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Proportionality in Election Systems

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Abstract

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Purpose: This essay introduces first an overview over the most common election systems and, in particular the Swedish election system. The main purpose is then to examine how well the current system in Sweden performs with respect to proportionality and then to analyse how a more proportional system can be achieved.

Method: The Swedish election system is analysed using a computer simulation method. Based on a large number of simulated election outcomes is the degree of proportionality examined. In the simulations take the main values of the Swedish election system, namely the number of constituencies, the numbers of adjustment seats and the most important value in the technique of distributing mandates, different values.

Conclusion: When the constituencies are of equal size, a number of 10 to 15 constituencies give the most proportional result. For the current distribution of constituencies, a value between 1.2 and 1.3 on the first divisor is preferred. A large number of adjustment seats leads in general to a more proportional result.

Keywords: Voting, Election system, Proportionality, Simulation

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1. Introduction

This chapter provides an introduction to the essay. After some general background information and presentation of the main problems, we present the purpose of the essay and also its delimitation.

1.1 Background

Voting systems are seldom called in questions, but in fact, paradoxes and impossible theorem are present in most of them. This is one of major problems with voting systems. If for example the number of mandates in the parliament increases and all the other factors are held constant; a specific constituency can then lose a mandate in the parliament. This seems like contradictory result. How can a constituency get one mandate less when the total number of mandates in parliament increases and all the other factors are held constant? This is just one example that can occur when a voting system is used. This is the so called Alabama paradox and can occur in a quota system, which uses a simple rounding technique to determine who gets the last remaining votes when the standard quota is fulfilled. A system that suffers from this is used in Sweden to determine the number of mandates each constituency should have. It has also been used in the United States during the nineteenth century. Through this essay, several similar examples of paradoxes will be presented and explained in more detail.

Using voting as a procedure for choosing among different alternatives has been done for several thousand years. Still there is no system that is sad to be superior to all the others; instead many different systems are used. Voting is used in many different aspects, for example, voting for candidates, voting for decisions. This essay focuses on voting procedure for parliamentary elections and the problem that is present in such a procedure, especially the Swedish election system.

How good is then the Swedish election system? After the 2010 election, a lot of debate occurred that criticised this system. The election system was created at a time when only five parties were present in the parliament and there were no indications that this number should increase. After the 2010 election, eight parties are present in the parliament, and it has been questioned whether the system is suited for the current conditions. Voices have been raised that the system is unfair and that it should be redesign. For example, there are complains that the number of adjustment seats is not enough (Sandberg 2010). The Election system has also been criticized for not leading to a distribution of mandates, which is sufficiently proportional to the number of votes. In the 2010 election the Social Democrat Party received 112 mandates and the Moderate Party got 107 mandates. With a fully proportional system[1] however, the Social Democrats would have received three mandates less and the Moderate Party one mandate less. Another conclusion from the last election is that the Alliance consisting of four parties (M, FP, KD and C)[2] needs two more mandates to receive an absolute majority. If a fully proportional were used, they would receive one additional mandate (Högström 2010). Consider the idea that the alliance instead would get one mandate more than they should have instead of one to few, then they would have obtained an absolute majority of mandates. This would have a significant effect on the decision making process in the parliament. When the election is very even can just a small factor in the election system lead to significant consequences. Since the alliance did not get an absolute majority in the last election they are dependent on other parties in order to get their propositions through. In particular, this also leads to a pivotal position for the right-wing nationalist party, the Swedish Democrats.

The Government have assigned a committee to investigate if the current system should be redesigned. The Swedish Minister of Justice minister Beatrice Ask has stated that since the last election did not give a proportional result an inspection of the system is needed. A decision from the committee is to be expected in December 2012 (The Ministry of Justice 2011).

1.2 The Problems Considered in this Essay

The main focus in this essay is on the voting system used in the Swedish parliament, which is a proportional system. This means that each party should receive the amount of seats in the parliament according to their share of total votes. The share of seats in the parliament can however only take integer values. A party cannot receive half a seat, but can get 32.76% of total votes for example. The decimals must be rounded in

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[1] A party should recieve number of mandates in the parliament similair to their share of total votes.

[2] M = Moderate Party
FP= Liberal People´s Party
KD= Cristian Democratic Party
C= Centre Party

some way. How to do this has been a subject for many controversies and different system have been used. This is the main discussion of problems that arises in proportional election systems.

1.3 Purpose

The first main purpose of this essay is to give a general overview of the different system that is used to distribute mandates in the parliament. It is important to note that many different systems can be used and no one is superior to all the others. To have a general understanding of the most important system will also help to recognize the benefits and disadvantages with the system that is studied. Another intention is to give a more detailed view of how the Swedish election system works. This system is probably much more technical than many Swedish voters suspect. It serves for that reason as a way to become more familiar with the Swedish election system.

The second main purpose is to improve the current election system. This means to find a system that decreases the difference between the number of received votes and the number of mandates in the parliament. To perform this are the most important variables in the Swedish election system varied. These are the number of constituencies, the number of adjustment seats and the most important value in the technique of distributing mandates. A comparison between the current system and the results from the analysis can then be made. This is based on the conditions that are present in the 2010 election in Sweden.

1.4 Delimitations

Of course there are some delimitations, which are necessary to be able to carry out the analysis in this essay. Voting systems are used in many different contexts. The focus is on voting systems that are used to distribute mandates in the election for the parliament. Due to the fact that most countries use different systems, some particular system must be chosen. In this essay, the focus is on the Swedish election system. Because the data in this essay is based on the distinctive features of the political system in Sweden, the results cannot easily been translated to other countries that uses a similar election system. There might for example be another number of parties in the parliament or another distribution of power, for example with three larger parties instead of two. It can however serve as a guide to understand possible effects that may occur if some aspects in the election system are changed. For example, the effect that occurs when the number of constituencies decreases, if a uniform improvement can be found, this can be used for other countries as well when these systems are analysed.

1.5 Acknowledgement

A special thanks is expressed to my supervisor Alexander Reffgen, which has contributed with a lot of important comments.

2. Method

This chapter gives firstly an overview over the structure of this essay. Then, it explains the choice of method and why this method is chosen. Lastly it presents which kind of data that is used and how this is collected.

2.1 Overview

This is an essay in Social Choice Theory, which is a sub discipline in Microeconomics. For that reason, a short introduction is presented of what social choice is and the most important result in that research area. This is the first part of chapter three. The remaining part of chapter 3 discusses voting systems in general. It gives an indication of how wide this area is. The reason for presenting this material is to give the reader a simple understanding of how the most important voting systems work, but also to show that there are many different ways to vote. As just states, this chapter is just an overview of some of all possible voting systems; it should therefore be read just as an introduction to the subject. The aim is just to give a quick and simple understanding of how different kinds of systems work, therefore are many examples used. It is not necessary for the remaining part of the essay to understand exactly how all the systems work.

Chapter four is the main part of the essay. It firstly presents the proportional voting systems, how the calculations in these systems are performed and how it works in practice. There are different methods within this system as well. The two most important ones are explained in more detail. After this, a detailed presentation of the Swedish election system is given. What are the main characteristics of this system, and which limitations are used? This is the foundation for how the analysis in the next subchapter is performed. There, are all the assumptions that are made in this study presented. It also presents exactly how the analysis is performed. This is quite technical and requires some understating in statistics. Even if it might be difficult to follow exactly how the analysis is performed is the result easier to interpret. The meaning of numbers is detailed descripted, which helps the understanding. Chapter four finishes with an interpretation and conclusion of the result. It also compares the system that is used today with alternative systems that can be used based on our analysis. How well does the system in use today actually perform? A summary of the essay is then given in a separate chapter.

An Appendix contains the data material from the study that is used throughout this study.

2.2 Choice of Method

The main results of this study are based on the outcome of two computer simulations, which were carried out using MATLAB, a scientific computation software. Chapter 4 presents the difference between the simulations and exactly how they are carried out. The main features of the election system is the number of constituencies, the numbers of adjustment seats and the most important value in the technique of distributing mandates. The simulation tries to figure out how well the systems perform when these variables are varied. In the simulations, some random variables are used to generate possible election result. For each scenario are 2500 simulations made. This creates many different outcomes for every combination of variables, out of which the average value is calculated. This is then used as the result for how proportional the system is when the variables take these particular values. For practical reason must some simplifying assumptions be made, the simulation follows still the actual system to quite a large extent.

2.3 Sampling and Processing of Data

The data in this study is based on the result in the 2010 election in the Swedish parliament. These numbers are collected from the Swedish election authority. The information about how this system works is gathered from the Swedish Constitution. Since the simulation is based on many features that are quite sophisticated the way to use the data is presented in chapter four. The main purpose of the simulations is to evaluate proportionality and for that reasons is a way to measure proportionality presented.

2.4 Source of Criticism

To perform a thorough analysis of the election system where all the aspects of the systems are taken into account is not possible given the delimitations with this essay. Therefore some assumptions are made which limit the essay. This of course affects the result, however, the analysis still take the most important circumstances into account and give hence a valuable result. Chapter four finishes with some of the limitations that had to be done to make the simulation possible. It also discusses the effects of these and some suggestions for future research are presented.

A comparison between election systems that are used in different countries is not possible due to the extent of essay. This is however not the purpose, since this essay mainly focus on the proportional election system. The method used to distribute mandates between different constituencies is neither taken into account in the simulations. Just as in the case for distributing mandates between different parties is a method for distributing mandates between the constituencies used. This aspect is however not considered. Instead, the size of the constituencies are either assumed to be equal or as in the current system. A change in the way to distribute mandates would affect the result to some extent. This question has for example been discussed in the United States in the last 300 years. As stated before, some delimitation must be made. This essay focuses instead on the way to distribute mandates between parties when the constituencies follow the distribution stated above. It is though important to consider the effect a change in this method has. This becomes clearer when a thorough description of the Swedish election system is presented.

3. Theory

This chapter provides an introduction to voting and election systems. Firstly, some general social choice theory is presented, and then we present the two main voting systems and discuss some problems related to them. Finally, we explain and compare the most common election systems that are used in practise.

3.1 Social Choice

Social Choice Theory is concerned with the question of how to aggregate individual preferences into a social preference in a good way in the sense that some reasonable normative restrictions are satisfied (Mas-Colell et al, 1995, p. 789). These restrictions can of course differ depending on who is asked and which preference to aggregate. Why is then the subject social choice needed? In places where market power is the dominant factor does not social choice seems to be superfluous? Consider for example free trade. Most people prefer this situation since it creates more competition and lower prices. However, this might also lead to some people losing their job, for example people with worse prerequisites than others. Cannot it be social optimal to give these people some subsidy to make them able to keep their business going? How can decision like this be made in a rational and transparent way? This is what social choice is about (Gaertner 2009, p.1). Free trade might not be social optimal under all circumstances, perhaps it can be better for the society in total if those with wore prerequisite get a chance to actually compete with established companies.

3.2 Voting Paradoxes

The pioneer in the area of social choice theory is Kenneth Arrow who in 1963 wrote the monograph Social Choice and Individual Values. This is still the foundation of social choice theory. Arrow use a function called the Social Welfare function, which is as a function that link, each individual preferences to a social preference. This is just a rule for how a social preference can be established. This seems not like an impossible function to create, however Arrow showed the opposite, instead he showed that it is impossible to find a function like this, if some of the attributes of such a rule are sad to be fulfilled

Arrow´s Impossibility Theorem says that there any social welfare function that satisfies the assumptions of Unrestricted Domain, Pareto Optimality, Independence of

Irrelevant Alternatives and No Dictatorship if the there is a finite number of individuals and at least three alternatives, exists. To understand what this means are the assumptions explained below.

- Unrestricted Domain (UD) – All possible preference profiles are allowed in the domain of the social welfare function. This mean that the function must account for all individuals preferences.
- Pareto Optimality (PO) - If all individuals prefer some alternative to another alternative, then should also the social preference prefer the former alternative to the latter.
- Independence of Irrelevant Alternatives (IEE) - It is only the information between the alternatives that are compared that should matter. The ranking between x and y should only be based on how the individual rank these alternatives to each other, not on how x and y are ranked to another alternative z.
- Non-Dictatorship (ND) - No individual should have absolute power to determine the outcome. This however does not say that all individual should have the same voting power (Gaertner 2009, p. 19-21).

Why are exactly these four conditions chosen? Arrow means that these are minimal requirements to guarantee a ration social preference, but also to guarantee that the members of the societies sovereignty and their possibility to have different values are respected (Arrow 1963, p. 31). Each member shall be able to make the decisions on his or her own and these shall be accepted by the social choice function, without any doubt.

It follows directly from Arrow´s theorem that if UD, PO and IRR are fulfilled than a social welfare function must be dictatorial, provided that there exist at least three alternatives. A rigorous proof will not be given here, but in case of interest it can be found in (Gaertner 2009, p. 21-25). In social choice theory there exist many other interesting impossibility theorems and voting paradoxes. Arrow's impossibility theorem is just one, another famous paradox is presented below. These paradoxes illustrate the difficulty to generate one final outcome out of many individual different preferences.

The Condorcet paradox

This is one of the most cited paradoxes in social choice theory, based on Marquis de Condorcet research in the 18th century. It states that the majority rule not always leads to a transitive[3] ordering of social preferences. The majority rule basically states that the alternative with most votes is the winner; this is presented in more detail later on. Consider this example, three agents are choosing among three alternatives x, y and z using the majority rule. The agents' preferences are as follows:

$$X_1 \succ Y_1 \succ Z_1$$
$$Z_2 \succ X_2 \succ Y_2$$
$$Y_3 \succ Z_3 \succ X_3$$

We can see that agent one and two prefer x to y, so by the majority rule must x therefore be socially preferred to y. For the same reason, must y be socially preferred to z since agent one and three prefers y to z. If the voting system is transitive then must x also be preferred to z, which however is not possible since we can see that agent two and three prefer z to x. This cyclic pattern that violates the transitivity condition is know as the Condorcet paradox and is a problem in many voting systems (Mas-Colell et al 1995, p. 796).

3.3 Voting Systems

Is there, as a consequence of Arrows Impossible theorem, no hope to find a good voting system? Even though Arrow states a negative message to this question, this argument depends on some normative conditions stated by Arrow. There exist a large literature, which tries to find voting systems that satisfies other normative conditions, not just those stated by Arrow. Below we present some voting systems and their underlying assumptions.

The majority rule is one of these voting systems and is easy to understand. It says that the alternative with most votes is the wining alternative. This majority can then be either simple or absolute. When absolute majority is used, the wining alternative must

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[3] Transitivity: If x is preferd to y and y is preferd to z, then must x also be prefered to z, for the relation to be transitive (Mas-Collel et al 1995, p.9)

receive at least half of the votes. With simple majority, the alternative with most votes is the winner, even if it gets less than half of the total number of votes. A famous theorem called May´s Theorem says that the three conditions: Anonymity, Neutrality and Positive Responsiveness[4] together with a condition of unrestricted domain are necessary as well as sufficient for the simple majority rule to be a social aggregation rule (Gaertner 2009, p. 40). A proof of this can be found in (Gaertner 2009, p. 40-43). It is also shown that positive responsiveness is not fulfilled in the absolute majority rule. The Condorcet paradox shows that the majority rule does not fulfil the criteria of transitivity. Is it a problem that the majority rule does not have this property? The main problem with a system, where the transitivity conditions is not fulfilled is that no best alternative can be stated. The probability for an intratransitive circle when there are three voters with three alternatives is 5.55%. This increases as the number of people and alternatives increase. Whether it is a problem, depends on the number of alternatives and voters (Gaertner 2009, p. 37-43).

Another kind of voting system that can be used is the Borda count. This is a bit more sophisticated method. Each voters rank their alternatives according to their preferences. These are then transferred into points, where the voters top alternative get most points. The alternative with most points in total is the winning alternative. Eurovision Song Contest, for example uses a system like this. This system is said to fulfil both the neutrality and anonymity condition (Gaertner 2009, p.102-105). An example illustrates the possibility to rank alternatives according to the Borda count method.

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[4] -Anonymity – All votes are treated equal in the sense that if all the voters in the set change preference with each other the outcome should still be the same.

-Neutrality (NE) - satisfies that all alternatives are treated equally. If all votes are inverted then should the outcome as well be inverted.

-Positive Responsiveness (PR) –If for example a preference relation is indifference between x and y an one person change his preference from y to x then the preference relation should now state that x is preferred to y (Gaertner 2009, p. 37-40). This means that if all individuals prefers two alternatives equally much, then if one individual change its preference from one of the alternative to the other, then the social preference should change in the same way.

Points \ Voter	x	y	z
1	A	B	C
2	B	A	B
3	C	C	A
	A =6	B =5	C =7

The interpretation of the example is that voter x gives three point to alternative C, voter y does the same, while voter z rank C as their worst outcome. Since C in total receives most points, this is the winning alternative.

If the voter prefers two alternatives over the other ones, however just slightly like one of these preferable alternatives more than the other one, this method give him or her the possibility to favour both by ranking these as the top alternatives. This is not possible in the majority system where each voter just can choose one alternative. If the voter, however just likes one alternative he is forced to give points to alternatives that is not preferred. Later on the Singe Transferable vote system is presented, which is based on the Borda count method.

3.4 Election Systems

To define different election systems and to explain the differences between them, some useful criteria can be used. The size of the constituencies, which defines the number of mandates each constituency has, is the first one. The main difference between the two major elections systems, the majority- and proportionality voting system, is based on this. In the majority voting system, it is most common to choose only one candidate in each constituency. This is used for example in the United Kingdom where the system is called the "First-Past-The-Post" (FPTP). In proportional election systems however, at least two candidates are usually elected. The second criterion states how much support a candidate needs to win a mandate in each constituency. At least relative majority is needed in a majority voting system, whereas in a proportional voting system the mandates are distributed according to the number of received votes. The design of the ballot is a third criterion. In some countries, it is only possible to mark one candidate on the ballot, whereas in some other countries the ballot requires the voter to rank different alternatives. A combination of the majority- and proportionality voting system is called a semi-proportional voting system, which tries to include the benefits from both systems (Choe 2003, p.18-19). These three main categories are presented below.

The majority voting system has two main specializations, either relative- or absolute majority. The relative majority system can be divided further into two groups. As presented above the FPTP system can be used, where just one candidate is chosen in each constituency according to simple majority. This is a straightforward and easy understandable system, which also tends to create strong governments. The reason for this is because the winner does not have to take into account the opinion of other parties. Due to the fact that just simple majority is needed this might lead to a distortion of the inhabitants political opinions. If the winning party receives 30% of the votes, this mean that 70% of the inhabitants do not share the winning party's political opinion, still this party has mandate to make decisions on their own. Another bias is that the party with most votes in the country, not necessarily has to turn out as the winning party. If a party receive just one vote to few in a constituency, these votes are completely worthless (Choe 2003, p.21-23).

Another form of relative majority is the Block Vote system (BV) in which each voter has as many votes as the number of mandates in the constituency. The voters can freely choose how to divide the votes, however, it is not possible for a candidate to receive more than one vote. Politicians from different parties can be chosen, which gives the voters the opportunity to choose exactly the candidates that is supported at most. The system basically states that if three mandates are on stake in a single constituency, the voter should cast votes on three different candidates (Choe 2003, p.21-23).

The absolute majority system can also be divided into two groups. The Two round system (TRS) is most common in use in presidential election, for example in France, Germany and Russia. The candidate with an absolute majority of votes is the winning alternative. If no alternative reach this amount, the two candidates with most votes will compete in a new round a couple of weeks later. Since just two candidates are left, one of these will get the majority, hence the name "Two round system". Each voter can in this election system support his favourite candidate, since it most probably will be another round later. The risk of tactical voting in the first round decreases as well, which is a main argument for using this system. When the first round finishes, the parties get the current status of each other strength in the opinion. When alliances should be established in the last round this is helpful information. The two remaining parties are now competing about the losing parties voters. An additional benefit is that it becomes more difficult for extremist parties to get a real influence in the parliament. If an extremist party performs well in the first election round but not receive an absolute majority, it becomes difficult for them to receive a majority in the second election round since most people will then vote for the alternative option. The main objective against this system is the cost of arranging it. To arrange one election is costly and time consuming for the government and a second election doubles this cost. This is especially crucial for countries with restricted budget. (Choe 2003, p.23-24).

A second majority voting system is the Alternative Voting system (AV). This gives the possibility to include a second and a third alternative if the first candidate is not elected. It differ between countries how many alternatives that have to be ranked. In a system like this is only one election round used, which decrease the administrational costs and transportation problems that are present in the TRS system (Choe 2003 p.24-26). To start with is only the first name on each ballot calculated. If some candidate receives an absolute majority is this the winner. In case no alternative gets an absolute majority, the candidate that receives least votes is eliminated. Instead, the second name on these ballots is used in a new calculation round. This is repeated until there is one candidate that receives an absolute majority (Janson 2012, p. 219). This system requires nevertheless more from each voter, who has to gather information from other parties than the most preferred one as well. It also requires more from the administration of the election, for example to recalculate votes from ballot where the top alternative is eliminated. David M Farrel introduced this system since he argued that the FPTP-system could possible lead to a negative outcome for parties with similar ideology if the voters for this ideology spitted their votes between similar parties. If there, for example are two conservative parties and one working class party, then even if the two conservative parties together have more votes than the working class party, the result can be that the working class party wins the election. The logic is that the similar parties “steal” votes from each other (Choe 2003 p.24-26).

The basic idea in the proportionality election system is that each party´s share of mandates should be equal to their share of votes in total. If a party receives 25% of the votes in the election, they should as far as possible also receive 25% of the mandates in the Parliament. There are three main factors that have an impact on this. The number of constituencies is the first one. If only one constituency exists then there is a small difference between the share of votes and the share of mandates in the parliament. In the Netherlands and in Israel this is the case for example. If on the other hand, many constituencies but only a small number of mandates in each constituency exists, then the risk that even large parties not gain any mandates even through a substantial amount of votes is received, is much larger. How the distribution of mandate is calculates is a second factor. This is discussed in more detail later in the chapter. The third main factor, which is also discussed in more detail later on, is the use of adjustment seats. These try to compensate for a possible distortion of the distribution of mandates, which main occur in a proportional system when many constituencies are used. Sometimes an election threshold is used as well, which also affects the distribution of mandate. The reason for using a threshold is to keep smaller parties outside the parliament because this is said to give a more efficient parliament. In Sweden for example, a party must receive at least 4% of the votes in the whole country and at least 12% in some constituency to qualify for the parliament (Choe 2003 p.24-28). This will however not be discussed in much more detail.

As for the majority election system there are different kinds of proportional election systems. Three examples are here presented. The first is used in many Scandinavian countries and is called the PR-system. Either one or many constituencies are used in this system, which is based on some quotas that decides the distribution of mandates. One example is the Hare-quota, which is defined as the number accepted votes in each constituency divided by the number of mandates in this constituency:

$$Hare\ Quota = \frac{Number\ of\ votes}{Number\ of\ mandates}$$

Each party that has the same or higher quota receives a mandate. If a party get twice the quota, one additional mandate is received. The remaining votes are then transferred to the next calculation round, where the parties receives the additional mandates according to their number of remaining votes in a decreasing scale. An example illustrates this method. Consider the matrix below.

Hare-Quota

Party	First round	Hare-Quota	Number of mandate	Second round	Number of mandate	Total mandate
SAP	2100	1200	1	900	1	2
M	1900	1200	1	700	0	1
C	800	1200	0	800	1	1
FP	750	1200	0	750	1	1
V	450	1200	0	450	0	0

Only SAP and M reach the Hare-Quota and receives one mandate each in the first round. The remaining votes are then used to distribute the remaining mandates in the second round. This type of system mostly gains smaller parties, as can be seen in the example above where the party FP, which only receive half as many votes as M still gets the same number of mandates. Modifying the Hare-quota can increase the benefits for the larger parties. The Imperiali quota replaces the denominator by the number of mandates plus two, which decreases the quota. This is a gain for the larger parties, since more mandates in the first round is received. A quota between the Imperiali- and the Hare quota is the non-rounded Droop quota, which instead of adding the number two in the denominator just adds the number one (Choe 2003 p.28-30).

The method is also called the largest remainder method. Instead of using different rounds as in the above example, each party's fair share can be immediately calculated according to the formula

$$\frac{Total\ number\ of\ seats}{Total\ number\ of\ votes} * party\ i\acute{}s\ number\ of\ votes$$

SAP´s fair share is for example 1.75. Each party then receives the number of mandates according to its lower integer value of its fair share. Then the party with the largest remainder receives the additional mandates, hence the name of the method

(Demange 2011, p 6-7). The result becomes the same as with the technique used in the example.

An alternative way is to use a divisor instead of a quota. There exist several different divisor methods. Two of them are the D´Hondt method, which is used in Spain and Portugal for example and the modified Sainte-Laguë method, used in Sweden, Denmark and Norway for example. D´Hondt method is based on the divisor series 1, 2, 3, 4… and so on, while the modified Sainte-Laguë series consists of the numbers 1.4, 3, 5, 7... and so on. The number of votes for each party is divided with a number in the divisor series, which gives the parties a value that is compared for each mandate. The party with the highest value receives the next mandate. When a party wins a mandate the next number in the divisor series is used. An example with the D´Hondt method clarifies this. The numbers are the same as in the Hare-quota example.

D´Hondt					
	SAP	**M**	**C**	**FP**	**V**
Number of votes	2100	1900	800	750	450
Frist ranking (number of votes/divisor 1)	2100 (1)	1900 (2)	800 (5)	750	450
Second ranking (number of votes/divisor 2)	1050 (3)	950 (4)	400	375	225
Third ranking (number of votes/divisor3)	700	633	267	250	150
Number of mandates	2	2	1	0	0

The first mandate is received by SAP since they has the highest value after the first ranking. In the second ranking, SAP divides their number of votes with the second number in the divisor series. The number 1050 is now used to compare with the other parties. Since M has 1900 votes according to their first ranking, they receive the second mandate. Party M does then just as SAP did previously and moves over to the second ranking. The third mandate is then handled to the party with highest remaining number, which in this case is the SAP at 1050. This process proceeds until all mandates are distributed. The numbers in the brackets shows which mandate they receive, number one stands hence for the first mandate and so on. Note the difference that occurs between this method and the Hare-Quota method. Party M receives two mandates instead of just one and the FP party gets zero mandates.

If the modified Saint-Laguë method is used instead, the numbers at each ranking decreases at a higher pace compared to the D´Hondt method, which result in a more proportional distribution (Choe 2003 p.30-31). A more thorough analysis of the divisor methods is presented in chapter 4.

Mixed Member Proportional (MMP) is a kind of voting system that is a mixture of the proportional and the majority voting systems. According to common practice it is placed under the category proportional election systems. The system has a part based on the proportional election system, for example can 50% of the mandates be distributed according to this method and the remaining part through a majority election system. The voters participate therefore in two elections. This is used for example in Germany and Japan. The argument for using a system like this it to get ride of the possible skewness the majority system cause with only one winner, but at the same time include the closeness between the inhabitants in the constituencies and the elected politicians, which is a major benefit in the majority voting system (Choe 2003, p.33-34). The most common connection between the systems is based on mandates, which is used in Germany for example. This means that the mandates the party wins through the majority system are subtracted from the mandates they win in the proportional system. This decreases the influence from the largest parties. An example illustrates the implementation. First the majority system distributes their share of mandates, which for example is 50% of the mandates. The remaining mandates are then distributed according to the proportional system, where the calculation is based on all mandates in the parliament. Note that the majority system is based on just 50% of the mandates in this case. The number of mandates each party receives through the proportional method is then decreased with the number of mandates gained through the majority system. Just half of the mandates are now distributed through the proportional method. In this way the systems interact with each other and compensate for the lack of proportionality the majority system suffers from but still take advantage of some of the positive effects (Anckar 2002, p. 19). For example in Germany, are firstly 299 out of 598 mandates distributed through the FPTP system. The proportional system distributes then 598 seats according to the

Sainte-Laguë method. The numbers of mandates gained through the FPTP system is then removed from this distribution so just 299 mandates remains. If a party receives more mandates through the FPTP system than is justified by the proportional system, these will be overhang seats. Different methods can then be used to solve the problems with the overhang seats. One example is that the party keeps their number of mandates and the total numbers of mandates in the parliament increases (Janson 2012, p. 271).

Single Transferable Vote (STV) is a kind of preference voting system where the voters rank their alternatives according to who they prefer at most, as is the Borda count method. Compared to most other proportional voting systems, the voter chooses among candidates rather than parties. It is possible to vote on politicians in the same party, but also to choose candidates from different parties if this is preferred. To ensure that the election is performed correctly it is important that sufficient information is available about all the candidates. At the same time put this responsibility on each voter to spend the time necessary to get an opinion about all candidates they are ranking. The design of the ballot is an important issue. It is possible that candidates at the top of the ballot receives many votes from people who do not know where to put their alternative votes and just select the first one in the list. The number of alternatives the voters must put on the ballot differs between countries. Five mandates are recommended to ensure it not becomes too difficult for the voters. When the mandates are distributed among the alternatives, a formula, which is similar to the Hare-quota, is used. It is called the Droop-quota and is defined as

$$Droop\ Quota = \left(\frac{Number\ of\ accepted\ votes}{number\ of\ mandates\ in\ the\ consistuency\ area + 1}\right) + 1$$

If a candidate reaches this quota then a mandate is received. The remaining votes for this candidate are then transferred to the next round where the second alternative on these ballots receives the remaining votes. This can however be performed quite arbitrary since it is just the redundant votes that should be calculated. Different methods are used to solve this problem. For example can there be some predetermined method, which selects the votes to calculate, or can all votes be transferred however to a lower value than the original vote. If the Droop Quota for example is 15 000 and a candidate receives 20 000 votes, then are 5000 redundant votes. Which 5000 out of these 20 000 ballots that should be transferred to the second alternative is then the problem. If no candidates reach the quota then the candidate with least votes is eliminated and the second alternative on these ballots receives their votes. These procedures are then repeated until all mandates are distributed (Janson 2012 p.151-156).

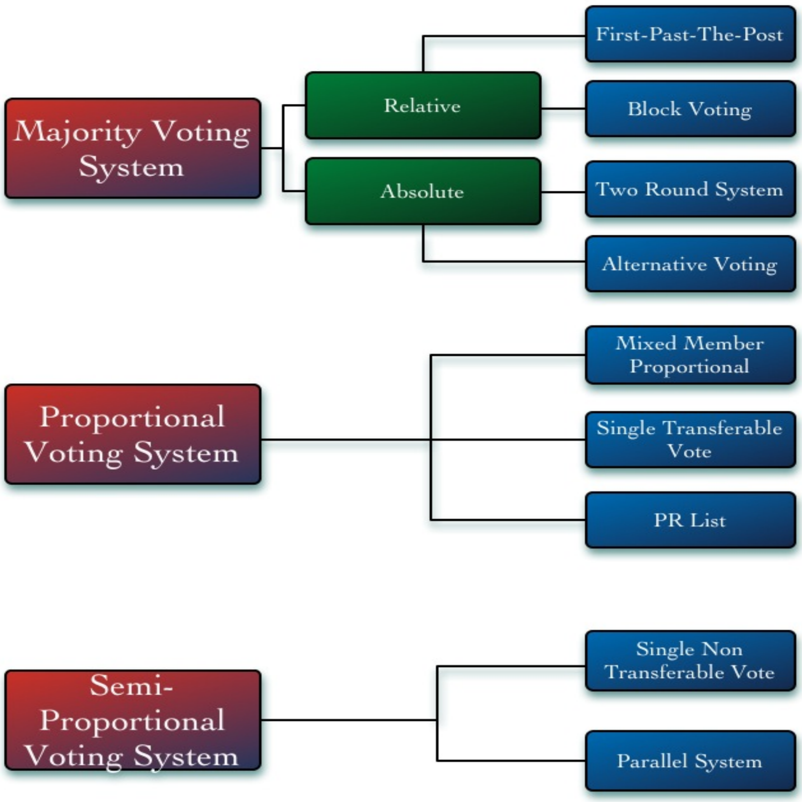
The third main category is the semi-proportional election systems, which as mentioned before is a mixture between the proportional and majority systems. Just as the MMP system does, it tries to combines the benefits from both kinds of systems but they do not interact with each other to the same extent as in the MMP system. There are two semi-proportional systems, which are the most common ones. The first is the Single Non-Transferable Vote (SNTV), where the voter can choose only one alternative, even if there are several mandates on stake. The candidates with most votes get the place in the parliament, even if he or she only receives 20 % of the votes. The vote is for a person, and a list is hence not used. This also puts pressure on the intern relationship in each party since the candidates are competing for the same voters. Smaller parties with a fragmented election support might be without mandates, since enough votes in a single district even if they in total get a substantial amount of votes cannot be gained. The main difference compared to the majority system is that more than one mandate are on stake in each constituency. The main difference to the proportional system is quite obvious since mandates are not distributed by share of votes but rather by majority (Anckar 2002, p.24). Consider the example where three parties exist: A, B and C. The constituency inhabits 10000 people and three mandates are on stake. Party A would like to receive two mandates but B and C are pleased if they only receive one mandate. A will therefore participate with two candidates, X and Y, while Party B and C only participates with one candidate each, Z and Q. The votes are as follows:

Party	Candidate	Votes	Seats
A	X	5000	1
	Y	1000	0
B	Z	2000	1
C	Q	2000	1
		10000	3

Party A is obviously the most popular party and receives 60% of the votes, however it receives only 33% of the mandates. The party must hence in advance evaluate how many candidates they should have and which persons that should represent them. If party A choose two more equally popular persons, they can get 3000 votes each and hence receive two mandates. A popular candidate can "steal" votes from a party comrade.

The second semi-proportional system is the Parallel System (PS). A close relation between this and the MMP system exists. The PS system does not have the mechanism from the MMP system that decreases the disproportionality caused by the majority election system. This is the reason for PS being called semi-proportional and MMP take place under the proportional election systems. The PS system does not decrease the number of mandates distributed through the proportional system with the number of mandates the party receives through the majority system. The two systems do not interact with each other as they do in the MMP system. The proportional part of the Parallel System is just based on their share of total mandates, while it is based on all mandates in the MMP system, which then is decreased with the number of mandates gained through the majority system. The MMP system tends therefore to give a more proportional distribution than the PS system (Choe 2003 p.38-41).

Below shows a diagram how the systems are connected to each other.



Majority Voting System
Relative
Absolute
First-Past-The-Post
Block Voting
Two Round System
Alternative Voting
Proportional Voting System
Mixed Member Proportional
Single Transferable Vote
PR List
Semi-Proportional Voting System
Single Non Transferable Vote
Parallel System

Figure 3.4.1

4. Analysis

This chapter starts with a more thorough description of the proportional election system than in the preceding chapter, and in particular, it explains the details of the Swedish election system. Then it presents a method to analyse the proportionality of the Swedish election system and how the main parameters of this system, namely the number of constituencies, adjustment seats and the value of the first divisor, should be chosen.

4.1 The Proportional Election Systems

As stated in the preceding chapter, Sweden uses a proportional voting system. The distribution of mandates was there presented in a general way. Either a quota or a divisor method can be used. The number of mandates in the examples was exogenously given, but when constructing a voting system the question concerning the number of mandates each constituency should get is crucial. Should it be equally distributed? In the European Union for example, Germany has 99 mandates while Malta has six. Germany´s population is however about 200 times larger then Malta´s. This gives Germany a population-to-seat ration of 1 mandate for each 826.285 inhabitants, while in the case of Malta it is 1 mandate per each 68.828 citizen (Demange 2011, p. 2). What is the consequence of this? If the purpose of the election system is to be as proportional as possible, then to give each constituency the same population-to-seat ratio is intuitively to prefer, which leads to more mandates for Germany, but also a significant amount of influence. Malta on the other hand becomes a minor actor in the parliament and loses their chance to affect the decisions.

The problem with constructing an assembly with different parties, and many minor constituencies is called the bi-apportionment problem. This is due to the geographical and the political aspect (Demange 2011, p 1). How the distribution of mandates between different constituencies should be made so the elections system becomes as proportional as possible has a long history and has been debated by many different professions such as mathematicians, statisticians and political analysts. In the United States for example, has this issue been debated for almost two hundred years (Young 2004, p. 3). This essay focuses however not on this important issue, but instead we focus on the question how to distribute mandates between parties based on their share of votes. The number of mandates for each constituency is taken as given.

If the purpose is to receive a result that is as proportional as possible, how should the election system be designed? This essay focuses on the Swedish election system, where PR-list system is used. The Swedish Election Authorities states “The Swedish electoral system should as fair as possible, reflect the people's political will. Therefore shall mandates (seats) in such parliament or council be distributed in relation to the number of votes that the parties receive in the election. If a party, for example, receives twenty per cent of the votes then they should also get about twenty per cent of the mandates. Such a voting system is called proportional.” (www.val.se A). The Hare quota, which also is known as the largest remainder method is in theory said to give the most proportional system. Is it not obvious to use this method then? Three famous paradoxes illustrate some substantial problem that can occur with this method.

The Alabama paradox shows that when the number of mandates in the assembly increases, while the other variables are constant, some constituencies may receive fewer mandates than before the increase. This was first discovered in Alabama during the nineteenth century. It shows that the state of Alabama receives eight mandates in the House of Representation when the number of mandates is 299, but when this number increases to 300, does Alabama only receive seven mandates. How can this contradictory result occur? The reason is because the largest remainder method is used. When the number of mandates changes, this affects the denominator in the quota for each constituency. This leads in some cases to another outcome. Consider the example, where the number of mandates in the assembly changes from 21 to 22 mandates.

Alabama Paradox		21 seat		22 seat	
	Pop	Quota	Mandate	Quota	Mandate
A	7270000	14,24	14	14,92	15
B	1230000	2,41	3	2,52	2
C	2220000	4,35	4	4,56	5

Figure 4.1.1

The example shows that B loses one mandate when the number of mandates in total increases (Young 2004, p 12).

A second paradox, called the population paradox defines the case when a constituency with larger population growth loses a mandate in the assembly to another constituency with lower growth rate. Consider a large constituency with lower growth rate than the national average and a smaller constituency with no growth rate at all. As time goes becomes the decrease in quota for the large constituency more substantial than for the smaller, even if it has a higher growth rate. Consider a small constituency A with the quota 1.541 in the year 1990 and a large constituency B with the quota 27.576 the same year. If a new calculation is done the next year, it is possible that A get the quota1.519 and B 27.350. The quota for both constituencies are now smaller because they grow less than the national average, constituency A however, receives the extra mandate according to the largest remainder system. This occurs because the quota for the larger constituency is affected more than for the smaller constituency when they grow below national average even if it has a higher growth rate (Young 2004, p 21-22).

The third paradox called the Oklahoma paradox or the New-State paradox describes the situation when a new constituency is added and the number of mandates in the assembly increases in relation with the new constituency´s population. When a new distribution of mandates is implemented because of this, a change in the number of mandates for some constituencies can occur, even though if the population in these constituencies is unchanged (Demange 2011, p. 7). An example illustrates, where a third constituency C and a fair amount of five mandates are added to the assembly.

Oklahoma Paradox

	Population	Quota	Mandates		
A	1045	10,45	10	Number of Mandates	100
B	8955	89,55	90		
Total	10000	100	100		
A	1045	10,43	11	Number of Mandates	105
B	8955	89,34	89		
C	525	5,24	5		
Total	10525	105	105		

Figure 4.1.2

C loses a mandate to constituency A even though the population is constant and a fair amount of five additional mandates are added. These paradoxes are presently as well when mandates between parties are distributed, rather than between constituencies as in in the examples above.

These paradoxes highlight some problems with the quota method, which are not desirable in a "good" election system. The main cause for these problems is because the remainders are adjusted differently depending on the size of the constituencies or parties, when changes occur, which are most significant in the population paradox. The divisor methods however, do not suffer from these problems, and the paradoxes just presented cannot occur in these methods (Demange 2011, p.7). This is one reason why they are more commonly in use than the Hare-quota (Choe 2003, p. 30)and (Anckar 2002, p.16). Previously have the two most commonly in use divisor methods, namely the D´Hondt and Sainte-Laguë method, been introduced. There exist other divisor methods as well. A few of them are mentioned later on.

The D´Hondt divisor is also known as the Jefferson method, actually Jefferson presented this more than 100 years before D´Hondt. Since quota methods contain some seriously problems, as shown above, a method that can create a proportional election result without these paradoxes occurring is preferred (Demange 2011, p.6-7). The mathematics behind the divisor models can be shown both by a multiplier and a divisor, and both methods results in the same outcome. The multiplier method is explained only briefly. In the following, denote by s_1 the number of mandates for party i.

Firstly is standard quota $q_i = \frac{S}{V} * v_i$[5], where S is the total amount of mandates, calculated, then multiply each q_i with a number $\alpha \geq 1$, which is chosen so $\Sigma\ (\alpha q_i) = S$ holds. Each party, p_i, receives (αq_i) mandates. When the value of α increases to satisfy the equation above, the value of αq_i tends to increase faster for larger then for minor parties. Large parties reach a new integer value and therefore a new mandate faster than minor ones. This is the reason why the method tends to favour larger parties and

[5] v = number of votes
s= number of mandates
p= party

coalitions. A short example illustrates this. Consider for example the two quotas: q_1=4.5 and q_2=5.5. For q_1 to reach the next integer it must be multiplied with approximately 1.11, while q_2 only needs to be multiplied with approximately 1.091. The larger quota reaches the next integer faster. The reason why the Alabama paradox not occurs in this method is because the lower quota condition is satisfied, which means that a party receives at least as many mandates as its lower quota, q_i. There is however, a possibility that a party gets more mandates than its upper quota, which corresponds to the upper integer of the standard quota. A party with a standard quota of five, can still receive seven mandates, the upper quota of six is then passed (Lanke 2012, p.3).

It is more usual and intuitive to describe this method with a divisor, rather than with a multiplier. The term $\frac{v_i}{\lambda}$ is used instead of (αq_i). Lambda indicates an unknown positive variable. Just as before must $\Sigma\left(\frac{v_i}{\lambda}\right) = S(\lambda)$ hold. Each party receives numbers of mandates according to $s_i = \frac{v_i}{\lambda}$. To explain the approach of this method, assume that $v_1 > v_2 >,,, > v_n$ holds. The first mandate is distributed when $\lambda = v_1$, p_1, receives then a mandate. The winner of the next mandate depends on the relationship between the numbers of votes for the two largest parties. Either when $\frac{v_1}{\lambda} = 2\ or\ \frac{v_2}{\lambda} = 1$, whichever happens first. Either party one wins its second mandate or the second largest party receives its first. The fractions can be rewritten to:

$$\lambda_1 = \frac{v_1}{1+1} \text{ and } \lambda_2 = \frac{v_2}{0+1}$$

The value of λ determines which party gets the next mandates. If the mandates are distributed among the parties according to $s_1, s_2, \ldots, s_p$ but these sum to less than S, who gets the next seat? A method for distributing the mandates is obtained by manipulating the lambdas. When $\frac{v_i}{\lambda}$ changes from s_i to (s_i + 1) the party p_i receives an additional mandate, this happens when $\lambda = \frac{v_i}{s_i+1}$. This is called the comparative figures. The rule for this method is to deliver the next mandate to the party with largest comparative figure, λ_i, and then to recalculate the party´s, λ_i, after each received mandate. The D´Hondt series with divisors 1,2,3,..,n, is now obtained (Lanke 2012, p 3-4).

This method is based on a way of rounding downwards. There are other ways to round as well. The Sainte-Laguë uses the standard-rounding principle, which is explained in the following.

The Sainte-Laguë method is also known as Webster method. As in the previous case was Webster about hundred years before Sainte-Laguë. In fact Webster and Jefferson competed against each other concerning which method to use for distributing mandates between states in the United States congress election (Young 2004, p.4-8). This method follow the same logic as D´Hondts method, however, it adds a half in the denominator for each comparative figure when a mandate is won, instead of the number one as in D´Hondts method. The comparative figures for the Sainte-Laguë method is presented in this figure:

$$\lambda_i = \frac{v_i}{(s_i + \frac{1}{2})}$$

The next mandate is thus given to the party with the largest value of λ_i. If all of the divisors $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, etc are multiplied with two, which does not affect the result, we obtain the following series of divisors 1,3,5, etc. Compared to D´Hondt method, Sainte-Laguë does not tend to favour larger parties as much because the divisors are increasing faster, which means that more votes are required to get the same number of mandates. To receive the second mandate according to D´Hondt method, the number of votes is divided with two, while in Sainte-Laguë method it is divided with three, which gives a smaller number to compare with other parties (Lanke 2012, p3-5).

An example illustrates how proportionality can be obtained within this method.

Suppose that three parties, with their numbers of votes in the brackets, A (333), B (237) and C (130), exist. For pedagogical reasons, we compare A and B. The advantage with the Sainte-Laguë method is otherwise that a pairwise comparison of parties is not necessary; instead the comparative figures can be used. Assume that there are seven mandates on stake in total, and that it is given that C receives one mandate for its share of votes. Party A and B shall divide the remaining mandates between them in a proportional way. Each mandate is worth 95 votes ($\frac{333+237}{6} = 95$). The diagram below operates a function for who is worth the additional mandate.

mandate	1	2	2,5	3	3,5	4	5	6
votes	95	190	237,5	285	332,5	380	475	570

A deserves a little more than 3.5 mandate while B deserves just below 2.5. The most proportional is, for A to receive four mandates and for B to get two mandates. Is there any way to motivate this? Note that $333 > \frac{333+237}{6} * 3.5$ and $237 < \frac{333+237}{6} * 2.5$, which is equivalent to $\frac{333}{7} > \frac{237}{5}$ (Linusson 2008, p 172). This is similar to the Sainte-Laguë formula, A divides their number of votes with the forth divisor, which is seven, and party B compares their number of votes with the third divisor. Because the comparative figure for A is larger then for B, they receive their fourth mandate, while B remains at two mandates.

In chapter three it is however a modified Sainte-Laguë method that is introduced. Instead of 1 as the first divisor, the value 1.4 is used. Why is this modification done? Obviously it has a negative effect for smaller parties since it requires more votes to get the first mandate. It keeps smaller parties out of the parliament. A similar effect as using a threshold is achieved. The reason for using a threshold is to create a stronger parliament, with more influence in the decision-making. If many minor parties exist, a situation where the larger parties are forced to negotiate with these to get the support they need, may occur. This makes the parliament and the decision-making more sluggish. To use a threshold in the proportional elections system, will include one of the advantages that are apparent in the majority election system, which creates stronger government (Anckar 2002, p.25).

Is it obvious to use the number 1.4 instead of 1? What if the number 1.2 is used? This question is analysed with the help of some simulations later on.

There exist other methods as well, which use different kinds of divisors, such as Huntington-Hills method. This is not presented in detail. It can though be mentioned that it has played a central role in the debate in the United States (Young 2004, p11, 16). Huntington-Hills divisor is used in the United States since 1930 for distributing mandates between constituencies. It use the divisor $\sqrt{n * (n+1)}$, which tends to favour smaller parties to a larger extent than D´Hondt and Sainte-Laguë methods (Demange 2001, p.8). The diagram below shows the most important systems and the formulas.

Methods	Divisors	Formula
D'Hondt (Jefferson)	1, 2, 3, 4, 5…	n+1
Sainte-Laguë (Webster)	$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$	$n + \frac{1}{2}$
	1, 3, 5, 7,…	2n+1
Modified Saint-Laguë	1.4, 3, 5, 7	
Huntington-Hill	"0", √2, √6, √12, √20	$\sqrt{n + (n+1)}$

Figure 4.1.3 (Linusson 2011, p. 36)

4.2 The Swedish Election System

As stated before, the Swedish election system shall be as proportional as possible. Several techniques to fulfil this are used. All the main steps are explained in this chapter. The first question concerns the number of mandates for each constituency. The Swedish parliament consists of 349 mandates, where 310 are so-called pre determined mandates that are distributed to the constituencies before the election. The remaining 39 mandates are so-called adjustment seats. These are distributed afterwards to compensate for the lack of proportionality, which may occur when the 310 mandates are distributed (www.val.se, B). The country is divided into twenty-nine constituencies. See Appendix A, for a list of these (Election law, chapter 4, §2). To determine the number of mandates for each constituency, the Hare-Quota is used. Each constituency receive a number of number mandates according to the formula:

$$\frac{\text{Number of people in the consistuency area}}{\left(\frac{\text{Number of people in the whole country}}{310}\right)} = \text{number of mandates}$$

Remember that the Hare-Quota states that if the number is not an integer, the quota is rounded downwards, and the largest remainder determines which constituency receives the additional mandates. The number 310 is used since there are 310 mandates determined in advance. If the remainder is equal in two constituencies, a lottery shall determine which constituency gets the additional mandate. The 30th of April the same year as the elections, determines the election authority the distribution of mandates between constituencies (Election law, chapter 4, §3).

After the voting process, the votes are counted and the distribution of mandates takes place. The number of votes each party received for all constituencies in the 2010 election is presented in appendix B. The example in this chapter is based on these numbers. First the 310 mandates are distributed to the constituencies. The example below shows the distribution of mandates in the constituency of Malmoe. The value in each cell corresponds to the value calculated by the modified Sainte-Laguë method. Party M receive 55160 numbers of votes in this constituency, which give them the value on the comparative figure $\frac{55160}{1.4} = 39400$ in the first round. Since this is the highest value, M gets the first mandate, which is marked with a red cell. When a party wins a mandate, it uses the next divisor in the next round. The comparative figure for party M is therefore $\frac{55160}{3} = 18386.67$ in the second round. This number is lower than the comparative figure for party S, which receive the second mandate for that reason (Election law, Chapter 14, §3).

Constituency of Malmoe	M	C	FP	KD	S	V	MP	SD	Total
mandate 1	39400,00	3425,00	8405,71	3767,14	34607,14	7227,14	10615,00	9468,57	
mandate 2	18386,67	3425,00	8405,71	3767,14	34607,14	7227,14	10615,00	9468,57	
mandate 3	18386,67	3425,00	8405,71	3767,14	16150,00	7227,14	10615,00	9468,57	
mandate 4	11032,00	3425,00	8405,71	3767,14	16150,00	7227,14	10615,00	9468,57	
mandate 5	11032,00	3425,00	8405,71	3767,14	9690,00	7227,14	10615,00	9468,57	
mandate 6	7880,00	3425,00	8405,71	3767,14	9690,00	7227,14	10615,00	9468,57	
mandate 7	7880,00	3425,00	8405,71	3767,14	9690,00	7227,14	4953,67	9468,57	
mandate 8	7880,00	3425,00	8405,71	3767,14	6921,43	7227,14	4953,67	9468,57	
mandate 9	7880,00	3425,00	8405,71	3767,14	6921,43	7227,14	4953,67	4418,67	
	3,00		1,00		3,00		1,00	1,00	9,00

Figure 4.2.1

To determine the distribution of the 310 pre-determined mandates, this process is performed in each constituency. Then, the distribution of adjustment seats takes place.

For pedagogical reasons is this called the "total distribution". This procedure is similar to the example above, but there is only one constituency, which is the whole country and the number of mandates is 349. To make the result as proportional as possible, each party should receive the number of adjustment seats so that the party´s share of mandates in the parliament equals the party´s share of votes in total (Election law, chapter 14, §4). To make this possible, the party then receives the additional number of adjustments seats, which is needed for the party to receive the number of mandates according to the total distribution. The party, which receive to few of the 310 mandates from the total distribution, is therefore entitled to receive some adjustment seats. If a party instead gets more of the 310 pre-determined mandates than they are entitled to regarding the total distribution, they still keeps its number of mandates. Instead a new total distribution of adjustment seats is obtained where these parties' mandates are removed. The same method is implemented if a party does not reach the threshold of 4% (Election law, chapter 14, §5). An example illustrates the method. In the 2010 election the parties received mandates according to the diagram below.

Party	Total distribution	First distribution	Adjustment seats
M	106	107	0
C	23	21	2
FP	25	17	8
KD	20	11	9
S	109	112	0
V	20	9	11
MP	26	19	7
SD	20	14	6
	349	310	43

Figure 4.2.2

The diagram shows that the parties M and S received more mandates according to the first distribution then they are obligated to according to the total distribution. They receive therefore zero adjustment seats. As the (Election law chapter 14, §5) states, these parties still keep their amount of mandates and a new total distribution is calculated where these parties are eliminated. When M and S are removed there are

130 mandates[6] left which shall be distributed to the remaining parties with the same method as before. Then the following result is obtained.

Party	Total distribution	First distribution	Adjusment seats
C	23	21	2
FP	24	17	7
KD	19	11	8
V	19	9	10
MP	25	19	6
SD	20	14	6
	130	91	39

Figure 4.2.3

The diagram shows the distribution of the 39 adjustment seats between the remaining parties. The adjustment seats are received in the constituency where the party has its highest remainder after the first 310 mandates are distributed. If a party has not received a mandate in a constituency, its remainder is equal to the number of votes they received in this constituency. For example gets the party C its adjustments seats in the constituency of Gothenburg and Södermanland, where the remainder are 12183, which is the same as the number of votes they received in this constituency.

4.3 Simulations

The main purpose with this essay is to examine how well the Swedish election system performs with respect to proportionality, and to analyse which changes that can be implemented to receive a system that gives a more proportional result. To analyse these questions are two simulations performed, which are presented below. To begin with, we present a short overview of over previous research in this field. It can be noted that none of these studies has the exact same framing as our study, at least to or knowledge.

Lars Davidsson examines the 2006 election in Sweden. He shows the difference in proportionality that may occur if some other election methods are used. The result is that if the first divisor takes the value 1 instead of 1.4, there is no difference. Another implication is that D´Hondts method gives a less proportional result than the Sainte-

6 349(Tot) -107(M) – 112 (S) = 130

Laguë, and that the Hare-Quota gives a more proportional result. This result tends hence to follow the theory in this area (Davidsson 2007, p. 31).

Jan Lanke investigates how a more proportional result can be achieved, by changing the election system. The problem, according to him, is that a party can receive more of the 310 pre-determined seats than they are obligated to according to the total distribution. Two factors are varied in this study, the number of adjustment seats and the value of the first divisor in the Sainte-Laguë divisor method. He shows the least amount of adjustment seats that are needed to give a result that can be called proportional. The research is performed for the period 1970-2010, and the first divisor takes values between 1-1.5 with a step size of 0.05. The result shows that the divisor 1.15 and 1.2 give the lowest value of maximum adjustment seats that is needed to ensure that no party receives more pre-determined seats than they are obligated to. This can imply that these systems give more proportional result (Lanke 2012, p.8).

Young shows the important result, that the Alabama, Population and Oklahoma paradoxes that can occur in the quota methods are not present in the divisor methods. This is a fundamental result and is one of main argument to use a divisor method instead of a quota method (Young 2004, p.22).

Below is the result of the 2010 Swedish parliament election shown.

2010 election

Party	number of mandates	Number of votes	% of mandaes	% of votes	% of votes, include others
Moderate Party	107	1791766	30,66	30,50	30,06
Centre Party	23	390804	6,59	6,65	6,56
Liperal Peoples Patrty	24	420524	6,88	7,16	7,06
Chrisitan Democratic Party	19	333696	5,44	5,68	5,60
Social Democratic Party	112	1827497	32,09	31,10	30,66
Left Party	19	334053	5,44	5,69	5,60
Green Party	25	437435	7,16	7,45	7,34
Swedish Democratic Party	20	339610	5,73	5,78	5,70
Others	0	85023	0,00	1,45	1,43
Total valid votes	349	5960408	100,00	101,45	100,00
Invalid Votes		68274		1,13	
Total participants in election		6028682		84,63	
Number of eligible voters		7123651			

Figure 4.3.1, source: www.val.se, C

Note that the percentage of votes is just based on the total number of votes for the parties that actually receive mandates. The reason is because these are the only parties that can receive mandates according to the threshold. The factor, others affects for that reason not the distribution of mandates and shall therefore not affect the share of votes when a comparison between these results are made. It becomes for that reason the relevant result to use, when a comparison with the simulations is done later on. This is why the percentage of votes sum to 101.45.

Is this result proportional? The social democrats gets 1% unit more mandates compared with its share of votes, which gives them 3.5 mandates to many. For most of the other parties there is just a small difference. It can be argued that since the largest difference for a party is just 1% unit, this system gives a proportional result, but when the election is very even, this small difference have a significant effect on the political decision-making. In the last election, three blocks received mandates: the Alliance (the right wing block), including M, FP, KD and C, the left wing block consisting of V, MP and S and a third block with the SD party. The Alliance won the last election even though they received only 49.57 % of the mandates, and hence not an absolute majority. If M instead of S would have received three mandates more than they were worth according to their number of votes, the Alliance had got 50.43% of the seats, which had given them an absolute majority in the parliament. This may seem like a naive comparison, it indicates though that just a small difference can determine whether a block gets an absolute majority in the parliament or not. The purpose with the proportional election system should then be to have as small difference as possible between the share of votes and the share of received seats in the parliament.

Is there any way to measure the proportionality of an election system and can any change in the current system be made that leads to a more proportional result? I have together with my supervisor, Alexander carried out two different simulations. These simulations examine what happens with the proportionality of the election system when some variable in the construction are changed. In following we describe first in detail the construction of the simulations, then is the result presented and interpreted.

4.3.1 Assumptions and Construction of the Simulations

The first simulation studies how the proportionality in the election result depends on the variables, adjustment seats and constituencies, while the second simulation examines the effect that, the numbers of adjustment seats and the value of the first divisor in the Sainte-Laguë method, have on proportionality. The main difference between the simulations is that simulation two uses the present classification of constituencies, while all constituencies in the first simulation are of equal size. The common assumptions for both simulations are presented below.

- p_i denotes party i, and eight parties are presently in total which all get at least 4% in each simulated election. It is also assumed that no other parties exist.
- These parties are divided into the same blocks as mentioned above

The left wing block (L):
1: Left Party (V)
2: Social Democratic Party (S)
3: Green Party (MP)

The Alliance or the right wing block (R):
4: Liberal Peoples Party (FP)
5: Centre Party (C)
6: Christian Democratic Party (KD)
7: Moderate Party (M)

Others:
8: Swedish Democratic Party (SD)

- This distribution between the blocks is used later in the simulation to calculate the number of votes for each party
- The measure for proportionality is:

$$\sum_{i=\text{party i}}^{8} |\%\ \text{mandate for}\ p_i - \%\ \text{votes for}\ p_i|$$

The absolute percentage difference between number of mandate and number of votes is summed for all parties, which gives a proportionality value of the election system. This measure is invented for this essay, to make it possible to perform the analysis. It is worth mentioning that a negative outcome is impossible, and a value of zero corresponds to a perfect proportional result. As the value increases, a worse representation of proportionality is achieved.

- The number of adjustment seats varies between 30 and 59
- The outcome of the 2010 election is used to determine the expected value and standard deviation, which the simulations are based on. This is explained in more detail later on.
- For each combination of variables (number of constituencies, adjustment seats and value of the first divisor) that is considered, are 2500 election outcomes simulated. It is a statistical reason to use 2500 simulations for each combination, which is based on the fact that an accuracy of 0.1 is used. If an accuracy level of 0.01 is used instead, an amount of 250000[7] observations are needed, which requires about 2 years of time for a computer to calculate.
- The election turnout is 80% on national level. This number varies though between the constituencies according to a normal distribution with a standard deviation of 0.03. Each fifth constituency acts as a stabilizer so the number 80% is reached. If the first four constituencies have turnouts of 0.78, 0.80, 0.81, 0.82, then the fifth constituency must have a turnout of 0.79, which result in an election turnout of 80% in total. This procedure is then repeated for all constituencies.

4.3.2 Simulation 1

The first simulation examines the impact the variables, adjustment seats and number of constituencies, have on the proportionality of the election result. Some additional assumptions are made:

- 7 017 479 peoples are entitled to vote in the election
- The number of constituencies takes the values 10, 15, 20, 21, …, 29, 30.
- The constituencies are of equal size. For example, if 20 constituencies are assumed, $\frac{7017479}{20} = 350\ 874$ people are entitled to vote in each constituency.

[7] To estimate an expected value with, n, numbers of observations with a standard error, α, the function: $\frac{\sigma}{\sqrt{n}} = \alpha$ is used. Note that σ is maximum of about 5 according to the tables in Appendix C

4.3.3 Execution of the Simulations

To begin with, the election result is calculated on the on national level according to the procedure presented below (note that the numbers indicate per cent of votes in total).

- Firstly receives the smallest block their share of votes, which is SD. This value has a normal distribution with expected value, μ equal to 5% and a standard deviation, σ of 0.75. This can also be denoted as $P_8 \sim N(5, 0.75)$. An interval between 4-7 %[8], limit the number of votes SD can get. This means that SD cannot receive less than 4% of the votes and be eliminated from the election according to the threshold; neither can they receive more than 7%. If no upper limit is assumed in this analyse, a chance that a party receive unreasonable large amount of votes may occur. SD could in a situation like that receive 100% of the votes.
- The remaining amount of votes is then distributed between the two major blocks.

A random number for the difference in per cent between the two other blocks is drawn. This random number denotes the difference in per cent of votes between the left and the right wing blocks and has a normal distribution with an expected value of 0 and a standard deviation of 2, in other words $(\mathrm{L} - \mathrm{R}) \sim \mathrm{N}(0,\ 2)$. This indicates the same opportunity to win for both blocks. According to this and the number of received votes for the SD party, a result for the remaining blocks can be calculated. Consider the outcome where SD receive 5.5% of votes, and the difference between the left and right wing blocks are 3 per cent, in favour for the right wing block. This results in 45.75% of the votes for the left wing block, and 48.75% of the votes for the right wing block.[9]

- Within the left block, the votes are then distributed as follows.

  $\mathrm{P}_1 \sim \mathrm{N}(5, 0.5)$, [4,8]

  $\mathrm{P}_2 \sim \mathrm{N}(10,2)$, [4,14]

  The Social democrats receives the remaining number of votes in the left block, $\mathrm{P}_3 = \mathrm{L} - \mathrm{P}_1 - \mathrm{P}_2$.
- For the right wing block are the votes distributed according to the same principle as above:

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[8] Can also be denoted [4,7]

[9] L+R= 94,5 and L-R= -3 <-> L+3=R
-> L+L+3=94,5 -> 2L=91,5 -> L=91,5/2 -> L=45,75

$P_4 \sim N(5, 1.5)$, [4,12]

$P_5 \sim N(6, 1)$, [4-9]

$P_6 \sim N(5, 0.5)$, [4-7]

As in the case for the left wing block, M which is the largest party gets the remaining votes: $P_7 = R - P_4 - P_5 - P_6$.

This shows the national distribution of votes. The votes are then distributed among the constituencies, which must be treated independently. In each constituency, each party gets a share of total votes according to the number calculated on national level (from the above section) with a random disturbance. To be precise, the party´s share of votes is multiplied with a random number from the uniform distribution on the interval from 0.9 to 1.1. If a party for example receives 5% nationally they can get a number of votes between 4.5-5.5 in a constituency, which are adjusted for every third constituency. When two constituencies are calculated, the third gets a number so these three constituencies get the same number as on national level. For example, if P_4 get 5% of votes nationally, and 4.5% respectively 5% in the first two constituencies, then it must in the third constituency receive 5.5% of the votes. This is analogous to the procedure for the election turnout of 80% in the assumptions.

According to the assumptions above, we implement a MATLAB-script that simulates 2500 election outcomes. Then we use another implemented MATLAB-script to distribute mandates in the same way that is used in Sweden and described in chapter 4.2. These values are then compared to each other according to the measure for proportionality in the assumptions. This indicates how proportional the election is. The procedure is repeated 2500 times, which gives 2500 numbers of proportionality, one for each simulated election. Finally, an average value of these 2500 numbers is calculated. This average number (the numbers in each cell in the appendix C matrices) shows how proportional the election system is based on the values the variables, adjustment seats and number of constituencies, takes. It is repeated for every combination of variables. The results are then placed in a matrix with adjustment seats on the vertical axis and the number of constituencies on the horizontal axis. An identical method is used to calculate the standard deviation, which is placed in another matrix below (these matrices are presented in Appendix C). This is performed for five different divisor methods:

- D´Hondt method (1,2,3,4,5…)
- Sainte-Laguë method (1,3,5,7…)
- Modified Sainte-Laguë method with 1.2 as the first divisor
- Modified Sainte-Laguë method with 1.4 as the first divisor (the system used in Sweden)
- Modified Sainte-Laguë method with 1.6 as the first divisor

It is worth mentioning that the intervals in the simulations are not exact. The reason is because it does not exist enough of reliable data to estimate reasonable values for the range within the parties can receive votes. Since it passes four years between each observation and the political landscape changes considerably during this time, it does not seems reasonable to include data from the last ten elections. The standard deviations are instead estimated on more vague grounds and we have used our common sense to determine these intervals, which may affect the result a bit, we believe however that it does not have a significant effect on the whole.

4.3.4 Simulation 2

The purpose with the second simulation is to study the effects the variables, adjustment seats and the value of the first divisor in the modified Sainte-Laguë method, have on the Swedish election system. The constituencies are set to have the present distribution. This implies that they consist of different amount of mandates, which is the main difference to the first simulation. In addition to the general assumptions, we assume that total amount of people which is entitled to vote are 7 123 651 and that the first divisor varies between 1-1.6 with a step size of 0.05.

The approach is similar to the first simulation. First, 2500 election results are simulated, and a distribution of mandates based on this performed. The same method and values on the normal distribution for each party as in the first simulation, is used for every combination of variables. This means, that a simulation with 30 adjustment seats and the first divisor 1.00 is performed, as well as one with 30 adjustment seats and the first divisor 1.05, and so on. The results are placed in one matrix with average values and one with standard deviations, with the number of adjustment seats on the vertical axis and the value of the first divisor in the modified Sainte-Laguë method on the horizontal axis. These are found in appendix C as well.

4.4 Results of the Simulations

The simulations above render out in six matrices with averaged summed values and six matrices with standard deviations. The full extension of these is found in the Appendix C.

What do the numbers in the matrices say? The proportionality measure shows the absolute percentage deviation between numbers of votes and share of mandates. For example, in the 2010 election the Moderate Party received 30.50% of votes and 30.66% of the mandates, which gives a proportionality value of 0.16. This is then summed for each party to a general measure of proportionality. It is worth noting that it is the absolute summed difference for all parties, which means that it has the same effect whether a party receives one to few as if they receive one to many mandates. A small value on the proportionality measure indicates that, a minor difference between the number of votes and share of mandates exist, which indicate that the result becomes more proportional. The standard deviation measures the average dispersion between each observation and the average value. A wide spread between each observation and the average value gives a high standard deviation. Because the deviation is squared, some extreme points can however have a disproportional large effect on the standard deviation.

The proportionality measure in the 2010 election is 2.30, which is received if the absolute difference between share of votes and share of mandates are summed in the figure 4.3.1. As stated in the introduction, there was an intense debate after the last election. A good system shall therefore give a number that is smaller than this. This number is used as a benchmark to evaluate the outcomes of the simulations.

In the first simulation, which is tested with different divisor systems, all of them show a similar pattern. They indicate that the result becomes more proportional as the number of adjustment seats increases. The same pattern occurs when the number of constituencies decreases, however just to some extent. The result tends to improve as the number of constituencies decreases to fifteen, then the improvement disappears. A reason can be that a limit is reached, where no further improvement are possible. This can easily be seen in the diagrams in appendix C, which shows decreasing numbers. A similar development occurs with the standard deviation, which is shown in the matrix below. A correlation between these two measures tends to present.

Exactly which divisor system gives the most proportional turnout, and which values shall the adjustable variables take? An exactly interpretation of the diagram shows that the largest number of adjustment seats, and ten or fifteen constituencies shall be used. The simulation is restricted to these values, there might for this reason be other values which give better result, for example, if 69 adjustment seats are used. The simulations illustrates that all the Saint-Laguë divisors results in a more proportional election with lower standard deviation than D´Hondt method. This becomes obvious just by looking at the diagrams. This confirms the literature, which states that D´Hondt method tends to favour larger parties, and create a less proportional system (Mattson and Petersson 2003, p 78). There is no system that gives better result in every cell than all the others, instead the values of the other variables is determining which divisor systems that is best suited. For example, if the current system with 29 constituencies and 39 adjustment seats are used, the system with a first divisor 1.2 results in the most proportional election outcome 5.75, which are far better than if the first divisor 1 or 1.6 is used. Still this value is worse then the benchmark value.

When many constituencies are present, for this situation more than 25, the system with the first divisor 1.2 or 1.4 gives better result than the others. As the number of constituencies decreases, the difference between the systems do the same, and in some cases the other two divisor systems becomes more proportional. For example, if 21 constituencies and 30 adjustment seats are used, a first divisor, 1.6, gives the best result. Exactly which divisor to use depends, as stated before, on which values the adjustable variables take. A system with 1.6 as the first divisor tends to become more proportional as the number of constituencies decreases at a higher pace than the other systems. This seems plausible, if many constituencies are present, then each constituency consist of less mandates. It becomes then harder for smaller parties to receive mandates if a large value on the first divisor is chosen. As the number of constituencies decrease, more mandates are available in each constituency, which gives smaller parties a possibility to receive mandates. A first divisor of one, which tends to favour smaller parties, is always inferior to another divisor system if more than 15 constituencies are chosen.

When 10 to 15 constituencies are used, just a small difference between the systems occurs. Even D´Hondt method gives a result similar to the four Sainte-Laguë methods, which divisor that is used does hence not have much of an impact, under this circumstance. If more constituencies are present, it depends on exactly which variables that are chosen to determine the system that is best suited. If more than 25 constituencies are used, the most proportional result is received with the first divisor 1.2 or 1.4.

The system currently in use in Sweden gives a proportionality value of 7.60 with a standard deviation of 3.14 according to simulation one. This can be compared with the best result of 0.59 with a standard deviation of 0.13, which is obtained when 59 adjustment seats and fifteen constituencies exists. As movement to the lower left corner in the diagram takes place, the result improves. The current system is placed in the upper right corner, which indicates that it is not optimal in order to achieve the most proportional result. If a change in the current system to the lower left corner in implemented, a much more proportional election result can be achieved.

In which sense is the first simulation a reasonable way to analyse the Swedish election system? At the moment, 29 constituencies and 39 adjustment seats are used. Is it reasonable to change the number of constituencies to 15 instead of 29? As stated before, the constituencies are in reality not of equal size. For that reason is simulation two performed, where the present size of the constituencies are used. Just as in the first simulation gets the result more proportional as the number of adjustment seats increases, independently of which first divisor that is used. The standard deviation decreases as well when the number of adjustment sets increase. A value of the first divisor, which is above or beneath the interval 1.2 to 1.3 gives a less proportional result with a larger standard deviation than values in this interval, except for a few situations with many adjustment seats where the value 1.35 give a better result. This is though just a small difference and can be disregarded from. On the interval 1.2 to

1.3, the number of adjustment seats determines which system that is most proportional. When a value above 45 adjustment seats is chosen however, the difference between the systems becomes insignificant small. When the variables take the 2010 election values, the result 3.92 with the standard deviation 2.23 is received. The difference to the most proportional result, 0.68 with standard deviation 0.25, when 59 adjustment seats and a first divisor of 1.3 is used, is significant large. Simulation 2 shows that a more proportional result can be obtained if the current system is changed. It is worth noting that the actual result in the 2010 election was 2.30, which are less than the comparable value received in this simulation. The difference is 1.62, this is however less than the standard deviation. The result in the simulations tends hence to give a good reflection of reality. This indicate as well that a worse outcome than in the last election can occur, and since a debate followed the last election, an even larger debate may occur if the result becomes even more disproportional.

4.5 Analysis of the Results

The conclusion from both simulations is that as the number of adjustment seats increases the result becomes more proportional. The standard deviation of the result decreases as well, which give a more stable result. The first simulation shows that the Sainte-Laguë in general gives a more proportional result with lower standard deviation than the D´Hondt method. The first devisor has a significant impact on the result. It cannot though be said that any of them is superior to all of the others, instead it depends on the values the adjustable variables take. If the current system with 29 constituencies and 39 adjustment seats are used, a system with the first divisor 1.2 is the most proportional. If more adjustment seats are allowed, the divisor 1.4 gives a similar result. The proportionality value is still around 4, which is a high value and above the benchmark value 2.3. Simulation two, which is based on 29 constituencies, presents lower values than simulation one. It indicates that a value between 1.2 and 1.3 is optimal and as the number of adjustment seats increases, the more proportional the result becomes, which is the same conclusion as simulation one when 29 constituencies are used. It is worth mentioning that as the number of adjustment seats decreases below 41, the first divisor 1.2 is superior to the other. When the first divisor 1.2 is used in the second simulation and the current amount of 39 adjustment seats are present, the proportionality measure gives the result 1.43. This is an improvement compared to the benchmark value from the 2010 election 2.30. Both simulations indicate hence that if 29 constituencies are used, a divisor value between 1.2 and 1.3 gives the best result, and as the number of adjustment seats increases, the more proportional the result becomes.

An important conclusion from the first simulation is the stronger impact the number of constituencies has on proportionality compared to the number of adjustment seats. It is not possible to get a more proportional result than the last election without decreasing the number of constituencies. If 10 to 15 constituencies are used the result gets more proportional independently on the number of adjustment seats. For example, when the first divisor 1.2 is used with fifteen constituencies, then a difference of 0.29 between the best and worst outcome depending on the number of adjustment seats, is received. In a best-case scenario, when either 15 or 30 constituencies are used, the difference is 3.31. Thus is can be claimed that the number of constituencies affects the proportionality to a larger extent than the number of adjustment seats. The simulation shows as well that in order to get a result that is more proportional than 2.30, the number of constituencies must decrease. If the number of constituencies decreases to a number between 10 and 15, has the value of the first divisor not much of an impact, neither has the number of adjustment seats.

The aspect to have a more local connection for the politicians is an argument for keeping the number of constituencies higher. Another reason might be the economical and practical aspects. The classification of constituencies mostly follows the geographical state boundary (Mattson and Petersson 2003, p. 70) and to change this requires substantial administrational and economic costs. A historical and political aspect is a third factor. Because this classification has been used for a long time it is difficult to decrease the number of constituencies without protests.

Why becomes the result more proportional as the number of adjustment seats increases? The reason for using adjustment seats is to correct for the discrepancy that may occur when mandates are distributed in many constituencies, instead of the country as one constituency. It seems quite logic that the proportionality measure decreases as the number of adjustment seats increases, which also is the case in both simulations. Does it exist any upper bound for the amount of adjustments seats that can be used? The simulations give no technical argument for this. It is limited to the number 59 by the assumptions. The upper limit seems instead to be restricted by other reasons, for example, that there must be reasonable many mandates available in each constituency so the system makes sense. The adjustment seats shall operate as the name indicates, to adjust; therefore it can be argued that no more adjustment seats than ordinary seats shall be present.

Based on our analysis it can be argued that the system used in Sweden today, not gives the most proportional result. Instead some of the following measures should be considered.

- Decrease the number of constituencies to a number between 10 and 15. Simulation 1 shows a significant improvement in proportionality when this is done. A further, however minor improvement can be achieved by increasing the number of adjustment seats, this improvement is though substantially small when less number of constituencies are used. By changing the first divisor under this circumstances have a similar independently minor effect.
- If a large amount of constituencies, for example the current 29, are used has the value of the first divisor a significant effect on proportionality. Both simulations indicate that a value between 1.2 and 1.3 is optimal to use under this circumstances. According to the second simulation, a value in this interval gives in general a more proportional result than the 2010 election, and as the number of adjustment seats increases the more proportional the result becomes. If 59 adjustment seats are used a value of 0.72 can be achieved, which is a major improvement compared to the 2010 election, but even with about 40 adjustment seats is a more proportional result most likely to occur.

Both the suggestions, which are based on the results from the simulations, indicates that the current system is not optimal, and by varying some of the variables a more proportional result can be obtained. They do however not give an exact result that says exactly which values to use, rather serves they as a guideline to create a more proportional result based on the actual circumstances. One reason is because they examine different aspects of the system. If it is not a realistic suggestion to decrease the number of constituencies then the result based on the second simulation is to prefer. If there are no obstacles, it seems to be recommendable to decrease the number of constituencies. Which simulation gives most relevant result is neither as clear. It can be argued that since simulation two is based on the current distribution and that this simulation gives result, which is more similar to the observed values in the 2010 election, this is the more important simulation. Simulation one should however not be condemned since there is no actual election to compare with when the number of constituencies is decreased, which is the main intention to examine in this simulation. Instead is the change in result when some variables are change the interesting part to look at, both simulation give for that reason valuable information. Even if one of them tends to reflect reality to larger extent, at least when it comes to numbers, shows the first simulation a uniform improvement in proportionality when the number of constituencies are decreased.

Why does the second simulation in most cases give better result than the first with the same number on the variables? One answer is because the second simulation uses the current distribution of constituencies. The second simulation gives also a result, which is closer to the actual result of the last election than the first simulation. The aspect that the constituencies are of different size in the second simulation seems to play a significant role, especially when many constituencies are used. If they are of equal the smaller parties might be just beneath to receive a mandate in each constituency. When some constituencies instead have more mandates to distribute it makes it possible for smaller parties to win mandates in some constituencies at least. They will though not receive mandates in the minor constituencies, however they might not have done that anyway if all the constituencies were of equal size.

The results in this essay are based on the circumstances in Sweden, the result can though be considered for other countries as well. D´Hondt method shows for example similar development as the Sainte-Laguë methods. That the result becomes more proportional as the number of constituencies decrease is hence an important result that other countries shall consider even if they use another divisor method. The same argument is relevant for the function that the adjustment seats has, an increase in these tends to establish better proportionality.

4.6 Shortcomings with the Simulations

To be able to perform these simulations some assumptions had to be made. One is the fact that the lowest number of votes a party can receive is 4%. No party can hence fall below the threshold, neither can any new parties enter. This is not a total reflection of reality. Sometimes a new party enters and sometimes some disappears, especially when a party is close to the 4% threshold. If 2500 elections are held, which are based on the last election it is most likely that some of the parties get to few votes in some elections. This can affect the result and the distribution of mandates.

The systems with the three blocks may not hold forever. Instead maybe all parties will compete individually. The left wing side is for example not a cooperation in the 2014 election in the same sense as in the election 2010. This is though not a major problem since the votes are based on the result for each party separately in the last election. The distribution between the blocks in this essay is just for technical reasons to perform the simulations. The essay examines just proportionality not who actually wins the election.

The number of mandates in the parliament is held constant in both simulations. If this numbers is changed, the result is of course affected. This aspect is not incorporated in the simulations. Exactly which impact an increase or a decrease has, is hypothetical. Has it the same effect as the numbers of adjustment seats? When the number of mandates increases gets the result more proportional as well? For that reason is a suggestion for future research to analyse in what way the number of mandates in the parliament affects the proportionality.

Some upper and lower limit of adjustment sets must also be considered and the interval 30-59 seemed like a reasonable choice from our side. To extend these numbers would also require more time to perform the simulations.

As stated before is neither the possibility to change the method for distributing mandates between the constituencies taken into account. Instead this distribution is taken as given. If it was possible to change this then another result would have been possible.

It would however be too extensive to take all of these aspects into consideration when programming the simulations. It took about one week for a computer to perform these. If some of the assumptions are relaxed, for example the possibility for a party to fall below the threshold, then it takes another week to perform the simulations. For that reason is it not possible to include these shortcomings in the essay. It is as stated before rather a suggestion for further research to investigate the effect if some of these restrictions are relaxed.

5. Summary

This chapter summarises the main aspects of this essay, namely the purpose, analysis and the results as well as our suggestions for measurements that make the Swedish election system more proportional.

In the first chapter it is stated that the system used for the Swedish parliament election is not without any criticism. Rather, an ongoing debate concerning the issue of proportionality is present. The purpose with this essay is therefore to examine how the system can be made more proportional. The second aim is to present a systematic overview over the most common in use elections systems. Since the Swedish election system is the main object with the analysis, a more detailed description of this system is presented. An important insight from chapter three is that many different systems can be used and many different systems are used around the world. No system can however be said to be superior to all the others. Why a specific system is used is dependent on historical, cultural, and political reasons.

Two different kind of simulations are performed to examine how proportional the Swedish election system is if some variables are adjusted. In the first simulation where the constituencies are of equal size it is clear that the outcomes becomes more proportional as the number of constituencies decreases. The reason for this is that when many constituencies are used there are fewer mandates in each constituency to distribute. In most cases these mandates are distributed to the largest parties, but as the number of constituencies decreases, more seats become available in each district and also the minor parties are able to win seats. Both simulations indicate that as the number of adjustment seats increases the more proportional the result becomes. This effect is however not as substantial as the effect from decreasing the number of constituencies, according to the first simulation. The second simulation where the constituencies have the current distribution of mandates shows that the value on the first divisor in the Sainte-Laguë divisor method has a major effect on proportionality. This effect is also more substantial than the effect of adjustment seats, just as in the first simulation. It is shown that a value between 1.2 and 1.3 gives the most proportional result.

Our result shows that the current elections system in Sweden is not optimal to achieve proportionality. If the adjustable variables in the simulations take the values used in the 2010 election, the result becomes not one of the most proportional ones. In the first simulation this is obvious only by looking at the diagrams, where these variables are in the upper right corner and the result tends to become more proportional as a movement to the lower let corner is performed.

A comparison between the benchmark value from the 2010 election and the outcomes from the simulations indicate that an election turnout, which is more proportional can be achieved by changing some of the variables, number of constituencies, adjustment seats and value of the first divisor, in the elation system. According to Simulation 2 is a first divisor value between 1.2 and 1.3 to prefer, and as the number of adjustment seats increases the more proportional the result becomes. The exact value can be discussed, however. No strict line exist, a value above 40 is though to prefer. Simulation 1 suggests that the number of constituencies should decrease to a value between 10 and 15. If this is implemented, has the value of the first divisor not a significant impact, neither has the number of adjustment seats. The simulations do not hence give one perfect solution, since different aspects of the election system are examined, instead they serve as guidelines to consider when the election system is analysed and perhaps a reconstruction is considered. These guidelines can be considered for other countries with a proportional election system as well, because the simulations show a uniform improvement in proportionality when the number of constituencies decreases and the number of adjustment seats increases.

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7. Appendix

7.1 Appendix A

1. Stockholms kommun,
2. Stockholms läns valkrets (Stockholms län med undantag av Stockholms kommun),
3. Uppsala län,
4. Södermanlands län,
5. Östergötlands län,
6. Jönköpings län,
7. Kronobergs län,
8. Kalmar län,
9. Gotlands län,
10. Blekinge län,
11. Malmö kommun,
12. Skåne läns västra valkrets (Bjuvs, Eslövs, Helsingborgs, Höganäs, Hörby, Höörs, Landskrona och Svalövs kommuner),
13. Skåne läns södra valkrets (Burlövs, Kävlinge, Lomma, Lunds, Sjöbo, Skurups, Staffanstorps, Svedala, Trelleborgs, Vellinge och Ystads kommuner),
14. Skåne läns norra och östra valkrets (Bromölla, Båstads, Hässleholms, Klippans, Kristianstads, Osby, Perstorps, Simrishamns, Tomelilla, Åstorps, Ängelholms, Örkelljunga och Östra Göinge kommuner),
15. Hallands län,
16. Göteborgs kommun,
17. Västra Götalands läns västra valkrets (Härryda, Kungälvs, Lysekils, Munkedals, Mölndals, Orusts, Partille, Sotenäs, Stenungsunds, Strömstads, Tanums, Tjörns, Uddevalla och Öckerö kommuner),
18. Västra Götalands läns norra valkrets (Ale, Alingsås, Bengtsfors, Dals- Eds, Färgelanda, Herrljunga, Lerums, Lilla Edets, Melleruds, Trollhättans, Vårgårda, Vänersborgs och Åmåls kommuner),
19. Västra Götalands läns södra valkrets (Bollebygds, Borås, Marks, Svenljunga, Tranemo och Ulricehamns kommuner),
20. Västra Götalands läns östra valkrets (Essunga, Falköpings, Grästorps, Gullspångs, Götene, Hjo, Karlsborgs, Lidköpings, Mariestads, Skara, Skövde, Tibro, Tidaholms, Töreboda och Vara kommuner),
21. Värmlands län,
22. Örebro län,
23. Västmanlands län,
24. Dalarnas län,
25. Gävleborgs län,
26. Västernorrlands län,
27. Jämtlands län,
28. Västerbottens län, och
29. Norrbottens län.

Appendix B

Number of votes election 2010

Consistuency Area	M	C	FP	KD	S	V	MP	SD	Other	BLANK	NG	Turnout
Blekinge län	27387	5771	5431	3973	36520	5075	5289	9830	886	1305	37	101504
Dalarnas län	44997	14086	8747	7925	67139	10533	10652	12470	2619	2722	52	181942
Gotlands län	9731	5657	1785	1128	12855	2342	3259	1225	658	571	46	39257
Gävleborgs län	41009	12982	9444	7235	67893	12814	10918	12616	2337	2359	104	179711
Göteborgs kommun	96981	12183	26829	19484	80543	27246	34205	15608	6223	3044	112	322458
Hallands län	67878	17178	15286	10994	52319	6904	11568	10507	2931	2458	61	198084
Jämtlands län	18193	10487	3155	2340	33013	5340	5339	3122	947	1251	16	83203
Jönköpings län	57901	16859	12134	27822	66316	8775	11438	13888	1431	2543	94	219201
Kalmar län	41631	13829	7847	9341	55116	7679	8713	8964	1616	2139	54	156929
Kronobergs län	34762	11559	6667	7111	35555	5380	7044	7424	1011	1523	53	118089
Malmö kommun	55160	4795	11768	5274	48450	10118	14861	13256	5426	1502	57	170667
Norrbottens län	26852	7618	7082	5388	85035	15240	8630	6309	1809	1354	86	165403
Skåne läns norra och östra	60930	12871	12677	9420	54529	6113	10195	21312	2107	2613	113	192880
Skåne läns södra	87893	12717	19622	9916	50557	7597	16176	19923	4125	2459	91	231076
Skåne läns västra	58628	8164	13967	6989	49900	5847	9869	17448	2623	1921	90	175446
Stockholms kommun	183421	33895	45939	28244	111688	39565	65351	16950	9834	3948	199	539034
Stockholms län	286249	41369	59461	44880	159222	31617	53788	29886	9877	6206	245	722800
Södermanlands län	47889	9850	11299	8095	59463	8637	13065	11370	1759	2053	71	173551
Uppsala län	64750	17838	16878	12265	58862	11845	18993	10003	3596	2481	84	217595
Värmlands län	45578	13379	10652	8312	68520	10231	9997	8502	2044	2115	46	179376
Västerbottens län	30184	12699	10296	9125	72008	17034	12246	4651	2355	1695	85	172378
Västernorrlands län	34550	11185	8253	6983	70341	9642	8757	7264	3000	1625	65	161665
Västmanlands län	43462	8266	12016	7406	58222	9154	9459	9992	2166	1941	82	162166
Västra Götalands läns norra	46582	11449	13393	11092	56060	9907	12003	10513	1861	2261	49	175170
Västra Götalands läns södra	34334	9273	8883	7745	37817	6136	7315	8350	1334	1447	39	122673
Västra Götalands läns västra	73853	13563	20194	16525	59477	10506	15794	12504	2621	2685	69	227791
Västra Götalands läns östra	47049	13914	10387	11092	57095	8223	9440	9725	1636	2297	52	170910
Örebro län	43791	9807	11415	11235	70818	10311	11846	11136	2019	2153	61	184592
Östergötlands län	80141	17561	19017	16357	92164	14242	21225	14862	4172	3267	123	283131
Sweden	1791766	390804	420524	333696	1827497	334053	437435	339610	85023	65938	2336	6028682

Appendix C

Simulation one

Divisor: 1 2 3 4 5 etc

Average summed percentage deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	1,01	1,14	6,00	8,11	10,43	12,73	14,72	16,84	18,66	20,24	21,76	23,64	24,55
31	0,98	1,09	5,77	7,85	10,16	12,43	14,43	16,50	18,34	19,88	21,41	23,21	24,14
32	0,97	1,05	5,51	7,58	9,88	12,12	14,12	16,16	18,02	19,51	21,06	22,80	23,75
33	0,96	1,02	5,27	7,32	9,60	11,80	13,82	15,83	17,70	19,15	20,72	22,39	23,36
34	0,95	1,00	5,03	7,08	9,32	11,51	13,53	15,50	17,37	18,79	20,37	21,99	22,97
35	0,95	0,98	4,79	6,81	9,05	11,22	13,22	15,17	17,05	18,43	20,02	21,59	22,59
36	0,95	0,97	4,57	6,54	8,76	10,92	12,92	14,84	16,74	18,07	19,69	21,19	22,20
37	0,95	0,96	4,34	6,28	8,50	10,63	12,63	14,52	16,42	17,73	19,34	20,80	21,82
38	0,95	0,96	4,12	6,02	8,23	10,34	12,30	14,19	16,06	17,38	19,01	20,41	21,44
39	0,95	0,95	3,91	5,78	7,94	10,04	11,97	13,86	15,71	17,03	18,66	20,02	21,04
40	0,95	0,95	3,71	5,53	7,69	9,76	11,64	13,55	15,35	16,69	18,32	19,64	20,66
41	0,95	0,95	3,51	5,28	7,42	9,47	11,32	13,23	14,99	16,35	17,98	19,26	20,28
42	0,95	0,95	3,33	5,04	7,15	9,19	10,99	12,91	14,64	16,00	17,58	18,88	19,91
43	0,95	0,95	3,16	4,81	6,88	8,91	10,67	12,59	14,27	15,67	17,18	18,50	19,55
44	0,95	0,95	2,97	4,58	6,61	8,63	10,35	12,27	13,92	15,33	16,79	18,12	19,18
45	0,95	0,95	2,82	4,37	6,33	8,35	10,03	11,96	13,57	15,00	16,39	17,74	18,81
46	0,95	0,95	2,67	4,15	6,06	8,07	9,72	11,64	13,22	14,68	16,01	17,36	18,44
47	0,95	0,95	2,52	3,94	5,80	7,81	9,41	11,32	12,87	14,34	15,62	16,98	18,07
48	0,95	0,95	2,38	3,74	5,54	7,54	9,10	11,01	12,52	14,00	15,23	16,61	17,71
49	0,95	0,95	2,25	3,54	5,28	7,28	8,81	10,70	12,17	13,67	14,86	16,24	17,34
50	0,95	0,95	2,12	3,34	5,03	7,02	8,50	10,35	11,82	13,34	14,49	15,87	16,92
51	0,95	0,95	2,02	3,15	4,78	6,71	8,20	10,01	11,48	13,01	14,11	15,50	16,51
52	0,95	0,95	1,90	2,97	4,53	6,41	7,90	9,66	11,12	12,68	13,73	15,13	16,11
53	0,95	0,95	1,80	2,81	4,31	6,13	7,61	9,32	10,78	12,31	13,36	14,77	15,70
54	0,95	0,95	1,71	2,65	4,09	5,84	7,31	8,99	10,44	11,93	12,99	14,41	15,30
55	0,95	0,95	1,62	2,51	3,87	5,56	7,03	8,65	10,11	11,55	12,62	14,05	14,90
56	0,95	0,95	1,54	2,35	3,66	5,29	6,74	8,32	9,78	11,18	12,27	13,69	14,50
57	0,95	0,95	1,47	2,20	3,47	5,03	6,47	8,00	9,46	10,81	11,90	13,33	14,12
58	0,95	0,95	1,40	2,07	3,27	4,77	6,20	7,68	9,13	10,44	11,55	12,97	13,73
59	0,95	0,95	1,34	1,95	3,10	4,53	5,94	7,35	8,81	10,09	11,19	12,62	13,34

Standard Deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	0,31	0,53	3,44	3,76	3,74	4,04	4,11	4,16	4,28	4,35	4,53	4,88	4,90
31	0,28	0,46	3,39	3,74	3,72	4,02	4,11	4,14	4,30	4,35	4,52	4,82	4,89
32	0,27	0,40	3,33	3,71	3,71	4,00	4,11	4,15	4,31	4,35	4,53	4,77	4,89
33	0,27	0,36	3,27	3,69	3,70	3,99	4,11	4,14	4,31	4,34	4,55	4,77	4,89
34	0,27	0,32	3,20	3,66	3,69	3,96	4,12	4,13	4,33	4,33	4,55	4,77	4,89
35	0,27	0,30	3,14	3,60	3,67	3,94	4,12	4,14	4,35	4,33	4,56	4,76	4,89
36	0,26	0,29	3,07	3,55	3,66	3,93	4,13	4,14	4,38	4,32	4,58	4,77	4,90
37	0,26	0,28	3,00	3,49	3,63	3,93	4,13	4,14	4,39	4,31	4,59	4,77	4,90
38	0,26	0,27	2,92	3,43	3,61	3,91	4,11	4,14	4,34	4,32	4,61	4,76	4,91
39	0,26	0,27	2,84	3,37	3,59	3,90	4,09	4,14	4,30	4,31	4,63	4,76	4,91
40	0,26	0,27	2,76	3,31	3,58	3,88	4,07	4,14	4,29	4,31	4,64	4,75	4,92
41	0,26	0,27	2,69	3,25	3,55	3,88	4,05	4,15	4,27	4,32	4,66	4,75	4,92
42	0,26	0,26	2,61	3,18	3,51	3,86	4,04	4,15	4,27	4,31	4,60	4,75	4,94
43	0,26	0,27	2,52	3,10	3,47	3,85	4,01	4,16	4,27	4,32	4,60	4,75	4,96
44	0,26	0,26	2,43	3,04	3,43	3,82	3,98	4,16	4,26	4,34	4,59	4,75	4,98
45	0,26	0,26	2,35	2,97	3,38	3,80	3,97	4,16	4,25	4,35	4,57	4,77	4,99
46	0,26	0,26	2,26	2,90	3,34	3,78	3,94	4,19	4,25	4,37	4,57	4,77	5,01
47	0,26	0,26	2,16	2,83	3,29	3,75	3,92	4,20	4,25	4,38	4,56	4,77	5,03
48	0,26	0,26	2,07	2,75	3,24	3,72	3,90	4,22	4,25	4,40	4,54	4,78	5,05
49	0,26	0,26	1,98	2,67	3,19	3,70	3,89	4,23	4,26	4,42	4,53	4,78	5,07
50	0,26	0,26	1,88	2,59	3,14	3,67	3,88	4,18	4,25	4,42	4,51	4,78	5,07
51	0,26	0,26	1,79	2,50	3,08	3,60	3,86	4,17	4,26	4,44	4,49	4,80	5,03
52	0,26	0,26	1,69	2,41	3,01	3,53	3,83	4,14	4,27	4,44	4,48	4,81	5,00
53	0,26	0,26	1,59	2,33	2,94	3,47	3,81	4,10	4,26	4,42	4,46	4,82	4,97
54	0,26	0,26	1,49	2,23	2,87	3,42	3,79	4,07	4,26	4,39	4,45	4,82	4,95
55	0,26	0,26	1,39	2,14	2,80	3,37	3,76	4,05	4,25	4,36	4,44	4,83	4,92
56	0,26	0,26	1,30	2,03	2,72	3,30	3,72	4,01	4,24	4,33	4,42	4,84	4,90
57	0,26	0,26	1,21	1,92	2,64	3,24	3,69	4,00	4,24	4,29	4,41	4,84	4,88
58	0,26	0,26	1,11	1,82	2,56	3,17	3,64	3,96	4,22	4,26	4,40	4,83	4,85
59	0,26	0,26	1,04	1,72	2,48	3,09	3,59	3,92	4,21	4,24	4,38	4,84	4,84

Divisor: 1 3 5 7 9 11 etc

Average summed percentage deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	1,05	0,96	5,33	7,14	8,76	10,02	10,59	11,26	11,60	11,53	11,55	12,08	11,29
31	1,00	0,94	5,32	7,14	8,72	9,89	10,51	11,09	11,49	11,31	11,41	11,77	11,12
32	0,96	0,93	5,29	7,12	8,67	9,77	10,41	10,92	11,35	11,12	11,27	11,51	10,94
33	0,92	0,91	5,27	7,12	8,61	9,63	10,35	10,75	11,24	10,91	11,12	11,31	10,78
34	0,89	0,90	5,26	7,11	8,56	9,51	10,28	10,61	11,14	10,73	10,99	11,10	10,61
35	0,86	0,87	5,24	7,05	8,51	9,40	10,19	10,44	11,02	10,54	10,86	10,89	10,46
36	0,83	0,84	5,24	7,00	8,46	9,28	10,12	10,30	10,92	10,36	10,72	10,70	10,32
37	0,81	0,82	5,23	6,95	8,41	9,18	10,07	10,17	10,84	10,18	10,59	10,50	10,17
38	0,79	0,80	5,21	6,88	8,37	9,08	9,87	10,02	10,59	10,00	10,47	10,32	10,03
39	0,78	0,78	5,20	6,83	8,31	8,97	9,67	9,90	10,38	9,84	10,36	10,14	9,90
40	0,77	0,77	5,20	6,78	8,27	8,89	9,48	9,78	10,17	9,70	10,25	9,95	9,77
41	0,75	0,76	5,18	6,71	8,24	8,81	9,29	9,63	9,97	9,54	10,14	9,78	9,65
42	0,72	0,75	5,17	6,67	8,11	8,71	9,11	9,53	9,77	9,39	9,87	9,61	9,53
43	0,70	0,74	5,17	6,62	7,97	8,63	8,94	9,42	9,58	9,26	9,66	9,47	9,43
44	0,69	0,73	5,15	6,55	7,83	8,56	8,77	9,30	9,40	9,12	9,43	9,32	9,32
45	0,67	0,73	5,15	6,51	7,71	8,47	8,61	9,20	9,23	9,00	9,22	9,17	9,22
46	0,66	0,72	5,14	6,47	7,59	8,40	8,45	9,12	9,06	8,89	9,03	9,03	9,11
47	0,65	0,72	5,13	6,41	7,47	8,34	8,28	9,01	8,90	8,77	8,84	8,91	9,00
48	0,65	0,71	5,12	6,36	7,35	8,24	8,13	8,92	8,74	8,67	8,65	8,78	8,91
49	0,64	0,71	5,12	6,32	7,23	8,19	8,00	8,84	8,58	8,58	8,47	8,65	8,83
50	0,64	0,71	5,09	6,27	7,13	8,13	7,86	8,63	8,43	8,46	8,29	8,55	8,63
51	0,63	0,70	5,07	6,23	7,03	7,94	7,74	8,42	8,30	8,37	8,13	8,43	8,43
52	0,62	0,70	5,01	6,19	6,93	7,79	7,63	8,22	8,16	8,28	7,97	8,33	8,23
53	0,62	0,70	4,96	6,15	6,83	7,61	7,49	8,03	8,03	8,08	7,82	8,22	8,05
54	0,62	0,69	4,91	6,12	6,75	7,43	7,39	7,83	7,91	7,88	7,67	8,11	7,86
55	0,61	0,69	4,86	6,09	6,65	7,27	7,29	7,65	7,78	7,67	7,52	8,02	7,68
56	0,61	0,69	4,81	5,94	6,58	7,11	7,18	7,48	7,67	7,47	7,39	7,92	7,52
57	0,61	0,69	4,76	5,82	6,51	6,96	7,10	7,31	7,57	7,28	7,26	7,82	7,36
58	0,61	0,69	4,69	5,70	6,43	6,82	7,01	7,15	7,44	7,10	7,13	7,74	7,19
59	0,60	0,69	4,65	5,57	6,37	6,67	6,91	7,00	7,34	6,92	7,02	7,65	7,05

Standard Deviation djustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	0,44	0,46	2,04	2,14	2,04	2,08	2,07	2,19	2,57	2,68	2,72	3,08	2,82
31	0,41	0,44	2,04	2,14	2,03	2,05	2,07	2,18	2,57	2,65	2,73	3,02	2,80
32	0,39	0,43	2,03	2,13	2,02	2,02	2,07	2,17	2,57	2,62	2,75	2,96	2,78
33	0,37	0,41	2,03	2,12	2,00	2,00	2,07	2,17	2,58	2,60	2,75	2,92	2,78
34	0,35	0,40	2,03	2,12	1,99	1,97	2,08	2,17	2,60	2,58	2,77	2,87	2,77
35	0,33	0,38	2,02	2,09	1,99	1,95	2,07	2,16	2,60	2,56	2,80	2,82	2,75
36	0,32	0,35	2,01	2,06	1,98	1,93	2,09	2,18	2,62	2,55	2,81	2,78	2,77
37	0,30	0,32	2,01	2,03	1,98	1,92	2,09	2,18	2,63	2,54	2,82	2,75	2,78
38	0,29	0,30	1,99	1,99	1,98	1,91	2,06	2,18	2,59	2,53	2,85	2,70	2,78
39	0,28	0,29	1,99	1,97	1,97	1,90	2,03	2,19	2,55	2,53	2,85	2,68	2,79
40	0,27	0,28	1,98	1,95	1,97	1,89	1,99	2,20	2,51	2,53	2,87	2,65	2,80
41	0,26	0,26	1,97	1,92	1,97	1,89	1,97	2,19	2,47	2,53	2,89	2,62	2,81
42	0,24	0,25	1,96	1,91	1,93	1,88	1,95	2,20	2,42	2,54	2,82	2,60	2,81
43	0,22	0,24	1,96	1,89	1,89	1,88	1,94	2,22	2,39	2,56	2,76	2,58	2,83
44	0,21	0,24	1,94	1,87	1,85	1,88	1,92	2,22	2,36	2,57	2,70	2,58	2,83
45	0,20	0,23	1,94	1,86	1,82	1,87	1,91	2,24	2,34	2,58	2,65	2,56	2,85
46	0,19	0,23	1,93	1,85	1,81	1,88	1,91	2,26	2,31	2,59	2,61	2,56	2,86
47	0,18	0,22	1,92	1,83	1,78	1,88	1,90	2,26	2,30	2,59	2,57	2,57	2,85
48	0,17	0,22	1,91	1,82	1,76	1,87	1,91	2,27	2,29	2,60	2,52	2,56	2,87
49	0,17	0,22	1,91	1,82	1,73	1,87	1,92	2,28	2,27	2,61	2,50	2,56	2,88
50	0,17	0,21	1,89	1,81	1,71	1,88	1,92	2,25	2,27	2,62	2,47	2,57	2,83
51	0,16	0,21	1,88	1,80	1,70	1,86	1,93	2,21	2,27	2,63	2,45	2,56	2,76
52	0,15	0,20	1,84	1,80	1,69	1,86	1,93	2,17	2,26	2,64	2,43	2,57	2,70
53	0,15	0,20	1,81	1,80	1,68	1,82	1,93	2,14	2,27	2,59	2,41	2,58	2,66
54	0,15	0,20	1,78	1,79	1,68	1,79	1,94	2,12	2,28	2,53	2,40	2,58	2,60
55	0,14	0,20	1,75	1,79	1,67	1,76	1,96	2,09	2,27	2,47	2,38	2,58	2,56
56	0,14	0,20	1,72	1,74	1,67	1,74	1,95	2,07	2,28	2,42	2,37	2,59	2,52
57	0,14	0,20	1,70	1,69	1,67	1,73	1,97	2,06	2,29	2,37	2,37	2,59	2,48
58	0,14	0,19	1,67	1,63	1,66	1,72	1,98	2,05	2,28	2,33	2,36	2,59	2,45
59	0,13	0,19	1,65	1,60	1,66	1,70	1,99	2,04	2,29	2,30	2,36	2,59	2,42

Divisor: 1,2 3 5 7 9 11 etc

Average summed percentage deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	1,07	0,98	4,22	4,88	5,29	5,86	5,68	5,99	6,25	6,13	6,33	7,19	7,15
31	1,02	0,96	4,16	4,84	5,20	5,72	5,59	5,85	6,16	5,97	6,23	6,96	6,98
32	0,98	0,94	4,06	4,77	5,10	5,59	5,49	5,71	6,05	5,83	6,13	6,79	6,81
33	0,94	0,92	3,98	4,73	5,01	5,44	5,42	5,58	5,96	5,70	6,02	6,62	6,67
34	0,91	0,91	3,91	4,70	4,93	5,31	5,34	5,45	5,87	5,57	5,93	6,45	6,53
35	0,87	0,87	3,83	4,55	4,85	5,19	5,25	5,33	5,77	5,44	5,84	6,30	6,40
36	0,84	0,85	3,76	4,42	4,76	5,04	5,19	5,22	5,68	5,32	5,74	6,14	6,28
37	0,82	0,82	3,70	4,29	4,70	4,94	5,12	5,11	5,60	5,21	5,66	6,00	6,16
38	0,80	0,80	3,62	4,15	4,63	4,83	4,97	5,00	5,45	5,10	5,59	5,87	6,05
39	0,78	0,78	3,56	4,03	4,56	4,71	4,81	4,90	5,35	5,00	5,50	5,75	5,95
40	0,77	0,77	3,51	3,92	4,50	4,63	4,66	4,81	5,20	4,90	5,43	5,63	5,85
41	0,75	0,76	3,44	3,81	4,45	4,54	4,53	4,71	5,07	4,81	5,37	5,51	5,75
42	0,73	0,75	3,39	3,71	4,33	4,42	4,40	4,62	4,94	4,72	5,23	5,40	5,67
43	0,70	0,74	3,34	3,63	4,18	4,34	4,27	4,54	4,81	4,64	5,11	5,30	5,59
44	0,69	0,74	3,28	3,53	4,05	4,25	4,14	4,46	4,69	4,57	4,98	5,20	5,49
45	0,68	0,73	3,24	3,45	3,93	4,16	4,03	4,39	4,58	4,49	4,86	5,10	5,42
46	0,67	0,73	3,20	3,38	3,79	4,08	3,93	4,32	4,46	4,42	4,74	5,01	5,35
47	0,66	0,72	3,14	3,29	3,69	4,02	3,82	4,25	4,36	4,35	4,63	4,93	5,28
48	0,65	0,72	3,11	3,23	3,58	3,94	3,72	4,19	4,27	4,30	4,52	4,85	5,22
49	0,64	0,71	3,08	3,18	3,46	3,88	3,64	4,13	4,18	4,24	4,42	4,78	5,15
50	0,64	0,71	2,98	3,11	3,37	3,82	3,55	4,03	4,09	4,18	4,33	4,70	5,01
51	0,63	0,70	2,92	3,05	3,28	3,71	3,48	3,90	4,01	4,13	4,24	4,63	4,87
52	0,62	0,70	2,80	3,01	3,18	3,65	3,40	3,78	3,92	4,08	4,15	4,56	4,73
53	0,62	0,70	2,71	2,95	3,10	3,52	3,32	3,67	3,84	3,97	4,07	4,50	4,60
54	0,61	0,69	2,63	2,91	3,03	3,40	3,25	3,54	3,77	3,86	4,00	4,44	4,49
55	0,61	0,69	2,55	2,87	2,94	3,29	3,19	3,45	3,70	3,75	3,93	4,38	4,37
56	0,61	0,69	2,47	2,74	2,88	3,18	3,12	3,36	3,63	3,65	3,86	4,33	4,26
57	0,60	0,69	2,40	2,63	2,82	3,09	3,07	3,26	3,57	3,56	3,80	4,27	4,17
58	0,60	0,69	2,31	2,52	2,74	3,00	3,02	3,18	3,51	3,47	3,73	4,22	4,08
59	0,60	0,69	2,25	2,41	2,69	2,89	2,96	3,10	3,45	3,38	3,68	4,17	4,00

Standard Deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	0,46	0,47	1,47	1,46	1,53	1,68	1,75	1,90	1,99	2,13	2,22	2,46	2,41
31	0,43	0,45	1,47	1,45	1,52	1,65	1,75	1,88	1,99	2,11	2,22	2,41	2,39
32	0,41	0,44	1,44	1,45	1,52	1,62	1,74	1,85	1,98	2,09	2,21	2,38	2,37
33	0,39	0,43	1,42	1,45	1,51	1,60	1,75	1,83	1,98	2,07	2,20	2,34	2,36
34	0,37	0,42	1,39	1,45	1,50	1,58	1,75	1,82	1,99	2,05	2,20	2,31	2,33
35	0,34	0,39	1,37	1,41	1,50	1,57	1,74	1,80	1,98	2,03	2,21	2,28	2,30
36	0,33	0,36	1,36	1,37	1,49	1,55	1,75	1,79	1,99	2,02	2,20	2,25	2,29
37	0,32	0,34	1,34	1,33	1,50	1,54	1,75	1,79	2,00	2,01	2,20	2,23	2,28
38	0,30	0,32	1,33	1,31	1,50	1,54	1,71	1,78	1,96	2,00	2,20	2,22	2,26
39	0,29	0,30	1,31	1,29	1,49	1,52	1,67	1,77	1,93	2,00	2,20	2,20	2,26
40	0,28	0,28	1,29	1,27	1,49	1,52	1,63	1,76	1,89	1,99	2,20	2,18	2,26
41	0,26	0,27	1,27	1,26	1,49	1,51	1,60	1,75	1,86	1,98	2,21	2,17	2,26
42	0,25	0,26	1,26	1,25	1,46	1,50	1,58	1,75	1,83	1,97	2,18	2,15	2,26
43	0,23	0,25	1,25	1,24	1,42	1,49	1,56	1,75	1,80	1,97	2,14	2,14	2,27
44	0,22	0,25	1,24	1,23	1,38	1,49	1,52	1,74	1,77	1,96	2,10	2,13	2,25
45	0,21	0,24	1,23	1,21	1,35	1,48	1,51	1,74	1,75	1,96	2,07	2,11	2,26
46	0,20	0,24	1,22	1,22	1,32	1,48	1,50	1,73	1,72	1,95	2,03	2,10	2,26
47	0,19	0,23	1,20	1,19	1,29	1,48	1,47	1,72	1,70	1,94	2,00	2,10	2,26
48	0,18	0,23	1,20	1,19	1,27	1,47	1,46	1,72	1,69	1,94	1,96	2,09	2,26
49	0,17	0,23	1,19	1,19	1,25	1,47	1,45	1,71	1,67	1,94	1,94	2,08	2,26
50	0,17	0,22	1,16	1,18	1,24	1,47	1,44	1,67	1,66	1,93	1,91	2,07	2,22
51	0,17	0,21	1,15	1,18	1,23	1,42	1,43	1,63	1,65	1,92	1,88	2,06	2,17
52	0,16	0,21	1,11	1,17	1,20	1,39	1,42	1,59	1,64	1,92	1,85	2,06	2,12
53	0,15	0,21	1,08	1,16	1,20	1,37	1,41	1,55	1,63	1,88	1,83	2,04	2,08
54	0,15	0,20	1,05	1,16	1,19	1,34	1,41	1,52	1,62	1,83	1,82	2,03	2,04
55	0,15	0,20	1,03	1,15	1,18	1,32	1,40	1,49	1,60	1,80	1,81	2,02	2,01
56	0,15	0,20	1,01	1,11	1,17	1,29	1,38	1,46	1,60	1,76	1,79	2,02	1,98
57	0,14	0,20	0,98	1,07	1,16	1,27	1,38	1,43	1,59	1,72	1,78	2,01	1,95
58	0,14	0,20	0,96	1,03	1,16	1,24	1,38	1,41	1,58	1,70	1,76	2,01	1,93
59	0,14	0,20	0,94	1,01	1,15	1,22	1,36	1,39	1,57	1,66	1,75	2,00	1,91

Divisor: 1,4 3 5 7 9 11 etc

Average summed percentage deviation djustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	1,07	0,95	2,20	2,27	2,61	3,07	3,28	3,99	5,20	6,32	8,08	10,41	11,58
31	1,02	0,93	2,17	2,23	2,54	2,95	3,19	3,84	5,04	6,06	7,83	10,05	11,25
32	0,97	0,91	2,07	2,17	2,47	2,84	3,09	3,68	4,87	5,80	7,58	9,72	10,94
33	0,93	0,90	1,98	2,13	2,39	2,73	3,02	3,54	4,73	5,57	7,33	9,39	10,63
34	0,90	0,89	1,88	2,10	2,33	2,63	2,95	3,41	4,59	5,34	7,11	9,08	10,34
35	0,87	0,86	1,79	1,99	2,28	2,54	2,87	3,28	4,45	5,11	6,90	8,77	10,05
36	0,84	0,83	1,72	1,90	2,21	2,45	2,80	3,17	4,33	4,90	6,68	8,47	9,78
37	0,82	0,80	1,65	1,81	2,15	2,37	2,75	3,06	4,21	4,70	6,49	8,18	9,51
38	0,79	0,78	1,57	1,72	2,11	2,29	2,63	2,96	4,00	4,51	6,29	7,88	9,24
39	0,77	0,76	1,52	1,65	2,05	2,21	2,51	2,87	3,84	4,34	6,09	7,60	8,99
40	0,77	0,75	1,46	1,59	2,01	2,15	2,41	2,78	3,67	4,18	5,92	7,35	8,74
41	0,75	0,73	1,41	1,52	1,98	2,09	2,31	2,69	3,51	4,02	5,76	7,09	8,49
42	0,72	0,72	1,37	1,46	1,93	2,01	2,22	2,62	3,37	3,87	5,48	6,83	8,26
43	0,70	0,72	1,34	1,42	1,84	1,97	2,14	2,55	3,22	3,74	5,23	6,59	8,04
44	0,69	0,71	1,29	1,36	1,76	1,92	2,05	2,47	3,09	3,59	4,97	6,37	7,80
45	0,67	0,70	1,26	1,33	1,67	1,87	1,98	2,41	2,96	3,47	4,73	6,14	7,60
46	0,66	0,69	1,23	1,29	1,59	1,83	1,92	2,36	2,84	3,37	4,50	5,93	7,41
47	0,65	0,69	1,20	1,25	1,53	1,79	1,86	2,30	2,74	3,25	4,29	5,73	7,19
48	0,65	0,68	1,17	1,22	1,48	1,74	1,81	2,24	2,64	3,15	4,09	5,53	7,01
49	0,64	0,68	1,16	1,20	1,41	1,71	1,76	2,20	2,55	3,06	3,91	5,35	6,84
50	0,64	0,67	1,13	1,16	1,37	1,68	1,71	2,12	2,47	2,96	3,74	5,18	6,53
51	0,63	0,66	1,12	1,14	1,33	1,64	1,67	2,03	2,39	2,89	3,58	5,00	6,19
52	0,62	0,65	1,07	1,13	1,28	1,61	1,63	1,94	2,31	2,82	3,42	4,85	5,87
53	0,62	0,64	1,02	1,10	1,25	1,54	1,58	1,86	2,24	2,68	3,28	4,70	5,58
54	0,61	0,64	0,98	1,08	1,22	1,48	1,55	1,79	2,18	2,54	3,15	4,54	5,28
55	0,61	0,63	0,94	1,07	1,18	1,43	1,53	1,72	2,11	2,41	3,02	4,41	5,00
56	0,61	0,63	0,91	1,02	1,16	1,37	1,49	1,65	2,06	2,29	2,91	4,28	4,74
57	0,60	0,62	0,88	0,97	1,13	1,32	1,46	1,59	2,01	2,19	2,80	4,13	4,49
58	0,60	0,62	0,85	0,93	1,10	1,27	1,44	1,54	1,96	2,09	2,69	4,01	4,25
59	0,60	0,62	0,83	0,90	1,08	1,22	1,41	1,50	1,92	2,01	2,61	3,90	4,03

Standard Deviation djustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	0,45	0,45	0,92	0,99	1,12	1,36	1,58	1,97	2,46	2,83	3,21	3,68	3,67
31	0,43	0,43	0,91	0,98	1,11	1,32	1,55	1,91	2,41	2,77	3,17	3,59	3,64
32	0,41	0,42	0,88	0,97	1,09	1,29	1,51	1,85	2,36	2,70	3,12	3,50	3,62
33	0,38	0,41	0,85	0,96	1,07	1,26	1,48	1,80	2,33	2,62	3,07	3,45	3,59
34	0,36	0,40	0,82	0,95	1,06	1,23	1,46	1,75	2,28	2,54	3,02	3,39	3,55
35	0,34	0,38	0,79	0,92	1,04	1,20	1,43	1,70	2,24	2,48	2,97	3,34	3,53
36	0,32	0,35	0,76	0,88	1,02	1,17	1,41	1,66	2,20	2,40	2,93	3,29	3,51
37	0,31	0,32	0,74	0,84	1,01	1,14	1,39	1,62	2,15	2,33	2,88	3,24	3,49
38	0,30	0,31	0,72	0,82	0,99	1,12	1,34	1,57	2,03	2,27	2,83	3,19	3,46
39	0,28	0,29	0,70	0,79	0,98	1,11	1,29	1,55	1,93	2,21	2,77	3,14	3,44
40	0,28	0,27	0,68	0,76	0,96	1,08	1,25	1,52	1,86	2,15	2,72	3,09	3,42
41	0,26	0,25	0,66	0,74	0,95	1,07	1,20	1,49	1,79	2,09	2,68	3,04	3,39
42	0,24	0,24	0,65	0,72	0,93	1,04	1,16	1,46	1,72	2,03	2,55	3,00	3,37
43	0,23	0,23	0,63	0,70	0,90	1,02	1,13	1,43	1,66	1,98	2,49	2,95	3,34
44	0,22	0,23	0,61	0,67	0,86	1,01	1,09	1,39	1,60	1,92	2,40	2,89	3,31
45	0,21	0,22	0,60	0,65	0,82	0,99	1,06	1,36	1,54	1,87	2,31	2,83	3,27
46	0,19	0,21	0,59	0,64	0,79	0,97	1,03	1,34	1,49	1,83	2,25	2,78	3,24
47	0,19	0,21	0,57	0,62	0,77	0,95	1,01	1,31	1,45	1,78	2,17	2,72	3,19
48	0,18	0,20	0,56	0,61	0,75	0,93	1,00	1,29	1,40	1,74	2,09	2,68	3,16
49	0,17	0,20	0,55	0,60	0,72	0,92	0,98	1,27	1,37	1,70	2,03	2,62	3,13
50	0,17	0,19	0,54	0,58	0,70	0,92	0,96	1,20	1,34	1,66	1,95	2,57	3,06
51	0,16	0,18	0,53	0,57	0,69	0,88	0,94	1,16	1,30	1,63	1,89	2,52	2,97
52	0,16	0,17	0,50	0,56	0,66	0,86	0,93	1,11	1,27	1,60	1,83	2,47	2,88
53	0,15	0,17	0,48	0,55	0,65	0,83	0,90	1,05	1,25	1,53	1,77	2,42	2,80
54	0,15	0,16	0,45	0,54	0,63	0,79	0,89	1,01	1,22	1,44	1,71	2,38	2,70
55	0,15	0,16	0,43	0,53	0,61	0,76	0,88	0,97	1,19	1,36	1,66	2,32	2,61
56	0,14	0,16	0,41	0,50	0,60	0,74	0,86	0,93	1,17	1,30	1,61	2,28	2,53
57	0,14	0,15	0,39	0,46	0,59	0,70	0,85	0,90	1,15	1,24	1,57	2,23	2,43
58	0,14	0,15	0,37	0,43	0,57	0,67	0,84	0,86	1,13	1,18	1,52	2,18	2,34
59	0,13	0,15	0,36	0,41	0,55	0,65	0,82	0,84	1,11	1,14	1,48	2,15	2,24

Divisor: 1,6 3 5 7 9 11 etc

Average summed percentage deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	1,05	0,86	1,58	2,04	3,06	4,83	6,54	8,76	11,15	13,19	14,96	17,15	18,47
31	1,01	0,85	1,54	1,96	2,91	4,59	6,29	8,46	10,86	12,83	14,63	16,73	18,08
32	0,96	0,83	1,47	1,88	2,77	4,37	6,05	8,16	10,58	12,50	14,30	16,32	17,70
33	0,93	0,81	1,40	1,81	2,64	4,15	5,84	7,87	10,30	12,15	13,97	15,94	17,32
34	0,89	0,80	1,35	1,75	2,52	3,95	5,62	7,57	10,03	11,81	13,64	15,54	16,94
35	0,86	0,77	1,30	1,66	2,40	3,74	5,39	7,29	9,74	11,48	13,33	15,15	16,58
36	0,83	0,75	1,25	1,57	2,30	3,55	5,18	7,02	9,48	11,14	13,00	14,77	16,21
37	0,81	0,73	1,21	1,50	2,20	3,38	4,97	6,75	9,22	10,82	12,70	14,39	15,85
38	0,79	0,71	1,18	1,43	2,12	3,21	4,72	6,49	8,87	10,49	12,40	14,01	15,50
39	0,77	0,69	1,15	1,37	2,02	3,05	4,47	6,23	8,54	10,18	12,08	13,64	15,14
40	0,77	0,68	1,12	1,33	1,95	2,91	4,24	5,99	8,21	9,87	11,79	13,26	14,79
41	0,75	0,67	1,08	1,28	1,89	2,78	4,01	5,73	7,88	9,56	11,50	12,89	14,44
42	0,72	0,66	1,06	1,23	1,79	2,64	3,80	5,50	7,55	9,25	11,09	12,53	14,10
43	0,71	0,65	1,04	1,20	1,70	2,52	3,60	5,29	7,24	8,95	10,72	12,17	13,76
44	0,69	0,64	1,02	1,16	1,61	2,41	3,40	5,06	6,93	8,64	10,34	11,82	13,42
45	0,68	0,64	1,00	1,13	1,54	2,30	3,22	4,85	6,63	8,35	9,96	11,48	13,09
46	0,67	0,63	0,99	1,10	1,47	2,20	3,05	4,66	6,35	8,07	9,60	11,15	12,78
47	0,66	0,63	0,97	1,07	1,41	2,12	2,89	4,46	6,06	7,78	9,23	10,82	12,45
48	0,65	0,62	0,95	1,04	1,36	2,03	2,73	4,29	5,79	7,51	8,87	10,48	12,15
49	0,65	0,62	0,94	1,02	1,31	1,96	2,59	4,12	5,52	7,26	8,51	10,16	11,85
50	0,64	0,61	0,92	1,00	1,26	1,89	2,45	3,85	5,26	6,99	8,17	9,84	11,45
51	0,63	0,61	0,91	0,99	1,22	1,77	2,33	3,61	5,01	6,75	7,83	9,53	11,04
52	0,63	0,60	0,89	0,97	1,18	1,69	2,22	3,38	4,78	6,51	7,51	9,23	10,65
53	0,62	0,60	0,87	0,95	1,15	1,60	2,11	3,16	4,56	6,18	7,19	8,94	10,26
54	0,62	0,60	0,85	0,94	1,12	1,52	2,01	2,96	4,36	5,85	6,88	8,64	9,88
55	0,61	0,60	0,83	0,93	1,09	1,44	1,93	2,78	4,16	5,53	6,57	8,36	9,51
56	0,61	0,60	0,81	0,90	1,07	1,38	1,84	2,61	3,97	5,22	6,28	8,10	9,14
57	0,61	0,60	0,80	0,88	1,05	1,32	1,77	2,45	3,81	4,91	6,00	7,83	8,79
58	0,60	0,59	0,78	0,86	1,02	1,27	1,71	2,32	3,62	4,63	5,72	7,58	8,43
59	0,60	0,59	0,77	0,84	1,01	1,22	1,63	2,19	3,46	4,35	5,47	7,33	8,09

Standard Deviation Adjustment seats \ Consistuency areas	10	15	20	21	22	23	24	25	26	27	28	29	30
30	0,43	0,43	1,10	1,51	2,18	2,93	3,46	3,60	3,74	3,80	3,95	4,39	4,30
31	0,41	0,41	1,06	1,44	2,10	2,86	3,41	3,54	3,73	3,78	3,96	4,33	4,29
32	0,39	0,40	1,02	1,39	2,03	2,78	3,37	3,52	3,72	3,78	3,97	4,27	4,29
33	0,36	0,39	0,98	1,33	1,95	2,70	3,31	3,48	3,72	3,77	3,98	4,27	4,29
34	0,34	0,38	0,93	1,28	1,88	2,61	3,25	3,43	3,70	3,76	3,99	4,26	4,29
35	0,32	0,35	0,90	1,21	1,81	2,54	3,19	3,40	3,69	3,75	4,00	4,24	4,30
36	0,30	0,33	0,86	1,15	1,74	2,45	3,13	3,35	3,68	3,75	4,00	4,24	4,30
37	0,29	0,30	0,83	1,09	1,69	2,37	3,07	3,30	3,66	3,73	4,01	4,23	4,30
38	0,28	0,28	0,81	1,03	1,63	2,29	2,98	3,26	3,59	3,73	4,02	4,23	4,32
39	0,27	0,26	0,78	0,99	1,57	2,20	2,89	3,21	3,53	3,72	4,02	4,23	4,33
40	0,27	0,24	0,76	0,95	1,51	2,12	2,80	3,16	3,50	3,70	4,02	4,23	4,33
41	0,25	0,23	0,73	0,90	1,47	2,04	2,72	3,12	3,46	3,69	4,03	4,22	4,35
42	0,23	0,22	0,71	0,87	1,40	1,97	2,63	3,06	3,42	3,68	3,97	4,22	4,36
43	0,22	0,21	0,69	0,84	1,33	1,89	2,54	3,01	3,39	3,68	3,96	4,22	4,37
44	0,21	0,20	0,67	0,80	1,27	1,83	2,45	2,96	3,35	3,66	3,93	4,21	4,38
45	0,20	0,19	0,66	0,77	1,21	1,76	2,36	2,91	3,30	3,65	3,91	4,22	4,39
46	0,19	0,18	0,64	0,75	1,15	1,69	2,27	2,85	3,26	3,63	3,89	4,21	4,40
47	0,19	0,18	0,62	0,72	1,10	1,64	2,18	2,80	3,23	3,61	3,87	4,19	4,40
48	0,18	0,17	0,61	0,70	1,06	1,57	2,10	2,75	3,18	3,60	3,83	4,19	4,41
49	0,18	0,17	0,59	0,68	1,01	1,52	2,02	2,68	3,13	3,57	3,80	4,18	4,41
50	0,17	0,16	0,58	0,66	0,97	1,47	1,94	2,58	3,09	3,54	3,77	4,15	4,39
51	0,17	0,15	0,57	0,64	0,93	1,37	1,87	2,49	3,02	3,49	3,73	4,14	4,35
52	0,16	0,14	0,55	0,63	0,90	1,30	1,79	2,41	2,97	3,45	3,69	4,11	4,32
53	0,15	0,14	0,53	0,61	0,86	1,24	1,71	2,32	2,90	3,39	3,64	4,09	4,30
54	0,15	0,14	0,51	0,60	0,84	1,17	1,64	2,24	2,84	3,31	3,58	4,07	4,26
55	0,15	0,13	0,49	0,59	0,81	1,10	1,58	2,15	2,77	3,23	3,53	4,03	4,22
56	0,14	0,13	0,48	0,57	0,79	1,05	1,51	2,07	2,71	3,15	3,48	3,99	4,19
57	0,14	0,13	0,45	0,54	0,77	1,00	1,45	1,99	2,65	3,06	3,41	3,96	4,15
58	0,14	0,13	0,43	0,52	0,75	0,95	1,39	1,90	2,57	2,97	3,35	3,92	4,11
59	0,14	0,13	0,41	0,50	0,73	0,91	1,33	1,81	2,50	2,87	3,27	3,87	4,07

Simulation 2

Average summed percentage deviation Adjustment seats \ First divisor	1	1,05	1,1	1,15	1,2	1,25	1,3	1,35	1,4	1,45	1,5	1,55	1,6
30	3,91	3,17	2,69	2,33	2,29	2,88	3,78	5,18	6,64	8,18	9,69	11,12	12,46
31	3,91	3,17	2,68	2,31	2,25	2,78	3,63	4,99	6,41	7,93	9,42	10,86	12,19
32	3,80	3,08	2,61	2,22	2,14	2,61	3,39	4,70	6,10	7,59	9,09	10,51	11,84
33	3,70	3,02	2,54	2,16	2,05	2,45	3,17	4,42	5,79	7,26	8,76	10,17	11,49
34	3,50	2,83	2,36	1,98	1,84	2,17	2,83	4,04	5,39	6,85	8,35	9,76	11,09
35	3,41	2,75	2,29	1,91	1,75	2,02	2,63	3,77	5,09	6,52	8,03	9,44	10,76
36	3,33	2,68	2,22	1,82	1,64	1,84	2,37	3,44	4,73	6,14	7,66	9,06	10,38
37	3,18	2,54	2,07	1,70	1,49	1,64	2,12	3,13	4,39	5,77	7,28	8,68	10,02
38	3,17	2,53	2,07	1,70	1,46	1,58	2,00	2,95	4,18	5,53	7,03	8,44	9,78
39	3,11	2,49	2,04	1,67	1,43	1,50	1,87	2,75	3,92	5,24	6,73	8,11	9,44
40	3,06	2,45	2,01	1,64	1,40	1,45	1,76	2,57	3,68	4,95	6,42	7,78	9,11
41	3,01	2,41	1,98	1,62	1,37	1,40	1,67	2,40	3,45	4,67	6,11	7,46	8,78
42	2,95	2,38	1,95	1,60	1,34	1,34	1,56	2,23	3,22	4,39	5,81	7,14	8,46
43	2,81	2,26	1,85	1,51	1,25	1,22	1,40	1,99	2,92	4,05	5,45	6,76	8,07
44	2,80	2,25	1,84	1,50	1,24	1,20	1,34	1,88	2,76	3,84	5,20	6,51	7,80
45	2,73	2,18	1,78	1,44	1,17	1,11	1,22	1,67	2,48	3,51	4,85	6,14	7,42
46	2,73	2,18	1,78	1,44	1,17	1,10	1,18	1,60	2,35	3,32	4,63	5,89	7,16
47	2,61	2,08	1,69	1,37	1,12	1,03	1,08	1,44	2,11	3,01	4,28	5,53	6,77
48	2,59	2,07	1,68	1,36	1,11	1,00	1,03	1,33	1,94	2,79	4,03	5,26	6,50
49	2,48	1,99	1,61	1,31	1,07	0,96	0,96	1,21	1,74	2,52	3,70	4,90	6,12
50	2,38	1,90	1,55	1,25	1,02	0,92	0,90	1,11	1,58	2,29	3,40	4,56	5,77
51	2,34	1,88	1,53	1,24	1,01	0,91	0,87	1,05	1,46	2,09	3,13	4,27	5,44
52	2,28	1,82	1,47	1,19	0,96	0,86	0,81	0,96	1,31	1,87	2,84	3,92	5,07
53	2,20	1,76	1,43	1,16	0,94	0,84	0,78	0,90	1,20	1,69	2,57	3,60	4,71
54	2,20	1,76	1,42	1,16	0,94	0,84	0,78	0,88	1,14	1,59	2,40	3,38	4,45
55	2,17	1,74	1,41	1,14	0,93	0,83	0,77	0,84	1,07	1,48	2,22	3,15	4,16
56	2,17	1,73	1,41	1,14	0,93	0,83	0,76	0,83	1,03	1,39	2,09	2,95	3,91
57	2,15	1,73	1,40	1,14	0,92	0,82	0,75	0,81	1,00	1,32	1,96	2,75	3,66
58	2,15	1,73	1,40	1,14	0,92	0,82	0,75	0,79	0,95	1,24	1,85	2,61	3,47
59	2,00	1,58	1,28	1,02	0,83	0,74	0,68	0,72	0,85	1,09	1,64	2,34	3,16

Standard Deviation Adjustment seats \ First divisor	1	1,05	1,1	1,15	1,2	1,25	1,3	1,35	1,4	1,45	1,5	1,55	1,6
30	1,42	1,28	1,14	1,02	1,20	1,62	2,03	2,38	2,53	2,59	2,64	2,75	2,68
31	1,42	1,28	1,14	1,01	1,17	1,59	2,01	2,37	2,55	2,62	2,66	2,78	2,70
32	1,42	1,28	1,14	1,00	1,11	1,50	1,94	2,33	2,54	2,61	2,65	2,76	2,68
33	1,41	1,28	1,14	0,98	1,07	1,43	1,87	2,29	2,52	2,60	2,64	2,75	2,67
34	1,37	1,22	1,08	0,92	0,97	1,30	1,76	2,20	2,45	2,54	2,58	2,68	2,60
35	1,36	1,20	1,07	0,90	0,92	1,23	1,69	2,15	2,42	2,53	2,57	2,66	2,60
36	1,32	1,17	1,02	0,86	0,86	1,14	1,59	2,07	2,37	2,50	2,54	2,64	2,58
37	1,28	1,12	0,96	0,81	0,78	1,04	1,48	1,98	2,29	2,44	2,50	2,59	2,54
38	1,28	1,12	0,96	0,81	0,75	0,99	1,42	1,93	2,27	2,44	2,51	2,61	2,55
39	1,28	1,12	0,95	0,80	0,72	0,93	1,34	1,85	2,23	2,42	2,51	2,60	2,56
40	1,28	1,11	0,95	0,80	0,70	0,88	1,26	1,78	2,18	2,39	2,52	2,62	2,58
41	1,28	1,11	0,95	0,79	0,69	0,84	1,19	1,70	2,13	2,37	2,52	2,63	2,60
42	1,28	1,11	0,95	0,79	0,67	0,78	1,12	1,63	2,08	2,34	2,50	2,63	2,59
43	1,24	1,07	0,90	0,76	0,63	0,71	1,01	1,52	1,97	2,26	2,44	2,57	2,55
44	1,24	1,07	0,90	0,76	0,62	0,68	0,96	1,46	1,93	2,23	2,44	2,58	2,55
45	1,21	1,03	0,87	0,71	0,58	0,61	0,86	1,34	1,80	2,14	2,38	2,54	2,52
46	1,21	1,03	0,87	0,71	0,57	0,59	0,82	1,30	1,77	2,10	2,38	2,55	2,54
47	1,18	0,99	0,84	0,69	0,55	0,54	0,73	1,19	1,65	2,01	2,31	2,49	2,49
48	1,18	0,99	0,84	0,68	0,54	0,52	0,67	1,11	1,57	1,94	2,27	2,47	2,48
49	1,15	0,97	0,81	0,67	0,53	0,49	0,59	1,00	1,44	1,84	2,19	2,41	2,44
50	1,11	0,93	0,78	0,64	0,50	0,46	0,52	0,90	1,34	1,74	2,12	2,34	2,40
51	1,11	0,93	0,78	0,64	0,50	0,45	0,47	0,82	1,25	1,66	2,07	2,32	2,39
52	1,07	0,89	0,74	0,60	0,47	0,41	0,42	0,72	1,13	1,53	1,96	2,24	2,33
53	1,05	0,88	0,73	0,59	0,46	0,39	0,38	0,63	1,00	1,39	1,85	2,15	2,28
54	1,05	0,87	0,74	0,59	0,46	0,39	0,36	0,60	0,95	1,33	1,80	2,13	2,27
55	1,05	0,87	0,73	0,59	0,45	0,38	0,35	0,56	0,88	1,25	1,74	2,08	2,24
56	1,05	0,87	0,73	0,59	0,45	0,38	0,34	0,53	0,84	1,18	1,68	2,04	2,22
57	1,05	0,87	0,73	0,59	0,45	0,38	0,33	0,51	0,80	1,13	1,61	1,99	2,20
58	1,05	0,87	0,73	0,59	0,45	0,37	0,32	0,47	0,75	1,05	1,54	1,93	2,16
59	0,99	0,80	0,66	0,52	0,38	0,31	0,25	0,38	0,61	0,89	1,37	1,77	2,01