Bachelor thesis

Supersymmetry and the Higgs sector of the Next-to Minimal SuperSymmetric Model

Jacob Winding

Theoretical High Energy Physics
Departement of Astronomy and Theoretical Physics
Lund University
Sölvegatan 14A
SE-223 62 Lund

Supervised by: Johan Rathsman

Abstract

The problems of the standard model are reviewed and the motivations for introducing supersymmetry are discussed. The basic theory behind supersymmetry is described briefly, followed by an introduction of two realistic supersymmetric models; the Minimal SuperSymmetric Model (MSSM) and its proposed extension, the NMSSM. Some details of the NMSSM are stated and some constraints on parameters are described. I then explore the Higgs boson masses and couplings for some interesting scenarios, including a few different ways of taking the limit where NMSSM reduces to MSSM.

Contents

1 Introduction

The standard model, despite being very successful and in overall very good agreement with experiments, has many obvious shortcomings. One of the leading contenders claiming to solve some of these is supersymmetry. As the perhaps most important example, we have the problem of the small Higgs mass in the standard model, called the hierarchy problem, to be explained further later on. This problem is automatically solved in supersymmetric theories, and there are many other problems which also goes away when we pass to a supersymmetric theory. From a theoretical viewpoint, supersymmetry is attractive not only because it solves some problems, but also since it can be said to be the most general possible extension of our normal spacetime symmetries. In string theory, supersymmetry is also a required ingredient to get a consistent theory containing fermions.

One of the purposes with this study, which is a Bachelor thesis at Lund university, is to give a taste of how supersymmetry works and how some of the problems in the standard model are solved. The other, more practical, purpose is to take a closer look at the Higgs sector in one of the proposed supersymmetric extensions of the standard model, and look at some of its qualitative features.

This paper is structured as follows: We start by very quickly reviewing the standard model, followed by a discussion on its deficits. The next section is an attempt to motivate the study of supersymmetry, which at first glance might seem slightly unrealistic but in fact fixes many of the problems of the standard model in a fairly natural way. In the next section, I try to introduce some basic concepts of supersymmetry at a hopefully understandable level, adopting the superspace and superfield view of the theory. The algebra and some of its simplest representations is presented, the concepts of superspace and superfields are introduced and finally some very simple global supersymmetric actions are written down. After this the simplest realistic model incorporating supersymmetry, MSSM, is introduced, and some of its problems are discussed. The simplest possible extension, NMSSM, which adds a singlet Higgs to the theory, is then proposed as a possible solution to the problems of MSSM, and the Higgs sector of this theory is studied in slightly more detail. Then the next section presents some numerical results investigating the Higgs mass spectrum for different parameter choices and in the MSSM limit, and finally some conclusions from these results are stated.

2 The Standard Model and supersymmetry

In this section I will first very briefly describe the standard model of particle physics, followed by a discussion of its deficits and problems. Then I will argue that there is good motivation to study supersymmetry and describe how supersymmetric extensions of the standard model solves some (although not all) of its problems.

2.1 Review of the standard model

The standard model (SM) is a highly successful theory of particle physics, and is over all consistent with all performed accelerator experiments (even though some precision measurements seem to disagree, such as the anomalous magnetic moment of the muon).

Name	Mass	Spin	Charge	Isospin I_z (L/R)
e^-	0.511 MeV	1/2	-e	$-\frac{1}{2}$ / 0
ν	$0 < m < 2$ eV	1/2	0	$\frac{1}{2}$
u	1.7-3.3 MeV	1/2	$\frac{2}{3}$e	$\frac{1}{2}$ / 0
d	4.1-5.8 MeV	1/2	$-\frac{1}{3}$e	$-\frac{1}{2}$ / 0
γ	$m < 1 \times 10^{-18}$ eV	0	$q < 1 \times 10^{-35}$e	0
$W^\pm$	80.40 GeV	1	$\pm$1e	0
Z	91.19 GeV	1	0	0
g (gluon)	0	1	0	0

Table 1: Properties of the particles in the first generation as well as the gauge bosons. Data taken from the Particle Data Group[1].

This section will be very brief, but there is of course a lot of reading material out there, so for a more complete introduction see for example [2]. Concisely put, the standard model is a relativistic quantum gauge field theory with gauge group SU(3) × SU(2) × U(1). The SU(2) × U(1) symmetry is spontaneously broken, leaving only the explicit remaining U(1) symmetry of electromagnetism, with the photon as the massless gauge boson. The symmetry breaking gives masses to the gauge bosons $W^\pm, Z$ which mediates the weak force. In order to break the symmetry, there needs to be a scalar field, called the Higgs boson, that when it acquires a nonzero vacuum expectation value (VEV) breaks the symmetry. It is through the couplings to this field that all particles acquire their mass. The SU(3) symmetry is left unbroken, and is carried by 8 massless gauge bosons called the gluons.

The remaining particles of the SM are all massive fermions, and seems to come "organized" in three identical, except for masses, generations. In every generation there are two *quarks* that carry the SU(3) charge, commonly called colour, as well as two *leptons* which doesn't carry colour. See table 1 for a summary of the particles in the first generation, as well as the gauge bosons and some of their properties. A curious fact about the standard model is that the weak force treats left and right handed fermions differently; it only interacts with the left handed particles. Thus, we have to treat for example the left handed electron e_L and the right handed e_R differently, and the same for the quarks. We organize the particles so that the left handed particles sits in SU(2) doublets, whereas the right handed are SU(2) singlets. For the first generation the particles are

$$L = \begin{pmatrix} \nu \\ e_L \end{pmatrix}, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad e_R, u_R, d_R \tag{1}$$

and analogously for the two other generations. The absence of a ν_R is due to the fact that the neutrino in the standard formulation is treated as massless and since the right handed neutrino doesn't have any gauge couplings, there is no reason for including it. However, since the neutrinos are not really massless, only very light, it is possible that a ν_R has to be added.

In a gauge theory such as the standard model, gauge symmetry doesn't allow us to directly add mass terms to the Lagrangian. This is solved by the idea of *spontaneous symmetry breaking*, where the vacuum becomes non-symmetric. In order to do this in the standard model, we add the *Higgs field*, which is a SU(2) doublet of complex scalar fields, and carries no colour. The parameters of the potential of this new field can then be chosen such that, in unitary gauge the stable minimum is one where the lower component of the doublet acquires a nonzero vacuum expectation value (VEV). This vacuum is not symmetric under the electroweak symmetry group SU(2) × U(1), but is symmetric under a (different) U(1) symmetry. This is the U(1) of electromagnetism, and we say that the electroweak symmetry has been spontaneously broken. Further, since the Z and the $W^{\pm}$ (which really are the linear combinations of the original gauge bosons of the unbroken electroweak symmetry that couples to the Higgs field) couple to the Higgs field and thus to its nonzero VEV, this gives them an effective mass term. In the same way we can add Yukawa couplings between the Higgs field and the massive fermions of the theory, giving them effective mass terms. That part of the standard model Lagrangian is called the Yukawa sector and is where most of the free parameters (the Yukawa coupling strengths, giving the masses) enters. An introduction at a very readable level to more exactly how the Higgs mechanism works can be found in [2], and for a slightly more technical review also discussing supersymmetry and technicolor ideas, see [3].

2.2 Problems of the standard model

The standard model, while highly successful and in good agreement with the absolute majority of performed experiments (there are however some persistent measured deviations, like the anomalous magnetic moment for the muon), still has some serious problems and is theoretically very unsatisfactory. For one thing, it has a large number (at least 19) of arbitrary parameters, including the particle masses, the three gauge couplings, the weak mixing angles and the CP-violating Kobayashi-Maskawa phase. Since it appears that neutrinos are not as previously believed massless, one must to the above ~ 19 parameters add three neutrino masses, three neutrino mixing angles and three different CP-violating phases. And to describe how the neutrinos acquire mass in a realistic way, even more parameters are necessary.

Another obvious deficit of the standard model is that it doesn't describe the fourth known force, gravity. What one would ultimately want is a theory that describes all the known forces in a unified way.

Other things that are observed but not described by the standard model is the large baryon asymmetry, i.e. why there is so much more matter than antimatter in the observed universe. There is a mechanism in the SM to create such an asymmetry but it is nowhere near strong enough to match what we observe. The large number of flavours in particle physics, that there are so many types of quarks and leptons, why they seem to come in generations and why the weak interaction mixes these generations the way it does is also examples of things that have no good explanations in the standard model.

The standard model also has to be modified in order to be consistent with the standard theories of cosmology. Especially, it cannot explain the observed cold dark matter, and when one calculates the vacuum energy in the standard model it gives contributions that are far too large to match the observed small, but nonzero, value of the cosmological constant. Another problem is that the standard model needs to be modified in order to be consistent with inflation.

Then there is the "hierarchy problem" concerning the smallness of gravity compared to electromagnetism. Since gravity couples to mass, this problem is equivalent to asking why the particle masses are so much smaller than the Planck mass scale ($M_P \sim 10^{19}$ GeV), i.e. why the ratio of the electroweak scale ($\sim M_W$) and the Planck scale is so tiny, $M_W/M_P \sim 10^{-16}$. From reasons stated above, we know that the standard model is an effective theory, at most valid up to the Planck scale. This means that when renormalizing the theory we must have a finite energy cutoff, that can at largest be the Planck mass. This finite cutoff is really what causes the problem. The masses of the particles come from the Higgs mechanism, so the question why the masses of the particles are small is in turn equivalent to asking why the Higgs has such a small mass. If one calculates loop corrections to the Higgs propagator, one finds that the corrections look like

$$\delta m_H^2 \approx \mathcal{O}(\alpha)\Lambda^2,$$

so the correction to the mass squared are proportional to the square of the UV-cutoff Λ, which by our previous reasoning is some large, finite energy, maybe of order M_P or at least of some large unification scale (called the GUT-scale, for Grand Unified Theory). So in order to give the Higgs its required mass, which of course is much smaller than either of these scales, these loop corrections has to be very precisely cancelled. This we can do by giving the tree level diagram exactly the required value. Technically, this is not a problem since there is no constraint on the value of the bare mass, but it introduces a very heavy fine-tuning into the theory, where the bare mass has to have exactly the correct value, and this is theoretically very unsatisfying.

A thing to note is that implicit in this line of reasoning is that you assume that there is no need for new physics below some very large energy, i.e. the cutoff scale is large. This is called the “big desert” assumption, and while many, holds this to be true, not all physicists agree. An argument for this assumption is that in the standard model, the running of the couplings is such that at a high energy scale, the grand unified scale (GUT-scale), of about $\sim 10^{16}$ GeV, all the known coupling become roughly the same, which implies that at least at this scale, our current physics should drastically change. Supersymmetry actually improves this a bit, making the couplings meet more closely than in the standard model. If one accepts the big-desert assumption, then the hierarchy problem is real, and some mechanism is needed to keep the Higgs mass small. Supersymmetry is the most popular proposal to solve this problem, but other theories exists, such as technicolor [4, 5] (in which the Higgs is a composite particle) and extra dimensions (for example the ADD model[6]). In the next section I will briefly explain how supersymmetry solves the hierarchy problem.

A final problem worth mentioning briefly is the strong CP problem. This comes from the observational fact that the strong interaction as described by QCD doesn’t seem to violate CP symmetry, in contrast with the weak force. This is a problem since there are natural terms in the QCD Lagrangian that violates CP conservation. To conform to the experimental data, a large amount of fine-tuning is again required. The most well-known proposition for solving this problem introduces new scalar particles, called axions, which can make an appearance in the NMSSM, as discussed later.

2.3 Motivation for supersymmetry

So what is supersymmetry and why would we want it? What it is, is a symmetry relating fermions to bosons, requiring that every particle has a supersymmetric partner with the same quantum numbers only differing in spin by 1/2. Since none of the known particles only differs in spin from another known particle, it might seem like a really bad idea to introduce a symmetry that more than doubles (we need at least one extra Higgs doublet, why is explained later) the number of particles needed. But despite this, there is still some compelling reasons for studying supersymmetric theories. From a phenomenological point of view, perhaps one of the stronger arguments is that supersymmetry (SUSY) offers a good candidate for dark matter, since it turns out that the lightest supersymmetric partner (which is a mixture of the superpartners of the photon, Z-boson and neutral Higgs bosons, called the neutralino) has to be stable, if we assume R-parity (which in turn is strongly implied from limits on the proton lifetime).

A slightly more theoretical argument concerns the gauge coupling unification at some high energy. In the usual standard model, the couplings run in such a way that they almost, but not exactly, all reach the same value at a high energy. Introducing SUSY gives exactly the contributions needed to make all the couplings meet.

Another attractive feature of supersymmetry is that it offers a way of constructing a theory of quantum gravity. The "ordinary" supersymmetry which we will study in this paper is a global symmetry, but if you gauge it, i.e. make it local, you necessarily get a theory of gravity, called supergravity[7]. Since, as we will see, supersymmetry is an extension of the ordinary spacetime symmetries rather than a new internal symmetry, it is natural to see how curved spacetime and thus gravity follows from a local supersymmetry.

As mentioned above, one other good reason to investigate supersymmetry is that it solves the hierarchy problem. Supersymmetry solves it since for every fermion that couples to the Higgs, it adds a scalar with the same quantum numbers. When calculating the loop corrections, the fermion loops and the scalar loops will be of the opposite sign and (if supersymmetry wasn't broken) be of the same size and thus cancel. Even when supersymmetry is broken, this removes the dependence on Λ^2 and reduces it to a logarithmic divergence[8], if supersymmetry is softly broken. It is worth pointing out that the problem isn't really the quadratic dependence but rather that we have good reasons to believe that the standard model only is valid up to some finite energy scale. This means that we cannot just send Λ to infinity and then remove the infinity in the Higgs mass by our usual renormalization methods. Instead we can only send Λ to some GUT-scale, since we expect the standard model to only be valid up to that scale. So if one believes that the standard model breaks down well before reaching the GUT-scale, in principle there is no problem. However we have no good theoretical or experimental reason to believe this to be the case, so the hierarchy problem remains.

Supersymmetry also lowers the vacuum energy you get from your quantum field theory, but nowhere near enough to explain the small cosmological constant. In fact, as is shown later, exact supersymmetry requires the cosmological constant to be exactly zero. Yet another reason is that string theory, which is one of the leading candidates for a "theory of everything" demands at least some kind of supersymmetry in order to consistently contain fermions.

However compelling these argument may seem, there is as of now no direct experimental indication that the world is supersymmetric. Since we do not see all the supersymmetric partners, it is obvious that they do not have the same mass, so if supersymmetry indeed is a symmetry of nature, it must be broken. Then one can ask, in what way is it broken and what is the breaking scale?

Supersymmetry has grown into a vast field, and the purpose of this section is merely to give the reader a flavour of the exciting ideas involved. For more complete expositions, there are numerous books and articles to consult, for example [9, 10, 11, 12].

3 Review of basic supersymmetry

In this section, I will briefly go through some of theory behind supersymmetry, stating the simplest possible algebra, look at its particle representations, introduce the concept of superspace and superfields, and finally look at how to use these concepts in order to write down a supersymmetric Lagrangian. Beware that this is a large and deep subject that branches of into many different areas of theoretical physics, and this brief review will only scratch the surface.

With that said, lets start by looking at how to characterize supersymmetry. In ordinary particle physics, we have three different kinds of symmetries:

- Poincaré invariance, or spacetime symmetry, i.e. symmetry under boosts, rotations and translations. This is generally described by the Poincaré algebra (which basically describes how "infinitesimal symmetry transformations", i.e. the generators, commute, like for example the relation $[J_i, J_k] = \epsilon_{ijk} J_k$ of rotation generators in quantum mechanics).
- Internal symmetries, for example the SU(2) $\times$ U(1) for the electroweak theory, or SU(3) for the strong force. These are local gauge symmetries, and their generators all commute with the Poincaré generators, which explains the name internal symmetries since they live in their own internal spaces without mixing with spacetime itself.
- The discrete symmetries C (charge conjugation), P (parity) and T (time reversal). In the standard model only the combination of all of them, CPT, is a real symmetry, and there is a theorem saying that this must be the case in any quantum field theory.

There is a famous theorem by Coleman and Mandula[13] that proves that under certain reasonable conditions these are all the possible symmetries we can have. In particular, it is not possible to extend the spacetime symmetry by adding new symmetry generators with non-vanishing commutators with the Poincaré group generators. However, one of the implicit assumptions of the theorem is that the algebra of the symmetry only involves normal commutators, or put in another way, that all generators are bosonic. It turns out that we can enlarge the symmetry by allowing the added generators to instead be anticommuting, or fermionic. Supersymmetry is then defined by introducing anticommuting symmetry generators which transforms in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representation of the Lorentz group, that is in the left and right handed Weyl spinor representation. Already here we see that since these generators are not scalar, they transform non-trivially under Lorentz transformations and are thus not the same as an additional internal symmetry.

In fact, in 1975, Haag Lopuszański and Sohnius [14] proved that supersymmetry is the most general symmetry allowed when including anticommutating generators. One might then think that further weakening of assumptions might lead to more interesting symmetries, but so far no compelling example of such a symmetry has been found. Thus, we can make a strong but not entirely unreasonable assertion that supersymmetry is *the only possible extension* of the ordinary spacetime symmetries.

Notational conventions

I here introduce some notation we need for stating the algebra and supersymmetric theories in a nice and clean way. For a left handed Weyl spinor with two components we will write ψ_α where $\alpha = 1, 2$ is a left handed spinor index, and for a right handed spinor I write $\bar{\psi}^{\dot{\alpha}}$, where the dot indicates a right handed spinor index. The conjugate of a left handed spinor is a right handed, $\bar{\psi}_{\dot{\alpha}} = (\psi_\alpha)^\dagger$. We use the two dimensional fully antisymmetric Levi-Civita symbols to lower and raise spinor indices, where $\epsilon_{12} = -1$ and $\epsilon^{12} = 1$. For example

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dot{\beta}}$$

and so on. Another definition that is needed to state the algebra in this formalism is of the matrices

$$\sigma^\mu = (1, \vec{\sigma}) = \bar{\sigma}_\mu \tag{2}$$
$$\bar{\sigma}^\mu = (1, -\vec{\sigma}) = \sigma_\mu \tag{3}$$

where 1 denotes the 2×2 identity matrix, $\vec{\sigma}$ is the ordinary Pauli matrices and μ is a Lorenz index taking values $0, 1, 2, 3$. Note that the bar here is part of the name and doesn't imply any kind of conjugation. Also note that in these conventions, the unbarred σ carries one undotted and one dotted index: $\sigma^\mu_{\alpha\dot{\beta}}$, whereas the barred has dotted-undotted indices: $\bar{\sigma}^{\mu\dot{\alpha}\beta}$.

We also need the Lorentz generators for left and right handed Weyl spinors, which are basically the commutators of σ^μ and $\bar{\sigma}^\nu$,

$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{1}{4}\left(\sigma^\mu_{\alpha\dot{\gamma}}\bar{\sigma}^{\nu\dot{\gamma}\beta} - (\mu \leftrightarrow \nu)\right) \tag{4}$$
$$(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} = \frac{1}{4}\left(\bar{\sigma}^{\mu\dot{\alpha}\gamma}\sigma^\nu_{\gamma\dot{\beta}} - (\mu \leftrightarrow \nu)\right) \tag{5}$$

Note here that $\sigma^{12} = \bar{\sigma}^{12} = -\frac{1}{2}\sigma_3$ so that the rotation generator $M^{12} = \frac{1}{2}\sigma_3$, as usual. As a warning to the reader, in the literature there are many different conventions for how to define various quantities, so these used here are in no way standard. In this study, we will follow the conventions of [10].

3.1 The general supersymmetry algebra

So we enlarge the Poincaré algebra with either left or right handed spinor generators, Q^I_α or $\bar{Q}^I_{\dot{\alpha}}$ where $I = 1, \ldots, N$ labels the different pairs of generators in case we add more than one pair. The new generators should commute with the translation generators, which is not directly obvious but can be shown using Jacobi relations, so

$$[Q^I_\alpha, P_\mu] = [\bar{Q}^I_{\dot{\alpha}}, P_\mu] = 0. \tag{6}$$

The fact that the added generators are spinors dictates how they transform under Lorentz transformations and thus their commutation relations with the generators $M_{\mu\nu}$ of the Lorentz transformations are

$$[Q^I_\alpha, M_{\mu\nu}] = \sigma^\beta_{\mu\nu\alpha} Q^I_\beta\,, \tag{7}$$
$$[\bar{Q}^{I\dot{\alpha}}, M_{\mu\nu}] = \bar{\sigma}^{\dot{\alpha}}_{\mu\nu\dot{\beta}} \bar{Q}^{I\dot{\beta}}\,. \tag{8}$$

Since Q^I is in the $(\frac{1}{2}, 0)$ representation and $\bar{Q}^I$ is in the $(0, \frac{1}{2})$, the anticommutator $\{Q^I, \bar{Q}^I\}$ must be in the $(\frac{1}{2}, \frac{1}{2})$, i.e. a fourvector. The natural candidate is P_μ. Further, by rotating the different Q^I generators, and rescaling them, we can always let $\{Q^I, \bar{Q}^J\} \propto \delta^{IJ}$, so the anticommutation relation becomes

$$\{Q^I_\alpha, \bar{Q}^J_{\dot{\beta}}\} = 2\delta^{IJ}\sigma^\mu_{\alpha\dot{\beta}} P_\mu\,. \tag{9}$$

Then, we can also take the anticommutator between Q^I_α and Q^J_β. Generally, we have for these

$$\begin{aligned} \{Q^I_\alpha, Q^J_\beta\} &= \epsilon_{\alpha\beta} Z^{IJ}, \\ \{\bar{Q}^I_{\dot{\alpha}}, \bar{Q}^J_{\dot{\beta}}\} &= \epsilon_{\dot{\alpha}\dot{\beta}} (Z^{IJ})^*\,. \end{aligned} \tag{10}$$

The Z^{IJ} are called the central charges, central because they commute with all the generators in the algebra. Because of the antisymmetric ϵ and the symmetry of the anticommutator, we must have $Z^{IJ} = -Z^{JI}$, so we see that with only one extra generator, $N = 1$ they must vanish because of this antisymmetry. This is obviously the simplest case, and is what we will study in slightly more detail in a later section. But first, one can note some general properties from the general algebra.

3.2 General properties of supersymmetric theories

Using the SUSY algebra, it is relatively easy to deduce some basic and general properties of supersymmetric theories. For one thing, since the usual Poincaré algebra is a subalgebra, an irreducible representation (irrep) of the full supersymmetric algebra will also be a representation of the Poincaré algebra, although not necessarily an irreducible one. As we will see, an irrep of the SUSY algebra will correspond to several different irreps of the Poincaé algebra, i.e. several particles. We call the irreducible representation of the SUSY algebra a *supermultiplet*, exactly because it contains multiple particles. The different particles in a supermultiplet are then related to each other through the action of Q^I and $\bar{Q}^J$.

Further, since $M_{12} = \frac{1}{2}\sigma_3 \equiv S_3$, the spin (or helicity) operator in the $x^3 = z$ direction, we have from the algebra above that $[J_3, Q_1^I] = \frac{1}{2}Q_1^I$ and $[S_3, Q_2^I] = -\frac{1}{2}Q_2^I$, and the exact same for $\bar{Q}_1^I, \bar{Q}_2^I$. This means that Q_1^I and $\bar{Q}_1^I$ raises the spin (helicity) in the z-direction by half a unit, and that $Q_2^I, \bar{Q}_2^I$ lowers it by half a unit. Combined with the reasoning above, this means that the particles in an irrep of the supersymmetric algebra have spins (helicities) differing by units of one half. This also means, by the spin-statistics theorem, that the Q and $\bar{Q}$ generators change bosons into fermions and fermions into bosons.

Another thing that is easy to prove using the algebra is that $P_\mu P^\mu = P^2 = m^2$ is a *Casimir* of the SUSY algebra. This means that it commutes with all the elements of the algebra, and thus, by Schurs lemma, that is has to be a scalar quantity, i.e. a multiple of the identity. This has the consequence that all the particles in a supermultiplet must have the same mass.

In a supersymmetric theory the energy (represented by P_0) must be positive definite. This follows from the algebra since, if $|\Omega\rangle$ is any state in the Hilbert room, which has a positive norm by definition, we have

$$\begin{aligned} 0 &\leq |Q_\alpha^I|\Omega\rangle|^2 + |\bar{Q}_{\dot{\alpha}}^I|\Omega\rangle|^2 = \langle\Omega|\bar{Q}_{\dot{\alpha}}^I Q_\alpha^I + Q_\alpha^I \bar{Q}_{\dot{\alpha}}^I|\Omega\rangle \\ &= \langle\Omega|\{Q_\alpha^I, \bar{Q}_{\dot{\alpha}}^I\}|\Omega\rangle = 2\sigma^\mu_{\alpha\dot{\alpha}}\langle\Omega|P_\mu|\Omega\rangle. \end{aligned} \tag{11}$$

Taking the trace of this, i.e. summing over $\alpha = \dot{\alpha} = 1, 2$ and using that all of the $\vec{\sigma}$ are traceless, we have Tr $\sigma^\mu = 2\delta^{\mu 0}$, and we get

$$0 \leq 4\langle\Omega|P_0|\Omega\rangle,$$

so the energy must be positive. This fact is very important when discussing the breaking of supersymmetry. In fact, it is easy to show that supersymmetry is spontaneously broken if and only if the vacuum energy is positive. A natural question is to wonder why the vacuum energy should matter at all, why can't we, as is usual when dealing with normal field theories, just shift the zero point energy in order to make it exactly zero? This we cannot do because the energy now is dictated by our symmetry, since the Hamiltonian appears in the algebra.

Another property is that a supermultiplet always contains the same number of fermionic and bosonic degrees of freedom. The degrees of freedom here means the number of different physical states. For example, a real scalar field has one degree of freedom, a photon has has two corresponding to its two helicities, and a spin 1/2 particle also has two corresponding to its different possible spins. To prove this, assign a fermion number N_F to each state in a supermultiplet, where $N_F = 1$ for a fermionic state and 0 for a bosonic state. Equivalently, this means that $(-1)^{N_F} = \mp 1$ for a fermionic respectively bosonic state. Then the statement that a given supermultiplet has the same number of fermionic and bosonic states can be stated

$$\sum_{states} (-1)^{N_F} = \mathrm{Tr}(-1)^{N_F} = 0\,, \tag{12}$$

where the sum runs over all states in the supermultiplet (which is the same as taking the trace over the representation). Since the supercharge Q turns bosons into fermions and vice versa, we have $(-1)^{N_F} Q = -Q(-1)^{N_F}$, that is, they anticommute. This we can use, by choosing some nonzero momentum p_μ and computing

$$\begin{aligned} 0 &= \mathrm{Tr}\left((-1)^{N_F}\bar{Q}_{\dot{\beta}}Q_\alpha - (-1)^{N_F}\bar{Q}_{\dot{\beta}}Q_\alpha\right) \\ &= \mathrm{Tr}\left((-1)^{N_F}\bar{Q}_{\dot{\beta}}Q_\alpha - Q_\alpha(-1)^{N_F}\bar{Q}_{\dot{\beta}}\right) \\ &= \mathrm{Tr}\left((-1)^{N_F}\{Q_\alpha, \bar{Q}_{\dot{\beta}}\}\right) = 2\sigma^\mu_{\alpha\dot{\beta}}\mathrm{Tr}\left((-1)^{N_F}P_\mu\right)\,, \end{aligned} \tag{13}$$

which since p_μ is nonzero implies the desired result. In the second equality, I use the cyclicity of the trace, followed by using the anticommutation of Q and $(-1)^{N_F}$ to get the anticommutator we need to use the algebra.

3.3 The D=4, N=1 SUSY algebra

The simplest of all possible supersymmetries in four dimensions are when we only add one new fermionic generator, this is known as $D = 4, N = 1$ supersymmetry. In this case there can be no central charges because of the fact that they are antisymmetric so $Z^{11} = -Z^{11} = 0$. The addition of multiple new generators and central charges do introduce some new things, but to discuss realistic supersymmetric models as we shall do here, it is enough to talk about $N = 1$. In fact, $N = 1$ is the only case that can be used as a simple low-energy extension of the standard model. This is because when $N > 1$ there is no supermultiplet containing only left or righthanded fermions, which we need since the weak force interacts differently with left and righthanded particles. Of course, at high enough energy we could still have a theory with $N = 2$ or higher, but then this symmetry has to be broken so that only a $N = 1$ supersymmetry remains closer to the electroweak scale.

The algebra in glorious detail is

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu \tag{14}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \tag{15}$$

$$[Q_\alpha, P_\mu] = [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0 \tag{16}$$

$$[\bar{Q}^{\dot{\alpha}}, M_{\mu\nu}] = \bar{\sigma}^{\dot{\alpha}}_{\mu\nu\dot{\beta}}\bar{Q}^{\dot{\beta}} \tag{17}$$

$$[Q_\alpha, M_{\mu\nu}] = \sigma^{\beta}_{\mu\nu\alpha}Q_\beta. \tag{18}$$

and the rest of the commutators involving only P_μ and $M_{\mu\nu}$ are as the usual Poincaré algebra. It turns out that the $N = 1$ supersymmetry has one additional symmetry, known as *R symmetry*. This is an internal symmetry, meaning that its commutators with the Poincaré generators disappears, and corresponds to a conserved quantity called R-charge which is +1 for the ordinary particles and -1 for their supersymmetric partners. That this charge is conserved is in essence what keeps the lightest supersymmetric particle from decaying. Worth mentioning is also that the R symmetry is Abelian and isomorphic to U(1). If we call this U(1) generator R one finds that

$$[Q_\alpha, R] = Q_\alpha, \quad [\bar{Q}_{\dot{\alpha}}, R] = -\bar{Q}_{\dot{\alpha}}.$$

3.4 Representations of the algebra

From the algebra above, the goal is to construct irreducible field representations of the algebra. I start with finding single particle representations of the $N = 1$ SUSY, which will tell us important facts about the number of fermionic and bosonic degrees of freedom and such.

The normal way of finding irreducible representations of the Poincaré group is through Wigners method of induced representations[15], also called the method of the little group. This works by finding representations of a subgroup, and then boosting these to find the full representations of the Poincaré group. In practice, one chooses a particular momentum p that has either $p^2 = 0$ or $p^2 = m^2$ depending on whether you want a massive or massless representation. Then one finds the subgroup that leaves p^μ unchanged, called the corresponding little group, and finds a representation of the little group on the states $|p^\mu\rangle$. This then induces a representation of the full Poincaré group. In fact, the fields that come out of this procedure will automatically fulfill the field equations of their respective representation. That is, a scalar field will fulfill the Klein-Gordon equation and a spin 1/2 field will fulfill the Dirac equation,

In the case of the Poincaré group, there are two Casimirs, the momentum squared $P^2 = m^2$ and W^2 where W is the Pauli-Ljubanskí vector defined by

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}.$$

W^2 has eigenvalues $m^2 s(s+1)$ where $s = 0, \frac{1}{2}, 1, \ldots$ is the spin, for massive states and $W_\mu = \lambda P_\mu$ where λ is the helicity for massless states (which of course implies $W^2 = 0$ for massless states). In supersymmetry $P^2 = m^2$ is still a Casimir, but W^2 is not. Instead, there is a second Casimir called C^2 which in general has a slightly complicated form combining both W,P and $Q, \bar{Q}$, which we won't need.

3.4.1 Representation of massive states

Since its enough in the method of induced representations to get a representation for a special value of p_μ, it's enough for our purposes to state the form of C in the rest frame where $p_\mu = (m, 0, 0, 0)$. There C^2 takes the form

$$C^2 = 2m^4 J_i J^i \,, \tag{19}$$

$$J_i = S_i - \frac{1}{4m}\bar{Q}\bar{\sigma}_i Q \,, \tag{20}$$

where S is the spin operator, σ_i are the Pauli matrices and $i = 1, 2, 3$ is a spatial index. From the algebra we see that the commutator between J_i and Q or $\bar{Q}$ is proportional to

$\vec{P}$ and thus disappears since we are in the rest frame. Further we see that since both S_i and $\bar{\sigma}_i$ fulfills the $SU(2)$ algebra, so does J_i. Thus, the eigenvalue of J^2 is $j(j+1)$ with j equal to integer or half integer values.

Further, from the algebra of the supersymmetry generators $Q, \bar{Q}$ in the rest frame we see that

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2m\sigma^0_{\alpha\dot{\beta}} = 2m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{21}$$

This is the algebra of two decoupled fermionic oscillators, with Q_1, Q_2 taking the role of annihilation operators and $\bar{Q}_1, \bar{Q}_2$ as the creation operators. Then, given any state with definite eigenvalues $|m, j\rangle$, we can define the new state $|\Omega\rangle$ such that

$$\begin{aligned} |\Omega\rangle &= Q_1 Q_2 |m, j\rangle, \\ Q_1|\Omega\rangle &= Q_2|\Omega\rangle = 0. \end{aligned} \tag{22}$$

Thus $|\Omega\rangle$ plays the role of a "vacuum" of our representation. Note that $|\Omega\rangle$ is degenerate, since it carries spin: for spin j it is $2j+1$ degenerate since j_3 as usual takes values $-j, \ldots, j$. Put another way, $|\Omega\rangle$ is a $2j+1$ dimensional $SU(2)$ multiplet. If we define normalized operators, $a_\alpha = \frac{1}{\sqrt{2m}} Q_\alpha$ and $a^\dagger_\alpha = \frac{1}{\sqrt{2m}} Q_{\dot{\alpha}}$ (abusing notation slightly and for the moment disregarding the different types of indices), then for a given $|\Omega\rangle$ the full irrep is

$$|\Omega\rangle, \quad a^\dagger_1|\Omega\rangle, \quad a^\dagger_2|\Omega\rangle, \quad a^\dagger_1 a^\dagger_2|\Omega\rangle = -a^\dagger_2 a^\dagger_1|\Omega\rangle. \tag{23}$$

This irrep has $4(2j+1)$ number of different states. To find the spins of the respective states, we use the commutators

$$[S_3, a^\dagger_2] = \frac{1}{2} a^\dagger_2\,, \tag{24}$$

$$[S_3, a^\dagger_1] = -\frac{1}{2} a^\dagger_1. \tag{25}$$

This means, that if we completely specify the vacuum, $|\Omega\rangle = |m, j, j_3\rangle$ the spins of the states listed above are $j_3, j_3 - \frac{1}{2}, j_3 + \frac{1}{2}, j_3$. So we get $2(2j+1)$ states with the same spin as $|\Omega\rangle$ and $2(2j+1)$ states that differs by $\frac{1}{2}$. This means that the number of bosonic and fermionic states match, in accordance to what we proved above.

The simplest example is the $j = 0$ or *fundamental* massive irrep. This irrep has a total of 4 states, with spins $0, -\frac{1}{2}, \frac{1}{2}, 0$ respectively. This correspond to one massive real scalar, one massive Weyl spinor, and one real pseudoscalar. The pseudoscalar is there because the parity transformation interchanges $a^\dagger_1$ with $a^\dagger_2$, which gives $a^\dagger_1 a^\dagger_2|\Omega\rangle$ an extra minus sign under parity. This supermultiplet is known as the *scalar* or *Wess-Zumino multiplet.*

3.4.2 Massless states

Instead of going to the rest frame, we can now go to the frame in which $P_\mu = (E, 0, 0, E)$. In this frame we find that $C^2 = 0$, and thus when we, in the same way as above, choose a vacuum state, it will not be degenerate. The anticommutation relations become

$$\begin{aligned} \{Q_1, \bar{Q}_1\} &= 4E \\ \{Q_2, \bar{Q}_2\} &= 0. \end{aligned} \tag{26}$$

From this we see that

$$\langle\Omega|Q_2\bar{Q}_2|\Omega\rangle = \langle\Omega| - \bar{Q}_2 Q_2|\Omega\rangle = 0\,,$$

which means that we can set $Q_2 = \bar{Q}_2 = 0$. So there really is just one pair of normalized creation/annihilation operators,

$$a = \frac{1}{2\sqrt{E}} Q_1, \quad a^\dagger = \frac{1}{2\sqrt{E}} \bar{Q}_1, \tag{27}$$

which obey

$$aa^\dagger + a^\dagger a = 1.$$

This algebra has the unique two dimensional representation with states $|\Omega\rangle, a^\dagger|\Omega\rangle$. As noted above, the ground state $|\Omega\rangle$ is now non-degenerate, and has helicity λ. The $a^\dagger$ operator will increase the helicity by 1/2, since it transforms in the $(0, \frac{1}{2})$ representation. So we have one state with helicity λ and one with helicity $\lambda + \frac{1}{2}$. However, on its own this representation cannot be part of a quantum field theory, since it isn't self conjugate under the CPT-transformation, which is required of any QFT. This is because CPT changes the sign of the helicity, so if a representation with helicity s appears, so must the representation with helicity $-s$. So, we have to pair two massless irreps together to obtain four states with helicities $-\lambda - \frac{1}{2}, -\lambda, \lambda, \lambda + \frac{1}{2}$.

If we for example set $\lambda = \frac{1}{2}$, then we will get two states with helicity $\pm\frac{1}{2}$ and two states with helicites ± 1. These are the states which appear if you write down a supersymmetric Yang-Mills theory, and the supermultiplet is therefore called the *gauge multiplet.* If we instead take $\lambda = \frac{3}{2}$ and add the CPT conjugate states, we get a supermultiplet with states of helicity $-2, -\frac{3}{2}, \frac{3}{2}, 2$. The states of helicities ± 2 has the degrees of freedom of a graviton, so this multiplet is called the *supergravity multiplet*, which appears in theories of supergravity.

3.5 Superspace and superfields

Just as we in order to easily write down Lorentz invariant theories introduce the concept of a four dimensional spacetime with coordinates x^μ, we can make it easier to write down theories that are explicitly supersymmetric by introducing the concept of *superspace.* To make the connection to ordinary spacetime, we can see that spacetime coordinates parametrise the space of right cosets of the Poincaré group modulo the Lorentz group. That is, we exponentiate the Poincaré algebra into a Lie group and then identify all points in the resulting space that are related by a homogeneous Lorentz transformation. In order to do the same thing with the supersymmetry algebra, one way to proceed is to write it as a Lie algebra (which it isn't in our standard notation, because of the anticommutators). If we introduce constant spinors $\theta^\alpha, \bar{\theta}_{\dot{\alpha}}$ whose components are anticommuting Grassmann numbers, which means that they fulfill

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\beta}}\} = 0. \tag{28}$$

Then, employing our sum conventions $\theta Q = \theta^\alpha Q_\alpha$ and $\bar{\theta}\bar{Q} = \bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$, and using the anticommutation of the Grassmann spinors, we can state the anticommutation relations of the supersymmetry algebra in terms of ordinary commutators:

$$[\theta Q, \bar{\theta}\bar{Q}] = 2\theta\sigma^m\bar{\theta}P_m\,, \tag{29}$$
$$[\theta Q, \theta Q] = 0\,, \tag{30}$$
$$[\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = 0. \tag{31}$$

Now when we have a Lie algebra, albeit one which contains anticommutating numbers, we can exponentiate to get a generic element of the corresponding group:

$$G(x,\theta,\bar{\theta},\omega) = \exp\left\{i\left[-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q}\right]\right\} \cdot \exp\left(-\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}\right), \tag{32}$$

where x is the four vector determining translations, ω is the parameters for ordinary boosts and rotations and the constant anticommuting spinors $\theta, \bar{\theta}$ are, loosely speaking, parameters for translations in the "anticommuting" dimensions. The minus in front of x^μ is a convention. If we, just as in the case of the normal Poincaré group, identify all the points which can be related by a Lorentz transformation, we get the space which is called $N = 1$ *rigid superspace*. The term rigid comes from that the supersymmetry parameters $\theta, \bar{\theta}$ are treated as constant, i.e. same all over space, which means that the supersymmetry we are discussing is a global symmetry. If we let the spinors depend on x we instead get a theory of supergravity, which falls outside the scope of this thesis. So the points in this superspace is in one-to-one correspondence with

$$\exp\left(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q}\right).$$

It is clear from this expression that the rigid superspace is parameterized by $(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$, which has $4+4$ parameters, the first 4 ordinary numbers and the next four Grassmann numbers.

Just as there are many advantages for constructing Lorentz covariant field theories using the language of spacetime, construction of explicitly supersymmetric Lagrangians become much easier when put into the language of superspace and fields on it, the so called superfields.

The idea is that we can expand the generic field in the Grassmann "coordinates" and since they anticommute we get an expansion with a finite number of terms. For example, the generic scalar field on superspace can be expanded as

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta n(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) \\ &\quad + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\psi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x),\end{aligned} \tag{33}$$

where f, m, v_μ, n, d are ordinary scalar fields and the fermionic components $\phi, \bar{\chi}, \psi, \bar{\lambda}$ are Grassmann valued spinor fields which anticommute with each other and $\theta, \bar{\theta}$. Different redundant terms has been removed using spinor identities. At first glance it might seem like terms on the form $\theta\theta$ should be zero from anticommutation relations, but the thing to remember is that in the summation convention we are using, the indices are lowered with the totally antisymmetric symbol $\epsilon_{\alpha\beta}$ so these kind of terms does not disappear.

To compute what happens when we apply an infinitesimal supersymmetry transformation on this field, we need a representation of the generators $Q, \bar{Q}$ as differential operators on superspace, the same way P_μ can be represented as $i\partial_\mu$ on spacetime. In order to do this we must first define what we mean with taking the derivative with respect to an anticommuting number. This is very logical, if we have a function $f(\xi)$ where ξ is an anticommuting number, the most general form of this function is $f(\xi) = a + b\xi$ and the derivative with respect to ξ is naturally defined as $\frac{\partial f}{\partial \xi} = b$. In our two component notation, we define

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \bar{\partial}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}, \tag{34}$$

and from the general definition we have

$$\partial_\alpha \theta^\beta = \delta_\alpha^\beta, \qquad \partial^{\dot\alpha}\bar\theta_{\dot\beta} = \delta_{\dot\beta}^{\dot\alpha}.$$

Using the antisymmetric symbol to lower and raise indices, one can work out various relations for derivatives, such as

$$\begin{aligned}\partial_\alpha(\theta\theta) &= 2\theta_\alpha, \quad \bar\partial_{\dot\alpha}(\bar\theta\bar\theta) = -2\bar\theta_{\dot\alpha},\\ \partial_\alpha\partial_\beta &= -\epsilon_{\alpha\beta}, \quad \bar\partial_{\dot\alpha}\bar\partial_{\dot\beta} = -\epsilon\dot\alpha\dot\beta,\end{aligned}$$

and so on.

To work out the action of a supersymmetry transformation on the scalar field, we can use the commutation relations to work out the action on a point in superspace and then use that for a scalar field the action on the field is inverse to that on points. The action of the ordinary Lorentz group on the superspace is as one would expect; x^μ transforms as a vector and $\theta, \bar\theta$ transforms as Weyl spinors. A Lorentz transformation doesn't mix the coordinates. A four vector translation also works as expected since P and Q commutes,

$$\exp(-\tau^\mu P_\mu)\exp(-x^\mu P_\mu)\exp(\theta Q + \bar\theta\bar Q) = \exp((\tau^\mu + x^\mu)P_\mu)\exp(\theta Q + \bar\theta\bar Q)\,,$$

but since $[\theta Q, \theta Q] \propto P$ a "supertranslation" from the left mixes the coordinates and doesn't only shift θ or $\bar\theta$ but also x. By using the commutation relations of the algebra and the Baker-Campbell-Hausdorff formula for products of exponentials of operators, one can find the left action on a point in superspace

$$\begin{aligned}G(y,\xi,\bar\xi)\cdot G(x,\theta,\bar\theta) &= \exp(-iy^\mu P_\mu + i(\xi Q + \bar\xi Q))\cdot\exp(-ix^\mu P_\mu + i(\theta Q + \bar\theta Q))\\ &= \exp(-i(x^\mu + y^\mu)P_\mu + i(\theta+\xi)Q + i(\bar\theta+\bar\xi)\bar Q\\ &\quad + \frac{i}{2}([\xi Q, \theta Q] + [\bar\xi\bar Q, \bar\theta\bar Q]))\\ &= G(x^\mu + y^\mu - i\theta\sigma^\mu\bar\theta + i\theta\sigma^\mu\bar\xi, \theta+\xi, \bar\theta+\bar\xi)\,, \end{aligned} \tag{35}$$

where $\xi, \bar\xi$ are any constant Grassmann-valued Weyl spinors. By linearising this action and using that the action on a scalar field is inverse to that on a spacetime point, one can work out that a good representation in terms of differential operators on a scalar superfield is

$$Q_\alpha \;:\; \partial_\alpha - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \quad \text{and} \quad \bar Q_{\dot\alpha} \;:\; \bar\partial_{\dot\alpha} - i\sigma^\mu_{\dot\alpha\beta}\theta^\beta\partial_\mu. \tag{36}$$

In precisely the same way we can work our way through finding the appropriate representation of action from the right, which is called the *supercovariant derivatives.* These do in fact have the same geometrical meaning as the normal covariant derivatives defined in general relativity, and since our superspace is rigid, one would naively think that it should have vanishing curvature and thus $D_\alpha = \partial_\alpha$. But in fact one can show that even rigid superspace has a non vanishing torsion, which makes the covariant derivatives nontrivial. The non vanishing torsion is a direct consequence of the existence of fermionic generators. Anyway, in our notation, the supercovariant derivatives are given by

$$D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \quad \text{and} \quad \bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu. \tag{37}$$

The minus signs in $\bar D_{\dot\alpha}$ comes from lowering the dotted index using the antisymmetric symbol. Since the ordinary spacetime translations doesn't mix the coordinates, the covariant derivative with respect to the x^μ is just the same as the partial derivative,

$$D_\mu = \partial_\mu.$$

3.5.1 The chiral superfield

From looking at the expansion for a scalar superfield, we see that if we take the lowest order component $f(x)$ to be a physical complex scalar field (which restricts what the rest of the components must be), there are too many degrees of freedom for the unconstrained superfield to constitute an irreducible representation of the Poincaré algebra. Thus we need to constrain it in some way to cut down the number of degrees of freedom. There are different such constraints, but a natural one is to require

$$\bar{D}_{\dot{\alpha}}\Phi = 0. \tag{38}$$

This defines a *chiral superfield*, which is important since it constitutes all the basic matter content in our supersymmetric extensions of the standard model. We can of course also require that

$$D_\alpha \Phi = 0 \tag{39}$$

which defines an *antichiral superfield*. The chiral field behaves very much like an analytic[1] function. Loosely speaking, since $\bar{\theta}$ is the complex conjugate of θ, the requirement in the definition of a chiral superfield can be seen as an analogue of the Cauchy-Riemann equations, which can be written $\frac{\partial f}{\partial z^*}$ =0. Another fact one can easily convince oneself of is that the complex conjugate of a chiral superfield is antichiral. From this it follows that if a chiral superfield is real valued, it is also antichiral. Thus both D_α and $\bar{D}_{\dot{\alpha}}$ makes the field vanish, and their anticommutator also vanishes, which is proportional to ∂_μ by the super Poincaré algebra, so the field must be constant. This again is just as for a real valued analytic function.

If we define new commuting, bosonic coordinates y^μ as

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \tag{40}$$

we can see that

$$\bar{D}_{\dot{\alpha}} y^\mu = 0$$

and almost by definition

$$\bar{D}_{\dot{\alpha}} \theta^\beta = 0.$$

This means that any function of y and θ (but not of $\bar{\theta}$) will fulfil the covariant constraint and thus be a chiral field. We thus have the expansion, for a general chiral superfield

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{41}$$

where A and F are complex scalar fields and ψ is a left handed Weyl spinor. Here we see a special feature of the $N = 1$ algebra: there is a supermultiplet only containing a left handed fermion. It is this that makes us able to use $N = 1$ supersymmetry to directly extend the standard model, as mentioned above.

The chiral superfield has 4 real bosonic degrees of freedom, and 4 real fermionic degrees of freedom, twice as many as we found in our fundamental one particle representation. This will be explained later, and the crucial thing is to look at how the component

[1] The word holomorphic is more frequently used when discussing supersymmetry. Strictly speaking, analytic means that the function can be expanded as a convergent powerseries around every point, whereas holomorphic is the weaker requirement that the function is complex differentiable in a neighbourhood of every point. For complex functions a major result in complex analysis is that in fact holomorphicity implies analyticity, which motivates why I can treat them as basically the same thing.

fields transform under a infinitesimal supersymmetry transformation. By using the Fierz identities to manipulate spinor sums and Taylor expanding the functions of y, again using that the series will terminate, one can expand the field given above in the ordinary coordinates $x, \theta, \bar{\theta}$. The result is

$$\Phi(x,\theta,\bar{\theta}) = A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) \\ + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2 A(x), \tag{42}$$

where $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu = \Box$.

It is appropriate here to say something about the dimensionality of the different components of the superfield. The dimensionality of the full chiral field will be the same as that of the dimension of the lowest component, the scalar field A. From quantum field theory we know that a scalar field has mass dimension 1. Further, from the supersymmetry algebra we see that the supercharges have mass dimension 1/2, so therefore the mass dimension of θ and $\bar{\theta}$ is $-1/2$ (just as the dimension of x^μ is minus that of P_μ). We thus see that the F field must have mass dimension 2, and the fermion field ψ must have dimension 3/2, just like in ordinary quantum field theory.

Using the definitions of derivatives, we can find how the components transform under a supersymmetry transformation, defined by the constant spinor ξ. We then find the variations of the component fields:

$$\begin{aligned} \delta_\xi A &= \sqrt{2}\xi\psi, \\ \delta_\xi \psi &= \sqrt{2}\xi F + i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu A, \\ \delta_\xi F &= -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi}. \end{aligned} \tag{43}$$

The most important thing to note here is that F transforms as a total derivative. Looking forward, this will mean that F will have a trivial equation of motion, not involving any derivatives, and be what is called an auxiliary field. We also see, as mentioned above, that the supersymmetry transformation mixes the boson A and the fermion ψ, which of course is the trademark sign of SUSY.

3.5.2 The vector superfield

Another way to restrain the general superfield is to impose a reality condition,

$$V(x,\theta,\bar{\theta}) = V^\dagger(x,\theta,\bar{\theta}). \tag{44}$$

This defines a *vector* superfield. In components, this means that all the scalar fields must be real, while the spinors must be related in the correct way, (using the same notation as in equation (33))

$$\begin{aligned} \phi^\dagger &= \bar{\chi}, \\ v_\mu &= v_\mu^*, \\ \psi^\dagger &= \bar{\lambda}. \end{aligned} \tag{45}$$

So in total we have 4 real scalar fields, 1 real vector and 2 complex Weyl spinors, for a total of $4 + 4 + 2 \cdot 4 = 16$ real degrees of freedom. The presence of the real vector field hints that we can use this to construct supersymmetric gauge theories, something that turns out to be true.

A thing to note is that if Φ is a chiral superfield, the real part of Φ, or equivalently $\Phi + \Phi^\dagger$, will be a vector superfield. The vector component of this field will be a derivative of a scalar field. This suggests that we can define a superfield analogue to the U(1) gauge transformation of the vector potential of electrodynamics, i.e. if V is a vector superfield we can make the transformation

$$V \mapsto V + (\Phi + \Phi^\dagger), \tag{46}$$

which will transform the vector component of V as

$$v_\mu \mapsto v_\mu + i\partial_\mu(\Phi + \Phi^\dagger),$$

which is just as the case of an ordinary U(1) transform of a vector potential. By working out how the different components are affected by the gauge transform, one can find that the combinations

$$\begin{aligned} \lambda_\alpha &= \psi_\alpha - \frac{i}{2}(\sigma^\mu)_{\alpha\dot{\beta}}\partial_\mu\bar{\phi}^{\dot{\beta}} \\ D &= d - \frac{1}{2}\partial^2 f \end{aligned} \tag{47}$$

are gauge invariant under the transformation in eq. (46).[2] Further, if one looks at how the other component fields transforms, one can see that a gauge can be chosen such that it eliminates most of them, which can simplify calculations a lot. In this gauge, known as the *Wess-Zumino gauge*, the vector field becomes

$$V = \bar{\theta}\sigma^\mu\theta v_\mu + (\bar{\theta}\bar{\theta})\theta\lambda + (\theta\theta)\bar{\theta}\bar{\lambda} + (\theta\theta)(\bar{\theta}\bar{\theta})D. \tag{48}$$

From this we see that the actual, physical degrees of freedom, those that can not be transformed away by a gauge transformation, are those of a real vector, one (complex) Weyl spinor and one real scalar field. A thing to note is that the number of bosonic and fermionic freedoms doesn't match. This is because the choice of gauge breaks supersymmetry. In fact, as we will see when constructing the abelian gauge theory Lagrangian, the D field will turn out to be unphysical.

We could now check how the components of the vector field transforms under a supersymmetry transformation, but we restrict ourselves to the case of the D field. By using the representations of the supercharges as differential operators on superspace we find

$$\delta_\xi D = \partial_\mu(\xi\sigma^\mu\bar{\lambda}(x) + \lambda(x)\sigma^\mu\bar{\xi}). \tag{49}$$

Just as for the F field in the chiral superfield, we see that the D field changes by a total derivative, a fact that is crucial to constructing a supersymmetric Lagrangian involving vector fields. This is really the next step in this review, but first we introduce some additional notation that is useful for writing down supersymmetric actions.

3.6 Superspace integrals

In order to later write down supersymmetric actions in terms of superfields, we introduce the concept of integrals over superspace. These are not integrals in a proper sense, there

[2] The naming of the D field coincides with the supercovariant derivative, but it will hopefully be clear from the context, and the name is used in all the literature so there is no sense in adopting a different name.

is not really any valid integration measure, but they are rather formal definitions, introducing convenient notational devices. The "usual" integral over Grassmann numbers is called the *Berezin integral*[16], and it is used, besides in supersymmetry, when we want to quantize fermions using the path integral method. When you define it, you want to preserve some basic properties of ordinary integrals, i.e. that it should be insensitive to shifting the integral variable by a constant, in the sense that

$$\int f(x+\eta)d(x+\eta) = \int f(x)dx,$$

where η is some constant, and it should be a linear operation. In order to keep these two properties we see that practically the only natural way to define it is

$$\int \mathrm{d}\xi(a+b\xi) = b. \tag{50}$$

If we compare this with taking the derivative, we see that they are the same:

$$\int \mathrm{d}\xi(a+b\xi) = \frac{\partial}{\partial\xi}(a+b\xi) = b. \tag{51}$$

This definition of the integral is easily generalized to integration over our superspace Grassmann spinors. If we use the notation

$$\mathrm{d}^2\theta = -\frac{1}{4}\mathrm{d}\theta^\alpha \mathrm{d}\theta^\beta \epsilon_{\alpha\beta}, \quad \mathrm{d}^2\bar{\theta} = -\frac{1}{4}\mathrm{d}\bar{\theta}_{\dot{\alpha}} \mathrm{d}\bar{\theta}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \tag{52}$$

$$\mathrm{d}^4\theta = \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \tag{53}$$

we get that

$$\int \mathrm{d}^2\theta(\theta\theta) = 1, \quad \int \mathrm{d}^2\bar{\theta}(\bar{\theta}\bar{\theta}) = 1, \quad \int \mathrm{d}^4\theta(\bar{\theta}\bar{\theta})(\theta\theta) = 1. \tag{54}$$

What the integrals here do, which is what makes them useful, is that they extract components out of a function of θ and $\bar{\theta}$. For example, the integral of some function $\Phi(\theta, \bar{\theta})$ over $\mathrm{d}^2\theta^2$ really only picks out the θ^2 component of Φ.

3.7 Supersymmetric Lagrangians

So far we have developed quite a bit of theory about abstract things like the supersymmetry algebra, superspace and superfields. In this section this formalism will be put to some use and we will see how (relatively) easy these concepts make it to write down supersymmetric Lagrangians. By a supersymmetric Lagrangian we mean one that changes by a total derivative under supersymmetry transformations, so that the action is invariant. Without the concepts of superfields, it's very hard to guess the form of the general supersymmetric action. We start by determining the simplest (sensible) Lagrangian involving a single chiral superfield, and then we find the most general Lagrangian involving an arbitrary number of chiral fields. Then we treat the supersymmetric extension of an abelian gauge theory, i.e. the supersymmetric version of QED. From here, one can continue to treat nonabelian gauge theories and talk about how renormalization works in supersymmetric theories and so on, but since this review was supposed to be brief, I will stop after that and in the next section instead introduce the simplest version of a supersymmetric extension of the standard model.

3.7.1 Lagrangians for chiral superfields

We first want a Lagrangian involving one chiral field Φ that is real, supersymmetric, Poincaré invariant and of dimension 4 (so that the action is dimensionless). In fact the dimensionality and reality pretty much determine what the action must look like. The dimension of the superfield is that of its lowest (in powers of θ) component, so as said above the chiral field will have mass dimension 1. We also know that the θ^2 term, F, has dimension 2, and remembering that the F component transforms as a total derivative under supersymmetry transformations (thus leaving the action invariant) we see that FF^* will be a real, globally supersymmetric action with the right mass dimension. Using the above defined integral over superspace, we can write this action as

$$\int \mathrm{d}^4x\mathrm{d}^4\theta\Phi\Phi^\dagger\,. \tag{55}$$

This action is real, invariant under supersymmetry transformations, is Poincaré invariant and has the correct dimension. By rewriting this in component form one can see that it does indeed contain the kinetic terms for the component fields A and ψ:

$$\frac{1}{2}\partial_\mu A\partial^\mu A^* - \frac{1}{4}(A\partial^2 A^* + A^*\partial^2 A) + FF^* - \frac{i}{2}(\xi\sigma^\mu\partial_\mu\bar{\xi} + \bar{\xi}\bar{\sigma}^\mu\partial_\mu\xi) \tag{56}$$

using some of the Fierz identities to simplify spinor expressions. We see here that the F field has no derivatives in the action, and therefore its equations of motion are trivial,

$$F = F^* = 0.$$

So we see that on shell, when the equations of motion hold, F is uniquely zero. F is what is called an auxiliary field.

In order to add interactions to this theory, we note that the F field transforms as a total derivative under supersymmetry transformations. This is in fact exactly what makes the action above supersymmetric, but we see that another possibility is to take an action that just takes out the F component and make it real by adding the complex conjugate:

$$\int \mathrm{d}^4x \left\{ \int \mathrm{d}^2\theta m^2\Phi(x,\theta) + \int \mathrm{d}^2\bar{\theta} m^2\Phi(x,\theta)^\dagger \right\}, \tag{57}$$

where the m^2 is there to give us the correct dimension. This also has all the desired properties we want. And since the covariant derivative obeys the product rule of derivation, any analytic (or, as discussed above, holomorphic, for the word more often used in the literature) function of a chiral field will again be a chiral field. This means that more generally the action

$$\int \mathrm{d}^4x \left\{ \int \mathrm{d}^2\theta W(\Phi) + \int \mathrm{d}^2\bar{\theta} W(\Phi)^\dagger \right\} \tag{58}$$

where W is any analytic function of Φ (but not of $\Phi^\dagger$), will be an acceptable supersymmetric action. This term adds mass and interaction terms to the Lagrangian, and W is called the *superpotential*. From dimensional grounds, we can see that W can be at most cubic in the superfield. This follows since we know that in order to get a renormalizable potential, all couplings must have non-negative dimension. The θ^2 component of $W(\Phi)$ have mass dimension of one more than W itself, so if W is cubic in Φ this component will have dimension 4 requiring the coupling to be dimensionless. Thus, W can be at most cubic.

The general supersymmetric theory only involving one chiral field is called the *Wess-Zumino model.* Its action is

$$\int \mathrm{d}^4x\mathrm{d}^4\theta\Phi\Phi^\dagger - \int \mathrm{d}^4x \left\{ \int \mathrm{d}^2\theta \left(\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 \right) + \text{h.c.} \right\} \tag{59}$$

and it describes a massive complex scalar field and a massive fermion, with some interaction terms. Still, the F field has no derivatives acting on it in the action, so its equations of motions will be trivial (although not as simple as in the free field case). By using the solution of F:s equations of motions, we can eliminate F from the action. We note that the bosonic part of the Lagrangian is

$$L_B = \partial_\mu A^* \partial^\mu A + F^* F - (mAF + gA^2F + \text{h.c}), \tag{60}$$

so the equations of motion for F and F^* are

$$0 = \frac{\partial L_B}{\partial F} = F^* - mA - gA^2, \tag{61}$$

$$0 = \frac{\partial L_B}{\partial F^*} = F - mA^* - g(A^*)^2. \tag{62}$$

$$\tag{63}$$

Using these, we can write the bosonic part of the action as

$$\begin{aligned} L_B &= \partial_\mu A^* \partial^\mu A + FF^* - (mAF + gA^2F + \text{h.c}) \\ &= \partial_\mu A^* \partial^\mu A + (mA - gA^2)(mA^* - g(A^*)^2) \\ &\quad - \big((mA(mA^* + g(A^*)^2) + gA^2(mA^* + g(A^*)^2)) + \text{h.c.}\big) \\ &= \partial_\mu A^* \partial^\mu A - (mA - gA^2)(mA^* - g(A^*)^2) = \partial_\mu A \partial^\mu A - |F|^2. \end{aligned} \tag{64}$$

So we see that the potential of the bosonic part of our Lagrangian, called V_F is given simply by $V_F(A, A^*) = |F|^2$. This also holds when we have a general superpotential, $W(\Phi)$. Then the scalar potential is given by the absolute square of the F-component, i.e.

$$V_F(A, A^*) = |F|^2 = \left.\frac{dW}{d\Phi}\right|_{\Phi=A}. \tag{65}$$

In this way, for a general supersymmetric potential, we can get part of our scalar potential. However, as will be described towards the end of next section, when we add gauge interactions these also contribute to the scalar potential in a rather similar way.

3.7.2 Supersymmetric, abelian gauge theory

When we looked at the vector superfield, we defined the Wess-Zumino gauge in which the vector field only had 3 components. However, this gauge is not preserved by supersymmetry transformations, which is disappointing since it means that the vector field V necessarily has a lot of components. We can however define another vector field that only contains the three components, which will then be used write down our field strength term in our gauge theory Lagrangian. We begin with defining the chiral and antichiral fields

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 D_{\dot{\alpha}} V \tag{66}$$

where all the D's are covariant derivatives and V is a vectorfield. W_α will obviously be chiral since

$$\bar{D}_{\dot{\beta}} W_\alpha = -\frac{1}{4}\bar{D}_{\dot{\beta}}\bar{D}^2 D_\alpha V = 0,$$

using that since $\bar{D}_{\dot{\alpha}}$ anticommutes and only has two components, $\bar{D}^3 = 0$. In the same way, $\bar{W}_{\dot{\alpha}}$ is an antichiral field. One can show that W_α and $\bar{W}_{\dot{\alpha}}$ both are invariant under the abelian gauge transformation (46), so there is no loss of generality in computing their components in the aforementioned Wess-Zumino gauge. Using this gauge and writing W_α as a function of the 'chiral' coordinate $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ introduced above, one finds that

$$W_\alpha(y,\theta) = -i\lambda_\alpha(y) + \theta_\alpha D(y) - \frac{i}{2}\sigma^\mu\bar{\sigma}^\nu\theta_\alpha f_{\mu\nu}(y) + (\theta\theta)\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{\lambda}^{\dot{\beta}}(y) \tag{67}$$

where $f_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$, D is a real valued scalar field and λ is a left handed spinor. $f_{\mu\nu}$ is, as can be suspected from its appearance, the ordinary field strength of our abelian gauge theory. The λ spinor has 4 real degrees of freedom, the D field has 1 real degree of freedom, and since the v_μ has 4 degrees of freedom, where 1 can be eliminated by a gauge transform, $f_{\mu\nu}$ which is gauge invariant must have 3 degrees of freedom, for a total of $4 + 1 + 3 = 8$ real degrees of freedom. This multiplet is called the *field strength multiplet* and is an off-shell irreducible representation of the super-Poincaré algebra.

So, since W_α is chiral, so is $W^\alpha W_\alpha$ which in addition will be Poincaré invariant. The chirality means that the θ^2 component will transform as a total derivative, and thus we can use its real part as a valid supersymmetric Lagrangian,

$$\int \mathrm{d}\theta^2 W^\alpha W_\alpha + \int \mathrm{d}\bar{\theta}^2 \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}. \tag{68}$$

If you work this out in components, you find that this is

$$(\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda) - \frac{1}{2}f^{\mu\nu}f_{\mu\nu} + D^2. \tag{69}$$

Just as for the F component in the chiral Lagrangian, there are no derivatives acting on the D field. Thus it is another auxiliary field, with an equation of motion that is simply $D = 0$. This Lagrangian then describes the free propagation of one gauge boson (through the $f_{\mu\nu}f^{\mu\nu}$ term) and its supersymmetric partner λ, called the gaugino.

Only having a free gauge boson and gaugino isn't very realistic or interesting, so the natural next step is to ask how we can add charged matter fields to the theory. For simplicity, start with a single chiral field Φ taking values in a one dimensional representation of the U(1) gauge group of our theory. That is, under a (global) gauge transform, $\exp(i\phi) \in U(1)$, Φ transforms as

$$\Phi \mapsto e^{ie\phi}\Phi, \tag{70}$$

e being the charge of Φ. Then clearly the term $\Phi^\dagger\Phi$ is gauge invariant (since $\Phi^\dagger \mapsto e^{-ie\phi}\Phi^\dagger$ under the same gauge transform). However, if we want to make the symmetry local and thus let $\phi \to \phi(x)$ be a function on spacetime, it is no longer guaranteed that $e^{ie\phi(x)}\Phi$ still is a chiral superfield (since the covariant derivative in the definition of a chiral field involves ∂_μ). So, we are in the uncomfortable situation that our local gauge transform violates supersymmetry. In order to escape this predicament, we can let $\phi(x)$ be, instead of a real valued function on spacetime, a full chiral field. Then $e^{ie\phi}\Phi$ will still be a chiral field, and instead the $\Phi^\dagger\Phi$ term will transform like

$$\Phi^\dagger\Phi \mapsto e^{ie(\phi-\bar{\phi})}\Phi^\dagger\Phi. \tag{71}$$

Now looking back at equation 46, we see that $(\phi - \bar{\phi})$ can be absorbed into a gauge transform of our vectorfield V. This suggests that a suitable, gauge invariant coupling between Φ and V is of the form $\Phi^\dagger e^{ieV}\Phi$, which indeed gives us back the correct coupling between a charged scalar field and a U(1) gauge field. So the Lagrangian looks like

$$\int \mathrm{d}^4\theta \Phi^\dagger e^{ieV}\Phi + \left\{ \int \mathrm{d}^2\theta \frac{1}{4} W_\alpha W^\alpha + \text{h.c.} \right\}. \tag{72}$$

We note here that this scalar chiral field has no mass, since gauge invariance forbids terms such as $m^2\Phi\Phi$. If we want massive chiral fields, we can instead take two different chiral fields Φ_+ and Φ_- with opposite charge, which allows us to add a gauge invariant mass term $m^2\Phi_+\Phi_-$ to the superpotential. So the full Lagrangian of what basically is the supersymmetric extension of scalar quantum electrodynamics looks like

$$\int \mathrm{d}^4\theta (\Phi_+^\dagger e^{ieV}\Phi_+ + \Phi_-^\dagger e^{ieV}\Phi_-) + \left\{ \int \mathrm{d}^2\theta \frac{1}{4} W_\alpha W^\alpha + m^2\Phi_+\Phi_- + \text{h.c.} \right\}. \tag{73}$$

If we look at the scalar potential of this Lagrangian, we find that in addition to the V_F term, that is the same as in the Wess-Zumino model, we also find a new term, coming much in the same way from the new auxiliary D component. This addition to the potential now looks like

$$V_D = \frac{1}{2g^2} D^2 \tag{74}$$

where D of course is rewritten using its equation of motion in terms of the other scalar fields in the theory. The full scalar potential thus is the sum $V = V_F + V_D$, something that will be used later when we look at the potential for the Higgs fields. This is indeed a general theme, the scalar potential of any supersymmetric theory is the sum of the squares of the auxiliary fields.

The simple abelian case can without too much trouble be generalized to the nonabelian case of Yang-Mills theory, but in order to keep this review brief, this won't be done here. The interested reader is invited to consult the vast literature, for example those reviews mentioned in the beginning of this section, for this and much more interesting theory about supersymmetry.

3.8 Concluding remarks

Supersymmetry is a vast field with many remarkable results, and this brief review has but scratched at the very surface. The approach followed here, using superspace and superfields, can be extended into a full fledged approach to quantum field theory, with Feynman rules and so on. By doing this, people have been able to prove many remarkable non-renormalisation theorems[17, 18]. Most important is the fact that supersymmetric theories have no quadratic divergences. This basically comes from the pairing of bosons with fermions, and since fermion masses only can be logarithmically divergent in a renormalizable theory and supersymmetry requires the boson mass to be equal to the fermion mass there can be no quadratic divergences. In fact, there is no renormalization of any of the parameters in the superpotential (i.e. masses and Yukawa couplings) apart from a global rescaling of the superfields. Gauge couplings are renormalized, however. If you go to theories with more supercharges, $N > 1$, even more divergences will disappear, and for $N = 4$ there will be no divergences at all. But as stated in section 3.2, $N > 1$ cannot be directly used to extend the standard model, which is what I next turn to.

4 Realistic supersymmetric models

In this section I will introduce the simplest way to extend the standard model into a supersymmetric theory, the minimal supersymmetric standard model (MSSM). Then an extension to this theory is presented, the next-to MSSM (NMSSM), mostly in order to solve a problem concerning the value of a dimensionfull parameter in the MSSM. Since nature obviously isn't supersymmetric at low energies, we also need to study how supersymmetry is broken. This is a large subject which I won't cover in any detail, only introduce the concept of how we can introduce so called softly breaking terms into the Lagrangian of our models, as discussed in the next section.

4.1 Softly breaking terms

The most popular ideas about how supersymmetry breaking works, is that it is spontaneously broken, in a manner similar to how the gauge symmetries are broken. In fact, supersymmetry is broken as soon as the vacuum gets a nonzero energy, as mentioned in section 3.2 . This fact means that the breaking of supersymmetry and the breaking of gauge symmetries are closely connected subjects. There are different additional terms you can introduce into your Lagrangian, such that you can give the fields in these terms a non-vanishing VEV, and thus give the vacuum a nonzero energy, breaking supersymmetry. For more details, see [20].

When constructing realistic models, we don't really need to care about the details of exactly how this happens. Instead, we can introduce terms into our Lagrangian that explicitly breaks supersymmetry, but at the same time preserves renormalizability and are such that at high energy, above the supersymmetry breaking scale, they become irrelevant. Such terms are called *softly breaking terms*, and are essentially things like scalar mass terms, gaugino masses or cubic scalar terms with dimensionfull couplings. For our purposes, analysing the Higgs sector at leading order, we only care about the scalar mass terms. So when we have the supersymmetric Lagrangian, we can then add all such allowed terms, and view them as an effective description of how supersymmetry is broken.

4.2 The MSSM

Just as it sounds, the MSSM is the model you get when you try to minimally extend the standard model to incorporate supersymmetry. Since none of the particles in the standard model have the same quantum numbers (excluding mass), one cannot let any of the known particles be each others superpartners. So instead we let every particle be a part of a corresponding superfield, and then put the superfields in the same $SU(2)_L$ doublets as in the SM. The same of course applies to the gauge fields, which now become part of gauge superfields.

The supersymmetric partners are sometimes called *sparticles* to distinguish them from the normal particles, and are given names by adding either an 's' at the beginning or putting '-ino' at the end. The superpartner to the electron is called *selectron*, the partners of the quarks are called *sqarks*, the W-boson has the *Wino* and we also have the *photino*, the *gluino* and so on. In formulae, these superpartners are usually denoted by putting a tilde on top of the symbol for the regular particle, so that for example the selectron is denoted $\tilde{e}$ and the photino $\tilde{\gamma}$.

As mentioned briefly above, we also have new particles called *neutralinos*, which are mixtures of the neutral superpartners of the Higgs bosons (which naturally are called the *higgsinos*) and the neutral gauginos. In the MSSM there are four different neutralinos, and the lightest one is usually assumed to be the lightest supersymmetric particle. In the same way, the charged higgsinos and winos ($\tilde{W}^{\pm}$) mix to form mass eigenstates with charge ± 1, called the *charginos*, of which there are two.

In a supersymmetric theory, only a Higgs with hypercharge $Y = 1/2$ can have the necessary Yukawa coupling to give masses to the up-type quarks with charge $+2/3$, and only a Higgs with hypercharge $-1/2$ can have the necessary couplings to give mass to the down-type quarks. This is because the superpotential is holomorphic, so the Higgs doublet giving mass to the up-type quarks cannot also give mass to the down-type quarks since we are not allowed to use the complex conjugate. Thus we at least need two different Higgs $SU(2)_L$-doublets in order to give mass to all the massive particles. We will call the $Y = 1/2$ doublet H_u, and the $Y = -1/2$ doublet H_d. Then the upper component of the $Y = 1/2$ will have isospin $T_3 = +1/2$ and therefore have electric charge $+e$. The lower component will be electrically neutral, and in the same way the upper component of the $Y = -1/2$ doublet will be neutral while the lower will have a negative electric charge. So the superdoublets look like

$$\hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \quad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix} \tag{75}$$

and the Higgs fields which gives masses to the fermions will be the corresponding scalar fields. We can then note that the Higgs superdoublet $\hat{H}_d$ has the same quantum numbers as left handed leptons (and sleptons). Therefore we can use it to give mass to the leptons as well as the down-type quarks, so we don't need another Higgs doublet for this purpose. There is another way to motivate the need of two different Higgs doublets which is based on anomaly cancellations, but this isn't logically needed.

The minimal superpotential involving these superfields which in a reasonable way extends the standard model is

$$W_{MSSM} = \sum_{i,j} y_u^{ij} \hat{u}_i \hat{H}_u \cdot \hat{Q}_j - y_d^{ij} \hat{d}_i \hat{H}_d \cdot \hat{Q}_j - y_e^{ij} \hat{e}_i \hat{H}_d \cdot \hat{L}_j + \mu \hat{H}_u \cdot \hat{H}_d \tag{76}$$

where the "hatted" letters denote the superfield doublets or singlets corresponding to the normal $SU(2)_L$ doublets/singlets in the standard model, and i, j are generation indices, $i = 1, 2, 3$. That is, for the first generation, $\hat{Q}_1 = (\hat{u}, \hat{d})^T$, $\hat{L}_1 = (\hat{e}_L, \hat{\nu}_e)^T$, $\hat{e}_1 = \hat{e}_R, \hat{u}_1 = \hat{u}$ and so on. The Higgs doublets are as described above, and the $\mathbf{y}^{ij}$ are the Yukawa couplings among generations. The products of $SU(2)_L$ doublets are given by

$$A \cdot B = \epsilon_{ab} A^a B^b$$

where ϵ_{ab} is the fully antisymmetric symbol in two dimensions with $\epsilon_{12} = 1$ and a, b are $SU(2)_L$ indices. In this superpotential the μ parameter has dimension mass and is what gives mass to the Higgs fields, and it is this simple fact which motivates the introduction of the next-to minimal supersymmetric standard model (NMSSM).

From this superpotential and the ordinary gauge couplings of the standard model, we can calculate the scalar potential, by calculating the F and D terms as described above. Doing this, and looking only at the Higgs sector of the potential, we find

$$V_F = \mu^2 (|H_u|^2 + |H_d|^2) \tag{77}$$

and

$$V_D = \frac{1}{8}g^2\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{1}{2}g_2^2|H_u^\dagger \cdot H_d|^2. \tag{78}$$

As described above, we can then add the soft supersymmetry breaking terms;

$$V_{\text{soft}} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + (m_3^2 H_u \cdot H_d + \text{h.c.}) \tag{79}$$

where the dimensionfull parameters $m_{H_u}^2, m_{H_d}^2$ and m_3^2 clearly have to be of the order of the weak or supersymmetric breaking scale. The total scalar potential is then the sum of these three terms. By letting at least one of $m_{H_u}^2$ and $m_{H_d}^2$ be negative, H_u and H_d acquires non-zero VEVs, breaking the symmetry.

From requiring vacuum stability we get some relations between $m_{H_u}^2, m_{H_d}^2, m_3^2$, the VEVs and μ; as described in more detail in the next section for the NMSSM. Using these, one can calculate and then diagonalize the mass matrices that describe the physical mass eigenstates in terms of the parameters of the model. The formulae are only stated here without much explanation, mostly for some completeness and comparison with the NMSSM. Hopefully, how the calculations are done will become clear after reading the next section as well as section 5, where the calculations leading to the mass matrices and mass eigenstates are explained in more detail.

In the MSSM, it turns out that we get one physical charged Higgs state, $H^\pm$, with a mass

$$m_{H^\pm}^2 = \left(\frac{2m_3^2}{v_u v_d} + \frac{1}{4}g_2^2\right)\frac{v^2}{2} \tag{80}$$

where $\langle H_u^0\rangle = v_u/\sqrt{2}$ and $\langle H_d\rangle = v_d/\sqrt{2}$, i.e. the VEVs, and $v^2 = v_u^2 + v_d^2$, which corresponds to the VEV of the Higgs in the standard model. We also get one neutral, pseudoscalar (CP-odd) Higgs, called A, with a mass

$$m_A^2 = \frac{2m_3^2}{\sin 2\beta}, \tag{81}$$

where the useful angle β is defined from $\tan\beta = \frac{v_u}{v_d}$. Finally, we also get two neutral scalar (CP-even) Higgses, H_1 and H_2 (where H_1 is lighter than H_2), that have the masses

$$m_{H_1,H_2}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta}\right]. \tag{82}$$

If we remember the relation $M_W^2 = \frac{1}{4}(v^2/2)g_2^2$ [3], and express $\sin 2\beta$ in terms of v_u and v_d, we see that

$$m_{H^\pm}^2 = m_A^2 + M_W^2. \tag{83}$$

We can also conclude that

$$m_{H_1}^2 + m_{H_2}^2 = m_A^2 + M_Z^2, \tag{84}$$

and the more striking inequality

$$m_{H_1} < \min(m_A, M_Z), \tag{85}$$

meaning that no matter how we choose our parameters, $m_{H_1} < M_Z$. This is a tree level prediction, and loop corrections can lift the mass of H_1 above the so far established limits, but this is still an important prediction of MSSM.

[3]The extra factor 1/2 coming from the factors of $\sqrt{2}$ in my definition of v_u and v_d.

As we are excluding larger and larger values of m_{H_1}, this is really a problem, called the little hierarchy problem. Just as the general hierarchy problem, this concerns the separation of mass scales, because in order to generate the large loop corrections needed to increase m_{H_1}, the other sparticle masses needs to become very large, again creating a new unexplained mass scale in the theory. The maximal bound one can get without adding new dynamics to the theory is something like $m_{H_1} \lesssim 135$ GeV. Thus results that exclude a Higgs mass lighter than that will exclude the whole of MSSM.

As mentioned above, the μ and m_3^2 parameters are dimensionfull. The m_3^2 parameter isn't a problem since it enters as one of the softly breaking terms, but μ enters through the ordinary Lagrangian, so the only natural values for it before SUSY breaking occurs, is either 0 or the Planck mass M_P. However, to be phenomenologically viable we must have a μ that is of similar size to the electroweak scale. Otherwise there would have to be miraculous cancellations between μ^2 and the soft supersymmetry breaking terms which I have not yet introduced. It is in order to solve this problem we motivate the study of the NMSSM, where by adding a singlet Higgs field and coupling it to the Higgs doublets, the μ parameter is generated through supersymmetry breaking. This breaking gives the singlet and the two Higgs doublets VEVs, and thus the singlet-doublet-doublet coupling gives us an effective μ. This explains why μ should be roughly the same scale as the electroweak breaking scale.

For a more complete discussion of MSSM, and some discussion about how it may be discovered, see for example [21].

4.2.1 R-parity

The superpotential described above is as stated the minimal one reproducing the results of the standard model. However, it is not the most general renormalizable superpotential we can write down; we could also include terms such as

$$\lambda^{ijk}\hat{L}_i \cdot \hat{L}_j\hat{e}_k + \omega^{ijk}\hat{Q}_i \cdot Q_j\hat{d}_k + \kappa^{ijk}\hat{L}_i \cdot \hat{Q}_j\hat{d}_k + \nu^i\hat{L}_i\hat{H}_u$$

where again i, j, k are generation indices. However, all these terms enable decays or production processes that violates the observed lepton and baryon number conservation, unless of course the couplings for some reason are very suppressed, which would be most unnatural. We could try and impose baryon and lepton number conservation directly, but this would be a step back compared to in the standard model, where this happens "automatically" since in the SM there are no allowed renormalizable terms that violate either baryon or lepton number conservation. Moreover, there are known non-perturbative effects that violates baryon and lepton number conservation[22], so they are probably not exact symmetries of nature. Of course, the simple observation that the universe we see today consists of only matter as opposed to equal parts matter and antimatter shows that such processes must be allowed.

In order to get rid of such terms, one imposes a new symmetry, called *R-parity* or equivalently, but slightly different, *matter parity*, that doesn't allow lepton or baryon number conservation. In addition, it has other nice consequences, discussed shortly below. Matter parity assigns to each particle in the theory a number

$$P_M = (-1)^{3(B-L)} \tag{86}$$

where B is the baryon number (quarks have $B = \frac{1}{3}$) and L is the lepton number. Gauge bosons carry neither number and so are assigned $P_M = 1$. It is easy to check that both the leptons and the quarks doublets have $P_M = -1$ and the Higgs, carrying none of the numbers has $P_M = 1$ just as the gauge bosons. Matter parity is then to be a conserved quantum number, and the condition to be imposed is that we only allow terms with a combined (i.e. the terms matter parity multiplied together) parity of $+1$. This forbids all the terms listed above.

An equivalent way of stating the symmetry that often is more useful is in terms of R-parity, defined instead for every particle as

$$P_R = (-1)^{3(B-L)+2s} \tag{87}$$

where s is the spin of the particle. As opposed to the matter parity, all particles in the same supermultiplet do not have the same R-parity, since they differ by (in $N = 1$ SUSY) half a unit in spin. It is simple to check that this means that all the normal particles has $P_R = +1$, while the sparticles (i.e. superpartners) have $P_R = -1$. Now the equivalent requirement is instead that all interaction terms should preserve P_R.

In the literature, one often encounters symmetries called "R-symmetries", which are things like additional U(1) symmetries you add to your model. These are not the same as R-parity, which is a discrete $\mathbb{Z}_2$ symmetry. Symmetries are called R symmetries when they do not commute with supersymmetry. We can easily see that this is the case with R-parity, since it depends on the spin and the supercharges change the spin by 1/2 unit. However, since R-parity is equivalent with matter parity, which does commute with supersymmetry, there really is nothing truly "R" about R-parity.

If R-parity is conserved, as we impose it to be in MSSM and NMSSM, this has a number of important consequences. It means that there can be no interactions with an odd number of sparticles, and that there can be no mixing between ordinary particles and sparticles. In particular, this means that supersymmetric particles has to be pair produced when we perform accelerator experiments with normal particles. It also means that the lightest supersymmetric particle (LSP) has to be stable, since there is no allowed decay process to only normal particles. This is how supersymmetry provides a possible explanation for the cold dark matter in the universe. So the imposing of R-parity seems to be well motivated from a phenomenological viewpoint. From a theoretical viewpoint, there are numerous proposed extensions[23] of MSSM where R-parity appears as a remnant of a continuous R-symmetry broken at some high energy scale.

4.3 The NMSSM

As stated above, the NMSSM [24, 25] is a proposed extension of the MSSM, which in a natural way solves the μ-problem. In order to get rid of the dimensionfull μ parameter, we add a new Higgs $SU(2)_L$ singlet $\hat{S}$ to the theory. Of course, in principle nothing forbids a μ term just because we add a new singlet, but we take it to have the "natural" value 0. The new superpotential looks like

$$W_{NMSSM} = W_{MSSM} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3, \tag{88}$$

where λ, κ are new, dimensionless parameters of the model. From this superpotential and the usual gauge couplings, the F and D part of the potential can be computed as described in section 3.7. The result looks very much like in MSSM, but with some extra terms;

$$V_F = |\lambda S|^2(|H_u|^2 + |H_d|^2) + |\lambda H_u \cdot H_d + \kappa S^2|^2 \tag{89}$$

$$V_D = \frac{1}{8}g^2(|H_u|^2 - |H_d|^2)^2 + \frac{1}{2}g_2^2|H_u^\dagger \cdot H_d|^2 \tag{90}$$

Since supersymmetry has to be broken, and we don't know nor care about the details of the breaking mechanism, we also have to add to the potential all possible terms which breaks supersymmetry in the acceptable, soft way explained in section 4.1. This soft potential looks like

$$V_{\text{soft}} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + \left(\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right). \tag{91}$$

Nothing is preventing us from adding the $m_3^2 H_u \cdot H_d$ term present in the MSSM case, but this would add an additional parameter with mass dimension, in conflict with the philosophy behind NMSSM, so we consider only the case $m_3^2 = 0$. The entire Higgs potential is then given by the sum of these,

$$V_{\text{Higgs}} = V_F + V_D + V_{soft}. \tag{92}$$

Then, as in the breaking of electroweak symmetry, we assume that $m_S^2 < \frac{1}{9}\kappa^2 A_\kappa^2$ so that $S = 0$ is an unstable state. If we define $\langle S \rangle = v_s/\sqrt{2}$ we see that when we expand the singlet field around its VEV we get a term $\mu = \lambda v_s/\sqrt{2}$ with mass dimension[4]. Since this term comes from supersymmetry breaking, it's natural for it to have a value of magnitude $|\mu| \lesssim M_{\text{SUSY}}$, where M_{SUSY} is the scale where supersymmetry is broken. This is the way in which NMSSM solves the μ-problem of MSSM.

Further, we also assume that at least one of the other Higgs mass parameters $m_{H_u}^2$ and $m_{H_d}^2$ are negative, so that H_u, H_d also get nonzero VEVs, as required to break the electroweak symmetry. We then have the gauge freedom to choose $\langle H_u^+ \rangle = \langle H_d^- \rangle = 0$, so that the vacuum is uncharged. In this treatment, I will discuss the vacuum obtained by further assuming all the remaining VEVs to be real, and described by

$$\langle H_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad H_d = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle S \rangle = \frac{1}{\sqrt{2}}v_s. \tag{93}$$

We then require this vacuum to be a stable local minimum of the potential, giving us three different relations of the type

$$\left.\frac{\partial V}{\partial S}\right|_{\text{vacuum}} = 0$$

relating the squared masses of the Higgs fields to the VEVs and the other parameters in the theory. The derivatives w.r.t. fields with zero VEVs are trivially zero. If solved for the masses, these three relations are

$$m_u^2 \equiv m_{H_u}^2 + |\mu|^2 = \frac{1}{8}g^2(v_d^2 - v_u^2) + \lambda\frac{v_s v_d}{2v_u}\left(\sqrt{2}A_\lambda + v_s\kappa\right) - \frac{1}{2}\lambda^2(v_d^2 + v_s^2) \tag{94}$$

$$m_d^2 \equiv m_{H_d}^2 + |\mu|^2 = \frac{1}{8}g^2(v_u^2 - v_d^2) + \lambda\frac{v_s v_u}{2v_d}\left(\sqrt{2}A_\lambda + v_s\kappa\right) - \frac{1}{2}\lambda^2(v_u^2 + v_s^2) \tag{95}$$

$$m_S^2 = -\frac{1}{2}v^2\lambda^2 - v_s^2\kappa^2 + \frac{1}{\sqrt{2}}A_\lambda\lambda\frac{v_u v_d}{v_s} + v_u v_d \kappa\lambda - \frac{1}{\sqrt{2}}A_\kappa v_s \kappa. \tag{96}$$

[4]Note that this definition of μ is a convention, which differs by a factor $\frac{1}{\sqrt{2}}$ from the most common one.

These new masses (m_u, m_d) are defined since when we give the singlet a VEV, effectively there will be an additional massterm of $|\mu|^2$ for the doublet fields, so we calculate the conditions for these effective masses. As above we define $v = \sqrt{v_u^2 + v_d^2}$. These requirements fixes the mass parameters. In order to further reduce the number of parameters, we can, as is usual, fix

$$v^2 \approx (\sqrt{2} \cdot 174 \text{ GeV})^2 \approx (246 \text{ GeV})^2,$$

where the factor of $\sqrt{2}$ comes from my definition of v_u and v_d. This just sets the scale of the symmetry breaking to be the electroweak one. As said above when discussing MSSM, v corresponds to the Higgs VEV in the standard model, and the numerical value of 174 GeV comes from the standard model relation

$$M_W^2 = \frac{1}{4} v^2 g_2^2.$$

Just as in the MSSM above, it is useful to define an angle β from

$$\tan\beta = \frac{v_u}{v_d}.$$

We now see that a full specification of the Higgs sector in the NMSSM requires six parameters:$\lambda, \kappa, A_\lambda, A_\kappa, \tan\beta$ and v_s. Conventions can be chosen such that $\lambda, \tan\beta$ and v_s are positive, and this is what I will do. For my purposes I also keep $\kappa > 0$, since switching this sign doesn't change any of my results. In my numerical studies I will replace A_λ by the physical mass of the charged Higgs, $m_{H^\pm}$, which of course must be positive, and we will see that the requirement of positive masses squared restrics A_κ to the negative range.

The "MSSM limit" can be approached smoothly by keeping the ratio $k = \kappa/\lambda$ fix and letting $\lambda \to 0$, while keeping $\mu = v_s\lambda/\sqrt{2}$ constant. Since the only couplings between the Higgs doublets and the new Higgs singlet are dependent on λ and κ, the singlet field decouples in this limit and one recovers the Higgs sector of MSSM. How this works will be explained in more detail in section 6.2.

Another thing worth mentioning about the NMSSM, is that it can be used to solve the little hierarchy problem of the MSSM. This is because if we let λ (or v_s) become larger, the mass of the lightest Higgs gets larger, so by having a large λ we can get a large Higgs mass. This approach is sometimes called λ-SUSY[26]. In this approach we however give up the requirement of perturbativity up to the GUT-scale.

5 Details of NMSSM

In this section I will go through some technical details about the Higgs sector of the NMSSM. First the mass matrices are described in some detail, and then the couplings of the Higgs to the W/Z and the quarks are briefly described, introducing the concept of reduced couplings in order to easily compare it with the standard model Higgs and the MSSM. Finally some theoretical and experimental limits on the parameter space are discussed. Some other articles discussing the Higgs sector of the NMSSM are [27, 28].

5.1 The mass matrices

Since mixing only can occur between states with the same quantum numbers, we get three different mass matrices, one for the charged Higgs states, one for the scalar or CP-even neutral states, and one for the pseudoscalar or CP-odd states. Since they are obtained by taking derivatives of the potential, they are all real and symmetric. This is at tree level, taking higher order corrections into account this is no longer the case. The realness also means that, at tree level, there is no CP violation.

5.1.1 Neutral scalar states

In the natural basis $\{H_{u,R}, H_{d,R}, S_R\}$ where the subscript $_R$ denotes the real part of the corresponding scalar field, we get the mass-squared matrix as follows

$$M^2_{s,11} = \frac{1}{4}g^2 v_u^2 + \frac{\lambda v_s v_d}{2v_u}(\sqrt{2}A_\lambda + v_s\kappa) \tag{97}$$

$$M^2_{s,22} = \frac{1}{4}g^2 v_d^2 + \frac{\lambda v_s v_u}{2v_d}(\sqrt{2}A_\lambda + v_s\kappa) \tag{98}$$

$$M^2_{s,33} = v_s\kappa\left(\frac{1}{\sqrt{2}}A_\kappa + 2v_s\kappa\right) + \frac{1}{\sqrt{2}}\lambda A_\lambda \frac{v_d v_u}{v_s} \tag{99}$$

$$M^2_{s,12} = v_d v_u(\lambda^2 - \frac{1}{4}g^2) - \frac{1}{2}v_s\lambda(\sqrt{2}A_\lambda + v_s\kappa) \tag{100}$$

$$M^2_{s,13} = \lambda\left(v_s(v_u\lambda - v_d\kappa) - \frac{1}{\sqrt{2}}A_\lambda v_d\right) \tag{101}$$

$$M^2_{s,23} = \lambda\left(v_s(v_d\lambda - v_u\kappa) - \frac{1}{\sqrt{2}}A_\lambda v_u\right) \tag{102}$$

where I've used the stability conditions to eliminate the mass parameters from the potential in favour of the vacuum expectation values and couplings. This matrix doesn't really lend itself to much further algebraic simplification, so it is evaluated in the form given here and numerical methods are used to find it's eigenvalues, which corresponds to the physical masses of the CP-even Higgs states, which are denoted H_1, H_2, H_3, ordered from the lowest mass to the highest.

5.1.2 CP-odd neutral states

In the natural basis $\{H^0_{u,I}, H^0_{d,I}, S_I\}$ we get the following mass matrix for the pseudo-scalar states:

$$M^2_{p,11} = \frac{1}{2}\frac{v_d}{v_u}v_s\lambda(\sqrt{2}A_\lambda + v_s\kappa) \tag{103}$$

$$M^2_{p,22} = \frac{1}{2}\lambda v_s\frac{v_u}{v_d}(\sqrt{2}A_\lambda + v_s\kappa) \tag{104}$$

$$M^2_{p,33} = -\frac{3}{\sqrt{2}}A_\kappa v_s\kappa + \frac{v_d v_u}{v_s}\lambda\left(\frac{1}{\sqrt{2}}A_\lambda + 2v_s\kappa\right) \tag{105}$$

$$M^2_{p,12} = \frac{1}{2}\lambda v_s(\sqrt{2}A_\lambda + v_s\kappa) \tag{106}$$

$$M^2_{p,13} = \frac{1}{2}v_d\lambda(\sqrt{2}A_\lambda - 2v_s\kappa) \tag{107}$$

$$M^2_{p,23} = \frac{1}{2}v_u\lambda(\sqrt{2}A_\lambda - 2v_s\kappa) \tag{108}$$

If the first two basis-elements are rotated with the angle β, a massless Goldstone mode decouples, and the new mass matrix (dropping the massless mode) in the basis (P_1, P_2)

becomes

$$
\begin{aligned}
M^2_{p',11} &= \frac{v^2}{2v_u v_d} v_s \lambda(\sqrt{2}A_\lambda + v_s\kappa) \equiv M_A^2 && (109)\\
M^2_{p',22} &= \frac{v_u v_d}{v_s}\left(\frac{1}{\sqrt{2}}A_\lambda + 2v_s\kappa\right) - \frac{3}{\sqrt{2}}A_\kappa v_s \kappa && (110)\\
M^2_{p',12} &= v\lambda(A_\lambda - 2\kappa v_s) && (111)
\end{aligned}
$$

where we introduce the mass parameter M_A^2. Note that this is not a physical mass, only a parameter which can be taken as one of the parameters instead of A_λ. It can be useful, because in the MSSM-limit, M_A becomes the physical mass of the pseudoscalar Higgs. The matrix that diagonalises this is of course a 2×2 orthogonal matrix, and can thus be parametrized by an angle θ_A. The new basis in which the mass matrix is diagonal is then

$$
\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}. \tag{112}
$$

5.1.3 Charged states

In the natural basis $\{H^+_{u,R}, H^-_{d,R}\}$, the mass matrix for the charged states looks like

$$
\begin{aligned}
M^2_{c,11} &= \frac{1}{4}v_d^2(g_2^2 - 2\lambda^2) + \frac{v_d v_s \lambda}{2v_u}\left(\sqrt{2}A_\lambda + \kappa v_s\right) && (113)\\
M^2_{c,12} &= \frac{1}{4}v_d v_u(g_2^2 - 2\lambda^2) + \frac{1}{2}\lambda v_s(\sqrt{2}A_\lambda + v_s\kappa) && (114)\\
M^2_{c,22} &= \frac{1}{4}v_u^2(g_2^2/2 - 2\lambda^2) + \frac{v_s v_u \lambda}{2v_d}(\sqrt{2}A_\lambda + v_s\kappa). && (115)
\end{aligned}
$$

By a rotation through the mixing angle β, this gives a mass matrix in a new basis $\{H^\pm, G^\pm\}$ with the only nonzero element

$$
\begin{aligned}
M^2_{c',11} &= \frac{v^2}{2v_u v_d} v_s\lambda(\sqrt{2}A_\lambda + v_s\kappa) + \frac{1}{4}g_2^2 v^2 - \frac{1}{2}\lambda^2 v^2 \\
&= M_A^2 + M_W^2 - \frac{1}{2}\lambda^2 v^2 && (116)
\end{aligned}
$$

To get the second equality, we use the definition of M_A^2 in addition to the previously noted relation $(g_2 v/2)^2 = M_W^2$. The state $G^\pm$ is a massless Goldstone mode. The charged Higgs state is denoted by $H^\pm$.

5.2 Reduced couplings

If we want to express how the physical Higgs particles, i.e. the mass eigenstates, couple to fermions and gauge bosons, what one needs to do is to express the original weak eigenstates H_u, H_d, S in terms of the mass eigenstates $H^\pm, A_1, A_2, H_1, H_2, H_3$. This is of course done by looking at the matrices that rotates the weak eigenstates into the mass eigenstates, i.e. the mixing matrices as defined above.

We are primarily interested in how the V-boson couples to the different Higgses, where V can be either $W^\pm$ or Z, and the couplings to quarks, since in the generic Higgs decay $H \to f\bar{f}$, there is a factor $\frac{m_f^2}{m_W^2}$ meaning that the heaviest fermion allowed dominates, i.e. either the top or bottom quark.

The way to find these couplings is to write down the relevant terms in the Lagrangian, which is originally in terms of the weak eigenstates, and then re-express it in terms of the mass eigenstates $H_i, H^{\pm}$ and A_j. The details can be found in [28].

In order to simplify the notation and keep it from getting unneedingly cluttered, we define so-called *reduced couplings*, where we take the full coupling and divide out the associated SM coupling,

$$G_{VVH_i} \equiv \frac{g_{VVH_i}}{g_{VVH}^{SM}}, \quad G_{ZA_iH_j} = \frac{g_{ZA_iH_j}}{G_{ZHH}^{SM}} = \frac{g_{ZA_iH_j}}{g/2},$$

where $g = \sqrt{g_1^2 + g_2^2}$, g_1 and g_2 being the gauge couplings of the electroweak force. Here, V can stand for either W or Z, the reduced coupling will be the same in either case. The reduced coupling we will look the most at is the H_iVV coupling, since this measueres how standard model like the scalar Higgses are. If we let $S_i = (H_{u,R}, H_{d,R}, S_R)$ be the scalar weak eigenstates, and $H_i = \sum_j S_{ij}S_j$, (i.e. S_{ij} is the mixing matrix), then this reduced coupling is defined as[27]

$$G_{H_iVV} = \sin\beta S_{i1} + \cos\beta S_{i2}. \tag{117}$$

The $H_iH^{\pm}W^{\mp}$ coupling is similarly given by

$$G_{H_iH^{\pm}W^{\mp}} = \cos\beta S_{i1} - \sin\beta S_{i2}. \tag{118}$$

Since these reduced couplings comes directly from orthogonal mixing matrices, we may conclude that they should fulfil certain sum rules. This is because of the sum rules that elements of orthogonal matrices fulfil: the sum of the squares of one row (or column) is equal to one. For the reduced couplings, this means

$$\sum_i G_{ZZH_i}^2 = 1, \quad \sum_i G_{H_iH^{\pm}W^{\mp}}^2 = 1. \tag{119}$$

In the same way, the reduced couplings of the Higgses to the top and bottom quarks also comes directly from an orthogonal matrix, but in this case a dependence on $\tan\beta$ also enters, since this describes how large the difference is between the two VEVs of the Higgs doublets. In this case the sum rules are

$$\sum_i G_{H_itt}^2 = \frac{1}{\sin^2\beta}, \quad \sum_i G_{H_ibb}^2 = \frac{1}{\cos^2\beta}. \tag{120}$$

We also have the sumrules from the columns, for example

$$G_{H_iVV}^2 + G_{H_iH^{\pm}W^{\mp}}^2 + S_{i3}^2 = 1, \quad i = 1, 2, 3 \tag{121}$$

where S_{i3} is the singlet component of H_i. If $S_{i3} \approx 0$, then H_i will be purely doublet and the corresponding sum rule $G_{H_iVV}^2 + G_{H_iH^{\pm}W^{\mp}}^2 = 1$ is recovered. Conversely, if $S_{i3} \approx 1$ then both the other couplings will be suppressed, which means that detection of H_i will be difficult. These sum rules are quite trivial in nature, but can be a useful check on the numerical methods used. They are also important phenomenologically, since they in effect is a good measure how standard model like the different H_i, A_i are. For example, in this last sum rule, if the first term is large it means that H_i is SM like, if the second term is large it means there's a large coupling between the doublets making H_i MSSM like, and the last term corresponds to how singlet-like H_i is.

Of course, since the standard model only has one (scalar) Higgs particle, only the H_1 couplings (assuming that the lightest Higgs also will be the standard model like) have a direct correspondence in the standard model. Nevertheless we can define reduced couplings by scaling away the gauge couplings and masses.

5.3 Constraints on the parameters

In this section I will explain some theoretical and experimental limits on the parameter space, and motivate the choices of parameters later used when studying some numerical results. I will discuss for which intervals it is sensible to choose values for the parameters, which I choose as $\lambda, m_{H^\pm}, \kappa, \tan\beta$ and A_κ. Some limits can be found from theoretical considerations and requirements, while others come from experiments at accelerators or astrophysics.

As said above, I use the value $v = \sqrt{2} \cdot 174 = 246$ GeV for the electroweak scale, and choose the 'standard' value $\mu = \lambda v_s/\sqrt{2} = 200$ GeV, which allows me to see v_s as a fixed value when λ is chosen. There is a restriction from LEP [29] on the minimal size in μ, requiring that $|\mu| > 100$ GeV, coming from lower limits on Higgsino masses, but this is really for MSSM. Nevertheless, a too small value of $|\mu|$ doesn't work.

Since my analysis is at tree level, we will not discuss parameters entering at loop level, where the Higgs masses get corrections depending on for example the top and the stop masses. There is probably many cases where even at tree level limits from measurements could be used to rule out large parts of the parameter space, but doing this in detail is regretfully beyond the limited scope of this study. Also, for such exclusions to be meaningful, at least the first loop level corrections should be included.

A general way to restrict the parameter space is to require that they fulfil some grand unified scenario where all couplings of the same type gets the same value at the GUT scale. This is called universal boundary conditions, and will not be required here.

5.3.1 λ and κ

We can see that with $\kappa = 0$, the Lagrangian, (88), has an additional $U(1)$ symmetry, called *Peccei-Quinn* symmetry[30] (henceforth called PQ-symmetry). This symmetry was proposed as a solution to the strong CP problem, i.e. the problem of explaining why QCD doesn't seem to violate CP symmetry like the electroweak interactions do. If this symmetry is exact, i.e. $\kappa = 0$, it will be spontaneously broken by the nonzero VEV of the singlet scalar, which will give rise to a massless Goldstone boson, called the Peccei-Quinn *axion*. This axion will show up as the extra pseudoscalar Higgs field (compared to in the MSSM). However, this case can in principle be ruled out since it would since there are lower bounds on allowed axionmasses[1] which only can be avoided if $10^{-16} < \lambda < 10^{-7}$[28]. Such a small value of λ would mean that v_s would have to grow very large, making the model unattractive as a solution to the μ-problem.

So from this we conclude that we need a nonzero value of κ, breaking the PQ symmetry. The size of the κ coupling will regulate how badly this symmetry is broken, and with a small value, only slightly breaking PQ symmetry, we will get a nonzero mass for the lightest pseudoscalar.

If one uses the requirement that λ, κ and the Yukawa couplings should stay small (eg. < 1) so that perturbation theory can be used up to the GUT scale, and uses the renormalization group flow, one can get the approximate limit at the electroweak scale[28]

$$\sqrt{\lambda^2 + \kappa^2} \lesssim 0.7 \qquad (122)$$

Also, from choosing a large number of different values of λ and κ at the GUT scale and using the renormalization group equations to run them down to the electroweak scale, one can see that the flow favours a small κ value.

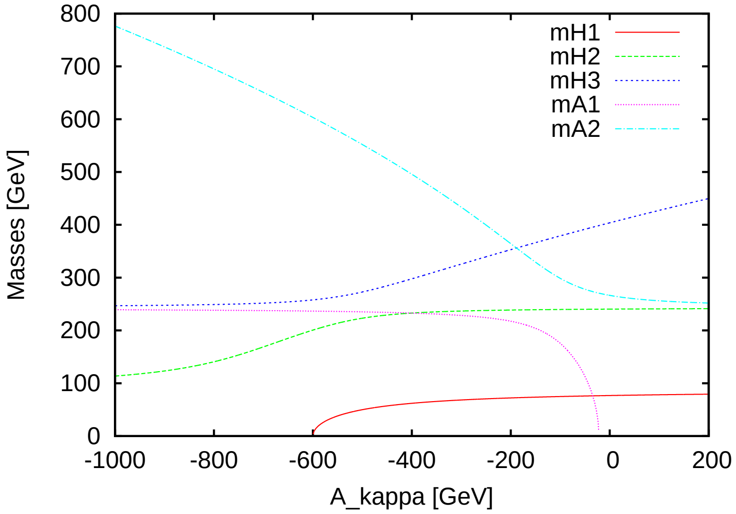


Figure 1: The masses as a function of A_κ, where $\tan\beta = 7, m_{H^\pm} = 250$ GeV, $\kappa = 0.3$ and $\lambda = 0.3$.

We also note that if λ gets too small, this forces v_s to become big, which means that the model no longer works well as a solution to the μ-problem. Even if we allow v_s to take a value of a few TeV, say 2 TeV, which is well over but still "close" to the electroweak scale in some sense, this places a limit on $\lambda \gtrsim 0.1$, so we get a rather stringent condition on λ.

If universal boundary conditions at the GUT scale are imposed (which gives us the so called constrained NMSSM[31]), we also get that the ratio λ/κ has to be close to 3. This is however not something that will be exclusively used since I don't in general impose universal boundary conditions.

5.3.2 $\tan\beta$ and A_κ

The range of A_κ is rather tightly constrained from the condition of vacuum stability. In figure 1, the only allowed range of A_κ is where all the masses are positive, i.e. $-600 \lesssim A_\kappa \lesssim -30$ GeV. In many cases the limits are a lot stricter than this. From such plots you can also see that for some choices of the other parameters there are no acceptable value of A_κ at all; in some cases the lightest pseudoscalar and the lightest scalar never both get a positive mass at the same time. For the coming plots where the A_κ dependence matters, the value of A_κ is chosen roughly in the middle of its allowed range, for a typical value of the running parameter. The typical values are -100 and -250 GeV.

As for $\tan\beta$, an analysis of the running couplings shows that a low value of $\tan\beta$ is favoured. However, experiments rule out a too small value, so a not so small value is required [27].

The theoretical upper bound is $\tan\beta \lesssim m_t/m_b \sim 50$, and I will briefly study what happens when you take a large value, $\tan\beta = 30$ in the model. See also [32] for a study of what happens when you saturate this upper bound. We also have a lower bound $\tan\beta \gtrsim 1.2$ from requiring λ_t, the top quark Yukawa coupling, to remain small up to the GUT-scale.

5.3.3 $m_{H^\pm}$ and the other Higgs masses

For the Higgs masses experiments have placed general lower limits. For the neutral Higgses, LEP has published negative search results[29] in some different decay channels, and depending on the precise branching ratios the limits looks a little bit different, but generally the lower bound from LEP is around $m_{H_i} > 80 - 90$ GeV, for the MSSM. In the NMSSM these limits can be avoided, but they still give some kind of general guidelines. In the numerical studies, a lower value of $m_{H^\pm} = 90$ GeV, and a higher value of 250 GeV will be used, when we don't let $m_{H^\pm}$ vary.

From different experiments at the Tevatron and LEP, rather strict limits can be placed on a SM-like Higgs[1], for example the Tevatron has excluded the range 158 GeV $< m_h <$ 173 GeV, and from LEP we have exclusions of a mass lower than 114 GeV, but since the couplings to fermions and gauge bosons of the MSSM or NMSSM Higgs bosons can be suppressed compared to the SM Higgs, these limits can be avoided.

6 Results

In this section I will present numerical results that explores some of the features of NMSSMs parameter space. First, masses and couplings are treated as a function of the charged Higgs mass. From this we see some possibly interesting features of the model. Then it is studied how the masses and reduced couplings (and thus the mixing) behave in some different kinds of MSSM limits.

Since my calculation is only at tree level, it is not sensible to compare directly with experimental limits. Even so, the general features are maybe even better understood at tree level, since it is easier to compare directly with the formulae without too much cluttering of the expressions. For my numerical results, I've written code in Java, using the basic linear algebra library JAMA to diagonalize and find eigenvalues of matrices.

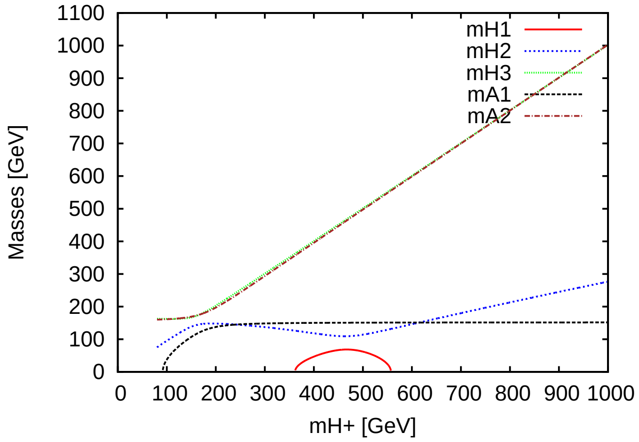
6.1 Varying the charged Higgs mass

In order to see how the NMSSM mass spectra behaves, it can be instructive to plot the masses as a function of the charged Higgs mass. From these plots, and the requirement of vacuum stability (i.e. $m^2_{H_1} > 0$, the lightest scalar mass positive) we can find limits on allowed values for $m_{H^\pm}$ for fixed values of the other parameters.

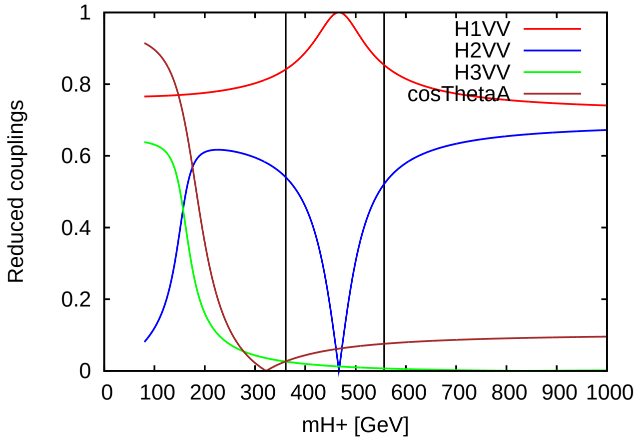
As an aside, just in order to confirm the theory and my numeric calculation, we can check that the sum rules for the reduced couplings actually holds in practise, which it turns out they do. A thing to note when looking at plots of reduced couplings and $\cos\theta_A$ is that what I really plot is the absolute value of the couplings. This is for two reasons, first of all that all we really care about is the strength of the different couplings, the sign can of course matter (mostly when you go to higher orders) but not for our purposes here. The other reason is that the numerical method used switched signs discontinuously, so without taking the absolute value the graphs looks very discontinuous and strange. This can probably be fixed rather easily, but since the sign doesn't matter for our purpose no effort was expended on this.

6.1.1 The NMSSM with a small κ

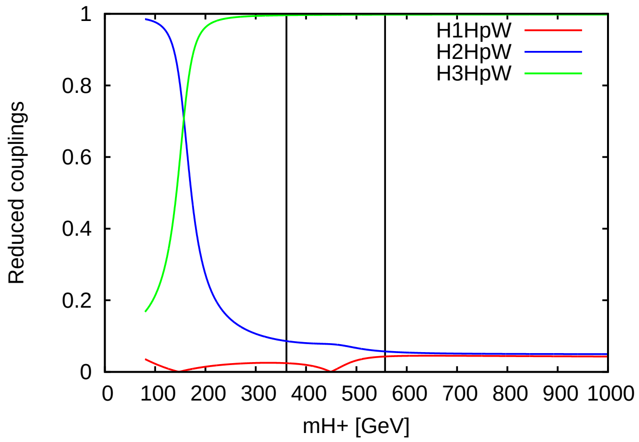
In figure 2 we can see that for the following choices of parameters, $\lambda = 0.3$, $\kappa = 0.1$, $\tan\beta = 2$, $A_\kappa = -100$ GeV, $m_{H^\pm}$ has to be between 360 and 550 GeV. This case is



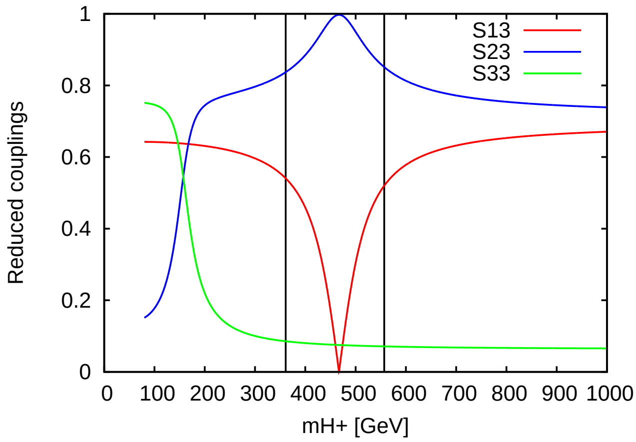
(a) Masses



(b) H_iVV-couplings and $\cos\theta_A$



(c) $H_iH^{\pm}W^{\mp}$-couplings



(d) S_{i3}

Figure 2: The Higgs masses, some different scalar couplings and $\cos\theta_A$ as functions of $m_{H^\pm}$, for $\kappa = 0.1, \lambda = 0.3, A_\kappa = -100$ and $\tan\beta = 2$. In (b,c,d), the physical range of the parameter space is inside the two black lines. S_{i3} is the singlet component of the H_i state.

representative for a small κ-value, which corresponds to a slightly broken PQ-symmetry. This is favoured by the renormalisation group flow. From the figure we also see that the mass of the lightest pseudoscalar almost doesn't change as soon as $m_{H^\pm}$ gets above $\sim$ 250 GeV. This is the singlet dominated pseudoscalar whose mass mainly comes from the $\frac{1}{3}\kappa S^3$ term in the Lagrangian. The heavier states that grows more or less linearly are the fields with little mixing with the singlet, (as seen from that S_{33} and $\cos\theta_A$ both are small) and behave the same way as in the MSSM. For these parameters, the next lightest scalar state, m_{H_2} also doesn't depend very strongly on $m_{H^\pm}$, and grows much slower than in MSSM.

So we see that even though the spectra of high mass states stay roughly the same, the three light Higgs states means that a NMSSM with parameters close to these will be easily distinguished from MSSM, even if we only find the lighter Higgses. Of course, this only works if the reduced couplings, G_{H_iVV} and $G_{A_1H_iZ}$ doesn't become too small to prevent detection, which happens in a small part of the relevant parameter space.

In figure 2.b the couplings of the scalar fields to the W/Z bosons as well as $\cos\theta_A$ are plotted as a function of the charged Higgs mass. Since the reduced couplings can be thought of as a measure of the mixing between weak eigenstates, this plot shows that the mixing depends on the charged Higgs mass in a slightly complicated way. The lightest Higgs is as one might guess the most standard model like, and it is the mixing with H_2 which raises $m_{H_1}^2$ above zero. For a specific value, $m_{H^\pm} = 466$ GeV, the coupling $G_{H_2VV} = 0$, so if this specific scenario is true, the next-lightest Higgs would be totally singlet-like (as we see in figure 2.d). In this case, the H_2 Higgs would be totally undetectable through the channels used to look for the standard model Higgs. We also see that in the physical range at least, the pseudoscalar mixing only varies a little. The $H_iH^\pm W^\mp$ couplings, which measure how doublet or MSSM-like the scalars are, vary very little in the physical range, but we do see that $H_1H^\pm W^\mp$ pass through zero when $m_{H^\pm} = 449$ GeV. We also see that the heavy H_3 is doublet-dominated.

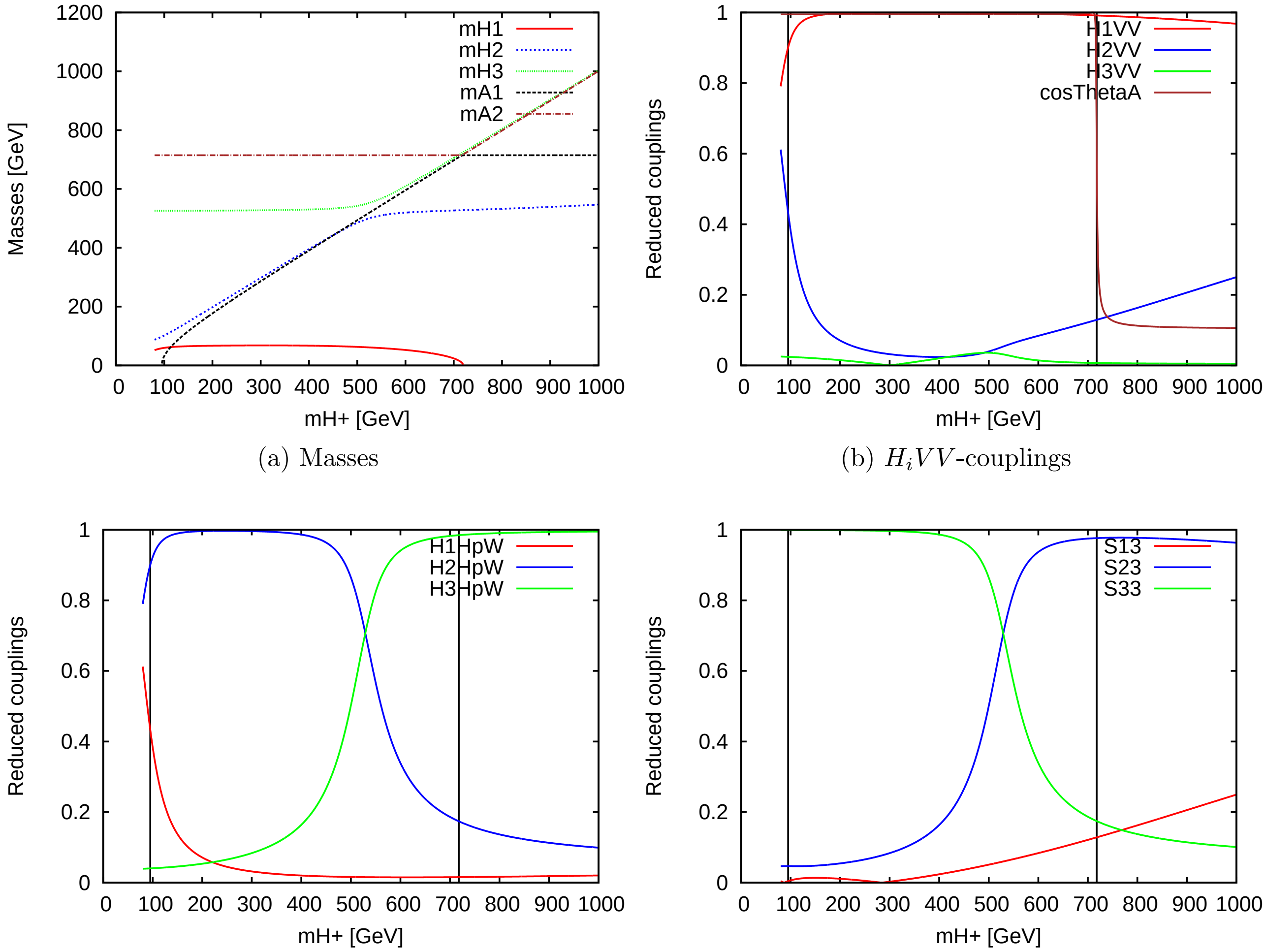
From the figure we can also see that m_{A_1} and m_{A_2} seems to switch behaviour with respect to the charged Higgs mass around $m_{H^\pm} \sim 150$ GeV. This switch is also apparent in how $\cos\theta_A$ behaves. After this however, the pseudoscalar mixing stays more or less constant, and doesn't vary rapidly in the physical region as the scalar mixing does. In the same way we see the switch in behaviour between m_{H_2} and m_{H_3} reflected in how all of the corresponding couplings switch, although this is outside the physical region.

6.1.2 Larger κ

If we let the value of κ get larger, the PQ symmetry is more badly broken and the lighter pseudoscalar gets a larger mass. This is not favoured by the renormalization group flow, but we have no a priori reason to exclude it. In figure 3 we have plotted the mass spectrum and couplings as functions of $m_{H^\pm}$ for $\kappa = 0.5$. In this case, the lightest Higgs is the most standard model like by far, and H_2, H_3 again switch behaviour, around $m_{H^\pm} \simeq 520$ GeV.

We also see that this large κ loosens the constraints on $m_{H^\pm}$ from vacuum stability. The value of $A_\kappa = -500$ GeV used in the figure has been chosen approximately in the middle of its allowed range for these parameters.

In this case, compared to the previous case with κ small, we see that apart from the lightest Higgs, the rest of the masses are significantly larger. However, they are not extremely heavy and are still very much within the range of detection, but the spectrum of light Higgses present in the previous case is absent. This would make it harder in this



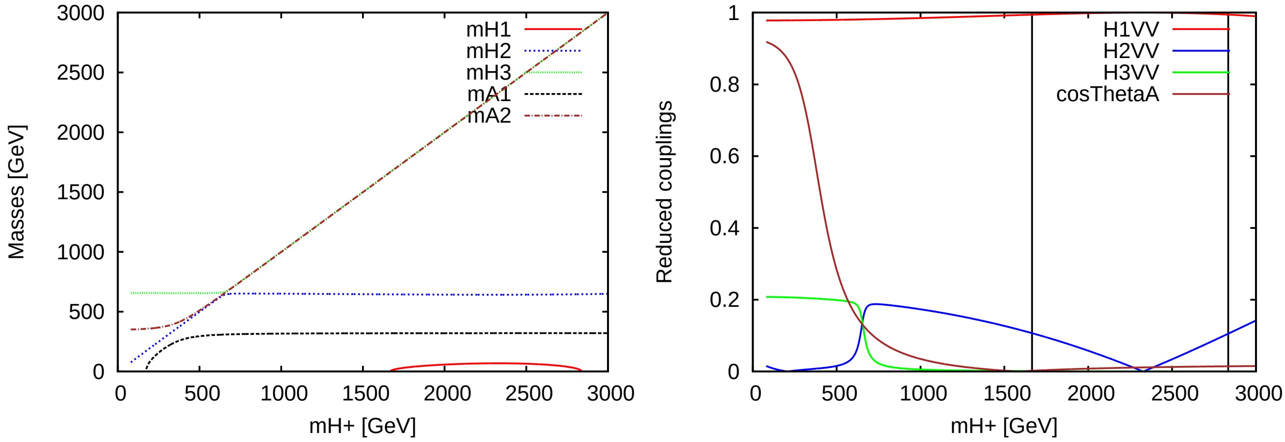
(a) Masses

(b) H_iVV-couplings

(c) $H_iH^{\pm}W^{\mp}$-couplings

(d) S_{i3}

Figure 3: The Higgs masses, some of the scalar couplings and $\cos\theta_A$, as functions of $m_{H^\pm}$, for $\kappa = 0.5, \lambda = 0.3, A_\kappa = -500$ and $\tan\beta = 2$. In (b,c,d) the physical range is again between the black lines.



(a) Masses

(b) H_iVV-couplings

Figure 4: The Higgs masses and reduced couplings of the scalar states to the Z and W bosons, as functions of $m_{H^\pm}$, for a higher value of v_s, equivalent to $\mu = 1000$ GeV, instead of the normally adopted value of $\mu = 200$ GeV. The rest of the parameters are the same as in figure 2. In (b), the physical range is inside the vertical black lines.

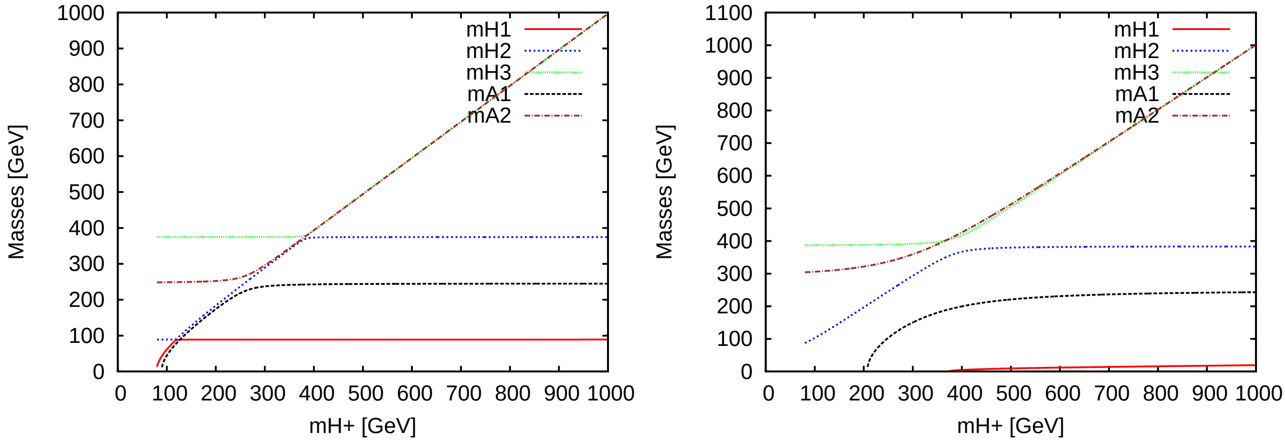
case to distinguish between the MSSM and the NMSSM if only the two lightest Higgses can be detected, compared to the case with smaller κ.

Just as in the previous case, the pseudoscalar mixing, i.e. $\cos\theta_A$, stays rather constant except at the place where the two pseudoscalar fields switch identity. In this case this switching behaviour is more distinct, something we see both in how the masses and how $\cos\theta_A$ behaves.

6.1.3 A larger v_s or μ-value

If we vary the expectation value of the singlet field, v_s, or equivalently the value of the effective $\mu = v_s\lambda$ parameter, this doesn't change the qualitative behaviour of the mass spectrum very much, but it changes the quantitative behaviour. All but the lightest Higgs gets heavier, including the charged Higgs, since the region of vacuum stability gets pushed upwards, see figure 4. We also see that the constraint from vacuum stability is relaxed (see for comparison figure 2), and that the charged Higgs mass, as well as the masses of H_3 and A_2 are now forced to be larger than ~ 1.5 TeV. Since the lightest of the Higgses remains light, it means that a higher v_s must make the H_1 more SM-like, which means that the H_2 becomes more singlet-like. That H_1 becomes doublet-like means that its coupling to W and Z should become large, and this is indeed also the case, as we can see in the right panel of figure 4.

If we make the same plots for a large value of κ, the effect on the mass spectrum will be bigger and the heavy states will thus be even heavier, since it gets amplified by the large κ value. In this case the heavy singlet dominated fields will decouple and the lower mass states will behave like in the MSSM, so in this case the distinction between NMSSM and MSSM will be hard to find.



(a) Small couplings (b) Large couplings

Figure 5: The Higgs masses as a function of $m_{H^\pm}$ for $\tan\beta = 35$. In (a),we have $\kappa = 0.1, \lambda = 0.1$, while in (b) $\kappa = 0.5, \lambda = 0.5$. In both cases $A_\kappa = -100$ GeV.

6.1.4 Large $\tan\beta$ values

What happens if you increase the value of $\tan\beta$, making one of the doublet VEVs much larger than the other? In figure 5, this is shown as a function of the charged Higgs mass for two different choices of the singlet couplings. From the figure we see that there seems to be rather distinct points where the identity of two different Higgses seem to switch. For example, the H_1 and the H_2 seem to switch behaviour with respect to the charged Higgs mass at around $m_{H^\pm} = 120$ GeV. This kind of behaviour really comes from the way the mixing matrices depends on $m_{H^\pm}$ (or equivalently A_λ) and from how we label the different states. A large $\tan\beta$ value means that the mixing with the H_u doublet will be much more important in terms of mass than the mixing with the H_d doublet. From this we can understand why a larger $\tan\beta$ value makes the identities of the Higgses more sharply defined.

What is perhaps more interesting to note is that we have three masses here that are almost independent of the charged Higgs mass, even though exactly what we call the state varies with $m_{H^\pm}$. This is also coupled to the fact that the couplings in this plot is rather small, $\kappa = \lambda = 0.1$. If we instead make them larger, we instead get the behaviour seen in figure 5, where the "switching" behaviour is not at all as sharp. Increasing the singlet couplings also pushes the lowest physically allowed value for $m_{H^\pm}$ upwards (i.e. the value where $m_{H_1} > 0$), and seems to give the lightest scalar Higgs a very small mass.

6.2 The MSSM limit

A few things can be noted analytically when we take the limit $\lambda, \kappa \to 0$ while keeping $\kappa/\lambda = k$ fixed. For example, we can see that the parameter m_s^2 in the Lagrangian will approach a fixed value as soon as λ and κ get small. This is seen by looking at the expression for m_s^2 we got from the requirement of vacuum stability, equation (96). Since we keep $\mu = \frac{1}{\sqrt{2}} v_s \lambda$ fixed, we have that

$$m_s^2 = -\frac{1}{2} v^2 \lambda^2 - v_s^2 \kappa^2 + \frac{1}{\sqrt{2}} A_\lambda \lambda \frac{v_u v_d}{v_s} + v_u v_d \kappa \lambda - \frac{1}{\sqrt{2}} A_\kappa v_s \kappa$$

$$\begin{aligned} &= -\frac{1}{2}v^2\lambda^2 - 2\mu^2k^2 + \frac{1}{2}A_\lambda\lambda^2\frac{v_u v_d}{\mu} + v_u v_d k\lambda^2 - A_\kappa k\mu \\ &\to k\mu(A_\kappa - k\mu) \text{ as } \lambda \to 0, \end{aligned}$$

also using the relation between κ and λ. If we instead keep κ fixed and let $\lambda \to 0$, we see that instead

$$\begin{aligned} m_s^2 &= -\frac{1}{2}v^2\lambda^2 - v_s^2\kappa^2 + \frac{1}{2}A_\lambda\lambda^2\frac{v_u v_d}{\mu} + v_u v_d\kappa\lambda - A_\kappa\kappa\frac{\mu}{\lambda} \\ &\to \infty, \text{ as } \lambda \to 0. \end{aligned}$$

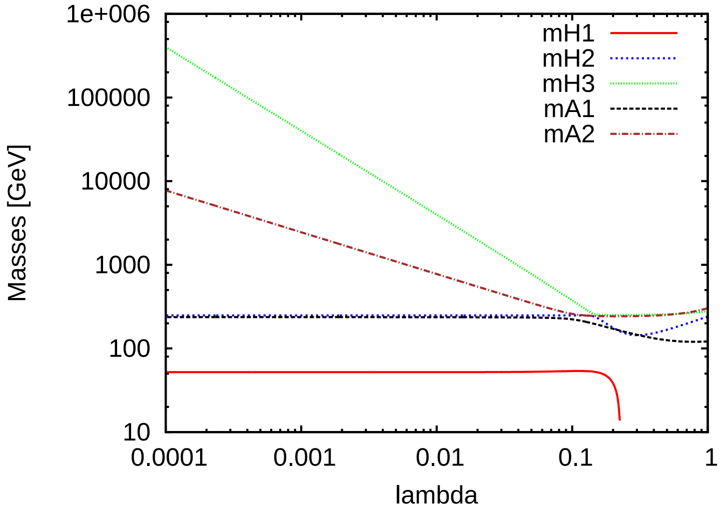


Figure 6: How the masses vary when λ goes to zero while keeping κ fixed. In this plot, $\tan\beta = 2, m_{H^\pm} = 250$ GeV, $\kappa = 0.1$ and $A_\kappa = -100$ GeV.

This behaviour is shown in figure 6, and it means that just as we can see from looking at the Lagrangian (88), as λ goes to zero the singlet field decouples and the masses of the singlet dominated states blow up.

Another way of taking an interesting limit is to let $\kappa \to 0$ keeping λ constant, as can be seen in figure 7. Here we see that in this limit (which really isn't a proper MSSM limit since λ stays large and thus the singlet doesn't fully decouple), the A_1 state becomes massless, which again is because we restore the PQ symmetry turning A_1 into the massless PQ-axion. However, it is seen that for all cases with fixed λ there is no way of keeping H_1 at a positive mass squared as κ goes to zero. For smaller values of λ, $m_{H_1}^2$ becomes negative for smaller values of κ, but any given value of λ ultimately restricts the lowest possible value of κ. So from looking at this in addition to the above discussed $\lambda \to 0$ limit we see that in order to decouple the singlet and reduce the theory to MSSM, one is in effect forced to take the simultaneous limit $\kappa, \lambda \to 0$ (or in addition take the limit $A_\kappa \to 0$ as studied later).

In figure 8, we see that when approaching the MSSM limit in the sensible way, none of the masses becomes large, which is understandable since we above showed that m_s^2 tends towards a finite value. However, also in this case the singlet decouples and does not mix with the other fields, which we see by looking at the reduced couplings and $\cos\theta_A$, seeing that the lightest Higgs state becomes completely standard model like, and also $\cos\theta_A$ goes to 1, so that both mixing matrices become block diagonal and there is no mixing between

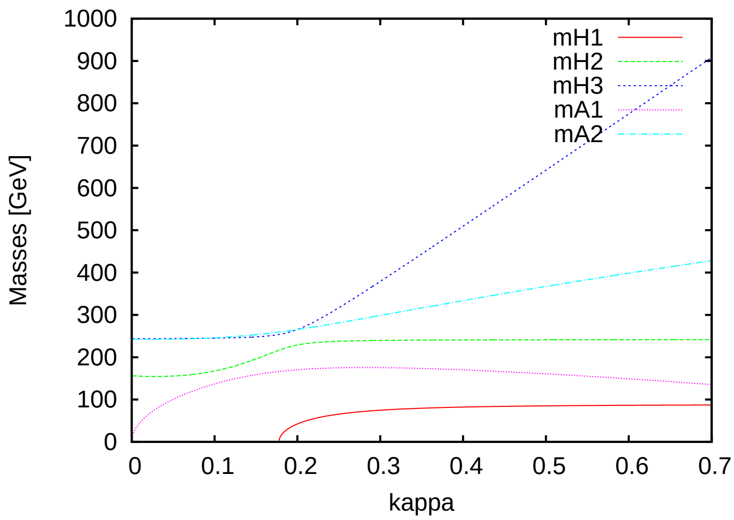


Figure 7: The masses as a function of κ, where $\tan\beta = 7, m_{H^\pm} = 250$ GeV, $A_\kappa = -100$ GeV and $\lambda = 0.3$.

the doublet and the singlet. That the lightest scalar becomes standard model like in this limit means that for small κ and λ of roughly the same size ($k \sim 1$) means that it could be detected as easily as in the standard model.

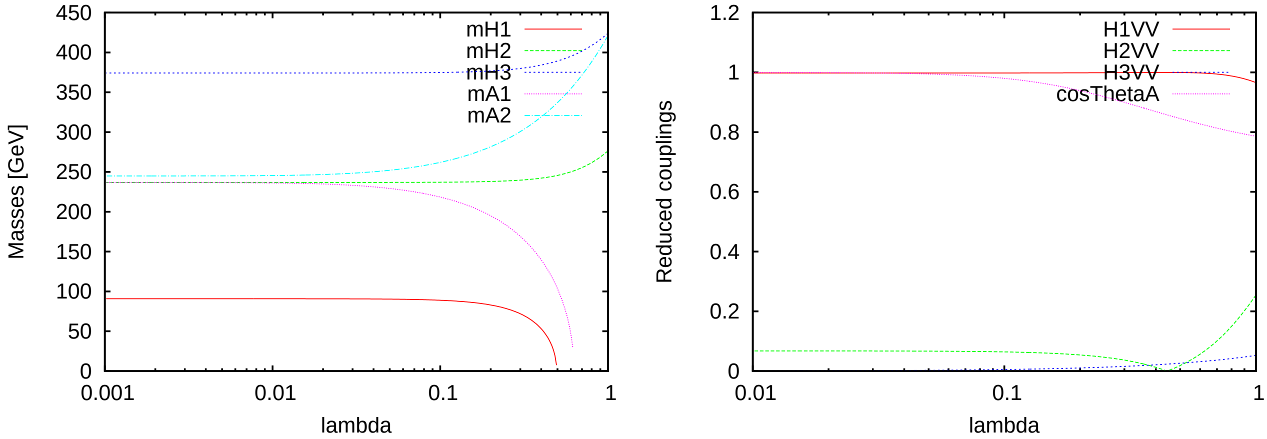
We also see that $\cos\theta_A$ goes to 1 much slower than the reduced scalar couplings. This is not a general feature but depends on the value of k. However, I've not found any cases with $k > 1$ where the pseudoscalar mixing disappears slower than the scalar mixing. This means that small values of λ and κ suppresses mixing between scalar singlet and doublet states much more than between the pseudoscalar states. Moreover, as λ becomes small the mixing between the scalar Higgs fields stop depending on λ and stays more or less constant. This is of course very reasonable since λ determines the coupling strength between the singlet and the doublets.

However, as we can see in figure 9, the reduced couplings (and thus the mixing) doesn't always vary slowly when we take the limit. This behaviour seems to occur only when $k \lesssim 0.5$, in the figure we have $k = 0.2$ as a representative case. In these scenarios, the couplings continue to vary very rapidly (considering the logarithmic scale) even when λ is very small, and the mixing only disappears when λ becomes really small. Differently from before, the lightest Higgs isn't the most standard-model like in this scenario. This role is instead filled by H_2. We also note that the masses of H_1 and H_2 gets very similar in the MSSM limit.

Since $\cos\theta_A$ approaches 0, we see that the lightest pseudoscalar state also decouples in the MSSM limit (that $\cos\theta_A = 0$ of course means that the off-diagonal element of the pseudoscalar mixing matrix $\sin\theta_A = 1$).

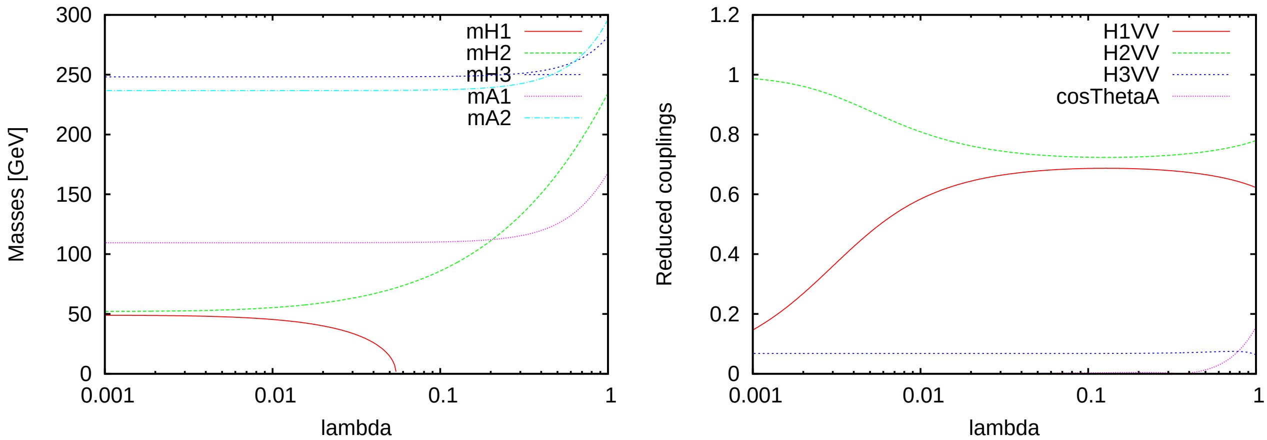
We however see that the masses of the two lightest Higgses are very small for these parameter choices, so this particular case is not realistic. For such small values of k the requirement that $m_{H_1}^2 > 0$ seems to rule out most of the parameter space, i.e. for many choices of other parameters, $m_{H_1}^2 < 0$. I've not found any case where the same thing happens for the pseudoscalar mixing, but no methodical search of such a scenario was carried out.

As we can see from the formula for m_S^2 in the MSSM limit, if we let the ratio $k = \kappa/\lambda$ get larger, then the masses of the singlet states should increase. And this is exactly



(a) Masses (b) H_iVV-couplings and $\cos\theta_A$

Figure 8: The masses, H_iVV-couplings and $\cos\theta_A$ as the MSSM limit is approached, using $k = 1$, $\tan\beta = 2, m_{H^\pm} = 250$ GeV and $A_\kappa = -100$.



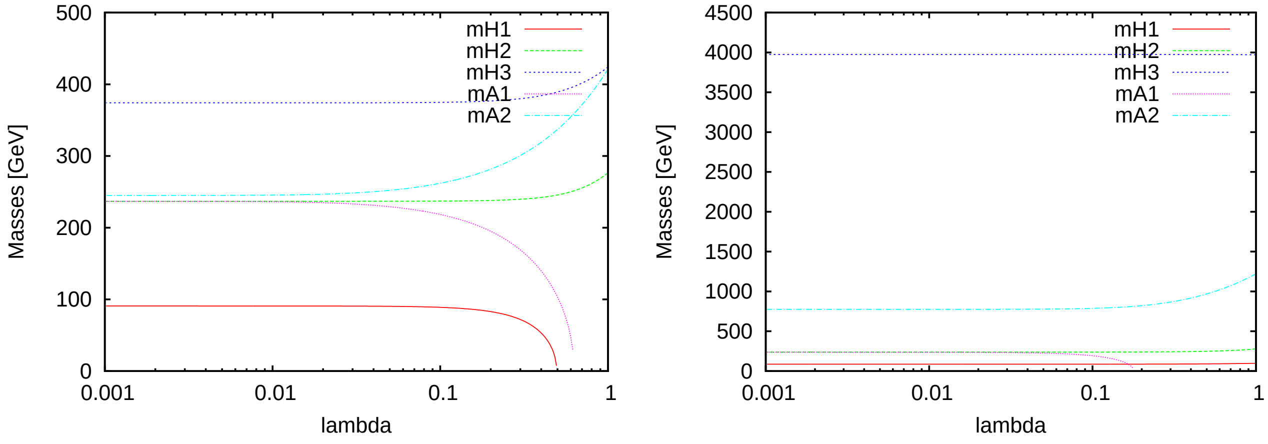
(a) Masses (b) Couplings and $\cos\theta_A$

Figure 9: How the masses, $\cos\theta_A$ and H_iVV couplings vary when approaching the MSSM limit, using $k = 0.2, \tan\beta = 2, m_{H^\pm} = 250$ GeV and $A_\kappa = -100$ GeV.

what happens as we can see in figure 10, where a larger k value is seen to push the heaviest scalar, the singlet dominated one, far up in mass, whereas the mass of the heavy pseudoscalar is also increased but not at all as much.

Another possible way of approaching the MSSM limit is to also send the A_κ parameter to zero, which effectively further suppress the S^3 term in the Lagrangian and thus restores the PQ-symmetry. And as we can see in figure 11, in this case the lightest pseudoscalar Higgs become massless in the limit, restoring the massless PQ axion. In this limit, we also see that the pseudoscalar mixing disappears ($\cos\theta_A \to 1$) and that the lightest scalar Higgs again is fully standard model like.

One could also imagine letting v_s go to zero smoothly, but in order to keep μ in an acceptable range this would mean that λ would have to become larger than allowed by the requirement of perturbation theory being valid up to the GUT-scale, i.e. $\lambda \lesssim 0.7$ as stated before. A small value of μ is excluded from bounds on the chargino masses[1] from



(a) $k = 2$

(b) $k = 10$

Figure 10: How the masses vary when approaching the MSSM limit for two different values of k. The rest of the parameters have the same value as those in figure 8.

direct searches. As said towards the end of section 4.3, we can give up the perturbative requirement and consider $\lambda \gg 1$ as in the λ-SUSY model [26], in which case this limit could be viable.

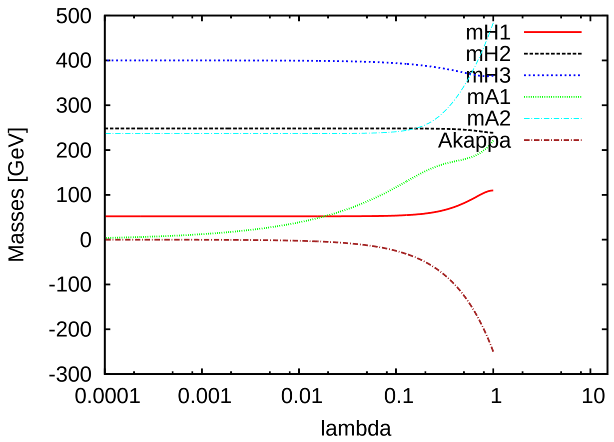
7 Summary and conclusions

In this study, we have briefly reviewed the motivation behind studying supersymmetry, followed by a short introduction to the very basics of supersymmetry in the superspace and superfields formulation. Then the MSSM was described, and its simplest extension, the NMSSM was introduced as a solution to the μ-problem. Some technical details of the Higgs sector of NMSSM was stated, including the mass matrices and definition of the reduced couplings. This was followed by some numerical studies of different choices of parameters, including a look at approaching the MSSM limit in some different ways.

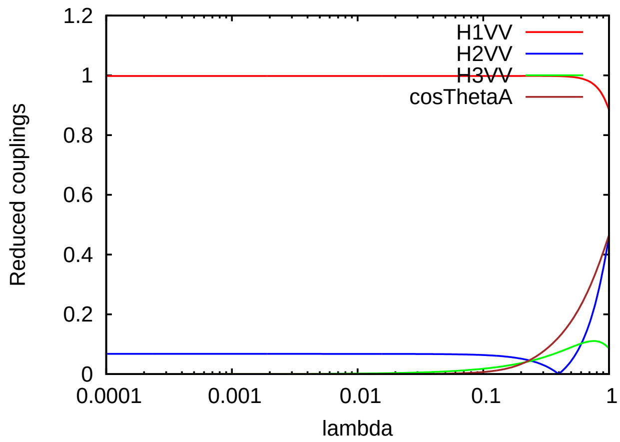
We find that in the by renormalization group flow favoured choice of κ, $\tan\beta$ and λ, the mass spectra with three different relatively light Higgs bosons should make the theory easy to distinguish from the MSSM even when not all the Higgses are detected. But in other perfectly allowed cases, the distinction might not be directly obvious. We also see that for this case some of the couplings to SM-particles pass through 0, so that it is possible that for example the H_2 state can be hidden and not interact in a standard model like way at all.

For a larger κ value, i.e. a more strongly broken PQ-symmetry, we see that the couplings behave in a qualitatively different way. In this case the switching behaviour takes place inside the physically sensible area, but the couplings of the H_1 state doesn't show any complicated dependence on $m_{H^\pm}$. Over the whole range, H_1 is the essentially fully standard model like.

If we want to study the limits where the singlet decouple, from looking at what happens when only one of κ or λ are sent to zero, we conclude that sensible limits exists only when both of them are decreased simultaneously. When we approach this MSSM limit, keeping the ratio κ/λ constant, the mixing with the singlet field disappears. Mostly this decoupling happens rather quickly, but in some cases, when $\kappa/\lambda \lesssim 0.5$, the mixing of the scalar states



(a) Masses



(b) H_iVV couplings and $\cos\theta_A$

Figure 11: How the masses, couplings and $\cos\theta_A$ vary when λ goes to zero while keeping $k = \kappa/\lambda$ fixed, in addition to also sending A_κ to zero. In this plot, $k = 1, \tan\beta = 2, m_{H^\pm} = 250$ GeV. A_κ begins on a value of -250 GeV, and is sent to zero in the same manner as λ and κ (i.e. we keep the ratio κ/A_κ constant).

depends strongly on the nonzero λ. We also see that in this case, the lightest Higgs is no longer necessarily the most standard model like, that role being taken by H_2. But for $\lambda > 0$, both H_1 and H_3 are slightly standard model like, so if this is the case (i.e. we have a small λ and a ratio as described), we could detect three relatively light Higgs bosons with different masses. However, these cases seem to depend very much on the ratio κ/λ having a specific value, and also seem to give the lightest states too low masses for it to be realistic, but since I have not scanned all of parameter space and in addition I am only doing tree level calculations, this kind of scenario cannot be altogether ruled out.

So we see from all this that the NMSSM model offers many interesting possibilities not seen in the MSSM.

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