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Master Thesis

Implementation of a Funds Transfer Pricing model with stochastic interest rates

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Abstract

The subject of *Funds Transfer Pricing* (FTP) is widely known within the banking industry, despite this there is a lack of consensus on how to allocate the costs and benefits to the users and suppliers of liquidity. A common practice in financial institutions, in particular before the financial crisis, was to charge business units a liquidity charge that was based on the average or the historic cost of funds, which did not properly reflect the liquidity risk for each specific business unit. Today practitioners are introducing more rigorous FTP approaches to better allocate the cost/benefit of liquidity arising products to business units[11].

The aim of this thesis is to implement and further develop a theoretical FTP model which will seek to, in as detailed manner as possible, transfer the liquidity costs/benefits arising from financial products back to the originator. This will result in a more transparent view of the liquidity costs/benefits associated with an institutions assets and liabilities and thus enhance its ability to take more informed decisions regarding the actual profitability of the products.

The focus will also be to model a benchmark rate which will serve as a proxy for the risk-free interest rate, which is one of the key underlying components in the total cost of funding, using a stochastic interest rate model. By examining the relationship between the FTP rate, the total cost of funding and the risk-free interest rate one is able to use the interest rate model together with interest rate scenarios to make predictions for future FTP rates and funding costs. The information provided by the simulation together with the scenarios can be an input for strategic funding decisions for the institution, e.g. how much is the expected cost of funding for a certain project under different scenarios looking two years ahead?

Incorporating this information when considering future business opportunities can help banks in assessing the risk when measuring the profitability of future funding agreements due to the uncertainty in the funding costs.

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Chapter 1

Introduction

1.1 Background

The foundation of the financial industry is based on the principle of borrowing funds at a certain cost and lend or invest those funds to customers or into business opportunities. To create value through these transactions, the total cost of borrowing, including the cost of doing business, should in the long term be strictly less than the return of the associated investment. The larger the difference between the cost of funding (borrowing) and the yield of the investment, the larger the profit generated. This difference becomes self evident if, in each case, the funds are mapped via a one to one map to an investment with the same characteristics in terms of maturity[1], cash-flow and riskiness.

In reality however, things are a bit more complex. Assets, i.e. investment and loans, are seldom matched with a specific liability or funding source, instead they are financed with a pool of liabilities, each with different characteristics. A loan can be financed partly through deposits with longer maturities and partly via interbank funding where the bank borrows funds from other banks at a certain yield which depends on the credit quality of the bank and the maturity of the loan. If the financing is comprised of only a few funding sources it is still fairly easy to calculate the profit generated from the transaction. In the everyday business of a large bank however, the liabilities used to finance assets can be manifold.

In particular when banks finance long term assets with short term liabilities, it takes on *liquidity risk* due to the mismatch in maturities which forces the bank to either roll over[2] liabilities that mature, raise additional funds or to liquidate balance sheet holdings. The

[1] Maturity refers to the final payment date of a financial product (e.g. a loan, bond, option etc) after which the product cease to exist, the product matures.

[2] Roll over is when a contract is renewed at maturity.

banks ability to do either of these things is heavily dependent on the current market conditions. During the recent financial crisis, which was largely a liquidity crisis, liquidity became a scarce resource and banks couldn't raise funds through some of its key funding channels, and to make matters worse, assets which previously were considered to be liquid became illiquid, forcing banks to liquidate positions at unfavorable prices in order to acquire funds.

In order to reduce the risk of a liquidity shortfall, financial institutions are increasingly looking to understand their liquidity exposure at a product level and to steer their businesses into a more sound liquidity practice. This can be achieved through an internal pricing system, often referred to as *Funds Transfer Pricing* where each balance sheet holding (assets and liabilities) is assigned a rate which is based on its usage/supply of liquidity. The FTP rate assigned to each product is constructed by a sum of variables each related to the cost of replicating the cash flow characteristics of the product. One of the key variables when estimating the cost of replication, regardless of the method used, is the underlying risk free rate, which can be seen as a base rate at which all banks cost of funding can be benchmarked. The development of this interest rate is therefore an important variable when estimating future funding costs for banks.

1.2 Aim of thesis

The goal of this thesis is to demonstrate how banks can use an interest rate model for simulating future interest rates together with three different historic scenarios for the risk-free interest rate curve and apply the result on an internal transfer pricing model to make projections for the funding costs of financial products in the future. The transfer pricing methodology can be applied to all assets and liabilities to measure their respective usage and supply of liquidity, business units responsible for issuing assets or raising liabilities are then charged or credited in proportion to the liquidity exposure of the transaction.
The transfer price is one part of the total cost/benefit of funds which is dependent on the evolution of the risk-free interest rate, if banks wants to be able to estimate funding costs for future time-periods, one of the key underlying components determining the cost is the risk-free interest rate. To implement a tool for doing future predictions, one has to look at both the evolution of the short term interest rate and the corresponding yield curve. In this thesis the short term interest rate is modeled using a stochastic interest rate model and predictions for the yield curve are constructed using historic interest rate curves describing different expectations for the future economy. Funding costs for different expectations regarding future developments of interest rates can then

be constructed and together with the transfer pricing method, projections for future funding costs of any product can be made.

1.3 Outline

The thesis will have the following structure: In chapter 2 the concept of liquidity and liquidity risk is described. In chapter 3 different sources of funding for banks is examined. Chapter 4 explain the concept of yield curves for interest rate securities. Chapter 5 examines one of the regulatory requirements surrounding liquidity for banks. In chapter 6 the interest rate model to be used in the simulation is presented. Chapter 7 introduce the subject of funds transfer pricing and presents a few different FTP-methods whereby the last method will be thoroughly examined and extended. In chapter 8 details for the interest simulation together with different scenarios for the future yield curve is presented. A generic product used in the presentation of the results will also be introduced in this chapter. Chapter 9 contains results. Chapter 10 concludes and in chapter 11 discussions and improvements are presented.

Chapter 2

Liquidity

Broadly defined, liquidity is the capacity to obtain cash when it is needed. Whether considering a financial or a non-financial enterprise, this objective is shared. This definition is not to be confused with the possession of liquid assets (e.g. Cash or bonds) that can be easily converted into cash, which is merely a subset of possible ways to obtain liquidity. Other sources such as the ability to get access to external funding must be included to estimate the total available capacity[19]. In fact, when examining the liquidity situation at a financial institution, it is often wise to divide the liquidity capacity into subcategories which separates the concept of being able to obtain liquidity by selling assets from the concept of raising more debt to stay liquid. These subcategories are referred to as *Market liquidity* and *Funding liquidity*. The subject for examination in this thesis is mainly linked to the concept of Funding liquidity, but for the sake of completeness a brief review of both concepts is presented below.

2.1 Market Liquidity

Market liquidity refers to the ability to sell an asset with short notice without incurring significant losses in its value. This is based on the notion that the current market price at any given time is the fair price at which a liquid asset should be traded at that time. When the traded price of an instrument is deviating from this market price the instrument is said to be traded with a *liquidity premium* (or "illiquidity premium"). The liquidity premium is dependent on a number of variables but it can be viewed, to some extend, as a reflection of the markets confidence around its fair price. A high liquidity premium indicates that the market is indecisive about the fair price of the instrument, a low premium reflect the opposite. The liquidity premium can be cumbersome to estimate due to its dependence on variables which are contingent on the specific transaction such as size and liquidation time. If the volume to be traded is large and the liquidation time (the time it takes to sell the given volume in the market) is fixed and short, then the liquidity premium is likely to have some impact on the final execution price of the entire position. It should be noted however that the measurability and even the existence of liquidity premiums have been the subject of a lively debate among practitioners for a couple of years because of the difficulty of separating the liquidity premium from the credit spread, so it is not self evident how the liquidity of an asset shall be measured[16].

2.2 Funding liquidity

The definition of funding liquidity varies slightly depending on the source, ECB (European Central Bank) defines *funding liquidity as the ability to settle obligations with immediacy* this definition is consistent with the one proposed by BIS (Bank for International Settlements)[9]. However the definition does not specify what means the bank have at their disposal to be able to settle these obligations, i.e. it does not say whether this includes selling assets to meet its obligations or if its referring only to the banks ability to raise funding through external funding channels. If it were to cover the banks ability to liquidate some of its holdings to fund itself, market liquidity would be merely a subset to funding liquidity, to avoid confusion in the subsequent parts of the thesis, funding liquidity will refer to the *ability to settle obligations with immediacy through external funding*. This will include the extension of existing contracts[1], e.g. lengthening of the time to maturity of outstanding liabilities. This distinction will be important when it comes to examining the different types of *liquidity risk* embedded in a transaction, and to be able to assign internal transfer pricing rates based on the liquidity risk characteristics of balance sheet holdings.

2.2.1 Funding Liquidity Risk

While funding liquidity is essentially a point-in-time binary concept, either the bank is able to settle its obligations with immediacy through external funding or not. *Funding liquidity risk*[2] however is a measure of the *risk* of not being able to settle its obligations in a future point in time. Hence, liquidity risk is no different from other risk measures such as Credit Risk and Market Risk, in the sense that it is a forward looking measure with a term structure and can take on infinitely many values, which broadly expressed, depends on its current funding position[3][9].

[1]Previously referred to as roll-over an agreement.

[2]Sometimes also referred to as "Mismatch or structural liquidity risk".

[3]The current funding position is referring to the banks ability to acquire funds in relation to future funding needs.

The problem with funding liquidity risk, as with liquidity risk in general, is the absence of a widely used risk measure to adequately quantify the aggregate liquidity risk. Although some methods for assessing these risks have begun to develop, recognized methods equivalent to *Value at Risk (VaR)* for Market risk and *The Standardized approach* for Credit Risk are still missing.

Despite the lack of a well accepted liquidity measure, some banks have developed ways of managing and to some extent measure the liquidity risk at product level simply by looking at the maturity and cash flow characteristics of balance sheet holdings. By doing so, the bank is able to get a better overview of the funding needs or funding contributions during the lifetime of a product. A simple example will illustrate the idea of comparing products from a liquidity perspective:
Consider the following two financial products, a 5yr loan with notional $100 000 and yearly principal payments of $20 000 paying 4% per year in interest on the outstanding debt. The second product is a 10yr loan with notional $100 000 and no yearly principal payments (the notional amount is payed back at maturity) paying 6% in annual interest. From a profit perspective the second loan is preferable, but from a liquidity perspective the first loan is better. The reason for this is that the outstanding amount needs to be funded during the lifetime of the loan, in other words the bank has to raise external funds or gather deposits to finance the loan until the loan matures (the whole amount is paid back). This implies that the longer the maturity the longer the financing commitment signed by the bank. This wouldn't be an issue if the bank always choose to finance loans or other products using funds with the same maturity as the originated transaction, but in reality, banks uses maturity transformation to increase the *Net Interest Margin (NIM)*[4] of its assets. Maturity transformation is when a bank borrows money on short-term and lend or invest the funds in long-term assets, since short-term funding normally is cheaper than long-term funding this method generates a higher NIM than if the funds used to finance the assets were borrowed at the same maturity as the assets itself. However, this makes the bank vulnerable when the liabilities used to finance longer term assets matures and the bank has to either roll over the debt or find a new source of funding, the bank is then exposed to funding liquidity risk. The risk can comprise of simply not being able to roll over the debt or raise new funds or it can only be done at an unfavorable price. The maturity transformation of the banks balance sheet holdings is the key driver for funding liquidity risk and in the aftermath of the financial crisis, measurement and management of liquidity risk became a top priority both from the financial industry itself and regulatory authorities. Funds Transfer Pricing (FTP), a tool, which was mainly developed to manage interest rate risk in US banks, with origins

[4]Net interest margin (NIM) is the difference between the interest income on an asset and the interest expense paid to lenders to finance the asset.

from the 70's, regained attention for its advantages in managing and pricing liquidity risk as well. In later sections, some of the most frequently used FTP methodologies will be presented together with some recent developments in the subject.

Chapter 3

Funding

3.1 Deposits

Deposits have long been the most important financing source for retail and commercial banks. It consists of funds that are put in the bank by customers, both consumers and enterprises, and in exchange the bank pays interest on the funds[1]. The deposit is a liability to the bank because it borrows money from the customer with the obligation to pay it back in the future. Deposits can have different characteristics in terms of term structure and optionality, i.e. the deposit can have fixed terms where the customer only is allowed to access their funds after a certain time or they can be able to withdraw them at any time. Optionality refers to the fact that there can be both a fixed term structure together with a limitation on the number of withdrawals during the lifetime of the deposit. Even though there exists a wide variety of deposits with different term structures, they are, as a liability class, generally considered to be a quite stable source of funding to the bank. In many western countries, governments protect depositors up to a certain amount in case the bank has to file for bankruptcy, which contributes to keeping the base of deposits stable since the customer doesn't need to worry about their savings if the bank is in financial distress.

3.2 Wholesale funding

Wholesale funding is an alternative way for the bank to obtain financing and accounts for an increasingly bigger part of banks funding[20]. Wholesale funding comprises of a number of different lending agreements banks use to distribute funds between each

[1] Nowadays when interest rate levels are on historically low levels, many banks doesn't pay any interest on its customers deposits.

other. These agreements can be short term or long term and/or secured or unsecured. A secured agreement is when the borrower posts collateral as a security for the loan, either to lower the interest rate paid for the loan or in some agreements pledging collateral is necessary to obtain the funds needed[2]. An example of collateralized loans are *repurchase agreement*, often referred to as repos, they are transactions in which the borrower sells securities to a lender in exchange for funds and repurchase these (or sometimes similar) securities, at a higher price, at a specific time in the future. The difference between the prices is effectively the interest paid for the loan. The market for secured loans, or repos, have been growing fast during the last decade and doubled in size between 2002 and 2007 with an outstanding gross amount of almost $10 trillion in the US just before the crisis in 2008[15] .

Another subset of wholesale funding is the interbank market, which is a vital source for the management of liquidity in many banks. The interbank market is a money market where banks extend short term loans to each other, either secured or unsecured. Most of the loans have a maturity of one week or less, and a large proportion is overnight loans, where banks fund their daily shortages of liquidity or, in the case of an excess, lend it to others. This source is also very important from a regulatory perspective since banks, nowadays, are required to hold an adequate amount of liquid assets to withstand sudden liquidity shocks, through the interbank market banks can raise the shortfall to fulfill the necessary liquidity requirements set by regulatory authorities.

The rate at which the banks lend to each other at different maturities depends on the specific interbank market, where benchmark rates are usually set by a panel of banks which are chosen to be representative for the current interbank market. The average rate, for a specific maturity, at which these panel-banks are willing to lend to each other determines the interbank rate for that maturity. This is done on a daily basis and these rates forms a *Yield Curve*[3] which shows the rate at which these panel-banks (and similar banks) are able to fund themselves (or invest excess funds) at different maturities[4]. There are many different interbank markets, one of the main is the London Interbank Market whose rate *LIBOR (London Interbank Offered Rate)* serves as an important benchmark rate for the pricing of numerous financial instruments. However the turmoil surrounding the LIBOR during the financial crisis in 2008 and the LIBOR scandal in 2012 have made practitioners shift to other, more stable and reliable sources to be used as benchmark-rates in pricing models.

[2]Collateral is a protection for the lender if the borrower is unable or unwilling to fulfill its obligations from the agreement. E.g. in case the borrower defaults on the loan, then the lender keeps the collateral

[3]Explanation follows in subsequent sections

Chapter 4

The Yield Curve

4.1 Yield Curve

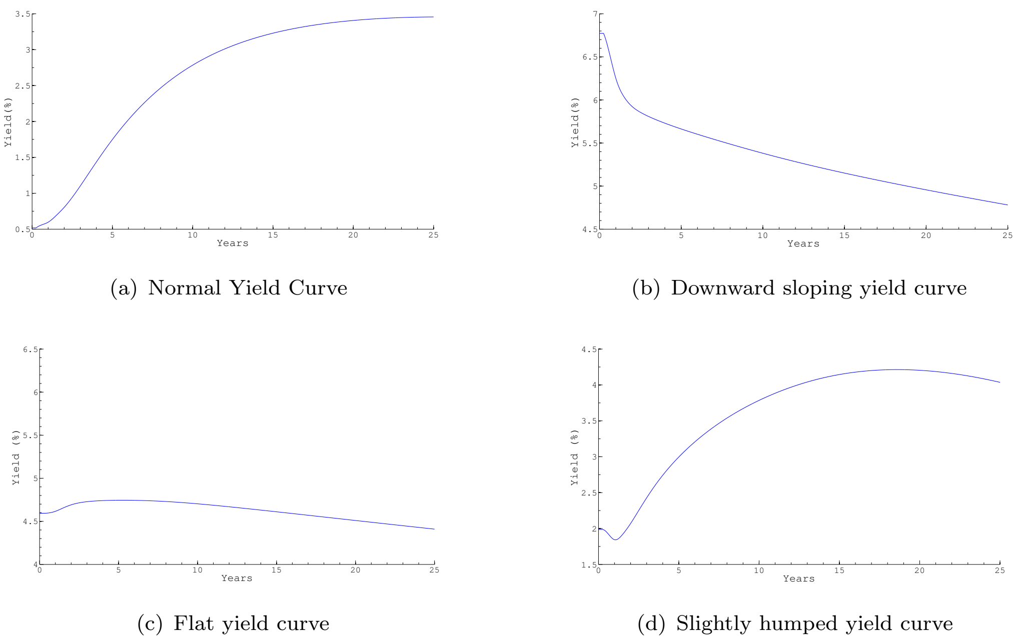
The *Yield Curve* describes the relationship between the yield of an interest bearing contract and the time to maturity. I.e. It depicts the interest rate (the cost of borrowing) for a specific borrower as a function of the contract length. If the yields are plotted for different maturities the yield curve is constructed. It is important that the yield curve is constructed from one type of asset. e.g. US government bonds. The yield curve of certain key securities works as a benchmark for many of the worlds funding agreements. One of the most influential key policy rates in the world today is the US treasury rates, which is the rates that an investor earns when investing in Treasury bills and Treasury bonds issued by the US treasury.

The shape of the yield curve is an important indicator of the market expectations of future interest rates, which in turn is an important indicator of the future economic activity. This makes the yield curve one of the primary tools in analyzing the future outlook for economic growth. But what determines the actual shape of the yield curve? The U.S treasury yield curve is shaped by the Federal Reserve and other market participants. Federal Reserve controls the federal funds rate, which is the rate at which banks can borrow funds overnight, this rate determines short-term interests rates[1]. All other interest rates on the yield curve are set in the market by auctioning so called treasury notes to the highest bidder. The final yield is set where supply for lending meets demand for borrowing for a number of different maturities and these are the interest rates used to construct the yield curve[7]. As a result the shape of the yield curve is highly dependent on investors expectations on future interest rates. If interest rates are expected to rise in the future, then the yields of long-term securities must be higher than short-term yields

[1] Short term interest rates refers to rates with maturities shorter than one year.

to attract investors, otherwise investors will not purchase long-term securities, instead they will invest on a short-term basis and then reinvest when interest rates have risen. Equivalently, if interest rates are expected to be lower in the future, borrowers will not borrow at long-term rates that are equal to or higher than short-term rates, instead they will borrow on a short term basis and wait for the rates to fall and then take out new loans. This is the reason why long-term interest rates must be lower than short term interest rates, if future short term interest rates are expected to fall. Figure 4.1 shows four yield curves for US treasury notes from different time periods each with different curvatures. Figure (a) below depicts a situation where interest rates are expected to rise in the future, this is the most common situation. Figure (b) illustrates the situation where interest rates are expected to fall, this was for example the case in mid 2007. In (c) the market expects rates to remain fairly constant for a long period and (d) shows a yield curve with a slight hump where the rates are expected to increase during a long period and then start to decrease.

The yield curve gives information about the *annual yield* an investor would earn if he were to invest in a security for a given maturity. It also gives information about future interest rate yields, i.e the market expectations of the annual yield of a security between two points in time in the future. The yield curve which illustrates the expectations for future yields is simply called the *forward yield curve* and is derived from the original yield curve.



(a) Normal Yield Curve

(b) Downward sloping yield curve

(c) Flat yield curve

(d) Slightly humped yield curve

FIGURE 4.1: US treasury yield curves from different time periods showing the current periods expectations on future interest rates[2].

4.1.1 Forward Yield Curve

The *forward yield curve* depicts the expectations regarding future yields on a security against time to maturity. It is derived from the current yield curve of a specific security by applying a bootstrap algorithm which gives the annual rate between two future points in time. The general expression for calculating the forward yield can be derived as follow: Suppose one would like to calculate the annual rate between two future time-points, t_1 and t_2, the spot rates from today up until these future time-points is given from the initial yield curve, r_1 for t=0 to t_1 (0,t_1) and r_2 for t=0 to t_2 (0,t_2). Then it is assumed, from market expectation theory, that longer term rates should be in line with markets expectations regarding future short term rates, i.e. investing in a long term contract should be expected to have the same return as investing in a series of short term contract. More formally:

$$(1 + r_1)^{d_1} \cdot (1 + r_{t_1,t_2})^{d_2 - d_1} = (1 + r_2)^{d_2} \tag{4.1}$$

$$\begin{aligned} Where&: \\ r_1 &= \text{Current rate for the time period } (0,\ \mathrm{t}_1) \\ r_2 &= \text{Current rate for the time period } (0,\ \mathrm{t}_2) \\ r_{t_1,t_2} &= \text{Forward rate for the time period } (\mathrm{t}_1,\mathrm{t}_2) \\ d_1 &= \text{Length between time 0 and } \mathrm{t}_1 \\ d_2 &= \text{Length between time 0 and } \mathrm{t}_2 \end{aligned}$$

This implies that an investor would be indifferent in choosing between an investment with a maturity of t_2 years, which gives an annual yield corresponding to the t_2 year yield on the initial yield curve, against investing in a series of forward rates from (0,t_1)[2] and from (t_1,t_2).

If 4.1 is solved for $r_{\mathrm{t}_1,\mathrm{t}_2}$ the following expression is obtained:

$$r_{t_1.t_2} = \left(\frac{(1 + r_2)^{d_2}}{(1 + r_1)^{d_1}} \right)^{\frac{1}{d_2 - d_1}} \tag{4.2}$$

[2]The spot rate from 0 to t_1 can be viewed as a forward rate starting from t=0.

From 4.2 the forward yield curve can be constructed for a specific fixed time interval between t_1 and t_2. Figure 4.2 plots the current yield curve for US treasury notes together with a corresponding forward yield curve.

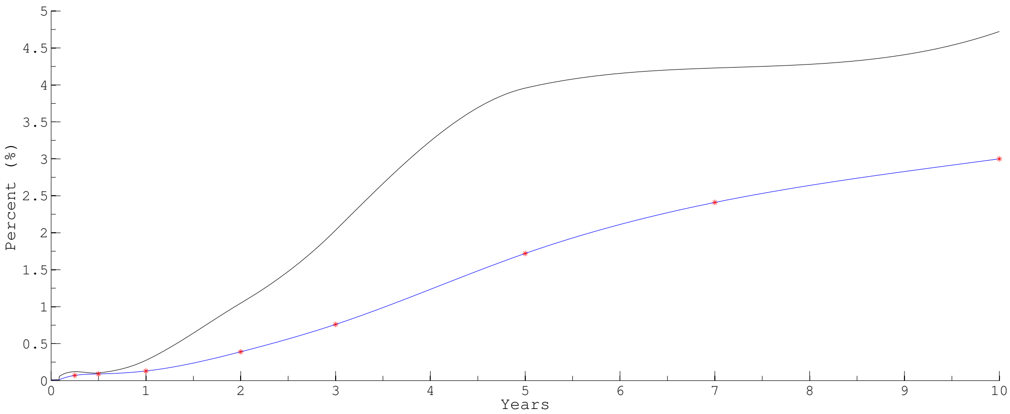


FIGURE 4.2: The blue curve shows the current yield curve which has been interpolated between quoted yields for specific maturities marked in red. The corresponding forward curve is illustrated by the black line. The fixed time interval chosen in this yield curve is one day. i.e t_2 - t_1 = 1 day

Although todays expectations of forward rates are not a perfect predictor of future rates, since it has no way of incorporating changes in the economic activity, it's still an important forward looking measure for future interest rates. In this thesis the forward yield curve will be used as an input for the simulation of future interest rates from a stochastic interest rate model through a Monte Carlo simulation.

Chapter 5

Regulation

5.1 BASEL - History

The foundation for the *Basel Committee on Banking Supervision* was established in 1975 in response to a series of disruptions in the foreign exchange markets in the early 70's. It comprised of central bank governors from the G10 countries who establish a forum for regular cooperation between the member states on matters in banking supervision. The main goal of this forum was:

> *To enhance financial stability by improving supervisory knowhow and the quality of banking supervision worldwide.*

Although the committee has changed in a number of ways since its foundation, e.g. the committee now consists of 27 member states, up from the initial ten and is reporting to an oversight body GHOS (Group of Central Bank Governors and Heads of Supervision), the objective still remain the same.

The decisions made at the committee are not legally binding, it is merely broad supervisory standards and guidelines together with recommended statements of best practice within banking supervision. It is then expected that authorities of the members states take responsibility in implementing these guidelines, at least partially, into their own national banking system[3].

Basel I was the first set of guidelines, presented in 1988 and is often referred to as the *Basel Capital Accord*. The incentives was to establish multinational guidelines to strengthen the stability of the international banking system and equal out the differences in competitive advantage due to different capital requirements. The accord was primarily focused on credit risk and left other risk like market risk and operational risk aside. The need for a better assessment of other risk factors together with a more comprehensive framework to cover financial innovations, e.g. derivatives, that became widely traded during the 90's was the main reasons for the introduction of a new regulatory framework in 2004, the Basel II.

Basel II was divided into three pillars:

- **Pillar 1:** Expansion of the capital requirements determined in Basel I.
- **Pillar 2:** Supervisory review of of the capital adequacy and internal assessment process of an institution.
- **Pillar 3:** A set of disclosure requirements which will allow market participants to better assess the risk of an institution.

Despite the extensive regulatory enlargements in Basel II the need for additional requirements became evident, especially when the global financial crisis erupted in 2008. The regulatory framework had clearly failed in assessing and controlling the risk linked to high leverage and insufficient liquidity buffers that forced many global actors into bankruptcy or bailouts[3]. During the crisis, in fact the same month that Lehman Brothers failed, the committee released *Principles for Sound Liquidity Risk Management and Supervision* which is a set of guidelines on how to improve liquidity risk management and supervision.

In November 2010 a new set of guidelines, which had been announced by the GHOS a couple of months earlier, were accepted in the G20 meeting in Seoul, the Basel III. The new regulatory package included additional capital requirements, stricter capital and liquidity requirements on banks whose failure threatens the entire system, and now there was also guidelines and risk measures to treat liquidity risk which had been omitted in the previous two accords.
The first liquidity risk measure that will be introduced in the beginning of 2015 which requires banks to hold a stock of liquid assets in order to secure short-term funding needs for a specific period is called *the Liquidity Coverage Ratio (LCR)* and will be the regulatory risk measure of consideration in this thesis due to its focus on funding liquidity risk. The full body of the Basel III framework is scheduled to be in operation in the beginning of 2019.

5.1.1 Liquidity Coverage Ration (LCR)

The Liquidity Coverage Ratio (LCR) is one of the quantifiable measures of funding liquidity risk proposed in Basel III[1]. Its main objective is to secure that the bank has enough liquid assets to fulfill its contractual payment obligations during 30-days of a significant liquidity stress scenario[12][2]. The LCR is defined as:

$$LCR = \frac{Stock\ of\ HQLA}{Total\ net\ cash\ outflows\ over\ the\ next\ 30\ calender\ days} \tag{5.1}$$

5.1.1.1 Stock of HQLA

The Numerator of the LCR is the *Stock of High Quality Liquid Assets* which obliges banks to hold a sufficient amount of unencumbered[3] HQLA to cover a certain portion of the net cash outflow over a 30-day period under a severe stress scenario[12]. The assets that are included in the stock of HQLA needs to fulfill certain requirements to be included, these requirements refer mainly to the market liquidity risk of the assets, i.e. how easily they can be liquidated under a defined stressed scenario at a relatively certain price[4]. Furthermore the regulators divide the stock of HQLA into mainly two subcategories, level 1 and level 2 assets.

Level 1is comprised of assets which are considered "the most liquid" in the event of a liquidity crisis, examples of level 1 assets are cash and central banks reserves[5]. The stock of HQLA is required to consist of at least 60 percent of level 1 assets.

The second category, level 2 assets, consists of assets which are considered to be less liquid than level 1 assets, but still qualifies to be included in the stock of HQLA albeit with a limit on the share to be included. Examples of level 2 assets is corporate bonds with rating AA- or higher, or Residential Mortgage Backed Securities (Mortgage loan). Since the minimum requirement on the share of level 1 assets in the stock of HQLA is 60 percent, the bank is limited to include at most 40 percent of level 2 assets in its stock of HQLA. The difference between the two categories for the inclusion in the LCR ratio is that the latter is subject to a haircut[6] in its value before it is included in the calculations for the HQLA. For example: If a bank wants to include a portfolio of AA rated corporate bonds in the stock of HQLA the total value of the portfolio is multiplied by a factor of

[1]The other is the *Net Stable Funding Ratio (NSFR) (coming into force in 2018) which intends to ensure the long term funding in banks. The NSFR will not be of subject for discussion in this thesis.*

[2]The scenario entails a combination of idiosyncratic and market-wide shocks. For the exact definition the reader is referred to http://www.bis.org/publ/bcbs238.pdf, p.6.

[3]An unencumbered asset is free of any encumbrances such as creditor claims or liens

[4]The reader is again referred to http://www.bis.org/publ/bcbs238.pdf, p.7 for a more comprehensive description of the criteria to be met for inclusion in the stock of HQLA.

[5]A complete list of qualifying level 1 assets is found in A.1.

[6]A haircut is a percentage that is being subtracted from the market value of an asset.

0.85 (i.e. a 15 percent haircut) before it is included[7]. This is because the market value of some assets are considered to be more volatile, especially under a stressed scenario, thus there is uncertainty regarding its liquidation value which is reflected in the haircut. In turn level 2 assets is divided into two subcategories, level 2A and level 2B assets, which can be seen in A.1.

The regulatory classification of assets is important for the subject of this thesis because investing in certain assets may involve additional costs for the bank linked to the regulatory risk measures like LCR. E.g. if a US bank has 1 million dollars invested in government bonds (qualified as level 1 assets), which it includes in its stock of HQLA. Then the bank decides to sell these bonds and use the funds to finance a mortgage for a private citizen which is classified as a level 2B asset subject to a 25 percent haircut. The bank is now "short" of \$ 1000 000 $\cdot$ 0.25 = \$ 250 000 in the stock of HQLA, this shortage is assumed to be funded via external funding e.g. through the interbank market. This thesis suggests that the cost of funds should be assigned to the asset subject to the haircut and thus reduce the profitability of the mortgage, something which must be taken into consideration when signing the loan if the bank seeks to get a full oversight of the total cost incurred by its assets. The methodology for assigning these "regulatory costs" to assets will be presented in section 7.3.5.

5.1.1.2 Total net cash outflows (TNCO)

The BCBS defines Total net cash outflows as: *Total expected cash outflows minus total expected cash inflows in the specified stress scenario for the subsequent 30 calendar days*[12]. More specifically:

Total net cash outflows over the next 30 calendar days =
Total expected cash outflows - Min{total expected cash inflows; 75% of total expected cash outflows}

The total expected cash outflows are calculated by multiplying the outstanding amount with the expected run-off[8] rate or yield of that liability. Similarly the total expected cash inflows are calculated by multiplying the outstanding notional of assets (after haircuts is applied) with its corresponding contractual rate at which they are expected to flow in under a, by the regulatory authorities defined, stressed scenario. The cash inflows is also provided with a cap of 75 percent of total cash outflows forcing banks to hold at least $25\% \cdot \varphi_{\text{reg}}$[9] of expected cash outflows in HQLA. The run-off rates applied to liabilities

[7] see A.1 for a complete list of level 2 assets and their respective haircuts
[8] The run-off rate is the rate by which a liability is expected (by the regulators) to be withdrawn from the banks balance sheet.
[9] φ_{reg} is the required ratio between the stock of HQLA and TNCO that is required by the regulatory authorities.

varies depending on how much of the outstanding notional of the liability is expected to be withdrawn over the next 30 days. Deposits for example is assigned different run-off rates depending on whether it is customer or retail deposits[10]. An important feature of the LCR is that it does not allow for double counting items, i.e. if an asset is included in the stock of HQLA, then the associated cash inflow is not permitted to be included in the TNCO[12].

[10] See A.1 for a list run-off rates applied to various liabilities.

Chapter 6

Interest rate models

6.1 The Vasicek Model

One of the earliest mathematical models for describing the evolution of interest rates was introduced in 1977 by the Czech mathematician Oldrich Vasicek. The model is a *one factor*, *mean reverting* short term interest rate model. One factor model refers to the movements in the interest rate which is described by the *market risk* as the only source of randomness. Mean reverting implies that the short term interest rate r will revert to a mean level in the long run for all future trajectories of r. The model is described by the following stochastic differential equation[1]:

$$dr(t) = a(b - r(t))dt + \sigma dW(t) \tag{6.1}$$

a: Speed of mean reversion. a controls the pace at which the process revert to the mean level b.
b: Mean reverting level. Long term mean for all future interest rates.
σ: Instantaneous volatility. A scalar that controls the amplitude of the randomness streaming from the Wiener process W_t.

The Vasicek model was the first to incorporate mean-reversion as a characteristic in interest rate models. When r is high, mean reversion creates a negative drift; when r is low mean reversion creates a positive drift. The intuition behind this is that interest rates cannot rise to infinitely high levels because high rates inhibits economic activity as there is a low demand for funds which would force a decrease in rates in the long run. Similarly, low rates creates a high demand for funds from borrowers, causing the rates

[1] This type of process is generally known as an Ornstein-Uhlenbeck stochastic process

to rise. As a result interest rates fluctuates within limited levels around a certain long term mean[17]. One of the major drawbacks of the Vasicek model is that the interest rate can become negative. An additional drawback is the poor fitting to the current term structure of interest rates, this issue was addresses by Hull & White in their 1990 paper, and they expanded the model so as to better fit the current term structure of interest rates.

6.2 Hull & White

As mentioned above, the need for a better fit to the currently observed yield-curve led Hull and White to derive an extension of the Vasicek model. The extension comprised of the introduction of a *time-varying mean reversion level*, often referred to as θ(t), which is derived from the initial forward yield-curve. This allows for the expectations of future interest rates, which are reflected in the forward yield curve, to be incorporated into the model and steer the output of the short term future interest to fluctuate around these expectations. The Hull & White model is described by the same expression as the Vasicek model with the modification of the mean-reversion level so as to be time-varying[17]:

$$dr(t) = a(\theta(t) - r(t))dt + \sigma dW(t) \tag{6.2}$$

$With:$

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})^{2} \tag{6.3}$$

$Where:$

$F(0,t)$: *Is the initial forward rate with maturity* t.

$F_t(0,t)$: *Is the partial derivative of F(0,t) with respect to* t.

Hull & White did go even further by making the volatility and mean-reversion rate time varying as well. This involves fitting the volatility parameter to a given term structure of volatilities observed in the market which is considered somewhat problematic since the volatility can be derived from many different instruments in the market (some more liquid than others) and the reliability of the result can be hard to determine[8]. This is why we will consider only the *mean reversion level* as being time-varying for the rest of the thesis.

[2] The interested reader is referred to Damiano, B & Mercurio, F. *Interest Rate Models - Theory and Practice* p.73 for the derivation of θ(t).

The Hull & White extension of Vasicek expressed in 6.2 is the model that will be used for the simulation of interest rates in subsequent sections and this is due to mainly two reasons:
First, the ability to incorporate the expectations regarding future interest rates by fitting the initial term structure makes the model suitable for future predictions through simulations. Secondly, the tractability of the model makes it convenient to calibrate its parameters, a and σ, using quoted market prices for traded securities e.g swaptions[3].
These are some of the reasons for the widespread use of the Hull & White model in risk management and in derivatives pricing, where the value of the underlying security is dependent the interest rate.
One of the main critics against the model is that H & W assumes that the interest rate is normally distributed, which allows for interest rates to become negative. For many years this has been considered somewhat unrealistic, but recent events like the global financial crisis and the (current) sovereign dept crisis has made practitioners revise this common notion that interest rates must stay positive. In fact, Bank of New York charged negative interests for a short period of time in mid 2011 on very large deposits[1].

[3] A swap is an agreement where two parties agrees to exchange future cash flows, an interest rate swaption gives the owner the right to enter into an interest rate swap.

Chapter 7

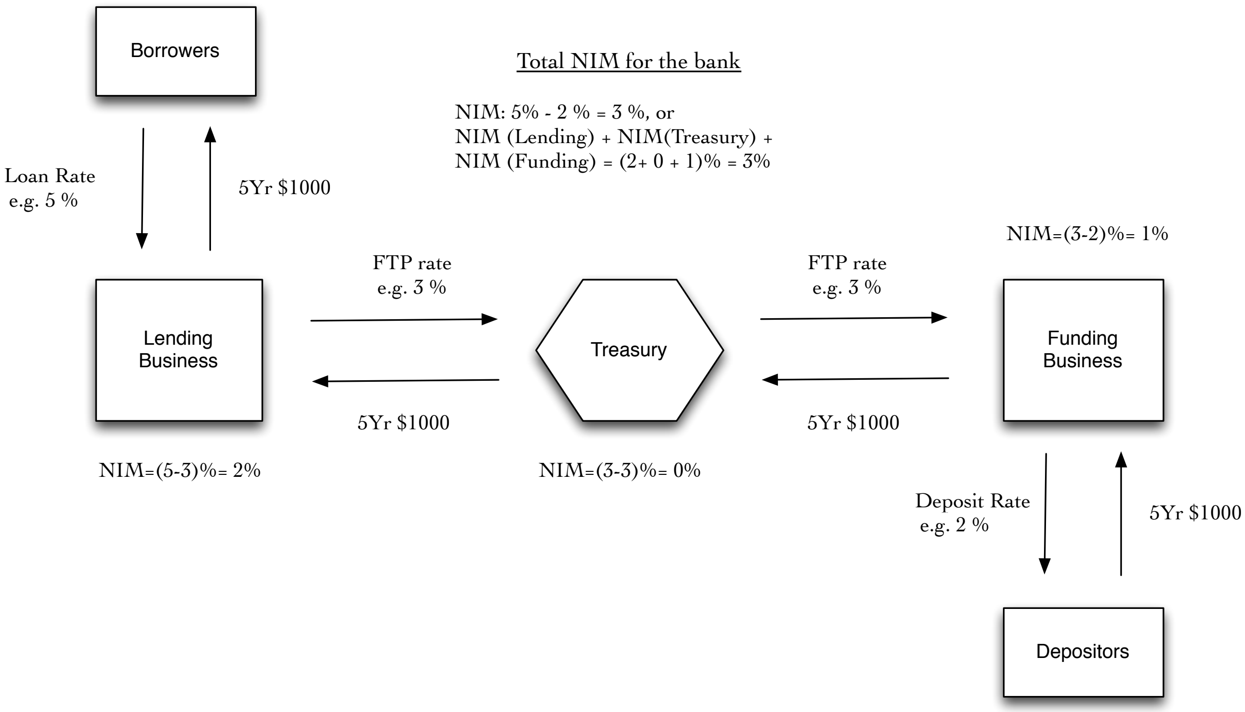
Funds Transfer Pricing

Funds Transfer Pricing (FTP) is an internal management information system and methodology designed to allocate the costs and benefits arising from the usage of funds to respective business units within a financial institution. This is done by allocating the net interest margin between fund users, such as lenders and investment units, and fund providers such as deposit- and wholesale raising units[14].

When utilizing FTP, financial institutions assign FTP rates to all earning assets to reflect the true cost of funding. On the liabilities side, FTP credits are applied to all interest-bearing liabilities reflecting the benefit to the institution for the raising of funds. The FTP rates are based on the institutions ability to raise funds e.g. either from depositors or in the wholesale money market (at the corresponding maturity).

In order to analyze the assets and the liabilities contribution to the net interest margin a profitability spread is calculated and assigned to each balance sheet item. For earning assets, the profitability spread is calculated as the difference between the yield (interest income) and the FTP charge. For interest-bearing liabilities, the profitability spread is calculated as the difference between the FTP credit (for raising funds) and the interest expense[10]. This means that lending units need to charge a rate higher than the FTP rate to make a profit and funds raising units need to raise money at a lower rate than the FTP rate to make a profit. This gives the institution a more transparent view of the actual profitability of their units and their respective balance sheet items. The total net interest margin for the bank can then be calculated by the sum of the individual contributions from the respective business units. Figure 7.1 describes the FTP process. So the basic question arises, how does the bank decide how much to charge and how much to credit for the usage/supply of funds, i.e how is the FTP rates determined?

FIGURE 7.1: Mechanics of FTP



7.1 Pooled average cost of funds

One method of allocating the cost/benefits arising from the usage/supply of funds is to use an average cost of funds approach (sometimes referred to as flat mixed rate) to calculate the FTP rate. In this approach the interest expenses from all the funding sources are summed up and divided by the total outstanding amount of liabilities which gives an average cost of funding rate. For example, if deposits were an institutions only source of funding, the FTP rate would be calculated as the sum of the total interest expenses for all deposits divided by average total deposits[1] [13].

$$r_{avg} = \frac{\sum_{i=1}^{n} r_i \cdot N_i}{\sum_{i=1}^{n} N_i} \tag{7.1}$$

$$r_i = interest\, rate\, for\, liability\, i = 1, ..., n$$

$$N_i = Notional\, outstanding\, for\, liability\, i$$

This implies that the rate applied to calculate the benefit or the cost of funds for balance sheet items is independent of their respective maturity, i.e. a deposit with a maturity of one year will receive the same credit as a deposit with a maturity of 10 years. As an

[1] Average since the balance of the institutions deposits is likely to vary since customers withdraw and deposit new funds continuously.

example of how this method is applied in practice, consider an FTP rate of 100 BPS[2] that is applied to all funding sources and assets. A loan with notional $1 million would then, irrespective of maturity and liquidity characteristic[3], receive an annual charge of $10000. The same rate is also used to credit fund providers for their supply of funds, in this case all deposit irrespective of their maturity would be rewarded 1 % of the notional amount[13]. This is shown in table 7.1.

The average cost of funds method is a favorable method because of its simplicity, however there are a couple of weaknesses with this approach. First, it does not account for the higher liquidity risk that longer-term assets imposes. If one charges the same FTP rate for the usage of funds the liquidity risk is assumed to be the same for all asset classes, irrespective of maturity. Equivalently it is assumed that liabilities is providing equally stable funding if one chooses to credit liabilities with the same FTP rate irrespective of their maturity and liquidity characteristics. This creates the wrong incentives at the business units responsible for investing and raising funds at the institution. Units responsible for issuing loans and investing in other assets is encouraged (or at least not discouraged) from writing longer term assets and thus using funds for a longer period of time which increases the liquidity risk of the institution. Conversely the business units responsible for raising funds is not awarded any premium for providing long-term stable funding instead of short-term funding, which tends to be more volatile, and thus further increases the liquidity risk of the institution. The limitations of the average cost of funds approach has made leading practitioners move towards more sophisticated methods of attributing the cost of liquidity to assets and liabilities, one type of methods that is considered to be good liquidity practice is called *Matched-Maturity* FTP methods. The level of sophistication and the details of the implementation varies between these methods but they all share the basic concept of addressing the unique characteristics of funds at the cash flow level and to use a matched-maturity transfer pricing[18].

Table 7.1: Cost and benefits of funds under the average cost approach [4]

Term in years	1	2	3	4	5
Loan/deposit principal	$ 1 million	$ 1 million	$ 1 million	$ 1 million	$ 1 million
Average cost of funds (BPS)	100	100	100	100	100
Charge for use of funds	$ 10000	$ 10000	$ 10000	$ 10000	$ 10000
Credit for benefit of funds	$ 10000	$ 10000	$ 10000	$ 10000	$ 10000

[2]BPS is a shortening for Basis Point and is equal to $\frac{1}{100}$th of a percent and is often used when pricing financial products because a small change in interest rate levels can have a big impact on the price when the notional amount is large.

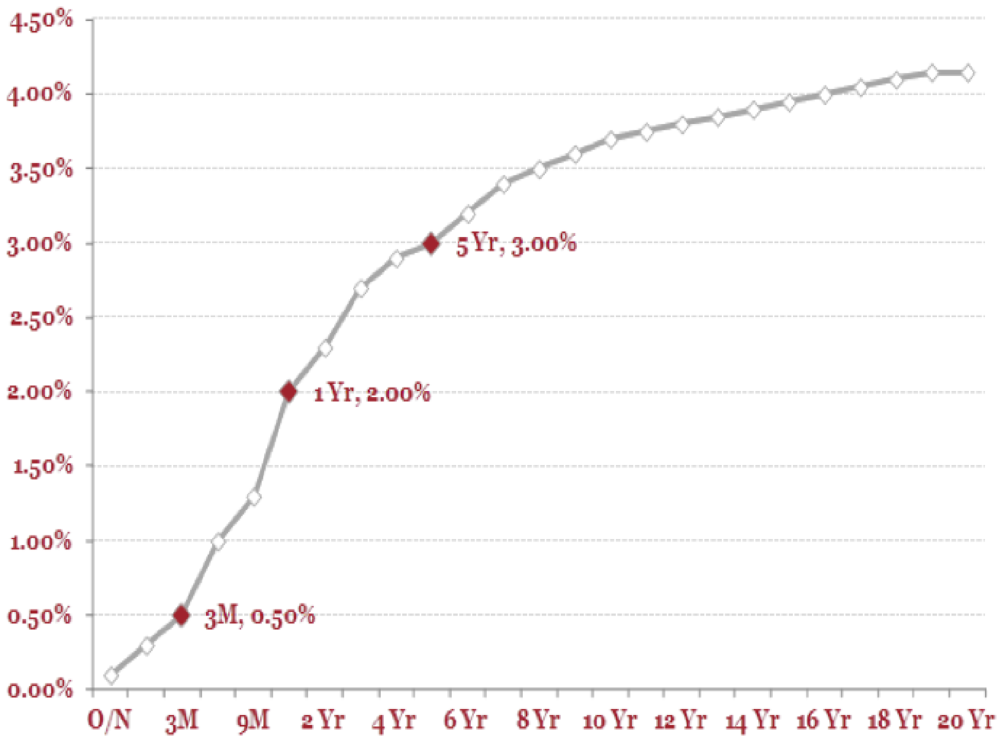
[3]Liquidity characteristic refers to the uncertainty (volatility) of the cash flow.

[4]The charges and the credits for the usage/benefit of funds is in annual basis

7.2 Matched-Maturity transfer pricing

The Matched-Maturity transfer pricing approach is by many considered as the most adequate one for creating a FTP framework that creates the right incentives for the usage and supply of funds[13]. The idea is to match the maturity profile of the expected cash flows streaming from the specific asset and liability instrument to the interest rate corresponding to the same maturity on the institutions funding curve. The rate is then charged/credited to the respective asset or liability which results in the instrument specific contribution to the net interest margin of the bank.
To explain this process more fully consider the following example: A lending division writes a new 5Yr loan to a customer which pays 7% in annual interest. The FTP rate that should be assigned to the loan is then derived from the institutions funding curve which is depicted in figure 7.2. It can be seen that the corresponding FTP rate for a maturity of 5Yr is in this case 3% which means that the contribution to the net interest margin from the loan is 7% - 3% = 4%. This transaction encumbers no interest rate risk nor liquidity risk because of the matching maturity.

FIGURE 7.2: Funding Curve [22]



Even though this approach better reflects the product specific impact on the liquidity risk and assigns the costs associated with this risk in a more comprehensive way then the pooled average approach, it fails to catch some of the behavioral aspects of some financial products. Consider for example a savings account where the customer is able to withdraw or deposit funds at any given time, i.e. there is no fixed time as to when the product matures. The question is then, which rate should you credit this savings account for the supply of funds? 1 month, 1 week or perhaps 1 day? Since the time of the funds remaining on the banks balance sheet is contractually unknown, one would have to perform some modeling to catch the behavior of the average customers savings account. If a run-off rate (i.e. a rate at which customers withdraw their money without depositing new ones) can be decided for the savings account, then the bank can estimate how much and for how long they can rely on savings deposits as a funding source and thus assign an appropriate FTP rate to reflect the benefit of supplying funds to the bank through savings accounts. The behavior of customers is, or at least should be, central for pricing many types of products offered by banks. This is why the matched-maturity approach should be extended so as to be able to incorporate some of theses aspects.

7.3 Advanced FTP approach

As a step towards a more realistic and complete model in which the optionality and product-specific characteristics can be captured and priced in an adequate way, a more economic approach presented by Christian Schmaltz in his book "A Quantitative Liquidity Model for Banks" is suggested. In this approach, the author distinguishes transfer prices for liquidity and liquidity risk. The transfer price for liquidity refers to the known (deterministic) cash flow component of a product and liquidity risk refers to the unknown (stochastic) component of the cash flow streaming from a product. The two contributions are calculated separately and summed together to form the FTP rate of the specific product. Additionally, upcoming regulatory requirements will effect banks balance sheets and force them to hold and invest partially in assets which are considered to be stable and liquid by the regulatory authorities. This thesis suggests that these requirements should be reflected in the internal pricing of holdings to further steer the incentives at the business units to assign new assets and liabilities that will help the bank to reach the regulatory requirements.

The outline of this chapter will be as follows: In the first section the process describing a products cash flow will be defined, this will be used in section 7.3.2 and 7.3.3 for the derivation of the transfer price of the deterministic and stochastic component of a product respectively. In section 7.3.5, the transfer price arising from regulatory requirements will be derived and the last section of this chapter will present an example of applying the methodology.

7.3.1 Product Cash flow

As stated above, the cash flow from a product is assumed to be described by two components, a deterministic and a stochastic component:

$$CF^i_{t_k} := \mu_{\mathrm{t_k}} + \sigma^i \cdot \Delta \mathrm{W}^i_{t_k} \tag{7.2}$$

Where $\mu_{\mathrm{t_k}}$ denotes the expected cash flow at time $\mathrm{t_k}$ and $\sigma^{\mathrm{i}} \cdot \Delta \mathrm{W^i}_{\mathrm{t_k}}$ is the product-specific uncertainty regarding future cash flows.

More specifically:

$$E[\mu_{\mathrm{t_k}}] = \mu_{\mathrm{t_k}}$$

$$Var[\mu_{\mathrm{t_k}}] = 0$$

$$E[\sigma^i \cdot \Delta \mathrm{W}^i_{t_k}] = 0$$

$$Var[\sigma^i \cdot \Delta \mathrm{W}^i_{t_k}] = (\sigma^i)^2 \cdot \Delta t$$

The uncertainty or variability in a products cash flow between two points in time is described by a *Wiener process*[5]. The choice of a Wiener process as the stochastic component is mainly due to one of its key characteristic that the change in process value between times s and t (s <t) is normally distributed with zero mean and variance equal to t-s=τ, i.e $\Delta \mathrm{W}_\tau \sim \mathrm{N}(0,\ \tau)$. This will be a useful property later on when trying to estimate the aggregate risk exposure of a portfolio of products.

Expression 7.2 implies that σ^{i} is an indicator of the sensitivity of a products cash flow with respect to liquidity shocks, e.g. The cash flow of a credit line[6] established to an enterprise is likely to have a different uncertainty in its cash flow than the cash flow from a mortgage loan.

Once the cash flow model of a product is established, the next step is to assign transfer prices to each of the components.

7.3.2 Deterministic component

The transfer price from the deterministic component of the product cash-flow is defined as a function of the contractual payments during the lifetime of the product. The function is defined as:

[5] Also referred to as Brownian motion.

[6] A credit line is a source of cash (normally with a limit) issued by a bank to a client that can readily be tapped when the borrower needs it.

$$TP^D(\mu_{\mathrm{t_k}}) := (r_{\mathrm{f}}(0, t_{\mathrm{k}}) - r_{\mathrm{b}}(0, t_{\mathrm{k}})) \cdot \mu_{\mathrm{t_k}} \cdot \Delta t \tag{7.3}$$

$$\begin{aligned} &Where: \\ r_{\mathrm{f}}(0, t_{\mathrm{k}}) &:= Funding\,Curve \\ r_{\mathrm{b}}(0, t_{\mathrm{k}}) &:= Benchmark\,curve\,. \\ \Delta t &:= Time\;step\;(days)\,. \end{aligned}$$

There is consensus in the literature on how to calculate the FTP of deterministic cash-flows, however different opinions exists regarding which curves to choose as parameters to the model. Many suggests that the risk-free interest rate curve is used as interest rate curve $\mathrm{r_b}$, this would imply that the funding spread equates the default risk premium. However in practice there exists no risk-free interest rate curve, instead liquid securities such as high rated government bonds are often used as approximations, which is why, later in this thesis, the yield curve for US treasury notes will be used as a benchmark curve[7] for calculating transfer prices.

Expression 7.3 describes the FTP rate assigned to a product with one cash flow μ that occurs at time $\mathrm{t_k}$. If a product consist of multiple cash flows n at times $\mathrm{t_1,..,t_n}$, the FTP rate assigned is calculated by the sum across all cash flows from the product. Expression 7.4 shows how the deterministic FTP rate component for **one unit** of a particular product is calculated.

$$TP^D(\mu_{\mathrm{t_0}}, .., \mu_{\mathrm{t_n}}) = \sum_{j=0}^{n} (r_{\mathrm{f}}(0, t_{\mathrm{j}}) - r_{\mathrm{b}}(0, t_{\mathrm{j}})) \cdot \mu_{\mathrm{t_j}} \cdot \Delta t \cdot (t_{\mathrm{j}} - t_0)\,, \tag{7.4}$$

7.3.3 Stochastic component

The contributions to the theory surrounding transfer prices of stochastic components is limited. The literature that does treat the subject agrees that the transfer prices for stochastic cash flows should be based on the cost of holding reserves against the deviations from the expected cash flow. The stochastic component that will be used in this thesis is described by the following function:

[7] Also referred to as *risk-free curve* or *base curve* in this thesis

$$
\begin{aligned}
&TP^{B}(\sigma^{\mathrm{i,p}}, \sigma^{\mathrm{i,m}}, n^{\mathrm{i}}{}_{2}, T^{\mathrm{i}}) \\
&where: \\
&\sigma^{\mathrm{i,p}} = unsystematic\, product\, risk\, for\, product\, i \\
&\sigma^{\mathrm{i,m}} = systematic\, product\, risk\, for\, product\, i \\
&n^{\mathrm{i}}{}_{2} = number\, of\, option\, exercises\, until\, maturity\, T \\
&T^{\mathrm{i}} = Time\, to\, maturity
\end{aligned}
$$

The unsystematic (product-specific) risk together with the systematic risk represents the total risk associated with product i. This is described using a factor model expressed in 7.5 with a common factor, the systematic risk together with a product-specific risk. The intuition behind this is that customers are assumed to be exposed to common liquidity shocks, e.g. economic downturn causing unemployment. The product-specific risk is motivated by the fact that different products attract different customers. This kind of factor model is also easy to extend to include multiple factors, if for example one wants to separate different systemic factors affecting cash flow of the product.

To be able to describe the risk exposure of a specific product between to points in time, a Wiener process is attached to each risk factor:

$$
\begin{aligned}
&\sigma^{i,p} \cdot \Delta \mathrm{W}_{t_k}^{i,p} + \sigma^{i,m} \cdot \Delta \mathrm{W}_{t_k}^{m} \qquad (7.5) \\
&being: \\
&\Delta \mathrm{W}_{t_k}^{i,p} = Product - specific\, liquidity\, shock \\
&\Delta \mathrm{W}_{t_k}^{m} = Systemic\, liquidity\, shock
\end{aligned}
$$

The normally distributed changes of a Wiener process makes it possible to measure the aggregate risk exposure of a portfolio of products between two points in time, given that one is able to make justified assumptions regarding the correlation between the individual risk factors of a product.
In our setup it is assumed that the systemic factor is independent on product-specific factors, i.e the systemic risk is not affected by liquidity shocks from individual products.

$$
\begin{aligned}
&\rho(\Delta \mathrm{W}_{t_k}^{i,p}, \Delta \mathrm{W}_{t_k}^{m}) = 0, \;\; \forall i = 1, .., d \\
&with\ d\ denoting\ the\ number\ of\ product\ categories.
\end{aligned}
$$

Furthermore it is assumed that there exists no interdependence between products.

$$
\rho(\Delta \mathrm{W}_{t_k}^{i,p}, \Delta \mathrm{W}_{t_k}^{j,p}) = 0, \;\; \forall i = 1, .., d, \;\; \forall j = 1, .., d, i \neq j
$$

This is a somewhat more simplified and problematic assumption, because it neglects the possible relation that might, or sometimes inevitably does exist between products. For example, it is reasonable to think, at least not unlikely, that there could be a dependency between a mortgage and a current account owned by the same customer, a sudden drop in the balance of the current account, close to an amortization of the mortgage, is likely to affect the cash flow of the mortgage payment. However, this simplification is necessary to be able to estimate the aggregate risk exposure for a portfolio of products. Furthermore, estimation of the interdependence between individual products would require huge amounts of data and extensive statistical analysis to be able make statements about their relationship.

The assumption about independence between risk factors enables the aggregate risk exposure, for a time interval of length Δt, of a portfolio of products to be derived as:

$$
\begin{aligned}
(\sigma^A)^2\Delta t &= Var[\sum_{i=1}^{d}\sigma^{i,p}\Delta \mathrm{W}_{t_k}^{i,p} + \sum_{i=1}^{d}\sigma^{i,m}\Delta \mathrm{W}_{t_k}^{m}] \\
&= Var[\sum_{i=1}^{d}\sigma^{i,p}\Delta \mathrm{W}_{t_k}^{i,p} + \Delta \mathrm{W}_{t_k}^{m}\cdot\sum_{i=1}^{d}\sigma^{i,m}] \\
&= Var[\sum_{i=1}^{d}\sigma^{i,p}\Delta \mathrm{W}_{t_k}^{i,p}] + Var[\Delta \mathrm{W}_{t_k}^{m}\cdot\sum_{i=1}^{d}\sigma^{i,m}] + 2cov[\sum_{i=1}^{d}\sigma^{i,p}\Delta \mathrm{W}_{t_k}^{i,p}, \Delta \mathrm{W}_{t_k}^{m}\sum_{i=1}^{d}\sigma^{i,m}] \\
&= \sum_{i=1}^{d}(\sigma^{i,p})^2\Delta t + (\sum_{i=1}^{d}\sigma^{i,m})^2\Delta t + 0
\end{aligned}
$$

Which gives:

$$
\sigma^A = \sqrt{\sum_{i=1}^{d}(\sigma^{i,p})^2 + (\sum_{i=1}^{d}\sigma^{i,m})^2} \tag{7.6}
$$

Expression 7.6 is an important component for determining the cost of holding a liquidity buffer against liquidity shocks which will be treated in the next section.

7.3.3.1 Funding Capacity

Based on the aggregate risk exposure in the previous section, a liquidity buffer to withstand liquidity shocks equal to this exposure can be derived for a given confidence level p.

FIGURE 7.3: Required liquidity buffer

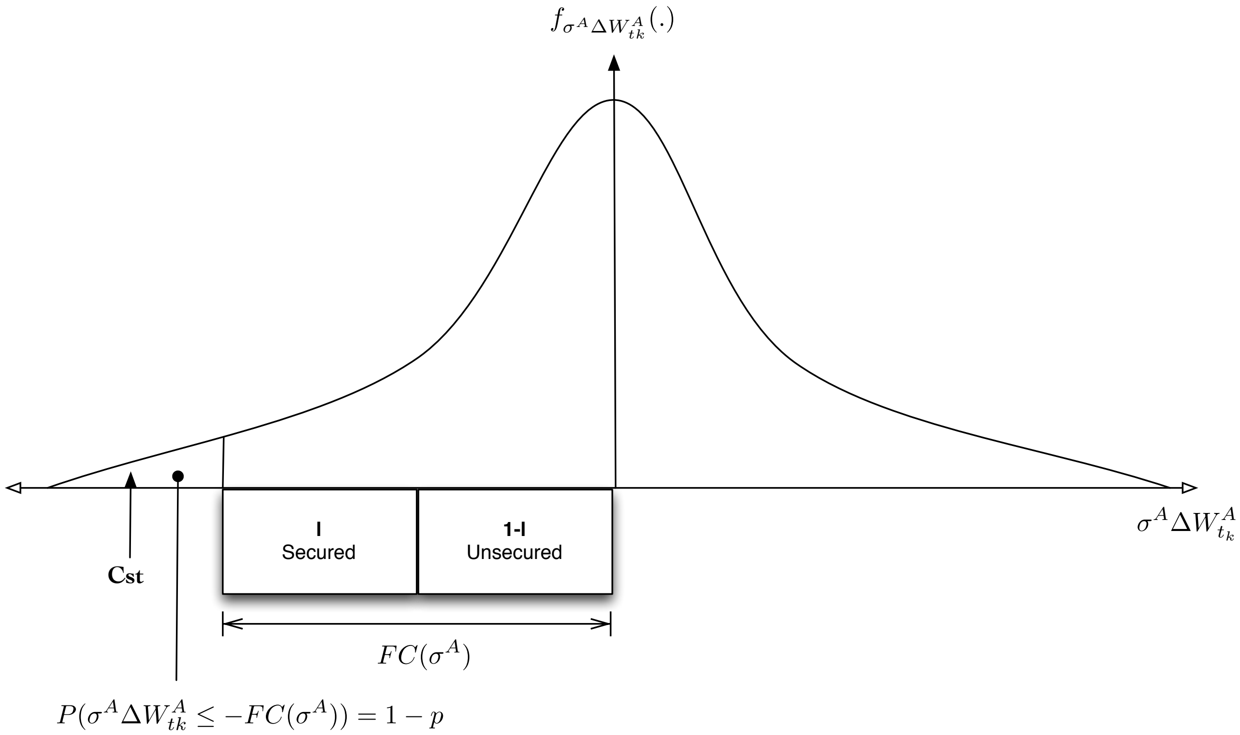


Figure 7.3 shows the setup. It plots the density function $f_{\sigma^A \Delta W^A_{t_k}}$ for the aggregate Brownian deviations.

Given that the banks aggregate risk exposure can be estimated, the necessary funding capacity (liquidity needed) to cover the risk exposure during Δt can be derived for different confidence levels, p. The bank is assumed to be able to raise the funds needed to cover the risk exposure via a combination of secured and unsecured funding, with l being the proportion of secured funding and 1-l corresponds to the proportion of unsecured funding[21]. Because the value changes in the Wiener process is normally distributed, it's possible to derive an expression for the required funding capacity needed to cover the aggregated risk exposure:

$$P(\sigma^A \cdot \Delta W^A_{t_k} \leq -FC(\sigma^A)) = 1 - p$$

$$P(\frac{\Delta W^A_{t_k}}{\sqrt{\Delta t}} \leq -\frac{FC(\sigma^A)}{\sigma^A \cdot \sqrt{\Delta t}} = 1 - p$$

$$FC(\sigma^A) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1 - p) \cdot \sigma^A \tag{7.7}$$

Expression 7.7 implies that there is a linear relationship between the required funding capacity FC and the risk exposure σ^{A}. The next step is to determine the costs associated with maintaining the funding capacity to cover the given risk exposure.

7.3.3.2 Implied cost of funding capacity

The cost of preserving a certain funding capacity is split between the cost of secured and unsecured funding. For secured funding, the cost is calculated as the difference between the yield on the liquid assets which is to be used as collateral in future secured funding agreements, and the cost of funding these assets (with unsecured funding). The cost function for the secured funding can be expressed as:

$$c_R(\sigma^A) = (-l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p)) \cdot \sigma^A \cdot \Delta Yield^{8} \tag{7.8}$$

Here it is assumed that the bank is able to obtain unsecured funding to finance these liquid assets to secure funding channels which cover the liquidity risk exposure arising from the banks holdings. It is also assumed that the bank is able to continuously raise funding to cover for the liquidity risk arising from new agreements irrespective of their maturity. It could be argued that this is an unrealistic simplification of the practical reality in a banks day-to-day business, and that it is not possible to raise funds immediately at a specific notional (to purchase liquid assets) on one agreement for a specific maturity. This problem can be overcome if it is assumed that the bank aggregates its funding needs at the end of each day and that these can be divided into buckets, where each buckets represent a set of risk exposure which needs to be funded with approximately equal maturities.

The unsecured part is a lot more cumbersome and the challenges surrounding it are somewhat at the core of measuring funding liquidity risk on a day to day basis. Obviously not carrying any liquid assets will incur zero costs, however this should be set against the risk of not being able to obtain unsecured funding in case of a liquidity shortfall. The financial crisis in 2008 has radically changed the perception regarding an institutions ability to obtain unsecured funding at a short notice. Before the crisis, institutions were able to fund their liquidity shortage through the interbank market which did not exhibit particularly high volatility, nor did the cost of short term funding exceed the cost of other liabilities. However when Lehman Brothers filed for bankruptcy on the night of 15th of September 2008, the overnight LIBOR went from 2,1% on the 14th to just over 6,4% on the 16:th of September. In addition some banks were not able to fund themselves at all via unsecured funding because of fear of counterparty credit risk. The key question is then how should this risk be model and dealt with? Should banks try to model the LIBOR rate by looking at explanatory variables? Or, should banks simulate scenarios like the 2008 crisis to measure the difference in funding availability compared

[8] The difference in yield is calculated by the difference between the cost of unsecured funding and the yield on level 1 assets, e.g. AAA-rated government bonds (e.g. German bonds).

with a normal ongoing business as usual scenario? At the moment there are no known solutions (at least not to the writer) to this problem. Regulators demand periodical testing of the unused funding capacity available, but it does not specify exactly how these tests should be conducted[6]. This thesis will not go any further in examining possible solutions but whats clear is that it would be of great value to look at the amount of liquid assets that banks should hold in relation to their cash flow in order to be able to manage the risk of a stressed liquidity scenario without severely affecting its profitability. For now, we assume that the unsecured part does not imply any additional costs for the bank, this means that expression 7.8 is the total cost of preserving a funding capacity to cover for the aggregate risk exposure σ^A.

7.3.4 Allocation of stochastic component to products

Once the cost of the required funding capacity for the aggregate risk exposure is calculated for a whole portfolio the remaining challenge is to allocate the costs to individual products. As stated in the beginning of section 7.3.3 the correlation between product-specific risk and market risk is assumed to be zero, the same is assumed for the interdependence between products. As mentioned this is a simplified and a somewhat unrealistic assumption and there is no simple approach to resolve this drawback unless one has access to necessary data for extensive data analysis and thereby determine the dependencies. However one slight improvement that can be made when allocating transfer prices to products is to account for a diversification effect[9] between products by looking at the relation of the required funding capacity under zero correlation to the required funding capacity under perfect correlation[21]. The economic reasoning is that the sum of each product specific funding requirements exceeds the aggregate funding requirements:

$$\sum_{i=1}^{d} FC(\sigma^{i,p}, \sigma^{i,m}) \geq FC(\sigma^A)$$

The objective is therefore to adjust the individual risk quantities $(\sigma^{i,p}, \sigma^{i,m})$ as to fit the relation:

$$\begin{aligned} FC(\sigma^A) &= \sum_{i=1}^{d} FC(\sigma^{i,p,adj}) + \sum_{i=1}^{d} FC(\sigma^{i,m,adj}) \\ &\leq \sum_{i=1}^{d} FC(\sigma^{i,p}) + \sum_{i=1}^{d} FC(\sigma^{i,m}) \end{aligned}$$

[9]Diversification effect is the reduced risk of a portfolio consisting of a variety of assets with non-perfect correlation where the risk of the portfolio is less than the sum of the individual risks.

Due to linearity in the risk quantity σ^A as can be seen in 7.7, the relation becomes:

$$FC(\sigma^A) = \sum_{i=1}^{d} FC(\sigma^{i,p,adj}) + \sum_{i=1}^{d} FC(\sigma^{i,m,adj})$$

$$\Leftrightarrow$$

$$-\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sigma^A = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sum_{i=1}^{d} \sigma^{i,p,adj} + -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sum_{i=1}^{d} \sigma^{i,m,adj}$$

$$\sigma^A = \sum_{i=1}^{d} \sigma^{i,p,adj} + \sum_{i=1}^{d} \sigma^{i,m,adj} \tag{7.9}$$

This implies that the allocation of the funding capacity is equivalent to the allocation of the aggregate risk quantity σ^A. To estimate the individual risk exposures $(\sigma^{i,p,adj}, \sigma^{i,m,adj})$ there exists a couple of different approaches whose benefits often depends on the data available and the simplicity of the calculations. In this thesis, due to the lack of data, a simple additive approach which estimates adjustment factors and attach these to $\sigma^{i,p}$ and $\sigma^{i,m}$ will be used. This will make the sum of the individual risk exposures equal to the aggregate risk exposure. The main advantage with the approach is that it does not require estimating the correlation matrix between products, furthermore when the number of risk factors is small the method is relatively simple and straightforward. Since our model contains two risk factors, product and market risk, there will be two adjustment factors: One for the diversification between the systemic risk factor and the product-specific factor and one for the diversification among products. The adjustment factors will be derived in the following sections, starting with the diversification between the systemic and product-specific factor.

7.3.4.1 Systemic/Product diversification

Combining 7.6 and 7.7 gives:

$$FC(\sigma^A) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2 + (\sum_{i=1}^{d} \sigma^{i,m})^2} \tag{7.10}$$

Then the sum of the individual risk measures p and m are replaced with a general Brownian risk across all products, P and M. i.e:

$$\sigma^P = \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2} \tag{7.11}$$

and

$$\sigma^M = \sqrt{(\sum_{i=1}^{d} \sigma^{i,m})^2} \tag{7.12}$$

If only product-specific risk is considered in 7.10, the required funding capacity becomes:

$$\begin{aligned} FC(\sigma^P) &= -\sqrt{\Delta t} * \Phi^{-1}(1-p) \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2 + 0} \\ &= -\sqrt{\Delta t} * \Phi^{-1}(1-p) \cdot \sigma^P \end{aligned}$$

Similarly if only systematic risk is considered, the funding capacity becomes:

$$\begin{aligned} FC(\sigma^M) &= -\sqrt{\Delta t} * \Phi^{-1}(1-p) \cdot \sqrt{0 + (\sum_{i=1}^{d} \sigma^{i,m})^2} \\ &= -\sqrt{\Delta t} * \Phi^{-1}(1-p) \cdot \sigma^M \end{aligned}$$

Under perfect correlation the risk measures are additive:

$$FC(\sigma^P + \sigma^M) = FC(\sigma^P) + FC(\sigma^M)$$

The relation funding capacity under zero correlation (which was assumed in the beginning) to funding capacity under perfect correlation gives the effect of diversification measured in percentage:

$$\begin{aligned} \kappa &= \frac{FC(\sigma^A)}{FC(\sigma^P) + FC(\sigma^M)} \\ &= \frac{FC(1) \cdot \sigma^A}{FC(1) \cdot \sigma^P + FC(1) \cdot \sigma^M} \\ &= \frac{\sigma^A}{\sigma^P + \sigma^M} \end{aligned}$$

Kappa (κ) is the diversification factor between the product-specific risk factor and the systematic risk factor. The product-only and market-only Brownian risks (σ^P, σ^M) are then adjusted for kappa:

$$\sigma^{P,adj} = \kappa \cdot \sigma^P$$
$$\sigma^{M,adj} = \kappa \cdot \sigma^M$$

After the adjustments, the sum of the Brownian risk exposures should be equal to the aggregate risk exposure:

$$\begin{aligned}
FC(\sigma^{P,adj}) + FC(\sigma^{M,adj}) &= FC(\kappa \cdot \sigma^P) + FC(\kappa \cdot \sigma^M) \\
&= FC(\frac{\sigma^A}{\sigma^P + \sigma^M} \cdot \sigma^P) + FC(\frac{\sigma^A}{\sigma^P + \sigma^M} \cdot \sigma^M) \\
&= FC(\sigma^A) \cdot (\frac{\sigma^P}{\sigma^P + \sigma^M} + \frac{\sigma^M}{\sigma^P + \sigma^M}) \\
&= FC(\sigma^A)
\end{aligned}$$

The next step is to allocate the funding capacity for Brownian risk across all products ($\sigma^{P,adj}$, $\sigma^{M,adj}$) to individual products. The necessary funding capacity for product i with respect to market risk can be derived as:

$$\begin{aligned}
FC(\sigma^{M,adj}) &= FC(\kappa \cdot \sigma^M) \\
&= FC(\kappa \cdot \sum_{i=1}^{d} \sigma^{i,m}) \\
&= FC(\sum_{i=1}^{d} \kappa \cdot \sigma^{i,m}) \\
&= FC(\sum_{i=1}^{d} \sigma^{i,m,adj})
\end{aligned}$$

The last two rows show that:

$$\sigma^{i,m,adj} = \kappa \cdot \sigma^{i,m}$$

Hence the market risk for product i ($\sigma^{i,m}$) has to be adjusted with κ to account for diversification. Similarly for the product-specific factor:

$$\begin{aligned}
FC(\sigma^{P,adj}) &= FC(\kappa \cdot \sigma^P) \\
&= FC(\kappa \cdot \sqrt{\sum_{i=1}^{d} (\sigma^{i,p})^2}) \\
&\leq FC(\kappa \cdot \sum_{i=1}^{d} \sigma^{i,p})
\end{aligned} \tag{7.13}$$

Since the main goal is to create additive risk factors for both market and product risk, an additional adjustment term has to be incorporated to account for inter-product diversification.

7.3.4.2 Inter-product diversification

The adjustment term to account for inter-product diversification is defined as:

$$\kappa^p := \frac{\sigma^P}{\sum_{i=1}^{d} \sigma^{i,p}}$$
$$\Leftrightarrow$$
$$\sigma^P = \kappa^p \cdot \sum_{i=1}^{d} \sigma^{i,p} \tag{7.14}$$

Inserting 7.14 into 7.13 from the previous section gives:

$$\begin{aligned} FC(\sigma^{P,adj}) &= FC(\kappa \cdot \sigma^P) \\ &= FC(\kappa \cdot \kappa^p \cdot \sum_{i=1}^{d} \sigma^{i,p}) \\ &= FC(\sum_{i=1}^{d} \kappa \cdot \kappa^p \cdot \sigma^{i,p}) \\ &= \sum_{i=1}^{d} FC(\kappa \cdot \kappa^p \cdot \sigma^{i,p}) \\ &= FC(\sum_{i=1}^{d} \sigma^{i,p,adj}) \end{aligned} \tag{7.15}$$

This shows that:

$$\sigma^{i,p,adj} = \kappa \cdot \kappa^p \cdot \sigma^{i,p}$$

The funding capacity to cover the aggregate risk exposure FC(σ^A) can now be written as the sum of the individual risk exposures from every single product, thus the risk measures is now additive:

$$\begin{aligned} FC(\sigma^A) &= FC(\sigma^{P,adj}) + FC(\sigma^{M,adj}) \\ &= FC(\sum_{i=1}^{d} \sigma^{i,p,adj}) + FC(\sum_{i=1}^{d} \sigma^{i,m,adj}) \\ &= FC(\sum_{i=1}^{d} \kappa \cdot \kappa^p \cdot \sigma^{i,p}) + FC(\sum_{i=1}^{d} \kappa \cdot \sigma^{i,m}) \end{aligned}$$

$$= \sum_{i=1}^{d} -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \tag{7.16}$$

This means that the required funding capacity for product i equals:

$$FC(\sigma^{i,p}, \sigma^{i,m}) = -\sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \tag{7.17}$$

As mentioned in the beginning of section 7.3.3, transfer prices of stochastic components should be based on the cost of holding reserves against deviations in the cash flow. The cost of holding such reserves were presented in section 7.3.3.2. Inserting the required funding capacity for an individual product, with the risk exposure ($\sigma^{i,p}$, $\sigma^{i,m}$), together with the proportion financed through secured funding l into the cost function for funding capacity 7.8 gives the transfer price for product i:

$$\begin{aligned} TP(\sigma^{i,p}, \sigma^{i,m}) &= c_R(\sigma^{i,p}, \sigma^{i,m}, l) \\ &= -l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield \end{aligned} \tag{7.18}$$

The expression above is limited in the sense that it only states the transfer price of a product with no optionality[10] and for a unit time length Δt. It would be necessary to expand the model to account for optionality and arbitrary maturities[21].

7.3.4.3 Incorporating optionality and varying maturities

The intuition behind pricing optionality is that the bank is exposed to sudden (expected) liquidity shocks if a product has a more frequent optionality, i.e. the bank has to hold reserves against this optionality which implies additional costs for the bank. Optionality and varying maturities can be incorporated into 7.18 fairly easy by extending the expression, which in its original form calculates the Brownian transfer price for a product during the time-step Δt, to account for longer time periods which can be seen as multiples of Δt. As a start consider the expression 7.18 to be a function of the time-step Δt, i.e.:

$$\begin{aligned} TP(\sigma^{i,p}, \sigma^{i,m}, \Delta t) &= c_R(\sigma^{i,p}, \sigma^{i,m}, l) \\ &= -l \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield \end{aligned} \tag{7.19}$$

If the Brownian transfer price for a product is to be calculated for the maturity (=T) of the product, one can write T as a multiples of Δt, i.e. T = $n \cdot \Delta t$, inserting into 7.18 gives:

[10] Optionality refers to the customers (the counterparty's) possibilities to affect its liquidity position in the bank.

$$\begin{aligned} TP(\sigma^{i,p}, \sigma^{i,m}, n \cdot \Delta t) &= c_R(\sigma^{i,p}, \sigma^{i,m}, l) \\ &= -l \cdot \sqrt{n \cdot \Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield \\ &= -l \cdot \sqrt{n} \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield \end{aligned} \tag{7.20}$$

To account for optionality during this time period the maturity has to be decomposed into two periods n_1 and n_2, where n_1 denotes the time period without exercises and n_2 denotes the number of exercise dates:

$$T = n_1 \cdot n_2$$

Referring back to 7.20, n_1 can be interpreted as the time-length of which the product has no optionality and thus is unable to change (e.g. the balance of a savings account with fixed withdrawal dates) and n_2 is the number of times the transfer price is summed up until the maturity. This allows for 7.20 to be extended further to be a function of T as well:

$$\begin{aligned} TP(\sigma^{i,p}, \sigma^{i,m}, T, n_1) &= TP(\sigma^{i,p}, \sigma^{i,m}, n_1) \cdot n_2 \\ &= \sqrt{n_1} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \cdot n_2 \end{aligned} \tag{7.21}$$

To make it more convenient, we can rewrite 7.21 to be a function of maturity (=T) and number of exercises (=n_2):

$$\begin{aligned} TP(\sigma^{i,p}, \sigma^{i,m}, T, n_2) &= TP(\sigma^{i,p}, \sigma^{i,m}, n_1) \cdot n_2 \\ &= \sqrt{n_1} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \cdot n_2 \\ &= \sqrt{\frac{T}{n_2}} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \cdot n_2 \\ &= \sqrt{T} \cdot \sqrt{n_2} \cdot TP(\sigma^{i,p}, \sigma^{i,m}) \end{aligned} \tag{7.22}$$

From 7.22 it can be seen that the Brownian transfer price grows linearly with maturity and number of exercises which is reasonable since the uncertainty around the product cash flow is likely to increase with longer maturity and a larger optionality. Note that T is expressed in units of Δt and n_2 is the **total** number of exercises until maturity.

7.22 is the final expression for the pricing of the Brownian risk exposure and together with the deterministic transfer price derived in section 7.3.2 we're approaching a complete model for the internal pricing of products. One final component remains before the model is complete and that is the part which takes regulatory requirements into account.

7.3.5 Regulatory component

To this author's knowledge there is no existing literature which treats the subject of incorporating regulatory requirements into a funds transfer pricing methodology. This is probably due to the fact that regulation surrounding liquidity has not been a current topic up until the financial crisis and the regulation which has been developed in recent years is pending to come into force. Nevertheless, most FTP methodologies, sophisticated or not, is striving to describe the liquidity characteristics of a product to be able to adequately assign FTP rates to reflect these characteristics. It is therefore natural to try and incorporate the coming regulatory requirements into our FTP framework to account for stricter regulatory liquidity rules ahead.

As described in chapter 5, regulatory requirements such as the Liquidity coverage ratio (LCR) which comes into force in the beginning of 2015 will require banks to hold a certain portion of high quality liquid assets in relation to their aggregate short term cash flow. This will affect the pricing of balance sheet holdings and in particular the asset side since the characteristics of the assets affect both the stock of High Quality Liquid Assets[11] (HQLA) and the total short-term net cash outflow. An example helps demonstrating the situation:
A bank gives a mortgage to a customer, the mortgage is solely financed by existing funds on the banks balance sheet. In essence this means that the bank is converting the cash from one asset class to another, the mortgage. If the cash used to finance the mortgage was included in the stock of HQLA then this will change due to the conversion since the mortgage is classified as a Level 2B asset subject to a 25 percent haircut[12], which means that the stock of HQLA will decrease by the notional (=N) $\cdot$ 0,25. The issuance will have no effect on the *Total net cash outflows over the next 30 calendar days (TNCO$_{30}$)* since, as mentioned in section 5.1.1.2, the regulation does not allow for double counting items, i.e. if an asset is included in the stock of HQLA the associated cash inflow cannot also be included in the denominator as part of the (TNCO_{30}). The impact on the LCR can formally be described as follows:

Before the issuance of the loan:

$$\frac{\text{Stock of HQLA}}{\text{Total net cash outflows over the next 30 calendar days}} := \varphi_{before}$$

[11] see section ?? for a detailed discussion.
[12] see Appendix A.1 for classification of assets.

After the issuance:

$$\frac{\text{Stock of HQLA} - N \cdot 0,25}{\text{Total net cash outflows over the next 30 calendar days}} := \varphi_{after}$$

If the bank wants to keep the LCR at the same ratio as before the issuance and include the new asset (the mortgage) in the stock of HQLA, it will have to borrow the deficit, which is equal to the notional times the haircut-factor (=hf), N · 0.25 during the lifetime of the mortgage. It is assumed that the bank is able to raise funds through unsecured funding to finance this deficit and invest these funds in level 1 assets which is to be included in the stock of HQLA. The difference between the cost of funding (r_f) and the yield from the level 1 asset (r_A) is effectively the cost of funding ($=cf$) the decrease, i.e.:

$$\begin{aligned}(r_f - r_A) \cdot N \cdot hf \cdot T &= \\ (r_f - r_A) \cdot N \cdot 0,25 \cdot T &= cf_{mortgage}\end{aligned} \tag{7.23}$$

To be consistent, this cost should be assigned to the mortgage, i.e. it should be added to its transfer price, since it is the characteristics of an asset in terms of regulatory requirements, yield etc that determines the change in LCR and thus the cost of maintaining the ratio at the desired level, this has to be reflected in the transfer price of that specific asset.

The example above illustrates the situation when the bank finance the asset by existing funds, often banks finance its businesses by raising additional funds and thereby expanding its balance sheet. In such a situation the stock of HQLA increases or remains unchanged depending on the quality of the asset and if the bank chooses to include the new asset as part of the stock of HQLA or its associated cash inflow in the TNCO_{30}. This optionality is undesirable when assigning internal rates to assets, since our main goal is to develop a consistent, general transfer pricing methodology which should only take into account the characteristics of the product from a regulatory perspective, not the choice of the bank. I.e. the pricing methodology should be indifferent on the banks decision to include the asset in the stock of HQLA or its corresponding cash flow in the TNCO_{30}. Furthermore the methodology should be independent on the associated funding used to finance the asset as the notion of FTP is based on assigning internal rates to assets and liabilities separately and thereby creating the right incentives at the respective business unit irrespective of other business units. This suggests that the pricing methodology should not distinguish between the case when the bank chooses to finance assets with existing funds or the case when funds is raised for financing.

An additional factor which should be taken into account is *the total available balance of high quality liquid assets* currently on the banks balance sheet. This is because the stock of HQLA used in the calculation of LCR can comprise of any subset of the banks assets as long as they fulfill the regulatory requirements. The relationship between this subset and the total available balance of high quality liquid assets is an indicator of how sensitive the bank is to trading highly liquid assets against less liquid assets[13]. This thesis suggests that this sensitivity should be reflected in the transfer prices of assets to make the internal pricing more dynamic and account for the current holdings of the bank. This suggests 7.23 to be modified as:

$$(r_f - r_A) \cdot N \cdot hf \cdot \delta_t \cdot T = cf \tag{7.24}$$

where:

$$\delta_t = \frac{\textit{Stock of HQLA}_{\mathrm{t}}}{\textit{Total available balance of high quality liquid assets}_{\mathrm{t}}}$$

Note: δ_{t} is provided with a subscript t to emphasize the ratios dependence on the time (date) of measurement.
7.24 implies a linear relationship between the relation 'Stock of HQLA' to 'Total available balance of high quality liquid assets' and the cost of funding the decrease. This means that if the bank has a large balance of available high quality liquid assets **not** included in the stock of HQLA then the bank is insensitive to investing in assets which are classified as less liquid since it has a backup capacity and is not forced to include the new asset in the stock of HQLA to keep the LCR at its previous level. In contrast, if the bank has a very low balance of available high quality liquid assets outside the stock of HQLA, i.e. $\delta_{\mathrm{t}} \approx 1$, the bank has to fund the entire decrease in the stock of HQLA to keep the LCR constant in case it decides to invest in less liquid assets.

The function δ_t is here specified as a relative measure between regulatory requirements and the holdings a bank has at its disposal for fulfilling these requirements. The linear incorporation of the function can be revised to non-linear relationships or other methods of inclusion to fit the business model of the individual bank regarding risk-appetite, profitability etc. Regardless, the quality of an asset with respect to regulatory requirements should be reflected in one way or another into its transfer price.

Finally, all the components to be included in the transfer price of a product have been derived, the final expression for the total transfer price of a product i looks like the following:

[13]Less liquid assets in this case is referring to assets with stronger regulatory restriction and larger haircuts, e.g. level 2 assets.

$$\begin{aligned} TP_i^{Tot}(\mu_{\mathrm{t_k}}, \sigma^{\mathrm{i,p}}, \sigma^{\mathrm{i,m}}, T^{\mathrm{i}}, n^{\mathrm{i}}{}_2, hf^i) = {} & TP_i^D(\mu_{\mathrm{t_k}}) \\ & + TP_i^B(\sigma^{\mathrm{i,p}}, \sigma^{\mathrm{i,m}}, n^{\mathrm{i}}{}_2, T^{\mathrm{i}}) \\ & + TP_i^{Reg}(hf^i) \end{aligned} \tag{7.25}$$

$$= \sum_{j=0}^{T} (r(0,t_{\mathrm{j}}) - r_{\mathrm{b}}(0,t_{\mathrm{j}})) \cdot \mu_{\mathrm{t_j}} \cdot \Delta t \cdot (t_{\mathrm{j}} - t_0)$$
$$+ (-l \cdot \sqrt{T^i} \cdot \sqrt{n_2{}^i} \cdot \sqrt{\Delta t} \cdot \Phi^{-1}(1-p) \cdot \kappa \cdot (\kappa^p \cdot \sigma^{i,p} + \sigma^{i,m}) \cdot \Delta Yield) + (r_f - r_A) \cdot hf^i \cdot \delta_t \cdot T$$

7.4 Product Example

To illustrate the methodology derived in the last section, the transfer price of a 5Yr car loan with a notional of \$50 000 and monthly principal payments of ($\mu_{\mathrm{t_j}}$) \$300 will be calculated. The risk parameters ($\sigma^{\mathrm{i,p}}$, $\sigma^{\mathrm{i,m}}$) together with the adjustment terms (κ, κ^p) are chosen arbitrarily due to the lack of data.

As a start, the product independent parameters are determined. It is assumed that the spread between the funding curve $\mathrm{r_f}$ and the benchmark curve $\mathrm{r_b}$ is constant at 60BPS (=0.6%). 50% of the Brownian risk exposure is backed by reserves and the exposure is calculated at a 99% confidence level. The cost of holding reserves to cover for the Brownian exposure is equal to the funding spread, i.e ΔYield $= \frac{60BPS}{s.t\ dev}$. The diversification factor between the product-specific and the systematic factor (κ) is chosen to be 0.5 and the diversification factor between products (κ^{p}) is set to 0.3. The final none product specific parameter is δ_{t} which is set to 0.5 during the lifetime of the product.

$$r_{\mathrm{f}}(0,t_{\mathrm{j}}) - r_{\mathrm{b}}(0,t_{\mathrm{j}}) = 60BPS$$

$$(t_{\mathrm{j}} - t_0) = 30 \cdot j$$

$$\mathrm{n_p} = 12 \text{ (nbr of yearly principal payments)}$$

$$\mu_{\mathrm{t_0}} = -50\ \ 000$$

$$\mu_{\mathrm{t_j}} = \frac{300}{50\,000} = 0,006 \ \ \forall t_j,\ j = 1, ..., T-1$$

$$\mu_{\mathrm{t_T}} = \frac{50\,000 - (12 \cdot 300 \cdot T)}{50\,000} \text{(Rest payment)}$$

$$l = 0,5$$

$$\Phi^{-1}(1 - 0.99) = -2.3263$$

$$\kappa = 0.8$$

$$\kappa^p = 0.3$$

$$\Delta Yield = \frac{60BPS}{s.t\ dev}$$
$$\delta_t = 0.5\ \ \forall t_k,\ k = 1, ..., T$$

The next step is to determine the product-specific parameters. First the product specific standard deviation ($\sigma^{\mathrm{i,p}}$) is set to 30% and the systematic standard deviation ($\sigma^{\mathrm{i,m}}$) is chosen to 20%. The maturity is 5Yr (1826 days) and the optionality (n_2^i) in this case refers to the number of total options the client has to **not** pay the principal payments which is 12 times per year due to the monthly payments, thus $n_2^i = 5{\cdot}12 = 60$. Finally, Car loans are not allowed to be included into the stock of HQLA which means that the haircut is equal to 100 %[14].

$$\sigma^{\mathrm{i,p}} = 0.3$$
$$\sigma^{\mathrm{i,m}} = 0.2$$
$$T^i = 5$$
$$n_2^i = 60\ (T \cdot 12)$$
$$hf^i = 1$$

Inserting these values in our framework yields the following:

$$\begin{aligned} TP_{Car\ loan}(\mu_{\mathrm{t_j}}, 0.3, 0.2, 5, 60, 1) &= \sum_{j=0}^{T^i \cdot n_p} \frac{60BPS}{365Days} \cdot \mu_{\mathrm{t_j}} \cdot 1 \cdot \frac{365Days}{12} \cdot j \\ &\quad + \frac{1}{365}(-0.5 \cdot \sqrt{5 \cdot 365} \cdot \sqrt{60} \cdot \sqrt{1} \cdot \Phi^{-1}(1 - 0.99) \cdot 0.8 \\ &\qquad \cdot (0.3 \cdot 0.3 + 0.2) \cdot 60BPS) + 60BPS \cdot 1 \cdot \frac{1}{2} \cdot 5 \\ &= [246, 9 + 9.7 + 150]\ BPS \\ &= 406, 6\ BPS \end{aligned}$$

Note that the transfer price is expressed as BPS (or percentage), by multiplying with the notional of the loan one retrieves the actual liquidity cost of the loan. The transfer price above is for the whole lifetime of the loan, often it is more useful to express it in annual terms which is retrieved by simple dividing the total transfer price with the maturity of the product.

$$\frac{406, 6\ BPS}{5\ Years} = 81, 32BPS/Year \approx 0.81\%$$

[14] The reason for this is most likely that the value of the underlying security (the car) is not considered to be stable.

This figure is an estimation of the annual liquidity cost of the Car loan for the bank. To get the total funding cost of the loan, which is essentially the cost of replicating its cash flow, the cost of funding each annual cash flow (=BC) at the corresponding benchmark rate for every year must be calculated and added to the annual transfer price. The cost is calculated as:

$$BC_j = \sum_{i=1}^{n_p} r_{\mathrm{b}}(0, t_{\mathrm{j,i}}) \cdot \mu_{t_{\mathrm{j,i}}} \cdot (t_{\mathrm{j,i}} - t_0),\ \forall j = 1, .., T \tag{7.26}$$

$$\begin{aligned} &where: \\ &\quad r_{\mathrm{b}}(0, t_{\mathrm{j,i}}) = \text{Benchmark rate at year j time i} \\ &\mu_{t_{\mathrm{j,1}}} = \text{Normalized principal payment year j at time i.} \end{aligned}$$

This cost is calculated for every year of the car loan and summarized in the left table, right column below.

US treasury yields

Maturity	rate(%)	BPS
1,0 year	0,13	0.936
2,0 year	0,39	5.616
3,0 year	0,76	16.416
4,0 year	1,24	35.712
5,0 year	1,72	612.32

+

Yearly TP

Maturity	TP(BPS)
1,0 year	81,32
2,0 year	81,32
3,0 year	81,32
4,0 year	81,32
5,0 year	81,32

=

Funding cost

Maturity	FC(BPS)
1,0 year	82.26
2,0 year	86.94
3,0 year	97.74
4,0 year	117.03
5,0 year	693,64

The column denoted FC(BPS) in the right table above shows the funding cost for every year of the car loan. The sum of the respective yearly costs yields the total funding cost during the lifetime of the product:

$$(82.26 + 86.94 + 97.74 + 117.03 + 693.64) = \mathbf{1074.61} \text{ BPS}$$

To conclude our example, let's assume that the product yields an annual interest of 4% =(400 BPS), giving a total of 400· 5 = 2000 BPS for five years. The total profit will simply be (2000-1075) BPS = 925 BPS per unit notional, which in our example becomes \$50 000 · 0.0925 = \$4 625[15].

The separation between the benchmark rate and the transfer price gives a transparent view of the part of total funding cost arising from the base rate and the part which arises from the liquidity characteristics of the product. Comparison of the profitability

[15] This example does not take into account any additional cost involved in the issuance of the loan such as administrative costs, fees etc.

between rates offered in the past with respect to liquidity risk becomes a lot more difficult if the total funding cost is not separated into a base rate and a liquidity premium. The funding cost of a loan granted in the past may prove to have been lower even though the base rate was at higher levels, a lower, and possibly underestimated liquidity premium charged to the business unit could be one reason. This would suggest that the business unit is not charged adequately for the liquidity risk arising from their assets.

Chapter 8

Simulation

As mentioned in previous sections the forward yield curve, derived from the current yield curve of any security, is one of the main indicators of the markets expectations regarding future yields for a specific security. On the basis of these expectations, we will implement the Hull & White one factor stochastic interest rate model discussed in section 6.2 and use it as a prediction tool for analyzing future possible outcomes of the short term interest rate. Then, to be able to make predictions for internal transfer prices based on the outcome of the simulation at a future point in time, one has to be able to say something about the curvature of the yield curve from this future point. To do this, three different scenarios for the future yield curve will be constructed using historical data from US treasury rates, these data will then be adjusted to the interest rate level from each simulation which creates a future benchmark curve to be used when determining future transfer prices and total funding costs. The first sections will present the data to be used for the interest rate simulations together with details for the implementation of the simulation tool in MATLAB. In section 8.3 details for the data chosen to construct the scenarios for the future yield curve. Finally, in the last section a generic product which will be used to facilitate comparison of the results from the simulations will be presented.

8.1 Simulation data

The interest rate which will be subject for simulation is the 1 month US treasury rate, thus the initial yield curve for US treasury notes will be used as an input in the model. The data is presented in table 8.1. The data submitted to construct the yield curve is consisting of yields for a limited number of predetermined maturities, 1 Month, 3 Months, 1Year,...,10Years, the simulation however will simulate interest rate paths with daily increments to reflect the daily changes in yields on treasury notes. To make the interest rate data applicable to the model, linear interpolation is performed on the data with increments equal to one day.

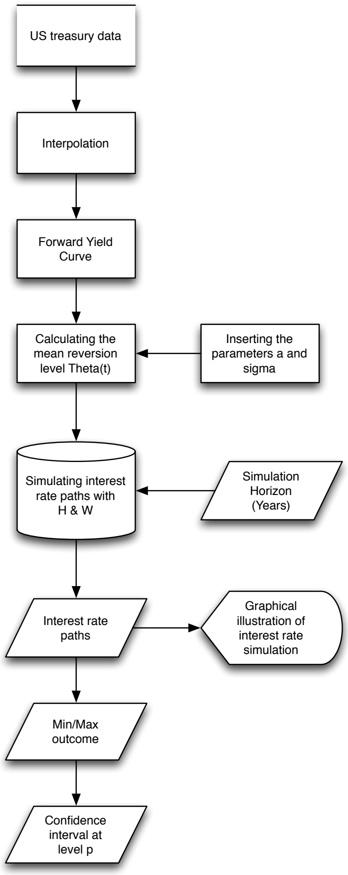
TABLE 8.1: US treasury yields

Maturity	Yield(%)
1 Month	0,01
3 Month	0,07
6 Month	0,09
1,0 year	0,13
2,0 year	0,39
3,0 year	0,76
5,0 year	1,72
7,0 year	2,41
10,0 year	3,00

8.2 Matlab implementation

Figure 8.1 shows the setup for the simulation of interest rate in Matlab.

FIGURE 8.1: Matlab scheme



At inception, data for US treasury notes are interpolated with daily (=Δt) increments. When the yield curve has been interpolated, the bootstrap algorithm in expression 4.2 is applied and the forward yield curve is constructed for time intervals equal to Δt. In the next step the parameters a and σ are estimated using an optimization scheme which calculates theoretical prices for interest rate swaptions and compare these with quoted market prices for swaptions[1]. By minimizing the difference in prices with respect to a and σ and averaging over the outcome one is able to extract a market consistent estimate of their respective values[2]. This way one is able to obtain one market approximation of the two parameters to be used in the interest rate model. Next, the drift parameter $\theta(t)$ is calculated with expression 6.3 for every t, up to the longest maturity of the dataset

[1]The interested reader is referred to Damiano, B & Mercurio, F. *Interest Rate Models - Theory and Practice* p 287 for details in calibrating interest rate models to swaption prices.

[2]The optimization scheme which has been used to approximate a and σ has been implemented by EY in Excel.

with a step length of Δt.

Finally the simulation is performed using algorithm 1 below. The additional input required is the simulation horizon expressed in years together with the number of simulation to be performed. The simulation stores the simulated interest rate paths in the variable r which becomes a matrix with dimension equal to [SimPaths] $\times$ [daysSim] from which the highest and the lowest value at the end of the simulation horizon is extracted. Finally a confidence interval for the outcomes of r at a chosen confidence level is constructed. In the next chapter results from interest rate simulations for different horizons are presented together with the corresponding effect in transfer pricing and total funding costs for a generic product.

T = Simulation Horizon (years);

dt = 1/365;

daysSim = T/dt;

SimPaths = 1000;

$\frac{\partial F}{\partial t}$ = ForwardRates(2:daysSim+1) - ForwardRates(1:daysSim);

θ(t) = $\frac{\partial F}{\partial t}$ + a $\cdot$ ForwardRates(1:daysSim) + $\frac{a^2}{2\cdot\sigma}$ $\cdot$ (1-e$^{(\text{-}2\cdot \text{a}\cdot \text{t})}$);

for *i:daysSim* **do**

  dW = normrnd(0, dt, [SimPaths,1]); `/* Changes in the Wiener process */`

  dr(:,i) = (θ(i) - a$\cdot$r(:,i))$\cdot$dt + $\sigma\cdot$dW(:,1); `/* Calculate the change in r */`

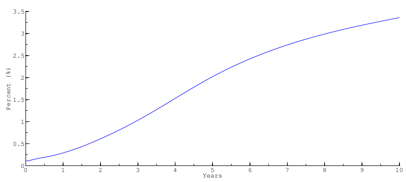
  r(:,i+1)=r(:,i)+dr(:,i); `/* Adding the change to r */`

end

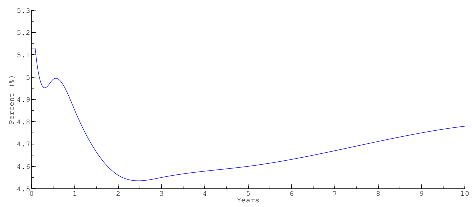
Algorithm 1: Simulation of interest rates

8.3 Future yield curves

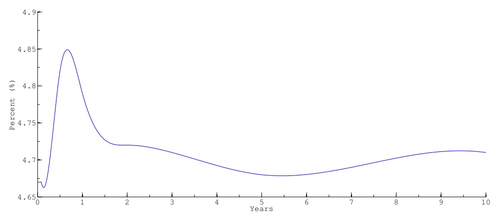
The three different yield curves which will be used is chosen to depict three different types of scenarios or future possible economic developments at the end of the simulation horizon. Each of these yield curves is chosen from historic US treasury rates and shifted vertically to fit the initial level of interest which is obtained through the simulation. The first scenario which can be seen in figure (a) below illustrates a situation where the interest rates are expected to rise steadily in the future. Figure (b) is from mid 2007 when the interest rates were expecting to decrease, and finally figure (c) is illustrating a situation where interest rates are expected to remain fairly constant within the next years.



(a) US treasury yield curve from 3/1 -2011



(b) US treasury yield curve from 31/7 -2007



(c) US treasury yield curve from 21/3 -2006

FIGURE 8.2: US treasury yield curves from different time periods[5]

8.4 Generic product

A generic product will be presented to facilitate comparison between scenarios and improve understanding of the costs attributable to the base funding (at the benchmark rate) and the liquidity characteristics of the product.

The product is comprised of a five year loan with equally sized yearly principal payments, i.e. 1/5 of the notional is payed every year until the end of year 5. This means that the product is exposed to optionality once every year, hence $n_2 = 5$. The funding spread is assumed to be constant at 60BPS during the lifetime of the product. The product is not qualified as a HQLA by the regulation. The remaining parameters is the same as in example 7.4. This gives:

$$r_{\mathrm{f}}(0,t_{\mathrm{j}}) - r_{\mathrm{b}}(0,t_{\mathrm{j}}) = 60BPS$$
$$n_{\mathrm{p}} = 1$$
$$\mu_{t_{\mathrm{j}}} = \frac{1}{5} \;\; \forall t_j, \; j = 1, ..., T$$
$$l = 0,5$$
$$\Phi^{-1}(1-0.99) = -2.3263$$
$$\kappa = 0.8$$
$$\kappa^p = 0.3$$
$$\Delta Yield = \frac{60BPS}{s.t\ dev}$$
$$\delta_t = 0.5 \;\; \forall t_k, \; k = 1, ..., T$$
$$\sigma^{\mathrm{i,p}} = 0.3$$
$$\sigma^{\mathrm{i,m}} = 0.2$$
$$T^i = 5$$
$$n_2^i = 5$$
$$hf^i = 1$$

The transfer price then becomes:

$$\begin{aligned} TP_{Gen}(\mu_{t_{\mathrm{j}}}, 0.3, 0.2, 5, 5, 1) = & \sum_{j=0}^{T^i \cdot n_p} 60BPS \cdot \mu_{t_{\mathrm{j}}} \cdot j + \frac{1}{365}(-0.5 \cdot \sqrt{5 \cdot 365} \cdot \sqrt{5} \cdot \sqrt{1} \\ & \cdot \Phi^{-1}(1-0.99) \cdot 0.8 \cdot (0.3 \cdot 0.3 + 0.2) \cdot 60BPS) + 60BPS \cdot 1 \cdot \frac{1}{2} \cdot 5 \\ = & \left[180 + 4.32 + 150\right] BPS \\ = & \; 334.24\, BPS \\ = & \; 3.34\% \end{aligned}$$

Chapter 9

Results

Results are presented in the following way: In section 9.1 results from the interest rate simulation described in section 8.2 are presented. In section 9.2 the results from section 9.1 are used together with the scenarios presented in section 8.3 generating three scenarios for every simulation horizon, the results are then used to calculate the cost of funding the generic product at the corresponding benchmark curve obtained from each scenario. Finally the results will be added to the transfer price of the generic product calculated in section 8.4 which will result in the total cost of funding for the product under different scenarios in the future.

9.1 Simulation of interest rates

The simulations have been conducted using data presented in section 8.1 as input. The simulation horizon ranges from T=1,2...,5 years, with 1000 simulated trajectories for each horizon. The mean reversion parameter a and the volatility parameter σ has been approximated to *0.44* and *0.30* respectively, these values will be constant during the simulations irrespective of simulation-horizon. From each simulation a 99 % level of confidence interval is constructed together with the highest and the lowest observed outcomes. The results of the simulations will be presented graphically for the years 1,3 and 5 and the results from all simulations will be summarized in a table at the end of the section.

9.1.1 One year horizon

In figure 9.1 the drift is not distinct, but a small positive drift can be identified and the mean of the process varies around slightly higher interest rate levels than current levels. The mean of the process at the end of year 1 was equal to 0.2854 %. The white area in the distribution depicted in figure 9.2 is representing 99 percent of the outcomes from the simulation together with a confidence interval for r represented between the blue intercepts along the horizontal axis. In this simulation a confidence interval at the 99 percent level was estimated to [-0.2711 0.8419] and the corresponding minimum and maximum outcomes was -0.4311 and 0.9512. As discussed when presenting the Hull & White model in chapter 6.2 negative interest rates are allowed in the model and represents a substantial part in our simulation when interest rates are expected to fluctuate around low levels in the near future.

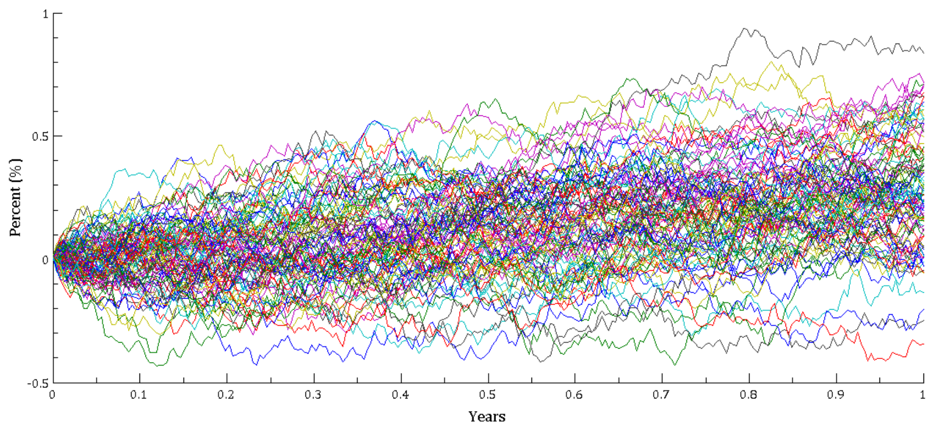


FIGURE 9.1: One year simulation of interest rates. The graphical illustration of the simulation contains only 100 trajectories for visibility purposes.

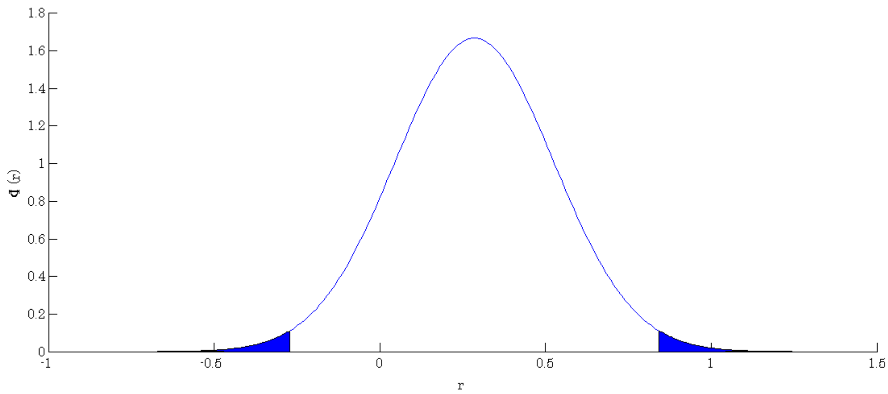


FIGURE 9.2: Normal plot of the distribution of the outcome of r. The confidence interval at the 99 percent level is obtained between the blue areas.

9.1.2 Three year horizon

In contrast to the one year simulation, where the drift was reasonably small, the simulation for the interest rate looking three years ahead shows a much more distinct positive drift during the whole simulation period. The mean of the simulation was 1.6153 % and the corresponding 99 percent confidence interval at the three year end was [1.0183 2.2022] with min/max values 0.8801/2.3263. Note that the confidence intervals has approximately the same width irrespective of their simulation horizon due to the constant variance.

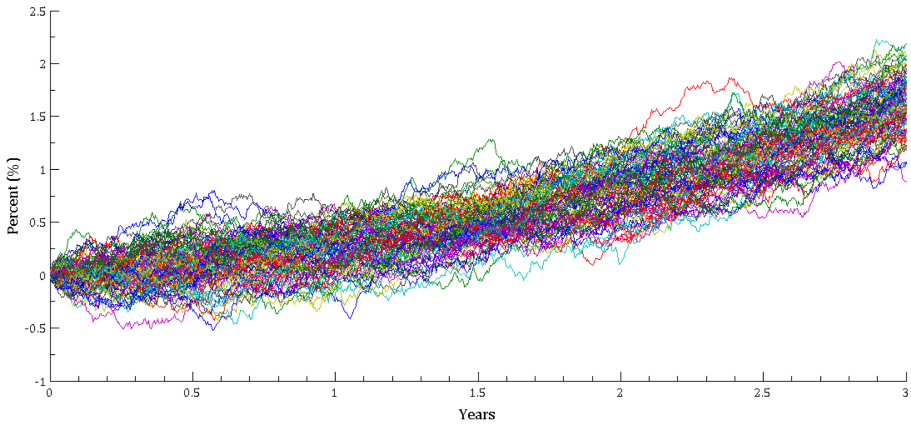


FIGURE 9.3: Three year simulation of interest rates

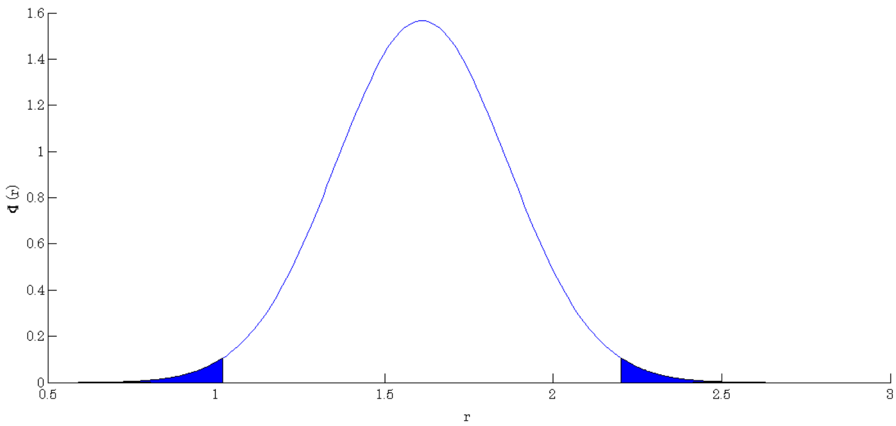


FIGURE 9.4: Normal plot of the distribution of the outcome of r for the three year simulation.

9.1.3 Five year horizon

The results of the five year simulation is similar to the three year simulation, with a distinct positive drift and interest rate levels which are considerably higher than todays levels with a mean of 3.7019 %. The 99% confidence interval is estimated to [3.2248 4.1790] and the minimum and maximum outcome of the simulation is 3.0143/4.4035. The effect of the mean reversion can be clearly seen for trajectories which deviates from the drift and is eventually pushed back to the expected levels.

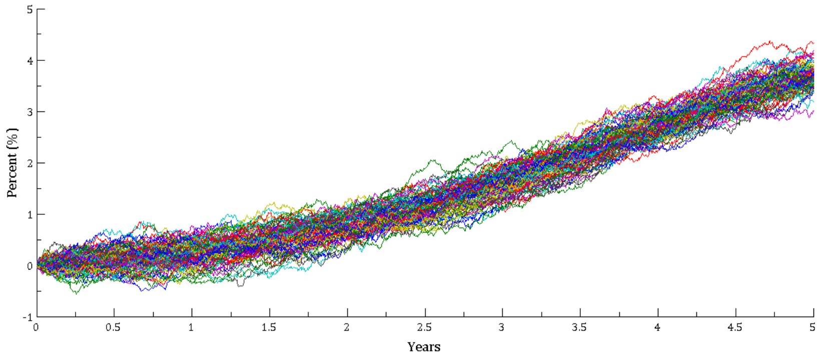


FIGURE 9.5: Five year simulation of interest rates

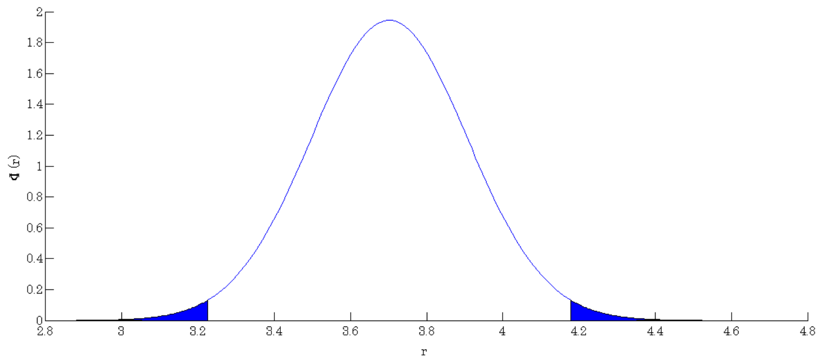


FIGURE 9.6: Normal plot of the distribution of the outcome of r for a simulation horizon of 5 years.

TABLE 9.1: Results from the simulation

Horizon	Mean(%)	Confidence Interval	Min/Max
1 Year	0.2854	[-0.2711 0.8419]	-0.4311/0.9512
2 Years	0.8164	[0.2698 1.3630]	0,1221/1.4911
3 Years	1.6153	[1.0183 2.2022]	0.8801/2.3263
4 Years	2.7133	[2.1868 3.2398]	1.9517/3.3322
5 Years	3.7019	[3.2248 4.1790]	3.0143/4.4035

9.2 Projections for future funding costs

The subsections below are divided by simulation horizon with each subsection containing three tables, one for each scenario with the corresponding yield curve for that scenario adjusted to the *mean interest rate level* of the interest rate simulation for the horizon which can be seen in table 9.1.
The cost of funding the generic product at the benchmark curve is calculated using 7.26 and presented at the bottom of each table of every scenario together with a confidence interval (=CI) for the cost. Finally the transfer price of the generic product is added to the base cost which yields the total funding cost of the product.

9.2.1 One Year

US treasury Yields (Rising)

Maturity	rate(%)
1,0 year	0.47
2,0 year	0.79
3,0 year	1.21
4,0 year	1.70
5,0 year	2.20
$\mathbf{BC_{Gen}}$	4.69
$\mathbf{CI_{BC_{Gen}}}$	[3.02 , 6.36]

US treasury Yields (Falling)

Maturity	rate(%)
1,0 year	0.01
2,0 year	-0.28
3,0 year	-0.29
4,0 year	-0.27
5,0 year	-0.24
$\mathbf{BC_{Gen}}$	-0.74
$\mathbf{CI_{BC_{Gen}}}$	[-2.42 , 0.92]

US treasury Yields (Flat)

Maturity	rate(%)
1,0 year	0.41
2,0 year	0.34
3,0 year	0.33
4,0 year	0.31
5,0 year	0.30
$\mathbf{BC_{Gen}}$	0.964
$\mathbf{CI_{BC_{Gen}}}$	[-0.72 , 2.62]

Scenario	$\mathbf{BC_{Gen} + TP_{Gen}}$	$\mathbf{CI_{BC_{tot}} + TP_{Gen}}$
Increasing	8.03	[6.36 , 9.70]
Falling	2.60	[0.92 , 4.26]
Flat	4.30	[2.62 , 5.96]

9.2.2 Two Years

US treasury Yields (Rising)

Maturity	rate(%)
1,0 year	1.00
2,0 year	1.32
3,0 year	1.74
4,0 year	2.23
5,0 year	2.73
$\mathbf{BC_{Gen}}$	6.28
$\mathbf{CI_{BC_{Gen}}}$	[4.64 , 7.92]

US treasury Yields (Falling)

Maturity	rate(%)
1,0 year	0.54
2,0 year	0.25
3,0 year	0.24
4,0 year	0.26
5,0 year	0.29
$\mathbf{BC_{Gen}}$	0.85
$\mathbf{CI_{BC_{Gen}}}$	[-0.79 , 2.48]

US treasury Yields (Flat)

Maturity	rate(%)
1,0 year	0.94
2,0 year	0.87
3,0 year	0.86
4,0 year	0.84
5,0 year	0.30
$\mathbf{BC_{Gen}}$	2.554
$\mathbf{CI_{BC_{Gen}}}$	[0.91 , 4.19]

Scenario	$\mathbf{BC_{Gen} + TP_{Gen}}$	$\mathbf{CI_{BC_{tot}} + TP_{Gen}}$
Increasing	9.62	[7.98 , 11.26]
Falling	4.19	[2.64 , 5.82]
Flat	5.89	[4.25 , 7.53]

9.2.3 Three Years

US treasury Yields (Rising)

Maturity	rate(%)
1,0 year	1.83
2,0 year	2.15
3,0 year	2.57
4,0 year	3.07
5,0 year	3.56
$\mathbf{BC_{Gen}}$	8.78
$\mathbf{CI_{BC_{Gen}}}$	[6.88 , 10.44]

US treasury Yields (Falling)

Maturity	rate(%)
1,0 year	1.37
2,0 year	1.08
3,0 year	1.07
4,0 year	1.10
5,0 year	1.12
$\mathbf{BC_{Gen}}$	3.35
$\mathbf{CI_{BC_{Gen}}}$	[1.45 , 5.00]

US treasury Yields (Flat)

Maturity	rate(%)
1,0 year	1.77
2,0 year	1.70
3,0 year	1.69
4,0 year	1.68
5,0 year	1.66
$\mathbf{BC_{Gen}}$	5.05
$\mathbf{CI_{BC_{Gen}}}$	[3.15 , 6.70]

Scenario	$\mathbf{BC_{Gen} + TP_{Gen}}$	$\mathbf{CI_{BC_{tot}} + TP_{Gen}}$
Increasing	12.12	[10.22 , 13.78]
Falling	6.69	[4.79 , 8.34]
Flat	8.39	[6.49 , 10.04]

9.2.4 Four Years

US treasury Yields (Rising)	
Maturity	rate(%)
1,0 year	2,89
2,0 year	3,21
3,0 year	3,63
4,0 year	4,13
5,0 year	4,62
$\mathbf{BC_{Gen}}$	11,96
$\mathbf{CI_{BC_{Gen}}}$	[10.39 , 13.55]

US treasury Yields (Falling)	
Maturity	rate(%)
1,0 year	2,43
2,0 year	2,14
3,0 year	2,13
4,0 year	2,16
5,0 year	2,18
$\mathbf{BC_{Gen}}$	6,53
$\mathbf{CI_{BC_{Gen}}}$	[4.95 , 8.11]

US treasury Yields (Flat)	
Maturity	rate(%)
1,0 year	2,83
2,0 year	2,76
3,0 year	2,75
4,0 year	2,74
5,0 year	2,72
$\mathbf{BC_{Gen}}$	8,23
$\mathbf{CI_{BC_{Gen}}}$	[6.66 , 9.82]

Scenario	$\mathbf{BC_{Gen} + TP_{Gen}}$	$\mathbf{CI_{BC_{tot}} + TP_{Gen}}$
Increasing	15.30	[13.73 , 16.89]
Falling	9.87	[8.29 , 11.45]
Flat	11.57	[10.00 , 13.16]

9.2.5 Five Years

US treasury Yields (Rising)	
Maturity	rate(%)
1,0 year	3,88
2,0 year	4,20
3,0 year	4,62
4,0 year	5,12
5,0 year	5,61
$\mathbf{BC_{Gen}}$	14,93
$\mathbf{CI_{BC_{Gen}}}$	[13.50 , 16.37]

US treasury Yields (Falling)	
Maturity	rate(%)
1,0 year	3,42
2,0 year	3,13
3,0 year	3,12
4,0 year	3,15
5,0 year	3,17
$\mathbf{BC_{Gen}}$	9,50
$\mathbf{CI_{BC_{Gen}}}$	[8.07 , 10.93]

US treasury Yields (Flat)	
Maturity	rate(%)
1,0 year	3,82
2,0 year	3,75
3,0 year	3,74
4,0 year	3,73
5,0 year	3,71
$\mathbf{BC_{Gen}}$	11,20
$\mathbf{CI_{BC_{Gen}}}$	[9.77 , 12.64]

Scenario	$\mathbf{BC_{Gen} + TP_{Gen}}$	$\mathbf{CI_{BC_{tot}} + TP_{Gen}}$
Increasing	18.27	[16.84 , 19.71]
Falling	12.84	[11.41 , 14.27]
Flat	14.45	[13.11 , 15.98]

Chapter 10

Conclusions

10.1 Interest rate simulations

The results from the simulation demonstrates one way of forecasting short term interest rates, which is one of the most vital variables for financial institutions, based on todays expectations. Here the simulation is used as a tool for predicting future funding costs of financial products which could be used in assisting bank management in strategic funding decisions today for upcoming or expected business opportunities in the future.

The results from the interest rate simulations are expected in many ways since the expectations for future interest rates are incorporated in the interest rate model through the forward yield curve. However a couple of interesting features can be extracted from the simulations. The first simulation, looking one year ahead, resulted in a confidence interval including negative interest rates which previously was considered unrealistic but in recent years have been seen as increasingly probable for some of the safest asset classes. In this context, negative interest rates are not likely to imply that borrowing funds would result in a positive revenue for the bank, however the spread between the risk-free interest rate and the banks funding rate may decrease when investors and institutions seeks alternative placements (with positive returns), forcing down interest rates at which banks fund themselves. The simulations looking 2-5 years ahead showed results in line with the expectations with only slightly lower means than the corresponding maturities from the forward yield curve. This is explained by the parameter estimation of a and σ from market securities which can vary depending on the securities used for calibration.

10.2 Projection of total funding cost

One of the key conclusions to be drawn when examining the internal transfer pricing methodology presented in this theses is that the transfer price does not depend on the interest rate level of the underlying risk-free rate due to the separation of the total funding cost into two components, the cost of funding at the risk-free rate and the cost of funding the liquidity characteristic of the product. The transfer price is only dependent on the spread between the banks funding rate and the underlying risk-free rate. The spread, in turn, depends on the credit worthiness, often measured as a credit rating of an institution. If the credit rating does not change there is no reason for the spread to change. This means that if the credit rating of the bank remains constant irrespective of movements in the risk-free rate, i.e. the funding rate shifts in parallel with the risk-free rate, the transfer price will remain constant. This implies that the transfer price for the generic product used in the projection of the total funding cost will generally remain constant for all the future yield curves and simulation horizons.

The results after the first simulation suggests that if the economic activity is expected to remain high after the first year, and thus interest rates are expected to rise further, funding costs will be considerably higher than if the economic activity is expected to slow down. If interest rates are expected to rise, funding costs is more than three times as high (8.03%) as if interest rates are expected to fall due to a sharp economic slowdown (2.60%) and nearly twice as high as if the rates are expected to remain constant (4.30%). The second scenario illustrates the situation mentioned previously where the funding spread is assumed to shrink due to the risk-free rate becoming negative which is likely to push other interest rates down as investors seeks alternative investments with positive returns. This results in a lower funding spread ($\mathrm{r_f}(0,\mathrm{t_j})$-$\mathrm{r_b}(0,\mathrm{t_j}$ $<$60BPS) which implies a lower transfer price and a cost for the benchmark curve close to zero.

After the two year simulation horizon, total funding cost for the first scenario (9.62%) is still more than twice as high as the scenario describing an economic downturn (4.19%) and more than one and a half time as high as if the economic outlook after two years is modest (5.89%). This means that if, for example an enterprise seeks financing for a project starting in two years time, the unit responsible for issuing a loan should analyze and incorporate its expectations regarding future economic activity to be able to give a competitive but still profitable offer for financing the future project. The difference between the highest and the lowest outcomes in the confidence interval is around 3.2-3.4 percent for each scenario and thus translates into a difference in yearly rates of 0.64 - 0.68 percent, which is a large relative difference if the general levels of interest rates are low.

The funding costs for the three and four year simulation horizon is significantly higher compared to todays levels for nearly all future expectations of the risk-free yield curve, it is only the second scenario in year three, in which we would expect a future fall in interest rates after three years, that the cost remains relatively low. Worth mentioning is the relative decrease in the proportion of total funding costs, for all scenarios, which constitutes of the transfer price. This is natural because of the assumption of a constant funding spread (except if interest rates become negative) which have a greater relative impact on the cost of funding when interest rates are low.

For the longest horizon, the interest rate levels for the short term interest rate are expected to be much higher than todays levels, thus affecting the future funding costs in the same direction irrespective of the forward expectations five years from now. The difference between the scenarios is still large in absolute terms but the relative difference has decreased compared to shorter horizons. The funding cost for the first scenario (18%) is now barely one and a half times as high as the second scenario and one fifth higher than if the future yield curve is expected to be flat after five years.

The projections could, as mentioned in the beginning, be a part of banks forward looking measures when planning for future funding decisions and to assist in highlighting risks involved in measuring the profitability of future business opportunities with respect to financing. Furthermore, banks will be able to get a more transparent view of the costs/benefits associated with financing assets and raising liabilities with respect to liquidity and liquidity risk, which is quantified through the FTP methodology, and the cost associated with the benchmark curve. The FTP method could also easily be extended to include individual dependencies between financial products and thereby get a more accurate estimate when assigning internal prices to products for different portfolios and balance sheets.

Chapter 11

Discussion and improvements

The objective when simulating the outcome of future interest rates is to be able to make predictions about its possible evolution over time. This is a cumbersome task for a variable like the short term interest rate (e.g. the US treasury rate) because of its possible dependency on numerous other variables, which in turn can be difficult to estimate and predict. Macro variables such as GDP, unemployment rates, indebtedness etc are along with other factors likely to have some impact on the evolution of interest rates. An interest rate model which dynamically is able to adapt to changes in every variable affecting the future outcome of interest rates would undoubtedly be very useful but hard to create.

The Hull & White one factor model used in this thesis has its limitations when it comes to adapt itself to different scenarios that might occur in the future because it only takes into account the current expectations of the forward yield curve as the main driver for future interest rates. Additional explaining variables which could explain sudden changes in the economic environment would be a step towards a more dynamic interest rate model.

The parameter estimation of the speed of mean reversion a and σ implies some uncertainty since calibration can be done via different market instruments whereby different estimations is obtained and its hard to determine which estimate is correct. An extension of the model is also possible whereby both a and σ is allowed to be time-varying, however this involves calibrating these parameters to a large number of historic quotes on interest rate derivatives, some more liquid than others which can result in unreliable estimates due to differences in market prices for non-liquid securities.

The FTP model examined in this thesis was chosen because of its generality and its ability to price all sorts of financial products and cash flows both on the asset side and the liability side. The deterministic part of the model is straightforward however the stochastic part require a thorough data analysis in order to estimate the number of risk factors, their dependencies and their respective parameter values adequately. It is assumed that the risk factors are normally distributed with zero correlation which is a simplified and to some extend unrealistic assumption. Unfortunately, due to the lack of data, an estimation of the risk factors and their dependencies were not possible which meant that arbitrary values and a generic product had to be used for illustration. Banks who have access to large databases for historic cash flows have an excellent opportunity to analyze the data and estimate the parameter values of the respective risk factors which could result in a profound understanding of the liquidity risk arising from its assets and subsequently accurate transfer prices could be assigned to its holdings.

Finally, the linear inclusion of the regulatory requirements into the transfer pricing method requires further investigations for determining a justified contribution to the transfer price.

Appendix A

A. Regulatory requirements

A.1 Classifications of assets and cash flow in LCR

Figure A.1

Illustrative Summary of the LCR

(percentages are factors to be multiplied by the total amount of each item)

Item	Factor
Stock of HQLA	
A. Level 1 assets:	
• Coins and bank notes • Qualifying marketable securities from sovereigns, central banks, PSEs, and multilateral development banks • Qualifying central bank reserves • Domestic sovereign or central bank debt for non-0% risk-weighted sovereigns	100%
B. Level 2 assets (maximum of 40% of HQLA):	
Level 2A assets	
• Sovereign, central bank, multilateral development banks, and PSE assets qualifying for 20% risk weighting • Qualifying corporate debt securities rated AA- or higher • Qualifying covered bonds rated AA- or higher	85%
Level 2B assets (maximum of 15% of HQLA)	
• Qualifying RMBS	75%
• Qualifying corporate debt securities rated between A+ and BBB-	50%
• Qualifying common equity shares	50%
Total value of stock of HQLA	

Cash Outflows	
A. Retail deposits:	
Demand deposits and term deposits (less than 30 days maturity) • Stable deposits (deposit insurance scheme meets additional criteria) • Stable deposits • Less stable retail deposits	 3% 5% 10%
Term deposits with residual maturity greater than 30 days	0%
B. Unsecured wholesale funding:	
Demand and term deposits (less than 30 days maturity) provided by small business customers: • Stable deposits • Less stable deposits	 5% 10%
Operational deposits generated by clearing, custody and cash management activities • Portion covered by deposit insurance	25% 5%
Cooperative banks in an institutional network (qualifying deposits with the centralised institution)	25%
Non-financial corporates, sovereigns, central banks, multilateral development banks, and PSEs • If the entire amount fully covered by deposit insurance scheme	40% 20%
Other legal entity customers	100%
C. Secured funding:	
• Secured funding transactions with a central bank counterparty or backed by Level 1 assets with any counterparty. • Secured funding transactions backed by Level 2A assets, with any counterparty • Secured funding transactions backed by non-Level 1 or non-Level 2A assets, with domestic sovereigns, multilateral development banks, or domestic PSEs as a counterparty • Backed by RMBS eligible for inclusion in Level 2B • Backed by other Level 2B assets • All other secured funding transactions	0% 15% 25% 25% 50% 100%
D. Additional requirements:	
Liquidity needs (eg collateral calls) related to financing transactions, derivatives and other contracts	3 notch downgrade
Market valuation changes on derivatives transactions (largest absolute net 30-day collateral flows realised during the preceding 24 months)	Look back approach
Valuation changes on non-Level 1 posted collateral securing derivatives	20%
Excess collateral held by a bank related to derivative transactions that could contractually be called at any time by its counterparty	100%
Liquidity needs related to collateral contractually due from the reporting bank on derivatives transactions	100%

Increased liquidity needs related to derivative transactions that allow collateral substitution to non-HQLA assets	100%
ABCP, SIVs, conduits, SPVs, etc:	
• Liabilities from maturing ABCP, SIVs, SPVs, etc (applied to maturing amounts and returnable assets)	100%
• Asset Backed Securities (including covered bonds) applied to maturing amounts.	100%
Currently undrawn committed credit and liquidity facilities provided to:	
• retail and small business clients	5%
• non-financial corporates, sovereigns and central banks, multilateral development banks, and PSEs	10% for credit 30% for liquidity
• banks subject to prudential supervision	40%
• other financial institutions (include securities firms, insurance companies)	40% for credit 100% for liquidity
• other legal entity customers, credit and liquidity facilities	100%
Other contingent funding liabilities (such as guarantees, letters of credit, revocable credit and liquidity facilities, etc) • Trade finance • Customer short positions covered by other customers' collateral	National discretion 0-5% 50%
Any additional contractual outflows	100%
Net derivative cash outflows	100%
Any other contractual cash outflows	100%
Total cash outflows	

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