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## SIMPLEX MINIMISATION FOR MULTIPLE-REFERENCE MOTION ESTIMATION

Mohammed E. Al-Mualla, Nishan Canagarajah and David R. Bull

Image Communications Group, Centre for Communications Research, University of Bristol,  
Merchant Venturers Building, Woodland Road, Bristol BS8 1UB, United Kingdom,

Tel: +44-117-954 5126, Fax: +44-117-954 5206,

e-mail: {m.almualla, nishan.canagarajah, dave.bull}@bris.ac.uk

### ABSTRACT

This paper investigates the properties of the multiple-reference block motion field. Guided by the results of this investigation, the paper proposes three fast multiple-reference block matching motion estimation algorithms. The proposed algorithms are extensions of the single-reference simplex minimisation search (SMS) algorithm. The algorithms provide different degrees of compromise between prediction quality and computational complexity. Simulation results using a multi-frame memory of 50 frames indicate that the proposed multiple-reference SMS algorithms have a computational complexity comparable to that of single-reference full-search while still maintaining the prediction gain of multiple-reference motion estimation.

### 1. INTRODUCTION

In video coding, the high correlation between successive frames can be exploited to improve coding efficiency. This is usually achieved using *motion compensated prediction* (MCP).

Among existing methods for motion estimation, the *block matching motion estimation* (BMME) algorithm has received considerable attention and has been, implicitly, incorporated into various video coding standards (e.g. MPEG 1-2, H.261 and H.263). This is mainly due to its simplicity and good compromise between prediction quality and motion overhead. In BMME, the current frame is divided into non-overlapping blocks and the motion of each block is represented by a single *two-dimensional* vector  $\mathbf{d} = (d_x, d_y)$ . This vector is the displacement between the current block and its *best match* block within a search window in a reference frame. This match is usually decided using a *block distortion measure* (BDM) such as the *sum of absolute differences* (SAD).

BMME has always been a bottleneck problem in many video applications, e.g. wireless video terminals and software-based video codecs, especially if real-time video coding is required. For a maximum displacement of  $\pm d_m$  pixels, a *full-search* (FS) BMME algorithm performs  $(2d_m + 1)^2$  block matches per block. This computational complexity is greater than that of all the remaining encoding steps combined. This has motivated the development of a number of fast BMME algorithms, e.g. [1]-[5].

Recently, a *multiple-reference* BMME (MR-BMME) algorithm has been reported in the literature [6]. In this algorithm, previous frames are assembled in a *multi-frame memory*. For each block in

the current frame, BMME is *extended* to search over all reference frames. In addition to the *spatial displacements*  $(d_x, d_y)$ , the estimated motion vector,  $\mathbf{d}$ , of the block will now include a *temporal displacement*,  $d_t$ , which is the value of the *index* into the multi-frame memory. The technique is reported to provide significantly improved prediction gain. This is, however, at the expense of a significant increase in computational complexity. For each block, *multiple-reference full-search* (MR-FS) performs  $R(2d_m + 1)^2$  block matches, where  $R$  is the size (in frames) of the multi-frame memory. This increase in complexity calls for further research into the area of reduced complexity motion estimation.

In [4] and [5] we proposed a very efficient BMME algorithm called the *simplex minimisation search* (SMS). In this paper we extend our SMS algorithm to the multiple-reference case. The paper is organised as follows. Section 2 briefly describes the single-reference SMS algorithm. Section 3 investigates the properties of the multiple-reference block motion field. Guided by the results of this investigation, section 4 proposes three multiple-reference SMS algorithms. Section 5 presents the results of testing the proposed algorithms. Finally, section 6 gives some concluding remarks.

### 2. THE SIMPLEX MINIMISATION SEARCH (SMS)

BMME can be formulated as a *two-dimensional constrained optimisation problem* [4][5]. The two dimensions are the horizontal,  $d_x$ , and vertical,  $d_y$ , displacements, the function to be optimised (or minimised) is the BDM, and the independent variables are constrained within a limited range,  $-d_m \leq d_x, d_y \leq +d_m$ , and are usually evaluated to a certain accuracy, e.g. half- or full-pixel accuracy. In this case, the search window represents a *search space* and each possible block within this window is represented by a *search location*. The corresponding BDM values form an *error surface* and the best match block represents the *global minimum* within this surface. The motion vectors resulting from this process form a *block motion field*.

Since BMME is an optimisation problem, then it can be solved with reduced complexity using a wealth of mature optimisation techniques. In [4] and [5] we proposed to solve this problem using the *simplex minimisation* (SM) optimisation method [7]. We called the resulting solution (or algorithm) the *simplex minimisation search* (SMS).

SM, as introduced by Nedler and Mead in [7], is a multidimensional unconstrained optimisation method. A *simplex* is a geometrical figure which consists, in  $N$  dimensions, of  $N + 1$  vertices and all their interconnecting line segments, polygonal faces,

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etc. A *nondegenerate* simplex is one that encloses a finite inner  $N$ -dimensional volume. To minimise a function of  $N$  variables, the SM method must be initialised with  $N + 1$  search locations defining an *initial* nondegenerate simplex in the search space. The method then takes a series of steps, *reflecting*, *expanding* or *contracting* the simplex from the location where the function value is largest, in an attempt to find a better location. This is repeated until a termination criterion is satisfied. For more details the reader is referred to [7].

The SMS algorithm uses a *two-dimensional constrained* version of the SM method. The initial *three* vertices of the simplex are set to the motion vectors of the blocks to the left of and above the current block, and to the  $(0, 0)$  vector. If this does not produce a nondegenerate simplex then a local search is applied around the best of those three. The initial nondegenerate simplex is then chosen from this new set of candidates. The search then proceeds as described in [7] subject to the constraints that any location produced by reflection, expansion or contraction must be *set to the required accuracy* and must also be *set to the nearest location within the search space*, before any BDM evaluation can take place. The search is terminated when *the three vertices of the simplex become neighbours*. The SMS algorithm is described in greater details in [4] and [5].

Simulation results showed that the SMS algorithm outperforms other fast BMME algorithms, providing a better prediction quality, a smoother motion field and a higher speed-up ratio.

### 3. PROPERTIES OF THE MULTIPLE-REFERENCE BLOCK MOTION FIELD

The design of the SMS algorithm was based on *two* important properties of the block motion fields of typical video sequences:

**Property 1** *The distribution of the block motion field is centre-biased. This means that smaller displacements are more probable and the motion vector  $(0, 0)$  has the highest probability of occurrence.*

**Property 2** *The block motion field is smooth and varies slowly. This means that there is a high correlation between the motion vectors of adjacent blocks.*

Before extending the SMS algorithm to the multiple-reference case, it is, therefore, important to ensure that the above two properties still hold for the multiple-reference block motion field. Figures 1–3 summarise the statistical properties of the multiple-reference block motion field (with  $R = 50$ ) and compare them to those of the single-reference field<sup>1</sup>.

Figure 1 shows the distribution of the relative frequency of occurrence of the spatial displacement  $d$ , where  $d$  here refers to both  $d_x$  and  $d_y$ . The figure indicates that *Property 1* still holds for the multiple-reference case, although longer displacements are slightly more frequent than in the single-reference case.

Figure 2 shows the correlation coefficients between the motion vectors of a block and its top and left neighbours. Again, this figure indicates that *Property 2* still holds for the multiple-reference case.

<sup>1</sup>The figures were generated using the luminance components of 6 QSF sequences with different motion content and different frame skips. Motion was estimated using full-search full-pixel block matching, SAD as the BDM,  $16 \times 16$  blocks,  $\pm 15$  maximum displacement, restricted motion vectors and original reference frames.

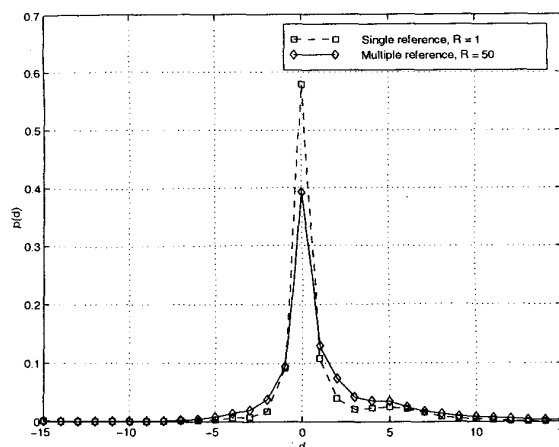


Figure 1: Frequency of occurrence of the spatial displacement,  $d$ , for the single-reference,  $R = 1$ , and multiple-reference,  $R = 50$ , cases.

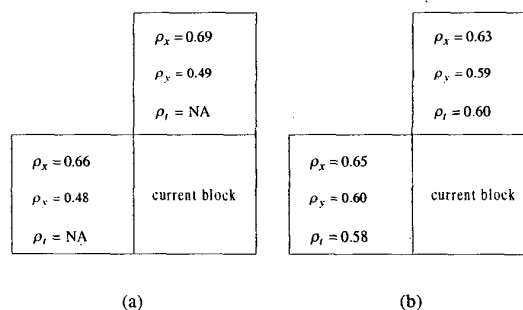


Figure 2: Correlation coefficients between the motion vector of a block and its top and left neighbours in (a) the single-reference,  $R = 1$ , case and (b) the multiple-reference,  $R = 50$ , case.

Figure 3 shows the distribution of the relative frequency of occurrence of the temporal displacement  $d_t$ . The temporal displacement  $d_t = 1$  (which refers to the most recent reference frame in memory) has the highest frequency of occurrence. This property will be exploited in the next section.

### 4. MULTIPLE-REFERENCE SMS

The above investigation indicates that *moving from a single-reference to a multiple-reference system does not significantly change the properties of the block motion field*. Thus, the SMS algorithm can be extended to the multiple-reference case. We propose three different extensions (or algorithms) as follows.

**MR-SMS** This is a direct extension of SMS. For each block, the SMS algorithm is used to search each frame in the multi-frame memory and produce a best match from that frame.

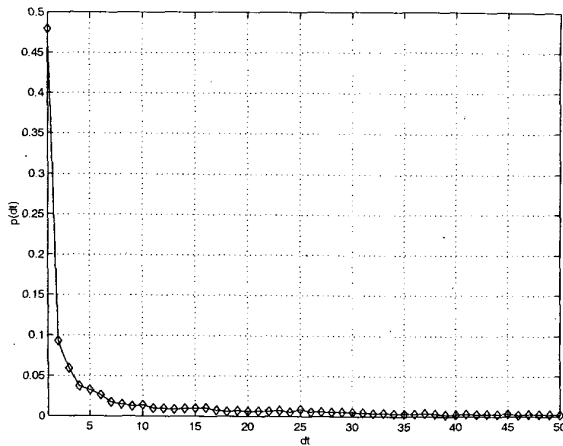


Figure 3: Frequency of occurrence of the temporal displacement,  $d_t$ , for the multiple-reference,  $R = 50$ , case.

The overall best match is then chosen from this set of  $R$  candidates.

**MR-FS/SMS** This is the same as MR-SMS but the most recent frame in memory (i.e. the frame for which  $d_t = 1$ ) is searched using full-search instead of SMS. This is motivated by the observation, made earlier, that the most recent frame has the highest probability of selection. Thus, it must be given more importance.

**MR-3DSM** The single-reference SMS search is based on a two-dimensional version of the SM optimisation method. Algorithm MR-3DSM, however, is based on a *three-dimensional* version. For each block, the initialisation procedure described in section 2 is applied to each frame in the multi-frame memory. This will generate an initial simplex of three vertices for each frame. The best *four* vertices, in terms of BDM value, are selected from this set of  $3R$  candidates. Those four vertices are then used as an initial simplex to a three-dimensional version of the SM optimisation method (the third dimension here is the temporal displacement). The same termination criterion described in section 2 is used, with the added condition that *the four vertices of the final simplex must have the same temporal displacement*.

## 5. SIMULATION RESULTS AND DISCUSSIONS

The proposed algorithms were tested using the luminance components of three QSIF sequences: AKIYO, FOREMAN and TABLE TENNIS<sup>2</sup>. Note that each sequence represents one of the classes defined by MPEG. AKIYO represents class A: low amount of movement and low spatial detail, FOREMAN represents class B: medium amount of movement and low spatial detail or vice versa, and TABLE TENNIS represents class C: high amount of movement

<sup>2</sup>Each sequence included 300 frames. AKIYO and TABLE TENNIS have luminance components of  $176 \times 120$  @ 30 fps, whereas FOREMAN has a luminance component of  $176 \times 144$  @ 25 fps.

Table 1: Comparison between different multiple-reference block matching algorithms in terms of prediction quality (average PSNR in dB) with a multi-frame memory of 50 frames.

	AKIYO	FOREMAN	TABLE
FS	45.93	32.20	32.17
MR-FS	46.55	33.97	32.87
MR-SMS	46.55	33.87	32.67
MR-FS/SMS	46.55	33.92	32.80
MR-3DSM	46.55	33.51	32.46

Table 2: Comparison between different multiple-reference block matching algorithms in terms of computational complexity (average searched locations/frame) with a multi-frame memory of 50 frames.

	AKIYO	FOREMAN	TABLE
FS	65,621	77,439	65,621
MR-FS	3,012,200	3,554,700	3,012,200
MR-SMS	38,880	106,830	69,443
MR-FS/SMS	103,820	183,240	134,270
MR-3DSM	35,867	66,357	45,518

and medium spatial detail or vice versa. The BDM was defined to be the SAD, the block size was set to  $16 \times 16$  pixels, and the maximum allowed motion displacement was assumed to be  $\pm 15$  pixels. The search was performed to full-pixel accuracy and motion vectors were restricted so that they don't point outside the frame. Motion was estimated and compensated using original previous frames. In addition to the proposed multiple-reference SMS algorithms, the single-reference full-search (FS) and the multiple-reference full-search (MR-FS) algorithms were also simulated. A multi-frame memory of size  $R = 50$  frames was employed.

Tables 1 and 2 summarise the results of the simulation. It is immediately evident that the proposed multiple-reference SMS algorithms provide significant reductions in computational complexity compared to the MR-FS algorithm. The proposed algorithms represent different degrees of compromise between prediction quality and computational complexity. At one extreme is the MR-3DSM. Compared to MR-FS, the MR-3DSM provides significant reductions in computational complexity (a speed up ratio of about 54–66) at the expense of a moderate reduction in prediction quality (about 0.41–0.46 dB loss). At the other extreme is the MR-FS/SMS algorithm. It uses full-search on the most recent frame in memory to provide a prediction quality which is almost identical to that of MR-FS (about 0.05–0.07 dB loss) and still achieves moderate reductions in computational complexity (a speed up ratio of about 19–22). Between the two extremes is the MR-SMS. Compared to MR-FS it achieves a speed-up ratio of about 33–43 with only a slight loss in prediction quality (about 0.1–0.2 dB loss). The above observations are further emphasised using Figure 4 which shows the prediction quality for the FOREMAN sequence at different frame skips.

A very interesting point to note is that the computational complexity of the proposed algorithms is comparable (and in some cases less than) that of single-reference FS and yet they still maintain the improved prediction gain of multiple-reference motion es-

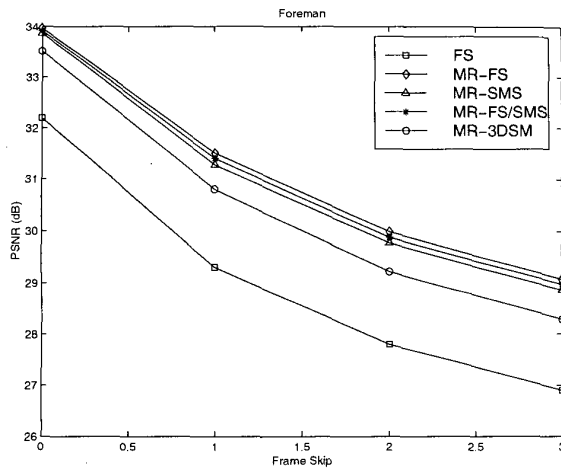


Figure 4: Prediction quality of the FOREMAN QSIF sequence at different frame skips. The multiple-reference (MR) algorithms use a multi-frame memory of 50 frames.

timation. This is also illustrated in Figure 5 which shows the subjective quality of the motion compensated 158<sup>th</sup> frame of FOREMAN. The uncovered background at the bottom-right corner of the frame is poorly compensated using single-reference FS (Figure 5(b)). This uncovered background is compensated with higher quality using the multiple-reference algorithms (Figures 5(c) and 5(d)). While the MR-FS achieves this improved prediction quality at the expense of about 50 orders of magnitude increase in computational complexity, our proposed MR-3DSM algorithm provides a similar improvement at no increase in computational complexity.

## 6. CONCLUSIONS

In this paper we investigated the properties of the multiple-reference block motion field. We found that moving from a single-reference system to a multiple-reference system does not significantly change the properties of the block motion field. Guided by the results of this investigation we extended the single-reference SMS algorithm to the multiple-reference case. Three multiple-reference SMS algorithms were proposed providing significant reductions in computational complexity compared to the multiple-reference full-search. The proposed algorithms represent different degrees of compromise between prediction quality and computational complexity. Simulation results using a multi-frame memory of 50 reference frames indicated that the proposed multiple-reference SMS algorithms have a computational complexity comparable to that of single-reference full-search while still maintaining the prediction gain of multiple-reference motion estimation.

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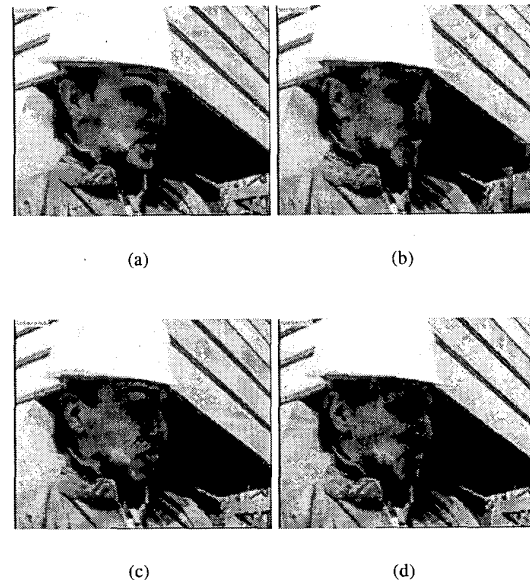


Figure 5: Subjective quality of the motion compensated 158<sup>th</sup> frame of the QSIF FOREMAN sequence @ 25 fps: (a) original frame, (b) compensated using single-reference FS; 28.24 dB and 77,439 searched locations, (c) compensated using MR-FS with  $R = 50$ ; 31.31 dB and 3,871,950 searched locations, and (d) compensated using MR-3DSM with  $R = 50$ ; 31.04 dB and 72,532 searched locations.

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