Short focal-length compound refractive lenses for x-rays

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ABSTRACT

We have fabricated and tested short focal-length compound refractive lenses (CRLs) composed of micro-bubbles embedded in epoxy. The bubbles were formed in epoxy inside glass capillaries. The interface between the bubbles formed 90 to 196 spherical bi-concave microlenses reducing the overall focal length inversely by the number of lenses. When compared with CRLs manufactured using other methods, the micro-bubble lenses have shorter focal lengths with higher transmissions and gains for moderate energy x-rays (e.g. 7 - 12 keV). We used beamline 2-3 at the Stanford Synchrotron Radiation Laboratory (SSRL) to measure focal lengths between 100-150 mm and absorption apertures between 90 to 120 μ m. Transmission profiles were measured giving, for example, a peak transmission of 27 % for a 126mm focal length CRL at 8 keV. The focal-spot sizes were also measured yielding, for example, an elliptical spot of 5 x 14- μ m² resulting from an approximate 80-fold demagnification of the 0.44 x 1.7 mm² source. The measured gains in intensity over that of unfocused beam were between 9 and 26. Theoretical gain calculations that include spherical aberrations show that these values are reasonable. The micro-bubble technique opens a new opportunity for designing lenses in the 8-9 keV range with focal lengths less than 30-40 mm.

Key Words: x-rays, compound refractive lenses, optics, synchrotron

I. INTRODUCTION

It is known for visible optics¹ that the focal length of a series of N lenses is reduced by 1/N. Tomie and A. Snigirev, V. Kohn, I. Snigireva and B. Lengeler showed this to be true for x-ray wavelengths using a series of holes drilled into a single substrate with the spaces between holes forming cylindrical lenses.²⁻³ These cylindrical biconcave lenses form a compound refractive lens (CRL) that is capable of focusing and collimating x-rays from 5 to 60 keV. Other CRLs were developed to obtain two-dimensional focusing that included the "coin" CRL⁵⁻¹² and the "microcapillary" or "bubble" CRL.¹³⁻¹⁷ The "coin" CRL is a series of thin disks with a spherical or parabolic dimple embossed on each side, stacked coaxially to form the CRL.⁶⁻¹⁰ The bubble lenses are a series of bubbles formed inside an epoxy-filled glass capillary, the material between the bubbles forming a bi-concave spherical lens.¹³ The bubble lens is based upon the fact that liquid between two bubbles in a glass capillary takes the form of a bi-concave lens under action of surface tension forces. Various polymers in the form of epoxy can be used. The polymers, as a rule, consist of carbon, hydrogen and nitrogen; each of them is characterised by a low absorption coefficient for 5-30 keV Xrays. The method of fabrication is given in ref. 14. The epoxy used in the lenses is C₁₀₀H₂₀₀O₂₀N and has a density of



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1.08 g/cc.

The bubble lens has five attributes that make it attractive for use for imaging and focusing: (1.) Because of the nature of the physics forming the bubble, the lens surface quality is extremely good, probably much better than any other method for forming the lens. (2.) The capillary "enclosure" insures that the series of unit lenses or bubbles are extremely well aligned coaxially. (3.) The minimum thickness of the lens, d, can be made extremely thin. (4.) The radius of curvature can be made to be much smaller than other methods of fabrication, resulting in fewer lenses required for a given focal length. Items 2, 3, and 4 all result in a high transmission low-loss CRL. Indeed, by increasing the number of unit lenses (or bubbles), the focal length can be made much smaller than other methods. Short focal lengths result in a high-gain lens for submicron spot sizes. (5.) The method and materials of fabrication result in an extremely robust and inexpensive CRL. The latter means that these CRLs can be disposable so that they can be used in high power single-pulsed operation.

As shown in Fig. 1, a series of N+1 bubbles forms N bi-concave lenses, each of focal length f_1 , resulting in a focal length f of:

$$f = \frac{f_1}{N} = \frac{R}{2N\delta} \tag{1}$$

The unit lens focal length f_l is given by:

$$f_1 = \frac{R}{2\delta} \tag{2}$$

where the complex refractive index of the unit lens material is expressed by:

 $n = 1 - \delta - i\beta \tag{3}$

and *R* is the radius of bubble which is equal to the radius of the capillary or D = 2R.

As with all CRLs, the aperture of the bubble lens is limited. This is due to absorption at the edges of the lens, making the CRL act like an iris as well as a lens. For a unit lens of bubble radius *R*, the absorption aperture radius r_a is given by Tomie and Snigerev et al. to be:^{2, 3}

$$r_a = \left(\frac{2R}{\mu N}\right)^{\frac{1}{2}} = \left(\frac{4\delta f}{\mu}\right)^{\frac{1}{2}},\tag{4}$$

where μ is the linear absorption coefficient of the lens material. This is the radius of the lens at which a ray passing through the lens would be attenuated by e^{-2} .



If absorption is neglected, only the central part of a spherical lens approximates the required parabolic shape of an ideal lens. The parabolic aperture radius r_p is:³

$$r_{p} = \left(4R^{2}\lambda r_{i}\right)^{1/4} = \left(\frac{2R^{3}\lambda}{\delta N}\right)^{1/4}$$

where r_i is the image distance and λ is the x-ray wavelength. Rays outside this aperture do not focus at the same point as those inside. The second half of eq. (5) is approximately true if $r_o >> f$, an accurate approximation for synchrotron sources, where the distance to the source, r_o , is quite large.

The effective aperture radius r_e is the minimum of the absorption aperture radius, r_a , and the parabolic aperture radius, r_p , and the mechanical aperture radius, r_m ; that is:

$$r_e = MIN(r_a, r_p, r_m).$$
(6)

The absorption aperture dominates the lens at the photon energies that we selected. The second term of (4) shows that the aperture, and consequently other important lens parameters such as resolution and depth of focus of a CRL, are solely determined by the required focal length and the choice of material of which the lenses are made. The calculated value of μ for the above discussed epoxy is equal to 5.8 cm⁻¹ for 8-keV x-rays, and the absorption radius $r_a = 58 \ \mu m$ for 103-unit epoxy lens (CRL 4-1; see table I below) formed inside of a D = 200 \ m capillary.

An important application of the bubble lens would be to increase the intensity of x-rays from a distant source at the image point. Gain is defined as the ratio of the x-ray intensity at the image plane of the lens to the intensity of x-rays that would have been obtained at the same plane without a lens.³ From energy-conservation arguments, an approximate expression of the gain of a two-dimensional lens is found to be:

$$G \approx \frac{A_L}{A_S} M^2 T \tag{7}$$

where *T* is the power transmitted through the lens over the incident power impinging on the aperture of the lens (for unit lenses with aperture radii $\approx r_a$, $T \approx 0.43 e^{-\mu Nd}$ where *d* is the minimum thickness of the unit lens), A_L and A_S are the area of the lens and of the source, respectively, and $M = r_o/f$ is the demagnification.¹¹ The gain is limited by the absorption aperture (2 r_a) and not the resolution.

The parabolic or spherical aberration aperture r_p is not appropriate for determining gain. However, when this aperture is exceeded, the primary effect is a loss of resolution and the gain is affected much less. We have found that using the Kirchhoff equation to determine gain for a spherical lens shows that eqn (7) is a good approximation (see J. T. Cremer et al in this volume).²⁰



(5)

Fig. 3. The focused beam profiles results from bubble CRL 4-1, (2R = 0.2 mm, N = 102 lens) for 7 and 8 keV.



As can be seen from (7), the gain is determined by the CRL transmission, *T*, and the square of the demagnification *M* or $1/f^2$. Thus the bubble lens with its short focal length and good transmission will give higher gain than most other CRLs. The transmission depends upon the thickness *d*, which in turn depends on the given lens material and on the diameter of the capillary channel. We have fabricated CRLs with minimum thicknesses *d* as small as 20-25 µm. Smaller thicknesses are expected in the future.

As an example, the gain for CRL 4-1 at 8-keV and the experimental conditions of the synchrotron at the Stanford Synchrotron Radiation Laboratory can easily be calculated. The approximate source size (FWHM) was $0.44 \times 1.7 \text{ mm}^2$ and its distance from the lens was 16.8 m.¹⁸ The lens focal length for lens 4-1 was 13 cm. Using (7) the expected gain was 28.9, which correspond to measured one of 25 (see Table I).

II. MEASUREMENTS

We fabricated and tested four bubble CRLs arbitrarily designated CRLs 1-1, 3-1, 3-4, 3-3, and 4-1. The calculated and measured characteristics of these CRLs are given in Table I below. Minimum lens thickness (*d*, the minimum distance between spheres) was determined by microscopy.

We used beamline 2-3 on the Stanford Synchrotron Radiation Laboratory's (SSRL's) synchrotron. This beamline possesses a double-crystal monochromator that was capable of giving x-rays from 2400 to 30000 eV with a 5 x 10^{-4} resolution. The approximate source size (full width half maximum, FWHM) was 0.44 x 1.7 mm², as given by SSRL.¹⁸ The experimental apparatus is shown in Fig. 2. The distance from the source to lens was 16.81 meters.¹⁸ The x-ray beam size from this source was approximately 2 x 20 mm² at the entrance to the experimental station; however, this size was reduced to approximately 0.4 x 0.4 mm² by Ta slits upstream of the monochromator. The CRL was placed in a goniometer head that could be manually tilted in three axes. The lens could also be remotely translated orthogonally (x and y) to the direction of the x-ray beam to maximize the x-ray transmission though the lens. An x-ray gas ionization detector was placed after a translatable slit for measuring the x-ray beam profile. This Ta slit was adjusted to below 6 µm by using a thin stainless steel



shim. It is likely that, as good as the Ta slit was, the jaws are not ideally parallel at these small dimensions and the slit width was minimally $> 3 \mu m$ when jaws appeared to be entirely closed.

After the slit width was adjusted, the Ta slit was then translated in the x direction across the focused x-ray beam and its profile obtained. We then manually moved the slits along the z-axis of the x-ray beam, measuring its vertical widths by scanning the slits over the beam at each location. Profiles of the focal spot size from lens 4-1 for 7 and 8 keV are

given in Fig. 3. The minimum waists of 4.5 and 5 μ m are seen to be at distances, r_i , of 10 and 13 cm, respectively, from the center of the lens.

Using these measured widths, we can profile the beam waist as a function of distance from the lens. The FWHM in the vertical direction is plotted as a function of distance from CRL 4-1 in Fig. 4 for 8 keV photons. This figure shows that the waist of the x-ray beam is converging to minimum at approximately $r_i = 14$ cm. The source size of 0.44 mm is focused to the minimum spot FWHM of 5 μ m, at distances of 13 cm from the CRL. Thus the demagnification is M = .011. The spot size in the horizontal was measured to be 19 μ m, which is larger because we are imaging a bigger source diameter (1.7 mm, FWHM) in that dimension.

The CRL has an aperture with an absorption profile that has a gaussian shape that causes stronger absorption of the extreme rays passing through the CRL outer radial regions, in contrast to the better-transmitted rays that pass through the central region. This aperture is much smaller than the source size, especially in the horizontal dimension. We obtained the transmission through the CRLs given in Table I by narrowing the x-ray beam to 50 x 50 μ m² and translating each CRL through the beam. This gave transmission profiles of the lenses. Examples of these profiles are given in Fig. 5 a, b, c for CRL's 4-1, 3-4, and 3-1, respectively. The absorption apertures (e⁻² points, not FWHMs) were obtained from these figures. For example, in Fig. 5 a, b, and c these measured values are approximately $2r_a = 119$, 147, and 245 μ m for 8, 9, 12 keV. The calculated values, 223, 262 and 300 μ m, are somewhat larger. The calculated and measured peak transmissions (transmission at the lens axis) for the lenses are given in Table I. These apertures are larger than what would be obtained by an Al lens and do not require higher photon energies in order to achieve adequate transmission.⁸

Given the measured transmissions and profiles, we can determine the gains of these lenses. Both measured and calculated gains are given in Table I for all these lenses. The gain values vary between 3.5 to 25.5. These gain values are primarily due to the large source size (see eqn. (7)). If the same lens is placed on a beam line using a third generation x-ray source, the gain of the CRL can be substantial. These sources can possess spot sizes a factor of 3 smaller (e.g. 0.5 by 0.5 mm^2). Also, typical distances from insertion devices to end stations can be around 51 m (as was used in ref. 2), compared to 16.8 m in our experiment, and would consequently increase the gain by $(51/16.8)^2$. For these source parameters, the gain at 11 keV from the same lens is 138, a sizable increase of the intensity when no lens is used. Although the gains of these CRLs were small, larger gains can be achieved using smaller source sizes, larger CRL apertures and longer object distances.

TABLE I

Measured and Calculated Parameters of bubble CRLs for SSRL BL 2-3 source (440 µm vertical x 1700 µm horizontal FWHM) at 16.8 m distance from CRL.

Bubble Lens designation	1-1	3-1	3-4	3-3	4-1	4-1
Photon Energy	12	12	9	8	8	7
Number of lenses	90	196	103	93	102	102
R, Radius of Curvature (μm)	165	250	100	100	100	100
Observed Lens Minimum Wall Thickness (µm)	76	41	37	36	23	36
Visual Lens Minimum Wall Thickness (µm)	10	50	25	25	10	10
Calculated Refractive Index Decrement	1.74E-06	1.74E-06	3.09E-06	3.91E-06	3.91E-06	5.12E-06
Tabulated Linear Attenuation Coefficient (cm-1)	1.49	1.49	3.77	5.50	5.50	8.40
Calculated Focal Length (cm)	52.8	37	15.7	13.8	12.55	9.6
Calculated Image Distance (cm)	54.5	37.8	15.8	13.9	12.6	9.7
Measured image Distance (cm)	32	36	17.5	13	14	10
Calculated Vertical Minimum Waist Diameter (µm)	15.1	10.4	4.4	3.9	3.2	2.7
Measured Vertical Minimum Waist Diameter (µm)	12.8	12	3.9	4.8	2.7	4
Calculated Horizontal Minimum Waist Diameter (µm)	64.1	44.0	18.8	16.6	13.5	11.5
Measured Peak Transmission	36%	30%	24%	16%	27%	5%
Calculated Attenuation Aperture Diameter (µm)	314	262	143	125	119	96.7
Measured Attenuation Aperture Diameter (µm)	321	245	147	150	149	149
Calculated De-Magnification	0.032	0.022	0.009	0.008	0.007	0.006
Calculated 2D-Gain	16.6	20.0	25.6	16.9	28.9	6.0
Measured 2D-Gain	8.9	3.5	13.4	**	25.5	**

III. DISCUSSION

CRLs have been made of bubbles embedded in epoxy with focal lengths between 9.6 to 53 cm for photon energies ranging between 7 to 12 keV with peak transmissions of up to 33 %.. These lenses give shorter focal lengths than previous methods. The use of the CRLs at these photon energies can have greater application in synchrotron and proposed coherent x-ray sources.¹⁹ The amount of absorption can be further reduced by decreasing the wall thickness, *d*, For photon energies above 30 keV epoxy lenses can achieve a performance comparable to those made of Be or Li, currently regarded as the best materials.^{11, 12, 20} Thus, one can expect CRLs made of epoxy to have an important future in x-ray optics.

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