FATIGUE CRACK PROPAGATION UNDER VARIABLE AMPLITUDE LOADING ANALYSES BASED ON PLASTIC ENERGY APPROACH

Sofiane Maachou[1,*] Mohamed Benguediab[1], Mohamed Mazari[1], Narayanaswami Ranganathan[2]

[1] Laboratory of Materials and Reactive Systems, Department of Mechanical Engineering, University djillali liabes, BP 89, city Ben M'hidi of Sidi Bel Abbes, Algeria.
[2] School Polytechnic University of Tours, 7 Avenue Marcel Dassault 37004 Tours, France.

[*] corresponding author: tel.: +213 552 880 859, fax.:+213 48 556 767, e-mail: Sofiane_8381@yahoo.fr

Resume

Plasticity effects at the crack tip had been recognized as "motor" of crack propagation, the growth of cracks is related to the existence of a crack tip plastic zone, whose formation and intensification is accompanied by energy dissipation. In the actual state of knowledge fatigue crack propagation is modeled using crack closure concept. The fatigue crack growth behavior under constant amplitude and variable amplitude loading of the aluminum alloy 2024 T351 are analyzed using in terms energy parameters. In the case of VAL (variable amplitude loading) tests, the evolution of the hysteretic energy dissipated per block is shown similar with that observed under constant amplitude loading. A linear relationship between the crack growth rate and the hysteretic energy dissipated per block is obtained at high growth rates. For lower growth rates values, the relationship between crack growth rate and hysteretic energy dissipated per block can be represented by a power law. In this paper, an analysis of fatigue crack propagation under variable amplitude loading based on energetic approach is proposed.

Available online: http://fstroj.uniza.sk/journal-mi/PDF/2014/11-2014.pdf

Article info

Article history:
Received 10 August 2013
Accepted 23 November 2013
Online 14 May 2014

Keywords:
Fatigue crack growth;
Correction factor;
Energetic approach;
Plasticity;
Crack closure.

ISSN 1335-0803 (print version)
ISSN 1338-6174 (online version)

1. Introduction

Fatigue crack growth resistance of a material depends upon a number of factors, such as its composition mechanical properties and heat treatment conditions, external loading and the ambient environment. The understanding of the mechanisms governing fatigue crack growth has made significant advances since the Paris law proposed about 40 years ago [1]. The effects of mechanical parameters are not put into evidence in Paris` relation. Elber [2] had shown that a crack stays closed during a certain period of a cycle (even if it is in tension) due to the existence of residual stresses in the plastic core at the crack tip he expressed by ΔK_{eff} the period of a cycle where the crack is totally open. Elber proposes the following relation:

$$\frac{da}{dN} = C \times \left(\Delta K_{eff}\right)^n \quad (1)$$

$$\Delta K_{eff} = K_{\max} - K_{op} \quad (2)$$

Where: N: a number of cycles; da/dN: Crack growth rate; ΔK_{eff}: Amplitude of the effective stress intensity factor; K_{max}: Maximum stress intensity factor; K_{op}: Stress intensity factor to the opening.

Besides this mechanically based concept other approaches based on energy consideration [3] or on micro-mechanisms acting at the crack tip had been developed.

In these approaches certain authors attempted to express the crack growth rate by explaining the effects of different parameters by means of theoretical models based on the crack tip opening theory and cyclic hardening.

The initial models on materials behavior are based on the determination of strain amplitude and damage accumulation in the crack tip plastic zone.

A new approach was proposed by Weertman [3] who considers that the crack advances when the accumulated plastic energy at the crack tip reaches a critical value.

Weertman [3] proposed to relate the crack growth rate to this energy by a law a type:

$$\frac{da}{dN} = \frac{A \times \Delta K^4}{\mu \times \sigma^2 \times U} \quad (3)$$

With: *da/dN*: Crack growth rate; ΔK: Stress intensity factor range; μ: Shear modulus; σ_c: Cyclic elastic limit; *U*: Surface energy creation; *A*: non - dimensional constant.

This model has gained a high interest; many authors have attempted to verify its validity and many experimental techniques had been developed to measure the energy of surface creation *U* which represents the dissipated energy in the plastic zone by unit created surface.

For ductile material the energy of plasticization is much higher than the theoretical surface energy creation γ. Ikeda and al. [4] had measured the quantity for steel of low carbon content and for high resistance aluminum alloy from hysteresis loops in the plastic zone using strain micro gages.

Davidson and al. [5] elaborated a method of evaluation of energy from measures of sub grain size. In addition to these methods, other techniques have been developed, such as microcalorimetry and infrared thermography. Recently Ranganathan and al. [6] have measured the hysteric work *U*, using the differential method of Kikukawa and al. [7].

On the basis of these results the authors show that over a certain value of ΔK called ΔK_{cr} (amplitude of the critical stress intensity factor) the value of *U* is constant and independent on both the ratio R and environment.

Klingbeil [8] had modified Weertman's relation [3] by introducing the total plastic energy dissipated per cycle *dW/dN* and had expressed the crack growth rate by:

$$\frac{da}{dN} = \frac{A \times \Delta K^4}{\sigma^2 \times E \times G_c} \times \frac{dW}{dN} \quad (4)$$

In this relation the crack growth rate is expressed in term of energy restitution ratio, the modulus of elasticity and the material resistance.

The constant *A* of equation (4) had been determined on the basis of experimental results of specific energy *U* and that Klingbeil [8] in this model assumed it to be equal to the restitution energy ratio *G* and by neglecting the surface energy creation γ (for the case of ductile materials). Following this assumption Klingbeil found *A* to be equal to 2.23×10^{-3} a closer value to that found elsewhere.

Recently Mazari and al. [9] proposed an energetic model for constant amplitude loading where the crack growth rate *da/dN* can be formulated in term of the total dissipated plastic energy per cycle Q_P. In this paper the model is applied in variable amplitude loading.

2. Experimental details

This study was conducted on the high strength aluminum alloy 2024 T351. The tests are conducted on compact tension specimens 12 mm thick and 75 mm wide with the crack growth in the LT orientation. The constant amplitude tests at five R ratios (0.01; 0.10; 0.33; 0.54 and 0.70) were carried out initially to characterize the material behavior. The variable amplitude loading are considered as representative of different aspects of the statistical spectrum (Fig. 1 and Table 1).

Table 1

Condition of loading for the spectrum.

Type of spectrum	Step 1	Step 2	Step 3	Step 4
	P_{min}=80 daN **P_{max}=150 daN** **R=0.53** **n1**	**P_{min}=160 daN** **P_{max}=392 daN** **R=0.41** **n2**	**P_{min}=323 daN** **P_{max}=600 daN** **R=0.54** **n3**	**P_{min}=138 daN** **P_{max}=323 daN** **R=0.43** **n4**
A	1	1	1	1
B	10	10	10	2
C	10	10	50	2
D	10	10	100	2

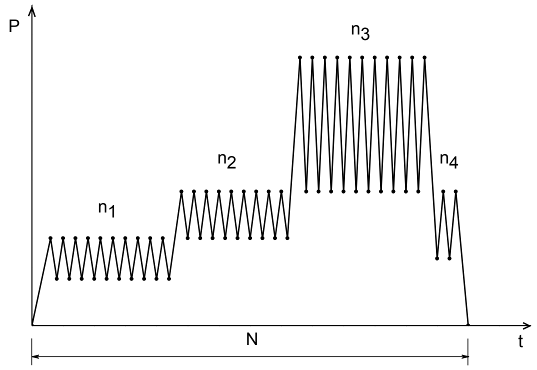

Fig.1. Configuration of the spectrum.

- Spectrum A consists of several load excursions which have a probability of one cycle per flight. This spectrum represents the most probable GAG cycle (Ground Air Ground) and consists of four mean levels.
- Spectrum B consists of 10 cycles in the first three load and two cycles in the fourth level.
- Spectrum C and D are similar to Spectrum B. with 50 and 100 cycles in the high load level.

All the tests were carried out under computer control at 20 Hz in ambient air and at selected crack length. The evolution of the crack mouth opening displacement δ (measured by a clip gage) and the differential displacement δ' with respect to the load P were recorded on a XY plotter at a frequency of 0.2 Hz. δ' is defined by:

$$\delta' = \delta - \alpha P \quad (5)$$

Where α is the specimen compliance at a particular crack length. The measurements were carried out during one cycle for constant amplitude tests and during one block for a reduced spectrum loading.

Typical δ vs P and δ' vs P diagrams for constant amplitude loading and reduced spectrum loading conditions are given in Fig. 2a - c.

The following parameters were measured from the diagrams.

The crack opening load P_{op} at the beginning of the horizontal portion of the δ'vs P diagram [7].

The energy dissipated per cycle Q which is the area enclosed by the δ' vs P diagrams after suitable corrections.

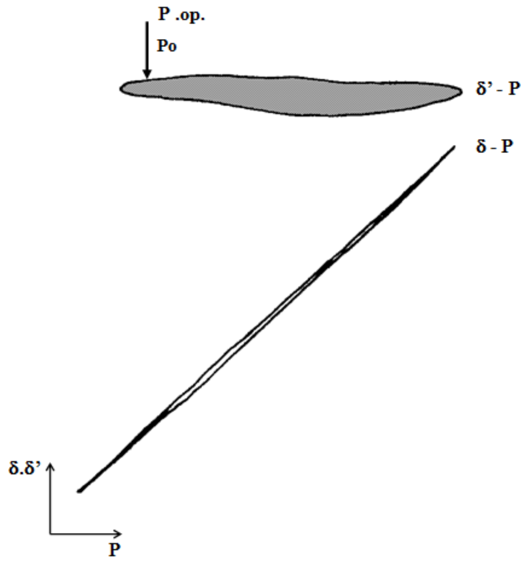

Fig. 2.a Diagram for CAL condition.

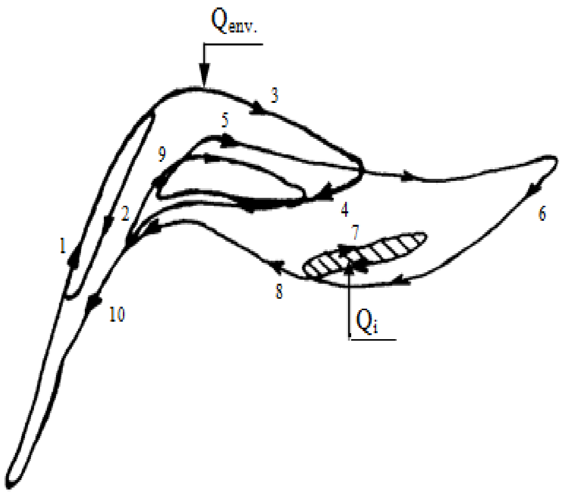

Fig. 2.b Diagram for VAL condition spectrum A.

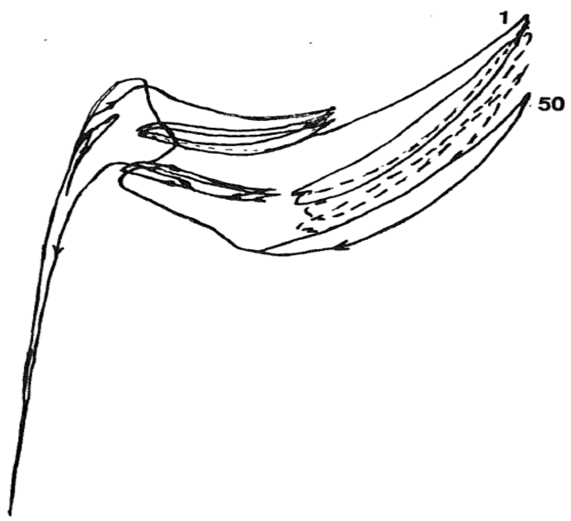

Fig. 2.c Diagram for VAL condition spectrum C.

The specific energy U_S is defined as:

$$U_S = \frac{Q}{2 \times B \times \frac{da}{dN}} \tag{6}$$

Where: B is the thickness of specimen and da/dN the crack growth rate.

For the VAL we consider two cases:

- **Case 1:** The crack advance is negligible during a block (Fig. 2b). Then we regard total energy as the energy of the envelope plus the elementary energies associated with each individual cycle as:

$$Q_{Total} = \sum Q_i \tag{7}$$

- **Case 2:** There are appreciable advance of the crack during each block (Fig. 2c). In this case we consider that the energies dissipated in each individual cycle as:

$$Q_{Total} = Q_{env.} + \sum Q_i \tag{8}$$

The analysis considers that in the first case the crack is almost stationary during the block of the charge; all hysteretic energy is dissipated in the same plastic zone, where as in the second case the energy of plastification is integrated at advanced crack with each cycle.

3. Experimental results

3.1 Constant amplitude loading

The evolution of da/dN with respect to K_{max} for five different R ratios is given in Fig. 3. The stress intensity factors K_{max} are determined according to ASTM standards.

The results are analyzed in terms of the energy parameters. The evolution of the crack growth rate with respect to the energy dissipated per cycle Q for different R ratios is presented in Fig. 4 In this figure we show the existence of two distinct stages.

At high growth rates values the relationship between da/dN and Q is linear and can be expressed as follows:

$$\frac{da}{dN} = A \times Q \tag{9}$$

With A above 1.8×10^{-4}.

For lower growth rates values the relationship between da/dN and Q can be represent by a power law of the type:

$$\frac{da}{dN} = B \times Q^n \tag{10}$$

With $B = 2.22 \times 10^{-5}$ and $n = 3.80$

The relationship between the specific energy U_S and K_{max} is shown in Fig. 5.

As has been shown previously by [8, 10], the same behavior was shown before by Ranganathan and col [10] for the same material and was attributed a crack advances step by step mechanism for lower growth rates and at high growth rates, the crack growth mechanism was characterized by striation formation during each cycle where the specific energy was constant.

3.2 Variable amplitude loading

3.2.1 Relation between the growth of crack da/block and K_{max}

The Fig. 6 shows the relationship between the crack advance per block and the maximum stress intensity factor K_{max} in the block for the different block load tests corresponding respectively to the load spectra A, B, C and D as defined in Fig. 1, compared to constant amplitude tests at R = 0.01 and R = 0.54.

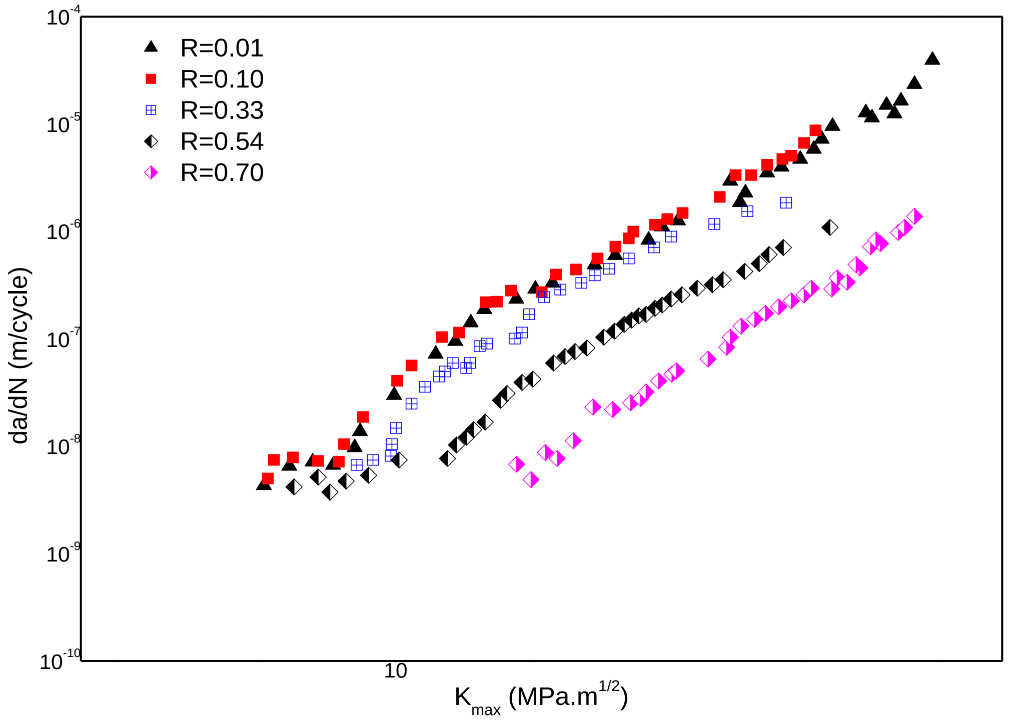

Fig. 3.Evolution of the crack growth rate da/dN with respect K_{max}.
(full colour version available online)

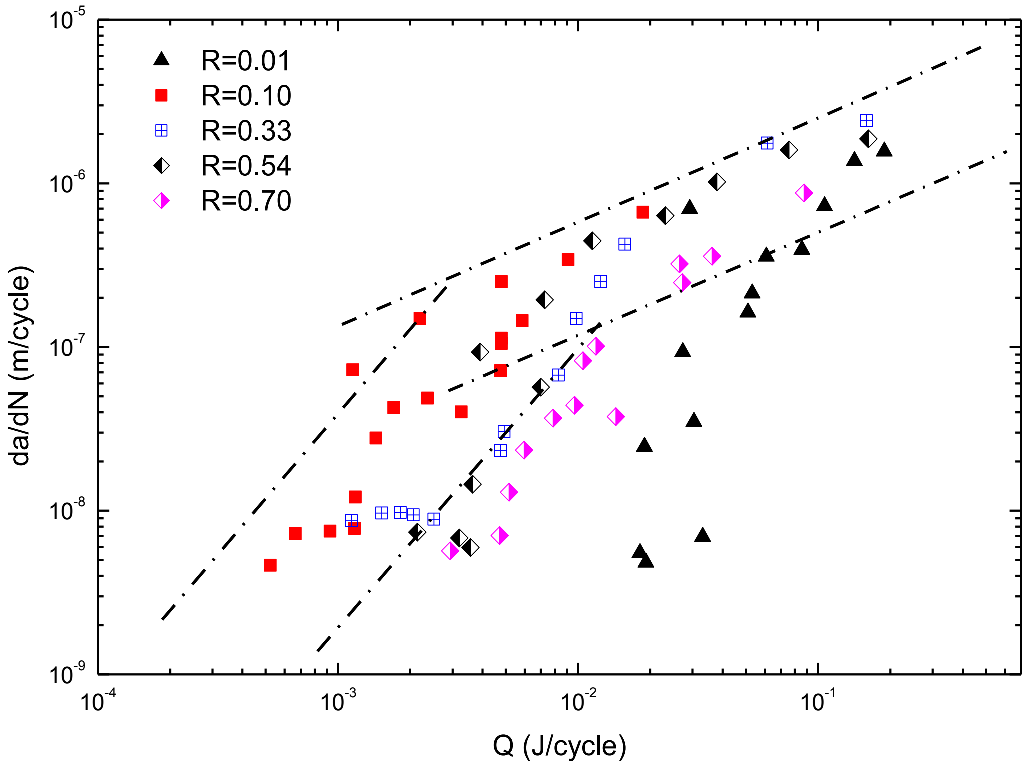

Fig. 4. Evolution of da/dN with respect to the energy dissipated by cycle Q.
(full colour version available online)

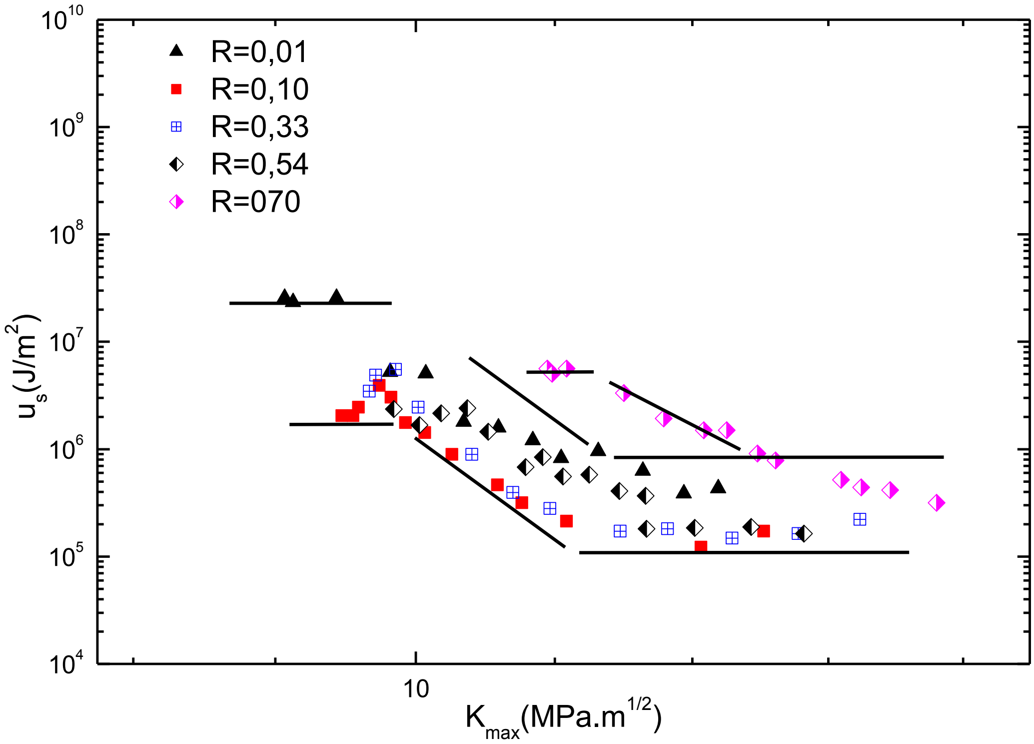

Fig. 5. Evolution of the specific energy U_S with respect to the stress intensity factor K_{max}.
(full colour version available online)

A detailed analysis of these experimental data has been previously made [11]. The following remarks can be made:

- The curve obtained for the spectrum A almost coincides with that for the test at R = 0.01.
- The crack growth rate for spectra A, B, C and D increases as and when spectrum severity (characterized by the number of high load cycles) increases.

The results presented above are now analyzed in terms of energy parameters.

3.2.2 Relation between da/block and energy hysteretic dissipated ΣQ_i

Fig. 7 shows the evolution the growth of crack *da/block* according to energy hysteretic dissipated per cycle ΣQ_i.

According to the results obtained, it is observed that crack growth *da/block* with respect to the dissipated energy per cycle Q changes according to a law of power of the type:

$$\left(\frac{da}{block}\right)_p = B \times Q_p^n \tag{11}$$

Where: Q_p: the exponential part of Q; n: exponent.

It is noticed that quad the growth rate of crack exceeds certain valuable energy dissipated per cycle becomes linear.

$$\left(\frac{da}{block}\right)_l = A \times Q_l \tag{12}$$

where: Q_l: the linear part of Q.

It was also observed that for lower crack growth rates different factors contribute to the increase of crack closing, such as roughness or oxidation [2]. Thus the energy dissipated per cycle Q is not useful for the creation of surfaces; part of this energy will be lost. From these conditions; we can establish an energetic model based on the correction of the evolution of the rates *da/block* of propagation of the crack in function Q.

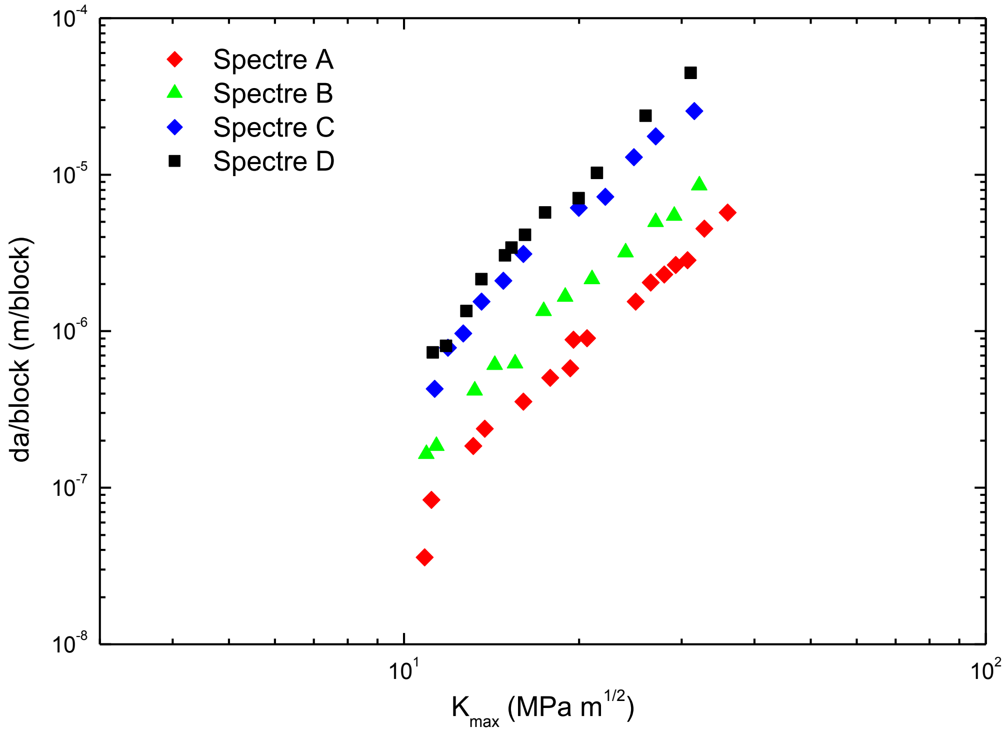

Fig. 6. Evolution of the crack growth rate da/block with respect to the stress intensity factor K_{max}.
(full colour version available online)

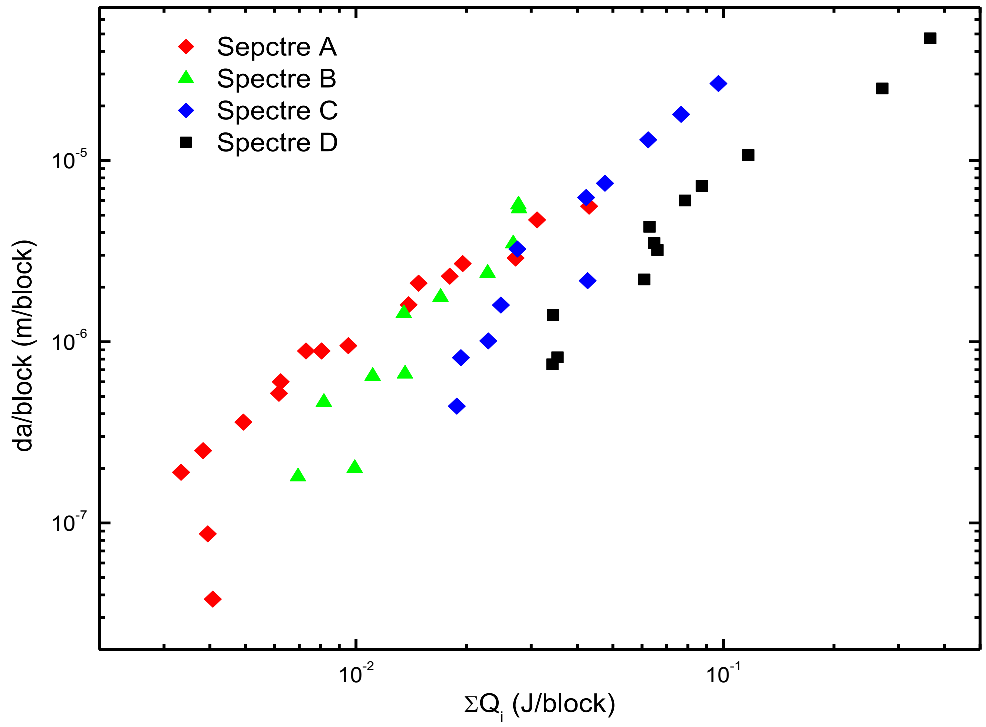

Fig. 7. Evolution of the crack growth rate da/block with respect to energy dissipated per cycle ΣQ_i.
(full colour version available online)

Considering that there is a continuity of the growth rate of the crack to the transitions. This continuity lets equalize relations (10) and (11) which give:

$$A \times Q_l = B \times Q_p^n \quad (13)$$

Thus:

$$Q_l = \frac{B}{A} \times Q_p^n \quad (14)$$

The equivalent energy dissipated by Q_{eq} cycle has the form:

$$Q_{eq} = K^* \times Q_p^n \quad (15)$$

With:

$$K^* = \frac{B}{A}$$

The procedure for the spectrum *A* is following:

For the growth rate higher than 1.6x10^{-6} *m/block* the energy dissipated per cycle is linear (Fig. 8).

For spectrum A, the relations (10) and (11) can be written as:

$$\left(\frac{da}{block}\right)_p = 13 \times 10^{-4} \times Q_p^{1.5279} \quad (16)$$

$$\left(\frac{da}{block}\right)_l = 10^{-4} \times Q_l^{1.026} \quad (17)$$

Equalizing (16) and (17), the equivalent energy dissipated per cycle Q_{eq} has the form:

$$Q_{eq} = 13 \times Q_p^{1.5279} \quad (18)$$

After correction the evolution between *da/block* and Q_{eq} is linear with coefficient K = 0.9x10^{-4} (Fig. 9) and can be written as:

$$\frac{da}{block} = 0.9 \times 10^{-4} \times Q_{eq}^{1.0436} \quad (19)$$

The same procedure is applied for the other spectrum (B; C and D).We obtained the following relation:

Spectrum B:

$$\frac{da}{block} = 3 \times 10^{-4} \times Q_{eq}^{1.0607} \quad (20)$$

Spectrum C:

$$\frac{da}{block} = 1 \times 10^{-4} \times Q_{eq}^{0.8443} \quad (21)$$

Spectrum D:

$$\frac{da}{block} = 2 \times 10^{-4} \times Q_{eq}^{0.9588} \quad (22)$$

The evolution of *da/block* with respect to the equivalent energy Q_{eq} is given in Fig. 10.

3.3 Estimation of equivalent load cycles and crack growth lives

The equivalent VAL for the studied spectra using the three methods presented here is given in Table 2 for all spectrums. For this studies each spectra there is a relation between the growth of crack *da/block* and energy hysteretic dissipated per block Q_{eq}. The crack length varied from 24 to 52 mm.

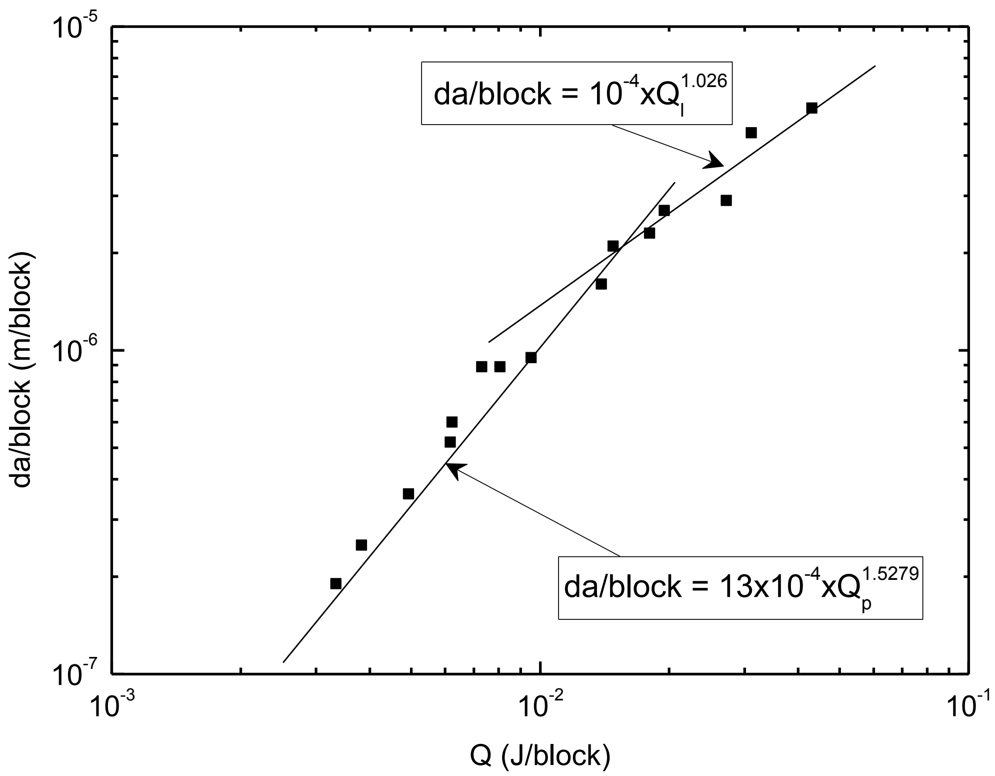

Fig. 8. Evolution of da/block with respect to Q for spectrum A.

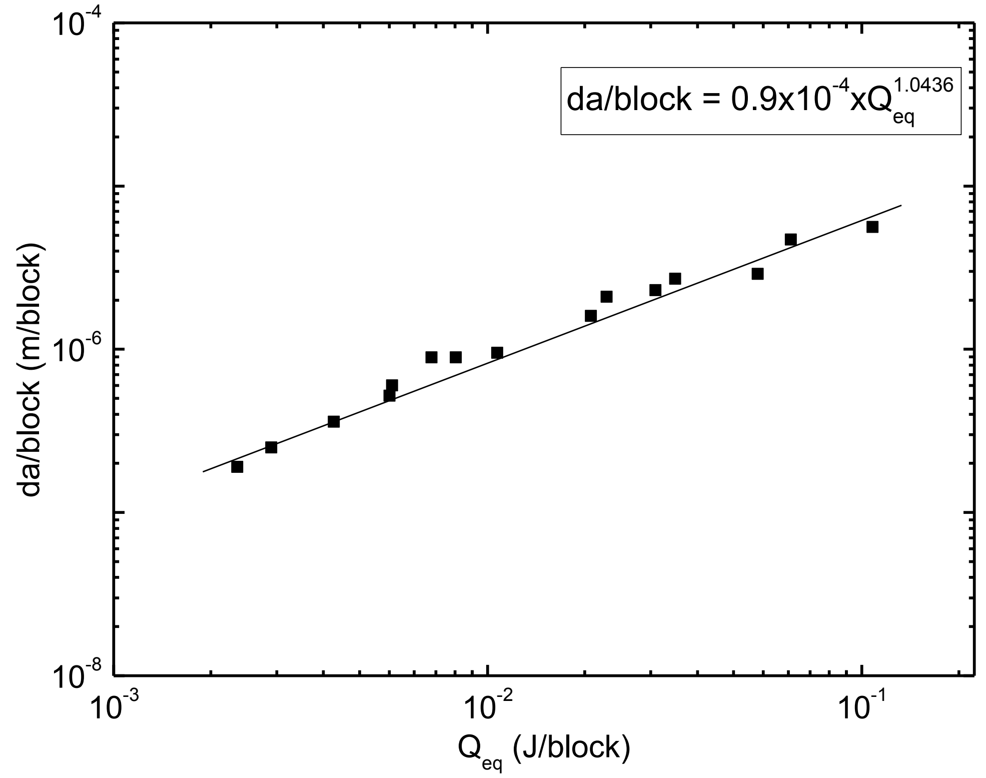

Fig. 9. Evolution of da/block with respect to Q_{eq} for spectrum A after correction

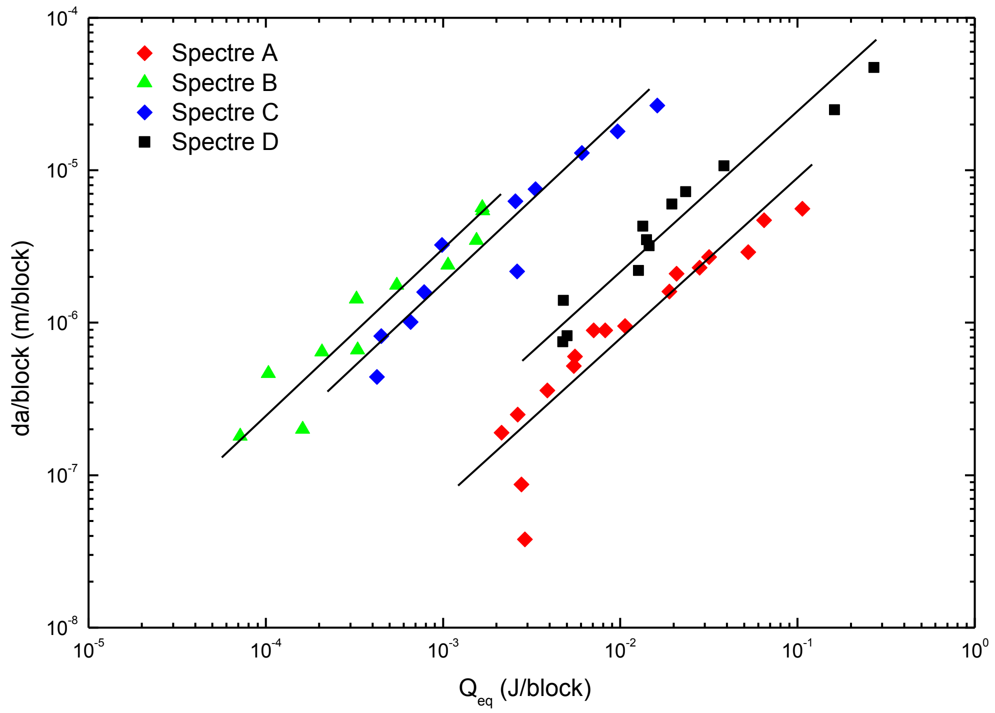

Fig. 10. Evolution of da/block with respect to Q_{eq} for spectrums A. B. C and D after correction
(full colour version available online)

The present study enabled us to evaluate the growth of crack with respect to the hysteretic equivalent energy dissipated by block for each spectrum with VAL through this evaluation we found the relations (19, 20, 21 and 22).

By examining these relations the following observations can be made with respect to the different methods.

Model

Based on these equations to create a program to estimate the life using MATLAB of each spectrum studied, the results give a life acceptable for spectrum A with respect to the value measured with an error of -9.04% from the spectrum against B has a negative error of 53.15% that is to say life almost half of the measured value. For spectrum C, this program provides a number of blocks a little larger than the number of block measured with 16.17% relative error; this is the method with respect to other method which gives a value closest to the measured value. In the case of spectrum D was found that the estimated useful life is less than the life measured.

Tracey method

This technique gives values of life much less than the lifetimes measured for all spectra with an average of -78.42% of the relative error.

Klingbeil method

This is the method that leads to generally good estimates of all spectra studied. The relative error is between -26.7% and -24% in comparison with experimental results and exception to the spectrum of A is -54.7% compared to the real values.

Table2
Comparison between numbers of blocks measured and obtained by other authors.

Block	number of blocks		Error %
A	Measured	75120	
	Model	68323	-9.04
	Tracey	10880	-85.5
	Klingbeil	34058	-54.7
B	Measured	23900	
	Model	11196	-53.15
	Tracey	5599	-76.6
	Klingbeil	17524	-26.7
C	Measured	9600	
	Model	11153	16.17
	Tracey	2315	-75.9
	Klingbeil	7242	-24.6
D	Measured	5500	
	Model	3670	-33.27
	Tracey	1336	-75.7
	Klingbeil	4179	-24.0

4. Conclusions

The study of fatigue crack propagation examines how a fatigue crack grows under cyclic load. This topic is the subject of considerable research, mainly dealing with the development of various models to better explain the crack propagation phenomenon. For the modern high performance structures designed for finite service life, fatigue crack growth occurs over a significant portion of the useful life of the structures.

It should be recognized that the effect of the stress ratio on fatigue crack growth behaviour is strongly material dependent.

The developed model allows linearising the relation between the crack growth rate and the hysteretic energy per cycle Q in all the spectra studied to describe the evolution of the growth rate of the crack in terms of consideration of energy. The obtained results are compared with those obtained numerically by Tracy and Klingbeil models.

References

[1] P.C. Paris, F. Erdogan: Trans ASME, D.J. Bas. Eng. 85(4) (1963) 528-534.

[2] W. Elber: Am. Soc. Testing Mats. (1971) 230–242.

[3] J. Weertma: Int. J. Fracture 9(2) (1973) 125-131.

[4] S. Ikeda, Y. Isumi, M. E. Fine: Eng. Fract. Mech. 9 (1977) 123-136.

[5] D.L. Davidson et al.: In: Proc. Symp. Env. Sens. Frac. Eng. Matls., TMS ASME Warendale, PA, U.S.A., 1980, p. 59.

[6] N. Ranganathan: Contribution to the development of an energy approach to the fatigue crack propagation (Thesis) Poitiers: University of Poitiers, 1985.

[7] M. Kikukawa, M. Jono and H. Hora: Int. J. Fracture 13(5) (1977) 699-701.

[8] N.W. Klingbeil: Int. J. Fracture 25(2) (2003) 117–128.

[9] M. Mazari: Comp. Mat. Sci. 41(3) (2008) 344–349.

[10] N. Ranganathan et al.: In: Proceedings of the 7th International Conference on strength of Metals and Alloys, Montreal, Canada 1986, pp 1267–1272.

[11] M. Benguediab, M. Mazari, N. Ranganathan and M. Zemri: Journal of Materials Science and Technology 9(2) (2001) 77-88.