represented in analytic form in terms of series with respect to certain group of symmetries which are obtained by using functional equation method.

Thus, at the study of properties of circular composites with constant conductivity of each component one can use an analytic formulas for the solution as well as analytic formulas for effective conductivity [1].

When the multiply connected domain tends to the limiting punctured domain the elliptic state equation becomes degenerating at a finite number of points. Thus we have to perform an asymptotic analysis of the solution to the corresponding boundary value problems. It is difficult to refer to all essential contribution to the asymptotic study of the solution to the boundary value problems for elliptic equations in singularly perturbed domain. Recently a new approach has been proposed (see, *e.g.*, [2]) based on the methods of nonlinear functional analysis. In our work we use also a special type of asymptotic analysis (see [3]) performed for the case boundary value problems for elliptic equations degenerating at a line (in three-dimensional case) or at a point (in two-dimensional case).

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UNDERDETERMINED LINEAR SYSTEMS IN THE SENSOR LOCATION PROBLEM

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We consider underdetermined linear systems and characteristics of optimal solutions in the sensor location problem.

For the finite connected directed symmetric graph $G = (I, U)$ let's consider following linear underdetermined system:

$$\sum_{(v,w)\in I^+(v)} x_{v,w} - \sum_{(w,v)\in I^-(v)} x_{w,v} = \begin{cases} S_v, & v \in S, \\ 0, & v \in I \setminus S, \end{cases} \tag{1}$$

where the sets $I^-(v)$ and $I^+(v)$ are of entering and proceeding arcs for the node v accordingly, x – vector of unknowns, $x = (x_{i,j},\ (i,j) \in U;\ S_v,\ v \in S)$, $S \subseteq I$.

Kronecker-Capelli theorem implies the following necessary and sufficient conditions of consistency for the system (1):

$$\sum_{v \in S} S_v = 0.$$

In the sensor location problem we shall assume, that the values of flows on all entering and proceeding arcs for the each node i of the set M, $M \subseteq I$ are known:

$$x_{i,j} = f_{i,j},\ (i,j) \in\ I^-(i) \bigcup I^+(i), \quad i \in M.$$

If the set M includes the nodes from the set S, that we known the values of flows on all entering and proceeding arcs for the nodes of the set M, $M \subseteq I$ and we known the values S_v, $v \in M \bigcap S$:

$$x_{i,j} = f_{i,j},\ (i,j) \in\ I^-(i) \bigcup I^+(i), \quad i \in M \ \text{ and } \ S_v = F_v, \quad v \in M \cap S.$$

Let's enter the set M^+. For that let's construct a cut $CC(M)$ concerning for the set of nodes M. Let's denote the set of nodes $I(CC(M))$, that are adjacent to nodes in M and the nodes of the set M. So, we construct the set $M^+ = I(CC(M)) \setminus M$. Let's denote $M^* = M \bigcup M^+$ and form the set $I \setminus M^*$.

Let's write down additional equations connecting arc flow for each proceeding arc from the node i, $i \in I \setminus M$ through the set split ratio coefficients p_{ij} on every arc $(i,j) \in U$ as follows:

- for each node i from the set $I \setminus M^*$ we shall carry out the following: we choose an any proceeding arc (i,j) from the node i, and we believe an arc flow on it arc equal unknown $x_{i,j}$. For each following proceeding arc (i,v) from the node i, let's express through the set split ratio coefficient on arc (i,v) as follows:

$$x_{i,v} = \frac{p_{i,v}\, x_{i,j}}{p_{i,j}} \tag{2}$$

  If for some node of set $I \setminus M^*$ exists the unique proceeding arc with a unknown flow in that case there is no additional equation connecting arc flow for proceeding arc from through the set split ratio coefficients.

- for each node i from the set M^+ we shall carry out the following:
  We choose an any proceeding arc (i,j) from the node i, for which arc flow it is known: $x_{i,j} = f_{i,j}$. It is obvious, that the node j belongs to set M. For each following proceeding arc (i,v) from the node i, with an unknown flow $x_{i,v}$ (v does not belong to set M) let's express through the set split ratio coefficient on arc (i,v) as (2).

Let's substitute the calculated arc flows for each proceeding arc from the node i, $i \in I \setminus M$ in the equations of the system (1). We delete from the graph G the set of the arcs $I^-(i) \bigcup I^+(i)$ for each node $i \in M \bigcup \widetilde{M}$, on which the arc flow and value S_i known, $\widetilde{M} \subseteq I$, $\widetilde{M}$ – some nodes from the set $I \setminus M$. Also, we delete from the graph G the set of the nodes $M \bigcup \widetilde{M}$. We shall denote the new graph $\widehat{G} = (\widehat{I}, \widehat{U})$. Graph $\widehat{G}$ will consist from a component of connectivity, and in some components of connectivity can not contain nodes of the set I^*. The system (1), (2) for the graph $\widehat{G}$ will be following:

$$\sum_{j \in \widehat{I}_i^+(\widehat{U})} x_{ij} - \sum_{j \in \widehat{I}_i^-(\widehat{U})} x_{ji} = \begin{cases} a_i, & i \in \widehat{I} \setminus I^*, \\ x_i \cdot sign[i], & i \in I^*, I^* \subseteq S, sign[i] = \pm 1, \end{cases} \tag{3}$$

$$\sum_{(i,j)\in\widehat{U}} \lambda_{ij}^{p} x_{ij} = \alpha_p, \quad p = \overline{1,q}, \tag{4}$$

where $\widehat{I}_i^+(\widehat{U}) = \left\{j : (i,j) \in \widehat{U}\right\}$, $\widehat{I}_i^-(\widehat{U}) = \left\{j : (j,i) \in \widehat{U}\right\}$; a_i, λ_{ij}^p, α_p – parameters of the system; q – the number of additional equation connecting arc flow for proceeding arc from through the set split ratio coefficients; $x = (x_{ij}, (i,j) \in U;\ x_i,\ i \in I^*)$ – vector of unknowns.

Calculation of a rank of the matrix of the system (3)–(4), building the algorithms for finding the solutions of the systems of the type (3)–(4) and characteristics of optimal solutions are investigated in [2]–[4].

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NUMERICAL MODELING OF A STATIC MAGNETIC FLUID SEAL SUBJECT TO DIFFUSION OF FERROMAGNETIC PARTICLES

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Because magnetic fluid is a stable colloidal suspension of small ferromagnetic particles in a carrier liquid, its macroscopic interaction with an external nonuniform magnetic field is determined by the force acting on each separate particle. The force causes a Brownian motion of the particles with respect to the carrier liquid, as a result of which the particle concentration increases in the places where the magnetic field intensity is higher. This leads to redistribution of the fluid magnetization M being of a basic magnetic characteristics of the fluid which is defined by the relation $M = M_s C L(\xi H)$ where M_s is the saturation magnetization of the fluid; C, the volume concentration of particles; $L(t) = \coth t - 1/t$, the Langevin function; H, the magnetic field intensity; $\xi = \mu_0 m/kT$; μ_0, the magnetic constant; m, the magnetic moment of a particle; k, the Boltzmann constant; T, the fluid temperature. The steady-state distribution of the concentration C in the fluid volume V is described by the equation $\nabla \cdot (\nabla C - \xi C L(\xi H)\nabla H) = 0$ with the Robin-type boundary condition $\partial C/\partial n - \xi L(\xi H)(\partial H/\partial n)C = 0$ and the condition of particle mass conservation $\int_V C dV = C_0 V$ where C_0 is a constant corresponding to a uniform distribution of particles. Exact solution of the problem is given in [1] and is of the form $C = \varphi C_0 V \big/ \int_V \varphi dV$, $\varphi = \sinh(\xi H)/(\xi H)$. A Stefan-type diffusion problem can arise if the fluid is under the action of a high-gradient magnetic field. The point is that the particles diffuse in the direction of magnetic gradient ∇H and if the gradient is sufficiently large, particle concentration in the magnet pole vicinity reaches a maximum possible value corresponding to the dense packing of the particles.