

# **A Black-Litterman portfolio allocation model combined with a Markov switching framework**

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# 1 Abstract

This is a M.Sc. thesis investigating the compatibility and performance of a regime switching framework as a complement to the Black-Litterman portfolio allocation model. Conclusively, it is considered to be a compatible match of models in terms of practical implementation and the results indicate that the model is performing well.

## 2 Introduction

### 2.1 Background

The Black-Litterman model is an asset allocation model developed by Fischer Black and Robert Litterman in the early 90's. It was published in the internal Goldman Sachs Fixed Income Research Note, Black and Litterman (1990) [Walters, J (2009)]. The primary reason for using the model is to overcome problems occurring with Mean Variance Optimization such as unintuitive, highly-concentrated portfolios, input-sensitivity, and estimation error maximization [Idzorek, Thomas M. (2005)].

The concept is that market data is complemented with investor opinions, i.e. views, to avoid these problems and create a better performing model. A Bayesian approach is used, implying that investors should be uncertain about market estimates and views and consequently express them as distributions [Walters, J]. In general terms, the market equilibrium data and investor opinions are expressed as distributions which are mixed together to create a new distribution. The parameters of the new distribution are then used to generate new portfolio weights by reversed optimization.

Since the views need to be expressed as distributions it is necessary to explain their uncertainty. It is originally suggested that the investor expresses an opinion about the return of an asset and also announces, measured in percentage, the confidence in that specific prediction. This has been widely known as one of the most unwieldy parts of the Black-Litterman model and in this thesis an alternative way is suggested in order to confront that specific problem [Idzorek, Thomas M. (2005)].

In this thesis it is investigated whether a regime switching model based on hidden Markov chains is appropriate to represent the investor views for the Black-Litterman model. More specifically, the regime switching model will state the probability of the market being in a certain regime at a certain point in time. Parameters such as mean and variance will be contingent on the models choice of regime and will be used as complementing views to the unprocessed market data in the Black-Litterman model.

### 2.2 Aim with thesis

The thesis is focusing on implementing the combined Markov switching and Black-Litterman model. The aim is to test and determine whether a Markov switching model is appropriate as a stock market predictive tool. More specifically it is investigated whether it can replace the subjective investor views normally used in the Black-Litterman model. This will be tested by building a model that uses market data from OMXS30 stocks to allocate a portfolio and compare it with a market weighted portfolio of the same asset universe.

## 3 Theory

### 3.1 Markov Switching model

#### 3.1.1 Fundamental understanding

Empirical experiments have led to a consensus regarding the behavior of financial time series saying that it can exhibit very different characteristics at different points in time. Instead of using a standard model with one set of variables, it is found natural to include several models to represent the different characteristics. In this thesis a Markov switching model is used as an application to analyze the data [Chung-Ming Kuan (2002)].

To discuss the application of a Markov switching model within economics and finance it is necessary to understand the fundamentals of a regime switching model. The concept is that the model incorporates several structures and thus in an improved manner models the statistical behavior of different states, in this context defined as regimes. A regime has specific statistical properties, such as variance and mean, that differs from the other regimes. One aspect of the Markov switching model is that it can switch in between these structures in order to capture the statistical behavior of the different regimes. The switching is regulated by a non-observable state variable with the statistical properties of a first order Markov chain. The current value of the state variable is only depending on the previous value and the time is considered as discrete. This means that a certain regime will appear during a random time frame, more specifically until a switching takes place within the Markov chain. The model is hence appropriate for describing correlated data that demonstrates clear patterns related to specific periods in time [Chung-Ming Kuan (2002)].

#### 3.1.2 A simple model applied to the stock market

A simple Markov switching model is a bivariate discrete time process consisting of an underlying Markov chain  $\mathbf{S}$  and the independent stochastic output variables  $\mathbf{Y}$ . The statistical distribution of the output variable  $\mathbf{Y}$  for a certain time  $\mathbf{T}$  is dependent on the state of the underlying variable  $\mathbf{S}$  for the current time  $\mathbf{T}$ . The state of the Markov chain itself is dependent on the previous state and the transition probability. We cannot observe anything but the output variables, which means it is impossible to do statistical inference on the Markov chain itself.

#### Application to the stock market

The simplest method for analyzing stock data would state that the statistical properties of the data can be explained by a single dependent variable  $\mathbf{Y}$ , the independent mean  $\mathbf{C}$  and independent variance  $\epsilon$  according to (1).

$$\begin{aligned} Y &= C + \epsilon \\ \epsilon &\sim N(0, \sigma^2) \end{aligned} \tag{1}$$

As explained in the previous section it is fair to assume that a model of such simplicity will not perform well in terms of predicting statistical properties. Let us instead introduce the two states  $\mathbf{S}_1$  and  $\mathbf{S}_2$  and describe the data with a Markov switching model according to (2).

$$\begin{aligned} Y_1 &= C_1 + \epsilon_1 \quad \text{if } S = S_1 \\ Y_2 &= C_2 + \epsilon_2 \quad \text{if } S = S_2 \end{aligned} \tag{2}$$

$$\begin{aligned} \epsilon_1 &\sim N(0, \sigma_1^2) \\ \epsilon_2 &\sim N(0, \sigma_2^2) \end{aligned}$$

In our model the current state  $\mathbf{S}$  for each time  $\mathbf{T}$  is decided according to a Markov chain with two states  $\mathbf{S}_1$  and  $\mathbf{S}_2$ . The probability distribution can be described by a 2x2 matrix according to (3).

$$\mathbf{P} = \begin{bmatrix} P(S_{T+1} = S_1 | S_T = S_1) & P(S_{T+1} = S_2 | S_T = S_1) \\ P(S_{T+1} = S_1 | S_T = S_2) & P(S_{T+1} = S_2 | S_T = S_2) \end{bmatrix} \tag{3}$$

The transition between states is a stochastic process and not deterministic, which means that one can never be certain that a transition will take place.

Important to notice after introducing these theoretical concepts is that the only observable variable is  $\mathbf{Y}$ . In order to create a model to depict stock market behavior the parameters  $\mathbf{C}$ ,  $\boldsymbol{\epsilon}$  and  $\mathbf{P}$  need to be estimated [Perlin, M. (2015)].

### 3.1.3 Estimating the parameters

Finding the parameters can be done with Maximum Likelihood estimation. In this section this will be demonstrated for a general case with N states assumed. A Hamilton filter will be used to calculate the filter probabilities in a recursive way.

The probability of being in state  $i$  at time  $t = 1$ , i.e. the start value, is expressed in (4).

$$\begin{aligned} \alpha_{i,1} &= P(S_1 = i | Y_1) = \\ &= \frac{f(S_1=i, Y_1)}{f(Y_1)} = \\ &= \frac{f(Y_1 | S_1=i)P(S_1=i)}{\sum_{j=1}^N f(S_1=j, Y_1)} = \\ &= \frac{f(Y_1 | S_1=i)P(S_1=i)}{\sum_{j=1}^N f(Y_1 | S_1=j)P(S_1=j)} = \\ &= \frac{f(Y_1 | S_1=i)p_i}{\sum_{j=1}^N f(Y_1 | S_1=j)p_j} = \\ &= [f(Y_1 | S_1 = i)p_i] \end{aligned} \tag{4}$$

$[f(Y_1|S_1 = i)p_i]$ , which is the final row in (4), is the normalized version of  $f(Y_1|S_1 = i)p_i$ . The same procedure is followed in (5) for time  $t - 1$  using the systems recursive properties.

$$\begin{aligned}
\alpha_{i,t} &= P(S_t = i|Y_{1:t}) = \\
&= \frac{f(S_t=i, Y_t|Y_{1:t-1})}{f(Y_t|Y_{1:t-1})} = \\
&= \frac{f(S_t=i, Y_t|Y_{1:t-1})}{\sum_{j=1}^N f(S_t=j, Y_t|Y_{1:t-1})} = \\
&= [f(S_t = i, Y_t|Y_{1:t-1})] \\
&= [f(Y_t|S_t = i, Y_{1:t-1})P(S_t = i|Y_{1:t-1})] = \\
&= [f(Y_t|S_t = i)P(S_t = i|Y_{1:t-1})]
\end{aligned} \tag{5}$$

As can be concluded from figure 1 the sole relation between  $S_1$  and  $Y_{1:t-1}$  is through  $S_{t-1}$ . Now, continuing with  $\alpha_{i,t-1}$ .

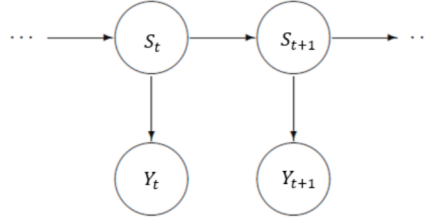


Figure 1: Markov property

$$\begin{aligned}
\alpha_{i,t} &= P(S_t = i|Y_{1:t-1}) = \\
&= \sum_{j=1}^N P(S_t = i, S_{t-1} = j|Y_{1:t-1}) = \\
&= \sum_{j=1}^N P(S_t = i|S_{t-1} = j, Y_{1:t-1})P(S_{t-1} = j|Y_{1:t-1}) = \\
&= \sum_{j=1}^N P(S_t = i|S_{t-1} = j)P(S_{t-1} = j|Y_{1:t-1})
\end{aligned} \tag{6}$$

In the last expression it can be seen that  $P(S_t = i|S_{t-1} = j)$  is the transition probability  $p_{i,j}$  and that  $P(S_{t-1} = j|Y_{1:t-1})$  is the filter probability at time  $t - 1$  which is  $\alpha_{j,t-1}$ . In combination with (5), (7) is reached.

$$\alpha_{i,t} = [f(Y_t|S_t = i) \sum_{j=1}^N p_{i,j} \alpha_{j,t-1}]; \tag{7}$$

### The Maximum likelihood formula

The set of parameters to estimate can be expressed as  $\theta$  in (8).

$$\theta = \{p_i, p_{i,j}, \mu_i^m, \Sigma_i^m, \Sigma_i^{m,n}\} \quad (8)$$

where  $i, j \in (1, 2 \dots N)$  and  $m, n \in (1, 2 \dots M)$  and

- $p_i$  = initial probability to be in state  $i$ .
- $p_{i,j}$  = transition probability from state  $i$  to  $j$ .
- $\mu_i^m$  = mean value for asset  $m$  in state  $i$ .
- $\Sigma_i^{m,n}$  = covariance between asset  $m$  and  $n$  in state  $i$ .

The conditional probability function  $f(y_t | S_t = i, \theta)$  is considered as the likelihood function for state  $i$  conditional on the parameters above. After the  $\alpha_{i,t}$  estimation, the full log likelihood function can be calculated as a function of the parameters in (9) which is a weighted average of the likelihood in each state using the state probabilities.

$$\ln L = \sum_{t=1}^T \ln \sum_{i=1}^N f(Y_t | S_t = i, \theta) P(S_t = i) \quad (9)$$

The final step is to maximize (9) and extract the parameters [Brandel, S. (2017)].



## 3.2 Black-Litterman model

The Black-Litterman model is a portfolio allocation model based on mean variance optimization. The main concept is to extract information from market data, transform it into distribution parameters and construct a weighted average distribution in combination with parameters from investor views. There are many different ways of doing this and this section explains the basic procedure developed by Fischer Black and Robert Litterman in the early 90's [Idzorek, Thomas M. (2005)].

### 3.2.1 Initial assumptions

The method starts with the normally distributed expected returns in (10). The mean  $\mu$  cannot be known with 100% certainty and therefore we will be using (11) where it is stated that the mean itself is normally distributed around the estimate  $\Pi$  with the variance  $\Sigma_{\Pi}$ . It is important to note that  $\mu$  and  $\Pi$  are uncorrelated. This implies (12) which is the reference model [Walters, J (2009)].

$$r|\mu \sim N(\mu, \Sigma) \quad (10)$$

$$\mu \sim N(\Pi, \Sigma_{\Pi}) \quad (11)$$

$$r \sim N(\Pi, \Sigma_r), \quad \Sigma_r = \Sigma + \Sigma_{\Pi} \quad (12)$$

$\Pi$  is the equilibrium return of the portfolio. Black and Litterman made a simplifying assumption stating that the covariance matrix  $\Sigma_r$  is proportional to the covariance matrix of the returns  $\Sigma$  according to (13) [Walters, J (2009)].

$$r \sim N(\Pi, \tau\Sigma) \quad (13)$$

The initial step of the Black-Litterman model is to generate a neutral starting point without views. Assuming CAPM theory the implied excess equilibrium returns will be derived from the market data and form the neutral starting point. This is achieved by reversed optimization finding the single global maximum of the convex quadratic utility function  $U$  in (14). No constraints are applied here. The solution in (15) can be reversed in order to find the portfolio weights  $w$  given a return vector  $\mu$  according to (16).

$$U = w^T \mu - (\lambda/2)w^T \Sigma w \quad (14)$$

$$\mu = \lambda \Sigma w \quad (15)$$

$$w = (\lambda \Sigma)^{-1} \mu \quad (16)$$

According to CAPM theory, the market capitalization weights of the risky assets constitute the equilibrium portfolio weights. Theoretically, the CAPM market portfolio contains all assets to be invested in, but investors generally simplify this to a smaller set in order to make the model easy to practically implement [Walters, J (2009)]. This simplification is used in this thesis and the implied excess equilibrium returns are calculated according to (17), where  $w_{mkt}$  is the market weights for the assets involved in our portfolio [Idzorek, Thomas M. (2005)].

$$\Pi = \lambda \Sigma w_{mkt} \quad (17)$$

An interpretation of the equilibrium return  $\Pi$  is that if all the investors follow the same strategy, the asset demand and supply are matching perfectly.  $\lambda$  is the risk aversion coefficient representing the average risk tolerance of the defined asset universe according to (18) where  $r_f$  is the risk free rate [Swedish Riksbank (2018)],  $\mu_p$  is the expected long term portfolio return and  $\sigma^2$  is its variance [He, G. and Litterman, R. (1999)].

$$\lambda = (\mu_p - r_f) / \sigma^2 \quad (18)$$

### 3.2.2 Views

Views are assigned to each asset by introducing the matrix  $Q$  and  $V$ . Each view can be expressed as (19).  $V$  is the matrix that identifies the asset involved in the different views,  $Q$  is the estimated return vector for every different view and  $\Omega$  is its variance. We are assuming  $K$  views and  $N$  assets in this theory section.

$$V\mu \sim N(Q, \Omega) \quad (19)$$

### 3.2.3 $\tau$ constant

The constant  $\tau$  is a scalar that is intended to regulate the influence of the uncertainty of each view and eventually the estimated equilibrium mean returns. There are different opinions of the best way to assign a value to this scalar. It is practically inversely proportional to the relative weight given by the Implied Equilibrium Return Vector.

- Black-Litterman (1990):  $\tau$  should be set close to zero since the uncertainty in the mean is smaller than the uncertainty in the return it self. Also Idzorek (2004) shares the same view.
- Lee (2000):  $\tau$  is set between 0.01 and 0.05.
- Blamont and Firoozye (2003):  $\tau = 1/N$ , where  $N$  is the number of observations.
- Meucci (2008):  $\tau = 1/T$  where  $T$  is the length of the time series [Seimertz, D. (2015)].

### 3.2.4 $V$ matrix

The  $V$  matrix connects the assets belonging to a specific view with the  $Q$ -vector. One row is corresponding to one view and the  $V$  matrix will hence be a  $K \times N$  matrix. Views can be absolute or relative. If a view is absolute, the row sums in the  $V$  matrix equal 1. If the view is relative, the row sums equal 0. The Black-Litterman model does not require the investor to specify views for all assets involved [Idzorek, Thomas M. (2005)].

### 3.2.5 Q vector

The Q-vector contains information regarding the excess returns of the views. Each row is corresponding to one specific view and hence the vector is of the form  $K \times 1$  [Idzorek, Thomas M. (2005)].

### 3.2.6 $\Omega$ matrix

The views are usually investor speculations and an element of uncertainty must hence be incorporated into the model. The  $\Omega$  matrix explains this. Every view could be expressed as (20), where the  $\epsilon$  vector elements are unknown, randomly, independently and normally distributed.  $\epsilon$  has a mean of 0 and a covariance matrix of  $\Omega$ .

$$V\mu = Q + \epsilon \quad (20)$$

In the Black-Litterman model it is assumed that views are uncorrelated and hence the  $\Omega$  matrix will be a diagonal  $K \times K$  matrix. If one of the diagonal elements equals 0, it means that the uncertainty of that specific view is zero and that the investor is 100% certain about that specific view. The variance of each error term  $\omega$  is the absolute difference from the error term epsilon's expected value of 0.

The calculation of  $\Omega$  is considered to be a complicated part of the Black-Litterman model. After defining V, the variance of each individual view portfolio can be calculated as follows where  $\Sigma$  is the covariance matrix of excess returns.  $v_k$  is a row vector found in matrix V.

$$\Omega = \begin{bmatrix} \tau v_1 \Sigma v_1^T & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & \tau v_k \Sigma v_k^T \end{bmatrix} \quad (21)$$

In this thesis, this particular part of the model will be specialized later on [Idzorek, Thomas M. (2005)].

### 3.2.7 Absolute and relative views

Views can be expressed as absolute and relative. An absolute view has information on the absolute return of one or several assets, while a relative view has information on the relation between two or more assets. Each absolute view will render a row in the V matrix with sum 1, while the row sum of each relative view equals 0.

### 3.2.8 An example of view allocation

An example of V and Q for a two view scenario with five assets is displayed in (22) and (23). Here it is assumed that the first view is an absolute view predicting asset 4 to increase by 5%. The second view is a relative view predicting asset 3 to outperform asset 4 by 3%.

$$V = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \quad (22)$$

$$Q = \begin{bmatrix} 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 3 & -3 & 0 \end{bmatrix} \quad (23)$$

### 3.2.9 Combined distribution

As mentioned earlier, the model is based on combining the distributions of the equilibrium returns and the views. The proof for the final formulas below will not be displayed in this thesis and can be found in [Satchell, S & Scowcroft, A (2000)]. Master formula for the BL returns, i.e. the combined distributions is (24):

$$\mu_{BL} = [(\tau\Sigma)^{-1} + V^T\Omega^{-1}V]^{-1}[(\tau\Sigma)^{-1}\Pi + V^T\Omega^{-1}Q] \quad (24)$$

The variance of the returns is expressed as (25):

$$\Sigma_{BL} = (1 + \tau)\Sigma - \tau^2\Sigma V^T(\tau V\Sigma V^T + \Omega)^{-1}V\Sigma \quad (25)$$

The BL weights can be found by inserting these parameters in (16) given no constraints on the model. With constraints such as (26) and (27) and an optimization tool must be used such as `fmincon` in MATLAB. (27) means that short positions are not allowed in the portfolio [He, G. and Litterman, R. (1999)].

$$\sum w_{BL} = 1 \quad (26)$$

$$w_{BL_k} \geq 0 \quad (27)$$

In figure 2 a perspicuous view of the Black-Litterman model is laid out [Idzorek, Thomas M. (2005)].

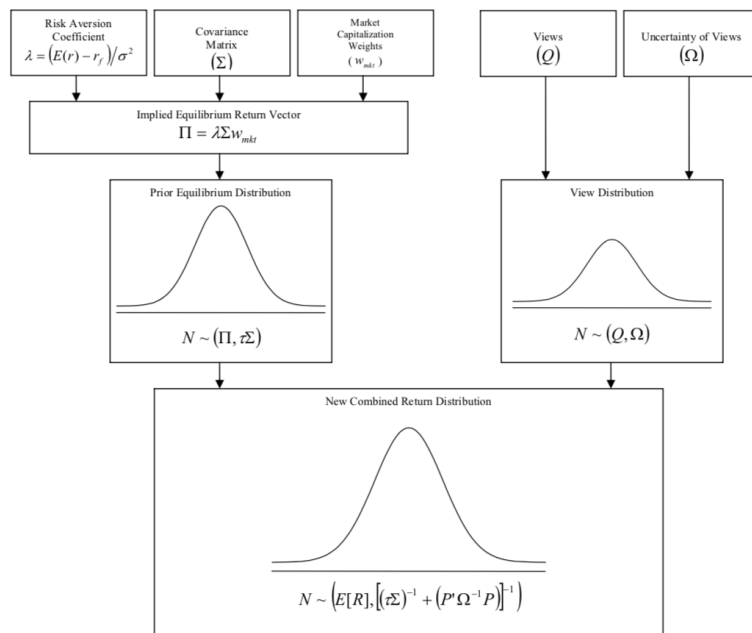


Figure 2: Black-Litterman model derivation visualized [Idzorek, Thomas M. (2005)]

### 3.3 Using the Markov switching results in the Black-Litterman model

The Black-Litterman model can be described as an advanced weighted average where the mean-variance market equilibrium portfolio is influenced by a view portfolio. It is very important to notice that the degree of influence is depending on the credibility of the views. A higher uncertainty factor of a view, in this case variance, will shift the final result closer to the neutral market equilibrium portfolio. In order to build a practical and accurate Black-Litterman model the chosen method of modeling the uncertainties must generate relevant information in a quick and comprehensible way. This thesis is exploring a model which uses a Markov switching framework to generate both the  $Q$  matrix and the  $\Omega$  matrix. More specifically, the  $Q$  matrix will be built using conditional mean data and the  $\Omega$  matrix will be calculated from the conditional covariance data. The probability of staying in the regime will be used as an additional measure of uncertainty to the covariance matrix. More details on how this is implemented will follow in the implementation section.

## 4 Implementation

In this section the combined Markov switching and Black-Litterman portfolio allocation model will be implemented on a set of historical data points in order to test the model.

### 4.1 Model settings

#### Markov Switching model settings

The MATLAB package MS\_Regress, created by Marcelo Perlin, is used to simulate and estimate the regimes [Perlin, M. (2015)]. Different ways of optimizing are available within the package, but following the theory section above, the optimization is performed with second partial derivatives of the log likelihood function. This is also known by the Hessian matrix. A multivariate analysis is being used and the distribution used in the maximum likelihood estimation is Gaussian.

#### Black-Litterman model settings

The BL model in this thesis is built from scratch and is adaptable to various types of portfolios with different constraints and allocation settings. The following setup is used for the results and analysis presented below.

- The model will use data for 10 assets during a test period of 800 days of exchange.
- The data will be divided into four different allocation periods reallocating the portfolio every 200:th day. Each time period will be denoted  $t_m$  with  $m = 1,2,3,4$ .
- The conditional mean output from the Markov switching model will be used as absolute views.
- The conditional standard deviation output from the Markov Switching model will be used as uncertainties for the views.
- $\tau$  is set to 0.025 in accordance with both Lee (2000) and the assumptions from Black and Litterman saying it should be set close to zero. This way, it is set according to a generally accepted standard.
- Only long positions are allowed.
- In order to find the sought Black-Litterman portfolio weights (24) and (25) are used in the MATLAB optimization function "fmincon" in order to optimize the Sharpe ratio in (28) where  $w_{BL}$  is the portfolio weight vector we are trying to find.

$$\frac{w_{BL} * \mu_{BL}}{\sqrt{w_{BL}' * \Sigma_{BL} * w_{BL}}} \quad (28)$$

- A market weighted portfolio is used as a reference portfolio. The same asset universe is used as in the Black-Litterman allocation.

## **Assets**

From a theoretical perspective it would be interesting to plug in all assets available to invest in and run the model. This is not realistic and since the Markov switching part of the model is particularly demanding from a time perspective only a small number of assets can be chosen. To be able to spot trends in the data, which is the main objective using the Markov switching function, assets which are susceptible to shifts in the economy need to be chosen. There are several different asset classes matching these criteria, but in this thesis the scope is narrowed down to stocks. From the other part of the spectrum, the assets chosen can't exhibit too much variance since the trends will be very difficult to distinguish with a regime switching model. Large cap stocks will hence be the primary target for analysis here and more specifically, in order for the computer to cope with the data, the portfolio will be restricted to OMXS30 stocks only.

## **Time frame**

One restriction of this framework is that the Markov switching model can't handle extreme shifts in the data series, even if it happens only once. This means that i.e. a 40% fall in stock price over a night will mess with the maximum likelihood estimation. More specifically the optimization function will suggest solutions outside the boundaries set by the constraints and the result will be non relevant. This will also cause the model to work extremely slowly. Consequently, the data must exhibit non-extreme behaviour and stocks that don't are neglected from the allocation.

## **Market data**

Market cap data must be collected for every allocation date. This is a cumbersome process since such historical data doesn't exist for Swedish stocks. The market cap for every date and company must be calculated manually by finding the amount of outstanding shares in quarterly reports and dates for emissions, splits and buybacks. If implementing this in real time with data including the present date, this is not a problem since today's number of outstanding shares is an accessible piece of information. In this case of back testing, stocks without clear information about the number of outstanding shares will be neglected.

## **The portfolio**

The portfolio chosen for the allocation is shown in figure 3. As mentioned, the actual returns of the portfolio are shown in figure 4. This data is analyzed in periods of 200 days at a time resulting in 4 different portfolio allocations. The daily closing prices displayed in figure 4 are transformed into log returns according to (29) for the upcoming analysis [Nasdaq OMX Nordic]. Since the Markov switching model uses information from 200 days prior to the allocation date, 1000 data points needed to be collected for the four allocations.



Name of asset
ABB Ltd
Alfa Laval
Autoliv SDB
Assa Abloy B
Atlas Copco A
Atlas Copco B
AstraZeneca
Boliden
Electrolux B
Ericsson B

Figure 3: List of stocks analyzed

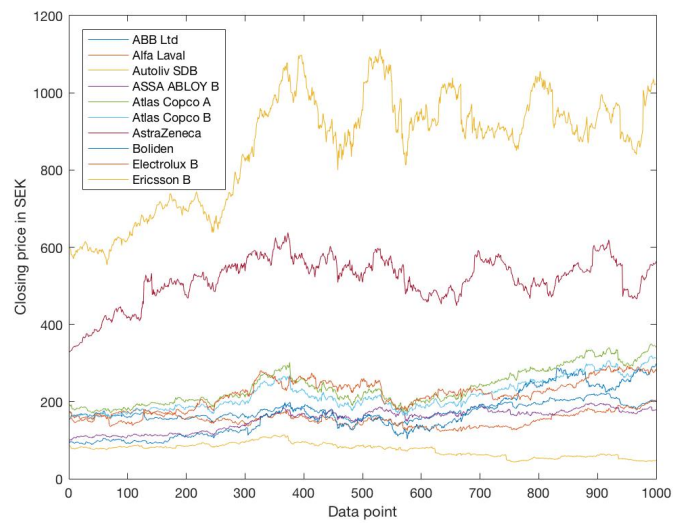


Figure 4: Portfolio returns [Nasdaq OMX Nordic]

$$r_{today} = \ln \left( \frac{Price_{today}}{Price_{yesterday}} \right) \quad (29)$$

The covariance matrices for each period are extensively used in the model and are shown in the Appendix.

## 4.2 Results from Markov switching model

In figures 5, 6, 7 and 8 the output from the Markov switching model is shown for all the four allocation periods. It is clear that there are different types of trends in the periods, but all periods have in common that they are dominated by a low volatility state, colored blue in the model.

In the third graph from the top the smoothing probability is displayed. The smoothing probability is using all the data within the sample, which means all the 200 data points are used regardless which point in time is analyzed. The filter probability, not used in this case, includes the data in the sample up to the point of analysis, in this case a maximum of 200 data points for each period.

The middle graph in each figure shows the conditional standard deviations for each asset and the top graph shows the raw log returns.

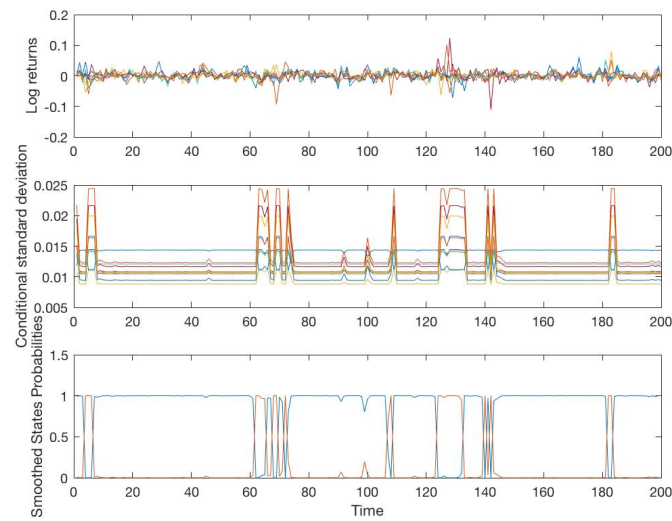


Figure 5: Regime statistics  $t_1$  showing smoothing probabilities, conditional standard deviation and raw log returns

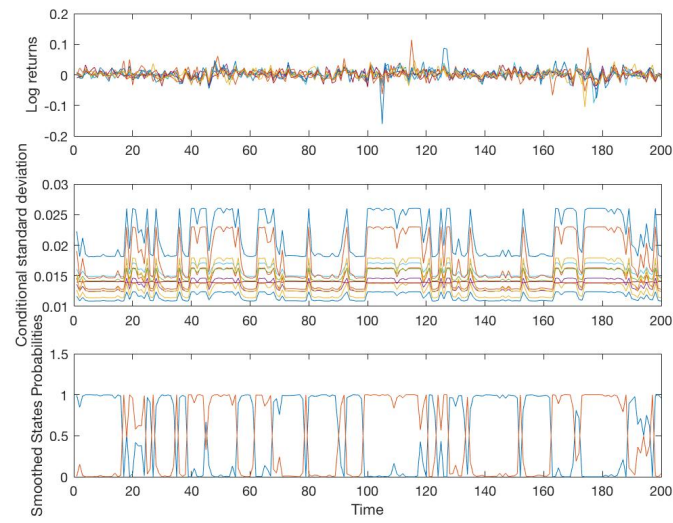


Figure 6: Regime statistics  $t_2$  showing smoothing probabilities, conditional standard deviation and raw log returns

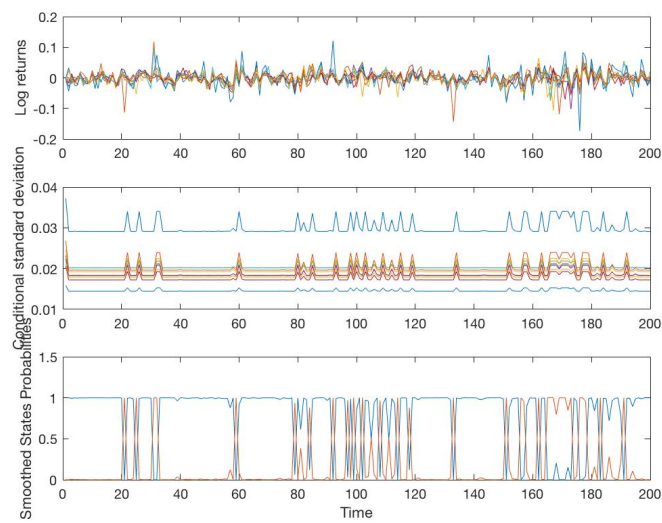


Figure 7: Regime statistics  $t_3$  showing smoothing probabilities, conditional standard deviation and raw log returns

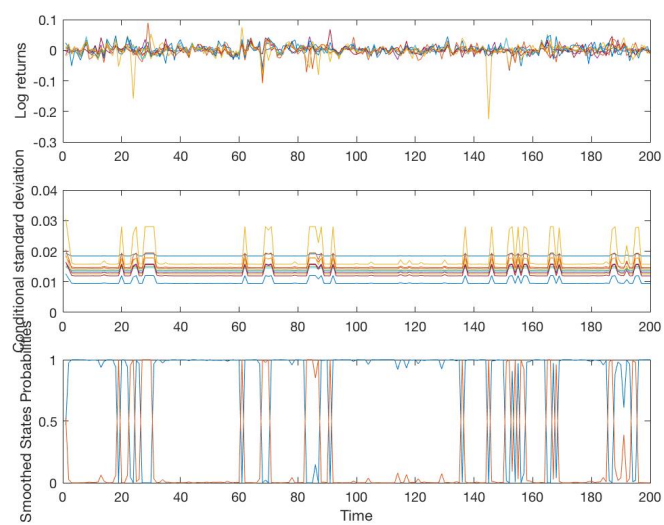


Figure 8: Regime statistics  $t_4$  showing smoothing probabilities, conditional standard deviation and raw log returns

### 4.3 The regime characteristics

The top graphs displaying the raw log returns in figures 5, 6, 7 and 8 are useful when analyzing the regime characteristics. The two regimes can be associated with certain behaviours in the return series and this can clearly be seen comparing the log returns to the state probabilities. One regime, blue color in the figure, is dominant when the returns exhibit stable behaviour and one is dominant when there are big shifts in the market. The shifts can be either positive or negative, it doesn't effect which state the model currently finds dominating. This model seems to identify high variance and low variance regimes and comparing with the log returns it seems like the high variance periods occur in combination with lower returns. This indicates a bear market state, in this case represented by the red probability graph. The bull market state is represented by the blue graph. This clear separation between different market behaviours might not always be the case though. Especially in the case of a model with three regimes or more, each state interpretation can be much less intuitive.

In the middle graphs it is clear that the model separates the assets within the portfolio and assigns different conditional standard deviations for each asset.

#### 4.4 Data results

In order to combine the Markov switching model with the Black-Litterman model the data in (30), table 1, table 2 and (31), was extracted from the regime switching model.  $t_m$  where  $m = (1, 2, 3, 4)$  is denoting which allocation period that is currently being analyzed.

- (30) shows the conditional covariance matrices for period 1, state 1 and state 2. Six more matrices are used for the other periods and states, but they are not displayed here. These are later used to construct the  $\Omega$  matrix.
- Tables 1 and 2 show the mean vectors for each state and each allocation period. These will be used to calculate the absolute views in the Q matrix in the Black-Litterman model.
- (31) shows the transition probability matrices for switching and these will also be used in determining the uncertainties modeled represented by the  $\Omega$  matrix.

$$\Sigma_{S_1 t_1} =$$

$$1e - 3 * \begin{pmatrix} 0.08 & 0.04 & 0.03 & 0.05 & 0.05 & 0.05 & 0.01 & 0.07 & 0.02 & 0.03 \\ 0.04 & 0.11 & 0.03 & 0.05 & 0.07 & 0.08 & 0.01 & 0.07 & 0.05 & 0.03 \\ 0.03 & 0.03 & 0.07 & 0.03 & 0.03 & 0.03 & 0.02 & 0.03 & 0.00 & 0.03 \\ 0.05 & 0.05 & 0.03 & 0.10 & 0.05 & 0.06 & 0.02 & 0.07 & 0.03 & 0.03 \\ 0.05 & 0.07 & 0.03 & 0.05 & 0.11 & 0.12 & 0.00 & 0.08 & 0.05 & 0.04 \\ 0.05 & 0.08 & 0.03 & 0.06 & 0.12 & 0.14 & 0.00 & 0.09 & 0.06 & 0.05 \\ 0.01 & 0.01 & 0.02 & 0.02 & 0.00 & 0.00 & 0.12 & 0.01 & -0.01 & 0.01 \\ 0.07 & 0.07 & 0.03 & 0.07 & 0.08 & 0.09 & 0.01 & 0.22 & 0.06 & 0.04 \\ 0.02 & 0.05 & 0.00 & 0.03 & 0.05 & 0.06 & -0.01 & 0.06 & 0.13 & 0.02 \\ 0.03 & 0.03 & 0.03 & 0.03 & 0.04 & 0.05 & 0.01 & 0.04 & 0.02 & 0.09 \end{pmatrix}$$

$$\Sigma_{S_2 t_1} = \tag{30}$$

$$1e - 3 * \begin{pmatrix} 0.45 & 0.14 & 0.04 & -0.07 & 0.10 & 0.14 & 0.03 & 0.03 & 0.02 & 0.01 \\ 0.14 & 0.40 & 0.10 & 0.06 & 0.09 & 0.09 & -0.01 & 0.13 & 0.07 & 0.06 \\ 0.04 & 0.10 & 0.16 & 0.06 & 0.06 & 0.05 & 0.01 & 0.04 & 0.00 & 0.01 \\ -0.07 & 0.06 & 0.06 & 0.29 & 0.08 & 0.03 & 0.05 & 0.06 & 0.05 & 0.02 \\ 0.10 & 0.09 & 0.06 & 0.08 & 0.27 & 0.25 & 0.01 & 0.07 & 0.07 & 0.09 \\ 0.14 & 0.09 & 0.05 & 0.03 & 0.25 & 0.25 & 0.01 & 0.06 & 0.07 & 0.05 \\ 0.03 & -0.01 & 0.01 & 0.05 & 0.01 & 0.01 & 0.79 & 0.05 & 0.01 & 0.03 \\ 0.03 & 0.13 & 0.04 & 0.06 & 0.07 & 0.06 & 0.05 & 0.08 & 0.09 & 0.00 \\ 0.02 & 0.07 & 0.00 & 0.05 & 0.07 & 0.07 & 0.01 & 0.09 & 1.05 & 0.18 \\ 0.01 & 0.06 & 0.01 & 0.02 & 0.09 & 0.05 & 0.03 & 0.00 & 0.18 & 0.68 \end{pmatrix}$$

	$\mu_{S1_{t_1}}$	$\mu_{S1_{t_2}}$	$\mu_{S1_{t_3}}$	$\mu_{S1_{t_4}}$
<b>ABB Ltd</b>	0.0005	0.0008	-0.0017	0.0010
<b>Alfa Laval</b>	0.0001	0.0003	-0.0013	0.0009
<b>Autoliv SDB</b>	0.0007	0.0034	0.0002	0.0008
<b>ASSA ABLOY B</b>	0.0003	0.0020	-0.0000	0.0005
<b>Atlas Copco A</b>	0.0003	0.0011	-0.0018	0.0018
<b>Atlas Copco B</b>	0.0003	0.0011	-0.0010	0.0016
<b>AstraZeneca</b>	0.0015	0.0005	-0.0004	0.0002
<b>Boliden</b>	0.0007	-0.0005	-0.0016	0.0012
<b>Electrolux B</b>	-0.0000	0.0001	0.0001	0.0003
<b>Ericsson B</b>	-0.0001	0.0007	-0.0005	0.0008

Table 1: Mean vector for state 1 for every allocation

	$\mu_{S2_{t_1}}$	$\mu_{S2_{t_2}}$	$\mu_{S2_{t_3}}$	$\mu_{S2_{t_4}}$
<b>ABB Ltd</b>	-0.0003	-0.0008	0.0007	0.0002
<b>Alfa Laval</b>	-0.0000	-0.0002	0.0004	0.0011
<b>Autoliv SDB</b>	-0.0003	-0.0008	-0.0008	-0.0007
<b>ASSA ABLOY B</b>	0.0017	-0.0000	-0.0000	0.0005
<b>Atlas Copco A</b>	-0.0002	-0.0014	0.0005	-0.0019
<b>Atlas Copco B</b>	-0.0001	-0.0017	-0.0016	-0.0016
<b>AstraZeneca</b>	-0.0019	-0.0003	0.0023	-0.0002
<b>Boliden</b>	-0.0016	0.0022	0.0054	0.0073
<b>Electrolux B</b>	-0.0001	0.0009	-0.0001	-0.0027
<b>Ericsson B</b>	-0.0000	-0.0011	-0.0011	-0.0048

Table 2: Mean vector for state 2 for every allocation



$$\begin{aligned} P_{t_1} &= \begin{bmatrix} 0.9498 & 0.3749 \\ 0.0502 & 0.6251 \end{bmatrix} P_{t_2} = \begin{bmatrix} 0.8219 & 0.2126 \\ 0.1781 & 0.7874 \end{bmatrix} \\ P_{t_3} &= \begin{bmatrix} 0.8543 & 0.6408 \\ 0.1457 & 0.3592 \end{bmatrix} P_{t_4} = \begin{bmatrix} 0.8994 & 0.5657 \\ 0.1006 & 0.4343 \end{bmatrix} \end{aligned} \tag{31}$$

#### 4.4.1 Modeling the views

The Markov switching model distinguishes what state we are currently in, which could be interpreted as a view for the assets included. The mean and uncertainty used in each view will be picked from the Markov switching data output used in the Black-Litterman model according to the reasoning below.

The Markov switching model estimates two return vectors, one for each state. In the same way it estimates two covariance matrices. Using the smoothing probability, it also estimates the transition probability matrix  $P$  for the underlying Markov chain describing the switching between the states. The idea in this model is to calculate weighted averages depending on the state probabilities for the mean vectors and covariance matrices. The  $P$  matrix (3) is available in the Markov switching model output and is a useful tool in order to assign weights for the calculation of  $\mu_{MS_{average}}$  and  $\Sigma_{MS_{average}}$ .

$$\begin{aligned}
\mu &= [\mu_{S_1} \quad \mu_{S_2}] \\
\mu_i &= E[r_k | X_k = i] \\
\Sigma_i &= Var(r_k | X_k = i) \\
Y_n &= \sum_{k=0}^n \ln\left(\frac{S_{k+1}}{S_k}\right) = \sum_{k=1}^n r_k \\
\tilde{X}_k &= \begin{cases} [1 & 0]^T & X_k = 1 \\ [0 & 1]^T & X_k = 2 \end{cases} \\
P_k &= [P(X_k = 1) \quad P(X_k = 2)] \\
\mu_{MS_{average}} &= E[Y_n] = \mu \sum_{k=1}^n E[\tilde{X}_k] = \mu \sum_{k=1}^n P_k^T = \mu \sum_{k=1}^n (P^T)^k P_0^T \\
&\approx n * \mu * \pi_{stat}^T \quad \text{if stationary distribution } \pi_{stat} \text{ is known}
\end{aligned} \tag{32}$$

A stationary distribution exists if the markov chain is ergodic. Looking at the properties of the  $P$  matrix for each time period  $t_m$  in (31) they are clearly representing ergodic Markov chains. The assumption is based on the properties of the environment where the model is applied, which is the stock market. If the stock market is assumed to have two states, bull and bear, both states will be reoccurring an indefinite amount of times. This means that stationary distributions exist which can be used to decide the probability of being in the different states. This is done by solving (33) or by finding one of the column vectors in (34) which both are equal to the stationary distribution. The last step approximation is allowed since the investments are held for a considerable amount of time and the  $P$  matrix is converging quickly to the stationary distribution  $\pi_{stat}$ .

$$\pi_{stat} = \pi_{stat}P \quad (33)$$

$$\lim_{k \rightarrow \infty} P^k \quad (34)$$

In this case (34) will be approximated by  $P^{1000}$ . The approximated results for the stationary distributions are shown in (35).

$$\pi_{stat_{t_1}} = \begin{bmatrix} 0.8819 \\ 0.1181 \end{bmatrix} \pi_{stat_{t_2}} = \begin{bmatrix} 0.5442 \\ 0.4558 \end{bmatrix} \quad (35)$$

$$\pi_{stat_{t_3}} = \begin{bmatrix} 0.8148 \\ 0.1852 \end{bmatrix} \pi_{stat_{t_4}} = \begin{bmatrix} 0.8490 \\ 0.1510 \end{bmatrix}$$

The predicted probability of the portfolio being in state  $k$  after period  $m$  is (36).

$$\pi_{stat_{t_m}}(k) \quad k = 1, 2 \quad (36)$$

The average weighted return vector is calculated according to (37).

$$\mu_{MS_{average}} = n(\pi_{stat}(1)\mu_{S_1} + \pi_{stat}(2)\mu_{S_2}) \quad (37)$$

The average weighted covariance matrix  $\Sigma_{MS_{average}}$  is calculated according to (38).

$$\begin{aligned}
\Sigma_{MS_{average}} &= Var(Y_n) \\
&= \sum_{k=1}^n P_k(1)\Sigma_1 + P_k(2)\Sigma_2 \\
&+ \mu \left( \sum_{k=1}^n \sum_{l=1}^n cov(\tilde{X}_k, \tilde{X}_l) \right) \mu^T \\
&= \sum_{k=1}^n P_k(1)\Sigma_1 + P_k(2)\Sigma_2 \\
&+ \mu \left( \sum_{k=1}^n \sum_{l=1}^n (P^T)^{(k-l)^+} Var(X_{min(k,l)}) P^{(l-k)^+} \right) \mu^T \\
&= \sum_{k=1}^n P_k(1)\Sigma_1 + P_k(2)\Sigma_2 \\
&+ \mu \left( \sum_{k=1}^n \sum_{i=-n+1}^{n-1} P_k(1)P_k(2) \det(P)^{|i|} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \mu^T \\
&= \sum_{k=1}^n P_k(1)\Sigma_1 + P_k(2)\Sigma_2 \\
&+ \mu \sum_{k=1}^n \left[ 2 \frac{1-\det(P)^n}{1-\det(P)} - 1 \right] P_k(1)P_k(2) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mu^T \\
&\approx n \left[ \pi_{stat}(1)\Sigma_1 + \pi_{stat}(2)\Sigma_2 \right] \\
&+ n \left[ 2 \frac{1-\det(P)^n}{1-\det(P)} - 1 \right] \pi_{stat}(1)\pi_{stat}(2)(\mu_{S_1} - \mu_{S_2})(\mu_{S_1} - \mu_{S_2})^T
\end{aligned} \tag{38}$$

The second row of the final formula in calculation (38) is a contribution to the variance coming from the switching. If the return vectors from the different states would have been the same, no states would be defined and the variance contribution from the switching would be zero. This is clearly seen in the formula.

There are several different methods of building the  $\Omega$  matrix from which one is explained in (20) in the theory section above. The method used in this thesis is a variant of (20) where the end formula in (38) is inserted instead of  $\Sigma$ . This basically means that the uncertainty of each view is represented by the variance for each asset implied by the Markov switching model.

## 4.5 Checking Regime switching model validity

To check the validity of the regime switching model the log returns  $r_k$  are normalized using the variance structure according to (39). Here, the returns for period 4 are tested.  $\tilde{r}_k$  represents the normalized log returns for the period and  $\Sigma_{t_4}$  represents the average weighted covariance matrix for the period.

$$\tilde{r}_k = \sqrt{\Sigma_{t_4}}^{-1}(r_k - \mu_{t_4}) \quad (39)$$

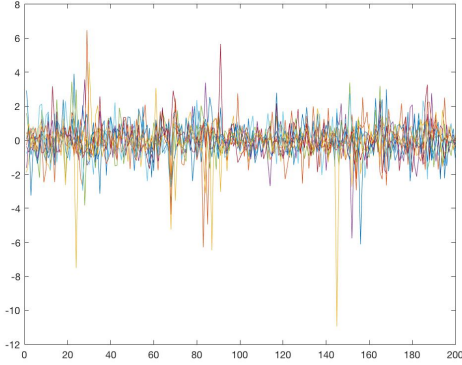


Figure 9: Normalized returns

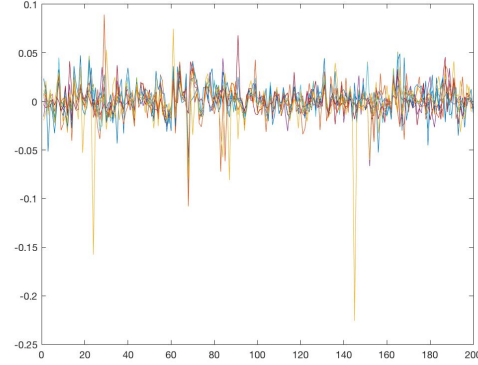


Figure 10: Raw return data

It is clear that the returns in figure 9 exhibits a behaviour more likely to come from a normal distribution compared to the raw data in figure 10. Returns from a normal distribution would exhibit no trends at all and no heteroscedascity. But it is definitely not a perfect normally distributed log return series, which indicates that the Markov switching model is not capturing all the trends that can be found in the data.

It should be mentioned that are a few large absolute returns spotted during certain parts of the period in figure 10, which would be extremely unlikely to happen under a normal distribution. There is without doubt no normally distributed model that can replicate a return series like this to perfection.

## 4.6 Results from Black-Litterman model

This section contains the main result of the thesis. It is partly a comparison between the Markov switching based Black-Litterman portfolio and a reference portfolio which is calculated from the market weight portfolio without views.

The absolute views are taken from the mean vectors in figures 1 and 2. The Q vector for each period is shown in figure 11.

	$Q_{t_1}$	$Q_{t_2}$	$Q_{t_3}$	$Q_{t_4}$
<b>ABB Ltd</b>	0.0004	0.0001	-0.0013	0.0009
<b>Alfa Laval</b>	0.0001	0.0001	-0.0010	0.0010
<b>Autoliv SDB</b>	0.0005	0.0014	0.0000	0.0005
<b>ASSA ABLOY B</b>	0.0005	0.0011	-0.0000	0.0005
<b>Atlas Copco A</b>	0.0003	0.0000	-0.0013	0.0013
<b>Atlas Copco B</b>	0.0003	-0.0002	-0.0011	0.0011
<b>AstraZeneca</b>	0.0011	0.0002	0.0001	0.0001
<b>Boliden</b>	0.0004	0.0007	-0.0003	0.0021
<b>Electrolux B</b>	-0.0000	0.0004	0.0001	-0.0002
<b>Ericsson B</b>	-0.0001	-0.0001	-0.0006	-0.0001

Figure 11: Absolute views for every time period

Results for the allocation in time period 1 are shown in figure 3.

	$PI_{t_1}$	$\mu_{BL_{t_1}}$	$w_{mkt_{t_1}}$	$w_{BL_{t_1}}$
<b>ABB Ltd</b>	0.000042	0.000072	0.201791	0.167054
<b>Alfa Laval</b>	0.000036	0.000062	0.037251	0.022406
<b>Autoliv SDB</b>	0.000026	0.000050	0.035972	0.097258
<b>ASSA ABLOY B</b>	0.000041	0.000076	0.067146	0.105849
<b>Atlas Copco A</b>	0.000048	0.000084	0.096229	0.086867
<b>Atlas Copco B</b>	0.000054	0.000095	0.040804	0.048357
<b>AstraZeneca</b>	0.000081	0.000147	0.335429	0.344750
<b>Boliden</b>	0.000047	0.000088	0.016871	0.060576
<b>Electrolux B</b>	0.000031	0.000045	0.027198	0.007091
<b>Ericsson B</b>	0.000039	0.000054	0.141309	0.059793

Table 3: Results from time period 1

The Black-Litterman portfolio weights for each period are displayed in figures 12, 13, 14 and 15.

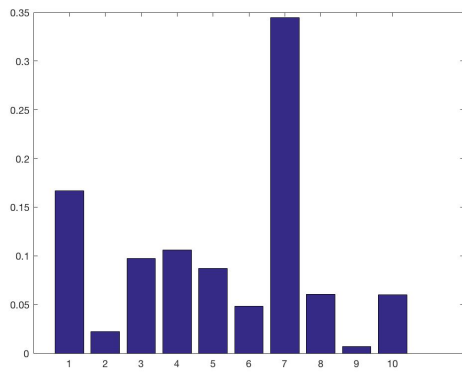


Figure 12: Weights time period 1

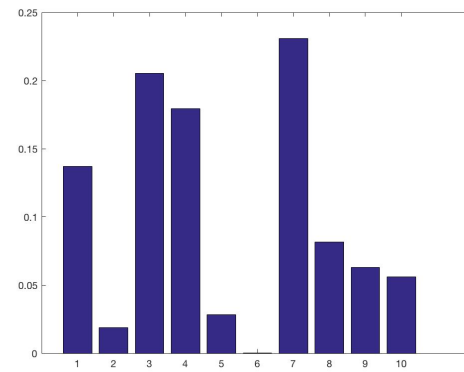


Figure 13: Weights time period 2

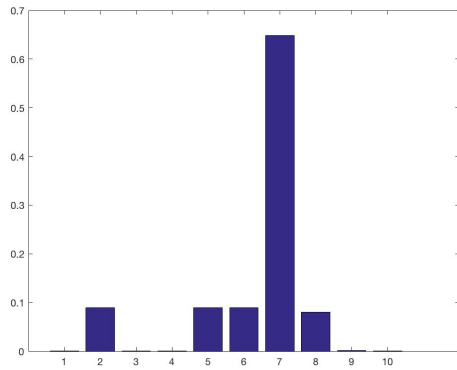


Figure 14: Weights time period 3

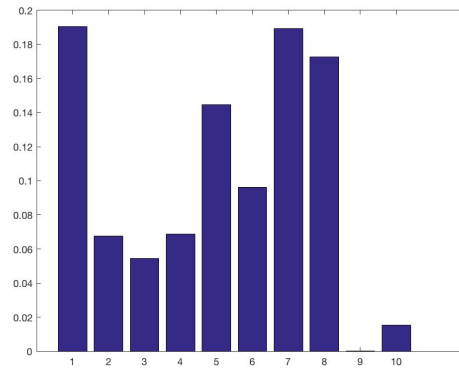


Figure 15: Weights time period 4

In figure 16 the final results are shown comparing the portfolio value for the Black-Litterman portfolio in blue and the market portfolio in red over the 800 days long test period. The Black-Litterman portfolio is outperforming the market weighted portfolio by 20% over the entire period.

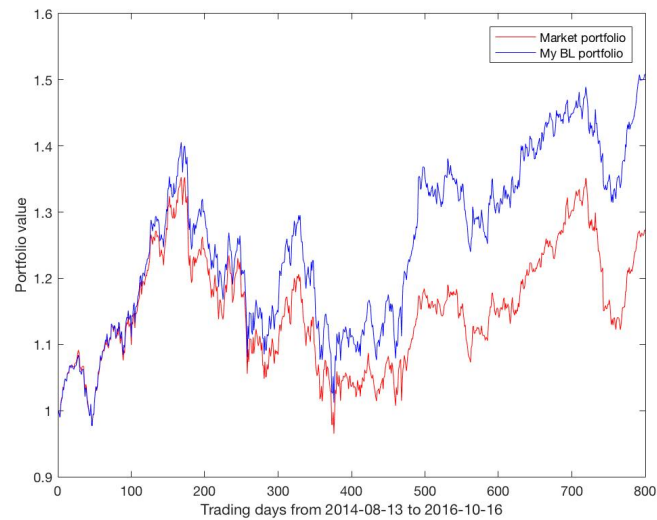


Figure 16: Portfolio comparison for 800 trading days reaching from 2014-08-13 to 2016-10-16



## 4.7 Analysis of results

### Sharpe ratio

The Sharpe ratio was calculated for every reallocation in order to compare the risk and return profiles of the Black-Litterman and the market weighted portfolios. The formula for calculating the ratio in this case is (40) with a scaling factor of  $\sqrt{252}$  since the data is collected daily and we want an annual ratio [Lo, Andrew W. (2002)].  $E[r]$  and  $\sigma_r$  are derived from the historical raw log returns. The results are shown in table 4.

$$\text{Sharpe ratio} = \frac{E[r] - r_f}{\sigma_r / \sqrt{252}} \quad (40)$$

Date	BL	Market
13/08/14	1.7772	1.4904
03/06/15	-0.7632	-0.9474
17/03/16	0.9970	0.5619
30/12/16	1.3853	0.9407

Table 4: Sharpe ratio at the four different allocation dates

### Consistency of the results

The outperforming of the market weighted portfolio in terms of absolute returns is a positive result. However the consistency of the result needs to be analyzed. The difference between the Black-Litterman and the market weighted portfolio returns is plotted and tested in a one-sided hypothesis test. Since the sample is large, a Z-test is used.

The hypothesis test is formulated according to (41).

$$\begin{aligned} H_0 : \mu &= 0 \\ H_1 : \mu &> 0 \end{aligned} \tag{41}$$

The Z-score is calculated according to (42) and equals 1.902.

$$Z = \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \tag{42}$$

The critical value for a one-tailed z-test with a significance level of 97% is  $\approx 1.89$ , which is smaller than our calculated Z-score. This means that the null hypothesis is rejected at the significance level of 97%.

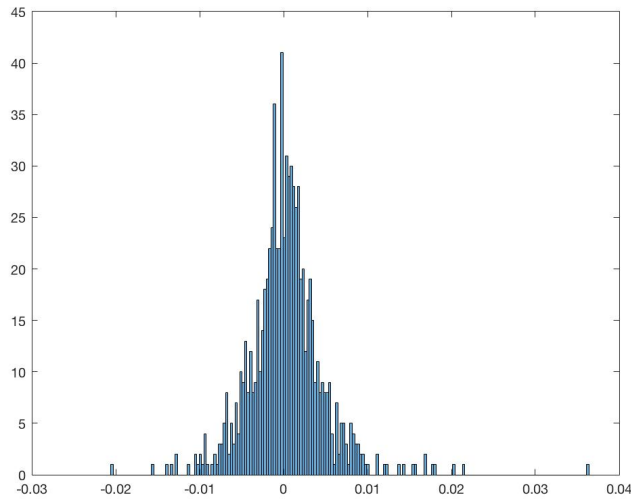


Figure 17: Distribution of difference between portfolio returns

## 5 Discussion

### 5.1 Discussion of results

As can be seen in figure 4 the Sharpe ratio for the Black-Litterman portfolio is higher in all four allocation cases. A higher Sharpe ratio indicates a better risk adjusted portfolio. This means that our model seems to be performing in terms of diversifying the risk. It is difficult to assess the credibility and performance of the model since only 4 Sharpe ratios could be calculated.

A requirement for the hypothesis test to be relevant is that the return differences are normally distributed, which they are probably not. The test should not be considered a strong proof of a functional investment model, but merely an indication that the positive results are reflecting the true model performance. The out-liars are not improving the credibility.

Consequently, the Sharpe ratios in combination with the hypothesis test indicate that the model is allocating a risk diversified portfolio which performs better than the market weighted portfolio.

### 5.2 Discussion of model settings and data selection

The constant  $\tau$ , which is a scaling factor closely related to the degree of view influence, was chosen early on in the testing process. While experimenting with the value, it was clear that a value too close to 0 would suppress the switching part of the model too much. A value closer to 1 generated portfolio weights that were influenced by the views to such a degree that extremely concentrated portfolios appeared. Setting such a value would obviously mean to disregard the main idea of the Black-Litterman approach. Consequently, the value 0,025 was chosen according to generally accepted standards.

There are a couple of reasons for using the range of 200 trading days for every re-allocation. Firstly, I believe that older data is less likely to contribute with relevant information about the current market trends. Secondly, a shorter time series wouldn't contain enough information for the Markov switching model to clearly distinguish market trends. Finally, the model package used in this project is unfortunately not appropriate for handling larger data sets.

The ten OMXS30 stocks in the portfolio were selected in alphabetical order from the OMXS30 list. This way, no intended selections of specifically high or low performing stocks were made and the stocks chosen were all extensively traded in order to reflect enough information about potential market trends.

## 6 Conclusion

In this master thesis a Markov switching framework has been applied to model the views in a Black-Litterman portfolio allocation model.

Firstly the Markov switching framework was established in order to calculate the parameters needed for the Black-Litterman allocation. The model parameters were tested and the results for for period 4 are visualized in figures 9 and 10. The Markov switching model does clearly not replicate the time series to perfection and this is indisputably a source of uncertainty for the entire investment model. Only the normal distribution assumption was tested for a couple of reasons. Firstly, I wanted to stick with the theoretical concept of normally distributed financial returns. Secondly, the Markov switching algorithm was working faster with the normal distribution assumption. Finally, the return series is clearly not originating from any known distribution type, which reduces the chances of improving the results by changing distribution. With that said, the model should still not be discarded since it is sufficient to identify a few distinct trends in order to increase the chances of allocating a high performing portfolio.

The Black-Litterman model was implemented with the mean and variance parameters from the Markov switching model used as views. The replacement of subjective investor views with information extracted from historical data was done in a very efficient way. However, the original Black-Litterman concept is entirely dependant on subjective views, which could raise the question whether a method of generating views from market data will enhance the performance or not. The argument for choosing a regime switching model is that certain market trends and situations can be quantified and applied in portfolio allocation very easily. These trends are assumed to be correlated to current investor opinions and market views. Conclusively, by recognizing the trends in the data, the investor views are still taken into consideration. It is definitely done in a less direct way, but it is technically easier to implement which makes it interesting.

The results were compared against a market weighted portfolio of the same assets. The Black-Litterman portfolio outperformed the benchmark portfolio by 20% over a period of 800 trading days. The result is considered positive.

Concluding the implementation phase of this thesis, the two models are compatible as the Markov switching model is generating data that is useful to create relevant views for the Black-Litterman model. Managing the Black-Litterman algorithms with parameters extracted from the Markov switching model is manageable, but generating those parameters takes time and computer power. That is the current bottleneck of the combined model.

As a final conclusion of the model performance, the Sharpe ratios in combination with the hypothesis test indicate that the model is allocating a risk diversified portfolio which performs better than the market weighted portfolio. More tests are required in order to

certify the model.

## **6.1 Improvement areas and further research**

As clarified earlier in the implementation section, a MATLAB package created by Marcelo Perlin was used to find the Markov switching parameters. This package was slightly modified in order to handle the format of the time series, but the computational algorithms remained untouched. The package had trouble handling large data sets, which was the main reason for the limited amount of tests. To program a Markov switching routine from scratch was outside the scope of the project this time, but it would be an interesting area to explore further. In order to improve the results of the model, other distributions than the normal distribution should be tested and the code needs to be reprogrammed accordingly.

Non-volatile securities such as obligations were not incorporated in the model and a step further would be to modify the model for the purpose of handling such assets. Since the model is adapted to optimize portfolios for risky assets, this would pose a challenge. Another way of improving the model would be to include trading with foreign securities. Foreign exchange costs would then be necessary to include and potentially an application of the Markov switching framework in order to track trends.

In order to fully capture the value of this investing model the transaction costs should be included for each reallocation. This is definitely possible to achieve, but was left outside the scope. The challenge is to find the right transaction cost information for each asset, potentially asset classes, and download it in an effective way. Integrating it in the Black-Litterman model would be a relatively easy task.

## References

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## 7 Popular science

### Portfolio allocation using short term predictions of business cycles

The Black-Litterman model is an asset allocation model developed by Fischer Black and Robert Litterman in the early 90's. It was published in the internal Goldman Sachs Fixed Income Research Note, Black and Litterman (1990). The model is an advanced Mean Variance Optimization framework, with an objective to maximize the return of a portfolio in relation to its risk. The primary reason for using the Black-Litterman model instead of a traditional Mean Variance Optimization model is to overcome problems such as unintuitive, highly concentrated portfolios, input sensitivity, and estimation error maximization. This is achieved by incorporating subjective investor opinions, called views, in the model, which usually works fine if the views are appropriately formulated. The problem with this concept is that it is not very easy to achieve consistency and operational efficiency when specifying the views. This thesis examines a model using historical data to find trends in the business cycle as a tool to extract the views. The model used is a regime switching model and more specifically a Markov switching model.

There are two reasons for choosing a model using historical data when creating views. Firstly, it is less time consuming to implement a model that automatically turns public information into views. Secondly, it formulates every view from a precise framework and is therefore more consistent in its allocation. However, the Black-Litterman model is successful because of its ability to use additional information not found in the historical exchange databases. This raises the question whether a method that is generating views from market data will enhance the performance or not. Someone once said investing this way is like "driving a car looking in the rear mirror". The response to people distrusting the concept is that a regime switching model can spot trends which are assumed to be related to market consensus and indirectly to the investor opinions.

In financial modelling a portfolio return series can traditionally be assumed to have a constant mean and a constant variance over a certain period of time. A two state Markov switching model instead calculates the probability for the portfolio to be in certain states during different stages of the same period. The model does this by assuming that the states can be described by a hidden Markov chain. The parameters of the hidden Markov chain cannot be observed and must be estimated from the market data. Each state has different statistical parameters, such as mean and variance. A high variance combined with a low mean could indicate that bear market conditions are dominating and vice versa could indicate a bull market. This market insight will be used to assign views.

In this thesis a model combining the Markov switching and Black-Litterman models is used to repeatedly reallocate a portfolio with 10 stocks listed on the OMXS30 during the period August 2014 to October 2017. Four reallocations are made and the result is compared to a market weighted portfolio. A positive conclusion is that the two frameworks are practically compatible and that the new portfolio outperforms the traditional



market portfolio in terms of absolute return and Sharpe ratio. The challenge lies in refining the Markov Switching model in order to let it handle larger data sets. It would be beneficial to test the entire model with different market conditions and other assets before it can be considered a reliable investment tool.

## 8 Appendix

The covariance matrices for the raw log returns during each period are displayed below.

$$\Sigma_{t_1} =$$

$$10^{-3} * \begin{pmatrix} 0.13 & 0.07 & 0.03 & 0.04 & 0.06 & 0.07 & 0.03 & 0.07 & 0.03 & 0.02 \\ 0.07 & 0.17 & 0.05 & 0.07 & 0.09 & 0.11 & -0.00 & 0.09 & 0.07 & 0.05 \\ 0.03 & 0.05 & 0.09 & 0.04 & 0.04 & 0.05 & 0.02 & 0.04 & 0.00 & 0.04 \\ 0.04 & 0.07 & 0.04 & 0.13 & 0.07 & 0.08 & 0.04 & 0.08 & 0.05 & 0.04 \\ 0.06 & 0.09 & 0.04 & 0.07 & 0.16 & 0.17 & 0.02 & 0.10 & 0.07 & 0.06 \\ 0.07 & 0.11 & 0.05 & 0.08 & 0.17 & 0.20 & 0.03 & 0.11 & 0.08 & 0.06 \\ 0.03 & -0.00 & 0.02 & 0.04 & 0.02 & 0.03 & 0.28 & 0.03 & 0.01 & 0.02 \\ 0.07 & 0.09 & 0.04 & 0.08 & 0.10 & 0.11 & 0.03 & 0.22 & 0.08 & 0.05 \\ 0.03 & 0.07 & 0.00 & 0.05 & 0.07 & 0.08 & 0.01 & 0.08 & 0.29 & 0.06 \\ 0.02 & 0.05 & 0.04 & 0.04 & 0.06 & 0.06 & 0.02 & 0.05 & 0.06 & 0.18 \end{pmatrix}$$

$$\Sigma_{t_2} =$$

$$10^{-3} * \begin{pmatrix} 0.13 & 0.11 & 0.07 & 0.10 & 0.11 & 0.12 & 0.08 & 0.15 & 0.08 & 0.09 \\ 0.11 & 0.22 & 0.07 & 0.12 & 0.15 & 0.16 & 0.08 & 0.16 & 0.08 & 0.12 \\ 0.07 & 0.07 & 0.16 & 0.07 & 0.09 & 0.10 & 0.06 & 0.10 & 0.08 & 0.08 \\ 0.10 & 0.12 & 0.07 & 0.18 & 0.15 & 0.15 & 0.10 & 0.18 & 0.11 & 0.11 \\ 0.11 & 0.15 & 0.09 & 0.15 & 0.25 & 0.26 & 0.11 & 0.21 & 0.12 & 0.13 \\ 0.12 & 0.16 & 0.10 & 0.15 & 0.26 & 0.28 & 0.12 & 0.22 & 0.13 & 0.14 \\ 0.08 & 0.08 & 0.06 & 0.10 & 0.11 & 0.12 & 0.20 & 0.12 & 0.07 & 0.11 \\ 0.15 & 0.16 & 0.10 & 0.18 & 0.21 & 0.22 & 0.12 & 0.53 & 0.18 & 0.12 \\ 0.08 & 0.08 & 0.08 & 0.11 & 0.12 & 0.13 & 0.07 & 0.18 & 0.40 & 0.09 \\ 0.09 & 0.12 & 0.08 & 0.11 & 0.13 & 0.14 & 0.11 & 0.12 & 0.09 & 0.24 \end{pmatrix}$$

$$\Sigma_{t_3} =$$

$$10^{-3} * \begin{pmatrix} 0.23 & 0.22 & 0.19 & 0.21 & 0.23 & 0.21 & 0.15 & 0.29 & 0.17 & 0.18 \\ 0.22 & 0.47 & 0.25 & 0.26 & 0.38 & 0.37 & 0.17 & 0.40 & 0.18 & 0.22 \\ 0.19 & 0.25 & 0.46 & 0.25 & 0.27 & 0.25 & 0.20 & 0.34 & 0.21 & 0.21 \\ 0.21 & 0.26 & 0.25 & 0.39 & 0.30 & 0.29 & 0.23 & 0.35 & 0.20 & 0.23 \\ 0.23 & 0.38 & 0.27 & 0.30 & 0.51 & 0.49 & 0.17 & 0.44 & 0.22 & 0.27 \\ 0.21 & 0.37 & 0.25 & 0.29 & 0.49 & 0.48 & 0.17 & 0.43 & 0.21 & 0.26 \\ 0.15 & 0.17 & 0.20 & 0.23 & 0.17 & 0.17 & 0.33 & 0.24 & 0.14 & 0.19 \\ 0.29 & 0.40 & 0.34 & 0.35 & 0.44 & 0.43 & 0.24 & 1.06 & 0.31 & 0.33 \\ 0.17 & 0.18 & 0.21 & 0.20 & 0.22 & 0.21 & 0.14 & 0.31 & 0.53 & 0.19 \\ 0.18 & 0.22 & 0.21 & 0.23 & 0.27 & 0.26 & 0.19 & 0.33 & 0.19 & 0.40 \end{pmatrix}$$

$$\Sigma_{t_4} = 10^{-3} * \begin{pmatrix} 0.12 & 0.11 & 0.08 & 0.07 & 0.11 & 0.10 & 0.05 & 0.09 & 0.09 & 0.09 \\ 0.11 & 0.28 & 0.11 & 0.12 & 0.18 & 0.18 & 0.06 & 0.17 & 0.16 & 0.17 \\ 0.08 & 0.11 & 0.24 & 0.07 & 0.11 & 0.10 & 0.01 & 0.10 & 0.10 & 0.10 \\ 0.07 & 0.12 & 0.07 & 0.21 & 0.13 & 0.13 & 0.07 & 0.12 & 0.13 & 0.11 \\ 0.11 & 0.18 & 0.11 & 0.13 & 0.25 & 0.23 & 0.07 & 0.20 & 0.14 & 0.12 \\ 0.10 & 0.18 & 0.10 & 0.13 & 0.23 & 0.23 & 0.07 & 0.19 & 0.14 & 0.12 \\ 0.05 & 0.06 & 0.01 & 0.07 & 0.07 & 0.07 & 0.18 & 0.04 & 0.06 & 0.09 \\ 0.09 & 0.17 & 0.10 & 0.12 & 0.20 & 0.19 & 0.04 & 0.37 & 0.11 & 0.13 \\ 0.09 & 0.16 & 0.10 & 0.13 & 0.14 & 0.14 & 0.06 & 0.11 & 0.31 & 0.11 \\ 0.09 & 0.17 & 0.10 & 0.11 & 0.12 & 0.12 & 0.09 & 0.13 & 0.11 & 0.67 \end{pmatrix}$$