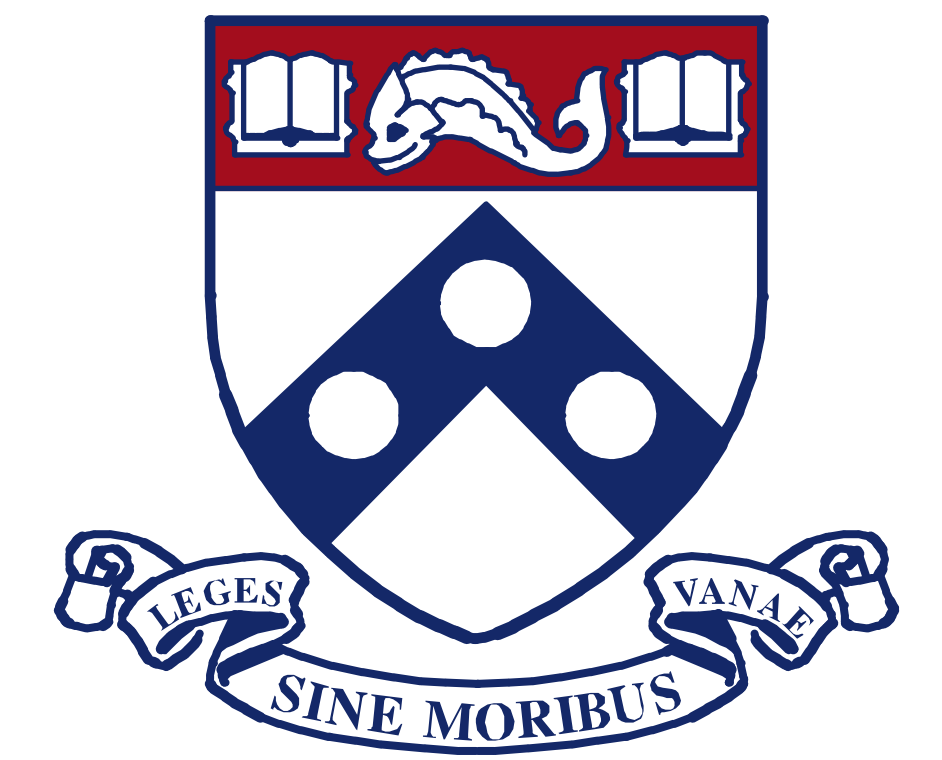


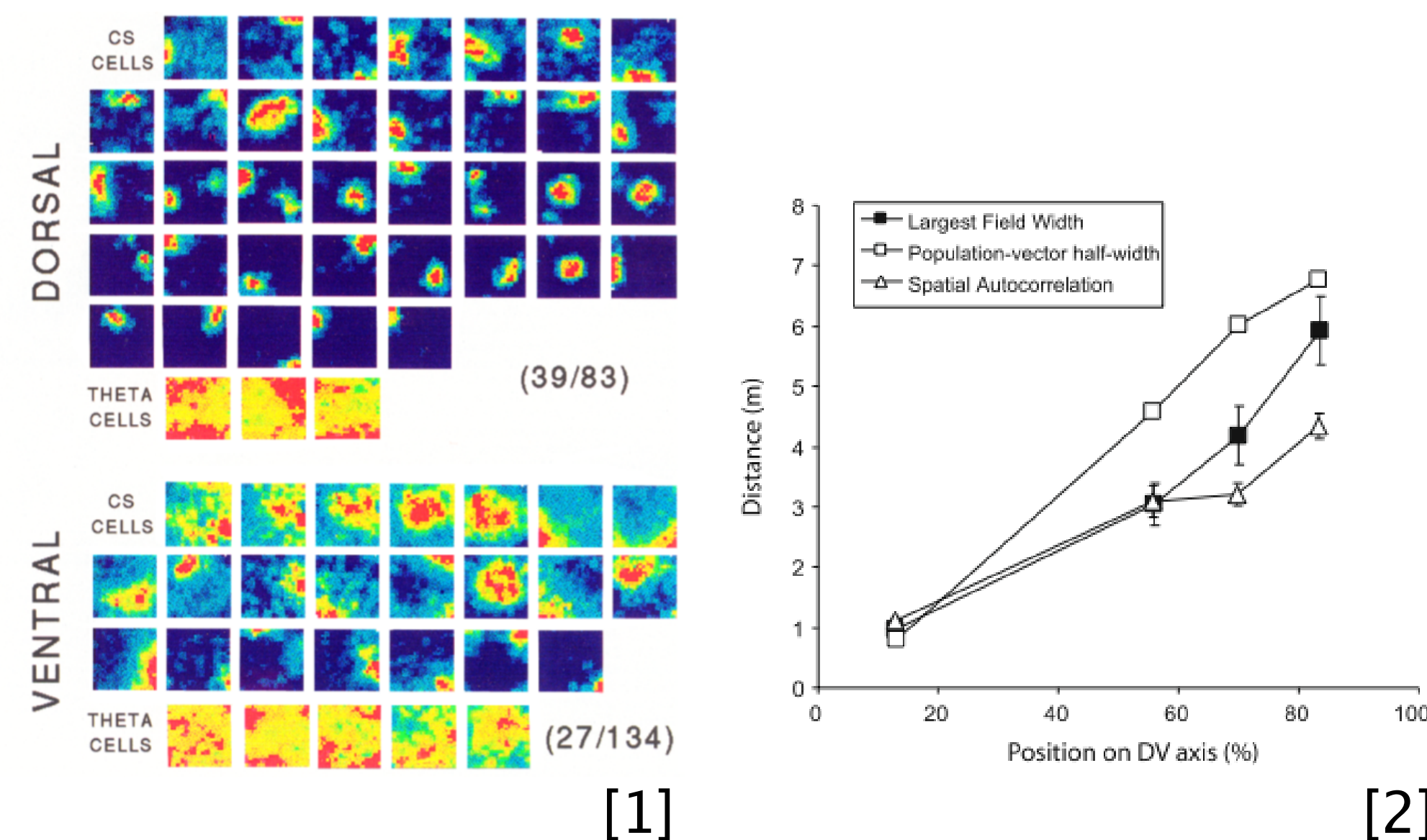
# Spatial navigation and multiscale representation by hippocampal place cells

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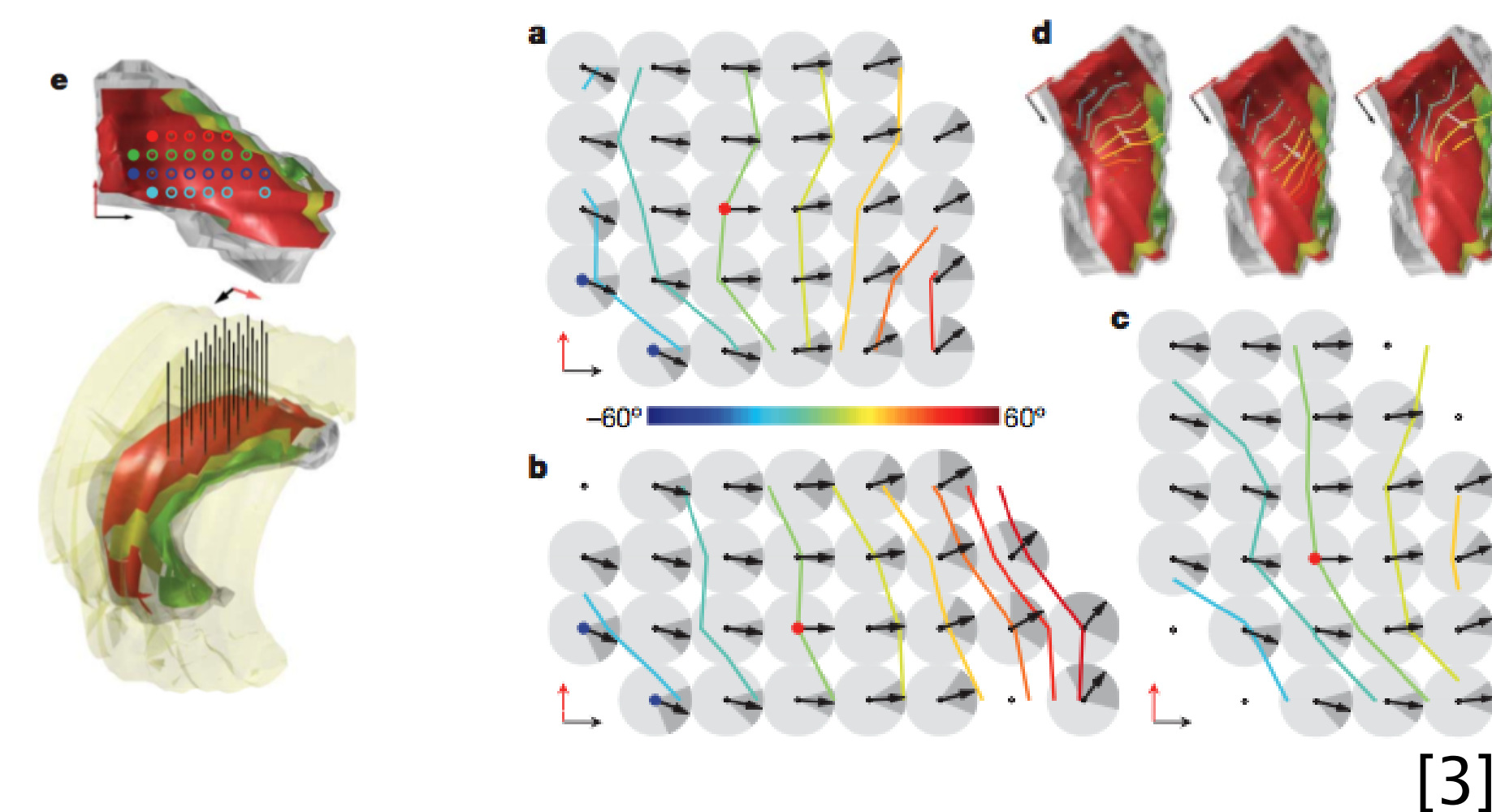


## 1. Background



Recordings of place cells at multiple points along the dorso-ventral axis of the hippocampus reveal a systematic increase in the spatial scale of the place fields [1],[2].

Hippocampal place cells fire at specific phases within a "theta" oscillation of ~10 Hz which has been linked to navigation behavior.



The theta oscillation has been shown to propagate as a travelling wave from dorsal to ventral hippocampus [3].

Thus, within every theta period, the spatial representation at a given theta phase cycles from fine- to coarse-grained.

**Question: is this multiscale dynamics useful for navigation?**

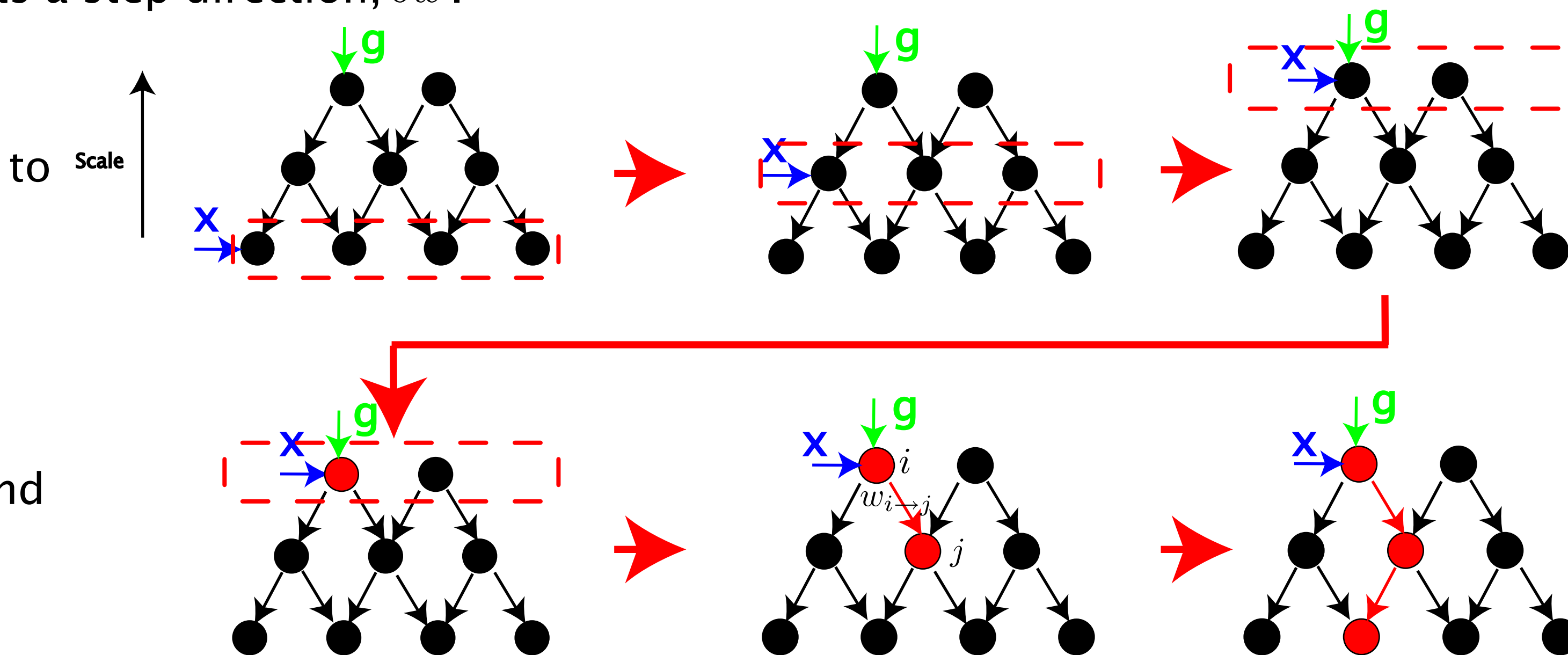
## 2. Model

Each place cell,  $i$ , has a Gaussian place field with center location  $\mu_i$  and scale  $\sigma_i$

During each theta cycle, the animal compares its current position ( $x$ ) to its goal position ( $g$ ) and selects a step direction,  $\delta x$ .

1. The different spatial scales are potentiated in sequence from small to large as the theta oscillation propagates through the network.

2. Once a place cell is identified whose place field overlaps both  $x$  and  $g$ , it becomes active.



3. Each edge in the tree has a weight  $w_{i \rightarrow j}$  that is proportional to the distance between the place field centers of parent and child. The path down the tree that minimizes the sum of the weights gives the "active set" of place fields which will be used to determine  $\delta x$ .

$$w_{i \rightarrow j} = \sigma_i^{-2} |\mu_i - \mu_j|^2$$

## 3. Model

### Determining the step direction

We model place cell activity within each iteration as binary:  $s_i = \begin{cases} 0 & \text{silent} \\ 1 & \text{spiking} \end{cases}$

To choose the step direction, we note that the "active set" of place cells are among those which are likely to fire in the next iteration if a step towards the goal is taken. Therefore,

$$\delta x(t) = \operatorname{argmax} P(\{s(t + \Delta t)\} | x(t), \delta x)$$

For independent place cells with Gaussian place fields,

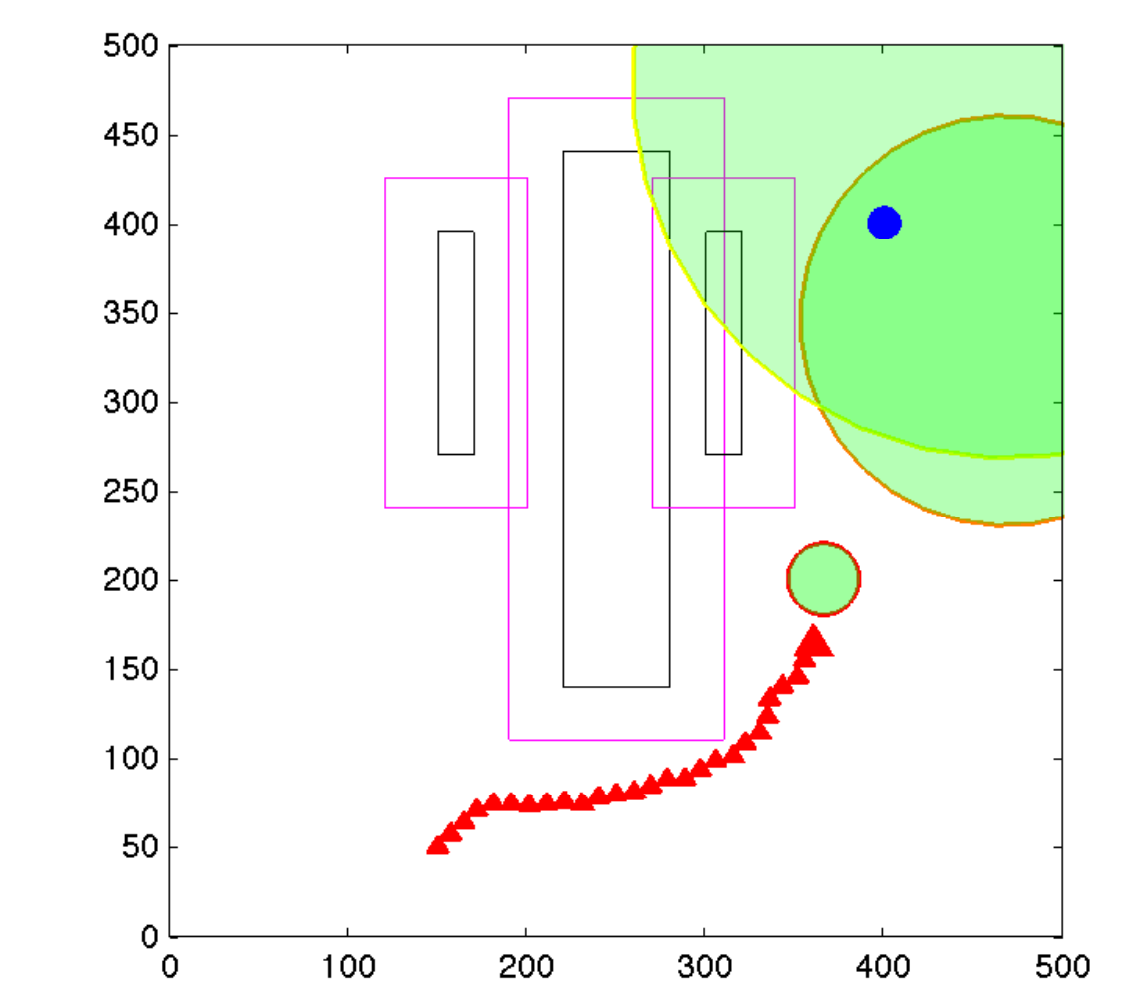
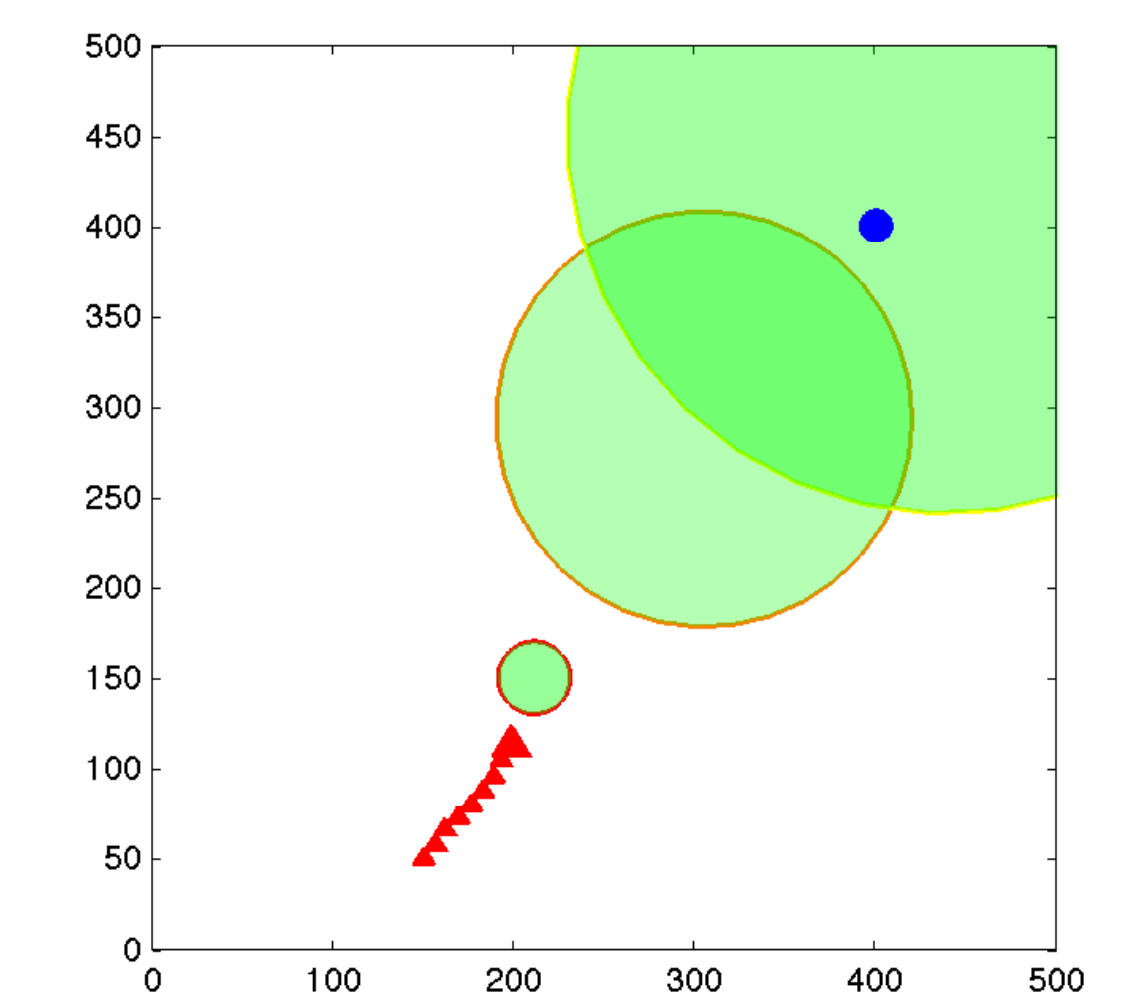
$$\delta x(t) = \frac{\sum_i s_i \sigma_i^{-2} \mu_i}{\sum_i s_i \sigma_i^{-2}} - x$$

### Navigating around obstacles

Inhibiting place cells whose fields overlap obstacles prevents them from entering the "active set" and biases the animal's path to avoid obstacles. Note that this information is available to the hippocampal formation in the form of "border cells" [4].

To prevent the animal from getting stuck in front of an obstacle, we include firing rate adaptation of the place cells.

Blue dot: goal  
Red triangles: path so far  
Green circles: 1 sigma contours of place fields in "active set"



## 4. Results

To extract a prediction from the model, we asked whether there is an optimal distribution of place cells across scales.

We assume that place field centers are randomly, uniformly distributed over the environment. For each scale,  $\sigma$ , we want the number of place cells  $n(\sigma)$  to assign to that scale, subject to a constraint on the total number of place cells,  $N$ :

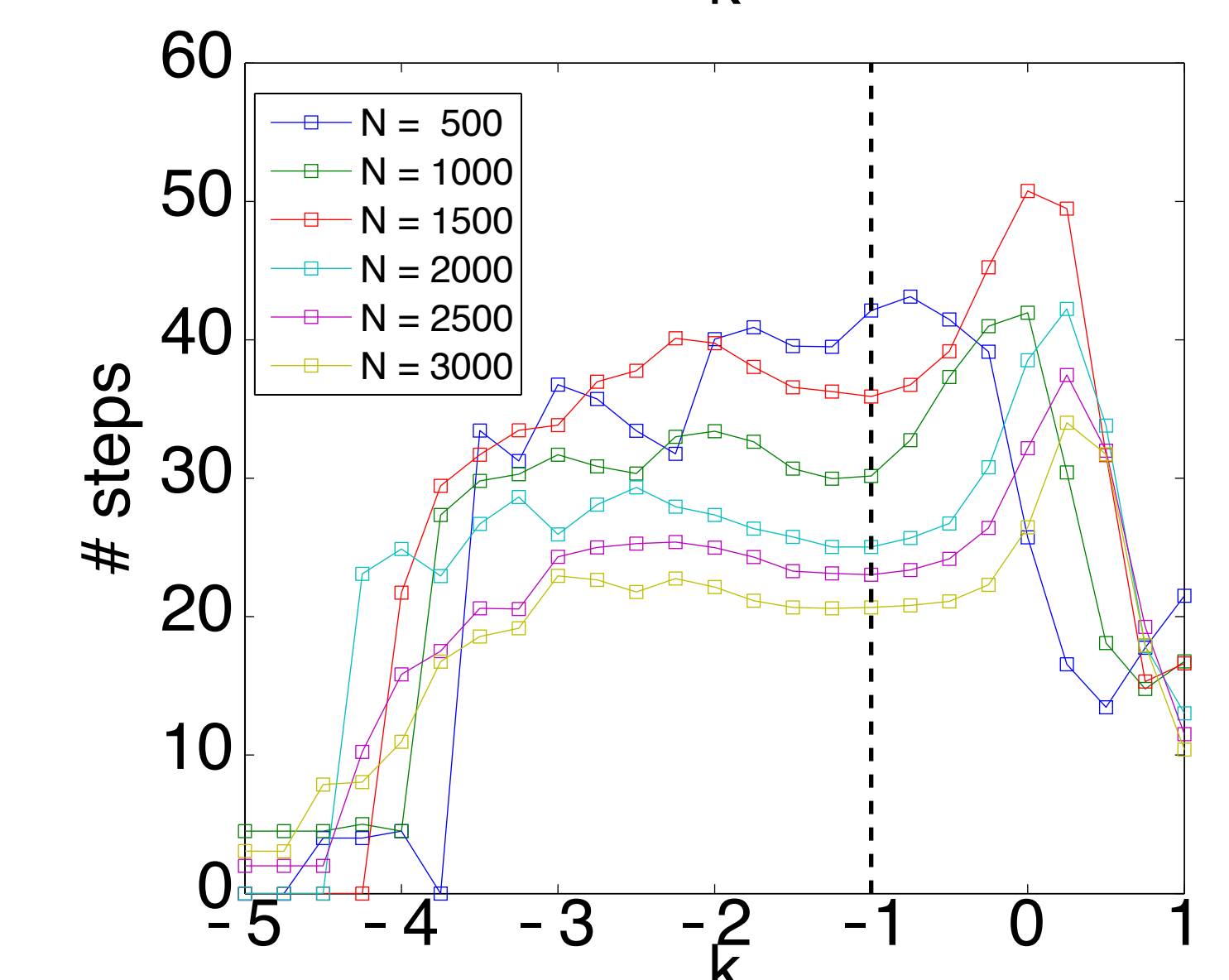
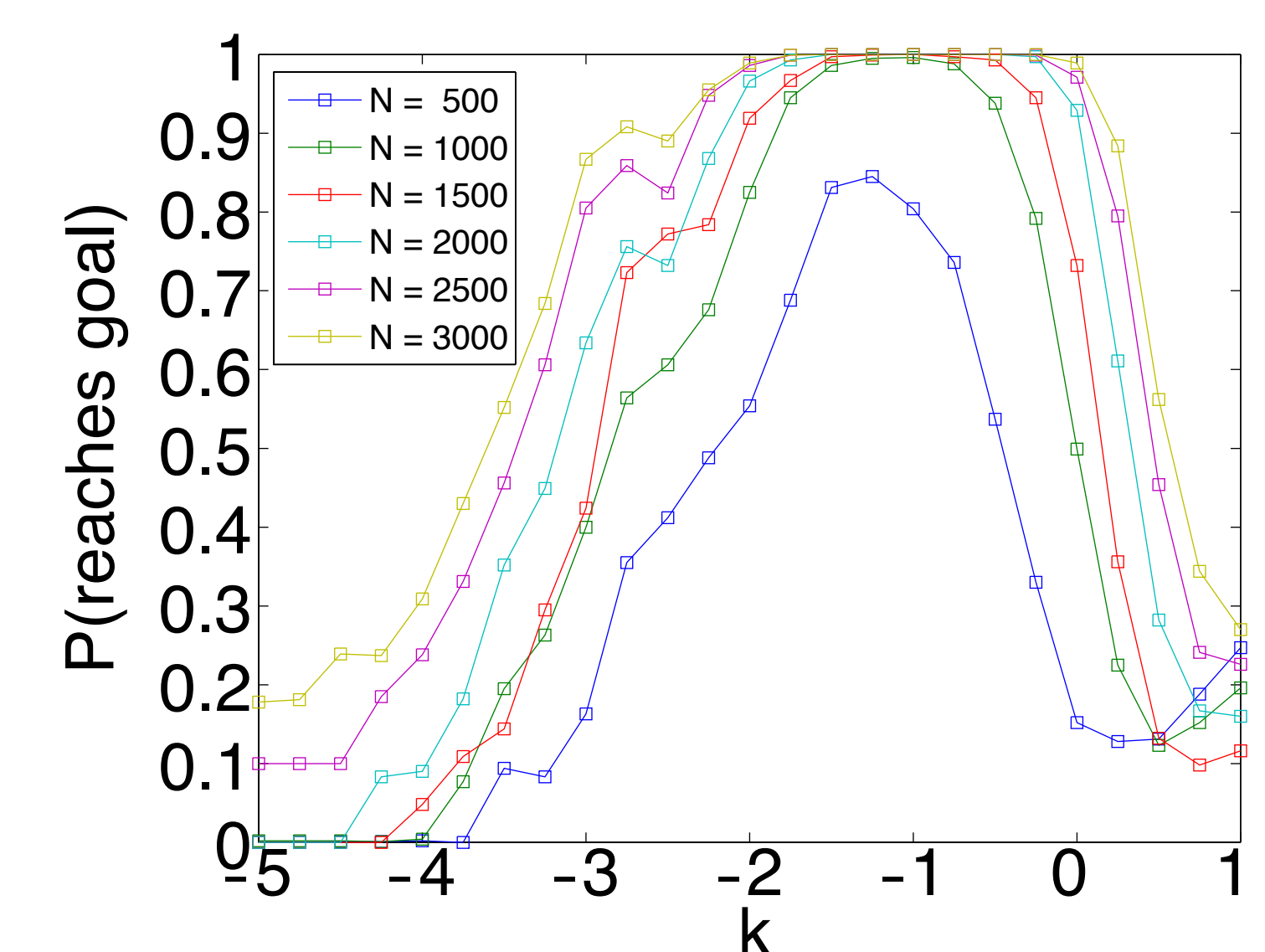
$$\sum_{\sigma} n(\sigma) = N$$

Demanding that the distribution  $n(\sigma)$  be scale-invariant (invariant under a uniform rescaling of all place field sizes and of the size of the environment) requires a power law:

$$n(\sigma) \propto \sigma^k$$

This scale-invariance approximation therefore reduces the problem to a single parameter,  $k$ .

Simulations suggest that the algorithm only produces successful navigation for  $-2 < k < 0$ , and that the shortest paths are generated for  $k = -1$ .



**References:** [1] Jung, Wiener, McNaughton, J. Neurosci. 14, 12 (1994).  
[2] Kjelstrup et. al. Science, 321, 5885 (2008).

[3] Lubenov and Siapas, Nature 459, 534 (2009).  
[4] Solstad et. al. Science, 322, 5909 (2008).