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Spatial instabilities of singular light beams in nonlinear cavity

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The problem of singular light beams interaction with nonlinear cavity has been studied analytically and numerically. Steady-state and transient modes of optical bistability for singular light beams with different topological structure have been described, and the possibility of realization and control over spatially localized and rotating diffractive optical patterns has been revealed and discussed.

Keywords:

1. Introduction

The singular light beams (optical vortices) are characterized by a presence of the screw dislocation in their wave front structure [1]. Optical vortices can be observed in the light fields with complex spatial structure [2], in laser cavities [3] or they can be obtained upon diffraction from computer-generated holograms [4]. Among the potential applications of singular light beams one can indicate performing of optical computing [5], development of optical tweezers [6], wave-guide technologies [7] etc.

Investigations of interaction processes between optical vortices and nonlinear medium are related with the stability problems for topological and spatial structure [8], formation of localized vortex beams [9], and development of the methods for transformation of their topological structure upon nonlinear interaction of the light beams [10, 11]. However, interactions of optical vortices with nonlinear interferometers as well as optical pattern formation with singular light beams are the open problems for present stage of research in this area.

In this work the theoretical model of vortex light beams interaction with nonlinear interferometer is proposed based on the mean-field limit for high-finesse optical cavity and some peculiarities of realization of optical bistability and spatial instabilities in the field of vortex light beams are discussed. The work is organized as follows: Section 2 presents the theoretical model of vortex light beam interaction with nonlinear cavity; the mode of optical bistability is thoroughly studied in Section 3 based on the result of numerical modelling; Sections 4 is devoted to the problem of diffractive patterns formation in conditions of modulation instability of spatial structure of the singular light beams; and Section 5 is devoted to the problem of diffractive patterns formation under two-wave mixing of counterpropagating light beams in ring-cavity.

2. Theoretical model

The theoretical model for investigation of optical bistability and pattern formation processes in spatially extended light beams interacting with a ring cavity filled with a nonlinear Kerr medium is

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based on the mean-filed limit. The dynamics of the light field passed though the cavity can be described by the following equation for the dimensionless complex amplitude [12]:

$$\frac{\partial E}{\partial t} = E_0 - \left[1 + i\eta\left(\Delta_0 - |E|^2\right)\right] E + i\Delta_\perp E \tag{1}$$

Here, E_0 is the input field, $\eta\Delta_0$ is the cavity detuning, $\eta = \pm 1$ determines the type of nonlinearity (+ corresponds to focusing, − corresponds to defocusing type), and $\Delta_\perp$ is the transverse Laplacian. Time is scaled to the cavity response time $\tau_R = L/vT$, where $v = c/n_0$ is the light velocity in the medium, L is the thickness of the cavity. Transverse variables are scaled to diffraction parameter $\sqrt{\beta} = \sqrt{(\lambda L/4\pi T x_0^2)}$. This model has been derived to describe high finesse cavities (transmission coefficient of the cavity mirrors $T << 1$), and assumes that only one longitudinal mode is excited.

In the following we will consider the problems of optical bistability and pattern formation in the field of the light beams with helical wave front (or with screw phase dislocation), the simples type of which can be mathematically described by a complex input light field E_0 in the form:

$$E_0(\rho, \varphi) = A_0[r/r_0]^{|m|} \exp[-(r/\sqrt{2}r_0)^2 + im\varphi] \tag{2}$$

Here, ρ and φ are polar coordinates, parameter r_0 is characterized the width of light beam, $m = \pm 1, \pm 2, ...$ is the so-called topological charge of optical vortex. Let us notice that $m = 0$ corresponds to the case of Gaussian light beam.

Numerical modelling of Eq.(1) was performed using absolutely stable two-step (three-layer) explicit method [13] that gives a possibility to calculate 2D spatial intensity and phase distribution of singular light beams for considerable time intervals.

3. Optical bistability and transient dynamics of optical vortices in nonlinear cavity

First, let us review the basic results for optical bistability mode in nonlinear interferometer driven by plane-wave illumination. The relation between the output and input optical intensities in Eq. (1) shows bistable behaviour for cavity detuning parameter $\Delta_0 > \sqrt{3}$ both for focusing ($\eta = +1$) and defocusing ($\eta = -1$) type of nonlinearity [14]:

$$I_0 = I_{OUT}\left\{1 + [3I_{OUT} - \Delta_0]^2\right\} \tag{3}$$

Investigation of optical bistability mode in the field of singular light beams has been carry out by means of numerical modelling of Eq.(1) with the initial conditions (2). The results are presented on Figs.1-3.

The dependences of transmitted power of optical beams $P_T = \int\int |E(x,y)|^2\, dxdy$ versus input power $P_0 = \int\int |E_0(x,y)|^2\, dxdy$ are presented on Fig.1 for three types of input light beams: Gaussian ($m = 0$, curve 1), first, and second order optical vortices ($m = 1$ curve 2, and $m = 2$ curve 3, consequently).

For used set of parameters (cavity detuning $\Delta_0 = 2.5$, defocusing nonlinearity, $\eta = -1$) the mode of optical bistability for Gaussian light beam takes place in very small range of input optical power $P_0 \approx 9.4 \div 10.4$. Contrary to this situation, when using the singular light beams with the same

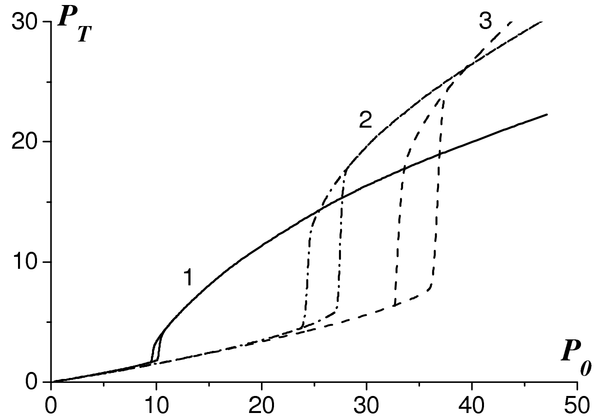

FIG. 1. Stationary transmission function of nonlinear cavity m=0 (curve 1), 1(curve 2), 2(curve 3)

cavity detuning one can considerably enlarge the region of realization of optical bistability mode (see curves 2 and 3). One can notice also that the range of input power, which leads to realization of optical bistability, is increased with the increasing of topological charge of optical vortex m, and the transmission of nonlinear cavity on the upper branch of bistable curve is increased comparing to the case of Gaussian beam. Thus, one can suggest that optical vortices have an advantage in realization of bistable devices with high values of transmission rate P_T. However, the drawback, which must be taken into account, is the considerable increasing of switching-on power $P_\uparrow$ with the transition from Gaussian to vortex light beams, which is mainly due to increasing of effective transverse width of optical vortices of the higher order comparing to Gaussian light beam.

Transient dynamics of output power of vortex light beams P_T is calculated on Fig.2 for different values of input power P_0. When the value of input power is lower than value of switching-on power, $P_0 < P_\uparrow \approx 27.6$, the transmission of nonlinear cavity is stabilized on low level (curve 1). After the power of input light beam is exceeding the switching-on threshold level ($P_0 > P_\uparrow$, curve 2) the process of formation of high transmission state of nonlinear interferometer is developing. It includes several characteristic steps: first step ($t < 3$) can be associated with local increasing of intensity in the maximum of vortex light beam, then ($t = 3 \div 15$) spatial enlargement of switching-on zone leads to formation of stationary transmission characteristic. Let us notice that the velocity of switching wave depends significantly on the input intensity (compare curves 2 and 3), and the steady-state can be reached for several tenth of cavity build-up times.

The dynamics of spatial intensity and phase distribution in the mode of optical bistability is presented on Fig.3, and it corresponds to results of Fig.2. One can see that spatial evolution of light beam can be characterised by the following typical stages: (1) - switching-on of the intensive part of optical vortex (Fig.3a-c) simultaneously leads to strong modulation of phase distribution, (2) - formation of spatially-locked phase state with ring-type intensity distribution (Fig.3d), and, finally, (3) - radial enlargement of switching-on area due to movement of switching wave till steady-state will be reached (Fig.3e). The velocity of radial switching wave is directly proportional to the intensity of input singular light beam.

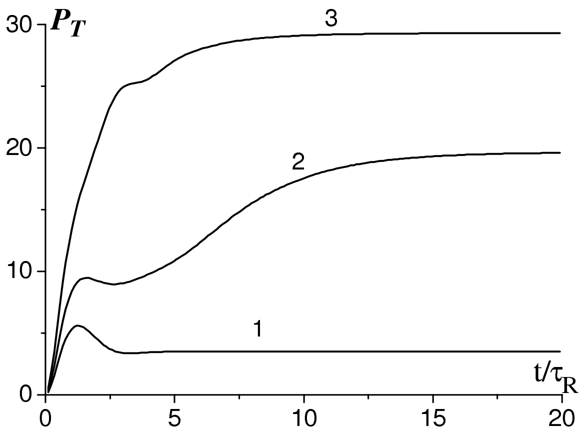

FIG. 2. Transient dynamics of nonlinear cavity switching in high transmission state $P_0 = 20$ (curve 1), 30 (curve 2), 45 (curve 3)

4. Modulation instability and optical pattern formation with singular light beams

Let us consider the stability of stationary solutions E_{OUT} of Eq. (1) with respect to spatially inhomogeneous perturbation of the light field $E = E_{OUT} + \delta E$, where $\delta E = A_q \exp(\lambda t)\cos(qx)$. Linearization procedure gives the following characteristic equation for the growth rate of spatially periodical perturbations $\lambda(q)$:

$$\lambda_{1,2}(q) = -1 \pm \sqrt{4I_{OUT}\Delta_0^q - 3I_{OUT}^2 - (\Delta_0^q)^2} \tag{4}$$

where $\Delta_0^q = \Delta_0 + \eta\beta q^2$. Taking into account the stationary transmission function (3), one can analyze the dependences of $\lambda(q)$ on the input intensity I_0 and detuning parameter Δ_0. The positive value of the growth rate ($Re(\lambda(q) > 0$) corresponds to increasing amplitude of the fluctuations A_q with spatial frequency q, and leads to formation of spatial periodical structure. This type of instability can be considered as an analogous of the Turing (modulation) instability for nonlinear optical system [12]. The physical reason of this instability is the concurrence of different transverse modes of nonlinear interferometer.

For the case of focusing nonlinearity ($\eta = +1$) the expression for $\lambda(q)$ reads as following:

$$\lambda_{1,2}(q) = -1\pm \\ \pm\sqrt{-(I_{OUT} - \Delta_0 - \beta q^2)(3I_{OUT} - \Delta_0 - \beta q^2)} \tag{5}$$

The dependences of $\lambda_1(q)$ for different value of the input intensity I_0 and for initial detuning $\Delta_0 = 0$ are presented on Fig.4a. One can see that the value $I_{OUT} = 1$ is the threshold level for arising of modulation instability. For the considered example the condition $I_{OUT} > 1$ leads to an exponential increasing in time of periodical fluctuations with spatial frequency q, which can be determined from Fig.4a.

Equalizing $Re(\lambda(q) = 0$ we can obtain the boundary region of the parameters where the modulation instability of stationary solution takes place, see Fig.4b. The growth rate of periodical fluctuations ($Re(\lambda(q))$ is positive for spatial frequencies from inside of the bounded domain.

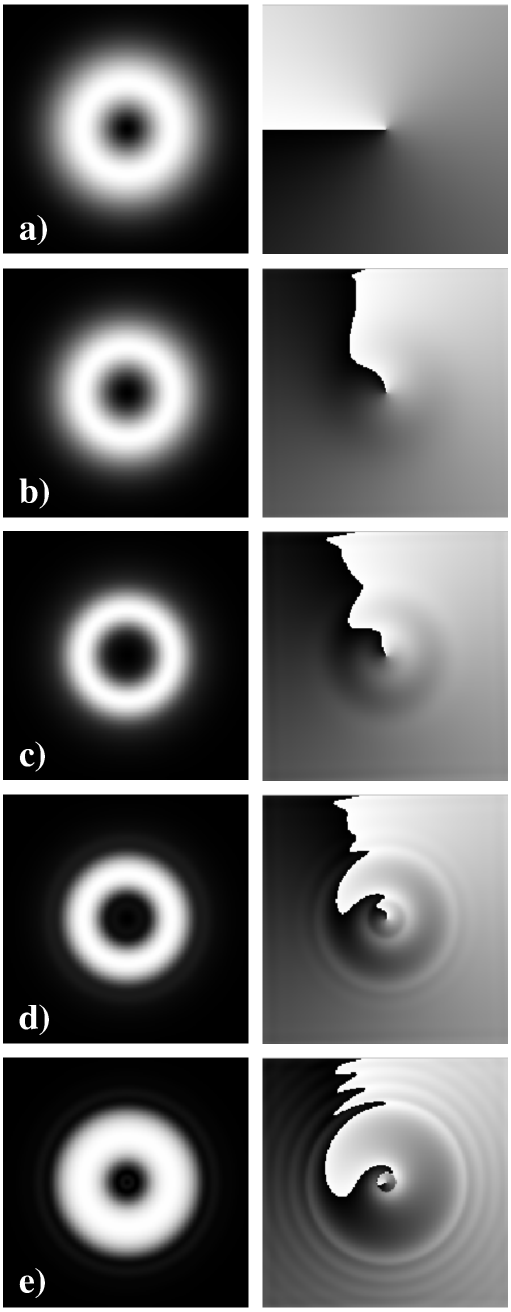

FIG. 3. Transient dynamics of spatial intensity (left row) and phase (right row) profiles in optical bistability mode. $t = 0(a), 1.5(b), 3(c), 6(d), 12(e)$, $P_0 = 20$, $\eta = -1$, $\beta = 0.01$, $\Delta = 2.5$. Only the central part of computational window is shown.

Summarizing the results of this task one can mention that in the self-focusing case of nonlinearity ($\eta = +1$), a pattern forming instability takes place for values of the input intensity above a certain threshold depending on the value of initial detuning parameter Δ_0. For the defocusing type of nonlinearity ($\eta = -1$), a pattern forming instability threshold is $\Delta_0 > 2$, and it is always frustrated by the

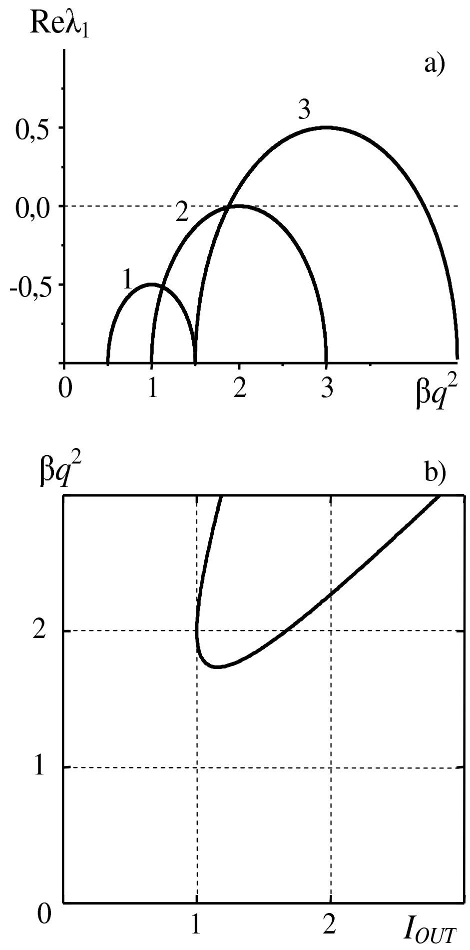

FIG. 4. a) - The growth rate of periodical fluctuation versus their spatial frequency, $I_{OUT} = 0.5(1), 1(2), 1.5(3)$; b) - The region of modulation instability on the parameters plane $I_{OUT} - \beta q^2$. $\Delta_0 = 0$.

bistable switching of interferometer [15].

Taking into account the results of linear stability analysis a numerical modelling of Eq.(1) in the mode of modulation instability was performed. To initialize the perturbations of spatial structure of the singular light beams the following procedure was realized. The complex amplitude of initial light beam was modulated with respect to radial and azimuthal perturbations with spatial frequency k_r and k_φ, consequently: $E_0^* = E_0 + \delta E_0$, where

$$\delta E_0 = a_0 \exp(im\varphi) \exp(i(k_r r - k_\varphi \varphi)) \quad (6)$$

Here, $a_0 \ll E_0$ is a small amplitude of initial perturbation. The typical results for realization of modulation instability mode in the field of singular light beam corresponding to self-focusing case of nonlinearity of intracavity material are presented on Fig.5. They were obtained for the beam with topological charge $m = +1$, and using azimuthal initial perturbation with spatial frequency $k_\varphi = 2$. The rest set of parameters is shown in figure caption.

Let us notice that the transmission function of nonlinear interferometer $I_{OUT}(I_0)$ is a single-valued for discussed case of initial detuning $\Delta_0 = 0$. When the intensity of input light beam is exceeded the threshold value $I_{thr}(k_{k_r,\varphi})$ a small initial perturbation of initial complex amplitude E_0 leads to spontaneous formation of spatially periodical set of localized bright spots in a transverse intensity profile of the output light beam.

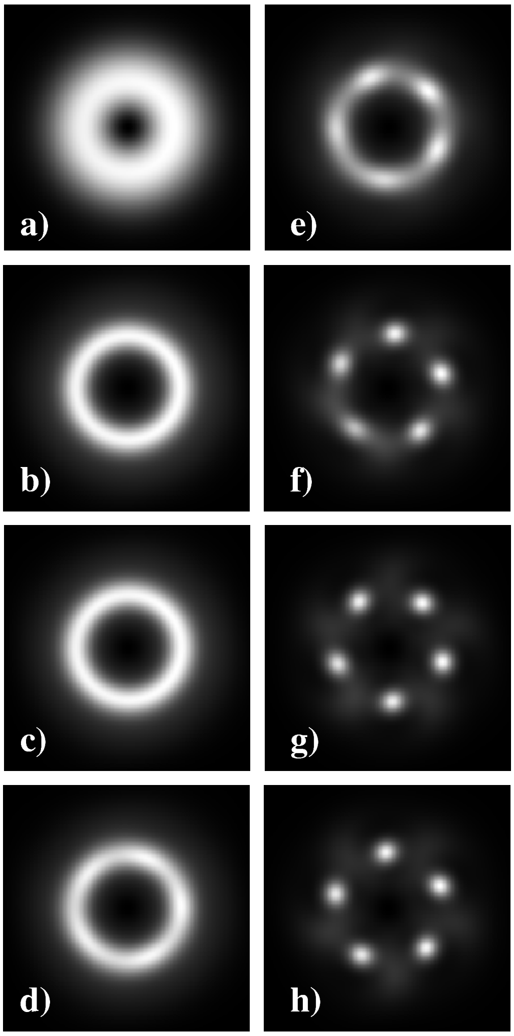

FIG. 5. Transient dynamics of spatial intensity profiles in modulation instability mode. $t = 0(a), 5(b), 10(c), 20(d), 25(e), 30(f), 35(g), 40(h)$, $I_0 = 5$, $\eta = +1$, $\beta = 0.1$, $\Delta = 0$. Only the central part of computational window is shown.

Figure 5 demonstrates an example of transient dynamics of spontaneous pattern formation in the field of singular light beam. One can see that after spatial intensity distribution has reached the quasi-stationary state (Fig.5a-d), inhomogeneous perturbation with definite spatial frequency growth in time and lead to formation of azimuthally periodical and counter clockwise rotating structure (Fig.5e-h). The velocity and direction of rotation are determined by phase gradient in the initial singular light beam, which depends in turn on the value of topological charge m. The number of localized peaks N is determined by initial intensity of singular light beam I_0 and its topological charge m. The results of numerical modelling show that for optical vortices with topological charge $m = 1$ the most probable are the spatial structures with $N = 5 \div 6$.

5. Pattern formation under two-wave mixing of counterpropagating optical vortices

In this section we will demonstrate that intracavity interaction of optical vortices in conditions of symmetry breaking instability leads to formation of different periodical diffractive patterns, which can be controlled by the change of topological charge of input light beams.

In the following, we analyze the problem of counterpropagating beams in a ring cavity [16]. When two coherent light beams are directed on the input mirror of a ring cavity in a such a way, that they are exactly counterpropagating inside an intracavity nonlinear medium, reflective grating of refractive index and (or) absorption coefficient of the medium is formed due to interference of counterpropagating beams. For theoretical description of this problem, we consider a passive ring resonator, filled by a medium with Kerr-like nonlinearity. The light fields transmitted by the system are proportional to the normalized functions E_1, E_2, which obey the following set of coupled-mode equations:

$$\frac{\partial E_1}{\partial t} = E_{10} + i\Delta_\perp E_1 + E_1 \left[i\eta \left(|E_1|^2 + 2|E_2|^2 - \Delta_{01} \right) - 1 \right] \tag{7}$$

$$\frac{\partial E_2}{\partial t} = E_{20} + i\Delta_\perp E_2 + E_2 \left[i\eta \left(2|E_1|^2 + |E_2|^2 - \Delta_{02} \right) - 1 \right] \tag{8}$$

where E_{10} and E_{20} are the complex amplitudes of the pump fields, which are determined by analogy with Eq.(2), $\eta\Delta_{01,02}$ are the cavity detuning parameters. The rest set of variables are the same as in Eq.(1). This scheme of interaction, contrary to Eq.(1), includes not only the self-modulation effects (the term $\sim E_i |E_i|^2$ in the right part of coupled-mode Eqs.(7-8)) but also cross-modulation interaction between two light waves (the term $\sim 2E_i |E_j|^2$), which are essentially due to dynamic grating of refractive index, formed in the nonlinear layer. We focus our attention to the problem of two-wave mixing with equal input amplitudes $E_{10} = E_{20} = E_0$ and detunings $\Delta_{01} = \Delta_{02} = \Delta_0$ with the aim to study the special bistable regimes, when expected equality of output amplitudes is broken. Moreover, we restrict ourself to the case of defocusing nonlinearity ($\eta = -1$).

We consider the set of input parameters, which leads to formation of nonreciprocal spatial patterns [16], and compare two types of interaction: (1) - interaction of oppositely charged optical vortices, and (2) - interaction of the light beams with the same topological charge. The system of coupled-mode Eqs.(7-8) has been solved numerically, and the formation and stability of optical patterns have been studying with the use of small amount of numerical noise in input light beams.

The dynamics of spatial intensity distribution for first type of interaction is presented on Fig.6. One can see that temporal evolution of intensity distribution passes the following typical stages: (1) - switching-on in the mode of high transmission in upper branch of optical bistability curve (Fig.6a-c), (2) - formation quasi-periodic patterns due to modulation instability of switching-on area (Fig.6d-e), and (3) - formation of periodic patterns and their torsion in counter clockwise direction under gradient of phase in the field of vortex light beams (Fig.6f). Let us notice that Fig.6 presents temporal evolution

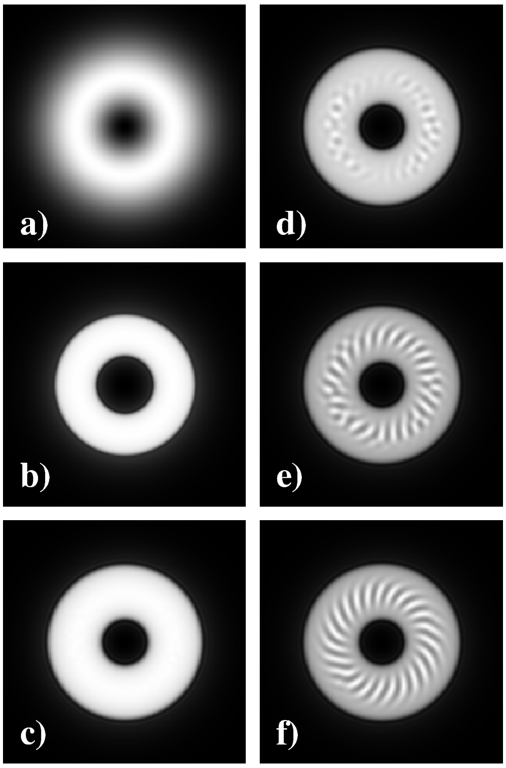

FIG. 6. Transient dynamics of spatial intensity profiles under formation of rotating periodical spatial structures. $t = 1(a), 12.5(b), 50(c), 375(d), 625(e), 1825(f)$, $I_{10} = I_{20} = 3.8$, $\eta = -1$, $\beta = 0.01$, $\Delta = 2.5$, $m_1 = +1$, $m_2 = -1$.

of intensity distribution in one of the interacting beams. Intensity distribution in second light beam is complimentary to the first one, i.e. the maximums in its periodic structure are coincided with the minimum in the field of the first beam, and vice versa. As it follows from numerical modelling, interaction of counterpropagating optical vortices with the same topological charge ($m_1 = m_2 = +1$) leads to formation of static periodical patterns in the switched-on area of light beams.

6. Conclusions

In this article, a model for studying of optical bistability and pattern formation under interaction of singular light beams with nonlinear interferometer has been developed. The problem of singular light beams illumination of nonlinear interferometer has been modelled based on absolutely stable two-step (three-layer) explicit numerical method. It was shown that the mode of optical bistability for optical vortices is characterized by the formation of ring-type intensity distribution in output light beam with spatially-locked inhomogeneous phase distribution, which leads to increasing of threshold for bistable mode, but gives the higher level of transmission on the upper bistable branch. In the problem of modulation instability of light beams in nonlinear interferometer it was shown that inclusion of phase singularity into the input light beams gives the possibility to obtain the set of spatial periodical and rotating localized structures, that permits the realization of additional control over the symmetry properties of diffractive optical patterns. As it follows from theoretical and numerical analysis, interaction of counterpropagating singular light beams in coniditions of symmetry breaking instability leads to formation of nonreciprocal static or rotating spatially periodical structures.

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