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BAUTIN-TYPE BIFURCATIONS AND STABILITY OF EMERGED SOLUTIONS FOR A DELAY DIFFERENTIAL EQUATION MODELING LEUKEMIA

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In [3], [4] a mathematical model of chronic myelogenous leukemia is considered. Its study may be reduced to that of the delay differential equation

$$\dot{x}(t) = -[\beta(x(t)) + \delta]x(t) + k\beta(x(t-r))x(t-r), \qquad (1)$$

where $\beta(x) = \beta_0/(1+x^n)$, β_0, n, δ, k, are positive parameters, and $r > 0$ is the delay. The significance of $x(\cdot)$ and of parameters is presented in the above cited works, and a brief study of equilibrium points and of their stability, as well as a comprehensive numerical exploration of the solution and its dependence on the parameters is performed there.

In [2] we make an extensive study of the stability of the two equilibrium solutions, completing (and, for a certain zone of the parameter space, correcting) the results of stability from [3], [4]. We indicate in [2] the situations when Hopf bifurcations occur and we study the orbital stability of the periodic solutions thus emerged, by computing the normal form of the restriction of the equation to the bi-dimensional center manifold, and by determining the sign of the first Lyapunov coefficient at the Hopf bifurcation point.

In the present work we show that this problem presents, for some values of the involved parameters, Bautin-type bifurcations. This implies that, for some zone of the parameters space, the problem has two limit cycles, one inside the other. Since the periodic solutions of the problem are important, this mathematical result might be valuable for the biologists. We present a numerical procedure to find Bautin-type bifurcation points. We study the stability of the periodic orbits emerged by this bifurcation, by computing the second Lyapunov coefficient at such a bifurcation point (by the method of [1]).

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