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Finite-Difference Time-Domain simulation of light propagation in 2D labyrinthine photonic structures

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The theoretical and numerical models for simulation of light propagation process through a medium with labyrinthine-like distribution of refractive index have been derived based on Finite-Difference Time-Domain (FDTD) method for direct solution of Maxwell's equations. It is shown that the distribution of the energy density of electromagnetic field is characterized by complex branching structure both for TE- and TM-polarized fields, and the transmission of structure is nonlinearly depended on the size of photonic cell.

1. Introduction

The finite-difference time-domain method (FDTD) is a very powerful numerical method for solving Maxwell's equations [1]. The potential of this method is given by its high accuracy and by the fact that it carries all the information about parameters of electro-magnetic field. Due to its accuracy, the FDTD method is widely used in simulating light propagation in optical waveguides like optical fibers or photonic crystals, for example [2, 3].

In this work, light propagation through two-dimensional (2D) inhomogeneous medium with labyrinthine-like distribution of refractive index will be analyzed by FDTD simulation. First of all, the theoretical model is presented in Section 2, in which the basic equations of the algorythm are shown both for TE mode (subsection 2A) and for TM mode (subsection 2B). Optical properties of the considered photonic structures are revised in subsection 2C. After that, the main results of the simulation are included in Section 3, and the main characteristics of continuous light propagation in a inhomogeneous dielectric structures will be analyzed from these results. As well as that, pulsed light beams propagation is also studied, analyzing the time dependece of the transmission through multi-layer labyrinthine photonic structures. Finally, in Section 4 we summarize the conclusions of this work.

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2. Theoretical model

Among the different methods to study light propagation through dielectric media, FDTD technique constitutes a very powerful one as long as it carries all the information about the time and space evolution of the electro-magnetic field. FDTD is a widely used technique that numerically solves Maxwell's equations with a high accuracy, entailing a considerable computing time. If we take Maxwell's equations for an dielectric medium

$$-\mu_0 \frac{\partial \vec{H}}{\partial t} = \vec{\nabla} \times \vec{E} \tag{1}$$

$$\varepsilon_0 \varepsilon(x, y, z) \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{H} \tag{2}$$

they can be decomposed into the three coordinate components to obtain a set of six differential equations:

$$-\mu_0 \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \tag{3}$$

$$-\mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \tag{4}$$

$$-\mu_0 \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \tag{5}$$

$$\varepsilon_0 \varepsilon(x, y, z) \frac{\partial E_x}{\partial t} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \tag{6}$$

$$\varepsilon_0 \varepsilon(x, y, z) \frac{\partial E_y}{\partial t} = \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \tag{7}$$

$$\varepsilon_0 \varepsilon(x, y, z) \frac{\partial E_z}{\partial t} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \tag{8}$$

The different components are graphically represented in the so-called Yee lattice, shown in Figure 1. Each surface of this lattice permits to perform the descritization of equations (3) - (8).

2.1. Space and time discretization of Maxwell's equations for TE mode

From now on, a 2D configuration of dielectric medium will be considered, in such a way that the electrical permitivity is reduced to the function of two coordinates only $\varepsilon = \varepsilon(x, y)$. In the following we consider the case, when electromagnetic wave falls normally to the surface

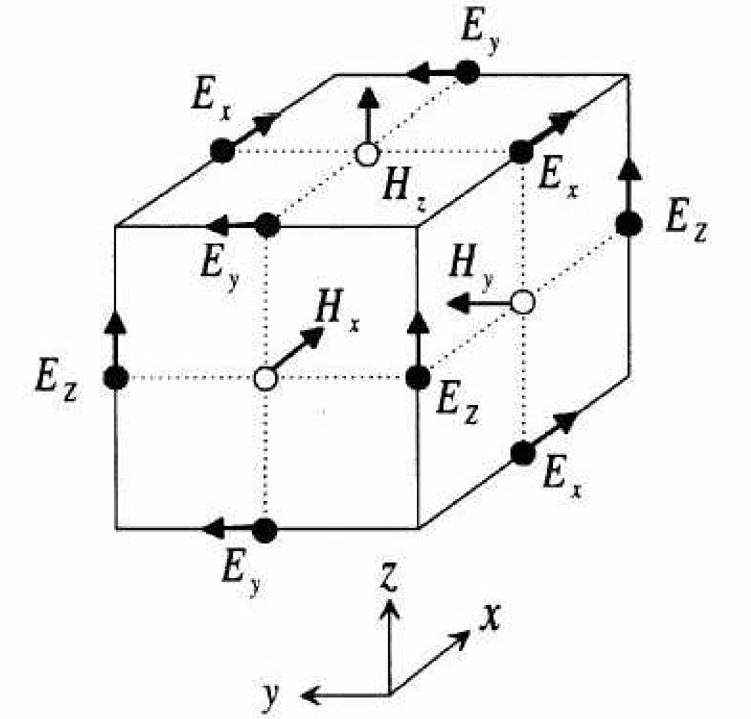

FIG. 1. Yee lattice showing the three spatial components of E and H fields.

$y = 0$, and the direction of vector $\vec{E}$ is orthogonal to the plane (x, y). With this situation, the differential equations for TE mode are:

$$-\mu_0 \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} \tag{9}$$

$$\mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} \tag{10}$$

$$\varepsilon_0 \varepsilon(x, y) \frac{\partial E_z}{\partial t} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \tag{11}$$

According to FDTD method, these differential functions are discretized in both space and time, so that they can be calculated as [1]:

$$H_x^{n+1/2}(i, j+1/2) = H_x^{n-1/2}(i, j+1/2) - \frac{\Delta t}{\mu_0 \Delta y} \left[E_z^n(i, j+1) - E_z^n(i, j) \right] \tag{12}$$

$$H_y^{n+1/2}(i+1/2, j) = H_y^{n-1/2}(i+1/2, j) - \frac{\Delta t}{\mu_0 \Delta x} \left[E_z^n(i+1, j) - E_z^n(i, j) \right] \tag{13}$$

$$E_z^{n+1}(i,j) = E_z^n(i,j) + \tfrac{\Delta t}{\varepsilon_0 \varepsilon(i,j)\Delta x}\left[H_y^{n+1/2}(i+1/2,j) - H_y^{n+1/2}(i-1/2,j)\right] - \\ -\tfrac{\Delta t}{\varepsilon_0 \varepsilon(i,j)\Delta y}\left[H_x^{n+1/2}(i,j+1/2) - H_x^{n+1/2}(i,j-1/2)\right] \tag{14}$$

where Δx and Δy are the characteristic steps of the $x-y$ spatial mesh, Δt is the time step, and the functions are discretized as $F^n(i,j) = F(i\Delta x, j\Delta y, n\Delta t) = F(x,y,t)$.

2.2. Space and time discretization of Maxwell's equations for TM mode

On the other hand, the equations for TM mode, which is realized, when vector $\vec{H}$ in the incident light field is orthogonal to the plane (x,y), follow from equations (3) - (8):

$$\varepsilon_0 \varepsilon(x,y)\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} \tag{15}$$

$$\varepsilon_0 \varepsilon(x,y)\frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \tag{16}$$

$$-\mu_0 \frac{\partial H_z}{\partial t} = \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_x}{\partial y}\right) \tag{17}$$

Again, discretizing these functions according to FDTD algorythm, we get [1]:

$$E_x^{n+1}(i+1/2,j) = E_x^n(i+1/2,j) + \frac{\Delta t}{\varepsilon_0 \varepsilon(i,j)\Delta y}\left[H_z^{n+1/2}(i+1/2,j+1/2) - H_z^{n+1/2}(i+1/2,j-1/2)\right] \tag{18}$$

$$E_y^{n+1}(i,j+1/2) = E_y^n(i,j+1/2) - \frac{\Delta t}{\varepsilon_0 \varepsilon(i,j)\Delta x}\left[H_z^{n+1/2}(i+1/2,j+1/2) - H_z^{n+1/2}(i-1/2,j+1/2)\right] \tag{19}$$

$$H_z^{n+1/2}(i+1/2,j+1/2) = H_z^{n-1/2}(i+1/2,j+1/2) - \tfrac{\Delta t}{\mu_0 \Delta x}\left[E_y^n(i+1,j+1/2) - E_y^n(i,j+1/2)\right] + \\ \tfrac{\Delta t}{\mu_0 \Delta y}\left[E_x^n(i+1/2,j+1) - E_x^n(i+1/2,j)\right] \tag{20}$$

In these equations Δx and Δy are the characteristic steps of the $x-y$ spatial mesh, Δt is the time step, and the functions are discretized as $F^n(i,j) = F(i\Delta x, j\Delta y, n\Delta t) = F(x,y,t)$.

2.3. Optical properties of 2D labyrinthine photonic structures

The labyrinthine structures that are typical for most nonequilibrium systems, such as the Belousov-Jabotinsky reaction [4] or magnetic liquids [5], have also been detected in many nonlinear optical self-organization processes, e.g., in the parametric interactions of light beams in nonlinear interferometers [6, 7]. As a rule, the mathematical models of the labyrinthine structure formation equations are related to analyzing the Ginsburg-Landau complex equation [8] or its particular cases such as the Swift-Hohenberg equation. The latter has been used by the authors [6] for explaining the labyrinthine-like light beam intensity distributions under the intracavity singular parametric interaction of the light waves. The labyrinthine structures are characterized by the presence of short-range and the absence of long-range order. Therefore, these structures can be considered as transient structures between photon crystals, complex-structured waveguides, and disordered systems.

There are many known nonlinear optical system configurations featuring self-organization effects that lead to the formation of labyrinthine structures in the light laser beam intensity profile [6, 7]. Among these systems, nonlinear interferometers stand out for their feedback process based on reflection from resonator mirrors, which amplifies the interaction effect of the radiation and nonlinear environment causing the optical modes competition. As a result, self-organizing diffraction structures of light fields are formed.

Further we will concentrate to the problem of counter-progagating two-wave mixing in the cavity with Kerr-like nonlinearity. The system of coupled-mode equations for this problem can be transformed [7] to the system of parametrically driven complex Ginsburg-Landau equations (PDCGLEs) using the following change of variables: $S = (E_1 + E_2)/2$ and $P = (E_1 - E_2)/2$. Thus, the resulting system of PDCGLEs can written as follows:

$$\frac{\partial S}{\partial t} = E_0 + \mu' S^* - \left(1 + i\eta'\Theta'\right) S + i\eta' S\,|S|^2 + i\Delta_\perp S \qquad (21)$$

$$\frac{\partial p}{\partial t} = \mu_0 + \left|\mu''\right| p^* - \left(1 + i\eta''\Theta''\right) p + i\eta'' p\,|p|^2 + i\Delta_\perp p \qquad (22)$$

where the summary input field is $E_0 = (E_{10} + E_{20})/2$, $\mu_0 = (E_{10} - E_{20})/2$ is a negligible quantity, $\eta' = \eta'' = 3\eta$, $\mu' = -i\eta P^2$ and $\mu'' = -i\eta S^2$ ($\mu'' = \left|\mu''\right| exp(i\varphi)$) are proportional to the strengh of driving forces, $p = Pexp(-i\varphi/2)$ is a phase shifted order parameter, $\Theta' = \frac{1}{3}\left(\Delta_0 - 2|P|^2\right)$ and $\Theta'' = \frac{1}{3}\left(\Delta_0 - 2|S|^2\right)$ are proportional to detuning parameters.

For the case of defocusing nonlinearity ($\eta < 0$) in the optical bistability region ($\Delta_0 \geq \sqrt{3}$) with exceeding of the threshold value of the light waves intensity ($I_0 = 8/\sqrt{3}$), spatially periodic, labyrinthine-like, and localized light field intensity structures are formed sequentially [7]. In particular case, at $I_0 = 1.6 - 1.8$, the spatially periodic structures are unstable and tend to developing complex-structured distributions of the light field.

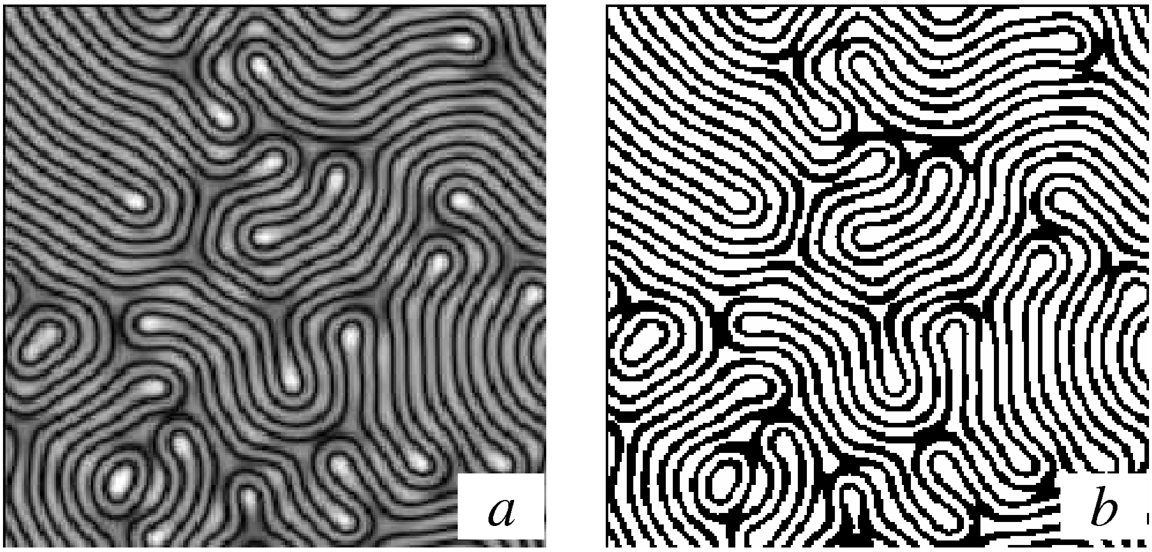

FIG. 2. Examples of labyrinthine-like distribution of refractive index. a) - Gradient-type structure; b) - Step-like structure.

Taking into account the dependence of the refraction index change on the intensity of the the light waves, which is defined for Kerr-type nonlinearity as $\Delta n \sim I_\Sigma$, the spatial distribution of the intensity of the light beams leads to a topologically similar structure of the nonlinear environment refraction index. An example of forming of such an optical structure is shown in Figure 1a. We can see that the distribution of the refraction index represents a complexly branched structure and the dielectric permitivity varies from $\epsilon_{min} = 1$ to $\epsilon_{max} = 4$, which corresponds to different levels of grey in the figure. This structure can be considered as a photonic structure for a probe light beam. We can see that the structure has areas with differently directed quasi-periodic distribution of the dielectric conductivity, as well as multiple curved waveguide structures and defects in the form of solitary areas of constant dielectric conductivity values. Although being valid for simulation purposes, the sample shown in the Figure 1a can not be generated in a realistic experimental setup, because the practical realization of the structures with a gradient distribution of the refraction index presents explicit technological difficulties; therefore, along with the given structures, further we will also use structures with a step-like profile (Figure 1b) having only two levels of values: $\epsilon_1 = 1$ and $\epsilon_2 = 4$.

3. Simulation results

Using the theoretical model described above, light propagation thorough a medium with labyrinthine-like distribution of refraction index was analyzed. In the numeric modeling, it was assumed that the source of the electromagnetic field in the form of a monochromatic wave with lateral dimensions of the same order as λ is placed on the left border of the structure and simulates the propagation of a spherical-like wave in the given structure. The structures spatial dimensions in our calculations are connected with the wavelength of radiation and for the considered case is $S = 17\lambda \times 17\lambda$. The FDTD results of the simulation of electro-magnetic waves propagation through this sample are shown in Figures 3. It presents the steady-state distribution of the density of electromagnetic field for different polarizations of the input wave. As we can see in Figures $3a - 3d$, after the transient process, the quasi-stationary distribution of the electromagnetic field energy is established in the structures of both types. This distribution is characterized by a complex branchy structure, which is formed as a result of the combination of the waveguide light propagation effects, the existence of differently directed photonic forbidden areas, and light localization on the solitary defects of the dielectric conductivity distribution.

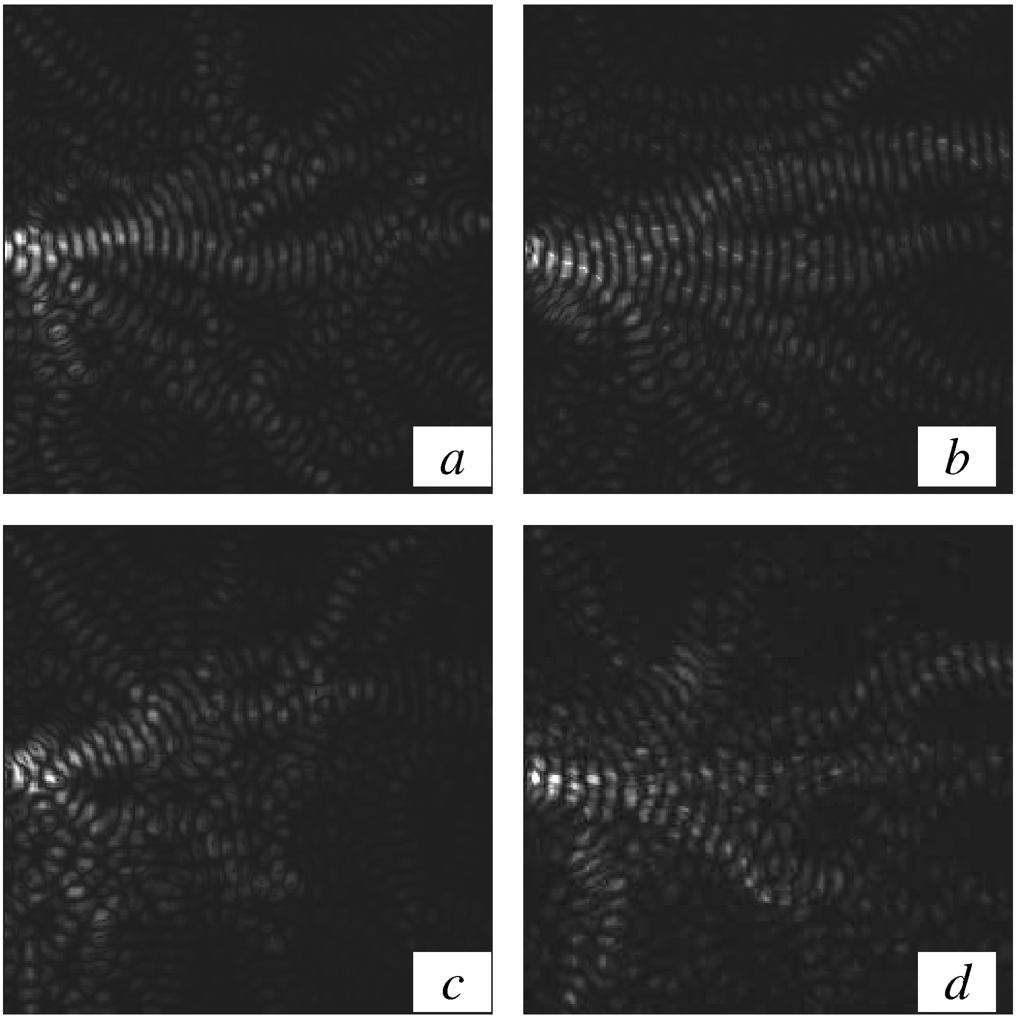

FIG. 3. Spatial distribution of the electromagnetic field density in gradient-type (a, b) and step-like (c, d) labyrinthine photonic structures. (a, c)TE mode, (b, d)TM mode.

The spatial transmission of the given structure allows considering electromagnetic waves propagation problems in labyrinthine structures of larger sizes. The examples of the constructed multilayer structures are presented in Figs. 4a-4c. According to the results of the extended numerical calculations, the proper branching degree provides equal probability of the transmission over each of the four edges of the considered labyrinthine structure cell. This allows evaluating the transit time of the radiation through the multilayer structure similarly to the diffusion theory: $\tau \sim n^2$, where n is the number of layers of the structure.

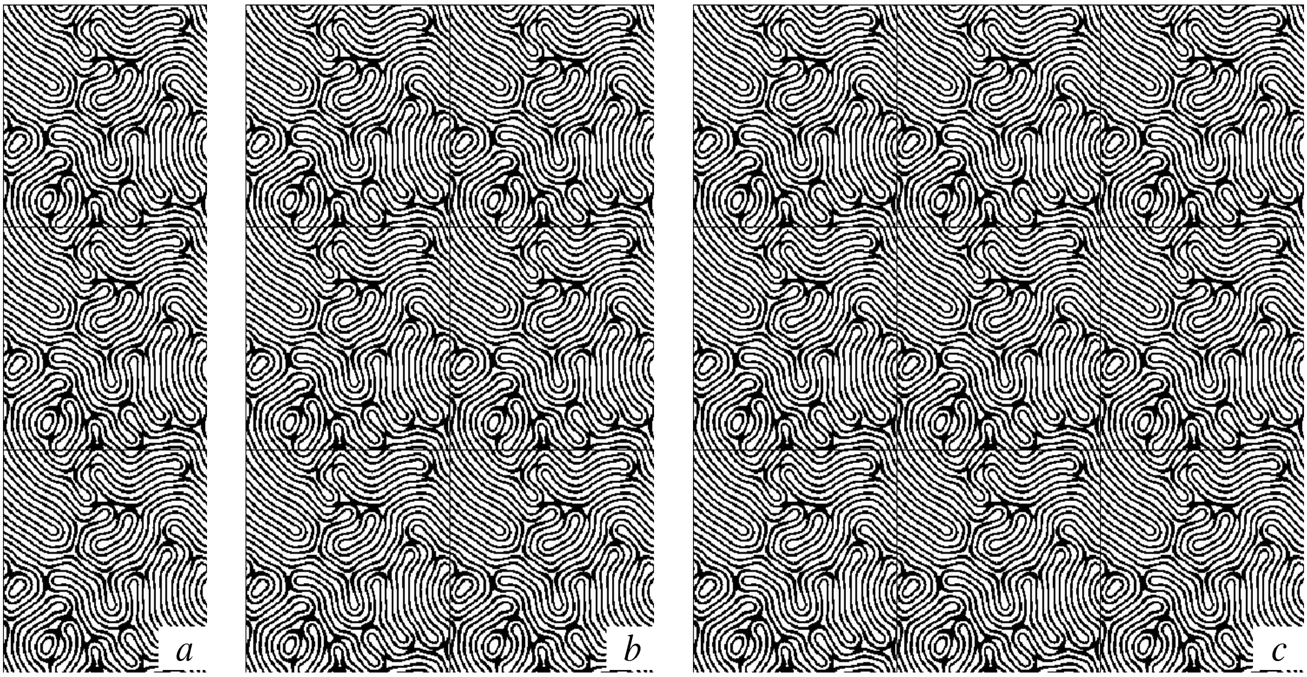

FIG. 4. 2D multilayer structures with labyrinthine-like distribution of refraction index. a - cell array 1×3, b - cell matrice 2×3, c - cell matrice 3×3

Figure 5a demonstrates the time evolution of transmission of considered structures, which can be obtained by solving the corresponding probabilistic problem. The curves $1-6$ correspond to the following structure size: 1 - cell array of dimension 1×3, 2 - cell array of dimension 2×3 , etc. The numerical analysis results of the quasi-monochromatic light impulses propagation kinetics in labyrinthine-like structures of different sizes are presented in Figure 5b. In this case, curve 1 corresponds to the case of light impulse propagation through a single cell with the labyrinthine structure shown in Figure 2a. Curve 2 is calculated for the case when the light impulse passes through a cell array of dimension 1×3, whereas curves 3 and 4 correspond to cell matrices of 2×3 and 3×3, respectively. Evidently, in the last two cases, the light impulse becomes considerably broadened. This can be explained by the meshing of the electromagnetic waves during the propagation through the optical labyrinths. The time evaluations of the light impulse transition through these structures performed with the help of exponential approximation for

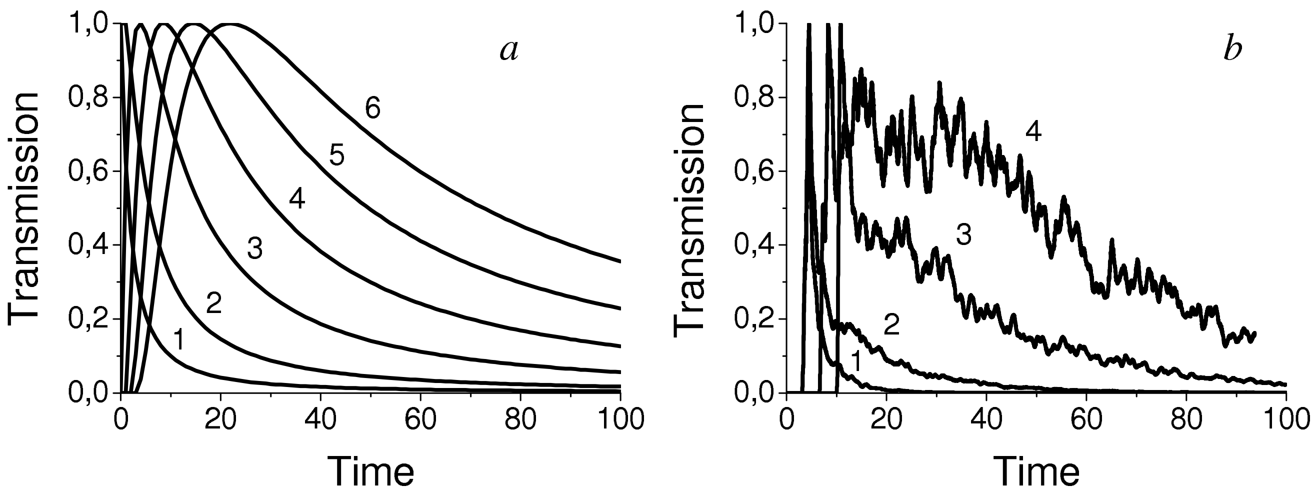

FIG. 5. Pulsed light propagation through multilayer structures with labyrinthine-like distribution of refraction index. a) - temporal dependence of transmission obtained by solving the corresponding probabilistic problem, b) - temporal dependence of transmission obtained by direct numerical modeling.

curves 2 − 4 ($\tau_2 \approx 6.6$, $\tau_3 \approx 26.8$, $\tau_4 \approx 60$), are in good agreement with those of the simple radiation diffusion model obtained above (Figure 5a). It is necessary to point out that this effect can be used in order to increase the optical information storage time and the localization of the electromagnetic field energy in the photonic structure.

4. Conclusions

In this work, we have analyzed optical beam propagation through a medium with labyrinthine-like distribution of the refraction index by the direct numerical solution of Maxwells equations using FDTD method. The effect of scattering in the transmitting beam has been studied from the simulated results. The established laws of light transition and refraction by 2D structures for differently polarized waves make it clear that these structures can be widely applied in realizing various optical systems for controlling the light beams propagation.

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