

ALHEP symbolic algebra program for high-energy physics

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Abstract

ALHEP is the symbolic algebra program for high-energy physics. It deals with amplitudes calculation, matrix element squaring, Wick theorem, dimensional regularization, tensor reduction of loop integrals and simplification of final expressions. The program output includes: Fortran code for differential cross section, Mathematica files to view results and intermediate steps and TeX source for Feynman diagrams. The PYTHIA interface is available.

The project website <http://www.hep.by/alhep> contains up-to-date executables, manual and script examples.

1 Introduction

The analytical calculations in high-energy physics are mostly impossible without a powerful computing tool. The big variety of packages is commonly used [1,2,3,4,5,6,7,8,9,10,11,12,13,14]. Some are general-purpose symbolic algebra programs with specific HEP-related plug-ins (REDUCE [15], Mathematica [16]), some are designed especially for particle physics (CompHEP [1], SANC [2], GRACE [3] etc.) and some are created for specific interaction class or specific task. Many of them uses external symbolic algebra core (Form [17], MathLink [16]). They can deal with matrix elements squaring (FeynCalc [7]) or calculate helicity amplitudes directly (MadGraph [4], CompHEP [1], O'Mega [5]). Some packages provide numerical calculations, some require external Monte-Carlo generator to be linked. Some programs contain also one-loop calculation routines (FormCalc [6], GRACE [3], SANC [2]). Nevertheless, there is no uniform program that meets all the user requirements.

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Every calculation requires the program-independent check. The optimal tactic is the simultaneous usage of two (or more) different symbolic algebra packages.

ALHEP is a symbolic algebra program for performing the way from Standard Model Lagrangian to amplitude or squared matrix element for the specified scattering process. It can also be useful for loop diagrams analysis. The basic features are:

- Diagrams generation using Wick theorem and SM Lagrangian.
- Amplitude calculation or matrix element squaring.
- Bondarev functions method for traces calculation.
- Tensor reduction of loop integrals.
- Dimensional regularization scheme.
- Generation of **Fortran** procedures for numerical analysis (PYTHIA [18] and LoopTools [19] interfaces are implemented).

The current ALHEP version have several implementation restrictions, that will be lifted in future. The following features are in progress of implementation:

- Bremsstrahlung part of radiative correction (the integration over real photon phase space).
- Complete one-loop renormalization scheme including renormalization constants derivation.
- Arbitrary Lagrangian assignment.

After these methods implementation the complete one-loop analysis will be available. Please refer to project website for program updates.

ALHEP website <http://www.hep.by/alhep> contains the up-to-date executables (for both Linux & Win32 platforms), manual and script examples. The mirror at <http://cern.ch/~makarenko/alhep> is also updated.

2 ALHEP Review. Program Structure

The ALHEP program internal structure can be outlined as follows:

- The native symbolic algebra core.
- Common algebra libraries:
 - Dirac matrices, tensor and spinor algebra, field operators & particle wave functions zoo.
- Specific HEP functions and libraries.
 - It include Feynman diagrams generation, trace calculations, helicity amplitudes method, HEP-specific simplification procedures, tensor integrals reduction and others.

- Interfaces to Mathematica, Fortran, TeX and internal I/O format.
Fortran code is used for further numerical analysis. Mathematica code can be used for viewing any symbolic expression in program. But no backward interface from Mathematica is currently implemented. TeX output can be generated for Feynman diagrams view. Internal I/O format is implemented for the most of symbolic expressions, allowing save and restore calculation at intermediate steps.
- Command script processor.
User interface is implemented in terms of command scripts. The ALHEP script language have C-like syntax, variables, arithmetic operations, function calls. All HEP-related tasks are implemented as build-in functions.

2.1 Getting Started

To use ALHEP one should download the pre-compiled executable for appropriate platform (Linux and Win32 are available) and write a control script to describe your task. ALHEP program should be launched with the single argument: script file name to be invoked The following steps are required to create a workspace:

- Download ALHEP executables from project website: `alhep.gz` (for Linux) or `alhep.zip` (Win32). Unpack executable, e.g.
`gzip -d alhep.gz`
- Download up-to date command list: `ALHEPCommands.txt`. The set of commands (or options) may be changed in future versions and this manual may be somewhat obsolete. Please refer to `ALHEPCommands.txt` file that always corresponds to the latest ALHEP version. The last changes are outlined in `RecentChanges.txt` file at website.
- Create some working directory and compose command script file therein. For example consider the $u\bar{u} \rightarrow W^+W^-\gamma$ process with $\{-+\}$ -helicities of initial quarks. To calculate amplitude we create the following script file (call it "test.al"):

```
SetKinematics(2, 3 // 2->3 process
, QUARK_U, "p\1", "e\1" // u
, -QUARK_U, "p\2", "e\2" // u-bar
, WBOZON, "f\1", "g\1" // W{+}
, PHOTON, "f\0", "g\0" // photon
, -WBOZON, "f\2", "g\2" ); // W{-}
SetDiagramPhysics(PHYS_SM_Q1GEN); // SM with 2 quarks only
SetMassCalcOrder(QUARK_U, 0); // consider massless
SetMassCalcOrder(QUARK_D, 0); // consider massless
diags = ComposeDiagrams(3); // create diagrams, e^3
DrawDiagrams(diags, "res.tex",
```

```

                DD_SMALL|DD_SWAP_TALES, FILE_START);
SetFermionHelicity(1, -1);
SetFermionHelicity(2, 1);
SetParameter(PAR_TRACES_BONDEREV, 1);
ampl = CalcAmplitude(RetrieveME(diags));
ampl = KinArrange(ampl);           // arrange result
ampl = Minimize(ampl);           // minimize result
SaveNB("res.nb", ampl, "", FILE_START|FILE_CLOSE);
f = NewFortranFile("res.F", CODE_F77);
CreateFortranProc(f, "UUWWA", ampl,
    CODE_IS_AMPLITUDE|CODE_COMPLEX16
    |CODE_CHECK_DENOMINATORS|CODE_PYTHIA_VECTORS);

```

The `SetKinematics()` function declares particles, momenta and polarization symbols. The physics is declared with `PHYS_SM_Q1GEN` option to restrict diagrams number. The amplitude for all (d, c, b) internal quarks can be obtained from generated here by quark mixing matrix replacement: $U_{ud}^2 \rightarrow U_{ud}^2 + U_{us}^2 + U_{ub}^2$ (chiral limit). Diagrams are created by `ComposeDiagrams()` call. `CalcAmplitude()` function creates the symbolic value for process amplitude. For detailed discussion of this example see sec. 15.1.

- Create simple batch file "run.me" like:

```
~/alhep_bin_path/alhep test.al
```

The ALHEP program creates some console output (current commands, scroll bars and some debugging data). If it is not allowed one should redirect console output to file here.

The batch execution with `test.al` command file takes about 1 minute at 1.8GHz P4 processor. The following files are created:

`res.nb`: Mathematica file containing symbolic expression of amplitude. Created by `SaveNB()` function.

`res.F`: F77 code for numerical analysis created by `NewFortranFile()` and `CreateFortranProc()` calls. The library file `alhep_lib.F` is required for code compilation and should be downloaded from project website. See sec. 11.1 for Fortran generation and compilation review.

`res.tex`: TeX source for 19 Feynman graphs generated. The AxoDraw [23] Latex package is used. The `res.tex` file should be included into your LaTeX document using `\input res.tex` command. The template document to include your diagrams can be found at program website. See fig. 1 in sec. 15.1 for diagrams generated.

`debug.nb`: Mathematica file with debugging information and some intermediate steps. The amount of debugging information is declared in `debug.ini` file in working directory. See sec. 13 for further details.

See sec. 15 and project website for another examples. It is convenient to use some example script as template and modify it for your purposes.

2.2 Calculation scheme

The usual ALHEP script contains several steps:

(1) Initialization section

Declaration of process *kinematics*: initial and final-state particles, titles for particle momenta & polarization vectors (`SetKinematics()`).

Physics model definition. The SM physics or part of SM Hamiltonian should be specified (`SetDiagramPhysics()`). The shorter Hamiltonian is selected, the faster is Wick theorem invocation.

Polarization declaration. Every particle is considered as polarized with abstract polarization vector by default. The specific helicity value can be set manually or particle can be marked as unpolarized. The several ways of polarization involving are implemented, see sec. 4.4 (`SetPolarized()`, `SetFermionHelicity()`, `SetPhotonHelicity()`, ...).

Setting *mass-order* rules for specific particles (`SetMassCalcOrder()`). One can demand the massless calculation for light particle, that greatly saves evaluation time. One can also demand keeping particle mass with specific order M^n and drop out the higher-order expressions like M^{n+1} . It allows to consider the leading mass contribution without calculating precisely.

(2) Diagrams generation

Feynman diagrams are generated for specific e^n order using the *Wick theorem* algorithm (`ComposeDiagrams()`) User may *draw diagrams* here (`DrawDiagrams()`), halt the program (`Halt()`) and check out if diagrams are generated correctly.

After the diagram set is generated one may *cut-off* not interesting diagrams to work with the shorter set or *select* the single diagram to work with (`SelectDiagrams()`). The loop corrections are calculating faster when processed by single diagrams.

Then *matrix element* is retrieved from diagrams set (`RetrieveME()`).

Before any operation with loop matrix element one should declare the N -dimensional space (`SetNDimensionSpace()`). Some procedures involve N -dimensional mode automatically for loop objects, but most functions (arranging, simplification etc.) don't know the nature of expression they work with. Therefore the dimensional mode should be forced.

The diagrams set and any symbolic expression may be saved and restored at next session to refrain from job repetition (`Save()`, `Load()`).

(3) Amplitude calculation

The amplitude evaluation (`CalcAmplitude()`) is the faster way for multi-diagram process analysis.

All the particles are considered polarized. The spinor objects are projected to abstract basis spinor. The basis vectors are generated in numerical code to meet the non-zero denominators condition. See sec. 7 for

method details.

(4) Matrix element squaring (coupling to other)

The squaring procedure (`SquareME()`) is controlled by plenty of options, intended mostly for the performance tuning and debugging.

It basically includes *reduction of gamma-matrix sequences* coupled to kinematically dependent vectors. It reduces the number of matrices in every product to minimum. The item-by-item *squaring* is followed. For $loop \times born^*$ couplings the virtual integrals are involved. See sec. 8 for details.

Amplitude calculation is fast, but its result may be more complicated than squaring expressions. Amplitude depends on particle momenta, polarization vectors and additional basis vectors. For unpolarized process the averaging cycle is generated in Fortran code, and complex-numbers calculation should be performed. The squaring result for unpolarized process is the polynomial of momenta couplings only. Hence there is no unique answer what result is simpler for 10-15 diagrams reaction. One should definitely use amplitude method for more than 10-15 diagrams squaring.

(5) Loop diagrams analysis

The *tensor virtual integrals* are reduced to scalar ones (`Evaluate()`, sec. 9). The scalar coefficients of tensor integral decompositions are also reduced to scalar integrals in the most cases (for 1 – 4 point integrals).

For *scalar loop integrals* the tabulated values are used. There is no reason to tabulate integrals with complicated structure. Hence scalar integrals table contain the A_0 and B_0 integrals with different mass configuration. It also contains a useful D_0 chiral decomposition. Other integrals should to be resolved using `LoopTools`-like [19,20] numerical programs.

The *renormalization* procedure is under construction now. The counter-terms (CT) part of Lagrangian leads to CT diagrams generating. In the nearest future the abstract renormalization constants (δ_m, δ_f etc.) will be involved and tabulated for the minimal on-shell scheme. The automatic derivation of constants is supposed to be implemented further. Please refer ALHEP website for implementation progress.

(6) Simplification

The *kinematic simplification* procedure is available (`KinSimplify()`, sec. 12). It reduces expression using all the possible kinematic relations between momenta and invariants. The minimization of $+/*$ operations in huge expressions can also be performed (`Minimize()`).

(7) Fortran procedure creation for numerical analysis

F90 or F77-syntaxes for generated procedures are used. Generated code can be linked to `PYTHIA`, `LoopTools` and any Monte-Carlo generator for numerical analysis.

3 ALHEP script language

The script syntax is similar to C/Java languages. Command line breaks at ”;” symbol only. Comments are marked as // or /*...*/. Operands may be variables or function calls. If no function with some title is defined, the operand is considered as variable. The notation is case-sensitive.

The script language have no user-defined functions, classes or loop operators. It seems to be useless in current version.

All ALHEP features are implemented as build-in functions. The execution starts from the first line and finishes at the end of file (or at `Halt()` command).

Variable types are casted and checked automatically (in run-time mode). No manual type specifying or casting are available. List of ALHEP script internal types:

- Abstract Symbolic Expression (*expr*).
Result of function operations. Can be stored to file(`Save()`) and loaded back(`Load()`). Basic operations: +,-,*. The division is implemented using `Frac(a,b)` function. Are not supposed to be inputed manually, although a few commands for manual input exist.
- Integer Value (*int*).
Any number parameter will be casted to integer value. Basic operations: +,-,*,|. The division is performed using `Frac(a,b)` function. No fractional values are currently available.
- String (*str*).
String parameters are started and closed by double quotes (”string variable”). Used to specify symbol notation (e.g. momenta titles), file names, etc. Basic operations: +(concatenate strings).
- Set of Feynman Diagrams (*diagrams*).
Result of diagrams composing function. Basic operations: `Save()`, `Load()` and `SelectDiagrams()`. Diagrams can also be converted into TeX graphics.
- Matrix Element (*me*).
Contains symbolic expression for matrix element and information on it’s use: list of virtual momenta (integration list) etc.

The total up-to-date functions list can be found in `ALHEPCommands.txt` file at project website.

4 Initialization section

4.1 Particles

Particles are determined by integer number, called **particle kind** (PK). The following integer constants are defined: ELECTRON, MUON, TAULEPTON, PHOTON, ZBOZON, WBOZON, QUARK_D, QUARK_U, QUARK_S, QUARK_C, QUARK_B, QUARK_T, NEUTRINO_ELECTRON, NEUTRINO_MUON, NEUTRINO_TAU. The ghost and scalar particles are not supposed to be external and their codes are unavailable. Antiparticles have negative PK that is obtained by "-PK" operation.

If kinematics is declared the particles can be selected using the **particle ID** (PID) number. Initial particles have negative ID (-1, -2) and final are pointed by positive numbers (1, 2, 3...).

4.2 Kinematic selection

Before any computations may be performed one needs to declare the kinematic conditions. They are: number of initial and final state particles, PK codes, symbols for momentum and polarization vector for every particle.

```
SetKinematics((int)N_Initials, (int)N_Finals,  
              [(int)PK_I, (str)momentum_I, (str)polarization_I, ...]);
```

Here `N_Initials` and `N_Finals` – numbers of initial and final particles in kinematics. The next parameters are particle kind, momentum and polarization symbols, repeated for every particle. For example, the $e^-(k_1, e_1)e^+(k_2, e_2) \rightarrow \mu^-(p_1, e_3)\mu^+(p_2, e_4)$ process should be declared as follows:

```
SetKinematics(2, 2,  
              ELECTRON, "k\_1", "e\_1",  
              -ELECTRON, "k\_2", "e\_2",  
              MUON, "p\_1", "e\_3",  
              -MUON, "p\_2", "e\_4");
```

4.3 Particle masses

All the particles are considered massive by default. Particle can be declared massless using the `SetMassCalcOrder` function.


```
SetMassCalcOrder((int)PK, (int)order);
```

PK: particle kind,
order: maximum order of particle mass to be kept in calculations. Zero value means massless calculations for specified particle. Negative value declares the mass-exact operations.

The mass symbols are generated automatically and look like m_e , m_W etc. Hence, all the electrons in process will have the same mass symbol. To declare unique mass symbols for specific particles the `SetMassSym()` function is used. The different masses for unique particles are often required to involve the Breit-Wigner distribution for particles masses.

```
SetMassSym((int)PID, (str or expr)mass);
```

PID: particle ID in kinematics (< 0 for initial and > 0 for final particles),
mass: new mass symbol, like "m\ \backslash $_X$ " for m_X .

4.4 Polarization data

All the particles are considered as polarized initially. The default polarization vector symbols are set together with kinematic data (in `SetKinematics()`).

To declare particle unpolarized the `SetPolarized()` function is used:

```
SetPolarized((int)PID, (int)polarized);
```

PID: particle ID in kinematics,
polarized: 1 (polarized) or 0 (unpolarized).

One can declare the specific polarization state (helicity value) for particles.

The photon (k, e) helicities are involved in terms of two outer physical momenta:

$$e_{\mu}^{\pm} = \left((p_{+} \cdot k) p_{-\mu} - (p_{-} \cdot k) p_{+\mu} \pm i \epsilon_{\mu\alpha\beta\nu} p_{+}^{\alpha} p_{-}^{\beta} k^{\nu} \right) / N_{k, p_{\pm}}. \quad (1)$$

```
SetPhotonHelicity((int)PID, (int)h, (str)"pP", (str)"pM");
```

PID: photon particle ID in kinematics,
h: helicity value: ± 1 or 0. Zero value clears the helicity information

pP, pM: p_{\pm} base vectors in (1) formula. The γ -coupled forms of (1) (precise or chiral [26]) are used automatically if available.

One may select the transverse unpolarized photon density matrix (instead of usual $-(1/2)g_{\mu\nu}$):

$$\sum e_{\mu}e_{\nu}^{*} \rightarrow -g_{\mu\nu} + \frac{P_{\mu}k_{\nu}}{(P \cdot k)} + \frac{k_{\mu}P_{\nu}}{(P \cdot k)} - \frac{k_{\mu}k_{\nu}k^2}{(P \cdot k)^2}. \quad (2)$$

```
SetPhotonDMBase((int)PID, (str)"P");
```

PID: photon particle ID in kinematics,
 "P": Basis vector P in (2).

The fermion (k,e) helicity is declared using:

```
SetFermionHelicity((int)PID, (int)h);
SetFermionHelicity((int)PID, (str)"h");
```

PID: fermion particle ID in kinematics,
 h: helicity value: ± 1 or 0. Zero value clears the helicity information. Density matrix is usual: $\sum u\bar{u} \rightarrow \hat{p}\gamma_{\pm}$ (massless fermion).
 "h": symbol for scalar parameter in the following density matrix (massless fermion): $\sum u\bar{u} \rightarrow (1/2)\hat{p}(1 + h\gamma_5)$.

Notes:

- `SetPolarized(PID, 0)` call will also clear helicity data.
- `SetXXXHelicity(PID, 1, ...)` also sets particle as polarized (previous `SetPolarized(PID,0)` call is canceled).
- `SetXXXHelicity(PID, 0, ...)` clears helicity data but does not set particle unpolarized.

4.5 Physics selection

One can either declare the full Standard Model (in unitary or Feynman gauge) or select the parts of SM Lagrangian to be used. The QED physics is used by default. The Feynman rules corresponds to [27] paper.

```
SetDiagramPhysics( (int)physID );
```

physID: physics descriptor, to be constructed from the following flags:
PHYS_QED: Pure QED interactions,
PHYS_Z and **PHYS_W:** Z- and W-boson vertices,
PHYS_MU, **PHYS_TAU:** Muons and tau leptons (and corresponding neutrinos),
PHYS_SCALARS: Scalar particles including Higgs bosons,
PHYS_GHOSTS: Faddeev-Popov ghosts,
PHYS_GAUGE_UNITARY: Use unitary gauge,
PHYS_QUARKS, **PHYS_QUARKS_2GEN** and **PHYS_QUARKS_3GEN:** $\{d, u\}$, $\{d, u, s, c\}$
 and $\{d, u, s, c, b, t\}$ sets of quarks,
PHYS_RARE_VERICES: vertices with 3 or more SCALAR/GHOST tales,
PHYS_CT: Renormalization counter-terms (implementation in progress),
PHYS_SM: full SM physics in unitary gauge (all the flags above except for
PHYS_CT),
PHYS_SM_Q1GEN: the SM physics in unitary gauge with only first generation
 of quarks (d, u),
PHYS_ELW: SM in Feynman gauge with no rare 3-scalar vertice,
PHYS_EONLY: no muons, tau-leptons and adjacent neutrinos,
PHYS_NOQUARKS: no quarks,
PHYS_4BOSONS_ANOMALOUS: anomalous quartic gauge boson interactions (see
 [28]), affects on AAWW and AZWW vertices.

The less items are selected in Lagrangian, the faster diagram generation is performed.

5 Bondarev functions

The Bondarev method of trace calculation is implemented according to [25] paper. The trace of γ_μ -matrices product becomes much shorter in terms of F -functions. The number of items for $Tr[(1 - \gamma^5)\hat{a}_1\hat{a}_2 \cdots \hat{a}_{2n}]$ occurs 2^n . For example, the 12 matrices trace contains 10395 items usually while the new method leads to 64-items sum.

The 8 complex functions are introduced:

$$\begin{aligned}
 F_1(a, b) &= 2[(aq_-)(bq_+) - (ae_+)(be_-)], \\
 F_2(a, b) &= 2[(ae_+)(bq_-) - (aq_-)(be_+)], \\
 F_3(a, b) &= 2[(aq_+)(bq_-) - (ae_-)(be_+)], \\
 F_4(a, b) &= 2[(ae_-)(bq_+) - (aq_+)(be_-)], \\
 F_5(a, b) &= 2[(aq_-)(bq_+) - (ae_-)(be_+)], \\
 F_6(a, b) &= 2[(ae_-)(bq_-) - (aq_-)(be_-)], \\
 F_7(a, b) &= 2[(aq_+)(bq_-) - (ae_+)(be_-)], \\
 F_8(a, b) &= 2[(ae_+)(bq_+) - (aq_+)(be_+)].
 \end{aligned} \tag{3}$$

The basis vectors q_{\pm} and e_{\pm} are selected as follows:

$$q_{\pm}^{\mu} = \frac{1}{\sqrt{2}}(1, \pm 1, 0, 0), \quad e_{\pm}^{\mu} = \frac{1}{\sqrt{2}}(0, 0, 1, \pm i). \quad (4)$$

The results for traces evaluation looks as follows:

$$\begin{aligned} Tr[(1 - \gamma^5)\hat{a}_1\hat{a}_2] &= F_1(a_1, a_2) + F_3(a_1, a_2), \\ Tr[(1 - \gamma^5)\hat{a}_1\hat{a}_2\hat{a}_3\hat{a}_4] &= F_1(a_1, a_2)F_1(a_3, a_4) + F_2(a_1, a_2)F_4(a_3, a_4) + \\ &+ F_3(a_1, a_2)F_3(a_3, a_4) + F_4(a_1, a_2)F_2(a_3, a_4). \end{aligned}$$

Please refer to [25] paper for method details.

```
SetParameter(PAR_TRACES_BONDEREV, (int)par);
```

par: 1 or 0 – allow or forbid Bondarev functions usage.

The numerical code for F -functions (3) is contained in `alhep_lib.F` library file. The code is available for scalar couplings only: $F_{\mu\nu}p^{\mu}q^{\nu}$. If vector Bondarev functions remain in result, the Fortran-generation procedure fails. One should repeat the whole calculation without Bondarev functions in that case.

6 Diagrams generation

The diagrams are generated after the kinematics and physics model are declared. The Wick-theorem-based method is implemented.

The only distinct from the usual Feynman diagrams is the following: the vertex rules have additional $1/2$ factors for every identical lines pair. It makes two effects:

- The crossing diagrams are usually involved if they have different topology from original. I.e. if two external photon lines starts from single vertex, they are not crossed. Nevertheless *all* the similar external lines have crossings in ALHEP.
- Some ALHEP diagrams have 2^n factors due to identical internal lines. For example, the $W^+W^- \rightarrow \{\gamma\gamma WW\text{vertex}\} \rightarrow \gamma\gamma \rightarrow \{\gamma\gamma WW\text{vertex}\} \rightarrow W^+W^-$ diagram will have additional factor 4. The result remains correct due to $1/2$ factors at every $\gamma\gamma W^+W^-$ vertex.

The diagrams are generated without any crossings. The crossed graphs are added automatically during the squaring or amplitude calculation procedures.

```
diagrams ComposeDiagrams((int)n);
```

n : order of diagrams to be created, $M_{X \rightarrow Y} \sim e^n$.

Uses current physics and kinematics information. Returns generated diagrams set.

One may select specified diagrams into another diagrams set:

```
diagrams SelectDiagrams( (diagrams)d, (int)i0 [, i1, i2...]);
```

d : initial diagrams set. Remains unaffected during the procedure.

$i0 \dots iN$: numbers of diagrams to be selected. First diagram is "0".

The new diagrams set is returned.

To retrieve matrix element from the diagram set the `RetrieveME()` is used.

```
me RetrieveME( (diagrams)d );
```

d : diagrams list.

7 Helicity amplitudes

The amplitudes are calculated according to [24] paper. Every spinor in matrix element is projected to common abstract spinor:

$$\bar{u}_p = \frac{\bar{u}_Q u_p \bar{u}_p}{\bar{u}_Q u_p} = e^{iC} \frac{\bar{u}_Q P_p}{(Tr[P_p P_Q])^{1/2}}, \quad u_p = \frac{u_p \bar{u}_p u_Q}{\bar{u}_p u_Q} = e^{iC} \frac{P_p u_Q}{(Tr[P_Q P_p])^{1/2}}.$$

The e^{iC} factor is equal for all diagrams and may be neglected.

The projection operator $P_Q = u_p \bar{u}_p$ is chosen as follows:

- $P_Q = \hat{Q}(1 + \gamma_5)/2$ – for massive external fermions,
- $P_Q = \hat{Q}(1 + \hat{E}_Q)/2$ – if one of fermions is massless.

The value of vector Q is selected arbitrary in Fortran numerical procedure. The additional basis vector E_Q (if exists) is selected meet the polarization requirements $((E_Q \cdot Q) = 0, (E_Q \cdot E_Q) = -1)$. The fractions like $1/Tr[P_Q P_p]$ may turn 1/0 at some Q and E_Q values. The denominators check procedures are generated in Fortran code (the `CODE_CHECK_DENOMINATORS` key should be used in `CreateFortranProc()` call). If the $|Tr[P_Q P_p]| > \delta$ check is failed, the another Q and E_Q values are generated.

```
expr CalcAmplitude( (me)ME );
```

ME: matrix element retrieved from the *whole* diagrams set.

The result expression is a function of all the particle helicity vectors. The averaging over polarization vectors is performed numerically. The numerical averaging procedure is automatically generated in Fortran output if unpolarized particles are declared.

The `CODE_IS_AMPLITUDE` key in `CreateFortranProc()` procedure declares expression as amplitude and leads to proper numerical code ($Ampl \times Ampl^*$).

8 Matrix Element squaring

The squaring procedure have the following steps:

- matrix elements simplification to minimize the γ -matrices number,
- denominators caching to make procedure faster,
- item-by-item squaring (coupling to other conjugated),
- saving memory mechanism to avoid huge sums arranging (`SQR_SAVE_MEMORY` option),
- virtual integrals reconstruction.

```
expr SquareME((me)ME1, [(me)ME2,] [(int)flags ]);
```

ME1: matrix element #1,

ME2: matrix element #2 (should be omitted for squaring),

flags: method options (defaults is "0"):

`SQR_CMS`: c.m.s. consideration (initial momenta are collinear). The additional pseudo-covariant relations ($p_1 \cdot \varepsilon_2 \rightarrow 0, p_2 \cdot \varepsilon_1 \rightarrow 0$) appear that simplify work with abstract polarization vectors.

`SQR_NO_CROSSING_1`: do not involve crossings for ME #1,

`SQR_NO_CROSSING_2`: do not involve crossings for ME #2,

`SQR_MANDELSTAMS`: allow Mandelstam variable usage,

`SQR_PH_GAMMA_CHIRAL`: tries to involve photons helicities in short chiral \hat{e}^\pm form (see sec. 4.4, [26]).

`SQR_PH_GAMMA_PRECISE`: involve precise \hat{e}^\pm form for photon helicities. If the polarization vectors are not coupled to γ_μ , the vector form e_μ^\pm is used (see sec. 4.4).

`SQR_SAVE_MEMORY`: save memory and processor time for huge matrix element squaring. Minimizes sub-results by every 1000 items and skips final arranging of the whole sum. No huge sums occurs in calculation in this mode, but the result is also not minimal. If result expression contains a

sum of $10^5 - 10^6$ items (when expanded), the arranging time is significant and `SQR_SAVE_MEMORY` flag should be involved. If no results are calculated in reasonable time the `CalcAmplitude()` (see sec. 7) procedure should be used.

If two matrix elements are given, the first will be conjugated, i.e. the result is $ME1^* \times ME2$.

9 Virtual integrals operations

The *tensor virtual integrals* are reduced to scalar ones using two methods.

If tensor integral is coupled to external momentum $I_\mu p^\mu$ and p -vector can be decomposed by integral vector parameters, the *fast reduction* is involved. The D_x integrals for $2 \rightarrow 2$ process contain the whole basis of 4-dimension space and D_x -couplings to any external momentum can be decomposed. It works well if all the polarization vectors are constructed in terms of external momenta.

The *common tensor reduction scheme* is involved elsewhere. Tensor integrals are decomposed by the vector basis like

$$I_{\mu\nu}(p, q) = I(p_\mu p_\nu, q_\nu q_\mu, p_{(\mu} q_{\nu)}, g_{\mu\nu}).$$

The linear system for scalar coefficients is composed and solved. Implemented for B_j and $C_{i,ij}$ integrals only. The other scalar coefficients should be calculated numerically [19].

```
expr Evaluate( (expr)src );
```

Evaluate: Reduction of tensor virtual integrals to scalar ones. The scalar coefficients in tensor VI decomposition are also evaluated (not for all integrals).

```
expr ConvertInvariantVI( (expr)src );
```

ConvertInvariantVI: Vector parameters are substituted by scalars. I.e.:

$$C_0(k_1, k_2, m_1, m_2, m_3) \rightarrow C_0(k_1^2, (k_1 - k_2)^2, k_2^2, m_1, m_2, m_3).$$

Note: A - and B - integrals are converted automatically during arrangement.

```
expr CalcScalarVI( (expr)src );
```

CalcScalarVI: Substitutes known scalar integrals with its values. The most of UV-divergent integrals (A_0, B_0) are substituted. The chiral decomposition for D_0 -integral is also applied. The complicated integrals should be calculated numerically [19].

The source expressions are unaffected in all the functions above.

10 Regularization

The dimensional regularization scheme is implemented. One may change the space-time dimension before every operation:

```
SetNDimensionSpace( (int)val );
```

val: 1 (n -dimensions) or 0 (4-dimensions space).

```
expr SingularArrange( (expr)src );
```

SingularArrange: turns expression to 4-dimensional form. Calculates $(n-4)^i$ factor in every item and drops out all the neglecting contributions.

```
SetDRMassPK( (int)PK )
```

SetDRMassPK: set the particle to be used as DR mass regulator.

PK: particle kind (see sec. 4.1). "0" value declares the default " μ " DR mass.

11 ALHEP interfaces

11.1 Fortran numerical code

The numerical analysis in particle physics is commonly performed using the Fortran programming language. Hence, we should provide the Fortran code to meet the variety of existing Monte-Carlo generators.

To start a new Fortran file the **NewFortranFile()** function is used:

```
int NewFortranFile( (str)fn [, (int)type]);
```

fn: output file name

type: Fortran compiler conventions: `CODE_F77` or `CODE_F90`. The `FORTAN 77` conventions are presumed by default.
 Function returns the ID of created file.

Then we can add a function to FORTAN file:

```
CreateFortranProc( (int)fID, (str)name, (expr)src [, (int)keys]);
```

fID: file ID returned by `NewFortranFile()` call.

name: Fortran function name.

src: symbolic expression to be calculated. The source may be $|M|^2$, $d\sigma/d\Gamma$ (use `CODE_IS_DIFF_CS` flag) or amplitude (`CODE_IS_AMPLITUDE` flag). The result is always $d\sigma/d\Gamma$ calculation procedure.

keys: option flags for code generation (default is `CODE_REAL8`):

`CODE_REAL8`: mean all the symbols in expression as `REAL*8` values.

`CODE_COMPLEX16`: declare variables type as `COMPLEX*16`.

`CODE_IS_DIFF_CS`: the source expression is differential cross section.

`CODE_IS_AMPLITUDE` the source expression is amplitude and a squaring code should be generated: `AMPL*DCONJG(AMPL)`.

`CODE_CHECK_DENOMINATORS` check denominators for zero. Used to re-generate free basis vectors in amplitude code (see sec. 7).

`CODE_LOOPTOOLS`: Create `LoopTools` [19] calls for virtual integrals.

`CODE_SEPARATE_VI` create unique title for every virtual integral function (according to parameter values). Do not use with `CODE_LOOPTOOLS`.

`CODE_PYTHIA_VECTORS` retrieve vector values from `PYTHIA` [18] user process `PUP(I, J)` array.

`CODE_POWER_PARAMS` factorize and pre-calculate powers if possible.

`CODE_NO_4VEC_ENTRY` do not create a 4-vector entry for function.

`CODE_NO_CONSTANTS` do not use predefined physics constants. All variables becomes external parameters.

`CODE_NO_SPLIT` do not split functions by 100 lines.

`CODE_NO_COMMON`: don't use `COMMON`-block to keep internal variables.

The scalar vector couplings, Bondarev functions (see sec. 5) and $\varepsilon_{abcd}p_1^a p_2^b p_3^c p_4^d$ objects are replaced with scalar parameters to be calculated once. These functions are calculated using `alhep_lib.F` library procedures.

Some compilers works extremely slow with long procedures. Therefore Fortran functions are automatically splitted after every 100 lines for faster compilation.

The internal functions functions have `TEMPXXX()` notation. To avoid problems with several ALHEP-generated files linking we should rename the `TEMP`-prefix to make internal functions unique.

```
SetParameterS(PAR_STR_FORTTRAN_TEMP, (str)prefix);
```

prefix: "TMP1" or another unique prefix for temporary functions name.

To obtain the better performance of numerical calculations ALHEP provides the mechanism for minimization of $+/\times$ operations in the expression. We recommend to invoke the `Minimize()` function before Fortran generation (see sec. 12).

11.1.1 *PYTHIA interface*

The PYTHIA [18] interface is implemented in terms of UPINIT/UPEVNT procedures and $2 \rightarrow N$ phase-space generator.

The momenta of external particles are retrieved from PYTHIA user process event common block (`PUP(I, J)`). The order of particles in kinematics should meet the order of generated particles, and the `CODE_PYTHIA_VECTORS` option should be used.

The template UPINIT/UPEVNT procedures for ALHEP \rightarrow PYTHIA junction are found at ALHEP website. However one should modify them by adjusting the generated particles sequence, including user cutting rules, using symmetries for calculation similar processes in single function etc.

The plane $2 \rightarrow N$ phase-space generator in `alhep_lib.F` library file is written by V. Mossolov. It is desirable to replace the plain phase-space generator by the adaptive one for multiparticle production process.

The more automation will be implemented in future. Please refer to ALHEP website for details.

11.2 *Mathematica*

The output interface to Mathematica Notebook [16] file is basically used to view the expressions in the convenient form. Implemented for all the symbolic objects in ALHEP.

```
SaveNB( (str)fn, (expr or me)val [, (str)comm ], (int)flags );  
MarkNB( (str)fn [, (str)comm ], (int)flags );
```

fn: Mathematica output file name.

val: expression to be stored.

comm: comment text to appear in output file (" if no comments are required).

flags: file open flags (default is "0"):

0: append to existing file and do not close it afterward,
FILE_START: delete previous, start new file and add Mathematica header,
FILE_CLOSE: add closing Mathematica block.

The `MarkNB` function is used to add comments only.

The Mathematica program can open valid files only. The valid `x.nb` file should be *started* (`FILE_START`) and *closed once* (`FILE_CLOSE`). It is convenient to insert `MarkNB("x.nb", "", FILE_START)` and `MarkNB("x.nb", "", FILE_CLOSE)` calls to start and end of your script file.

No backward interface (`Mathematica` \rightarrow `ALHEP`) is currently available.

11.3 *LaTeX*

The LaTeX interface in ALHEP is only implemented for Feynman diagrams drawing. The diagrams are illustrated in terms of AxoDraw [23] package.

```
DrawDiagrams( (diagrams)d, (str)fn [, (int)flags][, (int)draw]);  
MarkTeX( (str)fn, [, (str)comm][, (int)flags]);
```

`d`: diagrams set.

`fn`: output LaTeX file name.

`flags`: file open flags (0(default): append, `FILE_START`: truncate old):

`comm`: comment text to appear in output file (" if no comments are required).

`draw`: draw options (default is `DD_SMALL`):

`DD_SMALL`: diagrams with small font captions.

`DD_LARGE`: diagrams with large font captions.

`DD_MOMENTA`: print particles momenta.

`DD_SWAP_TALES`: allow arbitrary order for final-state lines (option is included automatically for $2 \rightarrow N$ kinematics).

`DD_DONT_SWAP_TALES`: deny the `DD_SWAP_TALES` option. The order of final lines is same to kinematics declaration.

11.4 *ALHEP native save/load operations*

The native ALHEP serialization format is XML-structured. It may be edited outside the ALHEP for some debug purposes.

```
Save( (str)fn, (diagrams or expr)val);
object Load( (str)fn );
```

fn: output XML file name.

val: diagrams set or symbolic expression to be stored.

12 Common algebra utilities

The following set of common symbolic operations is available:

- **Expand()**: expands all the brackets and arranges result. Works slowly with huge expressions.
- **Arrange()**: arranges expression (makes alphabetic order in commutative sequences). The most of ALHEP functions performs arranging automatically and there is no need to call **Arrange()** directly.
- **Minimize()**: reduces the number of "sum-multiply" operations in expression. Should be used for simplification of big sums before numerical calculations.
- **Factor()**: factorize expression.
- **KinArrange()**: arranges expression using kinematic relations,
- **KinSimplify()**: simplify expression using the kinematic relations. Works very slowly with large expressions. Mostly useful for $2 \rightarrow 2$ process.

```
expr Expand( (expr)src );
expr Arrange( (expr)src );
expr Minimize( (expr)src [, (int)flags]);
expr Factor( (expr)src [, (int)flags]);
expr KinArrange( (expr)src [, (int)flags]);
expr KinSimplify( (expr) src [, (int)flags]);
```

src: source expression (remains unaffected).

Minimize() function flags (default is **MIN_DEFAULT**):

MIN_DEFAULT = **MIN_FUNCTIONS** | **MIN_DENS** | **MIN_NUMERATORS**,

MIN_FUNCTIONS: factorize functions,

MIN_DENS: factorize denominators,

MIN_NUMERATORS: factorize numerators,

MIN_NUMBERS: factorize numbers,

MIN_ALL_DENOMINATORS: factorize all denominators $1/(a + b) + 1/x \rightarrow (x + a + b)/(x * (a + b))$,

MIN_ALL_SINGLE_DENS: factorize single denominators (but not sums, products etc.),

MIN_VERIFY: verify result (self-check: expand result back and compare to source).

Factor() function flags (default is 0):

FACT_NO_NUMBERS: do not factorize numbers,

FACT_NO_DENS: do not factorize fraction denominators,

FACT_ALL_DENS: factorize all denominators,

FACT_ALL_SINGLE_DENS: factorize all single denominators (but not sums, products etc.),

FACT_VERIFY: verify result (self-check: expand result back and compare to source).

KinArrange() function flags (default is 0):

KA_MASS_EXACT: do not truncate masses (neglecting SetMassCalcOrder() settings),

KA_MANDELSTAMS: involve Mandelstam variables (for $2 \rightarrow 2$ kinematics),

KA_NO_EXPAND: do not expand source.

KinSimplify() function flags (default is KS_FACTORIZE_DEFAULT):

KS_FACTORIZE_DEFAULT: factorize functions and denominators (the first two flags below),

KS_FACTORIZE_FUNCTIONS: factorize functions before simplification,

KS_FACTORIZE_DENS: factorize denominators (including partial factorization) before simplification,

KS_FACTORIZE_ALL_DENS: factorize all denominators,

KS_MASS_EXACT: do not truncate masses (neglecting SetMassCalcOrder() settings),

KS_MANDELSTAMS: involve Mandelstam variables (for $2 \rightarrow 2$ kinematics),

KS_NO_EXPAND: do not expand source (if no simplification are found).

13 Debugging tools

ALHEP allows user to control the most of internal calculation flow. The debug info is stored to `debug.nb` file (critical messages will also appear in console). The `debug.ini` file contains numerical criteria for messages to be logged, the debug levels for different internal classes. *Warning*: raising the `debug.ini` values leads to sufficient performance drop and enormous `debug.nb` file growth.

If one feels some problems with ALHEP usage, please contact author for assistance (attaching your script file). Do not waste the time for manual debugging using `debug.ini`.

There are some specific commands to view internal data. For example, the whole list of tensor virtual integrals reduction results is kept in internal *data*

storage and can be dumped to Mathematica file for viewing:

- `str ViewParticleData((int)PID)` returns the brief information on particle settings. Puts the information string to console and returns it also as a string variable.
PID: particle ID in kinematics.
- `ViewFeynmanRules((str)nb_file, (int)flags)` stores Feynman rules of current physics to Mathematica file.
- `ViewTensorVITable((str)nb_file, (int)flags)` stores tensor integrals reduction table to Mathematica file. The VI reduction table is filled during the `Evaluate()` operation.
- `ViewScalarVICache((str)nb_file, (int)flags)` stores scalar loop integrals values cache. The scalar VI cache is filled during the `CalcScalarVI()` invocation.
nb_file: output Mathematica file name,
flags: access flags to Mathematica file: 0,FILE_START and/or FILE_CLOSE.
Call without parameters turns output to `debug.nb` file.

14 System commands

The two system commands are useful:

- `Halt()`: stop further script processing. May be used to test the first part of script and save (`Save()`) internal result. If first part finished successfully, it may be commented (`*...*\`) and followed by loading procedure (`Load()`). Then script execution is restarted.
- `Timer()`: view time elapsed since the last `Timer()` call (from program start for first call).

15 Examples

15.1 Amplitude for $q\bar{q} \rightarrow W^+W^-\gamma$

Let's consider the example from sec. 2.1 in details. We also extend it for anomalous quartic gauge boson interactions [28]. And we don't use the Bondarev method for traces calculation this time. Please refer to sec. 2.1 for ALHEP installation notes.

We start `test.al` script from output files creation:

```

nbfile = "uuWVA_MPXXX.nb";           // Mathematica file name
MarkNB(nbfile, "", FILE_START);      // Create file
texfile = "res.tex";                 // LaTeX file name
MarkTeX(texfile, "", FILE_START);    // Create file

```

Then $2 \rightarrow 3$ process kinematics and physics are declared:

$$u(p_1, e_1) \bar{u}(p_2, e_2) \rightarrow \gamma(f_0, g_0) W^+(f_1, g_1) W^-(f_2, g_2). \quad (5)$$

```

SetKinematics(2, 3                    // 2->3 process
, QUARK_U, "p\1", "e\1"              // u
, -QUARK_U, "p\2", "e\2"            // u-bar
, WBOZON, "f\1", "g\1"              // W{+}
, PHOTON, "f\0", "g\0"              // photon
, -WBOZON, "f\2", "g\2" );          // W{-}
SetDiagramPhysics(PHYS_SM_Q1GEN|PHYS_4BOSONS_ANOMALOUS);

```

We declare physics with u - and d -quarks only. The amplitude will be summarized for all the possible internal quarks numerically. It requires the simple replacing of quark mixing matrix in resulting Fortran code: $U_{ud}^2 \rightarrow U_{ud}^2 + U_{us}^2 + U_{ub}^2$.

Next we declare the u - and d -quarks massless:

```

SetMassCalcOrder(QUARK_U, 0);        // consider massless
SetMassCalcOrder(QUARK_D, 0);        // consider massless

```

Set polarizations to "-+UUU":

```

SetFermionHelicity(1, -1);           // u
SetFermionHelicity(2, 1);           // u-bar

```

Create diagrams set and store it to LaTeX file:

```

diags = ComposeDiagrams(3);          //e^3 order
DrawDiagrams(diags, texfile);

```

Next we include the following lines:

```

Save("diags.xml", diags);           //save to XML file
//Halt();                             //stop execution
//diags = Load("diags.xml");         //load from XML file

```

We can save diagrams, stop the program now and view diagrams generated. To stop ALHEP session the Halt() line should be uncommented. Then we modify our script as follows:

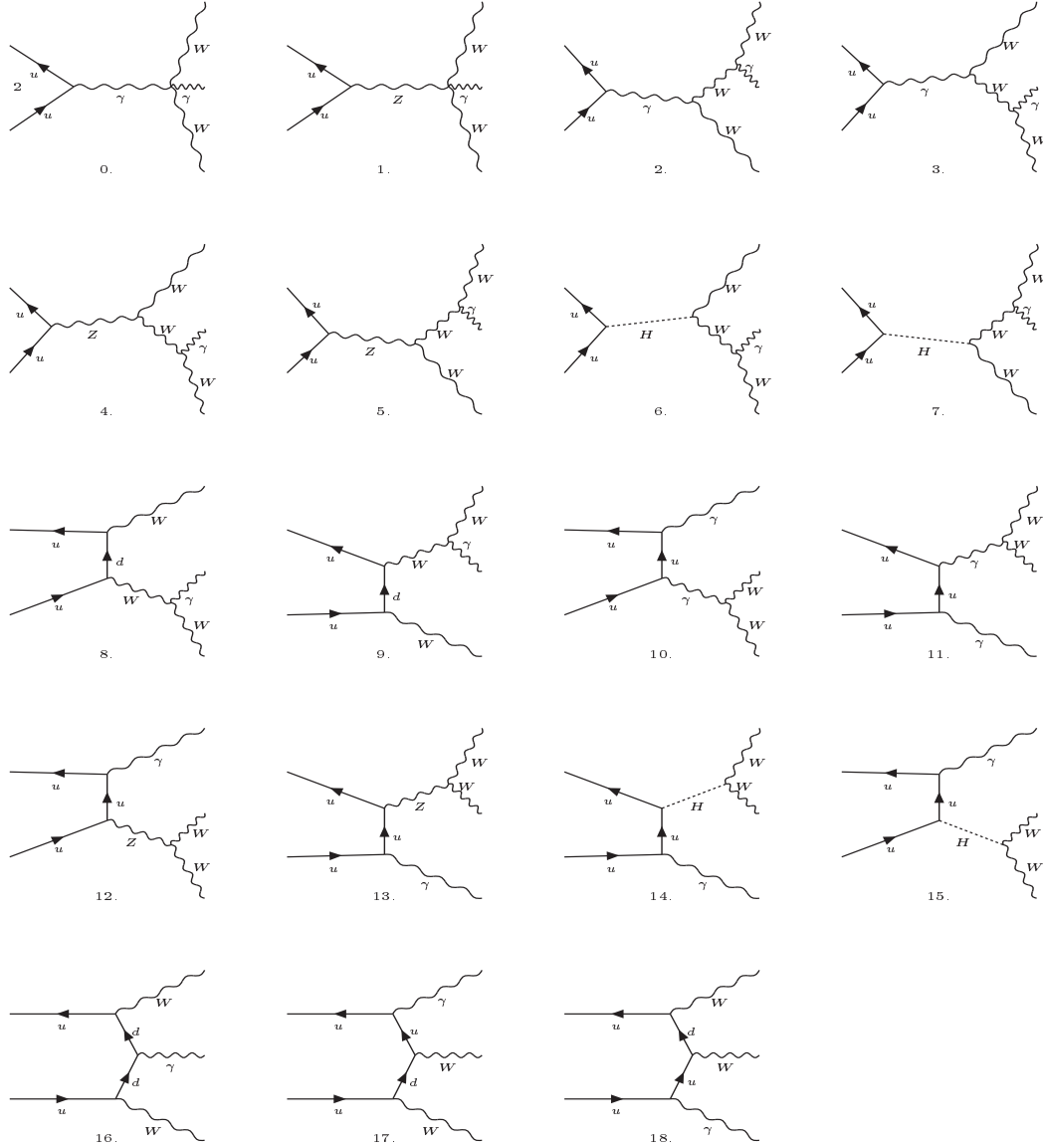


Fig. 1. The diagrams generated for $u\bar{u} \rightarrow W^-W^+\gamma$ process (see `res.tex` file). The anomalous quartic gauge boson interaction affects the first two diagrams.

```

/* diags = ComposeDiagrams(3);      // commented
...                                 // commented
Halt(); */                          // commented
diags = Load("diags.xml");         // uncommented

```

If we run the script again, it will skip the diagrams generation step and load diagrams from XML file.

Matrix element retrieval:

```

me = RetrieveME(diags);             //get matrix element
SaveNB(nbfile, me, "Matrix element"); //view

```


Calculate helicity amplitude, arrange result and minimize the $+/\times$ operations number:

```
ampl = CalcAmplitude(me);
SaveNB(nbfile, ampl, "Amplitude after CalcAmplitude()");
ampl = KinArrange(ampl);
SaveNB(nbfile, ampl, "Amplitude after KinArrange()");
ampl = Minimize(ampl);
SaveNB(nbfile, ampl, "Amplitude after Minimize()");
```

The another breakpoint can be inserted here. The result for amplitude is saved, the Halt and Load commands are commented for further use:

```
Save("ampl.xml", ampl);           // save amplitude
//MarkNB(nbfile, FILE_CLOSE); Halt(); // close NB and exit
//ampl = Load("ampl.xml");       // load amplitude
```

This breakpoint allows to repeat the next Fortran creation step without recalculating of matrix element.

Let's average over final state polarizations in further numerical procedure. Set final particles unpolarized:

```
SetPolarized(-1, 0);           // set unpolarized
SetPolarized(-2, 0);           // set unpolarized
SetPolarized(-3, 0);           // set unpolarized
```

The Fortran output for differential cross section:

```
SetParameterS(PAR_STR_FORTRAN_TEMP, "TMP1");
f = NewFortranFile("uuWWA.F", CODE_F77); //f77 file
CreateFortranProc(f, "uuWWA", ampl,
  CODE_IS_AMPLITUDE|           //square amplitude
  CODE_CHECK_DENOMINATORS|     //check 1/0 limits
  CODE_COMPLEX16|              //complex values
  CODE_POWER_PARAMS|           //F(M^2) instead of F(M)
  CODE_PYTHIA_VECTORS);        //use PYTHIA PUP(I,J) vectors
```

The SetParameterS call sets the unique notation for internal variables and functions. Please do not make it too long. The complex-type code is required for proper amplitude calculation.

Close Mathematica output file at the end of script:

```
MarkNB(nbfile, FILE_CLOSE);
```

The execution of this script takes less than 2 minutes at 1.8GHz P4 processor.

We will not discuss the structure of generated `uuWWA.F` file in details. But some remarks should be done:

Line 5: The main function call. The following parameters are declared (order is changed here): All the parameters (except the `kQOrig`) are of `COMPLEX*16` type. Ones the `CODE_COMPLEX16` option is set, all the real objects are treated as complex.

`kQOrig` (INTEGER): The ID of u (first)-quark in PYTHIA PUP(I,J) array.
Possible values: 1 or 2.

`PAR_a_0`, `PAR_a_c`, `PAR_a_n`, `PAR_ah_c`, `PAR_ahat_n`: Anomalous quartic gauge boson interaction constants $a_0, a_c, a_n, \hat{a}_c, \hat{a}_n$ [28].

`PAR_CapitalLambda`: Scale factor Λ for anomalous interaction [28].

`PAR_VudP2`: Quark mixing matrix element squared $|U_{ud}|^2$. The $U_{ud}^2 + U_{us}^2 + U_{ub}^2$ value may be passed to summarize the whole diagrams (neglecting quarks masses). The numbers for mixing matrix elements may be obtained using `QMIX_VAL(ID1, ID2)`, `QMIX_SQR_SUM(ID)` and `QMIX_PROD_SUM` functions of `alhep_lib.F` library.

Line 29: Internal COMMON-block with `PAR(XX)` array. All the scalar couplings and other compound objects are precalculated and stored in `PAR(XX)`.

Lines 49-55: External momenta initialization from PYTHIA PUP(I,J) array. The order of external vectors is expected as follows:

`PUP(I,1), PUP(I,2)`: initial particles. If `kQOrig=2` the order is backward: `PUP(I,2), PUP(I,1)`.

`PUP(I,3..N)`: final particles in the same order as in `SetKinematics()` call. One should modify this section (or `SetKinematics()` parameters) to make the proper particles order.

```
DO 10 I=1,4
  p_1(I) = DCMLX(PUP(I,kQ1Orig),OD0) // kQ1Orig = kQOrig
  f_0(I) = DCMLX(PUP(I,4),OD0)
  p_2(I) = DCMLX(PUP(I,kQ2Orig),OD0) // kQ2Orig = 3-kQOrig
  f_1(I) = DCMLX(PUP(I,3),OD0)
  f_2(I) = DCMLX(PUP(I,5),OD0)
10 CONTINUE
```

Lines 69-231: Polarization averaging and basis vector generation cycle. For any momenta set the `PAR(XXX)` array is filled. Then denominator checks and amplitude averaging are performed.

Lines 244-252, 438-444, ... Interaction constants and particle masses definitions in sub-procedures. The constants can be declared as main function parameters using `CODE_NO_CONSTANTS` option in `CreateFortranProc` function.

For the complete $p\bar{p} \rightarrow W^+W^-\gamma$ analysis the following steps are required:

- The PYTHIA client program should be written. The template files are available at ALHEP website.

- The another helicity configuration +-UUU should be calculated separately.
- The another channels $q_i \bar{q}_j \rightarrow W^+ W^- \gamma$ ($i \neq j$) should be calculated and included into generator.

15.2 Z-boxes for $e^- e^+ \rightarrow \mu^- \mu^+$

Let's calculate some box diagrams now. Consider the following process:

$$e^-(p_1, e_1) e^+(p_2, e_2) \rightarrow \mu^-(f_1, g_1) \mu^+(f_2, g_2). \quad (6)$$

As in previous example, we start command script from files initialization:

```
nbfile = "Zbox.nb";           // Mathenatica file name
MarkNB(nbfile, "", FILE_START); // Create file
texfile = "res.tex";         // LaTeX file name
MarkTeX(texfile, "", FILE_START); // Create file
```

The $2 \rightarrow 2$ kinematics declaration:

```
SetKinematics(2, 2,           // 2->2 process
  ELECTRON, "p\1", "e\1",    // e^{-}
  -ELECTRON, "p\2", "e\2",   // e^{+}
  MUON, "f\1", "g\1",        // mu^{-}
  -MUON, "f\2", "g\2");     // mu^{+}
```

The Standard model physics in Feynman gauge (we omit quarks for faster diagrams generation):

```
SetDiagramPhysics(PHYS_ELW|PHYS_NOQUARKS);
```

Set leptons massless and declare the N -dimensional space:

```
SetMassCalcOrder(ELECTRON, 0); // massless electrons
SetMassCalcOrder(MUON, 0);     // massless muons
SetNDimensionSpace(1);
```

Use Mandelstam variables throughout the calculation:

```
SetParameter(PAR_MANDELSTAMS, 1);
```

Consider unpolarized particles:

```
SetPolarized(1, 0);           // unpolarized e^{-}
SetPolarized(2, 0);           // unpolarized e^{+}
```

```

SetPolarized(-1, 0);           // unpolarized mu^{-}
SetPolarized(-2, 0);          // unpolarized mu^{+}

```

Compose born and one-loop diagrams:

```

diags_born = ComposeDiagrams(2); //e^2 order
DrawDiagrams(diags_born, texfile);
diags_loop = ComposeDiagrams(4); //e^4 order
Save("diags_loop.xml",diags_loop);
//diags_loop=Load("diags_loop.xml");
DrawDiagrams(diags_loop, texfile);

```

The 220 loop diagrams are created, saved to internal format and TeX file. The `ComposeDiagrams(4)` procedure takes 5-10 minutes here. It is convenient to comment `ComposeDiagrams(4)-Save()` lines at second run and `Load` loop diagrams from disk.

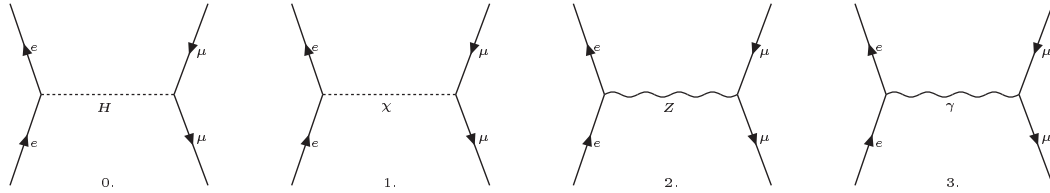


Fig. 2. Born level diagrams for $e^-e^+ \rightarrow \mu^-\mu^+$.

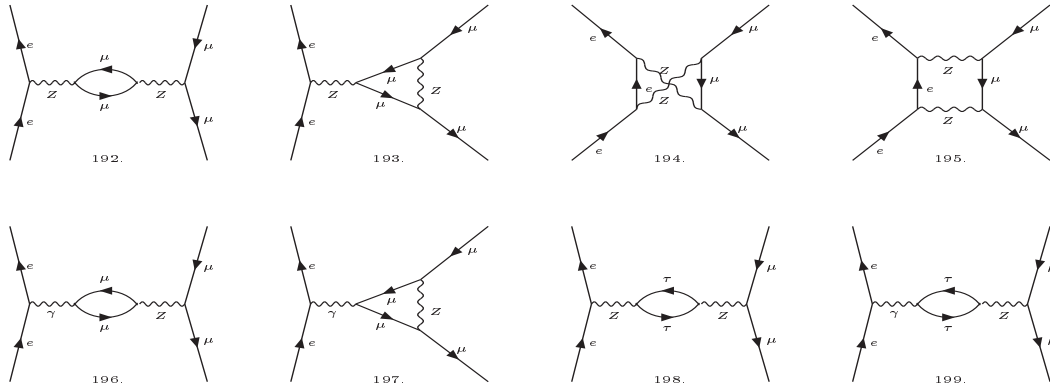


Fig. 3. Part of 220 loop diagrams stored to `res.tex` file

Let's select the double Z -exchange box graphs from the whole set (194 and 195 diagrams at fig. 3):

```
diag_box = SelectDiagrams(diags_loop,194,195);
```

Next we couple the loop and born matrix elements:

```

me_born = RetrieveME(diags_born);
me_box = RetrieveME(diag_box);
me_sqr = SquareME(me_box, me_born);

```

The simplification procedures are not included into **SquareME** implementation. It may take much time to arrange items in huge expression. Therefore all the simplification procedures are optional and should be called manually:

```
me_sqr = KinArrange(me_sqr);
me_sqr = KinSimplify(me_sqr);
SaveNB(nbfile, me_sqr, "squared & simplified");
```

The reduction of tensor virtual integrals follows:

```
me_sqr = Evaluate(me_sqr);
me_sqr = KinArrange(me_sqr);
me_sqr = KinSimplify(me_sqr);
SaveNB(nbfile, me_sqr, "VI evaluated");
```

Next we convert scalar integrals to invariant-dependent form and replace with tabulated values:

```
me_sqr = ConvertInvariantVI(me_sqr);
me_sqr = CalcScalarVI(me_sqr); // use pre-calculated values
me_sqr = KinArrange(me_sqr);
SaveNB(nbfile, me_sqr, "VI scalars ");
```

Turn to 4-dimensional space, drop out $(n - 4)^i$ items and final simplification:

```
me_sqr = SingularArrange(me_sqr);
SetNDimensionSpace(0);
me_sqr = KinArrange(me_sqr);
me_sqr = KinSimplify(me_sqr);
```

Save result and create Fortran code with LoopTools [19] interface:

```
Save("ZBox.xml",me_sqr); // save result
//me_sqr = Load("ZBox.xml"); // reload result
SaveNB(nbfile, me_sqr, "Z boxes result"); // view result
f = NewFortranFile("ZBOX.F", CODE_F77);
CreateFortranProc(f, "ZBOX", me_sqr,
                 CODE_POWER_PARAMS|CODE_LOOPTOOLS);
```

View tensor integrals reduction table and close Mathematica output file:

```
ViewTensorVITable(nbfile);
MarkNB(nbfile, FILE_CLOSE);
```

The script runs about 15 minutes on 1.8GHz P4 processor. The half of this time takes the `ComposeDiagrams(4)` procedure.

$$\begin{aligned}
& \frac{\alpha}{\pi} \frac{1}{e^2} \frac{1}{s} \\
& \left(- \left(- \left(m_z^2, t, m_z^2, s, m_e^2, m_z^2, m_z^2, m_z, m_z, m_e, m_z \right) \left(\left(\left(\frac{g_m^2}{(m_z^2 - s)} \left(m_z^2 t^2 + m_z^4 t + \frac{1}{2} m_z^4 u + \frac{1}{4} t u^2 + \frac{1}{4} t^3 \right) s - \left(m_z^2 t^2 + m_z^4 t + \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \frac{1}{2} m_z^4 u + \frac{1}{4} t u^2 + \frac{1}{4} t^3 \right) e^2 \right) \frac{g_p^3}{(m_z^2 - s)} s t^3 \right) \frac{g_m - g_p^2 e^2 t^3}{g_m} \frac{g^2}{g_m} + \right. \\
& \quad \left. \left(\frac{g_p^2}{(m_z^2 - s)} \left(m_z^2 t^2 + m_z^4 t + \frac{1}{2} m_z^4 u + \frac{1}{4} t u^2 + \frac{1}{4} t^3 \right) s - \left(m_z^2 t^2 + m_z^4 t + \frac{1}{2} m_z^4 u + \frac{1}{4} t u^2 + \frac{1}{4} t^3 \right) e^2 \right) \frac{g^4}{g_p} \right) - \\
& \quad \left(m_z^2, s, m_z^2, m_e, m_z, m_z \right) \left(\left(\left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) \frac{g_p^2}{(m_z^2 - s)} s - \left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) e^2 \right) \frac{g^4}{g_p} + \right. \\
& \quad \left. \left(\left(\left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) \frac{g_m^2}{(m_z^2 - s)} s - \left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) e^2 \right) \frac{g_m - g_p^3}{g_m} \frac{(m_z^2 + \frac{1}{2} s + u)}{(m_z^2 - s)} s \right) \frac{g}{g_m} + \right. \\
& \quad \left. \left. \frac{g_p^2}{g_p} \left(m_z^2 + \frac{1}{2} s + u \right) e^2 \right) \frac{g^2}{g_m} \right) s - \\
& \quad \left(m_e^2, s, m_e^2, m_e, m_z, m_z \right) \left(\left(\left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) \frac{g_p^2}{(m_z^2 - s)} s - \left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) e^2 \right) \frac{g^4}{g_p} + \right. \\
& \quad \left. \left(\left(\left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) \frac{g_m^2}{(m_z^2 - s)} s - \left(\frac{1}{2} m_z^2 - \frac{1}{4} s - \frac{1}{2} u \right) e^2 \right) \frac{g_m - g_p^3}{g_m} \frac{(m_z^2 + \frac{1}{2} s + u)}{(m_z^2 - s)} s \right) \frac{g}{g_m} + \right. \\
& \quad \left. \left. \frac{g_p^2}{g_p} \left(m_z^2 + \frac{1}{2} s + u \right) e^2 \right) \frac{g^2}{g_m} \right) s + \left(m_z^2, u, m_z^2, s, m_e^2, m_z^2, m_z, m_z, m_e, m_z \right) \\
& \quad \left(\left(\left(\left(2 m_z^2 u^2 + m_z^4 t + 2 m_z^4 u + \frac{1}{2} t^2 u + \frac{1}{2} u^3 \right) \frac{g}{g_m} \frac{g_p}{(m_z^2 - s)} s - \left(2 m_z^2 u^2 + m_z^4 t + 2 m_z^4 u + \frac{1}{2} t^2 u + \frac{1}{2} u^3 \right) e^2 \right) \right. \right. \\
& \quad \left. \left. \frac{g_p^2}{g_p} + \frac{1}{2} \left(\frac{g_m^2}{(m_z^2 - s)} s - e^2 \right) \frac{g^2}{g_m} u^3 \right) \frac{g^2}{g_m} + \frac{1}{2} \left(\frac{g_p^2}{(m_z^2 - s)} s - e^2 \right) \frac{g^4}{g_p} u^3 \right) - \left(m_z^2, t, m_e^2, m_z, m_z, m_e \right) \\
& \quad \left(\left(\frac{g_m^2}{g_m} \frac{(m_z^2 + \frac{1}{2} t - \frac{1}{2} u)}{(m_z^2 - s)} s - \left(m_z^2 + \frac{1}{2} t - \frac{1}{2} u \right) e^2 \right) \frac{g^4}{g_m} + \left(\frac{g_p^2}{g_p} \frac{(m_z^2 + \frac{1}{2} t - \frac{1}{2} u)}{(m_z^2 - s)} s - \left(m_z^2 + \frac{1}{2} t - \frac{1}{2} u \right) e^2 \right) \frac{g^4}{g_p} \right) t - \\
& \quad \left(s, m_z, m_z \right) \left(\left(\left(\frac{1}{2} \left(\frac{g_m^2}{(m_z^2 - s)} s - e^2 \right) \frac{g}{g_m} u - \frac{g_p^3}{(m_z^2 - s)} s t \right) \frac{g}{g_m} + \frac{g_p^2 e^2 t}{g_p} \right) \frac{g^2}{g_m} + \frac{1}{2} \left(\frac{g_p^2}{(m_z^2 - s)} s - e^2 \right) \frac{g^4}{g_p} u \right) + \\
& \quad \left(m_z^2, u, m_e^2, m_z, m_z, m_e \right) \left(\left(2 m_z^2 + s + 2 u \right) \frac{g}{g_m} \frac{g_p}{(m_z^2 - s)} s - \left(2 m_z^2 + s + 2 u \right) e^2 \right) \frac{g^2 g^2}{g_p} u - \\
& \quad \frac{1}{2} \text{Log} \left[\frac{1}{\mu^2} t \right] \left(\left(\frac{g_m^2}{(m_z^2 - s)} s - e^2 \right) \frac{g^4}{g_m} + \left(\frac{g_p^2}{(m_z^2 - s)} s - e^2 \right) \frac{g^4}{g_p} \right) u + \\
& \quad \text{Log} \left[\frac{1}{\mu^2} u \right] \left(\frac{g}{g_m} \frac{g_p}{(m_z^2 - s)} s - e^2 \right) \frac{g^2 g^2}{g_p} t + \\
& \quad \left((2 + \chi) \frac{g_p^2 e^2 t}{g_p} - \left((2 + \chi) \frac{g_p^3}{(m_z^2 - s)} s t - \left(\left(1 + \frac{1}{2} \chi \right) \frac{g_m^2}{(m_z^2 - s)} s - \left(1 + \frac{1}{2} \chi \right) e^2 \right) \frac{g}{g_m} u \right) \frac{g}{g_m} \right) \frac{g^2}{g_m} + \\
& \quad \frac{g^4}{g_p} u \left(\left(1 + \frac{1}{2} \chi \right) \frac{g_p^2}{(m_z^2 - s)} s + e^2 \left(- \left(1 + \frac{1}{2} \chi \right) \right) \right) \right) \right) \right) \right) \right)
\end{aligned}$$

Fig. 4. The result expression in `Zbox.nb` file. This result contains UV-regulator term χ , that should cancel one in $B_0(s, M_Z, M_Z)$ integral. It can be checked using `GetUVTerm()` function.

Code remarks:

Line 5: The main function call. The 3 parameters are usual Mandelstam variables (s, t, u , type is complex). The current ALHEP version does not care about interdependent parameters in Fortran output. And all the three Mandelstam variables may occur in parameters list. The future versions will

be saved from this trouble.

Line 14,50: Include LoopTools header file ("looptools.h"). See LoopTools manual [19] for details.

Line 20,21: Retrieve LoopTools values for UV-regulator `getdelta()` and DR-mass squared `getmudim()`.

The complete code of examples including scripts, batches and output files are available at ALHEP website.

16 Conclusions

The new program for symbolic computations in high-energy physics is presented. In spite of several restrictions remained in current version, it can be useful for computation of observables in particle collision experiments. It concerns both multiparticle production amplitudes and loop diagrams analysis.

The nearest projects are:

- Bondarev functions method improvement,
- Complete renormalization scheme for SM,
- Complete covariant analysis of the one-loop radiative corrections including the hard bremsstrahlung scattering contribution.
- Arbitrary Lagrangian assignment.

Refer ALHEP project websites for program updates:

<http://www.hep.by/alhep> ,

<http://cern.ch/~makarenko/alhep> .

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