Simulation Modeling of HM-Queueing Network With Limited Wait Time of Messages in Queueing Systems

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Abstract: *The Markov queueing networks with incomes, with limit wait of messages in the queue is considering. Such networks can serve as stochastic models of estimation and prediction of incomes of different objects. The algorithm of construction of simulation model of network with limit wait of messages is reduced.*

Keywords**:** HM-queueing network, expected incomes.

1. INTRODUCTION

For all real complex systems the dynamic time coherence between its objects, their current decisions and the subsequent events, and also uncertainty factors is characteristic. Use of analytical methods in practice for research and optimization of such systems is not always justified. The method of simulation modeling (SM) in comparison with the analytical has a number of advantages, namely, allows working with problems of the big dimension, to solve the problems containing stochastic variables, etc. If prototype of developed model is the real system, carrying out of experiments over which is inconvenient then using method of SM it is possible to draw various conclusions without intervention in its functioning. Besides, one of the feature of method of SM is possibility to predict behavior of system in the future.

Process of SM consists of three stages:

1) construction of simulation model and its verification;
2) carrying out of a series of simulating experiments;
3) statistical processing of results.

Check of conformity of model to the real object of research can be spent, for example, on the maximum value of absolute deviations of the parameters received by means of SM (average lengths of queues to queueing systems (QS), average times of stay of messages in QS, averages number of occupied service channels in the network systems) and similar characteristics of real objects.

Distinctive feature of simulation experiments from real tests is simplicity of repetition and reproduction of conditions of experiment. However at the research of complex systems it is necessary to remember about influence of an initial condition and take care to the further about reduction of its influence or full exception from results of modeling [1].

Cases in which the method of SM is applied are very much. One of them consists to possibility of check characteristics of the modeled system calculated by means of various approximated methods.

For check of theoretical results at research of complex qeueuing networks (QN) the method of SM allows investigating practically any QN. This method consists in modeling of process of functioning such network on any certain interval of time. However at program realization it is necessary to remember about an error of calculations which will increase with growth of an interval of research.

2. DESCRIPTION OF HM-QUEUEING NETWORK

Let's consider open HM-queueing network with single type messages that consist of n QS $S_1, S_2, \ldots, S_n$. State of such network could be described by the vector

$$k(t) = (k,t) = (k_1, k_2, \ldots, k_n, t), \qquad (1)$$

where k_i – number of messages in system S_i at the moment t, $t \in [0, +\infty)$, $i = \overline{1,n}$. For unification of designation let us introduce system S_0 (outside medium) from which the flow of messages with intensity λ comes into network. System S_i consist of m_i service channels, the service time in each of which has any distribution with intensity μ_i, $i = \overline{1,n}$; p_{0j} – probability of message entry from system S_0 to system S_j, $\sum_{j=1}^{n} p_{0j} = 1$; p_{ij} – probability of message transition from system S_j to the system S_i, $\sum_{j=0}^{n} p_{ij} = 1$, $i = \overline{1,n}$. Message during its transition from system S_i to system S_j brings to system S_j some random income and income of system S_i descend on this value correspondently, $i, j = \overline{0,n}$. Service rate of message occurs according to discipline FIFO.

Duration of stay of the message in queue i – th QS has any distribution with intensity θ_i and does not depend on other factors, for example, from time of staying in queue of other messages. Message, which waiting time in queue S_i has expired, transfers in queue of system S_j with probability q_{ij}, $i = \overline{0,n}$, $j = \overline{1,n}$. Matrixes $P = \|p_{ij}\|_{(n+1)\times n}$ and $Q = \|q_{ij}\|_{n\times(n+1)}$ are transition probabilities matrixes of irreducible Markovian chains[2].

Let's $v_i(k,t)$ – complete expected income, which system S_i obtains in time t, if at initial moment of time network was in state $(k,0)$. Expected incomes of network and systems from transitions of messages vary as follows: at transition of the message from i – th QS to j – th QS first system increases the expected income on random variable (RV) R_{ij} with distribution function $F_{ij}(x)$. Another QS decrease incomes on same RV accordingly. If duration of stay of message is terminated in queue of i – th QS then message according to transition probabilities matrix Q is transferred to j – th QS. Expected income of j – th QS increase on RV H_{ij} and i – th QS decrease on same RV H_{ij} with distribution function $Q_{ij}(x)$.

If the network does not make any transitions between states then each system S_i receives income r_i for a time unit during all period of stay in a current state, where r_i is RV with distribution function $F_i(x)$, $i=\overline{1,n}$.

The modeling initial data will be type of network – open or closed, number of QS n, transition probabilities matrixes P and Q, initial distribution of messages in queues of QS, number of service channels m_i, $i=\overline{1,n}$, partition laws of service and duration of stay of message in queues. Values of incomes R_{ij}, H_{ij} r_i, $i,j=\overline{1,n}$, from transition of messages between network states, if they are determined or depended on time. If incomes are RV then it is necessary to set their distribution functions.

3. SIMULATION MODELING OF NETWORK

The simulation model of QN represents the computer program reproducing process of functioning of network directed on reception of vector of states (1) during some discrete moments of time t_l, $l \geq 0$. At modeling the initial condition of the network at the moment of time t_0 is specify. Then according to mathematical model the trajectory of the vector of states on time intervals $[t_0,t_1)$, $[t_1,t_2)\ldots$ is under construction. The moment of time t_l correspond to the termination moments of service of messages in QS or to the termination moments duration of stay of the messages in queue of QS and called 0-moment [3]. Only two components of this vector are change. For example, the message pass from system S_i to system S_j, then $i-$th component decreases on 1 and $j-$th increase on 1. In time interval $[t_{l-1},t_l)$ changes does not occur. Thus, each 0-moment contain the information on what type of transition is realize (termination of service or termination duration of stay), from what QS in what transition what execute (actually in which queue of QS the new message was added) and the moment of time of arrival of this transition. All 0-moments are stored in program that ordered on the moment of approach of transition.

The algorithm of SM looks as follows: while modeling time has not reached the end of the period of modeling, we model transitions of messages which finished service and messages that not waited service at present time (0-moment) in others networks QS according to transition matrixes P and Q, i.e. we change vector of states. Incomes of network systems it is change on the corresponding values. Then we modeling time of service and duration of stay of newcomers messages in the network QS by partition law in this QS. The service time and duration of stay of message in queue can be distributed under any laws, for example, exponential, hyperexponential, normal or can be some mathematical functions or constants.

After all actions on processing of the current 0-moment calculation incomes of network QS is carried out. The considering cycle repeats with a new network state.

Let's consider each step of algorithm.

0. *Generation of initial network state at the time* $t=0$. At this step the given initial state of QS is analyzed. The messages are added according to the vector of state in queue of networks QS. The 0-moments which describe the termination moments of service of some messages or the termination moments duration of stay of services messages in queues of QS are created.

1. *Receiving of the first 0-moment.* At this step the program receives information about message type (termination of service or termination duration of stay), time and number of QS that finished work or terminator for the end of the modeling period. If the modeling period is terminating, then we process results of modeling and exit from research procedure. Otherwise we pass to the step 2.

2. *Processing of the current 0-moment.* On the given step all actions connected with transition of message, loading of others QS and calculation of new values of incomes caused by these changes are made.

2.1. Hit on the given step means that in one of channel of QS with number i service has terminated. Message according to transition probabilities matrix P is transferred to $j-$th QS:

if all service channels of $j-$th QS are occupied,
then program places the message in queue of this QS,
− duration of stay in queue of this QS is modeled according to the distribution law;
− termination time of this stay is remembered;
else message starts to be served by free channel of $j-$th QS,
− service time is modeling according to the distribution law in this QS;
− termination of this service is remembered;
if in the queue of $j-$th QS is still messages,
then first message from queue is taken and begins to be served by the released channel,
− termination of this service is remembered;

2.2. Hit on the given step means that in queue of $i-$th QS duration of stay of messages has terminated. Duration of stay of message in queue is modeled according to the distribution law. Message according to transition probabilities matrix Q is transfer to $j-$th QS; incomes of $j-$th QS at this moment of time increases on RV H_{ij}, incomes of $i-$th QS decreases on H_{ij}:

if all channels of $j-$th QS are loaded completely,
then program places the message in queue of $j-$th QS,
− duration of stay of message in queue is modeled according to the distribution law;
− termination of this duration of stay is remembered;
else message starts to be served by free channel of $j-$th QS,
− service time is modeled according to the distribution law in this QS;
− termination time of this service is remembered;
− message according to transition probabilities matrix P is transfer to $k-$th QS $k=\overline{1,n}$,

incomes of j – th QS at this moment of time decreases on RV R_{jk}, and incomes k – th QS decreases on RV R_{jk}.

3. Going to the step 1.

Modeling of work of QN on some interval of time is made several times. As result for each experiment the random trajectory of change of incomes in time turns out. For reception of resultant of trajectory for the expected income of network, trajectories received as result of imitating experiments are averaged.

4. CONCLUSION

Thus, the algorithm of simulation modeling for finding expected incomes of HM-network in a case when incomes from transitions between network’s states are random variables are received. The further investigations in this area are associated with simulation modeling of HM-queueing network with unreliable systems.

5. REFERENCES

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