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To the problem of absorption abilities of small mass extra dimensional black holes in the context of black holes production at the LHC

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We study interaction process of Primordial Black Holes (PBH) with such dense astrophysical objects as white dwarfs (WD) in the framework of extra dimensional cosmology. It is shown that in wide mass region PBHs are able to absorb WD for the time, considerably lower than the Hubble time. The efficiency of accretion onto such PBHs results in some new constraints on abundance of PBHs which are discussed here. The possibility to devour various astrophysical objects by small mass PBH is connected with the strongly discussed problem of possible absorption of our planet by black holes created in the LHC. It is shown that the possibility of this dramatic event is negligibly small even on the geological scale.

Keywords:

1. Introduction

Modern development of unified field theory is based on the assumption of existence of additional spatial dimensions. The latter are incorporated in the widely discussed at present braneworld model, which has strong motivation in the superstring and M-theory. According to this model all the ordinary matter is confined in a (3+1)-dimensional hypersurface, a 3-brane, wich is embedded in the higher dimensional space-time, the bulk, while gravity is able to propagate along all spatial dimensions [1]. Additionally to other significant consequences of the braneworld scenario which are important to both the particle physics and cosmology, the braneworld model predicts a possibility of mini black hole (BH) production in the process of hight energy proton collisions at the Large Hadron Collider (LHC). This "*black hole production may be the most spectacular physics at future colliders, perhaps even beginning with the 2008 startup of the LHC. And even if it is not phenomenologically accessible in the near future, it raises some very profound theoretical issues*" [2].

Recall that the D-dimensional braneworld approach leads to redefinition of the fundamental Plank mass M_p

$$M_p^{D-2} = \frac{(2\pi)^{D-4}}{4\pi G_D}, \tag{1}$$

where G_D is the D-dimensional gravitational constant [3] which enters the D-dimensional action of gravitational field

$$S_D = \frac{1}{8\pi G_D} \int d^D x \sqrt{-g} \frac{1}{2} \mathcal{R}, \tag{2}$$

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where $\mathcal{R}$ is the Ricci scalar. Extra dimensions extend the four-dimensional interval to

$$ds^2 = e^{2A(y)}dx^\mu dx_\mu + g_{mn}(y)dy^m dy^n, \tag{3}$$

where $A(y)$ and $g_{mn}(y)$ are a warp factor and a general form of compact metric, respectively.

The four-dimensional Planck mass M_4 which enters the four-dimensional gravitational action

$$S_4 = \frac{M_4^2}{2}\int d^4x\sqrt{-g_4(x)}R \tag{4}$$

is connected with the higher dimensional Planck mass M_p by the relation

$$\frac{M_4^2}{M_p^2} = M_p^{D-4}\int \frac{d^{D-4}y}{(2\pi)^{D-4}}\sqrt{g_{D-4}}e^{2A} \equiv M_p^{D-4}L_w^{D-4}, \tag{5}$$

where L_w is the typical size of compact extra dimensions.

If $M_p \sim M_4 \sim 10^{19}$ GeV, then obviously $L_w \sim 1/M_p$, but, however, if M_p is significantly lower, say $M_p \sim 1$ TeV, then $L_w \gg 1/M_p$. In other words, it is possible to solve the so-called *hierarchy problem* (according to which the characteristic scale of gravity, $M_4 \sim 10^{19}$ GeV, is 16 orders of magnitude larger than the Electroweak scale $M_{EW} \lesssim 1$ TeV) by means of introduction of extra spatial dimensions [4]. This fact open up the way to study possible manifestations of such quite exotic physical phenomena as quantum gravity and mini BH at the next generations of colliders. Namely, we will show in the next Section, that a higher dimensional BH has significantly large gravitational radius r_H than the four dimensional one of the same mass. As a consequence, the extra dimension presence makes it possible to bring colliding particles at distances comparable with the BH creation radius r_H at much lower, possibly, at TeV energies.

2. Properties of extra dimensional mini Black Holes

A basic parameter describing BH properties is its gravitational radius r_H. General behavior of a BH depends on the ratio between the latter and the size of extra dimensions L_w. Actually, if r_H is much larger than the size of extra dimensions, $r_H \gg L_w$, the produced BH is effectively a four-dimensional object. If, in the opposite, $r_H \ll L_w$, then this small BH is a higher-dimensional object which is completely submerged into extra-dimensional space-time [5].

In this last case one has to use D-dimensional generalization of the Schwarzschild metric [6]

$$\begin{aligned} ds^2 = \;& \left[1-\left(\frac{r_H}{r}\right)^{D-3}\right]dt^2 - \left[1-\left(\frac{r_H}{r}\right)^{D-3}\right]^{-1}dr^2 \\ & -r^2 d\Omega_{D-2}^2, \end{aligned} \tag{6}$$

where $d\Omega_{D-2}^2$ is the line element on the unit $(D-2)$-dimensional sphere. Gravitational radius

$$r_H = \frac{1}{\sqrt{\pi}M_p}\left(\frac{M_{BH}}{M_p}\right)^{\frac{1}{D-3}}\left(\frac{8\,\Gamma\left(\frac{D-1}{2}\right)}{D-2}\right)^{\frac{1}{D-3}}. \tag{7}$$

now is not directly proportional to the mass M_{BH} of BH (as it for $D = 4$).

Thermodynamical characteristics of extra dimensional BH, like Hawking temperature T_H and time t_H of complete Hawing evaporation take the form of

$$T_H = \frac{D-3}{4\pi r_H} \tag{8}$$

and

$$t_H = \frac{1}{M_p}\left(\frac{M_{BH}}{M_p}\right)^{\frac{D-1}{D-3}} . \tag{9}$$

As follows from Eqs. (7),(8) and (9), extra dimensional mini BHs are larger, colder and, thus, live longer compared with their four dimensional analogues with the same mass M_{BH} [5].

The fact that extra dimensional mini BHs have the larger horizon radius with respect to the four dimensional one means lower fundamental Plank mass M_p in Eq. (7). According to the Eq. (5) one can assume M_p to be in the range of Electroweak scale ~ 1 TeV due to possible existence of extra spatial dimensions. Thus, together with lowering Plank mass we obtain considerable growing of the horizon radius r_H of mini BH. The last circumstance makes possible a formation of mini BHs in collisions of particles with soon available energies of several TeV.

Indeed, one finds r_H to be of order 10^{-4} fm, slightly changing from $D = 5$ to $D = 11$ for $M_{BH} = 5$ TeV and $M_p = 1$ TeV. This is merely a subnuclear distance attainable in future LHC experiments. Recall, in the absence of extra dimensions the lightest semiclassical mini BH would have a mass of, at least, a few M_4, and creation of a mini BH in high-energy collisions would then need center-of-mass energies $\sqrt{s} > M_4 \sim 10^{16}$ TeV, which correspond to the horizon radius of order of the four dimensional Plank length 10^{-20} fm. Obviously, such distances and energies are far beyond any experimental investigations.

Notice, that in order to be able to ignore quantum corrections in our calculations the mass of BH must be, at least, a few times larger than the scale M_p of quantum gravity [5, 7]. We will assume bellow that this condition is satisfied.

3. Are mini BHs possibly produced at the LHC really dangerous?

The very possibility of mini BH production at the LHC caused for wide and strained discussion both in scientific and public spheres raising up a question "*Could such monstrous objects like mini black holes (once they are produced at LHC) eat up the entire world?*" This question was controversially discussed in blogs and online-video-portals (see, for instance [8], and for web references [9]).

At the same time, the LHC Safety Assessment Group prepared "*Review of the Safety of LHC Collisions*" [10]. Also a comprehensive and profound analysis of the same problem bringing various astrophysical arguments was published recently [11] with a conclusion: "*there is no risk of any significance whatsoever from such black holes*".

The crucial point here is the fact, that a process of collision of ultra high energy cosmic ray particles with cosmic objects permanently occurs in nature. It is well known that almost 90% of all the incoming cosmic ray particles are protons and events with total energy exceeding 10^{11} GeV (> 400 TeV center-of-mass system, two orders larger than the LHC limit) have been already registered. Moreover, the flux of Ultra High Energy Cosmic Rays with energy above 10^{10} GeV is about one event per kilometer

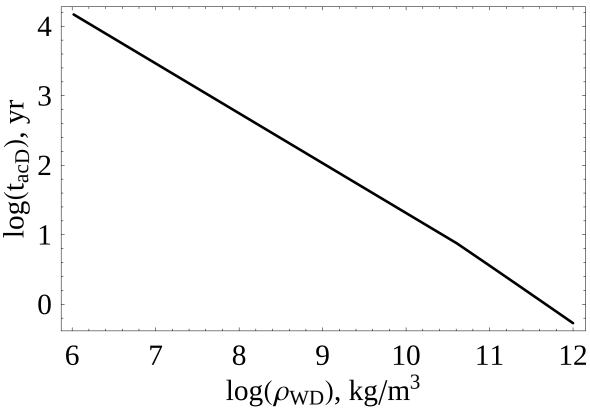

FIG. 1. Time of the PBH mass grow during accretion of WD matter until the exit from extra dimensional regime versus the central WD density.

squared per year [12]. Not a small value if planets or stars are considered. All this in general rises up a question of stability of celestial bodies with respect to extra dimensional BHs created by cosmic rays. If the stability is not absolute, tight constraints on the properties of these mini BHs can be established.

Alternative way to estimate the influence of the extra dimensional BHs on celestial bodies is to analyze the role of Primordial Black Holes. At least four dimensional PBHs could be formed at the early stages of cosmological evolution saving an information about the epoch of their formation till later, possibly, till present [13–15].

PBH with a single extra dimension and cosmological consequences resulting in accordance with the Randall-Sundrum II scenario are discussed in [16–18].

But if the number of extra dimensions is fixed in the previous scenario, one have to deal with two-parametric space (M_p, D) (strictly speaking, there is the third parameter - a warp factor A, see (5)). We will show in the next section that extra dimensional PBHs can both restrict a physically allowed region in this space and give another argument of the celestial bodies stability.

4. WD as a tool of multi dimensional PBH search

It is clear, that from the point of view of absorptional abilities of BHs the most preferred objects would have as large average density as possible. Most dense object in the Universe are neutron stars (NS). However, the latter are not only the most dense object, but also have extremely strong magnetic fields. Since the ultra high energy cosmic rays undergo deflection and deceleration in the large magnetic fields that surround NS, they are not appropriate object for mini BH creation by cosmic rays. But, nevertheless, the NS absorption by PBHs is possible [19, 20]. One can find the discussion of multi dimensional PBH capture by NS and some observational consequences in [11]. Note that the NS absorption by PBH can be accompanied by some type of gamma ray burst [19].

The next dense objects after NS are white dwarfs (WD). They are the most appropriate dense object to test accretion by PBHs [21]. The corresponding theory of WD matter accretion onto four dimensional PBH was developed in [22–24].

Results of WD absorption by the extra dimensional mini BH are presented in [11]. However the problem of WD absorption by the extra dimensional PBH has not been analyzed from all the sides. Below we pursue this analysis in some new directions.

Three parametric space (M_p, D, A) has some observational restrictions. Namely, unwarped $(A = 0)$ flat scenarios with $D = 5$ have macroscopic size of extra dimension, and are thus ruled out [11]. But with sufficient warping one finds some viable scenarios, such as that of [25]. The corresponding theory of accretion onto PBH has been proposed in [18].

Consequently, let us consider six-dimensional PBH. The Hawking mass or the formation mass of PBH which survives until now

$$M_{Haw} = (6.22 \times 10^{41})^{\left(\frac{D-3}{D-1}\right)} M_p^{\frac{2(D-2)}{D-1}} \quad GeV \tag{10}$$

can be found by equating a life time of extra dimensional BH (9) to the 13 Gyr age of the Universe. For $D = 6$ and $M_p = 1$ TeV one gets $M_{Haw} \simeq 10^3$ kg (4-D PBH has $M_{Haw} \simeq 10^{12}$ kg).

D-dimensional generalization of the Equation of accretion has the form [11]

$$\frac{dM}{dt} = \pi \lambda_D c_s R_B^2 \rho, \tag{11}$$

where

$$R_B = \left[\frac{D-3}{4c_s^2}\right]^{1/(D-3)} r_H \tag{12}$$

is so called Bondi radius,

$$\lambda_D = 4\left[\frac{2(D-3)}{D+1-\Gamma(7-D)}\right]^{[D+1-\Gamma(7-D)]/[2(\Gamma-1)(D-3)]}, \tag{13}$$

where Γ is the constant of a general polytropic equation of state $p = K\ \rho^{\Gamma}$, where p is pressure, ρ is density and K is a constant coefficient.

D-dimensional Bernoulli equation now takes the form

$$\left(1 - \frac{c_s(r)^2}{\Gamma - 1}\right)^2 \left(1 - \frac{D-7}{D-3} c_s(r)^2\right) = \left(1 - \frac{c_s(\infty)^2}{\Gamma - 1}\right)^2, \tag{14}$$

where $c_s(r)$ is the sound speed at distance r from the PBH. An important peculiarity of Bondi accretion model is the presence of so called sonic point r_s, where speed of matter motion towards PBH approaches the speed of sound [26]. Respective sound speed at the sonic point

$$c_s(r_s) = \sqrt{\frac{2(D-3)}{D+1-\Gamma(7-D)}}\ c_s(\infty) \tag{15}$$

can be easily obtained. The value $c_s(\infty)$ is determined by equation of state and the inner structure of WD. It can be written in the form

$$c_s^2(\infty) = \frac{m_e}{3m'} \frac{x^2}{\sqrt{1+x^2}}\ c^2, \tag{16}$$

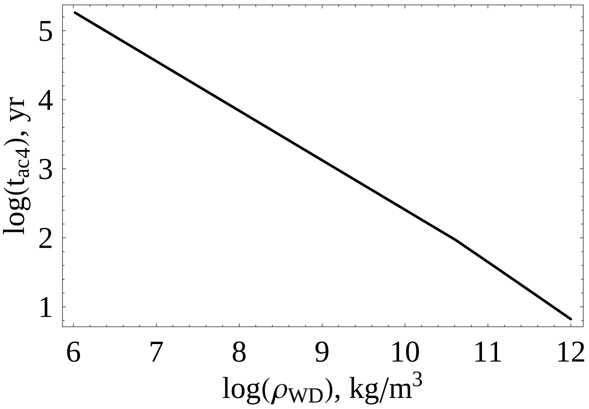

FIG. 2. Time of the PBH mass grow from the start of the four dimensional accretion regime until the total WD absorption versus the central WD density.

where c is the speed of light

$$x = 0.8\sqrt[3]{\frac{\rho}{10^9\ kg/m^3}}, \tag{17}$$

m_e is the electron mass and $m' \simeq 2 \times 1.66 \times 10^{-24}$ gr is a rest mass of nucleus per one electron for the widely distributed oxygen-carbon WDs.

Using Eqs. (12),(13),(15),(16) and (17) one can integrate accretion equation (11) and, in particular, find a time of WD matter absorption by PBH in different regimes.

Remind that we divide PBH growth process in two stages. The first one corresponds to the evolution of PBH mass from the initial value M_{Haw} up to the time of transition to the four dimensional regime. This accretion stage lasts for

$$t_{acD} = d_0 \frac{4c_s}{(D-5)\lambda_D k_D} \left(\frac{M_p}{1\ TeV}\right)^{(D-2)/(D-4)} \left(\frac{M_4}{1\ TeV}\right)^{2(D-5)/(D-4)},\ D > 5, \tag{18}$$

where

$$d_0 = \frac{(1\ TeV)^3}{\pi\rho} \tag{19}$$

is the characteristic distance in TeV units,

$$k_D = \frac{2(2\pi)^{d-4}}{(D-2)\Omega_{D-2}}, \tag{20}$$

and

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]} \tag{21}$$

is the volume of the unit $D-2$ sphere.

Resulting dependence of the time t_{acD} on the WD density is presented in fig.1 for the six dimensional accretion onto PBH.

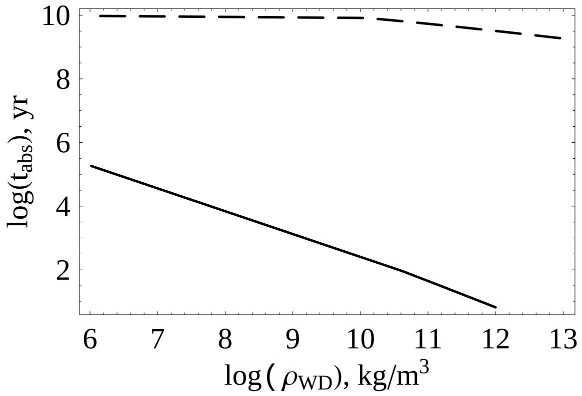

FIG. 3. Total time of WD absorption by PBH in six (solid line) and four (dashed line) dimensional cases.

Another important time scale

$$t_{ac4} = d_0 \frac{4c_s}{\lambda_4 k_4} \left(\frac{M_4}{1\ TeV}\right)^2 \left(\frac{1}{1\ TeV\ R_{B0}}\right),\ D = 4, \tag{22}$$

where R_{B0} is initial Bondi radius, is connected with the initial BH mass M_{Haw} through Eqs. (7) and (12) and characterizes the duration of the four dimensional accretion stage starting after the completion of six dimensional accretion and lasting until a complete WD absorption.

Dependence of t_{ac4} on the central WD density is presented in fig.2.

5. Conclusions

One can see that the timescale t_{ac4} exceeds t_{acD}. It will be also interesting to compare the total time of WD absorption by extra dimensional PBH with the case of extra dimension absence. According to the corresponding analysis [22–24] a four dimensional PBH is able to absorb typical WD for less than the Hubble time. The total time of WD absorption in the four and six dimensional cases is shown in fig.3. One can expect, that in the case of extra dimension presence the absorption time is sufficiently lower.

In contrast to extra dimensional mini BHs which are the only phenomenological possibility, PBH existence in general is strongly motivated by the theory of cosmological structure formation, independently of the presence of extra dimensions.

Thus the fact that modern populations of dense cosmic objects like WDs are stable and widely distributed proves the safety of LHC collisions and stability of celestial bodies with respect to the mini or primordial BHs absorption.

It should be stressed that the study of possible absorption of astrophysical objects by mini BHs allows to investigate much wider region of fundamental energy scale M_p than the LHC with $M_p < 14$ TeV.

Above results additionally supports conclusions which were made in [11].

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References

[1] R. Maartens, Living Rev. Rel. **7**, 7 (2004), gr-qc/0312059.

[2] S. B. Giddings, AIP Conf. Proc. **957**, 69 (2007), 0709.1107.

[3] S. B. Giddings (2001), hep-ph/0110127.

[4] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. **B429**, 263 (1998), hep-ph/9803315.

[5] P. Kanti, Int. J. Mod. Phys. **A19**, 4899 (2004), hep-ph/0402168.

[6] R. C. Myers and M. J. Perry, Ann. Phys. **172**, 304 (1986).

[7] S. B. Giddings and S. Thomas, Phys. Rev. **D65**, 056010 (2002).

[8] R. Plaga (2008), 0808.1415.

[9] B. Koch, M. Bleicher, and H. Stocker (2008), 0807.3349.

[10] J. R. Ellis, G. Giudice, M. L. Mangano, I. Tkachev, and U. Wiedemann, J. Phys. **G35**, 115004 (2008), 0806.3414.

[11] S. B. Giddings and M. L. Mangano, Phys. Rev. **D78**, 035009 (2008), 0806.3381.

[12] L. Anchordoqui, T. Paul, S. Reucroft, and J. Swain, Int. J. Mod. Phys. **A18**, 2229 (2003), hep-ph/0206072.

[13] J. B. Zel'dovich and I. D. Novikov, Astron. J. (USSR) **43**, 758 (1966).

[14] S. Hawking, Mon. Not. Roy. Astron. Soc. **152**, 75 (1971).

[15] B. J. Carr, Lect. Notes Phys. **631**, 301 (2003), astro-ph/0310838.

[16] D. Clancy, R. Guedens, and A. R. Liddle, Phys. Rev. **D68**, 023507 (2003), astro-ph/0301568.

[17] Y. Sendouda, K. Kohri, S. Nagataki, and K. Sato, Phys. Rev. **D71**, 063512 (2005), astro-ph/0408369.

[18] V. V. Tikhomirov and Y. A. Tsalkou, Phys. Rev. **D72**, 121301 (2005), astro-ph/0510212.

[19] E. V. Derishev, V. V. Kocharovsky, and V. V. Kocharovsky, Pisma Zh. Eksp. Teor. Fiz. **70**, 642 (1999).

[20] S. E. Yuralevich, Vestnik BGU **1**, 10 (2002), (Bulletin of Belarus State Unevercity, Series **1**, 1, p. 10 (2002), in russian).

[21] S. E. Yuralevich, Nucl. Phys. **B 118**, 509 (2003).

[22] V. V. Tikhomirov, S. E. Siahlo, and Y. V. Bondarenko, Nonlin. Phenom. Complex Syst. **7**, 347 (2004).

[23] V. V. Tikhomirov and S. E. Yuralevich, Phys. Atom. Nucl. **67**, 2041 (2004).

[24] V. V. Tikhomirov and S. E. Siahlo, Grav. Cosmol. **11**, 229 (2005).

[25] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), hep-ph/9905221.

[26] S. L. Shapiro and S. A. Teukolsky (1983), *Black holes, white dwarfs, and neutron stars: The physics of compact objects*, New York, USA: Wiley, 645 p.