Solutions' matching method for simulation of power and dynamical characteristics of diode pumped Er-Yb laser

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Possibility of the usage of matching solutions' method for simulation of spatially distributed models of diode pumped Er-Yb laser has been analyzed. The stability analysis near local equilibrium has proved the possibility of adiabatic elimination of the population densities variables as fast variables in the system. The results obtained have been compared to that known from other approaches.

1. Introduction

The solid-state erbium lasers radiate in region of 1.5 μm. This spectral range is eye -save and finds wide technical and technological applications. The configurations of such lasers are rather different, one should mention short length resonators (of about 100 μm) [1–5], fiber optical amplifiers of different lengths (from few cm up to several meters) [6–15]; different variants of glass lasers [2-5,7,10,12-14] and crystal active media lasers[16–19], which use the scheme of end [2,6,10–12,14] and transverse [2] pumping scheme, also variants of laser diode pump [5,14] or pump by Ti: sapphire laser [19].

Nowadays, the most popular system of this type is a diode pumped laser system on glass which is co-doped with erbium and ytterbium ions [5,14].

The process of laser generation is described from the point of two models. In the case of short resonator or dynamic characteristics description, concentrated model is used [5,13,21,22]. It is presumed that all principal lasers values (active medium population density and pump radiation density) are constant along the active medium and they are determined as medium averages.

In the case of long active medium (for example, fiber laser), longitudinal pump and for pump characteristic solving are used usually quasi-one-dimension distributed laser model [3,8,11,14,23].

As we will demonstrate the concentrated model is good only for high-quality resonator and when the energy distribution in the resonator is nearly uniform.

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In the case of high optically density of active medium on pump wavelength profiles of lasing variables distribution are very non-uniform. It leads to impossibility of application of variables averaged over active medium to describe laser system.

Therefore one can consider space-dependent generation models. It worth to mention that usually the one dimension models are considered. And the evolution of variables is studied just along -one-dimension space, co-directed with resonator axes. Most complex models, e.g., considering 3D distributions, are very seldom and used only in the context of temperature's influence effects simulation [24].

The attempts of usage of these distributed models for dynamics processes analysis encounter with considerable difficulties that have primarily mathematical character. That, in the first place binds with high stiffness of the mathematical model of Er-Yb laser system .

At numerically simulation (with using any methods, including special methods for stiff systems of differential equations) the oscillations arise in the moment of realization the condition of generation's beginning. They account by the high stiffness pf the system. They are exclusively artificially and do not correspond with real physic process.

So, in this report we analyze the space distributed model for quasi-steady state time dependence generation regimes .

2. Theoretical model.

We use one-dimensional laser model (fig.1a), when active medium is a glass matrix of codoped Er and Yb ions. We use the four-level schema for the descriptions of the dynamics of populations Yb^+ and Er^+ ions (fig.1b) [3,4].

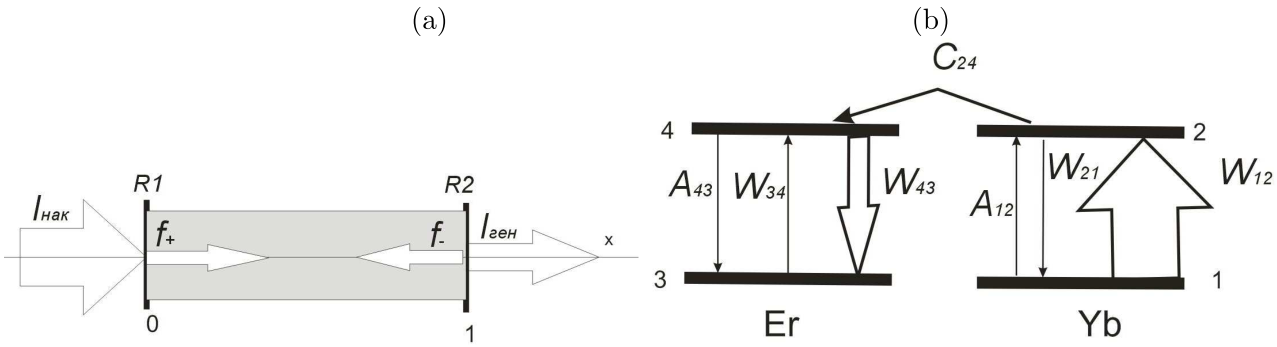

FIG. 1. The simplified level scheme of Er-Yb system used in the laser model (a). Scheme of the pump and generation waves propagation in an active medium (b).

In the context of this schema, the energy of resonance stimulation of Yb^+ ions in the region 980 nm give to short-living state $^4I_{11/2}$ of Er^+ ions due to cross-relaxation process. Then it transits to state $^4I_{13/2}$ (the level 4 on fig. 1b), that is a main laser level.

We neglect inverse cross-relaxation processes and up-conversion processes in consequence of fast relaxations of the state $^4I_{11/2}$. These processes influence on heating of an active medium [24], but not on generations efficiency.

So, the dynamics equations for population levels densities n_i normalized to concentrations of respective ions, can be presented in the form:

$$\frac{\partial n_2(x,t)}{dt} = \alpha_1 I_p(x,t)\,(1-n_2(x,t)) - I_p(x,t)\alpha_3 n_2(x,t) - \beta_1 n_2(x,t) - \gamma_1 n_2(x,t)\,(1-n_4(x,t)) \quad (1)$$

$$\frac{\partial n_4(x,t)}{dt} = (-\alpha_2 n_4(x,t) + \alpha_4(1-n_4(x,t)))\,(f_+(x,t)+f_-(x,t)) - \beta_2 n_4(x,t) + \gamma_2 n_2(x,t)\,(1-n_4(x,t)) \quad (2)$$

$$n_1 = 1-n_2, \qquad n_3 = 1-n_4, \quad (3)$$

where the time is normalizes on resonator round-trip time;
the dimensionless intensities of generation's waves $f^{\pm}$(going in positive (+) and in negative (-) direction along X axis) and the pump I_p are normalized on pump intensity on input of resonator; α_i, β_i and γ_i are dimensionless probabilities of stimulated, spontaneous emissions and cross-relaxation transitions respectively:

$$\alpha_1 = \frac{\sigma_{12} I_{nak}\xi \cdot L_{rez} n_{ref}}{h\,\nu_1 c} \quad \alpha_3 = \frac{\sigma_{21} I_{nak}\xi \cdot L_{rez} n_{ref}}{h\,\nu_1 c};$$

$$\alpha_2 = \frac{\sigma_{43} I_{nak}\xi \cdot L_{rez} n_{ref}}{h\,\nu_2 c}; \qquad \alpha_4 = \frac{\sigma_{34} I_{nak}\xi \cdot L_{rez} n_{ref}}{h\,\nu_2 c};$$

$$\beta_1 = \frac{A_{21} L_{rez} n_{ref}}{c}; \;\; ; \; \beta_2 = \frac{A_{43} L_{rez} n_{ref}}{c}$$

$$\gamma_1 = \frac{C_{24}\, N_{Er}\, L_{rez}\, n_{ref}}{c}; \;\; \gamma_2 = \frac{C_{24}\, N_{Yb}\, L_{rez}\, n_{ref}}{c};$$

$$l_p = N_{Yb} \cdot \sigma_{12}; \;\; \xi = \frac{1-e^{-L_{rez}\cdot l_p}}{L_{rez}\cdot l_p}.$$

We suppose, that the reflected pump wave is absent. Then the equation for the evolution of dimensionless intensities can be presented as:

$$\frac{\partial f_{\pm}(x,t)}{\partial t} \pm \frac{\partial f_{\pm}(x,t)}{\partial x} = (k_{g1} n_4(x,t) - k_{g2}(1-n_4(x,t)))\cdot f_{\pm}(x,t) - k_l \cdot f_{\pm}(x,t) + R_{lum} n_4(x,t) \quad (4)$$

$$\frac{\partial I_p(x,t)}{\partial t} - \frac{\partial I_p(x,t)}{\partial x} = -\alpha_1 I_p(x,t)\,(1-n_2(x,t)) + I_p \alpha_3 n_2(x,t), \quad (5)$$

where the values of space coordinate x is normalized on resonator length L_{rez}; α_1, α_3 are the dimensionless probabilities of stimulations transitions between itterbium ions; k_{g1}, k_{g2} are the dimensionless gain coefficients for transitions $2 \rightarrow 3$ and $3 \rightarrow 2$; k_l is the dimensionless coefficient of internal losses; R_{lum} is the dimensionless luminescence factor. So

$$k_{g1} = \sigma_{43} N_{Er} L_{rez}; \quad k_{g2} = \sigma_{34} N_{Er} L_{rez};$$

$$k_l = -ln\,(1-\rho)\,; \; R_{lum} = \frac{\delta\, A_{43}\, N_{Er}\, h_{Plank}\, \nu_2\, L_{rez}}{I_{nak} \cdot \xi}$$

The equations system (1) - (3) should be accomplished with boundary conditions:

$$f^+(0,\,t) = r_1 \cdot f^-(0,t); \qquad f^-(1,t) = r_2 \cdot f^+(1,t) \tag{6}$$

The parameters entered into the equations (1) - (3), for Er-Yb system have been chosen as in [3,4] and are shown in the table 1.

L, m	r_2	r_1	C_{24}, m^{-3}/c	A_{21}, $^{-1}$	A_{43}, c^{-1}	n_{ref}	λ_{pump}, nm	λ_{out}, nm	N_{Yb}, m^{-3}
0.02	0.7	0.999	$5 \cdot 10^{21}$	660	90.9	1.5	980	1550	$6.25 \cdot 10^{26}$

N_{Er}, m^{-3}	σ_{43}, m^2	σ_{34}, m^2	σ_{12}, m^2	σ_{21}, m^2	δ	ρ	d, m
$5 \cdot 10^{25}$	$5.7 \cdot 10^{-25}$	$6.6 \cdot 10^{-25}$	$2.0 \cdot 10^{-25}$	$5.0 \cdot 10^{-25}$	$3 \cdot 10^{-2}$	0.05	$2.16 \cdot 10^{-6}$

Table 1. Laser system's parameters

The correctness of the formulated model has been tested on the basis of data of papers [3-5]. So, we obtain the good results coincidence for time, power, and threshold dependencies.

3. The analysis of systems stability.

First, we perform the stability analysis of the concentrated system for populations n_2, n_4 and for total generations intensity $U = f^+ + f^-$. The dependencies of the roots λ of characteristic equation for linearized system in the vicinity of steady state versus dimensionless pump are presented on fig. 2 The vertical line corresponds to generations threshold. Red line - an imaginary part of λ, dark blue - the real one.

As one can see, λ_1 is real and negative after the threshold, then the respective direction (eigenvector v_1) is stabile. Two remainder eigenvalues λ_2 and λ_3 are imaginary and complex

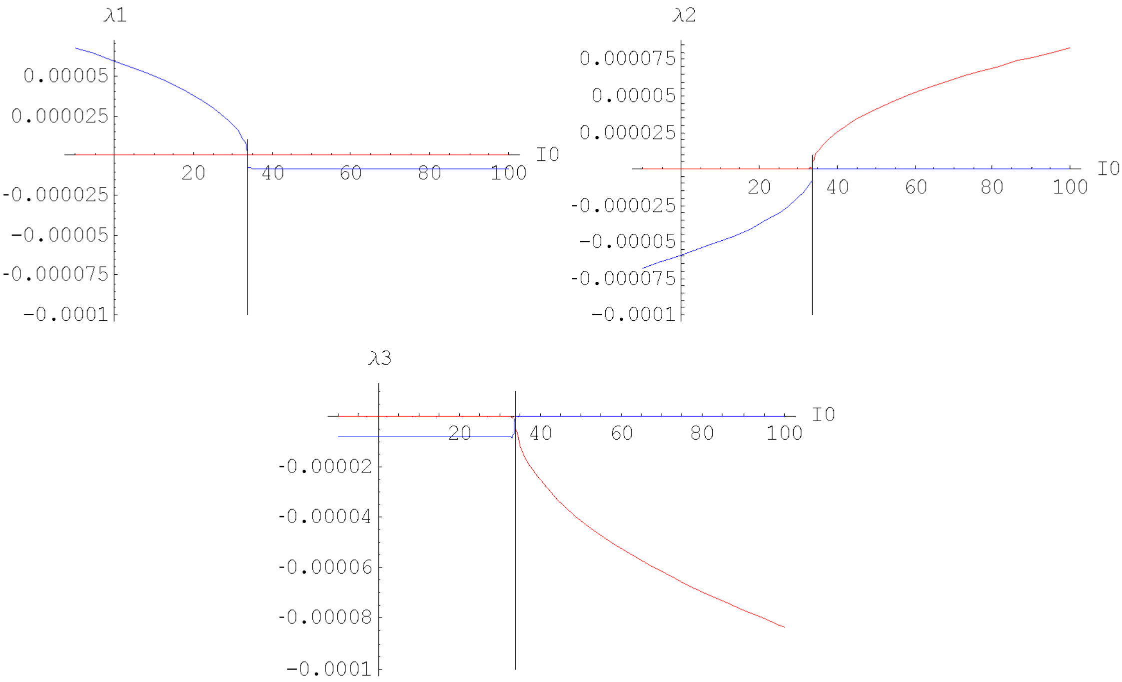

FIG. 2. The eigenvalues of linearized system versus dimensionless pump I_0

conjugated in the region, interesting for us. They describe periodic solutions in linearized system. Thus, in the region after the threshold the generation start as oscillations regime.

Then we determine eigenvalues and eigenvectors for our system near the threshold for the dimensionless pump power $I_0 = 40$. In this case the eigenvalues are:

$$(- 8.09788\ 10^{-6},\ - 2.25171\ 10^{-8} - 0.0000343939\ i,\ - 2.25171\ 10^{-8} + 0.0000343939\ i).$$

The eigenvectors is $(4.48306 \cdot 10^{-7}, -3.26166 \cdot 10^{-7}, 1.0)$ for the first eigenvalue and it directed at axis U. It means that U variable can be considered as a slow one (and stable). The variables n_2 and n_4 are fast in this system because of the real part of two residuary eigenvalues are practically zero. As has shown more a detailed analysis, the same situation occurs and for points in a phase space of the system relevant to transient regimes (but is higher than a threshold). It enables one to eliminate adiabatically variables of population densities from the set of equations with preservation of the phenomenon description accuracy for conditions, close to stationary. One should also remark, that singular points of the system are preserved as original ones at such a procedure.

4. The numerically schema and adiabatic elimination

. Direct numerically solution of the system (1)-(3) with the standard mathematical packages turned out to be impossible because of high stiffness of the system. So, it was necessary find special methods , which overcome this feature. The peculiarity of the system (1-3) is a relation between typical times scale for system. So, the length of the resonator is $L = 0.02\,m$ and the round trip time is $\tau \sim 0.1\ ns$. At the same time, the life times for excited states Yb and Er (2 and 4) are $\sim 0.1\ s$. Therefore the formation of spatial profiles of population densities and lasing densities becomes on times considerably smaller that the life times of the states. It leads in necessity of use of the algorithms maintaining prescribed accuracy, at numerical simulation of a system on very long time intervals.

This following numerical schema provides such accuracy: the equations of generation are solved with the usage of explicit two-layer schema for hyperbolic equations [.., ..]; the integration of the populations equations are performed with the modified Euler's method up to the moment of generation start given by the formula : $k_{Gain1}n_4(x,t) - k_{Gain2}(1 - n_4(x,t)) = k_{Loss}$.

At the moment of generation start the intensity of generation starts to grow up sharply then trends to decrease and so on - that is there is a transient regime corresponding to relaxation oscillation. In this moment the solving algorithm should be modified. Any standard scheme in this situation leads to artificial oscillation shown in fig.2a. Nevertheless,it worth to mention that transient oscillation in fact could be really observable in some laser system for strong excess of pumping above a threshold. It is also interesting that the problem of sharp change of a variables (and oscillations) exists only for generation intensity, dynamic behavior of population levels densities remains to be regular.

(a) (b)

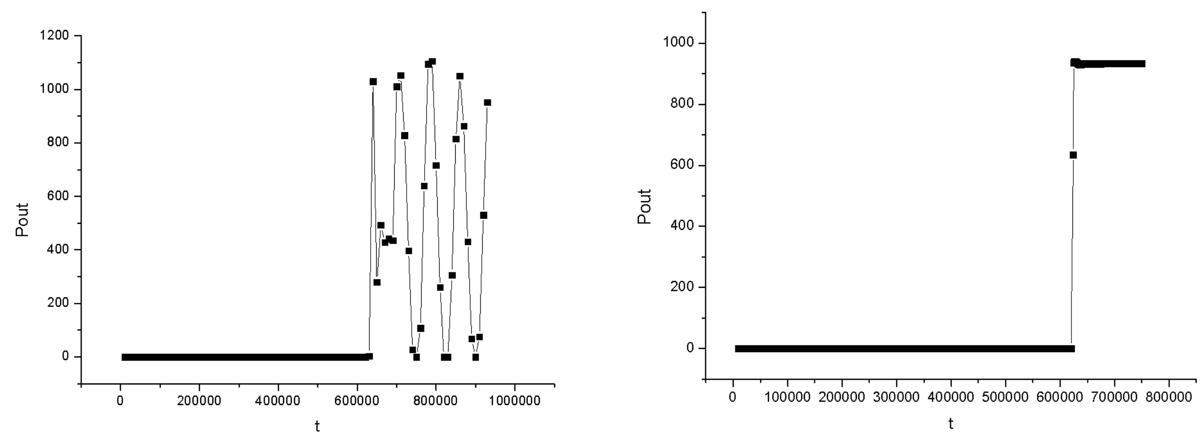

FIG. 3. Simulations of transient characteristics of a laser: (a) - without adiabatic elimination, (b) - with adiabatic elimination.

For a solution of this problem the method of adiabatic elimination of fast variables is proposed. The equations for densities of population of states n_2, n_4 are written in a quasistationary approximation. Then the obtained system is solved to find $n_2,\ n_4$ at a given value of the total intensity of generation $U = f_+ + f_-$. Two various solutions are gained, only one of which has physical sense (in view of a normalization: $n_i \in [0,1]$). Thus, for n_4 the relation is gained:

$$n_4(U) = \frac{1}{2(2U\alpha_2 + \beta_2)\gamma_1}\{2U\alpha_1\alpha_2 + 2U\alpha_2\beta_1 + \alpha_1\beta_2 + \beta_1\beta_2 + 3U\alpha_2\gamma_1 + \beta_2\gamma_1 + \alpha_1\gamma_2 -$$
$$\sqrt{(\beta_2(\beta_1+\gamma_1) + U\alpha_2(2\alpha_1 + 2\beta_1 + \gamma_1) + \alpha_1(\beta_2 - \gamma_2))^2 + 4\alpha_1(\alpha_1+\beta_1)(2U\alpha_2+\beta_2)\gamma_2}\} \quad (7)$$

Then the scheme of a numerical solution can be reorganized as follows: up to the moment of making of inverted population of the laser level the numerical scheme remains the previous. At the moment of realization of a requirement of the beginning of generation - intensities are still computed with use of the explicit two-layer plan, and the value n_4 which is included in it is calculated with the use of the equation (5). The dynamic characteristics of the scheme proposed gives the results compared with previous one shown in fig.2b.

The basic drawback of a method of isentropic elimination is inexact modeling of the transient regime. Nevertheless, this method allows to estimate simultaneously power as well as dynamic characteristics of a laser system.

We summarize the obtained results. The Yb:Er laser system within the considered distributed model is extremely stiff, that makes impossible its solution with use standard mathematical packages. Necessity of calculation dynamic characteristics of a system on long times superimposes an additional requirement of the precision of used numerical algorithm. It has been shown, that the satisfactory solution is possible with the use of the method of adiabatic elimination of fast variables which for our system appeared populations of states. As a results it is possible to obtain satisfactory solutions for a quasistationary condition of the distributed laser system. A principal shortage of a method is flattening at exposition of transient.

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