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Risk Management for Swedish Farmers

An empirical study on hedge ratios for Swedish wheat

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Abstract

The paper investigates data on purchasing price of wheat from Swedish grain buyer Lantmännen and MATIF future contracts on milling wheat in an attempt to replicate the conditions for a Swedish farmer trying to manage his risk on wheat by trading future contracts on the MATIF exchange. Two static linear regressions and four dynamic GARCH models are employed on a sample of 1679 daily returns and 339 weekly returns ranging from 2009-07-01 to 2016-01-11. All regressions are ran on both daily returns and weekly returns to investigate how the rebalancing frequency changes the outcome of the hedges. The correlation of spot and future price changes from 0.19 for daily returns to 0.49 for weekly returns and all weekly return hedges outperforms the daily hedges in variance reduction. It is however hard to find a general best model over both daily and weekly returns and for all samples. The simple OLS performs best in the daily sample with -3.19% in variance over the full sample compared to a no-hedge and in the weekly return the VECM-VECH reduces variance by -29% over the full sample compared to a no-hedge.

Keywords: Dynamic hedge ratio, Wheat futures, GARCH, BEKK, VECH

List of abbreviations and acronyms

AIC – Akaike information criterion

AR – Autoregressive

ARCH – Autoregressive Conditional Heteroskedasticity

BEKK-GARCH – Baba-Engle-Kraft-Kroner-GARCH

CAPM – Capital Asset Pricing Model

ECM – Error Correction Model

GARCH – Generalized Autoregressive Conditional Heteroskedasticity

GJR-GARCH – Glosten-Jagannathan-Runkle-GARCH

HQ – Hannan–Quinn Information Criterion

MGARCH – Multivariate GARCH

MV – Minimum Variance

OLS – Ordinary Least Squares

SC - Schwartz Information Criterion

TGARCH – Treshhold-GARCH

VAR – Vector Autoregressive

VECM – Vector Error Correction Model

VECH-GARCH – Vectorized GARCH

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1. Introduction

The Swedish agricultural market produces an average of 2-3 millions of ton of wheat per year and is a vital part of Sweden's food supply, behind this crucial industry is numerous farmers giving their every day to ensure that there is wheat to be harvested every autumn. To be able to provide Swedish wheat farmers a stable working environment and the access to capital needed to run a modern agricultural operation it is key to understand the relevance and importance of hedging. When investors look for reliable investments and banks for safe loans, both assess the potential risks and the potential profits involved with the business. The higher the risk associated with growing wheat, the higher the profit has to be to appeal investors to invest and banks to give loans with manageable interest rates. If a farmer is able to reduce the risk involved in growing wheat it's easy to think that not only would the farmers sleep better but also make his search for capital an easier endeavor. The Swedish market offers different options for managing risk for wheat growing, it is possible to turn to the large grain buyers in Sweden and opt to buy their many services and price insurances such as forwards. It also possible to turn to the open market and the exchange to buy and sell your own future contracts with whoever wants to acts as insurer on the global market. There exists multiple markets for such commodity trading, futures on wheat are sold and bought on markets not exclusively but largely on exchanges such as MATIF in Paris and CBOT in Chicago there is also over-the-counter trading were two parties deal with each other directly. The exchanges are the better choice for a farmer not well versed in the finance sector as the contracts are standardized and regulated and therefore not overly complicated, there is also the ease of trusting the other party of the future as the exchange operates as an intermediator. This paper aims to contribute evidence and understanding towards the positive effects of hedging for wheat farmers and to answer the question if it is reasonable for a Swedish farmer to turn to the international market to reduce price risk of wheat utilizing the MATIF-Paris exchange and its standardized future contracts. The paper will also try to answer over which frequency a farmer hedging wheat should re-evaluate and rebalance his hedge, daily, weekly or keeping it constant over time. Thirdly this paper aims to answer which of the tried models holds the best explanatory power for the variation of prices over time as this is crucial for keeping the optimal hedge ratio and giving the highest risk reduction. The remainder of the paper is organized as follows. Section 2 presents relevant literature and previous research, section 3 describes the background information on trading and hedging wheat on the international market, section 3 also discusses limitation of the paper. Section 4 describes the methodology used for the empirical results. Section 5 presents information about the data set and describes transformations made on the data. Section 6 presents the empirical results of the paper and section 7 concludes the paper with discussing the results and potential further studies.

2. Literature Review

Price volatility and the uncertainty it brings has given way for large literature on the topic of hedging. The traditional way and most intuitive way to envisage hedging is with the naive hedge. The naive hedge is a negative one position of an asset with a 1:1 movement against the underlying asset. The negative one hedge gives a total opposite movement in the hedge from the spot price thus eliminating any price risk. However for this total elimination of risk to be possible the underlying asset and the future must exhibit perfect correlation and this is very rarely the case in the market. When total elimination of risk by a naïve hedge is impossible to achieve there is a need for alternative ways of hedging assets. Early literature on optimal hedging ratios with futures was based on linear regression with OLS on cash and future prices assuming the conditional volatility on future and cash price to be time independent, such early literature includes Johnson (1960), Markowitz (1959) and Ederington (1979). Johnson (1960) introduced a strategy for hedging called the 'minimum variance hedge' (MVH) Johnson stated that the price risk was the variance of the return on a two asset hedged position for example a two asset portfolio holding a stock and a future on the same stock. The MVH is thus a way to rate different hedging portfolios based on the lowest total variance over time. The way to hedge the most efficiently was therefore according to Johnson (1960) to minimize the variance. Similarly Markowitz (1959) proposed that measuring hedging efficiency is best done by calculating the reduction in standard deviation. Ederington (1979) came to the same conclusion and proposed that the efficiency is best measured by the percent reduction in variance. Anderson and Daunthine (1981) evaluate hedging performances in the mean-variance framework since they argue that it better encompasses a larger part of investors since there is an obvious tradeoff between risk and return in hedging which is not taking into account in the minimum variance framework used by Johnson (1960), Markowitz (1959) and Edgerton (1979). Kahl (1983) finds results that optimal hedge ratios are independent of the investor's risk aversion but the hedge position itself is not independent of the risk. This means that regardless of a farmer's risk aversion, the same hedge ratio should always be chosen. Kahl further argues that the minimum variance framework is to be used to evaluate all hedges. Up to this point most papers had been looking for the optimal hedging ratios assuming the conditional variance to be constant over time. Cecchetti, Cumby and Figlewski (1988) tried to correct two major problems for at the time the standard hedging design. The first one was that in most cases risk was given consideration but the tradeoff between risk and return was ignored. Secondly contrary to previous studies, they allowed time variation in the joint distributions using an ARCH-model. By allowing time variation in the distribution it was possible to take into account the change in volatility of assets over time. Baillie and Myers (1991) extended the work of Cecchetti, Cumby and Figlewski (1988) by studying six different commodities spot and future prices using a Generalized ARCH-model which relaxes the constraints of independent distribution between future and cash prices allowing for weak dependence between successive prices changes in higher orders. Their result indicated that the distribution is well described by a GARCH-model. They could however not find a general model fitting best for hedging all six commodities and the results varied on the commodity and future being hedged. Bollerslev, Engle and Wooldridge (1988) expands the original CAPM model by using a MGARCH to estimate the returns of Bills, bonds and stocks on the US-market with expected returns proportional to the conditional covariance's of each return with that of the market portfolio. They simplified the MGARCH model by arguing that "A natural simplification is to assume that each covariance depends only on its own past values and surprises." They found that the conditionals covariances are varying over time and affects the time-varying risk premium. Floros and Vougas (2004) studies the Greek stock index and two different corresponding futures using OLS, ECM, VECM and M-GARCH. They find results that volatility of future prices and spot prices are varying over time. The results show that the M-GARCH is the superior hedging tool for both tried futures with the Greek stock index as underlying asset. More recent studies that is more agricultural centric is hedging ratios studies done on the volumetric volatility using weather derivatives. Weather is important when considering risk in the agricultural sector, weather derivatives is not designed to primarily manage price volatility and instead tries to lower volumetric volatility. Volumetric volatility would translate to the variation of the size of the harvest. Weather derivatives are complicated financial instruments as the weather conditions affect harvest size differently for almost all crops and regions. Manfredo and Richards (2009) studies a weather sensitive crop, nectarines and studies weather derivatives on nectarine yields in Fresno county, California. They find that with an option straddle position weather derivatives reduces basis risk. They also conclude that it decreases technical basis more risk than spatial basis risk. Golden, Zang and Zhou (2010) also studies the use of weather derivatives to reduce both basis risk and default risk. They find that their models work better in the winter season then the summer season. Extending price risk hedging with weather derivatives is an interesting topic but due to the lack of weather derivatives available for Swedish farmers and its complexity is outside of the scope for this paper. Iwarsson (2012) covers the Swedish agricultural market and helps to understand the Swedish agricultural market and to manage risk within it. His book covers different hedging strategies for Swedish farmers as well as guidance in the myriad of futures and options on the international markets. Iwarsson concludes that a Swedish wheat farmer stand to gain significantly in both risk reduction as well as increased returns by hedging using simple strategies such as using a naïve hedge with the most liquid wheat future contract on MATIF.

3. Background

3.1. Risk management for farmers

Farmers face different risks depending on what crops they are growing or their geological position however all farmers face, price-, quality- and volumetric risk. What price can the harvest be sold for, how large will the harvest be and of which quality. As stated above this paper focuses on price risk. Price risk can be thought of as the different price a farmer face between planting his crop and harvesting or how the price changes on wheat in the farmers silos waiting to be sold. This difference in price becomes a problem if the price decreases, not only because farmers want high profits but also because there is costs to cover from the growing season. If a farmer plants wheat in all fields because the market price is high at the time of planting, say €150 per ton and the price drops to €75 per ton when the harvest time comes. A farmer would face the difficult decision of storing the harvest in silos until the price rises and either keep running on saved money or take out a loan to cover costs. He could also sell for €75, half of the expected income. These options would all include some hardship and costs for the farmer. The uncertainty of future income would also make it harder for a farmer to make present time investments in machines and equipment.

This risk and uncertainty can be managed by buying or selling financial instruments such as futures, forwards and options, this paper investigate the futures alternative. By selling future contracts the price risk is reduced by the ensured future sell price, this is a form of hedging. However even if the farmer buys future contracts to cover the full expected harvest, risk still remains, basis risk. Anderson and Danthine (1981) acknowledged the fact that cash and future prices rarely are perfect substitutes or that the relationship of the cash price to the relevant future price at the time of delivery may not be predicted with certainty. This uncertainty is known as basis risk. Figlewski (1984) explains it as "The risk that the change in the futures price over time will not track exactly the value of the cash position." The basis is thus the difference or the basis of the spot minus the future $b_t = S_t - F_t$ which means that even if the farmer secures his future price on the whole harvest he is still exposed to the risk that the spot price rises above the future price. Because of the non-perfect substitute between cash position and future contracts for wheat it is not optimal to just hedge the full harvest one-to-one with futures and therefore it is important to estimate the optimal hedging ratio. Hedging ratio is the value of the underlying asset compared to the value of the futures sold or bought on the underlying asset. so for example a hedge ratio of 0.5 would mean that 50% of the estimated harvest is sold with futures and the other 50% are sold at the spot price. 50% is exposed to spot price risk and the other 50 % are exposed to basis risk. This paper uses different approaches to estimate this ratio. Static hedging that is a constant ratio. This ratio will be calculated using linear regression OLS and error correcting model, ECM. The second method is a dynamic hedge where the hedge ratio is changing over time. This will be calculated using the diagonal VECH-TGARCH model and the diagonal BEKK-TGARCH model.

3.2. Wheat trading on the exchange

Marketplaces for farmers to secure their future income can be traced all the way back to the ancient Greek and Roman time. Futures still principally work the same way today as then, farmers can sell futures to manage price risk and firms using crops for production such as a mills can buy futures to secure future purchase prices. Futures are also interesting for individuals and firms looking to speculate in commodities, investors will try to turn profit on the basis risk by trying to identify goods where the price will move in a certain direction. The reason futures are the main financial asset for commodity trading is because it has proven rather difficult for a stock exchange to offer commodities at spot prices with a buy and sell spread as is offered for listed stocks. In present time there are primary seven exchanges that are important for Swedish farmers looking for futures on agricultural product. (Iwarsson 2012)

- MATIF (NYSE Euronext)
- LIFFE (NYSE Euronext)
- The Chicago Board of Trade (CBOT)
- The Chicago Mercantile Exchange
- The Kansas City Board of Trade
- The Minneapolis Grain Exchange
- Eurex

In this paper we look specifically on MATIF as it is the largest European wheat trading platform. The standardized contract on milling wheat on MATIF is specified like so:

Table 1	**Milling Wheat Futures – MATIF Euronext**
Unit of trading	Fifty tonnes
Delivery months	September, December, March and May
Min price movement	25 Euro cents per tonne (€12.50)
Last trading day	18:30 on the tenth calendar day of the delivery month
Trading Hours	10:45 – 18:30 Paris time
Notice day/Tender day	The first business day following the last trading day
Origins tenderable	Milling Wheat from any EU origin
Price basis	Euro and euro cents per tonne, in an approved silo in Rouen (France) and Dunkirk (France)
Quality	Sound, fair and merchantable quality of the following standard: Specific weight 76 kg/hl, Moisture content 15%, Broken grains 4% Sprouted grains 2%, Impurities 2%

	Discounts apply to reflect any difference between the delivered and standard quality, in accordance with Incograin No.23 and the Technical Addendum No.2 Mycotoxins not to exceed, at the time of delivery, the maximum levels specified under EU legislation in force with respect to unprocessed cereals intended for use in food products
Tender period	Any business day from the last trading day to the end of the specified delivery month

As seen above the MATIF contracts expires 4 times a year and there is always multiple contracts out at the same time, at the time of February 2016 you can buy contracts reaching into May 2017 meaning that there is 6 contract in the open market at this time. This makes following the general price curve of the futures tricky. This paper solves this issue by utilizing an index tracking the most liquid future contract, i.e. the one with the largest open interest translating into the current most traded contract. All contracts on MATIF are standardized like specified above which simplifies the matching between buyer and seller.

3.3. Futures

Commodity Futures on wheat at the exchange is a standardized contract between two parties, a buyer and a seller. The buyer of a future on the exchange agrees to buy the good at the specified price, quantity, quality at the date of delivery. The seller of the future agrees to deliver the good as the conditions specify in the contract. Because it is often impractical to settle the contracts with physical trade there is also a second and more frequently used settlement, cash settlement. If a farmer wants to cancel a position it's as simple of just buying an identical contract in the opposite direction, a wash out. The value of the contract is the difference between the current future price and the price paid at the time of purchase. This paper theoretically utilizes cash settlement future as it is unlikely that a farmer would deliver physical wheat to France. A future contract could be bought from either a bank or the exchange. If the farmer buys from a bank there is normally a credit evaluation of the buyer and seller done by the bank. Doing credit evaluations on the exchange is neither plausible nor effective; therefore a different approach is needed. Both parties in a contract are required to put a security of around 10% of the underlying assets value to a third party, the exchange. The gain/loss from the future is transferred each day between the buyer and seller's account. If either runs out of his security deposit due to transfers to the other part they are forced to either put more money into the account or the exchange closes the future by buying/selling it back. The owner of a sold future is allowed as a last resort to deliver the crop to a storage facility approved by the exchange and will be paid by the exchange. In the case of MATIF-wheat-futures the delivery point is silos in Rouen or Dunkirk, the smallest amount allowed for MATIF deliverance is 10 contracts totaling 500 tons. This is unfortunately not a realistic option for single Swedish wheat farmers due to the very large volume needed to get the shipping cost reasonable. (Iwarson 2012)

Iwarson (2012) argues that Futures plays a pivotal role in the agricultural sector for three reasons

- Price discovery
- Price security
- The ability to deliver the crops to the exchange as a last resort

Price discovery means that with future contracts being traded on the exchange crop prices becomes public domain by looking at the current prices of futures at the exchange. Price security has been explained earlier in the paper and is the ability to secure future prices at present time.
Pricing for futures for physical goods such as wheat are determined by arbitrage pricing when the good is in good supply and can easily be bought on the market. Such a time is likely after the harvest. When an investor can turn to the market and buy physical wheat to exploit any incorrect prices to gain a risk free profit the price should adjust quickly to the arbitrage free price: $F_t = S_0 e^{(c-y)T}$ where F is the future price today S is the spot price discounted back to present value from time T, c is the cost of carry until time T and y the convenience yield until time T.
The convenience yield is the benefits the holder of the asset places on physically owning the good compared to the future contract and could for example be for the purpose of securing ones production or profit from temporary local shortages. The cost of carry is the cost of storing the good and the cost of financing it minus the income from owning the good.
When the good does not yet exist or is in low supply, like during the growing season the prices on the futures are based on supply and demand on the future contract. In the case of wheat futures where the liquidity normally is high the supply and demand price should reflect the unbiased expectation and be: $F_t = E_t[S_T]$. Today's future price would be the expected future value of the spot. As with all investment goods the price is also reflected by the risk associated with the asset, if negative systematic risk exists in the asset the future price will be higher than the expected future spot price to compensate for the systematic risk the holder must bear. The examples above are considered with zero systematic risk. (Hull 2012)

3.4. Static and dynamic hedge ratio

Two different assumptions about the covariance between spot and future prices are tried in this paper. Either that it is constant or that it changes over time. Assuming that the covariance is constant over time would lead a farmer to utilize a static hedge ratio, hence the ratio of his wheat the farmer decides to hedge for a static hedge ratio framework is determined based on the belief that the covariance of the spot and future is constant over time. The static hedge framework uses models such as OLS, VAR and ECM to calculate the historical optimal constant hedge ratio and uses this to forecast the optimal hedge ratio in the future. It is instinctively easy to see why such a method would be attractive for a farmer more interested in spending time on the core business then calculating advanced financial models. Assuming however that the variance and covariance changes over time, we look at the framework that arose from the work of Cecchetti, Cumby and Figlewski (1988) and Baillie and Myers (1991). For a hedge to be calculated with time varying covariance and variance of two assets a Bivariate GARCH model would be adopted allowing the historical covariance and variance to vary over time to forecast future covariance and variances. Bollerslev et al (1988) shows why it is reasonable to utilize dynamic models when working with economic time series data "Of course the GARCH specification does not arise directly out of any economic theory, but as in the traditional autoregressive moving average time-series analogue, it provides a close and parsimonious approximation to the form of heteroscedasticity typically encountered with economic time-series data" The quotation is from page 119 in Bollerslev et al (1988) which utilizes a MGARCH model to expand on the original CAPM-model (Sharpe (1964) and Lintner (1965)) by looking at conditional variances of returns of bills bond and stocks in the US-market. Practically the real difference between the static and dynamic approach in this paper lies in the way they evaluate historical data to forecast the best future hedge ratio. A farmer could adopt a rolling window forecast for static models and rebalance his hedge daily just as he could by using a dynamic approach. Hence the difference between the static and dynamic hedge is not the frequency of rebalancing the hedge but how it evaluates and utilizes historical data. This paper evaluates which of the two approaches best evaluate historical data and which model best explain the historical variation.

3.5 Limitations and assumptions

The paper only considers in-sample estimation and thus does not try to perform any Out-of-sample forecasting of the variance and covariance. This In-sample estimation doesn't present how well the models forecast future variance and covariance. The transaction cost and brokerage fee for buying and selling futures is considered to be zero, meaning that there is no penalty for trading more frequently on daily returns then on weekly returns. The calculations are based on the fact that there is always wheat owned by the farmer needing to be hedged regardless of growing and harvesting seasons.

4. Methodology

There are a number of different hedging models employed in this paper, both static and dynamic. In this chapter the models used are further explained to gain an intuitive understanding.

4.1 Naïve hedge and No hedge

The naïve hedge has the hedge ratio of 1. In the case of wheat farming this means that the farmer would sell future contracts for the same quantity as he is expecting to harvest or has in his silos. The reason it can be seen as naïve is since it doesn't take into account the underlying components characteristics and might lead to increased risk in the case of the future and spot market not being perfectly correlated. The No hedge is quite self-explanatory and is when a farmer decided to not buy and future contracts to manage price risk, simply keeping a hedge ratio of 0.

4.2. Static hedge

4.2.1. Ordinary Least Squares

Modeling static hedges with Ordinary least squares (OLS) has been done for a long time, Johnson (1960) was among the first to use OLS to model a static hedge. OLS is employed in an attempt to use linear approximation to find the best fit to the data resulting in the lowest sum of squared error terms which is the difference between the approximation and the data. The linear approximation looks like:

$$\Delta lnS_t = \beta_0 + \beta_1 \Delta lnF_t + \varepsilon_t \qquad [1]$$

$$\beta_1 = \frac{\sigma_{fs}}{\sigma_f^2} \qquad [2]$$

In [1], ΔlnS is the logged spot return, ΔlnF is the logged future return, β_0 is the constant, ε_t is the error term and β_1 show how much of the return of the spot that is explained by the future returns and is the optimal hedge ratio h^* suggested by this model. The ratio h^* is not time dependent and is constant over the whole sample. For OLS to be the best linear unbiased estimator (BLUE) the estimation has to fulfill the six Gauss Markov assumptions (Westerlund 2005). The Gauss Markov assumptions of heteroscedasticity and autocorrelation has been questioned by Baillie and Myers (1991) and Cecchetti, Cumby and Figlewski (1988). They assumed that the error terms were varying over time and criticized the use of OLS as it would not be the most effective model instead favoring ARCH and GARCH models allowing for time-varying error terms and volatility clustering commonly found in financial time series. Baillie and Myers (1991) also argued that a more robust model would be found using GARCH because it would allow for weak dependency for the error terms in higher terms explained as the GARCH ability to let the current error terms interact with past variances and covariances. Serial autocorrelation will be tested for using Breusch-Godfrey test and heteroscedasticity with Whites. As above mentioned OLS has received some well-founded critique (Baillie and Myers, 1991) and (Cecchetti, Cumby and Figlewski, 1988), Nevertheless OLS has the usefulness of being easy to calculate and interpret.

4.2.2. Error Correction Model

The Error correction model (ECM) is similar to the OLS model. The difference however is that the short-term relationship between the spot and the future can be evaluated while still maintaining the long-run relationship. This long-run relationship is estimated by running an OLS on the non-stationary series of the spot and future. By saving the residuals from this long-run OLS model and incorporating them with one lag into the ECM we can see how fast shocks revert back to the long run mean while still maintaining the short run dynamics. The formula looks like the following:

$$\Delta lnS_t = \beta_0 + \beta_1 \Delta lnF_t + \gamma e_{t-1} + \varepsilon_t \qquad [3]$$

$$e_{t-1} = lnS_{t-1} - \alpha - \beta lnF_{t-1} \qquad [4]$$

The Model is similar to the OLS but has the added term γe_{t-1}, e_{t-1} reflects the long term relationship and γ indicates how long it takes a shock to the spot price to return to the long-term-mean. The rest is as in the OLS ΔlnS is the logged spot return, ΔlnF is the logged future return and β_1 show how much of the return of the spot that is explained by the future returns and is the optimal hedge ratio h^* suggested by this model.

4.3. The dynamic hedge

Dynamic hedging will be evaluated by two different GARCH extensions, The Diagonal VECH-TGARCH and the Diagonal BEKK-TGARCH, these two models are tested to hopefully gain a more robust result and to be able to see if the stricter parameter restriction in the covariance matrix in BEKK improves the variance reduction over the VECH model. To better understand the multivariate GARCH models it is helpful to first explain the univariate GARCH were we only have one variable. The univariate GARCH models the volatility of the data by allowing for time varying variance. For explanatory reasons an AR (2)-GARCH (1,1) model is further investigated. The mean equation is an autoregressive model where the dependent variable is dependent on two past lags and an error term. The error term is in turn described by a generalized auto regressive conditional heteroscedasticity (GARCH) process were the error term is affected by the weighted average of the long-run average variance, the variance predicted for this period and the new information in this period that is captured by the most recent squared residual (Engle 2001). The AR(2)-GARCH(1,1) model is:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + u_t \quad u_t|\Omega_{t-1} \sim N(0, \sigma_t^2) \qquad [5]$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b\sigma_{t-1}^2 \qquad [6]$$

Equation [6] describes the GARCH part of the model and how it affects the error term u_t from equation [5]. The error term u_t is allowed to depend conditionally on all information up to time t-1, this is indicated by Ω_{t-1}. u_t is thus dependent on the weighted function of the long-time average variance, a_0, conditional information on volatility from the former period, σ_{t-1}^2 and the new information from the error term u_{t-1}^2. With other words The GARCH model forecasts the variance of date t returns as the weighted average of the constant, yesterday's forecast, and yesterday's squared error. (Engle et al 2008) a_0, a_1, b are weighted summing to one to keep the GARCH function weakly stationary. To further demonstrate how the GARCH function utilizes historical information we extend equation [5]

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b\sigma_{t-1}^2 \qquad [7]$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b((a_0 + a_1 u_{t-2}^2 + b\sigma_{t-2}^2) + b\sigma_{t-2}^2) \qquad [8]$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b((a_0 + a_1 u_{t-2}^2 + b\sigma_{t-2}^2) + b(a_0 + a_1 u_{t-3}^2 + b\sigma_{t-3}^2) \qquad [9]$$

In a summarized form this becomes:

$$\sigma_t^2 = \frac{a_0}{1-b} + a_1 \sum_{n=1}^{\infty} b^{n-1} u_{t-n}^2 \qquad [10]$$

Equation [10] show that in a GARCH function the further back in time the shocks are the less impact it has on present volatility. The parameters in the mean and variance equation are estimated by maximizing the log likelihood function:

$$L_t = \frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{(y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 y_{t-2)})}{\sigma_t^2} \qquad [11]$$

Having briefly explained the Univariate GARCH we return to the Multivariate GARCH (MGARCH). The MGARCH model is similar to the univariate counterpart, both are maximized by the log-likelihood function and both show how the variance changes over time. In the multivariate GARCH however there are two or more variables, naturally it also express how the covariance between the different series evolves over time.

The multivariate GARCH log-likelihood functions in turn looks like:

$$L_t = \frac{N\,x\,T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(|H_t|) - \frac{1}{2}\varepsilon_t' H_t^{-1}\varepsilon_t \qquad [12]$$

Where H_t is the covariance-variance matrix and ε_t the error term, with the bracket $|H_t|$ indicates that H_t is a determinant and as an example if we have a 3x3 $|H_t|$ of 9 parameters a to I we get:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \qquad [13]$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg) \qquad [14]$$

$$= aei + bfg + cdh - ceg - bdi - afh \qquad [15]$$

Different mean equations can be used for a MGARCH model process. In this paper two different approaches are tried, the Vector auto regression model (VAR) which utilizes past lags of both series and Vector error correction model (VECM) that utilizes past lags for both series but also the long-run equilibrium similar to the ECM model explained earlier. Written with two lags VAR and VECM can be expressed like:

VAR(2)

$$\Delta lnS_t = \beta_0 + \beta_1 \Delta lnS_{t-1} + \beta_2 \Delta lnS_{t-2} + \alpha_1 \Delta lnF_{t-1} + \alpha_2 \Delta lnF_{t-2} + \varepsilon_t \qquad [16]$$

VECM(2)

$$\Delta lnS_t = \beta_0 + \gamma u_{t-1} + \beta_1 \Delta lnS_{t-1} + \beta_2 \Delta lnS_{t-2} + \alpha_1 \Delta lnF_{t-1} + \alpha_2 \Delta lnF_{t-2} + \varepsilon_t \qquad [17]$$

To investigate if there is any asymmetry in the GARCH process, if negative returns have larger impact then positive returns, we add the GJR/TGARCH extension to the tried GARCH models. The TGARCH model was developed by Jean-Michel Zakoian in 1994. This extension enables us to examine if there is any leverage effect in the data (Brooks 2002). The extension to the GARCH models is the addition of the term:

$$+D(u_{t-1}^2 I_{t-1}) \text{ where } I = 1\ if\ u_{t-1} < 0\ and\ 0\ if\ u_{t-1} \geq 0 \qquad [18]$$

According to Engle, Focardi and Fabozzi 2008 there are three major challenges with Multivariate GARCH models.

1. Determining the conditions that ensure that the variance-covariance matrix H_t is positive definite for every t
2. Making estimation feasible by reducing the number of parameters to be estimated
3. Stating conditions for the weak stationarity of the process

As variance cannot be negative there is a problem if the maximization from the log-likelihood function estimate gives a negative definite H_t matrix. The number of parameters from a GARCH function increases very rapidly with the amount of variables included resulting in very noisy and hard to interpret estimates. The third part is keeping the model weakly stationary this is done by restricting$a_{i,j,0} + a_{i,j1}, b_{i,j} \leq 1$ when maximizing the likelihood function. The 3 challenges expressed by (Engle et al 2008) have been tried to be solved for by expanding on the original GARCH model. Numerous different GARCH models extensions have been created, in this paper the VECH and BEKK extensions are examined and utilized.

4.3.1. VECH-GARCH

The original VECH-GARCH introduced by Bollerslev, Engle and Wooldridge 1988 utilizes the VECH operator that stacks the lower part of a $N\,x\,N$ matrix as a $N(N+1)/\,2\,x\,1$ matrix, this converts the matrix of variances and covariances into a vector of variances and covariances (Wang 2003):

$$VECH\begin{bmatrix} u_{1,t-1}^2 & u_{1,t-1}u_{2,t-1} \\ u_{1,t-1}u_{2,t-1} & u_{2,t-1}^2 \end{bmatrix} = \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} \qquad [19]$$

VECH-GARCH is quite general in the sense that every conditional variance and covariance in the output is a function of all the lagged conditional variances and covariances, as well as the cross-products of returns and lagged square returns, hence the model exhibits full parameterization (Wang 2003). For a VECH-GARCH(1,1) we have:

$$\mathrm{u}_t|\psi_{t-1} \sim N(0, H_t) \qquad [20]$$

$$Vech(H_t) = A_0 + A_1 Vech\big(\mathrm{u}_{1,t-1}u_{2,t-1}\big) + BVech(H_{t-1}) \qquad [21]$$

Equation [20] and [21] shows the GARCH part of the equation without the mean-equation, the same goes for equation [22] and [23] below which is an extended version of [22] with 2 assets examined.

The VECH-GARCH model with 2 assets will in full become:

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} a_{11,0} \\ a_{12,0} \\ a_{22,0} \end{bmatrix} + \begin{bmatrix} a_{11,1} & a_{12,1} & \alpha_{13,1} \\ a_{21,1} & a_{22,1} & \alpha_{23,1} \\ a_{31,1} & a_{32,1} & \alpha_{33,1} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11,1} & b_{12,1} & b_{13,1} \\ b_{21,1} & b_{22,1} & b_{23,1} \\ b_{31,1} & b_{32,1} & b_{33,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \qquad [22]$$

$$h_{11,t} = \alpha_{11,0} + a_{11,1}u_{1t-1}^2 + a_{12,1}u_{1,t-1}u_{2,t-1} + a_{13,1}u_{2,t-1}^2 + b_{11,1}h_{11,t-1} + b_{12,1}h_{12,t-1} + b_{13,1}h_{22,t-1}$$

$$h_{12,t} = \alpha_{12,0} + a_{21,1}u_{1t-1}^2 + a_{22,1}u_{1,t-1}u_{2,t-1} + a_{23,1}u_{2,t-1}^2 + b_{21,1}h_{11,t-1} + b_{22,1}h_{12,t-1} + b_{23,1}h_{22,t-1}$$

$$h_{22,t} = \alpha_{22,0} + a_{31,1}u_{1t-1}^2 + a_{32,1}u_{1,t-1}u_{2,t-1} + a_{33,1}u_{2,t-1}^2 + b_{31,1}h_{11,t-1} + b_{32,1}h_{12,t-1} + b_{33,1}h_{22,t-1}$$

[23]

If this full parameterization VECH-GARCH model [22] is run on a spot and future price series, the parameters for equation [23] can be explained as h_{11t} describes the variance of the spot over time, h_{22t} the variance of the future over time and h_{12t} the covariance between the spot and the future over time. As seen above in equation [23] the original VECH-GARCH becomes very large even for 2 assets with 21 parameters and will quickly become infeasible when adding more assets (Brooks 2002). To simplify the model and reduce the number of parameters estimated Bollerslev, Engle and Wooldridge (1988) developed an alternative modeling approach where the matrices A_1 and B from VECH-GARCH is made diagonal, this new diagonal matrix model is called the DVECH-GARCH model, this mean each covariate only depends on own past lags and surprises. If we look at equation [22] and apply the diagonal approach, A_1 and B transforms into:

$$\begin{bmatrix} a_{11,1} & 0 & 0 \\ 0 & a_{22,1} & 0 \\ 0 & 0 & a_{33,1} \end{bmatrix} \text{and} \begin{bmatrix} b_{11,1} & 0 & 0 \\ 0 & b_{22,1} & 0 \\ 0 & 0 & b_{33,1} \end{bmatrix}$$

This means that the number of parameters to be evaluated in the diagonal case is $3N(N+1)/2$ parameters. In the bivariate case of the DVECH-GARCH 9 parameters are estimated down from the 21 parameters from the VECH-GARCH. The 9 parameters in the DVECH-GARCH is 3 for α_1, 3 for β and 3 for α_0. Since this paper also examine asymmetrical effects of the data 3 parameters for the GJR/TGARCH extension, parameter matrix D is added. The DVECH-TGARCH for a bivariate model is thus written as:

$$h_{11,t} = \alpha_{11,0} + a_{11,1}u_{1,t-1}^2 + b_{11,1}h_{11,t-1} + d_{11}u_{1,t-1}^2 i_{11,t-1} \quad [24]$$

$$h_{12,t} = \alpha_{12,0} + a_{22,1}u_{1,t-1}u_{2,t-1} + b_{22,1}h_{12,t-1} + d_{12}u_{1,t-1}u_{2,t-1}i_{12,_{t-1}} \quad [25]$$

$$h_{22,t} = \alpha_{22,0} + a_{33,1}u_{2,t-1}^2 + b_{33,1}h_{22,t-1} + d_{22}u_{2,t-1}^2 i_{22,_{t-1}} \quad [26]$$

The downside of the Diagonal transformation of the VECH-GARCH is that it restricts interaction between different series in the way that it does not allow interaction between different variances and covariances. The upside is the large reduction in the number of parameters being estimated and hence the simplicity of estimation. A further issue with both the VECH- and DVECH-GARCH is that it cannot guarantee positive-definite matrices. This issue can be addressed by either utilizing a Cholensky factoring on the matrices or to restructure the model entirely to get squared parameter values as done in the model developed by Baba, Engle, Kraft and Kroner 1990 and published by Engle and Kraft in 1995 called the BEKK-GARCH.

4.3.2. BEKK-GARCH

The BEKK-GARCH model addresses the issue of non-positive semidefinite of the covariance-variance matrix and guarantees a positive semi-definite covariance matrix by a very simple idea of the structure of the matrixes which ensures quadratic term in the RHS (Brooks 2001). The BEKK-GARCH:

$$H_t = A_0{}'A_0 + A_1{}'\mathrm{u}_{t-1}u_{t-1}{}'A_1 + B'H_{t-1}B + ID'u_{t-1}u_{t-1}{}'D \qquad [27]$$

For a BEKK-GARCH(1,1) A_0, A_1 and B are 2x2 matrixes and the model in matrix form is as follows:

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$$
$$= \begin{bmatrix} \alpha_{11,0} & \alpha_{12,0} \\ \alpha_{21,0} & \alpha_{22,0} \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 & u_{1,t-1}u_{2,t-1} \\ u_{1,t-1}u_{2,t-1} & u_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \qquad [28]$$
$$+ \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix}$$

The key feature of model The BEKK-GARCH [27] is that builds in sufficient generality and allowing the variances and covariances to interact without requiring the estimation of to many variables. (Wang 2003) However even if BEKK-GARCH reduces the number of parameters compared to the DVECH-GARCH it still requires the estimation of a large number of parameters, $N^2 + N(N+1)/2$ which is why a diagonal version of the BEKK-GARCH was introduced were the number of parameters reduces to $N + N(N+1)/2$. (Engle, Focardi and Fabozzi 2008). Due to the relative low amount of observation in the weekly sample (338 observations) my focus is on parsimony over flexibility, parsimony in this case is achieved by reducing the number of parameters estimated and hence trade the upside of cross examination of variances and covariances for a reduction on parameters as in the DVECH-GARCH and Diagonal BEKK-GARCH.

4.4. Evaluation of hedge performance

Only the variance is optimized in the model used in this paper, the minimum variance model by Johnson (1960). The trade-off between risk and return are obvious but for comparing different hedges, the minimum variance hedge offers easy calculation and interpretation and is thus the method of choice. The return of the hedges will still be assessed in the results but the variance reduction is the main goal. The hedge ratio is calculated by the covariance between the two assets divided by the variance of the future .The optimal hedging ratio according to the different models is denoted h^*. For the static hedges the ratio h^* is:

$$h^* = \frac{\sigma_{sf}}{\sigma_f^2} \qquad [29]$$

And for the dynamic hedges h^* becomes:

$$h_t^* = \frac{\sigma_{sf,t}|\Omega_{t-1}}{\sigma_{f,t}|\Omega_{t-1}} \quad [30]$$

The return for the unhedged portfolio is given by equation [31] and the hedged portfolio is given by equation [32] or [33] Where $\Delta S_t = (S_t - S_{t-1})$ and $\Delta F_t = (F_t - F_{t-1})$ is the return of respective series between time period t-1 and t:

$$R_{unhedged} = \Delta S_t \quad [31]$$

$$R_{hedged} = \Delta S_t - h^* \Delta F_t \quad [32]$$

For the Dynamic hedge models the return equations changes into:

$$R_{hedged} = \Delta S_t - h_{t-1}^* \Delta F_t \quad [33]$$

This time parameter added to h* in the dynamic models indicates that the farmer at time period t-1 decides how many future contract he holds between t-1 and t based on the information available up to t-1. To calculate the variance of the different hedges we use the equations:

$$Var_{unhedged} = \sigma_s^2 \quad [34]$$

$$Var_{hedged} = \sigma_s^2 + h^* \sigma_f^2 - 2h^* \sigma_{sf} \quad [35]$$

$$Var_{dynamic} = \sigma_{s,t}^2 + h_{t-1}^* \sigma_{f,t}^2 - 2h_{t-1}^* \sigma_{sf,t} \quad [36]$$

Where $\sigma_f^2$is the variance of the future and $\sigma_s^2$is the variance for the spot, $\sigma_{sf}$is the covariance of the two assets.

The Sharpe ratio is a way to compare risk/return ratios for investment performance. This ratio is calculated for the full sample hedge models. Since return is not the objective of the optimization function in the minimum variance model, emphasis will not be placed on this ratio but since it is a commonly used evaluation of portfolios it is also included for comparison. The ratio is calculated as follows.

$$Sharpe\ ratio = \frac{R_h - R_f}{\sigma_h} \quad [37]$$

This ratio is the annualized return of the hedge, R_h minus the risk-free interest rate R_f divided by the standard deviation of the hedge, σ_h.

5. Data

For the empirical study of hedging to be possible a minimum of two time series are required, a spot price series and a future price series. The data for the spot selling price for a farmer in Sweden is obtained by using the purchasing price to Swedish farmers from the largest grain buyer in Sweden, Lantmännen. The price of purchase from Lantmännen is very good indicator of what price a farmer can expect to be able to sell his crop for at any given day. The data for the purchasing price consists of 1641 daily observations of wheat prices from 2009-07-01 to 2016-01-12. The Future price on the MATIF exchange is collected from D*atastream* and consists of 1641 daily observation from 2009-07-01 to 2016-01-12. The chosen time period is based on availability of data. The Future contracts on the MATIF exchange expires 4 times a year, March, May, September and December (delivery months). A consistent time series of futures is created by ordering the prices of the futures over time and using the price of the future contract with the largest open interest translating to the most traded contract. When a new future contract gains a larger open interest then the current contract followed by the series, it switches to the new contract and follows its price until a yet newer contract in time gains a larger open interest. The data is adjusted by removing weekend and large Swedish public holidays such as christmas and new year's eve as very little to none trading is done these days. This adjustment is done to remove false-zero return. A number of econometric tests are performed to establish the data series properties. Table 2 shows the results from a number of tests on the data. First up is a Dickey-Fuller test for unit root on the price series at level. The results indicates that all series contain a unit root at the 1% level. Table 2 also shows the result from Engle Granger unit root test for first differences were we are able to reject the null hypothesis of unit root on the 1% level, meaning that all series individually or jointly has no unit root nor is non-stationary at first difference. As all series are I(1) according to the Engle-Granger test it is interesting to see if the two daily and two weekly series exhibit a long-term relationship. We test for this running an Engle-Granger test for cointegration. The test is the third test in the table 2, the results shows that both daily and weekly returns are cointegrated. This indicates a long-run relationship.

Table 2	1-Day Spot	1-Day Future	Week Spot	Week Future
Augmented Dickey-fuller test at Level	0.5482	0.3169	0.4739	0.3206
Engle granger unit root test at first diff	0.0000	0.0000	0.0000	0.0000
Engle granger test for cointegration	0.0002	0.0004	0.0000	0.0000

Because non-stationarity time series causes issue when regressing time series we convert the I(1) series to stationarity series. The daily prices are converted to daily returns by taking the natural logarithm of price changes between trading days.

For the spot prices:

$$ln\left(\frac{S_t}{S_{t-1}}\right) = \Delta lnS_t \quad [38]$$

For future price:

$$ln\left(\frac{F_t}{F_{t-1}}\right) = \Delta lnF_t \quad [39]$$

Similarly the weekly prices are made stationary by taking the log return of the prices from Monday to Monday the following week. Next we examine if the four series are normally distributed by looking at the Jarque-Bera statistics in table 3. Both series and horizons reject the null hypothesis that the series are normally distributed at the 1% level. All series suffers from Leptokurtosis effect which means that the distribution has fat tails compared to a normal distribution. Fat tails means that a larger part of the returns have more extreme returns as in the data contain more large movements compared to a normal distribution. These extreme movements are more frequent in daily returns, the spot kurtosis goes from 54 in daily returns to 10 in weekly and from 19 to 8 in future returns. This seems logical since the weekly return have more time to absorb shocks. Daily spot and future as well as weekly spot returns are negatively skewed which mean that the three series have more negative movements then a normal and symmetric distribution. Theese 3 series exhibits a larger left tail. The weekly future return however, have a fatter right hand side tail. This indicated that it is composed by more positive returns.

Table 3	Mean%	Median %	Max %	Min %	Std.Dev %	Skewness	Kurtosis	Prob	No obs
1-Day S	0.0083	0.02	11.394	-21.81	1.32	-2.6248	54.127	0.00	1678
1 Day F	0.014	0	11.058	-17.03	1.54	-0.7668	19.828	0.00	1678
Week S	0.013	0.01	2.277	-4.73	0.66	-1.0067	10.865	0.00	338
Week F	0.0193	0	4.114	-2.93	0.68	0.3953	8.303	0.00	338

The correlation between the spot and future log returns for daily and weekly sample is 0.19 and 0.49 respectively.

6. Empirical results

6.1. Regression results

The Ordinary least square estimation is presented below in table 4. The OLS estimation is done using the transformed log return stationary series to avoid the problem of spurious regressions due to both series containing a unit root.

Table 4: **OLS Estimation**

Variable	Daily	Weekly
β_0(x100)	0.0059 (0.03)	0.0254 (0.16)
β_1	0.1654*** (0.0205)	0.4775*** (0.0458)
R-Square	0.0372	0.2443

The OLS-model gives an estimate of 0.1654 for β for daily returns translating into a hedge ratio of 0.1654 over the whole period likewise, the weekly return suggest a hedge ratio of 0.4775. The R^2 value can be seen as a measurement for goodness of fit (Ederington, 1979) how good the model specification fits with the data tested. For the daily returns it is 0.0372 thus only 3.72% of the variation is explained by the model. The weekly return works better and roughly 24% of the data is explained by the model indicating that OLS have a greater fit for the weekly return. Heteroscedasticity is tested for with White test and is displayed in appendix A, Table 12. For neither daily nor weekly are we able to reject the null hypothesis of homoscedasticity. This suggests that we have heteroscedasticity in the data. Autocorrelation is tried for both horizons using a Breusch-Godfrey serial correlation LM test. For the daily sample the null hypothesis of no autocorrelation is rejected and for the weekly returns the null hypothesis is not rejected, hence the weekly returns doesn’t seem to suffer from autocorrelation which is reasonable since there is 5 days’ worth of daily-returns integrated in the weekly return. Neither the daily nor weekly OLS model is BLUE. While the weekly and daily OLS may still be unbiased, they are not the most efficient linear estimator.

Table 5: **ECM Estimation**

Variable	Daily	Weekly
β_0(x100)	0.0059 (0.0314)	0.0223 (0.1637)
β_1	0.1722*** (0.0204)	0.4769*** (0.0459)
γ	-0.0260*** (0.0043)	0.053635 (0.0548)
R-Square	0.0576	0.2804

The ECM model has a higher R^2 value then OLS for both the daily and weekly returns and hence seem to explain the data better. β_1 explains the movement in the future price relative to the spot in the short run and is thus our hedge ratio. For the daily it is 0.17 and for the weekly it is 0.47. Compared to the OLS hedge ratios, the ECM suggests slightly higher ratio for both the daily weekly returns. γ gives information on how quickly the spot price return to the long-run mean after a shock in the future price, this long run negative coefficient is the reason the suggested hedge ratios grow somewhat between the OLS and ECM as the OLS cannot separate the short-run relation and long-run information. In appendix A, Table 12 we see the results from white's heteroscedasticity test and Breuch-Godfrey serial correlation LM test. For autocorrelation the null hypothesis cannot be rejected of either test at the 5% level indicating that we have no autocorrelation for either sample. The heteroskedasticity test indicates heteroskedasticity for the weekly sample but not the daily.

In the dynamic models we first need to evaluate how to specify the mean equations, VAR and VECM. A different number of lags can be chosen to be included and this is evaluated with VAR-lag-length-criteria seen in appendix A, table 13 for daily returns and table 14 for weekly returns. Both samples suggest the same amount of lags, AIC and HQ suggested 2 lags while SC suggest 1 lag, Since 2 of 3 suggested 2 lags, a VAR(2) and VECM(2) model is utilized as mean in the dynamic GARCH models. Below in table 6 is the DVECH-TGARCH results presented. 4 different specifications are displayed. The DVECH-TGARCH with VAR(2) as mean equation run on both weekly and daily returns. The DVECH-TGARCH with VECM as mean equation run on both daily and weekly returns.

Table 6: **Diagonal VECH**

GARCH = A0 + A1.*RESID(-1)*RESID(-1)' + D1.*(RESID(-1)*(RESID(-1)<0)) *(RESID(-1)*(RESID(-1)<0))' + B1.*GARCH(-1)

Transformed Variance Coefficients	Mean Eq. VAR(2)		Mean Eq. VECM(2)	
	Daily	Weekly	Daily	Weekly
A0(1,1)	2.27E-05***	0.0006	2.24E-05***	0.0002**
	(1.63E-06)	(0.0007)	(1.85E-06)	(8.78E-05)
A0(1,2)	1.66E-07	0.0002	1.75E-07	4.89E-05**
	(1.37E-07)	(0.0001)	(1.42E-07)	(1.90E-05)
A0(2,2)	1.30E-05***	0.0001***	1.34E-05***	0.0001***
	(2.27E-06)	(4.16E-05)	(2.35E-06)	(4.58E-05)
A1(1,1)	0.0804***	-0.0025	0.0827***	0.2365**
	(0.0180)	(0.0542)	(0.0183)	(0.1039)
A1(1,2)	0.0056**	0.0634	0.0058**	0.1027**
	(0.0027)	(0.0553)	(0.0027)	(0.0495)
A1(2,2)	0.2274***	0.1585***	0.2233***	0.1744***
	(0.0197)	(0.0408)	(0.0201)	(0.0430)
D1(1,1)	0.3536***	-0.0094	0.3487***	-0.1587
	(0.02668)	(0.0538)	(0.0360)	(0.1028)
D1(1,2)	0.0004	-0.0008	8.43E-05	-0.0763
	(0.0055)	(0.0550)	(0.0054)	(0.0503)
D1(2,2)	-0.0860***	-0.0073	-0.0821***	-0.0916*
	(0.0227)	(0.0643)	(0.0227)	(0.0543)
B1(1,1)	0.6656***	0.5148	0.6663***	0.6645***
	(0.0187)	(0.5340)	(0.0202)	(0.1111)
B1(1,2)	0.9866***	0.6667**	0.9864***	0.8502***
	(0.0074)	(0.3202)	(0.0077)	(0.0460)
B1(2,2)	0.7885***	0.7523***	0.7883***	0.7678***
	(0.0192)	(0.0571)	(0.0197)	(0.0596)

* Coefficient matrix B and D is not PSD for 1 day return.
* Coefficient matrix A and D is not PSD for weekly return.

For Daily returns The DVECH-TGARCH with VAR and VECM mean equation get same amount of significant variables, all coefficients are significant in the conditional variance-covariance matrix except D(1,2) and A_0(1,2). Looking at the TGARCH extension coefficients for the daily sample and both mean equations we can see that for lagged spot returns D(1,1), negative shocks affect the spot returns by 0,35 more than positive shocks, for lagged future returns D(2,2) however the negative shocks seems to affect the future returns less then positive with 0,08. D(1,2) is insignificant and we cannot draw any conclusions on asymmetrical effects on the covariance. For the weekly returns, the mean equation VECM seems to perform a lot better than the VAR as it contains 10 out of 12 significant coefficients compared to the 4 out of 12 in the VAR mean equation weekly sample. Little can be said for the asymmetrical properties of the weekly sample as only one of the six possible TGARCH coefficients are significant, D(2,2) for the VECM mean equation. The model has a few non-positive semidefinite matrixes that would pose a problem if we were to do an out-of-sample forecast but for in sample estimation it poses no issue. In Table 7 the Diagonal BEKK-TGARCH results are displayed in the same fashion as in table 6. 4 diagonal BEKK-TGARCH models with 2 different returns and 2 different mean equations.

Table 7: **Diagonal BEKK**

GARCH=A0+A1*RESID(-1)*RESID(-1)'*A1+D1*(RESID(-1)*(RESID(-1)<0))*(RESID(-1)*(RESID(-1)<0))'*D1+B1*GARCH(-1)*B1

Transformed Variance Coefficients	Mean Eq. VAR(2)		Mean Eq. VECM(2)	
	Daily	Weekly	Daily	Weekly
A0(1,1)	2.50E-05***	5.80E-06	2.48E-05***	2.58E-05***
	(1.68E-06)	(4.53E-06)	(1.92E-06)	(3.51E-06)
A0(1,2)	6.16E-06***	1.38E-05***	6.27E-06***	1.46E-05***
	(1.39E-06)	(3.52E-06)	(1.41E-06)	(3.88E-06)
A0(2,2)	1.54E-05***	2.51E-05***	1.57E-05***	5.71E-06
	(2.12E-06)	(3.24E-05)	(2.18e-06)	(5.03E-06)
A1(1,1)	0.2283***	0.0300	0.2315***	0.6181***
	(0.0269)	(0.1292)	(0.0272)	(0.0539)
A1(2,2)	0.4248***	0.6226***	0.4187***	0.0162
	(0.0188)	(0.0527)	(0.0192)	(0.1350)
D1(1,1)	0.6136***	4.34E-05	0.6100***	1.65E-06
	(0.0202)	(88.2879)	(0.0281)	(2285.5610)
D1(2,2)	-0.0411	7.13E-07	-0.0475	1.55E-05
	(0.0661)	(1225.515)	(0.0652)	(177.1511)
B1(1,1)	0.8197***	0.9235***	0.8159***	0.2095
	(0.0109)	(0.0622)	(0.0120)	(0.1888)
B1(2,2)	0.8832***	0.2230	0.8845***	0.9236***
	(0.0098)	(0.1811)	(0.0100)	(0.0700)

The diagonal BEKK model has the diagonal matrix restriction of A, B and D and is more restrictive then the DVECH, This model is positive semidefinite by the quadratic nature of the model specification. For the daily returns, both VAR and VECM has significant D(1,1) and insignificant D(2,2). This indicates that negative return seems to have a larger impact then positive for spot returns then for future returns. Otherwise then D(2,2) the daily model has all significant variables. For the weekly returns sample VAR and VECM looks to have gotten opposite results, for VAR A(2,2) and B(1,1) is significant and for VECM A(1,1) and B(2,2) are significant.

6.2. Hedging ratios and performance

The different hedge ratios for the full sample can be seen graphically in Appendix B. Figure 1 to 4 for the daily return hedge ratios and figure 5-8 for the weekly return hedge ratios. In the table presented below I refer to variance reduction as a lower total variance to the benchmark, the no hedge. A negative percentage displayed in the *variance red %* row is a reduction in variance, a better performing hedge. A positive percentage in this same row is thus a worse hedge and increases variance. The return and variance for the different hedge ratios over the full daily sample is seen in the table below:

Table 8	No hedge	Naïve hedge	OLS	ECM	VAR-VECH	VAR-BEKK	VECM-VECH	VECM-BEKK
Yearly Return	3.08%	-1.25%	2.36%	2.45%	2.78%	2.68%	2.76%	2.65%
Reduction In % Points	-	-4.33	-0.71	-0.63	-0.30	-0.40	-0.32	-0.42
Yearly volatility	0.2099	0.2910	0.2065	0.2065	0.2070	0.2074	0.2070	0.2074
Variance red %	-	92,3%	-3.19%	-3.14%	-2.68%	-2.36%	-2.70%	-2.38%
Sharpe ratio	0.1466	-0.0429	0.1145	0.1185	0.1341	0.1292	0.1333	0.1280
Avgerage hedge ratio	0	1	0.165	0.148	0.143	0.156	14.5	15.8

Looking at the daily returns for the full sample the different hedges have poor hedging performance regardless of which model used to estimate the optimal hedging ratio. The two static hedges OLS and ECM perform very similar and the increased complexity of the ECM seems to make little difference and the variance reduction is low for both models at about -3% while lowering the return by about -0.6-0.7 percentage points. All four dynamic models decrease variance by between 2-3% and lower the yearly return by roughly 0.3-0.4 percentage points. All in all, if an investor or farmer is interested in trading on daily returns to hedge wheat he would experience a negligible reduction in variance for a small loss in return regardless of which of the included model he would choose to use. The naïve hedge performs the absolute worst with a variance increase of 92.3% and a reduction in return by 4.33 percentage points. Looking at the Sharpe ratios none of the included hedges outperform the no-hedge in a risk/return context. The return and variance for the weekly returns and hedges are the following:

Table 9	No hedge	Naïve hedge	OLS	ECM	VAR-VECH	VAR-BEKK	VECM-VECH	VECM-BEKK
Avg. Yearly Return	3.14%	-1.25%	0.80%	0.77%	3.34%	1.32%	-0.47%	-1.11%
Reduction In % Points	-	-4.39	-2.35	-2.38	0.19	-1.82	-3.61	-4.25
Yearly Volatility	0.2099	0.2910	0.2065	0.2065	0.2070	0.2074	0.2070	0.2074
Variance red. %	-	5%	-24%	-24%	-21%	-24%	-29%	-27%
Sharpe Ratio	0.1267	-0.0491	0.03678	0.0354	0.1511	0.0614	-0.0224	-0.0522
Average hedge ratio	0	1	0.48	0.49	0.54	0.44	0.49	0.52

The hedging performance for the weekly return in the table above offer better hedging results compared to the daily return sample. All hedges except the naïve hedge give substantial reduction in variance ranging from -24 % to -29% compared to the no-hedge. As expected this variance reduction comes for the cost of lower return and the reduction ranges from -1.82 to -4.25 percentage points, the exception being the VAR-VECH which features both a variance reduction and a return increase. The Naïve hedge is as in the daily return sample the worst performing hedge and both gives the highest percentage point reduction in return of -4.39 and is also the only hedge increasing the variance, in this case by 5% which of course is widely better than the daily return sample naïve hedge with its 92.3% increase. Looking once again at the Sharpe ratio only the VAR-VECH offers a higher Sharpe ratio compared to the no hedge, 0.1511 and 0.1267 respectively.

Table 10 presents the results for a robustness test done by dividing the daily return sample into 4 time periods each consisting of 420 observations and running the same regressions as with the full sample. This is done to see if the models give the same results with different data samples.

Robustness test – Daily returns

Table 10	No Hedge	Naive Hedge	OLS	ECM	VAR-VECH	VAR-BEKK	VECM-VECH	VECM-BEKK
Sample 1	01/07/2009 to 17/02/2011							
Yearly returns	42%	1%	33%	33%	37%	36%	36%	31%
Red. Percentage points	-	-41	-9	-10	-5	-6	-6	-12
Yearly Volatility	22.2%	29.6%	21.5%	21.5%	21.8%	21.8%	21.8%	23.9%
Variance reduction	-	77.1%	-6.8%	-6.8%	-3.5%	-4.1%	-3.8%	15.3%
Avg. Hedge ratio	0	1	0.223	0.230	0.214	0.196	0.223	0.176
Sample 2	21/2/2011 to 01/10/2012							
Yearly returns	0.5%	-2.3%	0.0%	3.8%	2.5%	-5.3%	-5.3%	-5.3%
Red. Percentage points	-	-2.8	-0.4	-0.4	3.4	2.0	-5.7	-5.7
Yearly Volatility	21.7%	33.5%	21.2%	21.2%	21.0%	21.1%	22.5%	22.5%
Variance reduction	-	139.0%	-4.2%	-4.2%	-5.9%	-5.7%	7.5%	7.5%
Avg. Hedge ratio	0	1	0.149	0.153	0.180	0.184	0.399	0.399
Sample 3	04/10/2012 to 27/05/2014							
Yearly returns	-16.3%	0.6%	-15.5%	-15.4%	-20.3%	-20.1%	-14.6%	-20.1%
Red. Percentage points	-	16.9	0.8	0.9	-4.0	-3.9	1.7	-3.9
Yearly Volatility	20.6%	28.1%	20.6%	20.6%	21.9%	22.0%	21.1%	22.0%
Variance reduction	-	86.5%	-0.2%	-0.2%	13.4%	14.2%	5.3%	14.2%
Avg. Hedge ratio	0	1	0.049	0.054	0.017	0.015	0.049	0.015
Sample 4	29/05/2014 to 12/01/2016							
Yearly returns	-10.4%	-3.4%	-8.8%	-8.8%	-15.1%	-11.8%	-15.9%	-11.4%
Red. Percentage points	-	7.0	1.6	1.6	-4.7	-1.3	-5.5	-1.0
Yearly Volatility	19.0%	24.2%	18.6%	18.6%	18.7%	18.6%	18.6%	18.6%
Variance reduction	-	62.2%	-3.7%	-3.7%	-2.5%	-3.8%	-3.5%	-3.9%
Avg. Hedge ratio	0	1	0.201	0.205	0.173	0.166	0.187	0.175

From table 10 we can see that the no hedge and in turn the Swedish wheat price behave differently over the 4 samples with the annualized returns ranging from -16.3% to 42% while the volatility goes between 19% and 22.2%. Another interesting thing to notice is the fact that the hedge ratios vary so greatly with the samples. In sample 3 where we have the biggest down turn of the spot prices with -16.3% annualized returns in this period the hedge ratios shrinks down to almost 0 while in sample 1,2 and 4 when the returns are generally positive 0.5-42% we instead have around 0.15 to 0.20 in average hedge ratios for the different hedge models. This could suggest that the correlation between the spot and future market is higher during bull markets then bear markets. It is complicated to pick out a clear best hedge model over the 4 samples as none of the hedges perform especially well over any sample period although the static models, OLS and ECM seems to perform more consistent.

Table 11 presents the results for a robustness test on the weekly return sample. The weekly returns sample is only divided into two subsamples, sample 5 consisting of 218 observation and sample 6 consisting of 120 observations. As with the daily return sample robustness test, all the regressions from the full sample are run.

Robustness test – Weekly returns

Table 11	No Hedge	Naive Hedge	OLS	ECM	VAR-VECH	VAR-BEKK	VECM-VECH	VECM-BEKK
Sample 5	3/07/2009 to 16/09/2013							
Yearly returns	8.35%	-0.88%	4.20%	4.18%	7.22%	7.21%	6.30%	5.93%
Red. Percentage points	-	-9.2	-4.1	-4.2	-1.1	-1.1	-2.0	-2.4
Yearly Volatility	27%	28%	24%	24%	24%	24%	24%	24%
Variance reduction	-	10.8%	-22.6%	-22.6%	-20.5%	-20.0%	-18.5%	-18.2%
Avg. Hedge ratio	0	1	0.450	0.451	0.492	0.464	0.472	0.480
Sample 6	23/09/2013 to 11/01/2016							
Yearly returns	-6.96%	-1.68%	-4.17%	-4.19%	-0.73%	-1.73%	-0.89%	-1.37%
Red. Percentage points	-	5.3	2.8	2.8	6.2	5.2	6.1	5.6
Yearly Volatility	20.5%	18.8%	17.0%	17.0%	16.8%	16.8%	16.7%	16.8%
Variance reduction	-	-16.0%	-30.8%	-30.8%	-32.9%	-33.1%	-33.6%	-32.9%
Avg. Hedge ratio	0	1	0.529	0.525	0.509	0.566	0.522	0.562

From table 11 we can see in Sample 5 that the hedging ratios averages between 0.45 and 0.48 and generally all hedges excluding the naïve hedge perform well with variance reductions of -18.2% to -22.6%. The annualized return reduction is rather large in sample 5 where the VAR-VECH and VAR-BEKK models reduces the return the least with -1.1 percentage points in annualized return compared to the no hedge. For sample 6 we have a general down turn in the price level and the annualized return for the no hedge is -6.96% in annualized returns. Here all hedges including the naïve hedge reduce variance while increasing the returns relative to the no hedge. Over sample 5 and 6 the VAR-VECH model look to be the best choice as it has among the best annualized returns while still reducing variance by a comparable amount to the other hedges. The weekly return sample 6 ranging from 23/09/2013 to 11/01/2016 is perhaps the most relevant results in this paper as they are the most recent in time and also produces the best hedge performance.

7. Conclusion

This paper examines the possibility of using MATIF-futures on milling wheat to hedge the price movement of Swedish wheat. Two static models, OLS and ECM are tried as well as four dynamic models, The VAR-VECH, the VAR-BEKK, the VECM-VECH and the VECM-BEKK all four with the threshold dynamic TGARCH extension to examine asymmetric in the data. The hedges are compared to the base lines of a no hedge and a naïve hedge with hedge ratios of 0 and 1 respectively. The hedges are tried for both daily returns and weekly returns. The daily returns are run over both the full sample 2009-07-01-2016-01-12 as well as over 4 different subsamples for robustness test, the weekly return are run over the full sample and over 2 subsamples. All hedges are optimized using the minimum variance framework and the focuses of the results is therefore on the variance reduction. The results suggest no clear answer of the best general model over both daily- and weekly returns and all samples. The correlation estimation shows that the daily returns have a correlation of 0.19 while the weekly has a larger correlation of 0.49. This large increase in correlation between daily and weekly returns indicate that the spot and future price follow each other more closely over a week then over a day. This is also reflected in the results of the different hedges where overall the weekly return hedges outperform the daily return hedges. The largest variance reduction of the most effective daily return hedge, the OLS features a -3.19% variance reduction over the full sample while the most effective weekly return hedge, the VECM-VECH offers a -29% variance reduction over the full sample. One of the likely reasons this correlation difference exist between daily and weekly returns is the fact that a lag exist between the Future price and Swedish wheat price. According to Food and Agricultural Organization of the United Nations, Sweden produced roughly 3 million tons of wheat in 2014 compared to the international market of 729 million tons or European market of 150 million tons. The occasional need for Swedish wheat to be exported or foreign wheat to be imported mean the prices has to be competitive and the relative small share of Swedish wheat production compared to the international market is indication of Sweden being a price follower of the international price. This lag is also visible when looking at a graphical representation of the two series as seen in appendix B figure 9. This lag seems natural as the prices on the very liquid future contracts on MATIF should rebalance to the true international price faster than the Swedish price which is in this paper is represented by the Swedish grain buyer Lantmännens buying price from farmers in Sweden. This price lag is better incorporated over one week. For the full sample the empirical results suggest that a farmer would have done well for himself historically between 2009-07-01-2016-01-12 turning to MATIF and trading MATIF futures utilizing a VECH-TGARCH with a VAR mean equation to rebalance his hedge each week keeping an average of 52% of his wheat hedged. This would not only have kept his variance 21% lower over time but would also have increased his return by 0.19 percentage points. If the sole goal was managing the risk, a weekly traded VECM-VECH would have reduced the variance by -29% with a reduction in return by -3.61 percentage points. For all its complexity the dynamic hedges offers little improvement over the simplest of models, OLS over the full sample. The OLS has comparable variance reductions over all samples and over both daily and weekly returns. In the daily return full sample it is the best performing hedge in term of variance reduction with -3.19% and in the weekly full sample the OLS has a variance reduction of -24%, which is only 5 percentage points worse than the best performing hedge variance wise, the VECM-VECM with -29%. The OLS great simplicity and its ease of understanding would most likely make it the model of choice for many farmers. However there is still some merit to use the more advanced GARCH functions, turning to subsample 6 for weekly returns between the dates 23/09/2013 to 11/01/2016 we find the best performing hedge, results show that a farmer would have lowered his variance by -33.6% while increasing his annualized return relative to the no hedge by 6.1 percentage points by using a VECM-VECH. From a practical viewpoint it is more likely to convince a farmer to start hedging using recent in time results compared to results ranging many years back in time as future years are more likely to be similar to close past year then more distant ones. This makes perhaps the weekly results in subsample 6 the best argument for showcasing the upside to hedging wheat utilizing MATIF Future milling wheat contracts. Further studies on this subject could include trying different variations of the GARCH framework to try and find a better fit for the data. The inclusion of an out-of-sample forecast of the variances and covariances with the GARCH models might be beneficial to uncover additional information about the different strategies employed in this paper. There exist numerous wheat and grain future contracts traded on exchanges in the world and it's not certain that the MATIF future is the best fit for Swedish wheat, trying different future contracts for wheat such as the on the Chicago board of trade (CBOT) would perhaps yield greater variance reduction. Another interesting further study would be to examine different future contracts and compose a weighted basket of future contracts to find a better correlation with Swedish wheat prices.

Reference list

Andersson, R & Danthine, J-P. (1981) Cross hedging. *Journal of Political Economy*, Vol. 89, No.6, pp. 1182-1196.

Baillie, R & Myers, J. (1991) Bivariate Garch Estimation of the Optimal Commodity Futures Hedge. *Journal of Applied Econometrics,* No.6, No. 2. pp. 109-124.

Black, F & Scholes M. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, Vol 81, No. 3, Pp. 637–654.*

Bollerslev T, Engle, R & Wooldridge, J. (1988) A capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy*, Vol 96, No. 1. Pp 116-131.

Cecchetti, S, Cumby, R & Figlewski, S. (1988) Estimation of the Optimal Futures Hedge. *The Review of Economics and Statistics,* Vol. 70, No.4, pp.623-630.

Ederington, L. (1979) The Hedging Performance of the New Futures Markets. *The Journal of Finance,* Vol. 34, No.1, pp. 157-170.

Engle, R (2001). GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics. *The Journal of Economic Perspectives*, Vol. 15, No. 4, pp. 157-168.

Engle, R, Foccardi, S & Fabozzi, F. (2008) ARCH/GARCH Models in Applied Financial Econometrics. *Handbook of finance,* Wiley online library.

Engle, R & Kroner, K. (1995) Multivariate Simultaneous Generalized ARCH. *Economic Theory,* Vol. 11, No.1, pp. 122-150.

Food and Agricultural Organization of the United States, FAOSTAT Database, http://faostat3.fao.org/home/E, viewed: 2016-04-03.

Figlewski, S. (1984) Hedging Performance and Basis Risk in Stock Index Futures. *The Journal of Finance,* Vol. 39, No.3, pp. 657-669.

Floros, C & Vougas, D. (2004) Hedge ratios in Greek stock index futures market. *Applied Financial Economics,* Vol. 14, No.15, pp. 1125-1136.

Floros, C & Vougas, D. (2006) Hedging Effectiveness in Greek Stock Index Futures Market, 1999-2001. *International Research Journal of Finance and Economics,* Vol. 5, pp. 17-18.

Golden, L, Yang, C & Zou, H. (2010) The effectiveness of Using a Basis Hedging strategy to Migigate the Financial Consequences of Weather-Related Risks. *North American Actuarial Journal,* Vol. 14, No.2, pp. 157-175.

Hull, John C. (2012) *Options, Futures, And Other Derivates*. Pearson Education Limited, Harlow.

Iwarson, T. (2012) *Bättre betalt för skörden*. Steiners förlag AB, Vaxholm.

Johnson, L. (1960) The Theory of Hedging and Speculation in Commodity Futures. *The Review of Economic Studies,* Vol. 27, No.3, pp. 139-151.

Kahl, K. (1983) Determination of the Recommended Hedging Ratio. *American Journal of Agricultural Economics,* Vol. 65, No.3, pp.603-5.

Kumar, B & Pandey, A. (2009) Role of Indian Commodity Derivatives Market in Hedging Price Risk: Estimation of Constant and Dynamic Hedge Ratio and Hedging Effectiveness. *22nd Australasian Finance and Banking Conference 2009*. Available at SSRN: http://ssrn.com/abstract=1452881.

Kumar, B, Singh, P & Pandey, A. (2008) Hedging Effectiveness of Constant and Time Varying Hedge Ratio in Indian Stock and Commodity Futures Markets, working paper no.2008-06-01, Indian institute of management, Ahmedabad.

Manfredo, R & Richards, T. (2009) Hedging with weather derivates: a role for options in reducing basis risk. *Applied Financial Economics,* Vol. 19, No. 2, pp. 87-97.

Markowitz, H. (1959) *Portfolio selection: efficient diversification of investments*. New York, John Wiley.

Stein, J. (1961) The simultaneous Determination of Spot and Futures Prices. *The American Economic Review,* Vol. 51, No.5, pp. 1012-1025.

Wang P. (2003) *Financial Econometrics: Methods and models*, New York, Routledge.

Zakoian, J-M. (1994) Treshold heteroskedasticity models. *Journal of Economic Dynamics and Control*, Vol. 18, No.5, pp. 931-955.

Appendix A

Test for Autocorrelation and Heteroscedasticity for OLS and ECM

Table 12	OLS		ECM	
P-values in table	Daily	Weekly	Daily	Weekly
Autocorrelation	0.0063	0.3484	0.0558	0.1317
Heteroskedasticity	0.7299	0.8245	0.1064	0.0000

Table 13 - Daily sample VAR-Length-Lag

VAR Lag Order Selection Criteria
Endogenous variables: LNRSPOT LNRFUTURES
Exogenous variables: C
Date: 01/27/16 Time: 14:18
Sample: 7/01/2009 1/12/2016
Included observations: 1670

Lag	LogL	LR	FPE	AIC	SC	HQ
0	9477.903	NA	4.04e-08	-11.34839	-11.34189	-11.34598
1	9529.516	103.0423	3.82e-08	-11.40541	-11.38593*	-11.39819
2	9538.857	18.62534	3.79e-08*	-11.41180*	-11.37935	-11.39978*
3	9540.495	3.261781	3.80e-08	-11.40898	-11.36353	-11.39214
4	9543.852	6.678308	3.81e-08	-11.40821	-11.34978	-11.38656
5	9546.331	4.924704	3.81e-08	-11.40638	-11.33498	-11.37992
6	9548.409	4.123379	3.82e-08	-11.40408	-11.31969	-11.37281
7	9555.021	13.10555*	3.81e-08	-11.40721	-11.30983	-11.37113
8	9556.750	3.422497	3.82e-08	-11.40449	-11.29413	-11.36360

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level), FPE: Final prediction error
AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

Table 14 – Weekly VAR-Length-Lag

VAR Lag Order Selection Criteria
Endogenous variables: LNRSPOT LNRFUTURE
Exogenous variables: C
Date: 02/16/16 Time: 12:16
Sample: 7/13/2009 1/11/2016
Included observations: 330

Lag	LogL	LR	FPE	AIC	SC	HQ
0	2419.722	NA	1.48e-09	-14.65286	-14.62983*	-14.64367
1	2424.474	9.419306	1.48e-09	-14.65742	-14.58835	-14.62987
2	2434.688	20.11724*	1.42e-09*	-14.69508*	-14.57995	-14.64916*
3	2435.959	2.487648	1.45e-09	-14.67854	-14.51736	-14.61425
4	2437.464	2.929443	1.47e-09	-14.66342	-14.45620	-14.58076
5	2441.706	8.201125	1.47e-09	-14.66489	-14.41161	-14.56386
6	2443.276	3.014856	1.49e-09	-14.65016	-14.35083	-14.53076
7	2444.444	2.229826	1.51e-09	-14.63299	-14.28762	-14.49523
8	2447.542	5.876872	1.52e-09	-14.62752	-14.23610	-14.47139

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level), FPE: Final prediction error
AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

Appendix B

Figure 1: VAR-VECH, ECM and OLS for Daily returns full sample

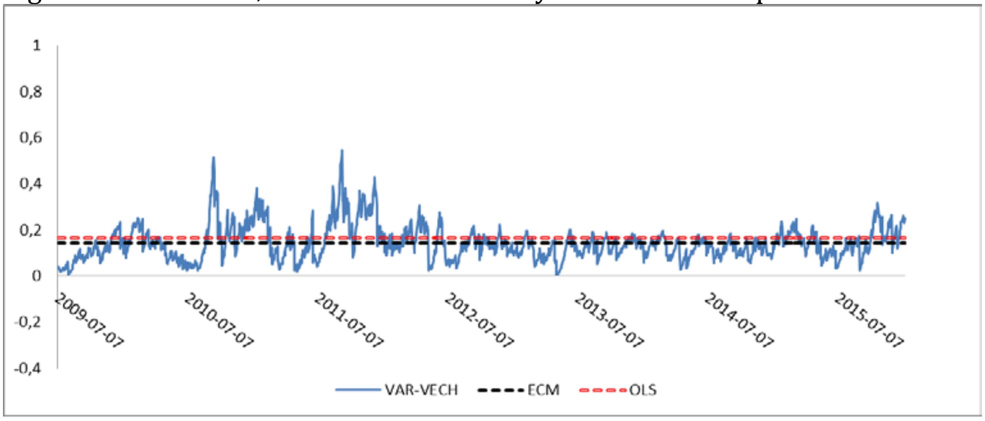

Figure 2: VAR-BEKK, ECM and OLS for Daily returns full sample

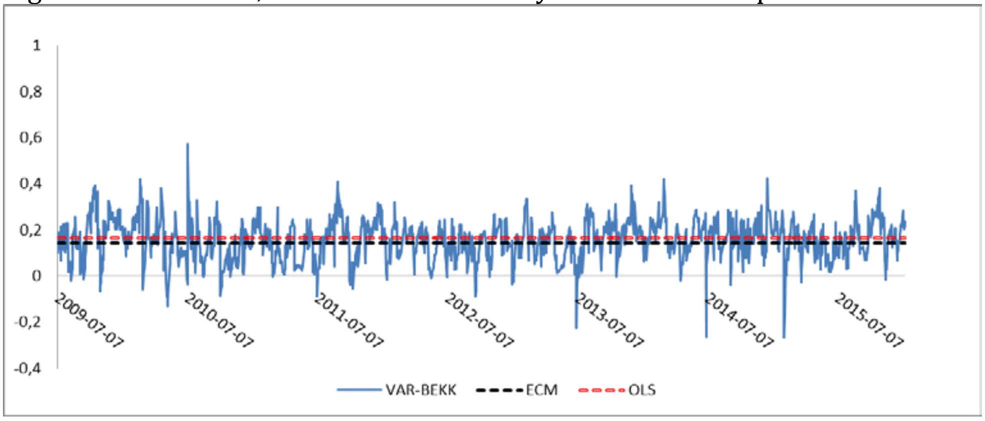

Figure 3: VECM-VECH, ECM and OLS for Daily returns full sample

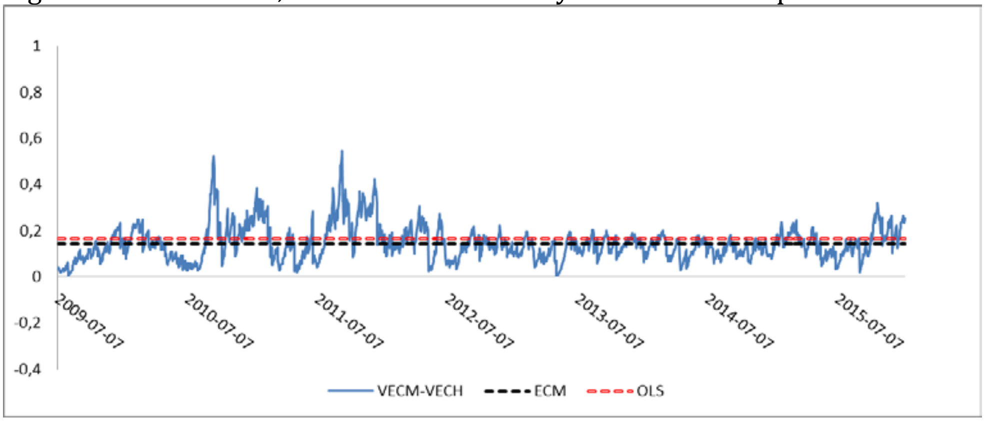

Figure 4: VECM-BEKK, ECM and OLS for Daily returns full sample

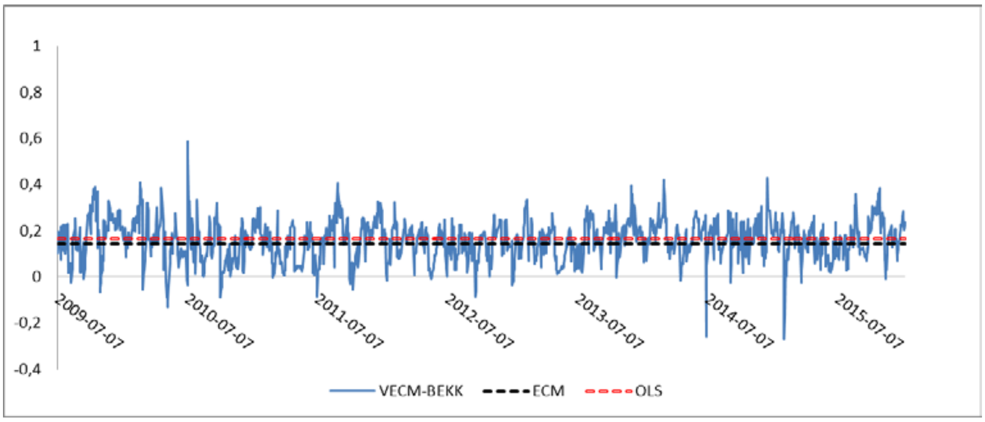

Figure 5: VAR-VECH, ECM and OLS for Weekly returns full sample

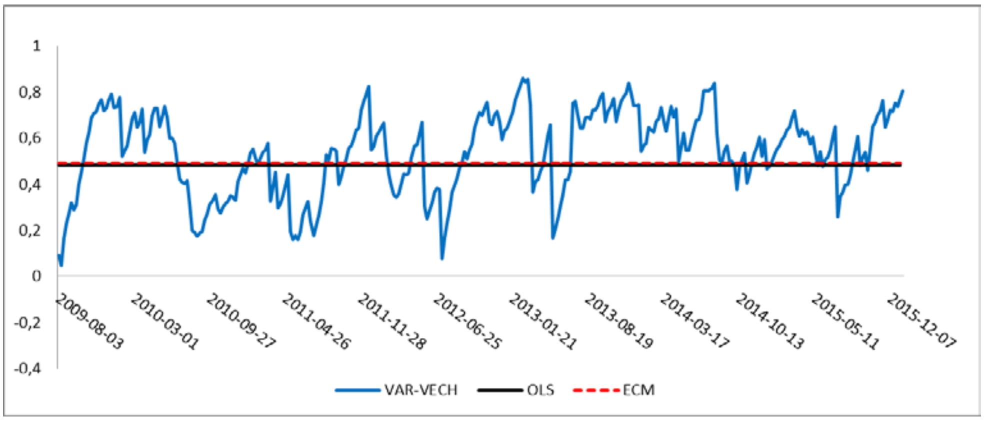

Figure 6: VAR-BEKK, ECM and OLS for Weekly returns full sample

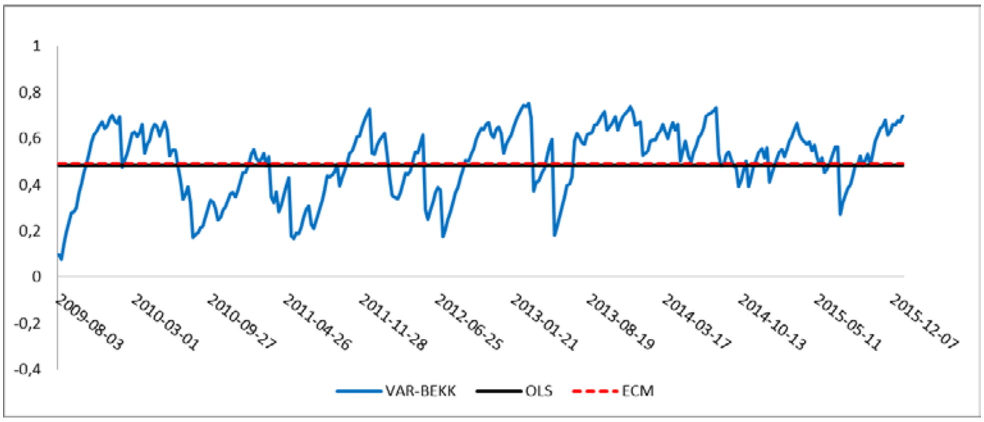

Figure 7: VECM-VECH, ECM and OLS for Weekly returns full sample

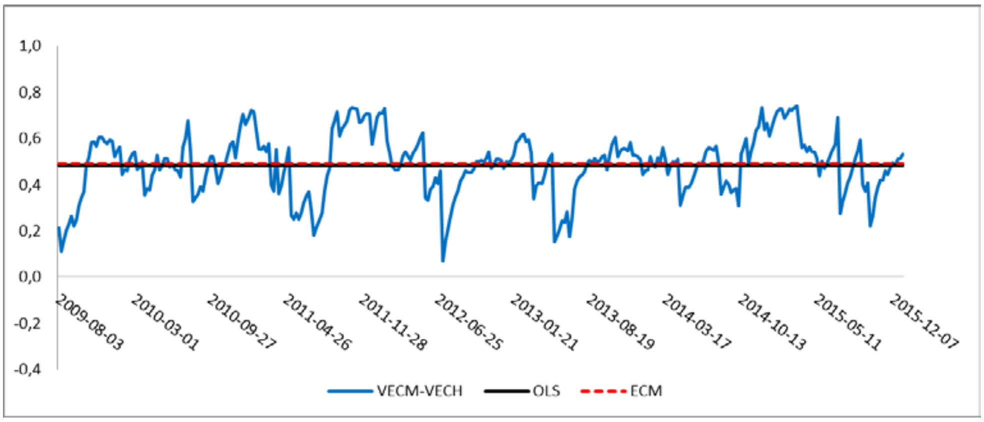

Figure 8: VECM-VECH, ECM and OLS for Weekly returns full sample

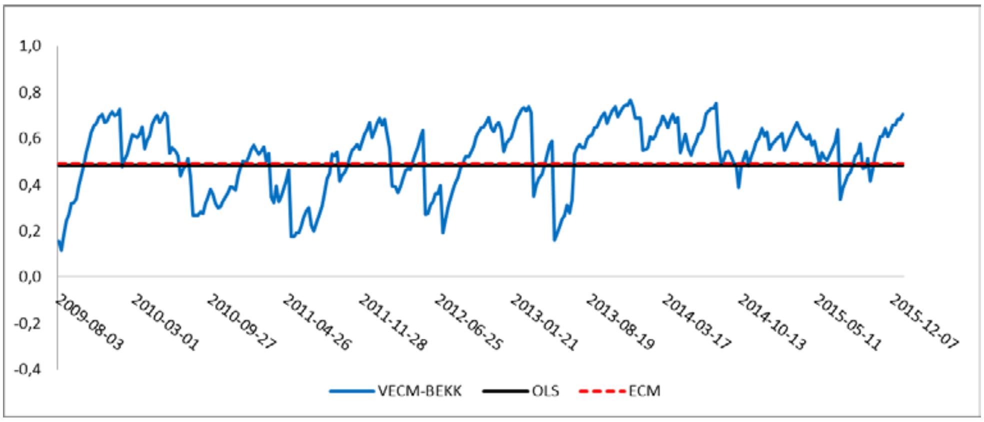

Figure 9: Visualized price levels of spot and future price