"SIRIUS-T" PACKAGE FOR CHARACTERISTICS EVALUATION OF THE QUEUEING SYSTEMS

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This paper contains description of new features of software package "Sirius-T" as an elaboration of "Sirius++" and "Sirius-C" packages. "Sirius-T" introduces new models for calculation characteristics of tandem queues allowing performance evaluation and capacity planning of the telecommunication networks and their fragments. Additionally, the software facilitates a study of queueing models under consideration of correlated service and evaluation of the algorithms' numerical complexity.

Keywords: Telecommunications, Computer Networks, Performance Characteristics, Queueing Systems, Tandems.

1. INTRODUCTION

The described software "Sirius-T" is the next generation of the software "Sirius++" that is described in [5]. Apart minor extensions that facilitate study of the queueing systems under consideration of the correlated service (described in terms of semi-Markovian process) some major improvements have been made.

One of such improvements allows analysis of numerical complexity of algorithms. Due to approximate nature of the implemented algorithms there is no analytical way known to perform such analysis. However, with minor performance loss it is possible to gather the information during calculation.

Another improvement allows using calculation under different platforms (e.g. different operating systems or hardware). This is important while calculation precision differs depending on the development tools shipped with the platform.

Finally, the most significant improvement was achieved by adding support of new queueing systems. In particular several tandem queues have been added to a set of supported models. This allows studying of more complex networks and their fragments.

The software "Sirius-T" has been developed in C++ using Microsoft Visual Studio 2008 and GNU GCC 4.5.1.

2. DESCRIPTION OF THE TANDEM QUEUES

Before describing tandem models supported by the "Sirius-T", let us briefly specify common aspects of studied tandem queues. The key difference of tandems from single server queues is that it has two service devices (servers). Service time duration at the first server has general distribution with the first moment b_1, $0 < b_1 < \infty$. The first server has a buffer of size N (two cases are considered $N < \infty$ and $N = \infty$).

The second server is characterized by the PH-type service time distribution having an irreducible representation (β, S). Here β is the stochastic row-vector of dimension K and S is $K \times K$ matrix having the negative diagonal and non-negative non-diagonal entries, such as the column vector $S_0 = -S\mathbf{e}$ is non-negative and has at least one positive entry. The average service time is defined as $\beta(-S)^{-1}\mathbf{e}$. Here and in the sequel $\mathbf{e}$ is column vector of appropriate size consisting of units. For more information about PH see [7]. The buffer before the second server has size $M-1$, $M < \infty$.

Each request needs to be sequentially served by first and second servers.

"Sirius-T" continues utilizing batch Markovian Arrival process ($BMAP$) as an input flow. The $BMAP$, a special class of tractable Markov renewal process, is a rich class of point processes. It includes many well-known processes such as stationary Poisson, PH-renewal process, Markov Modulated Poisson Process ($MMPP$) and others. The epochs of customers' arrival (possibly in batches of a random size) coincide with the transition epochs of some continuous-time Markov chain $\nu_t, t \geq 0$ called as the directing process of the BMAP. This process has a finite state space $\{0, \ldots, W\}$ and it is completely defined by some set of $(W+1) \times (W+1)$ matrices $D_k, k \geq 0$. The matrix D_k consists of intensities of transitions of the process $\nu_t, t \geq 0$ that are accompanied by arrival of a batch of size k into the system. The BMAP was introduced by D.Lucantoni in [6] as a more nice form of versatile input process introduced earlier by M.Neuts in [8]. The BMAP is recommended by many researchers as a good descriptor of flows in the modern telecommunication networks. It takes well into account the bursty, correlated nature of these flows. It makes its exploiting to model the real life flows being attractive.

For understanding the examples from the following section we need to describe some additional characteristics of the BMAP flow. A matrix generating function

$$D(z) = \sum_{k=0}^{\infty} D_k z^k, \qquad |z| \leq 1,$$

introduced by D.Lucantoni in [6] facilitates analytical study of queueing systems. The vector $\boldsymbol{\theta}$ of the process ν_t, $t > 0$ stationary distribution satisfies equations $\boldsymbol{\theta} D(1) = \mathbf{0}, \boldsymbol{\theta}\mathbf{e} = 1$. Here $\mathbf{0}$ is zero row vector of appropriate size. The average intensity λ (fundamental rate) of the $BMAP$ is defined as

$$\lambda = \boldsymbol{\theta} D'(z)|_{z=1}\mathbf{e}.$$

The intensity λ_g of groups arrival is defined as

$$\lambda_g = \boldsymbol{\theta}(-D_0)\mathbf{e},$$

variance c_{var} of intervals between the groups arrival is calculated as:

$$c_{var} = 2{\lambda_g}^{-1}\boldsymbol{\theta}(-D_0)^{-1}\mathbf{e} - {\lambda_g}^{-2},$$

the correlation coefficient c_{cor} of intervals between the successive groups arrival is calculated as

$$c_{cor} = ({\lambda_g}^{-1}\boldsymbol{\theta}(-D_0)(D(1) - D_0)(-D_0)^{-1}\mathbf{e} - {\lambda_g}^{-2})/v.$$

We consider partial admission losses at first server if $N < \infty$, i.e. the only requests from batch arrival will be lost that exceed the free buffer capacities.

We can now describe and illustrate with numerical examples models supported by "Sirius-T".

3. TANDEM QUEUES WITH LOSSES

We consider $BMAP|G|1|N \to \cdot|PH|1|M$ tandem with possible losses at the first and the second server (please see the figure 1 for illustration). This model was studied in paper [2].

In case the entering batch of customers finds insufficient number of places in a buffer (or the buffer is already full at all), the appropriate number of customers from the batch joins a queue while the rest (or even the whole group) leaves the system forever (i.e. is lost at the first server). We denote the probability $P^{(1)}_{loss}$ of the request to be lost at the first server. In case the customer completes the service at first server and meets the buffer before the second server be busy, this customer leaves the system forever and is considered to be lost at the second server. We denote the probability $P^{(2)}_{loss}$ of the request to be lost at the second server. If buffer before first server is infinite, then requirements can be lost at the second server only.

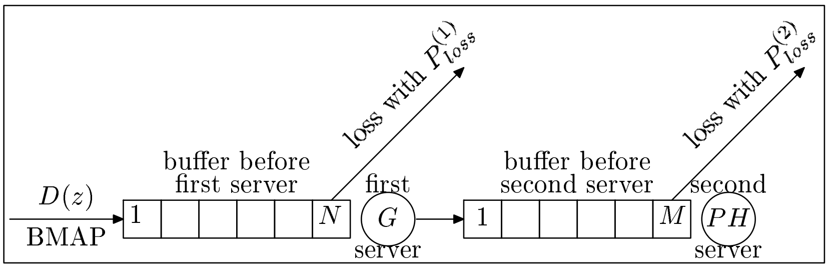

Fig. 1. Structure of the tandem queue with losses

Let us illustrate the module with some example.

The input parameters of the tandem are defined as specified below.

Table 1 $BMAP$**-flow of intensity** $\lambda = 10$**, intensity of groups** $\lambda_g = 5$**, with correlation** $c_{cor} = 0.2$**, and variation** $c_{var} = 12.2732$

D_0		$D_1 = D_3$		D_2	
-6.74538	5.45412×10^{-6}	2.01021	0.0134084	2.68027	0.0178778
5.45412×10^{-6}	-0.219455	0.036728	0.0291068	0.0489707	0.038809

Service time at the first server is degenerate. To vary service time the following values are used $\{0.01, 0.03, 0.05, 0.07, 0.08, 0.085, 0.09, 0.095, 0.1, 0.12, 0.14\}$.

Phase-type service time distribution at the second server has the following parameters

$$S = \begin{bmatrix} -20 & 0 \\ 0 & -80 \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}.$$

These parameters make mean service time at the second phase equal to 0.03875.

The buffer capacity M at the second server is 2.

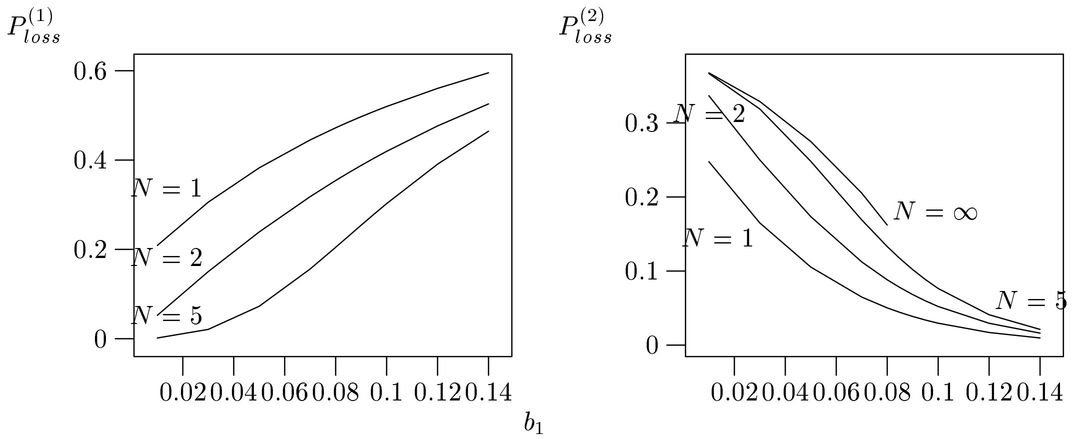

Fig. 2. Loss probability $P^{(1)}_{loss}$ and $P^{(2)}_{loss}$ at the first and at the second servers depending on average service time b_1 at the first phase

4. TANDEM QUEUES WITH BLOCKINGS

We consider $BMAP|G|1|N \to \cdot|PH|1|M$ tandem with possible losses at the first server and blockings of the first server if there is no free space in a buffer at the second server (please see the figure 3 for illustration). This model was studied in paper [1].

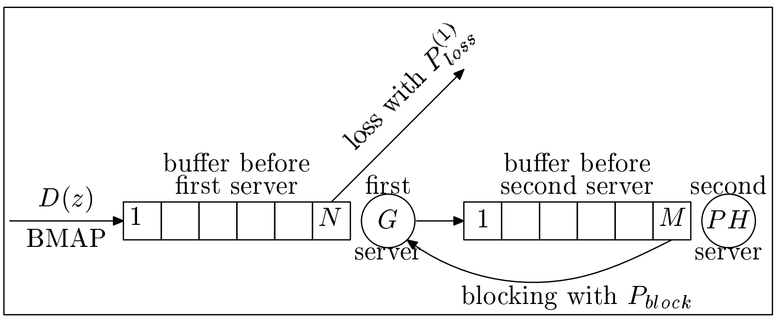

Fig. 3. Structure of the tandem queue with blockings

Let us illustrate the module with some example. We will use input data from the previous section. However, due to ergodicity condition, in case of an inifinite first buffer, we need to limit service time at the first server to the following values $\{0.01, 0.03, 0.05, 0.07, 0.08\}$.

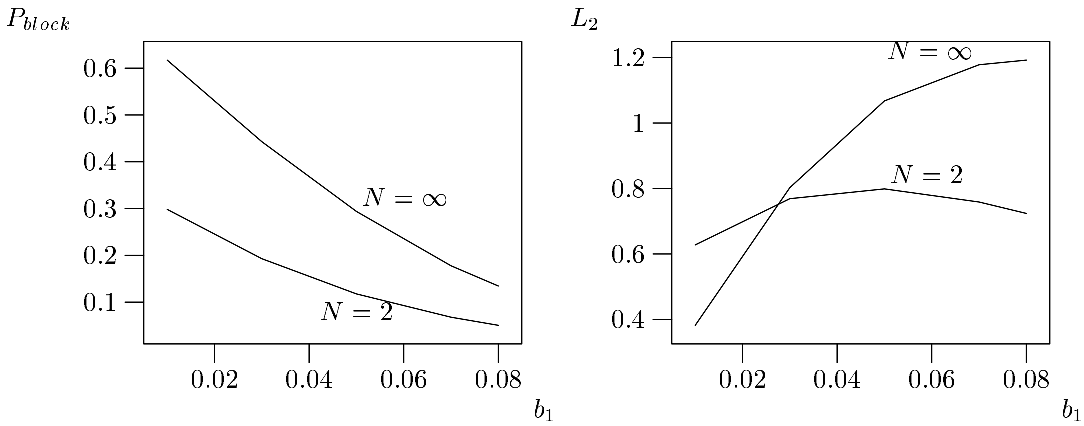

Fig. 4. Blocking probability P_{block} and average queue length L_2 at the second server depending on average service time b_1 at the first phase

5. TANDEM QUEUES WITH FEEDBACK AND LOSSES

We consider $BMAP|G|1|N \to \cdot|PH|1|M$ tandem with possible losses at the first server and at the second server as well as feedback mechanism (please see the figure 3 for illustration). By feedback we meen, that any request with probability p after been served at the second server can return to the buffer at the first server, and with probability $1 - p$ can leave system forever. This model was studied in paper [4].

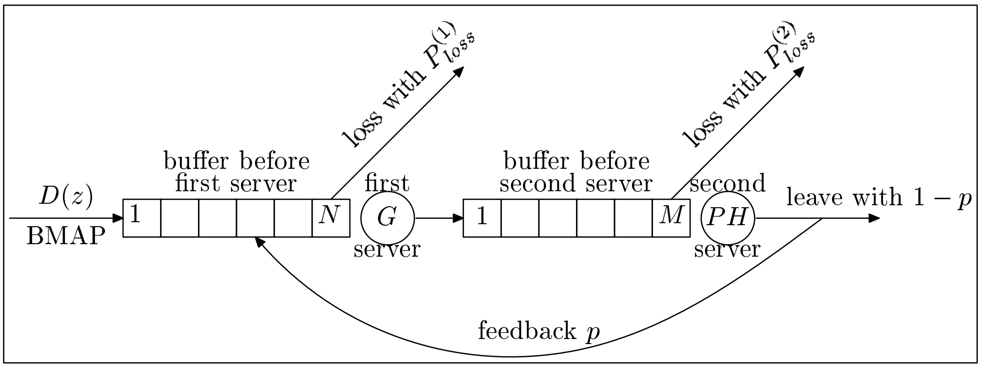

Fig. 5. Structure of the tandem queue with feedback and losses

Let us illustrate the module with some example. We will use input parameters as in the second example, but the service time distribution at the first server we consider deterministic with $T = 0.05$.

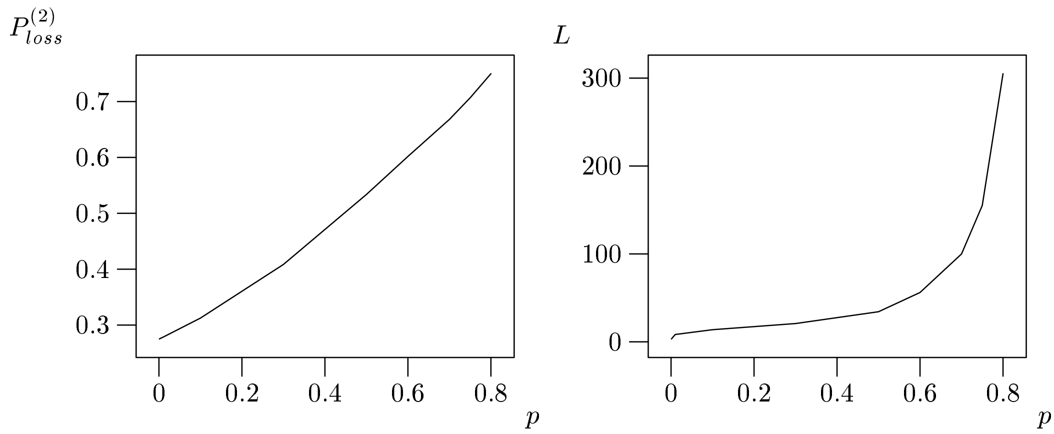

Fig. 6. Loss probability $P_{loss}^{(2)}$ and average queue length L_2 at the second server depending on feedback probability p

6. TANDEM QUEUES WITH RETRIALS AND LOSSES

We consider $BMAP|G|1 \to \cdot|PH|1|M$ tandem with retrials at the first server and possible losses at the second server. The orbit before the first server is infinite. If an arriving request finds the first server busy it joins the orbit and tries to get service in exponentially distributed time intervals with intensity $\alpha_i = i\alpha + \gamma$, where i denotes number of customers at the orbit. This model was studied in paper [3].

We will use the same input data as in previous section, but the retrial intensity in this example is considered to be linear one, i.e.

$$\alpha_i = i\alpha + \gamma, \qquad i \geq 1,$$

where i denotes number of customers in an orbit and $\alpha = 3$, $\gamma = 5$.

The buffer size at the second server as well as the calculated performance characteristics are given in the table 2 below.

Additionally, probability $P_0^{(2)}$ denotes that the second server is idle at arbitrary epoch.

Table 2 **Results of some performance characteristics calculations for different input parameters**

Parameters	Case 1	Case 2
M	9	4
L_1	1.96517	1.13647
L_2	2.33711	1.098326
P_{loss}	0.178394	0.254642
$P_{0,0}$	0.279899	0.286573
$P_0^{(2)}$	0.308468	0.281546

7. CONCLUSION

The new “Sirius-T” software package allows study of new models to calculate performance characteristics of the tandem queueing systems as a adequate models of the fragments of the telecommunication networks. Apart that, it extends algorithms with additional characteristics allowing investigation of numerical complexity. All this makes “Sirius-T” application valuable not just for sceintists, but also for engineers who needs to perform analysis of some telecommunication networks or their fragments.

The development of the “Sirius-T” will be continued to allow handling more complex systems and algorithms.

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