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THE STABILITY OF THE SWIRLING FLOWS WITH APPLICATION TO HYPERBOLIC TURBINES

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The presence of a large variety of vortex flows in nature and technology has raised many theoretical and numerical problems concerning the stability of such structures. In these conditions, in order to minimize the simulation requirements for nonlinear time-dependent problems, stability analyses of vortex motions are of main importance in flow control problems. A particular case arises in the Francis turbines operate at partial discharge. The swirling flow downstream the runner becomes unstable inside the draft tube cone, with the development of a precessing helical vortex and associated severe pressure fluctuations [1].

The main goal of this paper is to develop a methodology for analyzing the swirling flows with helical vortex breakdown by using the linear stability analysis of axisymmetrical swirling flow fields. Obviously, the axial symmetry hypothesis is a major simplification having the main benefit of dramatically reducing the computational cost [2]. On the other hand, it introduces important limitations as far as the three-dimensionality and unsteadiness of the flow are concerned. Essentially, an axi-symmetric flow solver provides a circumferentially averaged velocity and pressure fields that it why is used as a basic flow for linear stability analysis. However, this axisymmetric flow field provides a good indicator for the vortex breakdown occurrence and development through the extent of the central quasi-stagnant region, although the unsteady velocity and pressure fluctuations cannot be recovered.

First, the eigenvalue problem governing the linear stability analysis of the axi-symmetric Batchelor vortex against normal mode perturbations is investigated for the case of high Reynolds numbers using a spectral collocation technique [3]. The accuracy of the method for is assessed underlying the necessity for the construction of a certain class of orthogonal expansions functions satisfying the boundary conditions. Graphical representations of the spectra are given pointing out the most unstable mode. Comparison of the eigenfunctions amplitudes with the ones from [4] are presented proving that the obtained results agree very well with the existing ones.

Next, the linear stability analysis is applied to the circumferentially averaged flow field downstream to the hydraulic turbines runner operating at partial discharge. The circumferentially averaged flow field cannot capture the unsteadiness of the 3D flow, it is used for stability analysis. In particular, the methodology developed in this paper is particularly useful for assessing and optimizing various techniques to stabilize the flow. The numerical evaluations prove that the distribution of the radial eigenfunctions is similar to the one obtained by numerical simulation in [1].

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BAUTIN-TYPE BIFURCATIONS AND STABILITY OF EMERGED SOLUTIONS FOR A DELAY DIFFERENTIAL EQUATION MODELING LEUKEMIA

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In [3], [4] a mathematical model of chronic myelogenous leukemia is considered. Its study may be reduced to that of the delay differential equation

$$\dot{x}(t) = -[\beta(x(t)) + \delta]x(t) + k\beta(x(t-r))x(t-r), \tag{1}$$

where $\beta(x) = \beta_0/(1+x^n)$, β_0, n, δ, k, are positive parameters, and $r > 0$ is the delay. The significance of $x(\cdot)$ and of parameters is presented in the above cited works, and a brief study of equilibrium points and of their stability, as well as a comprehensive numerical exploration of the solution and its dependence on the parameters is performed there.

In [2] we make an extensive study of the stability of the two equilibrium solutions, completing (and, for a certain zone of the parameter space, correcting) the results of stability from [3], [4]. We indicate in [2] the situations when Hopf bifurcations occur and we study the orbital stability of the periodic solutions thus emerged, by computing the normal form of the restriction of the equation to the bi-dimensional center manifold, and by determining the sign of the first Lyapunov coefficient at the Hopf bifurcation point.

In the present work we show that this problem presents, for some values of the involved parameters, Bautin-type bifurcations. This implies that, for some zone of the parameters space, the problem has two limit cycles, one inside the other. Since the periodic solutions of the problem are important, this mathematical result might be valuable for the biologists. We present a numerical procedure to find Bautin-type bifurcation points. We study the stability of the periodic orbits emerged by this bifurcation, by computing the second Lyapunov coefficient at such a bifurcation point (by the method of [1]).

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