$$\sum_{(i,j)\in\widehat{U}} \lambda_{ij}^p x_{ij} = \alpha_p, \quad p = \overline{1,q}, \tag{4}$$

where $\widehat{I}_i^+(\widehat{U}) = \left\{j\colon (i,j) \in \widehat{U}\right\}, \quad \widehat{I}_i^-(\widehat{U}) = \left\{j\colon (j,i) \in \widehat{U}\right\}$; a_i, λ_{ij}^p, α_p – parameters of the system; q – the number of additional equation connecting arc flow for proceeding arc from through the set split ratio coefficients; $x = (x_{ij}, (i,j) \in U;\ x_i,\ i \in I^*)$ – vector of unknowns.

Calculation of a rank of the matrix of the system (3)–(4), building the algorithms for finding the solutions of the systems of the type (3)–(4) and characteristics of optimal solutions are investigated in [2]–[4].

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NUMERICAL MODELING OF A STATIC MAGNETIC FLUID SEAL SUBJECT TO DIFFUSION OF FERROMAGNETIC PARTICLES

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Because magnetic fluid is a stable colloidal suspension of small ferromagnetic particles in a carrier liquid, its macroscopic interaction with an external nonuniform magnetic field is determined by the force acting on each separate particle. The force causes a Brownian motion of the particles with respect to the carrier liquid, as a result of which the particle concentration increases in the places where the magnetic field intensity is higher. This leads to redistribution of the fluid magnetization M being of a basic magnetic characteristics of the fluid which is defined by the relation $M = M_s C L(\xi H)$ where M_s is the saturation magnetization of the fluid; C, the volume concentration of particles; $L(t) = \coth t - 1/t$, the Langevin function; H, the magnetic field intensity; $\xi = \mu_0 m/kT$; μ_0, the magnetic constant; m, the magnetic moment of a particle; k, the Boltzmann constant; T, the fluid temperature. The steady-state distribution of the concentration C in the fluid volume V is described by the equation $\nabla \cdot (\nabla C - \xi C L(\xi H) \nabla H) = 0$ with the Robin-type boundary condition $\partial C/\partial n - \xi L(\xi H)(\partial H/\partial n) C = 0$ and the condition of particle mass conservation $\int_V C dV = C_0 V$ where C_0 is a constant corresponding to a uniform distribution of particles. Exact solution of the problem is given in [1] and is of the form $C = \varphi C_0 V \big/ \int_V \varphi dV$, $\varphi = \sinh(\xi H)/(\xi H)$. A Stefan-type diffusion problem can arise if the fluid is under the action of a high-gradient magnetic field. The point is that the particles diffuse in the direction of magnetic gradient ∇H and if the gradient is sufficiently large, particle concentration in the magnet pole vicinity reaches a maximum possible value corresponding to the dense packing of the particles.

Taking into account the particle diffusion process is a distinctive feature of a contemporary mathematical simulation of hydrostatics problems for a magnetic fluid. The subject of present study is the problem on stability of a static magnetic fluid seal under the action of external pressure drop allowing for the particle diffusion phenomenon. Notice that magnetic fluid sealing is the most known application of magnetic fluid in engineering. Its idea is that the sealing of a gap between solid surfaces, e.g. between a rotating shaft and a body of machine, is realized by means of filling the gap with a magnetic fluid which is held by a nonuniform magnetic field. We consider a behavior of a magnetic fluid plug in a narrow profiled gap between two coaxial fixed cylindrical surfaces under the action of an external pressure drop and magnetic-and-capillary forces. To the outer cylindrical surface, an annular axially magnetized permanent magnet with a hyperbola-shaped pole head is attached. A magnetic fluid between the shaft and the pole head forms a hermetic barrier that isolates regions with different pressures. The plug is retained in the gap by the high-gradient magnetic field directed across the gap, which opposes a destructive action of pressure. The stability limits of the magnetic fluid plug are determined by deformation of the disconnected free surface that consists of two asymmetric fragments with no points of contact. It should be noted that the problem was solved in [2] under the assumption of a uniform particle concentration, i.e. with neglect of the particle diffusion influence on stability of the seal.

It is essential that although the problem is of the coupled type and its physical statement is sufficiently complicated, but its mathematical model admits an explicit analytical solution even if the particle diffusion effect is taken into account. Application of numerical methods is limited to calculation of definite integrals and the dichotomy procedure.

A free surface shape and a plug location in the gap are determined mainly by the fluid magnetization M depending on the magnetic field intensity and the particle concentration on the free surface. It is found numerically that redistribution of concentration due to the magnetophoretic transfer of particles results in the change of sealing ability of the magnetic fluid seal. The particle concentration in the sealing layer is appreciably nonuniform up to formation of the domain with the dense particle packing that leads to the Stefan-type problem on concentration and influences appreciably on the burst (critical) pressure drop at which the seal is broken. If the magnetic fluid volume V is small, diffusion of particles results in decreasing the critical pressure drop. A contrary situation is observed at large volumes corresponding to real magnetic fluid seals: in consequence of the diffusion effect, the critical pressure drop increases considerably in comparison with the critical value obtained under the uniform concentration approximation. These results are in good agreement with experimental data [3]. They are of interest not only to the hydrodynamic magnetic fluid theory but also from the viewpoint of their practical use in the computerized optimization of magnetic fluid sealing devices.

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