Control of a High Voltage Power Supply for a Traveling Tube Transmitter

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Title and subtitle
Control of a high voltage supply for a traveling wave tube transmitter

Abstract

The goal of the master thesis is to upgrade the control of a power supply for a radar system. A controller based on Kalman filtering and state feedback has been developed for this two inputs two outputs system.

Key words

Classification system and/or index terms (if any)

Supplementary bibliographical information

Control of a high voltage Power supply for a Traveling Wave Tube Transmitter

Rapport de projet de fin d'études

Abstract

Key words

Multi-input Multi-output system, Linear quadratic regulator, Optimum observer, Uncertainties on models, Disturbances, Implementation.

The goal of the Master Thesis is to upgrade the control of a power supply for a radar system. An observer-controller has been developed for this 2 inputs 2 outputs system.

With the simulations which finally fit specifications and a satisfactory robustness analysis, it seems reasonable to consider an implementation of the controller on the real system.

Abstract in French / Résumé

Mots clés

Système Multi-entrées Multi-sorties, Régulateur Linéaire quadratique, Observateur optimal, Incertitude de modèle, Perturbations, Mise en œuvre.

Ce projet de fin d'étude a pour but d'améliorer le contrôle d'une alimentation pour un système Radar. Une commande du type observateur-contrôleur a été développée sur un système 2 entrées 2 sorties.

Avec des simulations qui atteignent finalement les spécifications et une analyse de robustesse satisfaisante, il semble envisageable de procéder à une mise en œuvre du contrôleur sur le système réel.

Supervisors

Bo Bernhardsson and Tore Hägglund (Automatic Control Department, Lund Institute of Technology)
Leif Carlsson (Ericsson Microwave Systems FX/NT in Mölndal, Sweden)
Smail Tedjini (ESISAR / INPG Grenoble)

Preface

Studying or working abroad is a wonderful and enriching experience. You are in a new environment, with a language and cultural challenge and you learn how to deal with it.

Sweden is one of the leading countries in high technologies. It has numerous, large or small, companies dedicated to the new technology, Internet, defense electronics, electrical engineering and so forth. Although most Swedes can speak English, especially at University or in companies, you find yourself surrounded by a completely new culture and a new language, which is quite exciting.

This master thesis is the second part of my final year of engineering that I decided to accomplish in Lund (Sweden) as an exchange student from ESISAR/INPG Grenoble (France) in order to specialize myself in Automatic Control.

This work will be accomplished in the department of Automatic Control of Lund, world famous by the presence of Karl Johan Åström in its ranks, for Ericsson Microwave Systems in Mölndal near Gothenburg, part of one of the major Swedish company in engineering.

Acknowledgments

First, of course, I would like to thank my supervisors Bo Bernhardsson and Tore Hägglund, for the automatic control department in Lund, and Leif Carlsson for Ericsson Microwave Systems FX/NT in Mölndal. They continuously provided me with help and valuable comments.

Personally, I learned a lot on this project about topics such as optimal control and robust control. It was also a pleasure to work for EMW inside the Automatic Control department in Lund, sign of a good partnership.

I would like also to thank Karl Johan Åström to have arranged the project with EMW. Everybody at the departments both in Lund and in Mölndal has been most helpful and friendly.

I also thank Smail Tedjini my supervisor for ESISAR / INP Grenoble for permitting me to do this year abroad. Didier Georges lured me into the field of Automatic Control and encouraged me to come in Lund for my final year.

Finally, I would like to thank my family and all my friends for their encouragement and their support.

Franck

Contents

Chapter 1

Introduction

1.1 The subject of the Master Thesis

The Goal of the master thesis is to design a regulator to improve the control of the cathode voltage power supply for the Traveling Wave Tube. The Traveling Wave Tube (TWT) is used in the radar system to amplify the radar wave.

A radar system needs a high voltage power supply to be able to amplify these waves to the right power.

The problem is that the quality of the radar system depends on the quality of the wave emitted. The radar system analyzes the signal from the echo of the emitted wave reflected on the target. From this analysis, the radar system can determine where the target is positioned and in which direction and speed the target is moving. The target could be an aircraft or a missile.

Improving the stability of the cathode voltage of the TWT will improve the performance of the radar system.
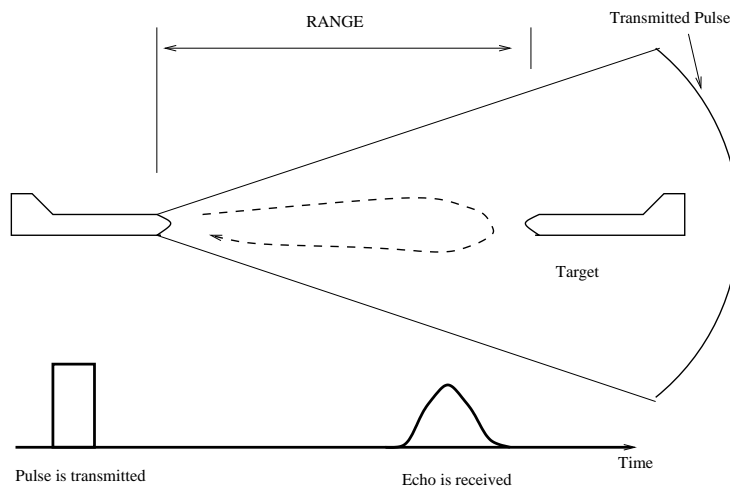
Figure 1.1: Usually, a target's range may be most easily determined by measuring the time between transmission of a pulse and reception of its echo.

The system will take place for example in a weapon locating system such as ARTHUR described further.

The possibility to improve this system with the Department of Automatic Control of Lund came up during the Swedish control meeting of 1998 that took place in Lund. Karl Johan Åström got interested in this problem and after a meeting with Ericsson Microwave Systems (EMW) in 1999, it was decided to involve a Master Thesis student to study the problem.

For EMW, this Master thesis is a way to put some advanced control design in the system and also a way to learn more about automatic control theories and the Matlab/Simulink software.

The work has been executed in Lund and in Mölndal near Gothenburg.

1.2   About Ericsson Microwave Systems AB (EMW)

Specialist in Microwave Technology

Ericsson Microwave Systems is Ericsson's core company for microwave communications and defense electronics, as well as the Research and Development center for microwave technology. It brings together the main specialist skills - high-frequency technology, signal processing and high-speed electronics - in products and systems within the defense electronics and the telecom areas. The company is divided into three business units; Microwave Radio, Base Stations, Defense Electronics and a unit for Core Technologies. Ericsson Microwave Systems is a wholly owned subsidiary of Telefonaktiebolaget LM Ericsson (100%). The head office is located in Mölndal outside Gothenburg, Sweden.

Full Advantage of Technical Interaction

Ericsson Microwave Systems is the world's leading supplier of microwave radio links for telecom networks and a leading supplier of advanced antenna systems and radio base stations for most mobile telephony standards. The defense electronics products are intended to provide information superiority for airborne, ground-based and naval sensor systems as well as customer-adapted tactical communications systems. Microwave technology and sophisticated signal processing are vital in defense electronics and more and more important in communications systems. Ericsson Microwave Systems takes full advantage of technical interaction in the development of telecom and defense products.

Production

Production is primarily performed at two locations - Borås and Mölndal. The main production of defense electronics is in Mölndal. The Borås factory, with over 1,300 employees, has been designated as the most efficient Ericsson plant in comparison with all Ericsson plants worldwide. Both plants are characterized by organizations with considerable flexibility and with goal-oriented groups in each production chain. In these goal-oriented groups, work with quality and improvement is performed continuously.

Geographic Locations

In Sweden Ericsson Microwave Systems operates in Borås, Lysekil, Kista, Skövde and Mölndal (head office). Ericsson Microwave Systems AB has a subsidiary in Norway, Ericsson Radar A.S (60%). Associated companies are Ericsson Saab Avionics AB (49.9%) and Saab Ericsson Space AB (40%).

Facts and Figures

Ericsson Microwave Systems invoicing amounted to 7.3 billion SEK in 1999. The number of employees was 4000 at the end of 1999. The average age is 37 years. 23% are female and 77% are male. Almost two thirds are engineers or technicians, and the majority of these are graduates, with over 70 qualified to PhD level. Ericsson Microwave Systems' accumulated expenses for research and development, including certain customer order related costs, totaled 591 million SEK in 1999.

1.3 Ericsson Radar AS - Radars Systems

Ericsson Radar develops and produces radar systems for defense programs all over the world. Ericsson Radar is a member of the Ericsson Group, a world supplier in telecommunications and electronic defense systems. In particular, Ericsson Radar supplies sensors and command systems for both army and naval use in artillery and air defense applications, as well as electronic warfare (EW) systems. These compact radar systems can be fitted in small, lightweight vehicles to produce highly mobile and rugged military equipment for operation in extremely harsh environments. Ericsson Radar offers a wide variety of services and works closely with the customer to ensure a fully integrated, functional and cost-effective system solution.

ARTHUR Weapon Locating System mounted on a Hägglund BV-206 tracked vehicle. ARTHUR is the world's first truly mobile stand-alone weapon locating and artillery fire control radar.

Weapon Locating and Fire Control Radar

The Artillery Hunting Radar ARTHUR is Ericsson Radar's main product. It is the world's first truly mobile, stand-alone weapon locating and artillery fire control radar. The design is very flexible and the equipment may be fitted in a wide variety of vehicles. The Norwegian and Swedish armies mount ARTHUR on a Hägglund BV 206 tracked vehicle. For the Danish army ARTHUR is installed in a 13' ISO corner framed container to be mounted on a 4x4 wheeled off-road truck. Both solutions provide high mobility and the ability to be transported by a Hercules C-130 aircraft. ARTHUR is designed to be operated in severe environments, and uses a high-precision phased array antenna with state-of-the-art techniques to suppress clutter and bird echoes. ARTHUR automatically detects shells from mortars, tube- and rocket artillery. The exact weapon position of the artillery is determined from measured ballistic data which is also used to estimate the impact area of both incoming and outgoing shells. Data from the radar is automatically transmitted to a combat center through an integrated command, control, communication and information system, where the information is further evaluated and in hostile situations used to direct counter-battery fire.
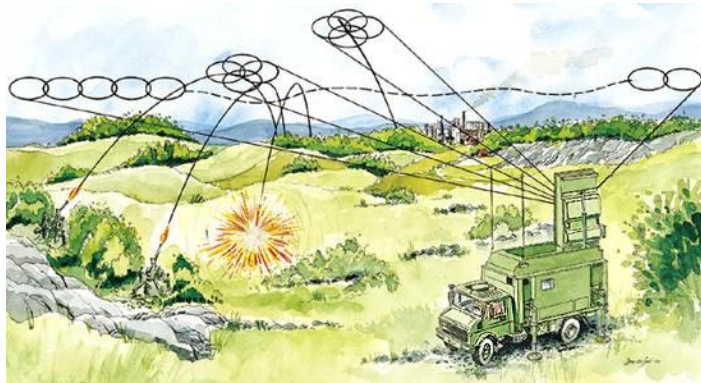
The container version of ARTHUR in position, covering a 90 degree sector in search of rising projectiles for weapon location and impact point estimation and falling projectiles for fire control.

1.4 The Automatic Control Department in Lund Institute of Technology

The department was formed in 1965. It has now about 40 people. There are eight professors, one guest researcher, some twenty PhD students, secretaries and a technical support staff.

The goal of the department is to provide students with a solid theoretical foundation combined with a good engineering ability. This is reflected in the research program which, broadly speaking, is divided into theory and applications.

Technology transfer takes many forms. One is to take results from research and present them so that they are easy to use. Probably the best way to do this is through personal exchange between industry and university. Students are a very effective vehicle for the transfer.

The major research areas are:

- Tuning, adaptation, and robust control

- Computer aided control engineering

- Applications

Projects are typically done together with external partners. Current projects include robotics, real-time systems, supervisory control, fuzzy control, and bio technology.

The department participates also actively in international programs for student exchange. This is facilitated by the fact that all advanced courses are given in English when students who do not speak Swedish participate.

Chapter 2

Description of the system and its control requirement

2.1 Description of the process and the control problem

The Traveling Wave Tube (TWT) gets a disturbance on its cathode voltage every time that it emits a radar pulse. A current is drawn for every pulse. This current is loading the cathode power supply and phase compensator system via output impedances of the system, mainly two capacitors. The result of the loading is a disturbance on the cathode voltage.

The Power Supply (PS) system is composed mainly by two plants called :

- the Cathode Voltage Power Supply

- the Phase Compensator

The control problem is to reject these disturbances as completely as possible in order to keep the same voltage when the next pulse of current, hence the next disturbance, is coming. The arrival of these pulses is supposed unknown and random. The time between two pulses is assumed to be between 200 and 600 microseconds and the length of the current pulse is 10 microseconds and the maximum average duty cycle is 2.5%.

To solve this control problem, we are able to measure :

- a fraction, via a sensor, of the sum of the output of the Cathode Power Supply voltage and the phase compensator voltage that is called **the Cathode Voltage** (the main voltage that we have to care about).

- a fraction, via another sensor, of **the phase compensator voltage**.

The cathode voltage has to be controlled to a nominal value of around -24000 Volts and the phase compensator voltage has to be kept at around 200 Volts.

There are -12V to +12V limitations on the control signals. The phase compensator voltage is also limited between 0 and 400 Volts.

It follows that the system is a MIMO system with two inputs and two outputs with some non-linearities and some very specific performance requierements.

2.2 Background

Ericsson Microwave Systems used a proportional integral controller for a slow
loop to compute the cathode power supply control signal. To help this regulator
reject fast disturbances, they have added a phase compensator in order to have
a better accuracy on the cathode voltage. The phase compensator has quite a
"special design" that I will not go into in details. But the idea is to add this
phase compensator voltage via a capacitor to the cathode voltage to compensate
for the residual slope that will occur.
The idea held by Ericsson Microwave is to control independently both signals,
phase compensator voltage and cathode voltage around a reference, 200V and
-24 000 kV in order to reject the disturbances.
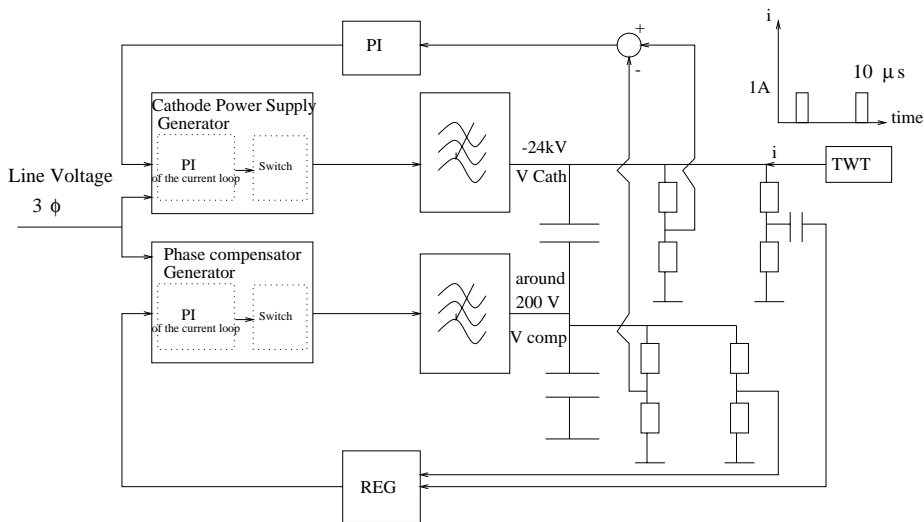
Figure 2.1: A proportional Integrator (PI) controller takes care of the Cathode
Voltage and another controller takes care of the Phase Compensator Voltage.
Two others PI controllers are inside the two plants to control the current.

Ericsson Microwave Systems is quite satisfied with the results obtained by
this design of regulator because the system works quite well. But they want to
improve the design in order to have more freedom on the radar system.

2.3 Power Electronics Description

See Figure 2.3 on page 14.
The power supply system of the cathode voltage for the TWT (Traveling Wave
Tube) is composed of a chain of blocks.
There are 14 different blocks in the block diagram, numbered as follow :

1. **Line Relay**
   The power to the plant is taken from a three phase line via relays.

2. **Six Pulse Rectifier**

   The function of this block is to rectify the three phase voltage to get a DC voltage.

3. **Filter**

   The function of the low pass filters is to smooth the DC voltage attenuating the high frequencies. The cut frequency of this LC-filter is around 140 Hz with L = 3 mH and C = $1100\,\mu$F.

4. **Switch Regulator**

   The switch regulator is commanded by the regulator amplifier in order to control the cathode voltage. The switch regulator operates in current controller Buck configuration. The duty cycle of the Buck switch regulator is determined by a Pulse Width Modulator (PWM) and is depending on the value of the control signal. The frequency of the switch regulator is 80kHz for the Cathode Power Supply system. This frequency will be a limit in the design of the new regulator.
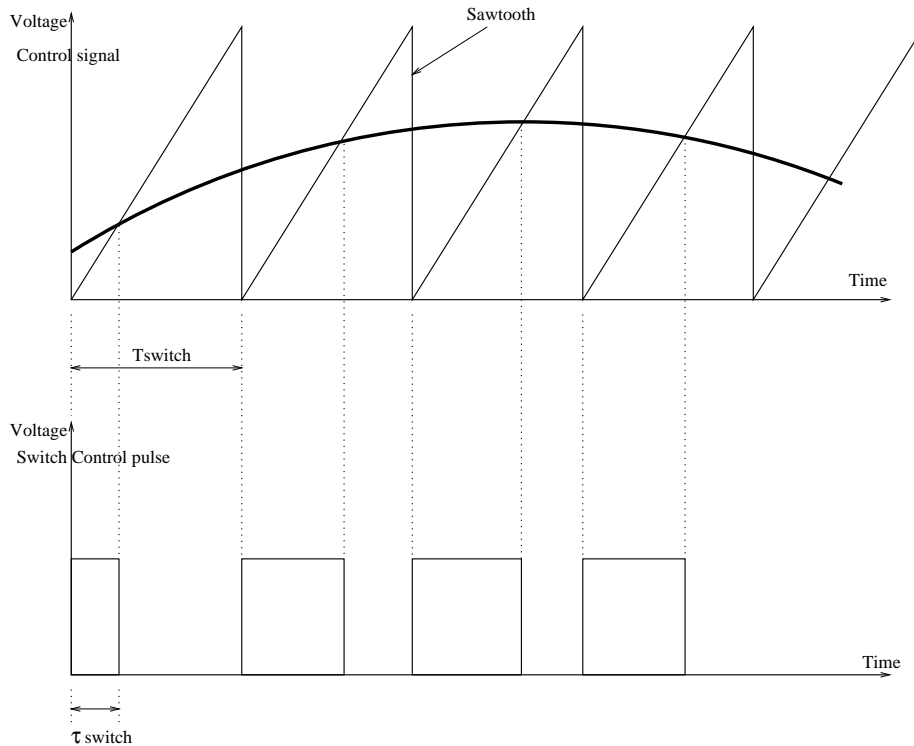
Figure 2.2: The Pulse Width Modulator transforms a DC voltage to a pulse voltage, the duty cycle of which is proportional to the DC voltage. T_{switch} is equal to 12.5 μs.

5. **Current Mode Sensor**

   This sensor enables the PWM to have information of the current. This kind of control loop is internal in the switch regulator and is not to be taken into account in my work.

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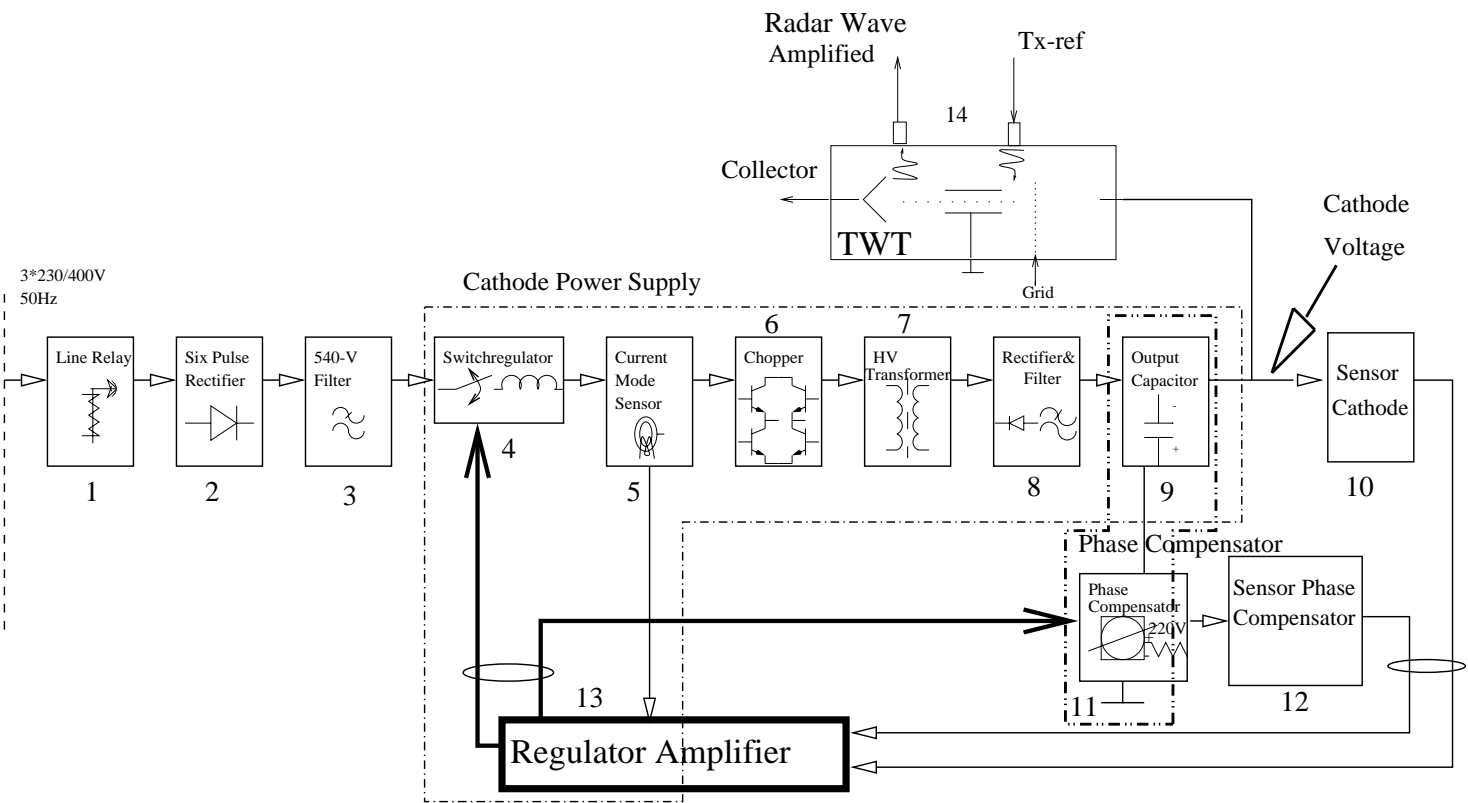
Figure 2.3: The Block diagram of the complete system shows how the control of the cathode voltage is performed from the three phase line via the Cathode Power Supply and the Phase Compensator.

3*230/400V
50Hz

Cathode Power Supply

Radar Wave
Amplified

Tx-ref

14

Collector

TWT

Grid

Cathode
Voltage

Line Relay
1

Six Pulse
Rectifier
2

540-V
Filter
3

Switchregulator
4

Current
Mode
Sensor
5

Chopper
6

HV
Transformer
7

Rectifier&
Filter
8

Output
Capacitor
9

Sensor
Cathode
10

Phase Compensator

Phase
Compensator
220V
11

Sensor Phase
Compensator
12

Regulator Amplifier
13

6. **Chopper**
   The frequency of chopper is 40 kHz. This circuit makes it possible to transform the switch regulator output voltage (DC) to high voltage by the High Voltage Transformer.

7. **High Voltage Transformer** (HV Transformer)
   It transforms the signal from -500 V +500 V to -12 kV +12 kV.

8. **Rectifier and Filter**
   Voltage doublers, rectifiers and filter result in a smooth DC voltage of -24 000 V, the cathode voltage for the TWT.

9. **Output Capacitor**
   The disturbances of the current pulses coming from the TWT are loading this capacitor and thus create a voltage disturbance that the regulator has to reject.

10. **Sensor of the Cathode Power Supply**
    This sensor is a voltage divider. It is necessary to measure a sample of the cathode voltage.

11. **Phase Compensator**
    The output voltage of the Phase compensator is limited between 0 and 400V. It should be controlled around 200 V in order to be able to reject disturbances. Roughly, the fast disturbances will be rejected by this system.

12. **Sensor of the Phase Compensator**
    The sensor has almost no dynamics and is also a voltage divider. Through this sensor we are able to measure the Phase Compensator voltage.

13. **Regulator Amplifier**
    The new designed regulator is a part of the existing regulator amplifier. The purpose of this work is not to redesign completely this amplifier regulator, only the control part.

14. **Traveling Wave Tube (TWT)**
    The TWT is a pulsed microwave amplifier. In order to maintain a good quality of the microwave signal, the tube requires a very stable and smooth power, the cathode voltage.
    A current is drawn in the TWT from the power supply during the microwave pulses. This output current creates the disturbances that the regulator needs to be able to reject as fast as possible.

2.4   System requirement and control concept

To improve the performance of the radar system in general, the power supply of the amplifier system of the radar waves has to be as stable as possible. The most critical system is the cathode voltage.
The main disturbances that we note in this cathode voltage is the one coming from the variation of the output current due to the pulsing. In a first approximation, we can consider this variation of the output current as pulses of 10

Current pulses

1 A

τ 10μ s

T as small as possible (around 200 microseconds)

Ecath

80V

-24 000V

Δ E cath

Cathode Voltage difference between two pulses
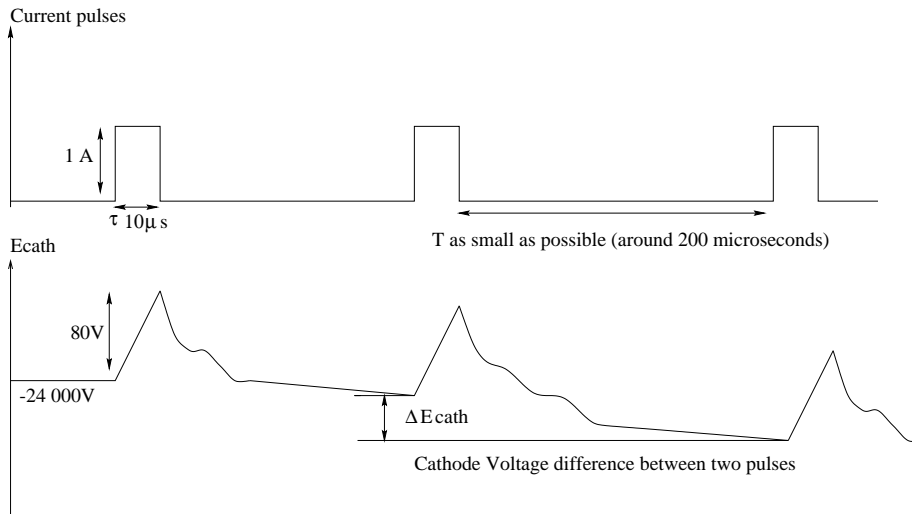
Figure 2.4: The control problem is to design a controller in order to particularly reject this kind of disturbances with a very good accurancy before the next pulse.

microseconds. But through the output impedance of the phase compensator and the cathode power supply, these disturbances add a voltage on the Cathode Power Supply voltage and on the Phase Compensator Voltage.
So the problem is to reject the perturbation as fast as possible in order to control the cathode voltage at -24 000 Volts with the best possible accuracy.

The automatic control study will be done in continuous-time.

It is most likely that the future implementation of the controller will be in continuous-time because of the cost of the analog-digital converter at the frequency required and the cost of development of a computer controlled system.

2.5   Performance - Robustness Specification

I do not specify any details about the accuracy for instance because of confidentiality. But the performance of the new regulator depends on the ability to restore the cathode voltage E_{cath} after each pulse disturbance. For the case of this work, an RMS average will be computed on 10 pulses after the steady state has been reached. This average should be inferior to a limit that remains confidential, so that the implementation of the new controller can be considered by EMW.
It is also required to be robust to slight variations of the plant because of the uncertainties in the model and to variations of the controller because of the approximation that will be made during its implementation. A robustness analysis should be performed with the appropriate tools needed for a multivariable or MIMO (Multi Input Multi Output) system in order to conclude on the possi-

Time to next pulse
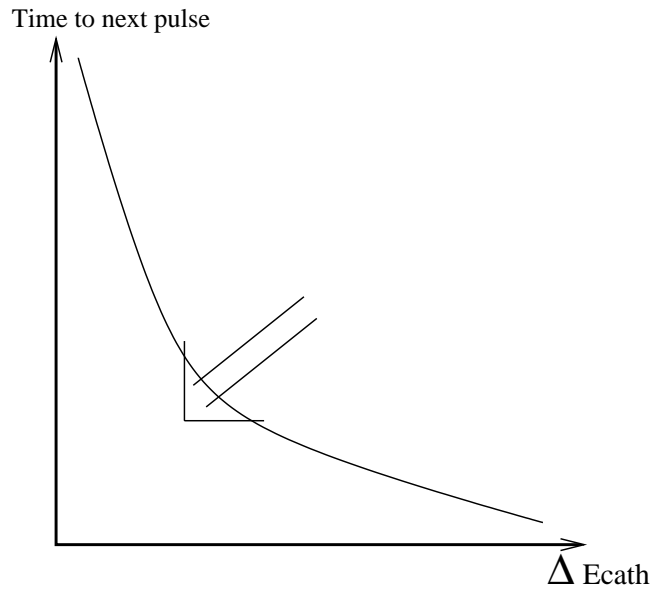
Δ Ecath

Figure 2.5: The task is to try to decrease ΔE_{cath} to the next pulse to be able to use as short time between pulses as possible.

bilities of implementation of this controller.

2.6 Design Process

Extracts from Control Design System Bernard Friedland pages 326-327.
"At this point, it is appropriate to view the system design procedure that will be developed in the next chapters. In order to determine the control and the observer gains a mathematical model of the plant must be developed. And even before that is done, an overall system concept must be developed in which it is decided by what means the control is to be accomplished and what variables are to be measured. At about this time it is also necessary to determine performance specifications: characteristics of the desired system behavior and the quality of components available that might be able to achieve the desired performance.

The design steps, moreover, do not end with the specification of the compensator. In almost every case the process is nonlinear; there are usually limits on the control signals; the process may be of higher order than that used in the model. The only way that these issues can be solved is by a very thorough simulation of all the effects that the system designer thinks might conceivably influence the system behavior. If the results of a comprehensive simulation are favorable, the system would finally be fabricated and tested. If the steps leading to this last step are done skillfully, and if fortune smiles, the system will work as expected."
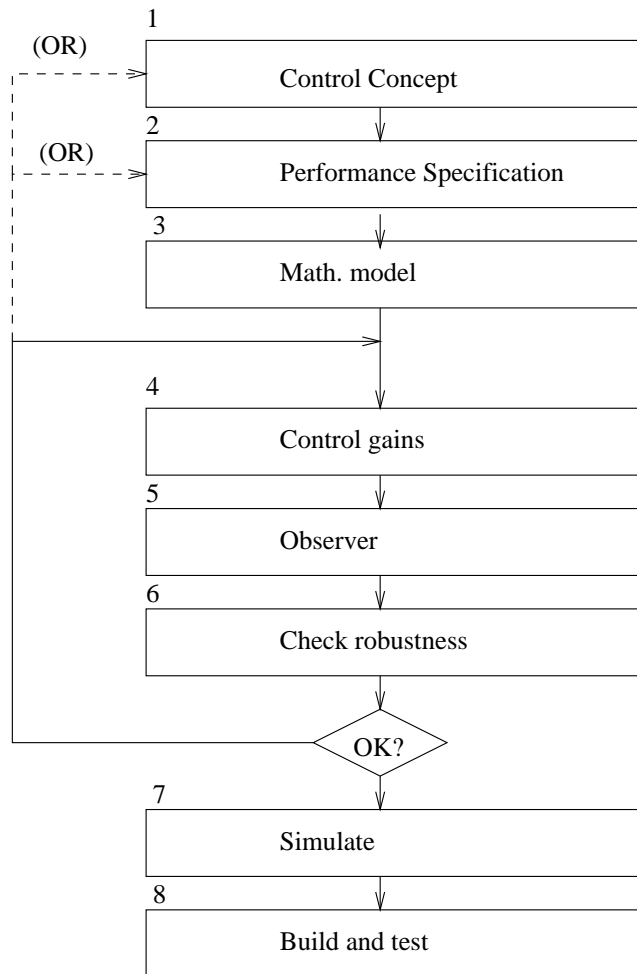
Figure 2.6: The steps sequence in control system design shows that it is necessary to have a mathematical model before searching the optimum control and observer gains that meet the specification.

Chapter 3

Modeling of the complete system

The modeling of the system will be conservative in order to be sure that the results can correspond to the real process.

All the modeling has been made from the Bode diagrams obtained from the software used by Ericsson Microwave Systems to study the behavior of the power electronics components. Its name is Simplis and it is running in a Unix environment.

From the Bode diagrams of the Cathode Voltage Power Supply and the Phase Compensator, the models of the plant and the one used in the observers will be deduced.

From a comparison of the results of Impedance Output frequency response, a model of the output disturbances added on the Cathode Voltage will be performed.

A model of the sensor will be computed directly from the schematics of the system.

All these tasks will permit to construct the Simulink's block diagrams in order to be able to simulate our system, to design a controller, and finally to control the system and try to fit the specifications.

3.1 Precisions about the numerical values of the transfer function

All the numerical values given in this report are in the international measurement system. But during the simulation, because of some numerical problems, the scaling of time used was different.

For instance, all the transfer functions were rescaled. The Laplace operator s becomes s'.

With
$$s' = 10^{-5} * s$$

The consequence is that the time unit of the simulation becomes $10\,\mu$s. This is not very convenient but it is the only way we were able to perform simulation.

3.2 Modeling of the plant

In this section, the presentation of an extended model and a reduced model will be made.

The extended model tries to fit with the best possible accuracy the Bode diagram that Ericsson's simulation software gives. This model will permit to simulate the plant in Matlab / Simulink.

The plant is composed of two parts : the Cathode Power Supply and the Phase compensator.

In this section, the model of Cathode Power Supply will be called model Cath and the model of the Phase Compensator model Comp.
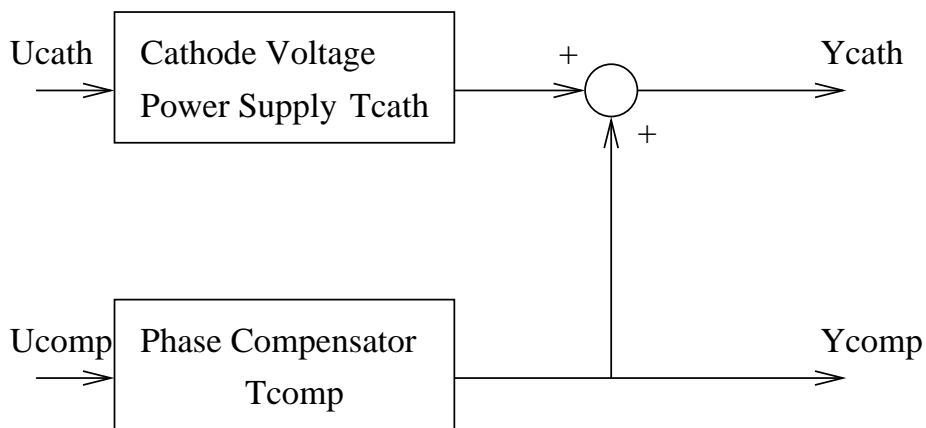
Figure 3.1: The total plant is constructed by adding the output of the two plants to get Y_{cath}, the cathode voltage, and by keeping as output the Phase Compensator Voltage Y_{comp}. U_{cath} and U_{Comp} are the control signal of respectively the Cathode Voltage Power Supply and the Phase Compensator.

3.2.1 Models of Cathode Power Supply

Data from the EMW computer simulation software

The data that we got from Ericsson goes from 0.1Hz to 1MHz.
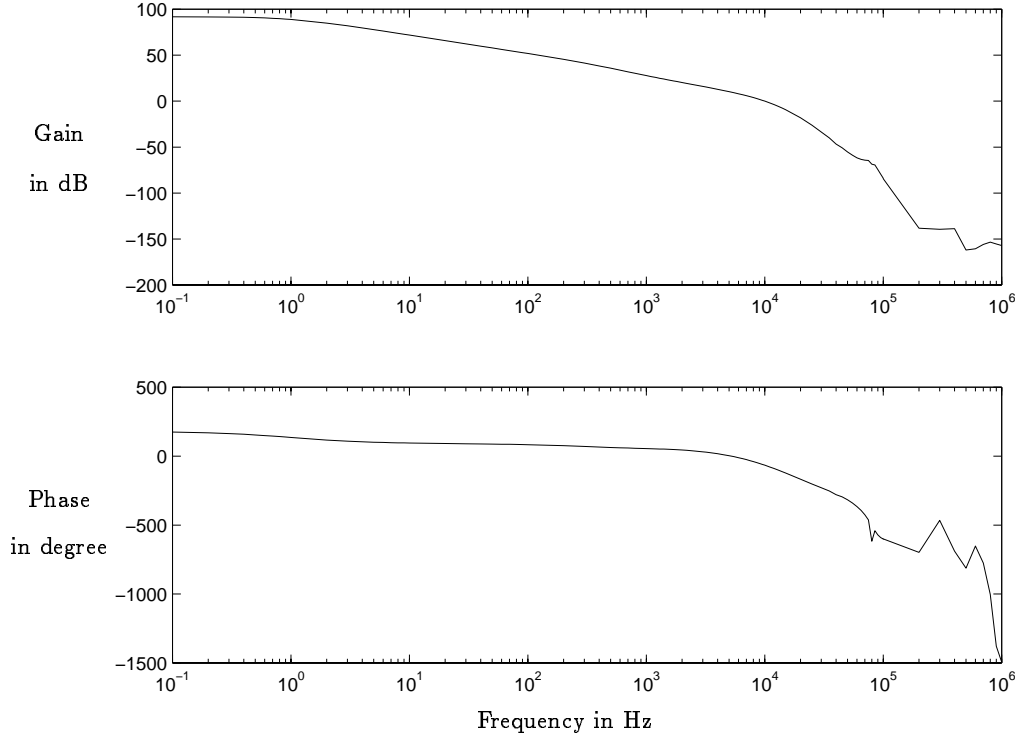
Figure 3.2: The line shows the Bode diagram of the Cathode Power Supply given by EMW. Above 80 000 Hz, the data are reasonably not reliable.

The clock frequency of the Pulse Width Modulator of the Cathode Power Supply is 80 kHz so the data are assumed to be reasonably reliable up to 80 kHz.

Complete Model

The data that we got from Ericsson permit to draw a good model up to 80 000 Hz. Beyond 80 000 Hz, the system becomes more and more nonlinear because of the Pulse Width Modulator so it is not possible to do a linear model.
By assumption on the comportment of the plant, a 4th order Butterworth filter will be added at high frequency on our model. A Pulse Width Modulator above its clock frequency works quite randomly and the delay is also increasing a lot. Also, it does not make too much sense to model a part of the system that we can not use to control the plant. But the problem is to be sure not to use this frequency domain in the simulation to control the system.

$$\mathbf{T_{Cath}} = \frac{G_{0_{Cath}}(s^2 - 2as + a^2 + b^2)(s + 2\pi f_3)(s + 2\pi f_6)(s + 2\pi f_7)}{(s^2 + 2as + a^2 + b^2)(s + 2\pi f_1)(s + 2\pi f_2)^4(s + 2\pi f_4)(s + 2\pi f_5)}B_{filter}$$

21

With

$$G_{0_{Cath}} = -9.1887 * 10^{18}$$

$$B_{filter} = \frac{\omega^4}{(s^2 + 2\zeta_1 \omega s + \omega^2)(s^2 + 2\zeta_2 \omega s + \omega^2)} \quad \omega = 2\pi * 80000 \text{rad}.s^{-1} \quad \zeta_1 = 0.38 \quad \zeta_2 = 0.92$$

$$a = 2\pi * 21600 \text{rad}.s^{-1} \qquad b = 2\pi * 100320 \text{rad}.s^{-1}$$

$$f_1 = 1 \text{Hz} \qquad f_2 = 15500 \text{Hz} \qquad f_3 = 840 \text{Hz}$$

$$f_4 = 440 \text{Hz} \qquad f_5 = 20000 \text{Hz} \qquad f_6 = 50000 \text{Hz} \qquad f_7 = 80000 \text{Hz}$$

Figure 3.3: The crosses show the Bode diagram of the model used to simulate the Cathode Power Supply plant in the Matlab / Simulink simulation. There is some uncertainty on this model above 70 kHz. The two curves can not be superposed anymore.

3.2.2 Models of Phase Compensator

Data from the Ericsson MW computer's simulation software

The data that we got from Ericsson goes from 0.1 Hz to 100 kHz. The simulation software failed to compute for higher frequencies.
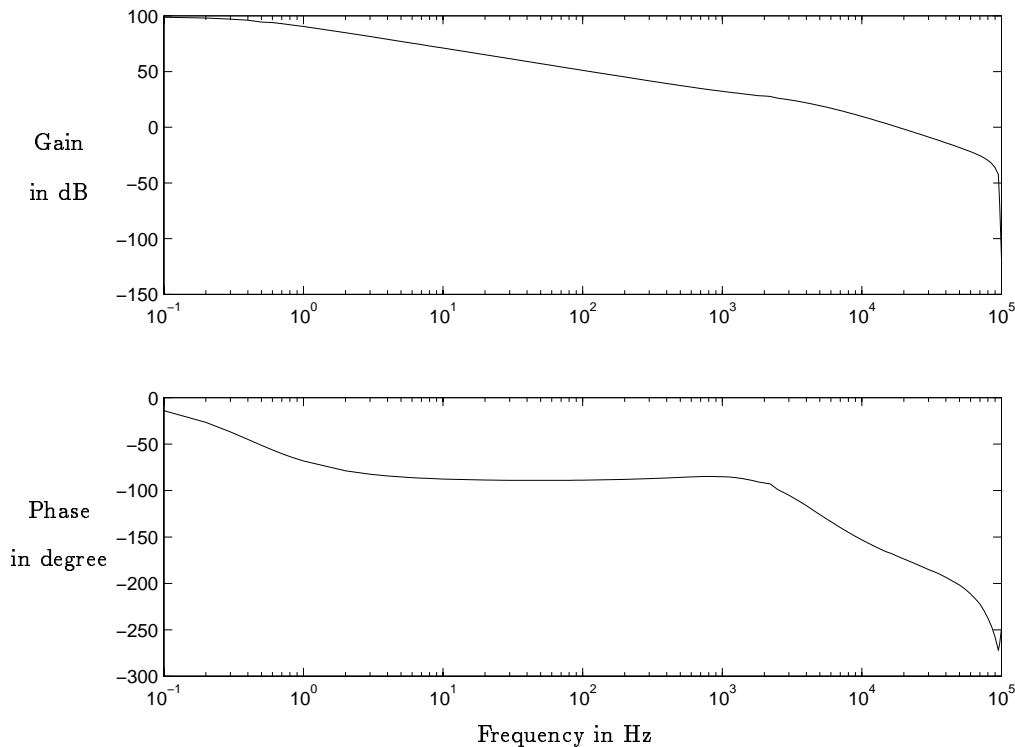
Figure 3.4: The line shows the Bode diagram of the Phase Compensator given by EMW. Above 100 000 Hz, there is no data.

The result is reliable up 100 kHz since the clock frequency of the Pulse Width Modulator of the Phase Compensator is 100 kHz.

Complete Model

The data that we got from Ericsson permit to draw a good model up to 100 000 Hz. After 100 000 Hz, the system becomes more and more nonlinear because of the Pulse Width Modulator so that it is not possible to do a linear model.
By assumption on the comportment of the plant, a 4th order Butterworth filter will be added at this frequency on our model for the same reason as for the Cathode Power Supply.

$$\mathbf{T_{Comp}} = G_{0_{Comp}} \frac{(s + 2\pi f_{10})}{(s + 2\pi f_{11})(s + 2\pi f_{12})(s + 2\pi f_{13})(s + 2\pi f_{14})} B_{filter}$$

23

With

$$G_{0_{Comp}} = 9.9176 * 10^{15}$$

$$B_{filter} = \frac{\omega^4}{(s^2 + 2\zeta_1 \omega s + \omega^2)(s^2 + 2\zeta_2 \omega s + \omega^2)} \quad \omega = 2\pi * 85000 \text{rad}.s^{-1} \quad \zeta_1 = 0.38 \quad \zeta_2 = 0.92$$

$$f_{10} = 1.1 * 10^3 \text{Hz} \qquad f_{11} = 2.1 * 10^3 \text{Hz} \qquad f_{12} = 3.9 * 10^{-1} \text{Hz}$$

$$f_{13} = 5 * 10^3 \text{Hz} \qquad f_{14} = 1.2 * 10^5 \text{Hz}$$

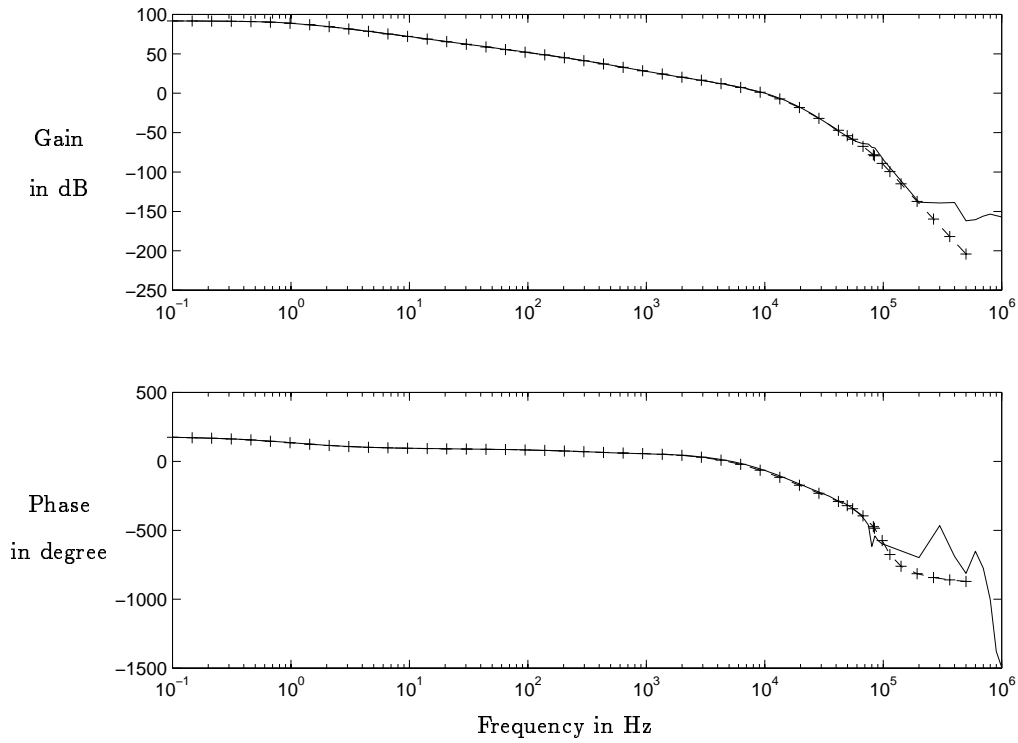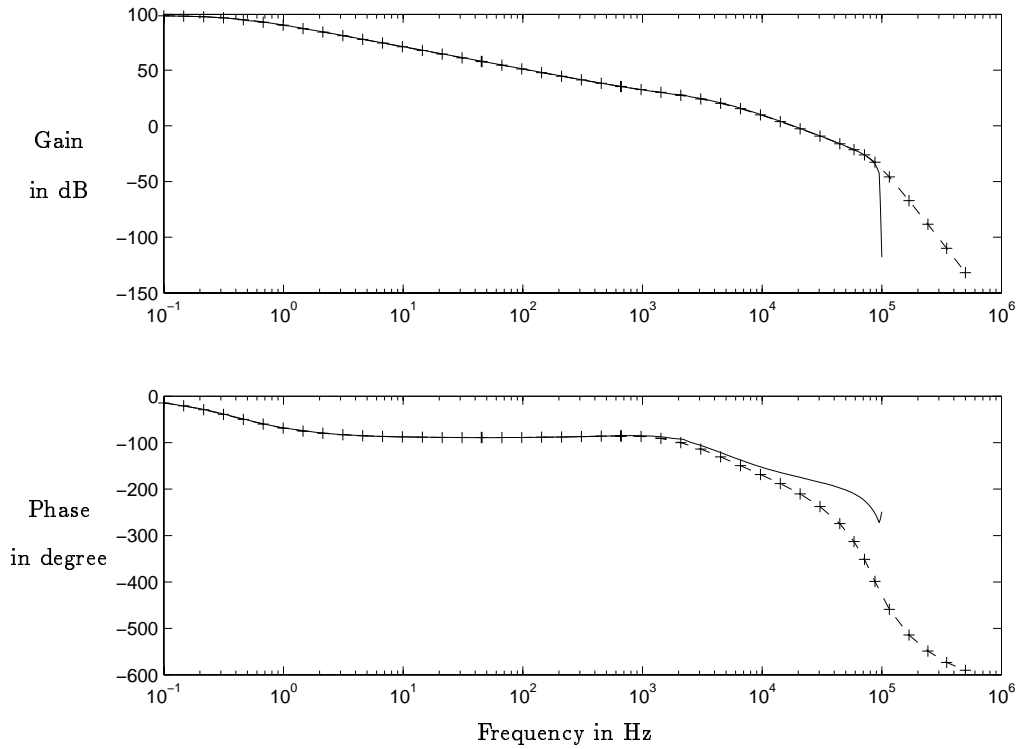Figure 3.5: The crosses show the Bode diagram of the model used to simulate the Phase Compensator plant in the Matlab / Simulink simulation. There is some uncertainty on this model above 1 kHz. The two curves of phase can not be superposed anymore.

3.2.3 Nonlinearities and uncertainties

Nonlinearities taken into account during the simulation

Each control signal has limitations before acting on the plant. The limitations are -12V +12V.
On the output of the Phase Compensator, there is also a limitation of 0 V to 400 V.
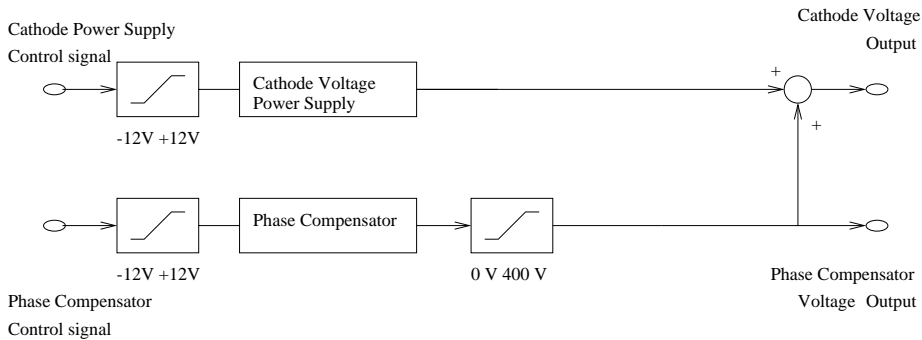
Figure 3.6: The construction of the plant with the identified nonlinearities : the control signal limitations and the saturation on the output of the Phase Compensator.

Nonlinearities that are not completly taken into account during the simulation

Beyond 100 000 Hz, the Phase Compensator system becomes more and more nonlinear because of the Pulse Width Modulator. The same thing happens after 80 000 Hz for the Cathode Power Supply. These uncertainties are known but it is very difficult to completely take care of them during the simulation. The model of the plants includes a Butterworth filter that permits to simulate the fact that the gain and the phase go down quite quickly.
The models used do not take care of the nonlinarities created by the several feedback loops that are inside the system. For instance, a current is sensed in the system to also adjust the control voltage in the regulator. Only a linear model of the two plants was given by EMW hence the nonlinearities inside these two plants are a bit neglected.
To care of these nonlinearities, a huge and time consuming work could be done to do a model of each "object" of the system. This job was not included in the master thesis.

3.3 Modeling of the disturbances

3.3.1 Theory

Here, we need to model the disturbances that we get on the outputs. With this model, we will be able to simulate the disturbances that we get on the outputs in Matlab/Simulink in order to be able to reject them.
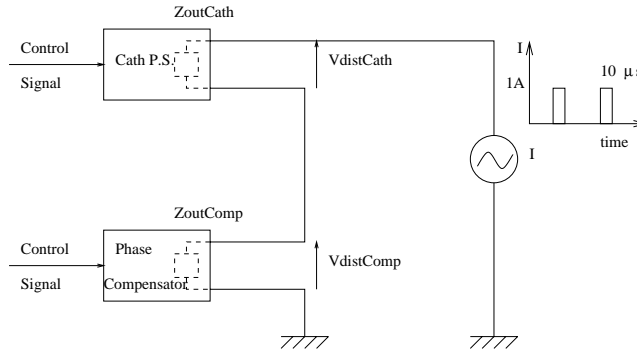
Figure 3.7: The pulses of current adding a disturbance to the outputs of the system.

3.3.2 Electronic Schematic
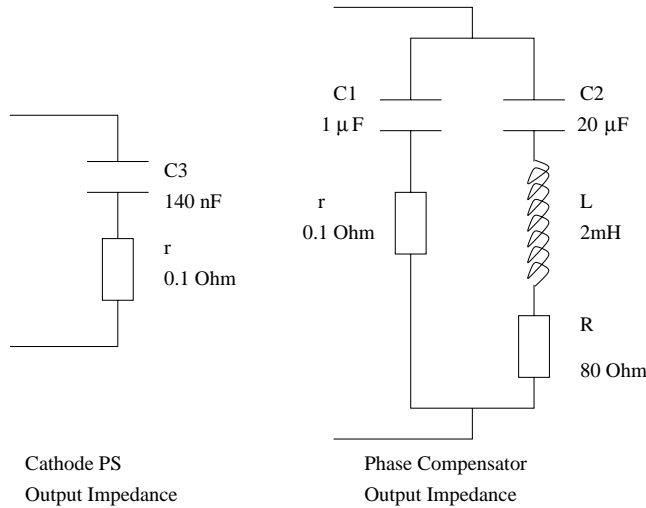
Figure 3.8: On the schematic of output impedances, the Cathode P. S. impedance is almost a capacitor, the Phase Compensator is much more complicated.

The simplification of the output impedance follows some simulations of output impedances that have been done in Ericsson Microwave Systems with the Simplis simulation software.

From the electronic schematic below, we will be able to compute the transfer function.

3.3.3 Computation of the transfer function of the output impedance of the cathode power supply

$$\mathbf{Z_{out_{cath}}}(\omega) = \frac{U_{cath}}{I} = r + \frac{1}{jC_3\omega} \qquad \text{Electrical equation}$$

$$\mathbf{Z_{out_{cath}}}(s) = \frac{U_{cath}(s)}{I(s)} = \frac{rC_3s + 1}{C_3s} \qquad \text{Transfer function}$$

With
$$\begin{cases} C_3 = 140 \quad \text{nF} \\ r = 0.1 \quad \Omega \end{cases}$$

$$\mathbf{Z_{out_{cath}}}(s) = \frac{1.4 * 10^{-8}s + 1}{1.4 * 10^{-7}s}$$

Output impedance versus frequency
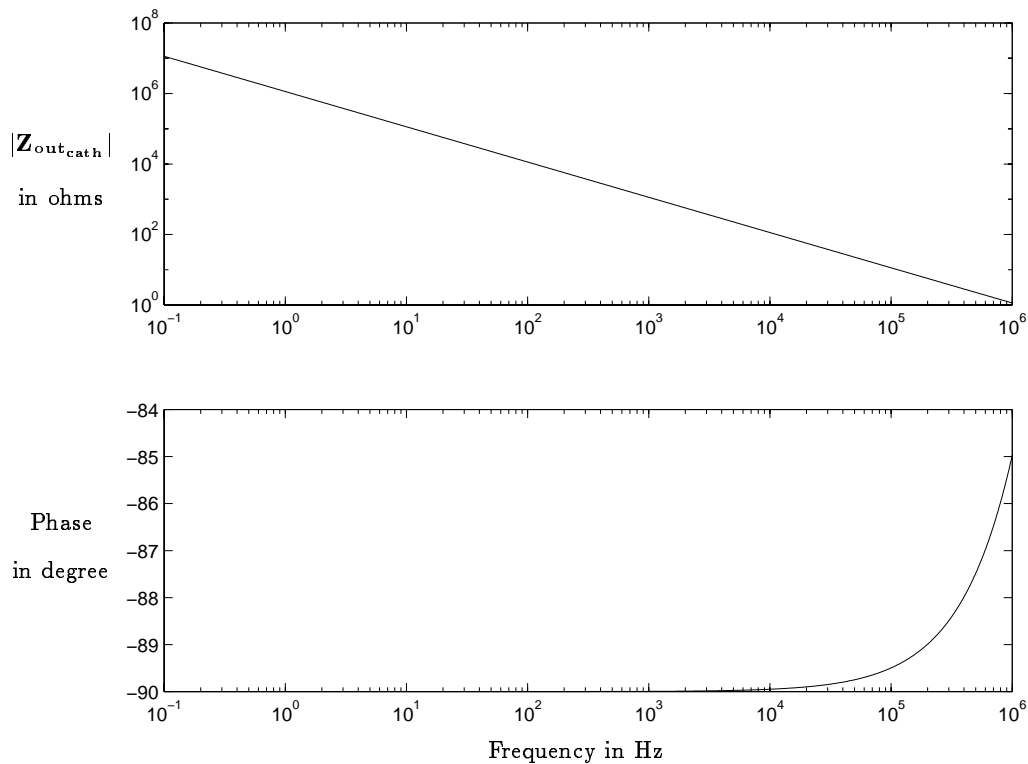
Figure 3.9: The output impedance versus frequency graphic of the cathode power supply shows that the current will be integrated to draw the disturbance.

Disturbances in time domain

The current pulses of 10 μs and 1 A load the output impedances of the system and are alternatively separated by 200 μs and 600 μs so that the duty cycle average is 2.5 %.

I : current
in ampères

Time in seconds
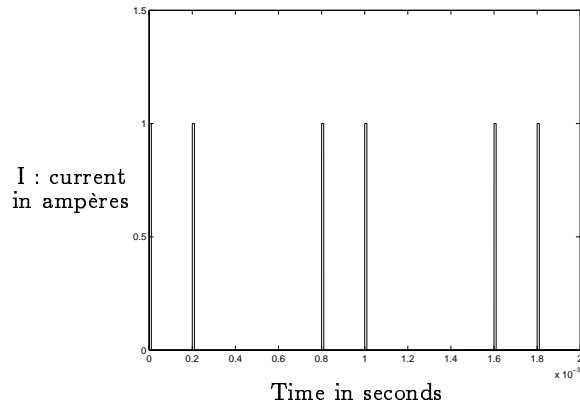
Figure 3.10: The amplitude of current pulses are considering to be of 1 A.

From a current pulses of 10 μs, the voltage disturbance gets the following shape through the output impedance of phase compensator.

$\mathbf{V_{dist_{cath}}}$
in volts

Time in seconds
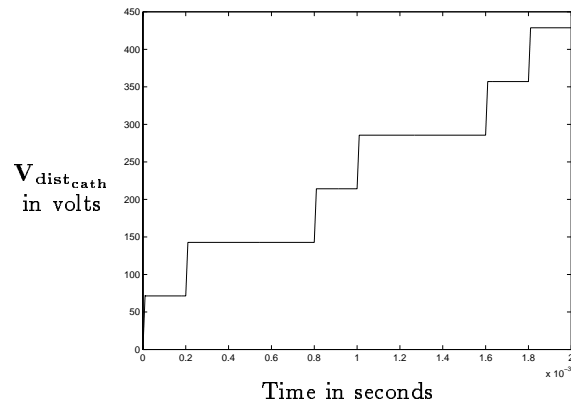
Figure 3.11: The disturbance on the cathode power supply output is almost a load (no control).

3.3.4 Computation of the transfer function of the output impedance of the phase compensator

From the electrical equations below :

$$\mathbf{Z_{out_{comp}}}(\omega) = \frac{U_{comp}}{I} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

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With

$$\begin{cases} Z_1 = r + \frac{1}{jC_1\omega} \\ Z_2 = \frac{1}{jC_2\omega} + jL\omega + R \end{cases}$$

We get the following transfer function :

$$\mathbf{Z_{out_{comp}}(s)} = \frac{U_{comp}(s)}{I(s)} = \frac{rC_3s + 1}{C_3s}$$

With

$$\begin{cases} C_1 = 1 & \mu F \\ r = 0.1 & \Omega \\ L = 2 & mH \\ C_2 = 20 & \mu F \\ R = 80 & \Omega \end{cases}$$

$$\mathbf{Z_{out_{comp}}(s)} = \frac{4 * 10^{-15}s^3 + 4.016 * 10^{-8}s^2 + 0.0016s + 1}{4 * 10^{-14}s^3 + 1.602 * 10^{-9}s^2 + 2.1 * 10^{-5}s}$$

Output impedance versus frequency
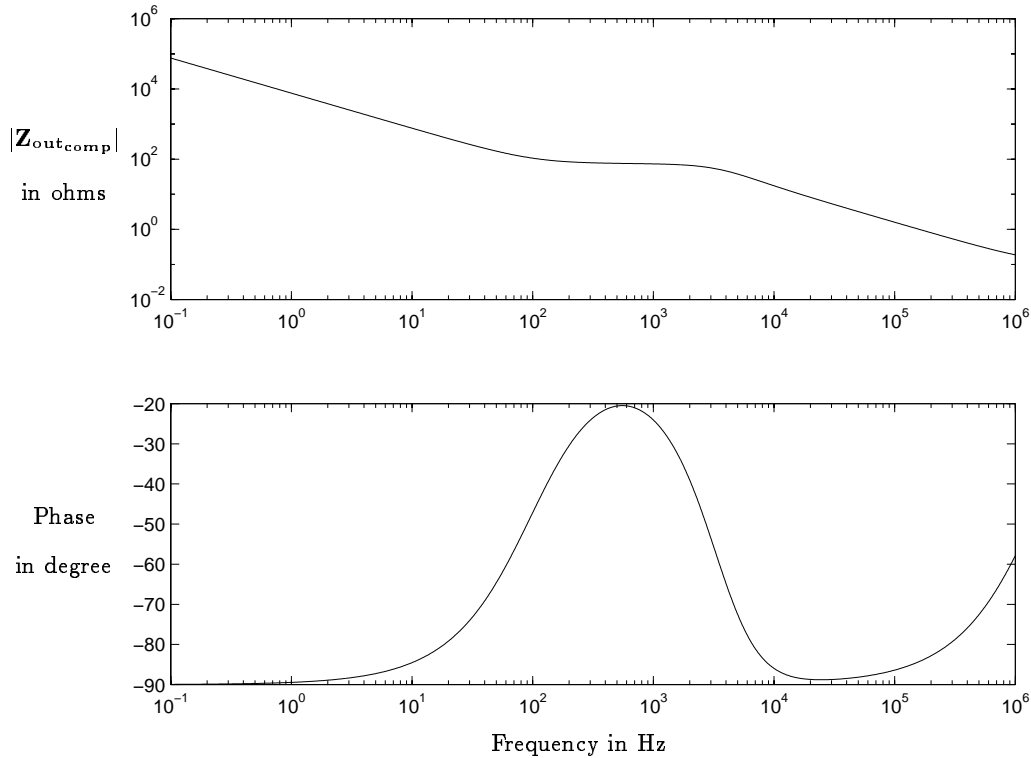
Figure 3.12: The output impedance versus frequency graphic of the phase compensator shows that the dynamics are more complex than for the cathode.

Disturbances in time domain
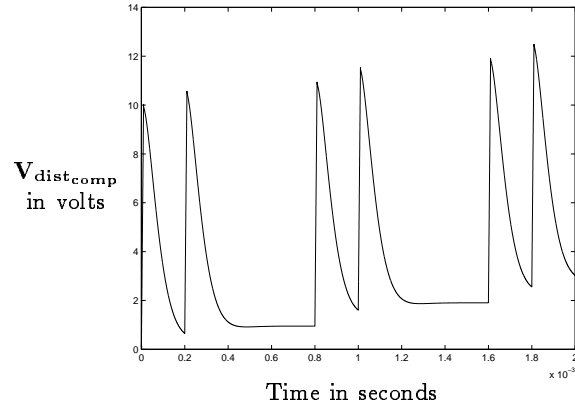
Figure 3.13: The disturbances on the Phase Compensator voltage are smaller than on the Cathode Power Supply.

From current pulses of 10 μs, the voltage disturbance has the following shape through the output impedance of phase compensator.

3.3.5 Resulting disturbances

Output impedance versus frequency

This is the output impedance versus frequency graphic of the resulting output impedance of the entire plant.
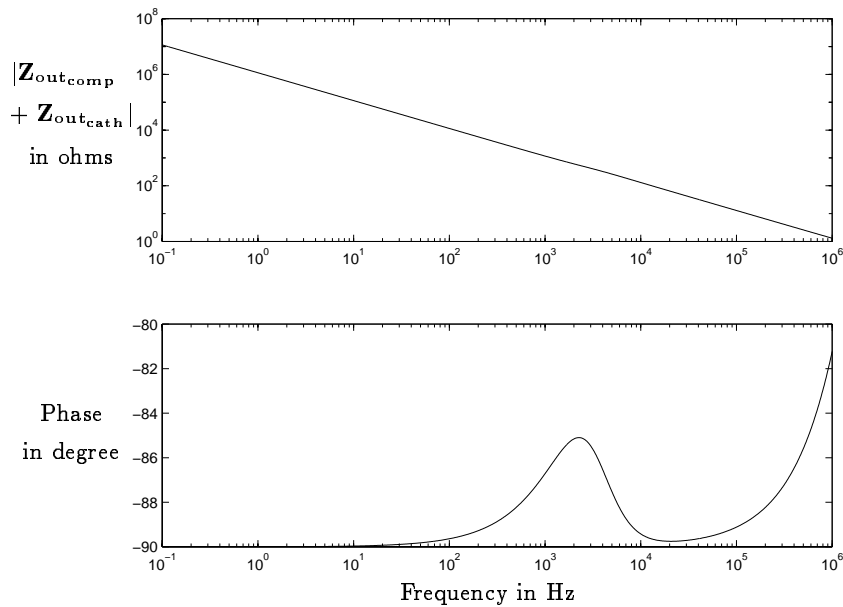
Figure 3.14: The output impedance versus frequency graphic of the parallel sum of the two output impedances shows that the impedance is mainly a capacitor.

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Disturbances in time domain

This graphic shows the total disturbance that we finally get in the cathode voltage -the main output that we have to care about-.
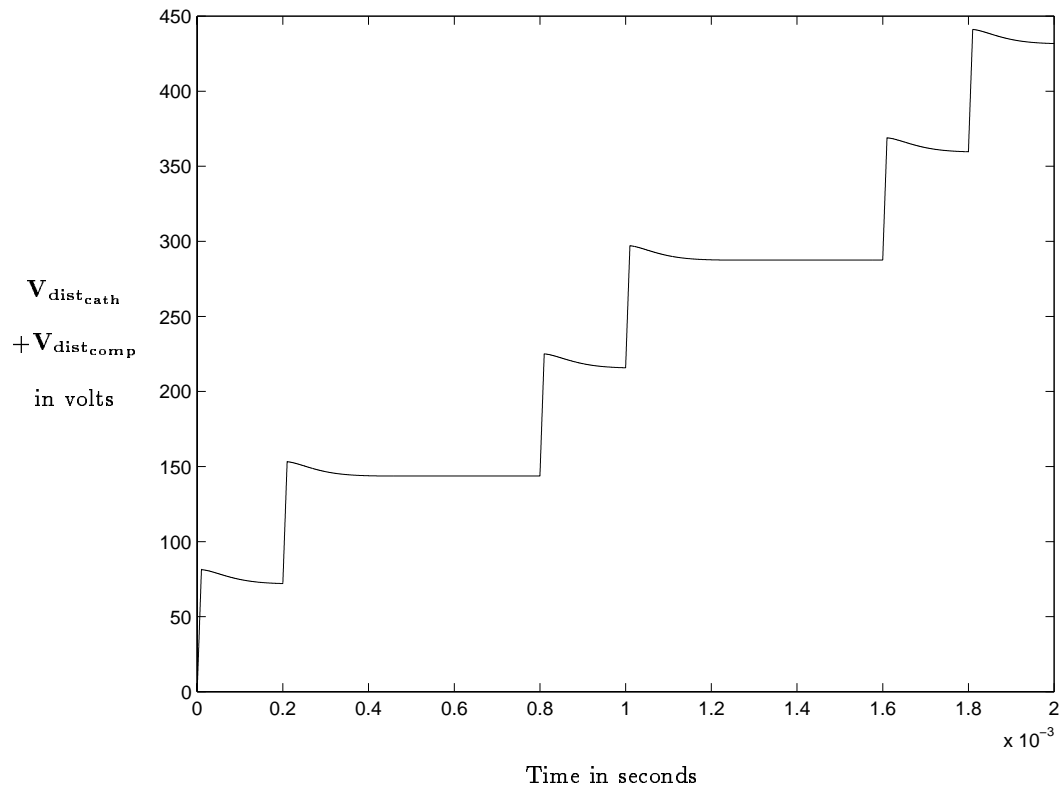
Figure 3.15: The resulting disturbance in time domain on Cathode Voltage is mainly a load. There is so far no control of the plant.

3.4 Model of the sensors

It is only possible to measure the voltages through the sensors described further.
The sensors transform the output voltages from around -24000 V and around
200 V to voltage between -12 V and +12 V so that they are usable by the control
electronic part.
The study of the behavior of the sensors in the frequency domains will permit
to see if they will be a problem in the future design of the controller or not.

3.4.1 Model of the Phase Compensator Voltage sensor

Schematics

The schematic of the sensor permits to see that only few dynamics will be
generated.
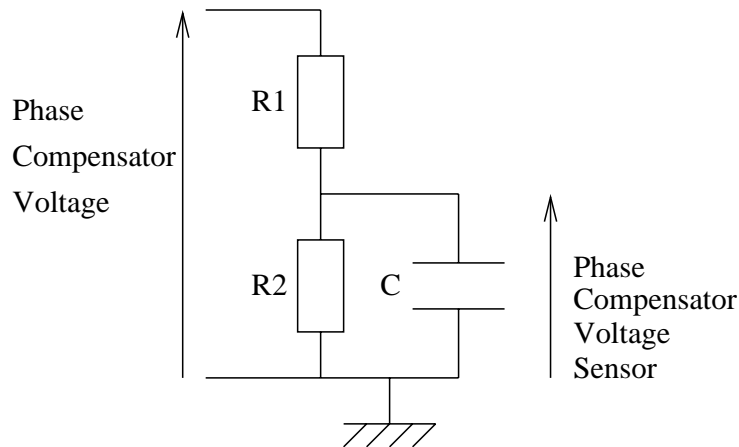
Figure 3.16: The schematics of the sensor used in order to measure the Phase
Compensator Voltage is quite simple. It is almost a voltage divider.

Transfer functions

$$V_{Comp_{sensor}} = \frac{R_2}{jR_1R_2C\omega + R_2 + R_1} \cdot V_{Comp} \qquad \text{Electrical equation}$$

$$\Leftrightarrow T_{Sensor_{Comp}}(s) = \frac{R_2}{R_1R_2Cs + R_2 + R_1} \qquad \text{Transfer function}$$

With
$R_1 = 300000 \quad \Omega$
$R_2 = 2150 \quad \Omega$
$C = 680 \quad \text{pF}$

$$T_{Sensor_{Comp}}(s) = \frac{4902}{s + 6.889 \times 10^5} \qquad \text{Transfer function}$$
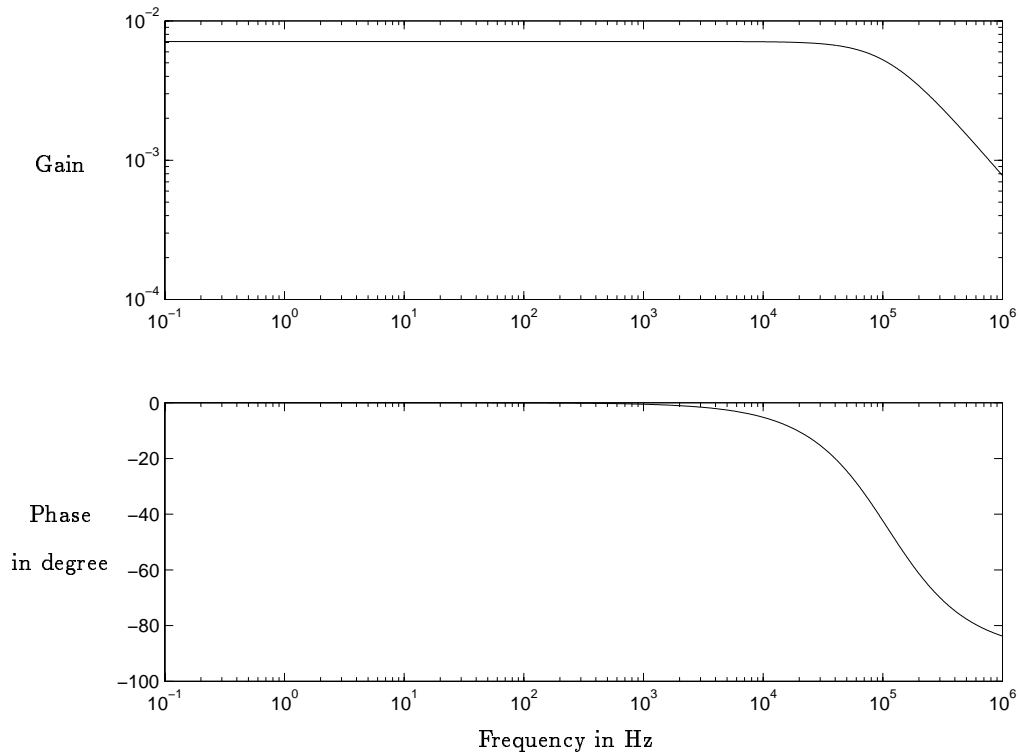
Bode diagram

Figure 3.17: The Bode diagram of the Phase Compensator Voltage sensor shows that the ratio is around 0.007. The high frequency above 10^5 Hz will be attenuated.

3.4.2 Model of the Cathode Voltage sensor

Schematics
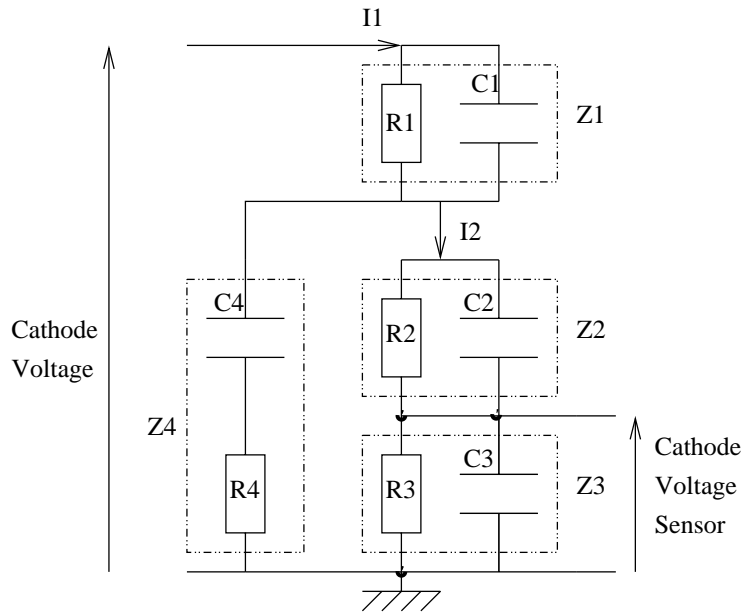
Figure 3.18: The schematics of the sensor used in order to measure the Cathode Voltage is much more complicated.

This schematic permits to see that the sensor is almost a voltage divider but with a lot of dynamics. We can count 4 capacitors.

Transfer functions

$$\begin{cases} V_{Cath} = I_1\left(Z_1 + Z_4\right) - I_2 Z_4 \\[2mm] 0 = I_1 Z_4 - I_2\left(Z_4 + Z_2 + Z_3\right) \end{cases}$$

$$\Rightarrow \begin{cases} I_2 = I_1 \frac{Z_4}{Z_4 + Z_2 + Z_3} \\[2mm] I_1 = \frac{Z_2 + Z_3 + Z_4}{Z_4 Z_1 + Z_2 Z_1 + Z_2 Z_4 + Z_3 Z_1 + Z_3 Z_4} \cdot V_{Cath} \\[2mm] V_{Cath_{sensor}} = Z_3 I_2 \end{cases}$$

$$\Rightarrow V_{Cath_{sensor}} = \frac{Z_4 Z_3}{Z_4 Z_1 + Z_2 Z_1 + Z_2 Z_4 + Z_3 Z_1 + Z_3 Z_4} \cdot V_{Cath}$$

With

$$
\begin{cases}
Z_1 = \frac{R_1}{jR_1C_1\omega+1} \\[2mm]
Z_2 = \frac{R_2}{jR_2C_2\omega+1} \\[2mm]
Z_3 = \frac{R_3}{jR_3C_3\omega+1} \\[2mm]
Z_4 = \frac{jC_4R_4\omega+1}{jC_4\omega}
\end{cases}
\qquad
\begin{aligned}
C_1 &= 100 && \text{pF} \\
R_1 &= 100 && \text{M}\Omega \\
C_2 &= 10 && \text{nF} \\
R_2 &= 1 && \text{M}\Omega \\
C_3 &= 500 && \text{nF} \\
R_3 &= 20 && \text{K}\Omega \\
C_4 &= 10 && \text{nF} \\
R_4 &= 1.5 && \text{M}\Omega
\end{aligned}
$$

$$
T_{Sensor_{Cath}}(s) = \frac{0.000198 s^2 + 0.033 s + 1.32}{s^2 + 234 s + 6667}
\qquad \text{Transfer function}
$$
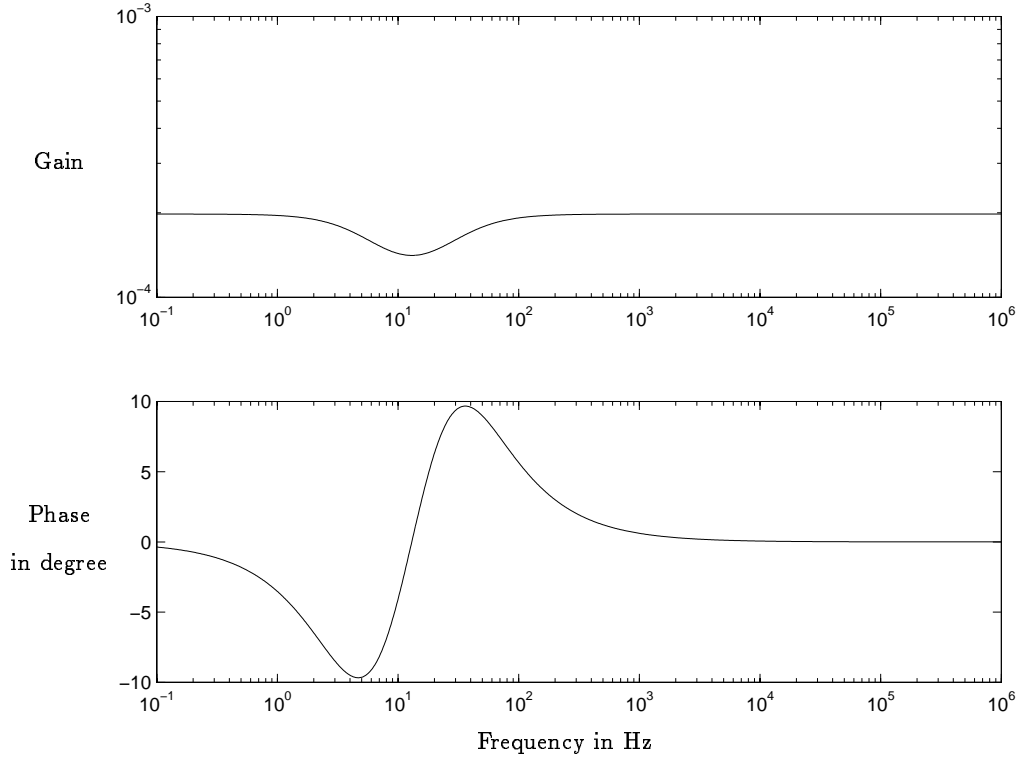
Bode diagram

Figure 3.19: The Bode diagram of Cathode Voltage Sensor shows that the ratio is 0.000198 for low frequency. The dynamic can be neglected.

3.4.3   Remarks about the sensors

The sensors are inserted into the simulation schematic. But in the design of the controller, the small dynamics inside the sensors are considered as unmodeled dynamics, hence uncertainties. The models of the sensor are not used to compute the observer and the feedback of the controller.
In conclusion, one sensor (Phase Compensator Voltage Sensor) is almost a voltage divider and it is also reasonable to compare the other one (Cathode Voltage Sensor) to a voltage divider.

This implies that a gain will be added before the controller to compensate for the attenuation due to these sensors.

3.5   The construction of the complete system

The disturbances from the current pulses are added to the Cathode Voltage Power Supply output and the Phase Compensator output via the output impedances. The Cathode Voltage, the sum of the Cathode Voltage Power Supply output, the Phase Compensator output and their disturbances, and the Phase Compensator Voltage, the sum of the Phase Compensator output and its disturbance, are measured via two sensors condered as voltage dividers.

The regulator will apply the control signal on the system and sense the two outputs in order to reject the disturbances.
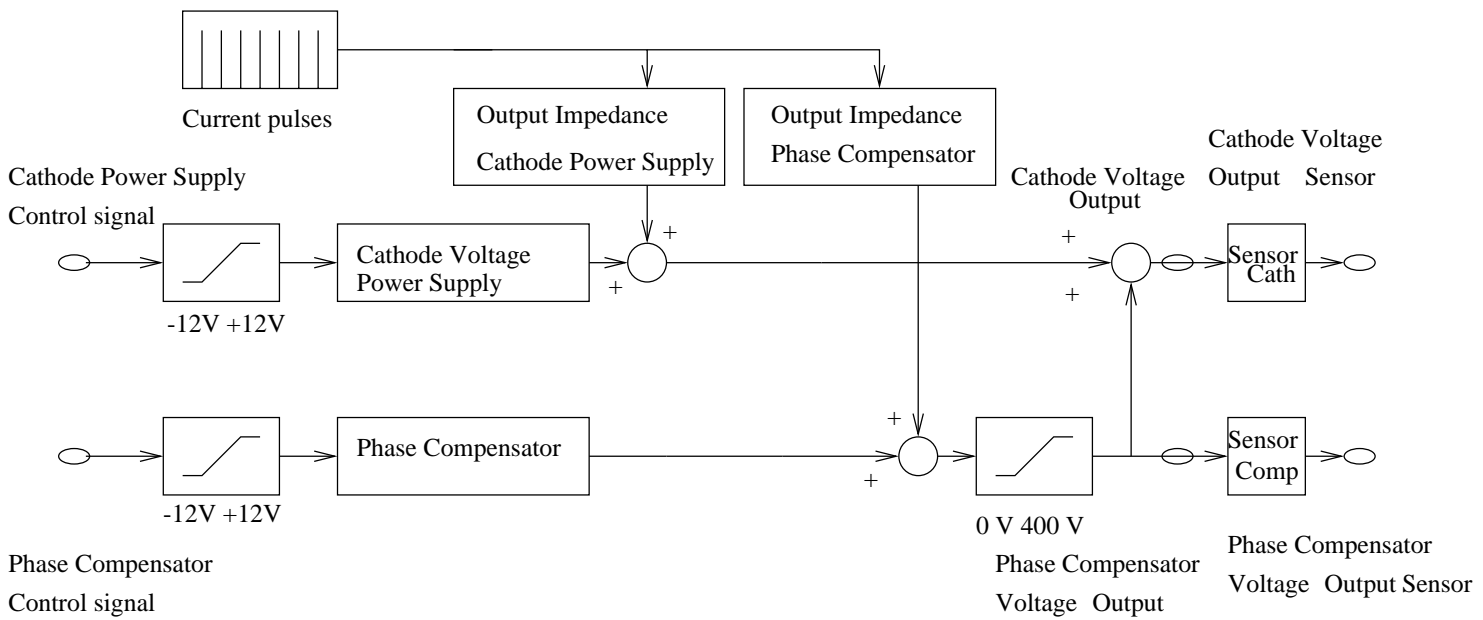
Figure 3.20: The disturbances and the sensors are added to the outputs to form the complete system.

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Chapter 4

Design of the regulator

4.1 Introduction to the state space representation

Physical notion of system state

Extracts from Control Design System, Bernard Friedland pages 16-17.
" *"The state of a dynamic system is a set of physical quantities, the specification of which completely determines the evolution of the system."* is an abstract definition of a state of a dynamic system.

Behavior of dynamic systems is represented by systems of ordinary differential equations. The differential equations are said to constitute a mathematical model of the physical process.

In the state-space approach, all the differential equations in the mathematical model of a system are first-order equations: only the dynamic variables and their first derivative (with respect to time) appear in the differential equations. Since only one initial condition is needed to specify the solution of the first-order equation, it follows that the number of first-order differential equations in the mathematical model is equal to the order of the corresponding system."

Electronic circuit example
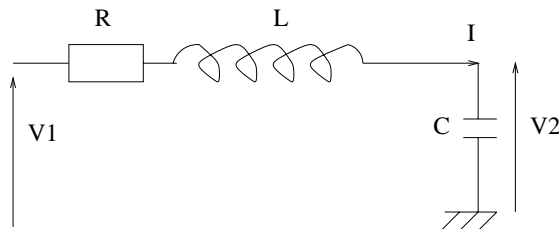
Figure 4.1: I and V_2 are the states of this simple electrical circuit.

$$\begin{cases} V_1 = Ri + L\frac{di}{dt} + V_2 \\ i = C\frac{dV_2}{dt} \end{cases}$$

The general rule of analysis of circuits is: "The states of an electrical circuit are the current in the self and the capacitor voltages."

$$\begin{cases} x_1 = i \\ x_2 = V_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} \dot{x}_1 = \frac{-R}{L}x_2 - \frac{1}{L}x_2 + \frac{1}{L}V_1 \\ \dot{x}_2 = \frac{1}{C}x_1 \\ y = x_2 \end{cases}$$

Notion of State Representation

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \ldots + a_n y = bu$$

Order n differential equations :

$$\begin{cases} x_1 = y \\ x_2 = \dot{y} \\ x_3 = \ddot{y} \\ \vdots \\ x_n = y^{(n-1)} = \frac{d^{n-1}y}{dt^{n-1}} \end{cases} \qquad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -a_1 x_n - a_2 x_{n-1} \ldots - a_n x_1 + bu \end{cases}$$

State representation → Matrix form

$$\mathbf{X}_{\in \mathbb{R}^n} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad \dot{\mathbf{X}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{pmatrix}$$

$$\dot{\mathbf{X}} = \begin{pmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -a_n & -a_{n-1} & \ldots & \ldots & \ldots & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ b \end{pmatrix} u$$

$$n*n \qquad\qquad\qquad n*1$$

$$y = \begin{pmatrix} 1 & 0 & \ldots & \ldots & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} + 0*u$$

$$\Leftrightarrow \begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u & \text{State Equation} \\ y = \mathbf{C}x + \mathbf{D}u & \text{Output Equation} \end{cases}$$

Generalization to Multi-Inputs Multi-Outputs

We are now considering a system at m inputs and p outputs and n states.

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u \\ y = \mathbf{C}x + \mathbf{D}u \end{cases} \quad \begin{cases} x : \text{state vector} \\ u : \text{input vector} \\ y : \text{output vector} \end{cases}$$

$$\begin{cases} \mathbf{A} : \text{state matrix} \\ \mathbf{B} : \text{input matrix} \\ \mathbf{C} : \text{output matrix} \\ \mathbf{D} : \text{Input/Output coupling} \end{cases}$$

Input-Output Relations :
State representation $\rightarrow$ Transfer Functions

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u \\ y = \mathbf{C}x + \mathbf{D}u \end{cases}$$
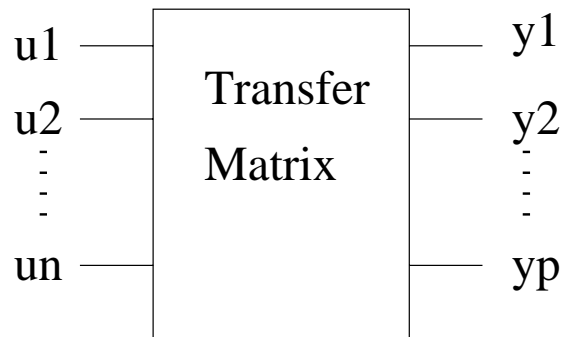
Figure 4.2: The transfer Matrix draws the relationship between inputs and outputs.

The computation of the transfer Matrix

$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

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4.2 Compensator design by the separation principle

Design of regulators for MIMO systems

Extracts from Control Design System, Bernard Friedland pages 222.
"It is possible to place the closed-loop poles anywhere we wish in the complex s
plane. This means that we can, in principle, completely specify the closed-loop
dynamic performance of the system. In principle , we can start with a sluggish
open-loop system and force it to behave with alacrity ; in principle, we can start
with a system that has very little open-loop damping and provide any amount
of damping desired. Unfortunately, however, what can be attained in principle
may not be attainable in practice. Speeding the response of a sluggish system
requires the use of large control signals which the actuator (or power supply)
may not be capable of delivering. The consequence is generally that the actuator
saturates at the largest signal that it can supply. In some instances the system
behavior may be acceptable in spite of the saturation. But in other cases the
effect of saturation is to make the closed-loop system unstable."

We consider the following system:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u & \text{State Equation} \\ y = \mathbf{C}x + \mathbf{D}u & \text{Output Equation} \\ u = -\mathbf{L}x & \text{Control Law} \end{cases}$$

If the dynamic system under consideration has more than one input, that
is, $\mathbf{B}$ has more than one column, then the gain matrix $\mathbf{L}$ in the control law has
more than one row.
If the dynamic system under consideration has more than one output, that is,
$\mathbf{C}$ has more than one row, then the gain matrix $\mathbf{L}$ in the control law has more
than one column.

The need for observers

Extracts from Control Design System, Bernard Friedland pages 259.
"[...], we studied methods for shaping the dynamic response of the closed-loop
system by selecting the feedback gains to "place" the resulting poles at desired
locations. In order to place the poles at arbitrary locations, it is generally
necessary to have all the state variables available for feedback. There are many
systems in which acceptable performance can be achieved by feeding back those
state variables that are accessible to measurement. But often it is not possible
to achieve acceptable performance using only those state variables that can be
measured."
"If the system is observable, it is possible to estimate those state variables that
are not directly accessible to measurement using measurement data from those
state variables that are accessible."

The separation principle

**Extracts from Control Design System, Bernard Friedland pages 290-
291.**

"The separation principle is so disarmingly simple that it almost comes as a surprise: it is hard to imagine that the observer designed for a known input can serve to estimate the state of the process for the purpose of generating the control input. But, it does work."

"Underlying the separation principle is a critical assumption, namely that the observer includes an exact dynamic model of the "plant" : the process under control. This assumption is almost never valid in reality. In practical systems, the precise dynamic model is rarely known. Even that which is known about the real process dynamic model is often too complicated to include in the observer. Thus the observer must in practice be configured to use only an appropriate model of the plant. This encounter with the real world does not vitiate the separation principle, but it means that the effect of an inaccurate plant model must be considered. If the design achieved through use of the separation principle is robust, it will tolerate uncertainty of the plant dynamics. The robustness tests (gain and phase margins, ...) can be used to assess robustness of control laws designed by use of the separation principle."

We consider the following system :

$$\dot{x} = \mathbf{A}\,x + \mathbf{B}\,u$$

With observations given by :

$$y = \mathbf{C}x$$

Where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are the model of the plant used for the observer.

If we design a "full-state feedback" control law by :

$$u = -\mathbf{L}x$$

And also suppose we have designed an observer :

$$\dot{\hat{x}} = \mathbf{A}\,\hat{x} + \mathbf{B}\,u + \mathbf{K}(y - \mathbf{C}\hat{x})$$
where $\hat{x}$ are the state estimate

If the states can not be measured the control law will be :

$$u = -\mathbf{L}\hat{x}$$

The observer-controller design

The *observer* permits to estimate the states of the system so that it will be possible for the states feedback to control the *system* and hence to reject the disturbances, see figure 4.3 on page 43.

The *Matrix of reference* permits to adjust the steady state of the system to the wanted reference, see section 4.6 on page 56.

The *gains* in front of the observer permit to reconstitute the signals, Cathode Voltage and Phase Compensator Voltage, that the observer-controller has to take care of.

The *model of the limitations of the control signal* permits to design an anti-windup, see section 4.8 on page 59.
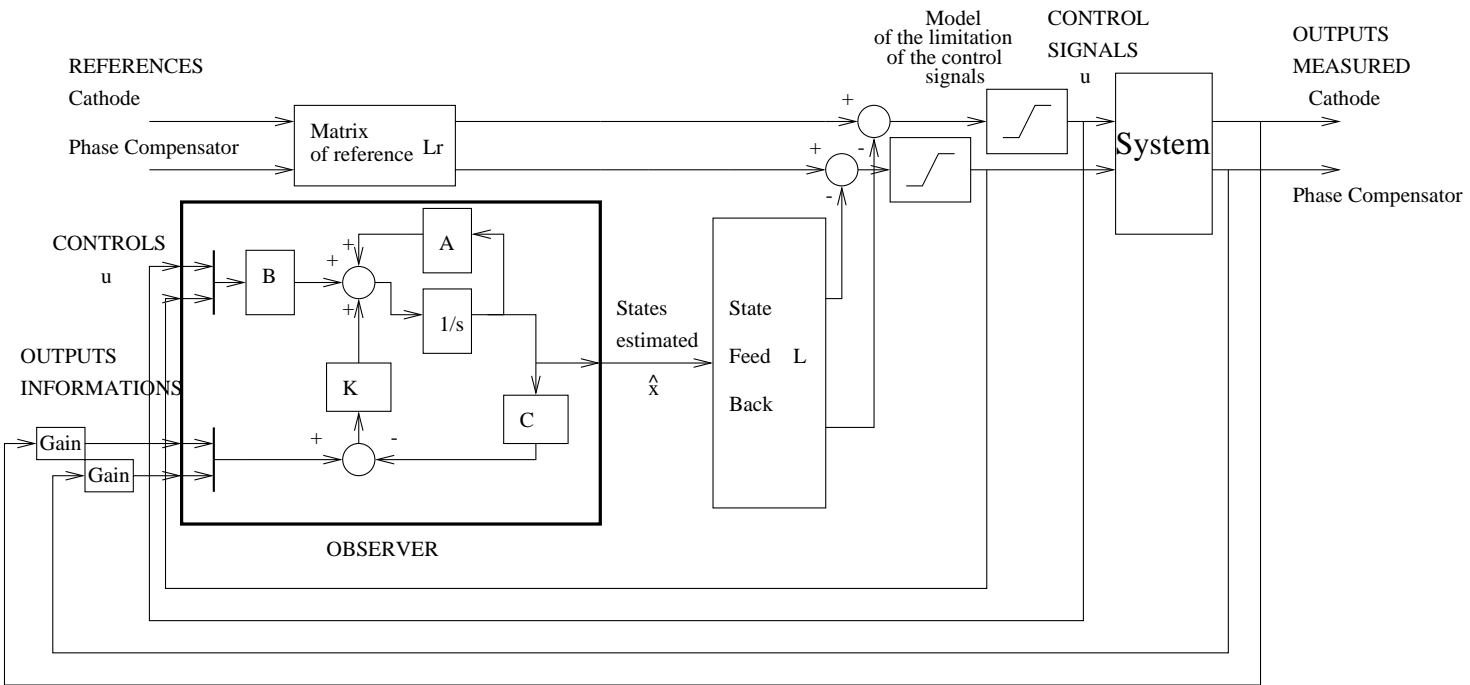
Figure 4.3: The observer-controller design will be able to estimate the states of the system and hence to control it in order to reject the disturbances.

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4.3 Models of the plant used for the observer

The model of the plant used for the observer is a reduced model which takes care of the important features of the Bode diagram that Ericsson's simulation software gives. This reduced model is set up in order to design the observer controller in a simplified form.

First, this is necessary to limit the complexity of the analog controller and to limit the time of simulation inside a reliable range. Second, it also permits to tune it easily and have a better understanding of its behavior.

The observer model is also composed of two parts : the Cathode Power Supply and the Phase compensator.

In this section, the model of Cathode Power Supply will be called model CathObs and the model of the Phase Compensator model CompObs.

4.3.1 First Reduced Model

To simplify the model, the slow poles are neglected. Instead of them, integrators are used. The second idea is to neglect the small ripple on the gain and phase curve (poles and zeros close for example).

Models of Cathode Power Supply
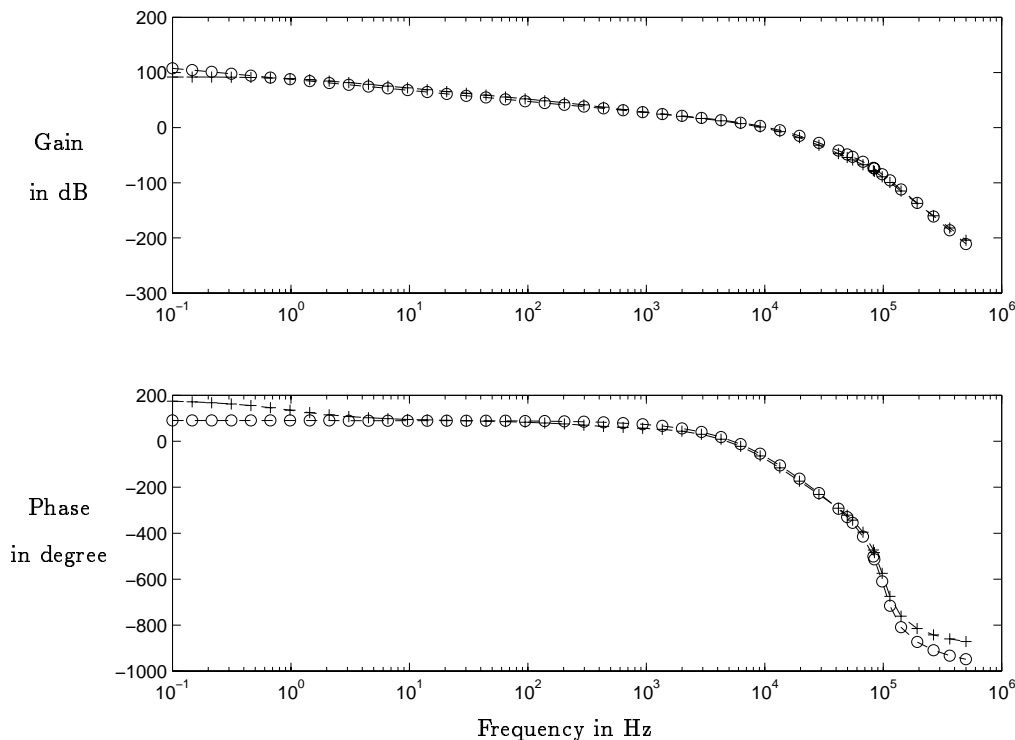
Figure 4.4: The circled line shows that the first reduced model used for the Cathode Power Supply is quite similar for all frequencies except at low frequency with the integrator instead of the slow pole.

$$\mathbf{T_{Cath_{Obs1}}} = G_{0_{Cath_{Obs1}}} \frac{\left(s^2 - 2as + a^2 + b^2\right)}{s\left(s^2 + 2as + a^2 + b^2\right)\left(s + 2\pi f_{21}\right)\left(s + 2\pi f_{20}\right)^4}$$

With

$$G_{0_{Cath_{Obs1}}} = -2.7131 * 10^{30}$$

$$a = 2\pi * 12000 \text{rad}.s^{-1} \qquad b = 2\pi * 70400 \text{rad}.s^{-1}$$

$$f_{20} = 1.55 * 10^4 \text{Hz} \qquad f_{21} = 3.2 * 10^4 \text{Hz}$$

Models of Phase Compensator
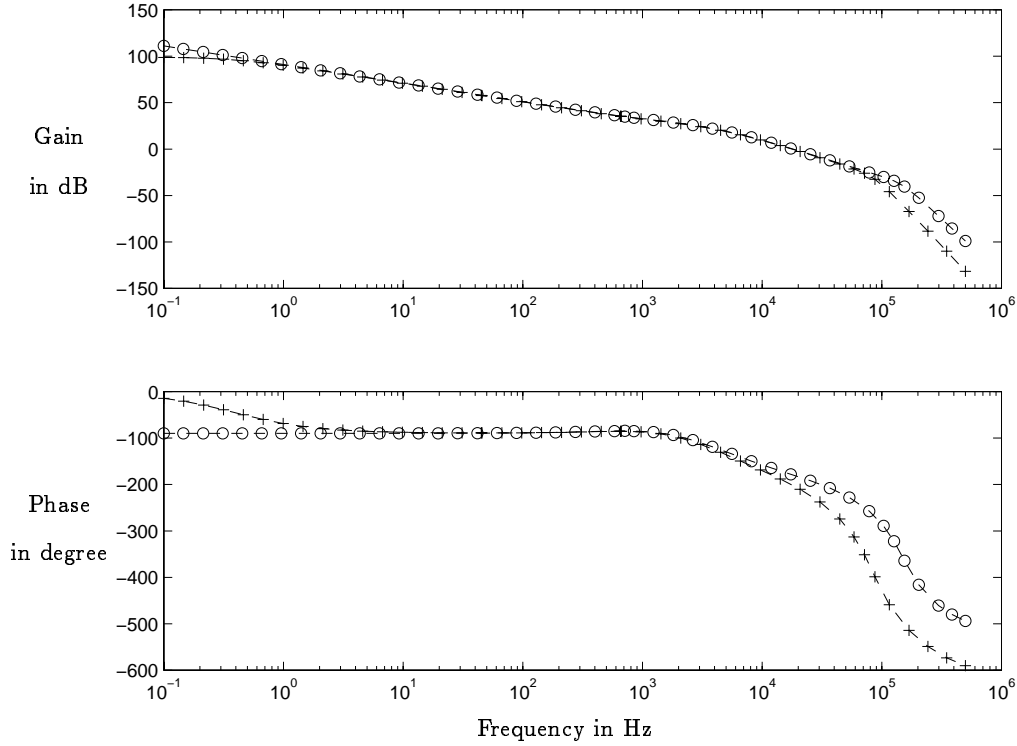
Figure 4.5: The circled line shows that the first reduced model used for the Phase Compensator is a bit different also at high frequencies.

$$\mathbf{T_{Comp_{Obs1}}} = G_{0_{Comp_{Obs1}}} \frac{s + 2\pi f_{40}}{s\left(s + 2\pi f_{41}\right)\left(s + 2\pi f_{42}\right)\left(s + 2\pi f_{43}\right)}$$

With

$$G_{0_{Comp_{Obs1}}} = 1.0175 * 10^{16}$$

$$f_{40} = 1.1 * 10^3 \text{Hz} \qquad f_{41} = 2.1 * 10^3 \text{Hz}$$
$$f_{42} = 5 * 10^3 \text{Hz} \qquad f_{43} = 1.2 * 10^5 \text{Hz}$$

4.3.2 Second Reduced Model

The second reduced model is constructed so that the model has a smaller order while taking into account the main frequency behavior.

By comparing the control results of the two different models, it will be possible to conclude on the necessity to keep a controller of the highest order or not.

Models of Cathode Power Supply
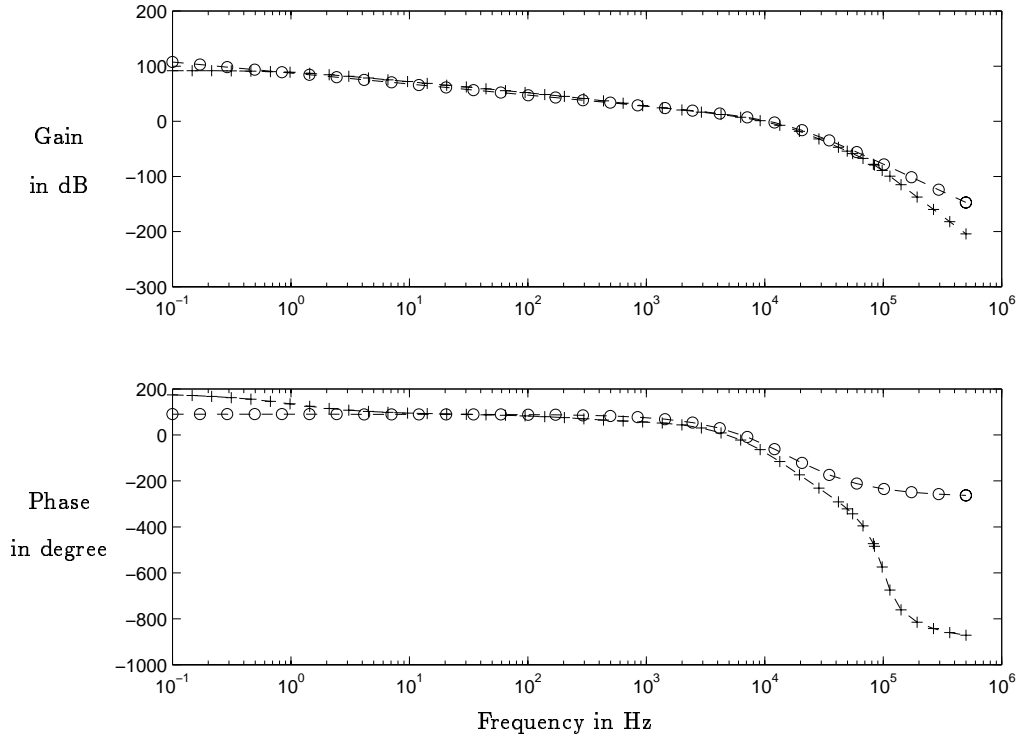
Figure 4.6: The circled line shows the second reduced model used for the Cathode Power Supply which comports much more differences than the first mainly above 50 000 Hz.

$$\mathbf{T_{Cath_{Obs2}}} = G_{0_{Cath_{Obs2}}} \frac{1}{s(s + 2\pi f_{30})^4}$$

With

$$G_{0_{Cath_{Obs2}}} = -1.3494 * 10^{25}$$

$$f_{30} = 1.55 * 10^4 \text{Hz}$$

Models of Phase Compensator
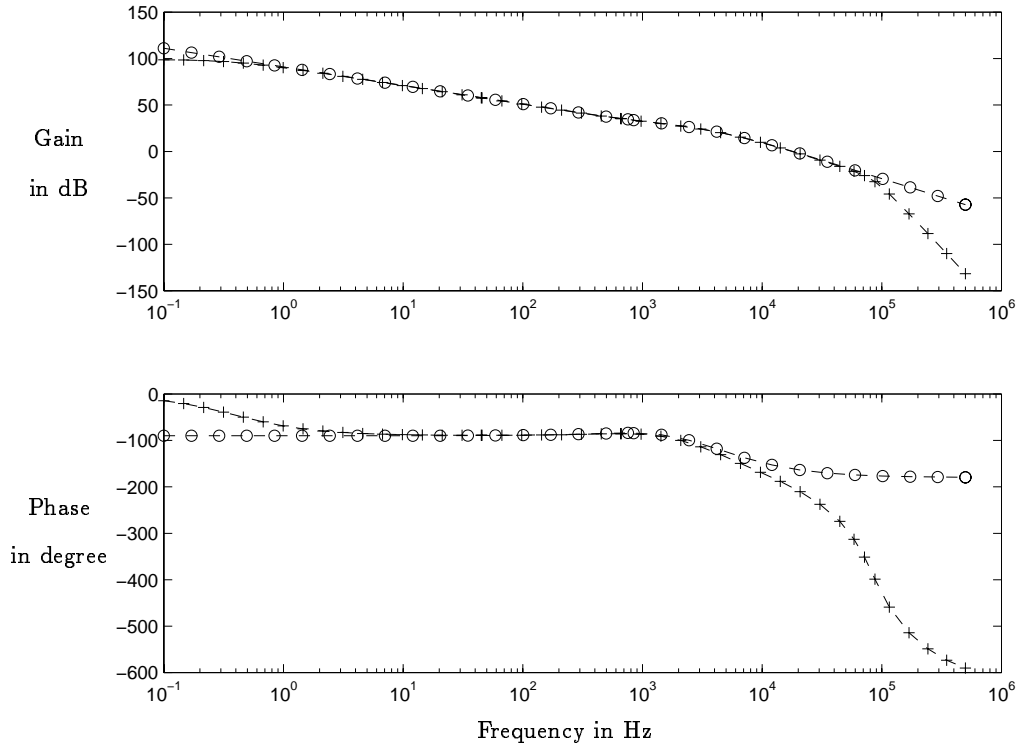
Figure 4.7: The circled line shows the second reduced model used for the Phase Compensator which also comports much more differences than the first mainly above 20 000 Hz for the phase.

$$\mathbf{T_{Comp_{O\,bs2}}} = G_{0_{Comp_{Obs2}}} \frac{s + 2\pi f_{50}}{s\left(s + 2\pi f_{51}\right)\left(s + 2\pi f_{52}\right)}$$

With

$$G_{0_{Comp_{Obs2}}} = 1.3495 * 10^{10}$$

$$f_{50} = 1.1 * 10^3 \mathrm{Hz} \qquad f_{51} = 2.1 * 10^3 \mathrm{Hz} \qquad f_{52} = 5 * 10^3 \mathrm{Hz}$$

4.3.3 Observability problem

In order to keep the system observable, it is necessary to construct the model in an unordinary way.

If there are the same poles in Model Comp and in Model Cath, the model built will not be observable.

In this case, the slow poles of the system are replaced by some integrators. The model drawn in 1 is unobservable; the model drawn in 2 is observable.
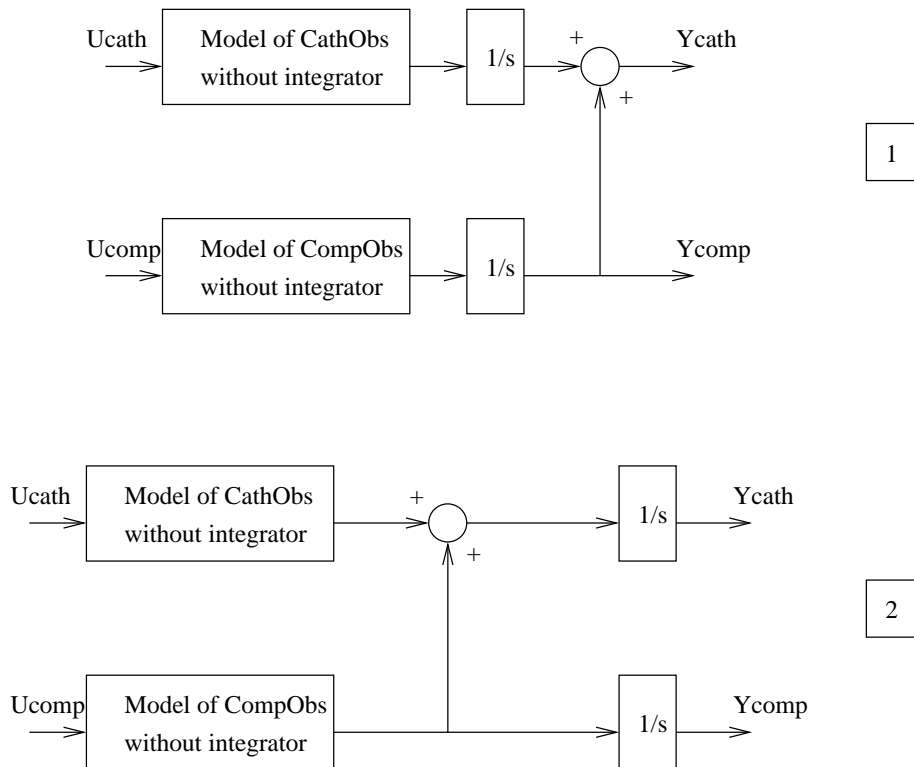
Figure 4.8: The construction 2 of the model used for the observer permits to keep the model observable.

4.3.4 State space representation

We are considering $\mathbf{T_{Cath_{Obs}}}$ and $\mathbf{T_{Comp_{Obs}}}$, previously computed and *where the integrator is removed from the transfer function because of observability problem introduced previously.*

The following equations define the relationship between $\mathbf{A_{Cath_{Obs}}}$, $\mathbf{B_{Cath_{Obs}}}$, $\mathbf{C_{Cath_{Obs}}}$, $\mathbf{A_{Comp_{Obs}}}$, $\mathbf{B_{Comp_{Obs}}}$, $\mathbf{C_{Comp_{Obs}}}$ and $\mathbf{T_{Cath_{Obs}}}$ $\mathbf{T_{Comp_{Obs}}}$:

$$\mathbf{T_{Cath_{Obs}}} = \mathbf{C_{Cath_{Obs}}} (s\mathbf{I} - \mathbf{A_{Cath_{Obs}}})^{-1} \mathbf{B_{Cath_{Obs}}}$$
$$\mathbf{T_{Comp_{Obs}}} = \mathbf{C_{Comp_{Obs}}} (s\mathbf{I} - \mathbf{A_{Comp_{Obs}}})^{-1} \mathbf{B_{Comp_{Obs}}}$$

With $\mathbf{A_{Cath_{Obs}}}$, $\mathbf{B_{Cath_{Obs}}}$, $\mathbf{C_{Cath_{Obs}}}$ the state space representation of the reduced model of the Cathode Power Supply and with $\mathbf{A_{Comp_{Obs}}}$, $\mathbf{B_{Comp_{Obs}}}$, $\mathbf{C_{Comp_{Obs}}}$ the state space representations of the reduced model of the Phase Compensator introduced previously, we can construct the state space representation ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) of the model used for the observer :

$$\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}u$$
$$y = \mathbf{C}x$$

$$\dot{\hat{x}} = \begin{bmatrix} \mathbf{A_{Cath_{Obs}}} & 0 & 0 & 0 \\ 0 & \mathbf{A_{Comp_{Obs}}} & 0 & 0 \\ \mathbf{C_{Cath_{Obs}}} & \mathbf{C_{Comp_{Obs}}} & 0 & 0 \\ 0 & \mathbf{C_{Comp_{Obs}}} & 0 & 0 \end{bmatrix} \hat{x}$$

$$+ \begin{bmatrix} \mathbf{B_{Cath_{Obs}}} & 0 \\ 0 & \mathbf{B_{Comp_{Obs}}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} u_{Cath} \\ u_{Comp} \end{pmatrix}$$

$$y = \begin{pmatrix} y_{Cath} \\ y_{Comp} \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \hat{x}$$

The two last states are introduced to take care of the observability problem.

4.3.5 Integration of the Cathode Voltage sensor in the model of the plant used for the observer

Why is there the need to integrate the sensor model in the model of the plant used for the observer ?

When we look at the Bode diagram of the Cathode Voltage sensor presented previously in section 3.4.2 on page 34, it is not expected to limit the performance. But let's have a look at the step response.

Such a dynamic can not be neglected, therefore it should be included in the
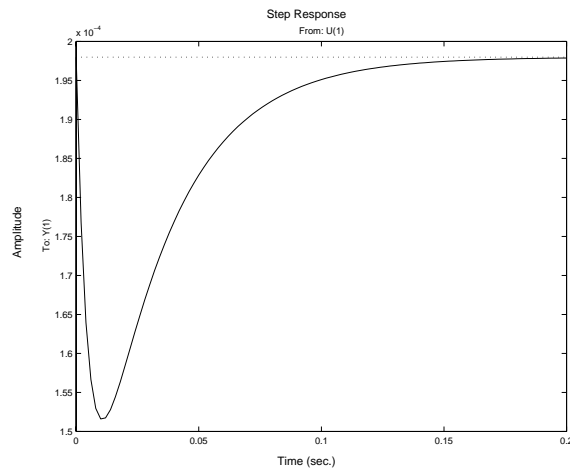
Figure 4.9: The step response shows that the Cathode Voltage sensor need 200 ms to reach its stationnary value.

model to be able to estimate the Cathode Voltage in order to be able to control it. In this way, the sensor is a limitation of the performance.

For comparison, the step response of the Phase Compensator Voltage sensor reaches its stationnary value after 10 μs.
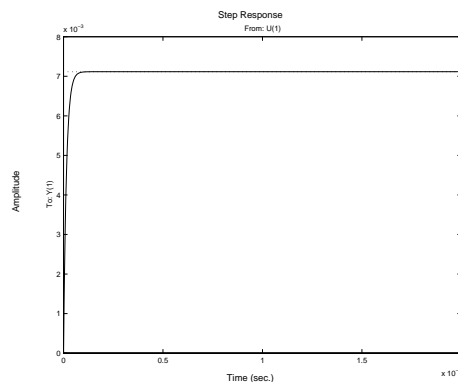
Figure 4.10: The step response shows that the Phase Compensator sensor needs 10 μs to reach its stationnary value.

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State space representation including the sensor model

From $\mathbf{T_{Sensor_{Cath}}}$ computed in section 3.4.2 on page 34, its state space representation ($\mathbf{A_{Sensor_{Cath}}}$, $\mathbf{B_{Sensor_{Cath}}}$, $\mathbf{C_{Sensor_{Cath}}}$, $\mathbf{D_{Sensor_{Cath}}}$) is derived :

$$\mathbf{T_{Sensor_{Cath}}} = \mathbf{C_{Sensor_{Cath}}} \left(s\mathbf{I} - \mathbf{A_{Sensor_{Cath}}}\right)^{-1} \mathbf{B_{Sensor_{Cath}}} + \mathbf{D_{Sensor_{Cath}}}$$

Hence, the state space representation of the model used for the observer including the Cathode Voltage sensor model is derived :

$$\dot{\hat{x}} = \mathbf{A}\,\hat{x} + \mathbf{B}\,u$$
$$y = \mathbf{C}x$$

$$\dot{\hat{x}} = \begin{bmatrix} \mathbf{A_{Cath_{obs}}} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{A_{Comp_{obs}}} & 0 & 0 & 0 \\ \mathbf{C_{Cath_{obs}}} & \mathbf{C_{Comp_{obs}}} & 0 & 0 & 0 \\ 0 & \mathbf{C_{Comp_{obs}}} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{B_{Sensor_{Cath}}} & 0 & \mathbf{A_{Sensor_{Cath}}} \end{bmatrix} \hat{x}$$

$$+ \begin{bmatrix} \mathbf{B_{Cath_{obs}}} & 0 \\ 0 & \mathbf{B_{Comp_{obs}}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} u_{Cath} \\ u_{Comp} \end{pmatrix}$$

$$y = \begin{pmatrix} y_{Sensor_{Cath}} \\ y_{Comp} \end{pmatrix} = \begin{bmatrix} 0 & 0 & \mathbf{D_{Sensor_{Cath}}} & 0 & \mathbf{C_{Sensor_{Cath}}} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \hat{x}$$

Remarks in this case about the state feedback

The computation of the state feedback will be performed with the following control law :

$$\text{if} \quad \hat{x} = \begin{pmatrix} \hat{x}_1 \\ \hat{x}_{Sensor} \end{pmatrix} \qquad u = -L\begin{pmatrix} \hat{x}_1 \end{pmatrix}$$

$\hat{x}_{Sensor}$ corresponds to the states of the Cathode Voltage sensor.
Since we are interested in controlling not the Cathode Voltage sensor output but the Cathode Voltage (which is estimated), we do not need this state in the state feedback but just in the observer.

4.4 The linear, quadratic optimal controller

Extracts from Control Design System, Bernard Friedland pages 337-338.

"The first reason for seeking an optimal controller is that in a multiple-input or multiple-output system, the pole-placement technique does not completely specify the controller or compensator parameters (gain). Consider, for example, a kth-order plant with m inputs and the entire state vector accessible for feedback. A non dynamic controller has km parameters to be determined, but only k possible closed-loop pole locations. Thus we have to set m times as many parameters as there are poles; there are infinitely many ways by which the same closed-loop poles can be attained. Which way is best? What algorithm can be used to determine the feedback gains?

The absence of a definitive algorithm for determining a unique control law is a detriment to the system designer who does not know how to handle this *"embarrassment of riches"*.

By choosing a control law to optimize performance, this embarrassment is avoided.

A more cogent reason for seeking an optimum controller is that the designer may not really know the desirable closed-loop locations."

Formulation of the optimum problem

$$
\begin{cases}
\dot{x} = \mathbf{A}x + \mathbf{B}u & \text{Vector-matrix differential equation} \\
& \text{where } x \text{ are the process states} \\
\\
u = -\mathbf{L}x & \text{Linear control law} \\
& \text{where } \mathbf{L} \text{ is a suitable gain matrix}
\end{cases}
$$

Here, however, instead of seeking a gain matrix to achieve specified closed-loop pole locations, we now seek a gain to minimize a specified performance criterion V (or cost function). The chosen cost function is the following:

$$
V = \int_t^\tau \gamma_1^2 y_{Cath}^2 + \gamma_2^2 y_{Comp}^2 + \gamma_3^2 u_{Cath}^2 + \gamma_4^2 u_{Comp}^2 \quad dt
$$

With
$$
\begin{cases}
y_{Cath} & \text{Cathode Voltage Output} \\
y_{Comp} & \text{Phase Compensator Voltage Output} \\
\\
u_{Cath} & \text{Cathode Power Supply Control Signal} \\
u_{Comp} & \text{Phase Compensator Voltage Control Signal}
\end{cases}
$$

And the weight γ_n will indicate the penalty on each signal (control signal, output) that we have.

Now, a translation in ordinary cost function to be used in the MATLAB command *lqr* has to be done.

$$\begin{cases} V = \int_t^\tau \gamma_1^2 y_{Cath}^2 + \gamma_2^2 y_{Comp}^2 + \gamma_3^2 u_{Cath}^2 + \gamma_4^2 u_{Comp}^2 \quad dt \\[2mm] V = \int_t^\tau x^T \mathbf{Q} x + u^T \mathbf{R} u + 2x^T \mathbf{N} u \quad dt \\[2mm] \text{Where} \\[1mm] \mathbf{Q} = \mathbf{C^T Q_0 C} \quad \text{with} \quad \mathbf{Q_0} = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \\[4mm] \mathbf{R} = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \\[4mm] \text{and} \quad \mathbf{N}, \text{ the cross term matrix, is a zero matrix} \end{cases}$$

The *lqr* MATLAB command computes the optimum gain by solving the following equation :

$$\begin{cases} \text{The Riccati equation with } \hat{S} \text{ the optimal solution when } \dot{\hat{S}} = 0 \\[1mm] -\dot{\hat{\mathbf{S}}} = \hat{\mathbf{S}} \mathbf{A} + \mathbf{A^T} \hat{\mathbf{S}} - \hat{\mathbf{S}} \mathbf{B} \mathbf{R^{-1}} \mathbf{B^T} \hat{\mathbf{S}} + \mathbf{Q} \\[3mm] \text{For the control law to be optimal, we must have:} \\[1mm] \hat{\mathbf{L}} = \mathbf{R^{-1}} \mathbf{B^T} \hat{\mathbf{S}} \end{cases}$$

How to set the Q, R Matrix?

First, it is necessary to rescale the signal so that we weigh signals of comparable size.
It is then possible to put more or less penalty on one signal so that the corresponding optimal gain matrix will be found for the corresponding cost function. In our case we would like to put more penalty on the Cathode Voltage Output than on the Phase Compensator Ouput.
The following weights can be done, for example, where the denominators are the rescaling terms :

$$\begin{cases} \gamma_1 = \frac{1\ 000}{24\ 000^2} \qquad \text{The reference of the Cathode Voltage is 24 000 V.} \\[3mm] \gamma_2 = \frac{1}{200^2} \qquad \text{The reference of the Phase Compensator Voltage is 200V.} \\[3mm] \gamma_3 = \frac{1}{12^2} \ \gamma_4 = \frac{1}{12^2} \ \text{The saturation of both control signals are -12V +12V.} \end{cases}$$

With those weights, a **L** matrix will be computed by the *lqr* command which solves this optimum problem.

4.5 The optimal observer, the Kalman Filter

The Kalman filter permits to place the poles of the observer such that the errors on the states estimates will be white noise, see figure 4.3 on page 43.

What is an Optimal Observer?

The Kalman filter is an optimal observer in the way that the poles are placed optimally deriving from a cost function.

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u + v \\ y = \mathbf{C}x + w \end{cases}$$

where u is known input and v and w are *white* noise. The solution is the following observer where $\hat{\mathbf{K}}$ is optimally chosen :

$$\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}u + \hat{\mathbf{K}}(y - \mathbf{C}\hat{x})$$

The chosen cost function is the following :

$$V = \int_t^\tau x' \begin{bmatrix} \gamma_{11} & 0 & \dots & 0 \\ 0 & \gamma_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \gamma_{1n} \end{bmatrix} x + \gamma_{21}y_{Comp}^2 + \gamma_{22}y_{Cath}^2 \quad dt$$

With

$$\begin{cases} y_{Cath} & \text{Cathode Voltage Output} \\ y_{Comp} & \text{Phase Compensator Voltage Output} \\ \\ x & \text{States that we have to estimate} \end{cases}$$

The weights γ_n will indicate the penalty on each signal (states, output) that we have.

Now, a translation in ordinary cost function to be used in the MATLAB command *lqe* has to be done. It would have been also possible to use the MATLAB command *kalman*.

$$\begin{cases} V = \int_t^\tau x' \begin{bmatrix} \gamma_{11} & 0 & \dots & 0 \\ 0 & \gamma_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \gamma_{1n} \end{bmatrix} x + \gamma_{21}y_{Comp}^2 + \gamma_{22}y_{Cath}^2 \quad dt \\ \\ V = \int_t^\tau x'\mathbf{Q}x + y'\mathbf{R}y + 2x'\mathbf{N}y \quad dt \\ \\ \text{Where} \\ \mathbf{Q} = \begin{bmatrix} \gamma_{11} & 0 & \dots & 0 \\ 0 & \gamma_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \gamma_{1n} \end{bmatrix} \\ \\ \mathbf{R} = \begin{bmatrix} \gamma_{21} & 0 \\ 0 & \gamma_{22} \end{bmatrix}, \\ \\ \text{and} \quad \mathbf{N}, \text{ the cross term matrix, is a zero matrix} \end{cases}$$

The *lqe* MATLAB command computes the optimal gain by solving the following equation :

$$\begin{cases} \text{The Riccati equation with } \hat{P} \text{ the optimal solution when } \dot{\hat{P}} = 0 \\ -\dot{\mathbf{P}} = \mathbf{A}\hat{\mathbf{P}} + \hat{\mathbf{P}}\mathbf{A}' - \hat{\mathbf{P}}\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\hat{\mathbf{P}} + \mathbf{Q} \\ \\ \text{For the control law to be optimal, we must have:} \\ \hat{\mathbf{K}} = \hat{\mathbf{P}}\mathbf{C}'\mathbf{R}^{-1} \end{cases}$$

How to set the Q, R matrix?

First, it should be necessary to rescale the signal so that we weight signals of comparable size.
And after it is possible to put more penalty on a signal so that the corresponding optimal gain matrix will be found for this cost function. In our case we would like to put more penalty on the Cathode Voltage Output than on the Phase Compensator Ouput.
The following weights can be done, for example, where the denominators are the rescaling terms :

$$\begin{cases} \gamma_{21} = \frac{10000}{24000^2} & \text{The reference of the Cathode Voltage is 24 000 V.} \\ \\ \gamma_{22} = \frac{1}{200^2} & \text{The reference of the Phase Compensator Voltage is 200 V.} \\ \\ \gamma_{1n} = \frac{1}{\overline{x_{1n}}^2} & \text{Where } \overline{x_{1n}}^2 \text{ is the average of the square value of the state} \end{cases}$$

But to set those matrices is not as easy as for the optimal state feed back. To tune a Kalman filter, many iteration steps are necessary.

4.6 The computation of the matrix of reference $(\mathbf{L_r})$

The idea is to compute the matrix $\mathbf{L_r}$ so that the static errors between the references (set points) and the outputs are equal to zero, see figure 4.3 on page 43 The total system becomes the following:

$$\begin{cases} u = -\mathbf{L}\hat{x} + \mathbf{L_r}y_r \\ \dot{x} = \mathbf{A}x + \mathbf{B}u = \mathbf{A}x + \mathbf{B}\left(-\mathbf{L}\hat{x} + \mathbf{BL_r}y_r\right) \\ \dot{\hat{x}} = (\mathbf{A} - \mathbf{KC} - \mathbf{BL})\hat{x} + \mathbf{KC}x + \mathbf{BL_r}y_r \\ y = \begin{pmatrix} \mathbf{C} & 0 \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} \end{cases}$$

The state space representation is the following :

$$\frac{d}{dt} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & -\mathbf{BL} \\ \mathbf{KC} & \mathbf{A} - \mathbf{KC} - \mathbf{BL} \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} \mathbf{B} \\ \mathbf{B} \end{pmatrix} \mathbf{L_r}y_r$$

We get the transfer matrix as follows :

$$y = \bar{\mathbf{C}}(s\mathbf{I} - \bar{\mathbf{A}})^{-1} \begin{pmatrix} \mathbf{B} \\ \mathbf{B} \end{pmatrix} \mathbf{L_r}y_r$$

with

$$\bar{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & -\mathbf{BL} \\ \mathbf{KC} & \mathbf{A} - \mathbf{KC} - \mathbf{BL} \end{pmatrix} \quad \text{and} \quad \bar{\mathbf{C}} = \begin{pmatrix} \mathbf{C} & 0 \end{pmatrix}$$

To have unitary stationary gain $(\frac{y}{y_r} = 1)$, we must have with $s = 0$:

$$\mathbf{Id} = -\bar{\mathbf{C}}(s\mathbf{I} - \bar{\mathbf{A}})^{-1} \begin{pmatrix} \mathbf{B} \\ \mathbf{B} \end{pmatrix} \mathbf{L_r} \quad \text{with} \quad \mathbf{Id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{for the two outputs}$$

If we assume that $\bar{\mathbf{A}}$ is inversible, therefore :

$$\mathbf{L_r} = \left[\bar{\mathbf{C}}\bar{\mathbf{A}}^{-1} \begin{pmatrix} \mathbf{B} \\ \mathbf{B} \end{pmatrix} \right]^{-1}$$

4.7 The addition of Integral action

See figure 4.11 on page 58.
In a first approach, since the system is quite complex and non-linear, it was decided not to use any integral action.
Since the outputs are not so stable, there was a drift on one of them (the Phase Compensator voltage) so the need of an integrator was expressed.
A drift on the Phase Compensator voltage means that this output could hit the saturations later and hence lose a part of the performance.

First, it was only added an integral action for the Phase Compensator voltage but it gave a drift on the other one. So a second integral action was set up finally on the second outputs to be sure to reject all drifts. Since the two new states of the integral action are known, it is not neccessary to estimate them.

The resulting equations are the following:
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are the matrices used as models to compute the estimated states in the observer and $\mathbf{K}$ is the observer matrix.

$$
\begin{vmatrix}
\dot{\hat{x}} = (\mathbf{A} - \mathbf{KC})\hat{x} + \mathbf{B}u + \mathbf{L}y \\
\dot{z}_{Cath} = y_{Cath} - y_{r_{Cath}} \\
\dot{z}_{Comp} = y_{Comp} - y_{r_{Comp}}
\end{vmatrix}
$$

$$
y = \begin{pmatrix} y_{Cath} \\ y_{Comp} \end{pmatrix} = \begin{pmatrix} C & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{x} \\ z_{Cath} \\ z_{Comp} \end{pmatrix}
$$

To compute the new controller matrix of the state feed back, it is neccessary to add the representation of the new state in the feedback. The matrix $\mathbf{Q}$ of the LQR algorithm becomes the following in order to take into account the weight of integral action we want to have in the controller.

$\mathbf{Q_{old}}$ is the matrix neccessary to the LQR algorithm without any integral action. $\mathbf{Q_{int^2}}$ is the new $\mathbf{Q}$ matrix.

$$
\mathbf{Q_{int^2}} = \begin{pmatrix} \mathbf{Q_{old}} & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{pmatrix}
$$

REFERENCES yr
Cathode

Phase Compensator

Matrix
of reference Lr

reconstituted
OUTPUTS

Observer

States
estimated

$\hat{x}$

CONTROLS

1/s

zCath

1/s

zComp

State
Feed L
Back

CONTROL
SIGNALS

u

System

OUTPUTS y

Cathode

Phase Compensator

+

-

+

-

+

-

+

Gain

Gain

The gains
permit to
reconstitute
the outputs

REFERENCES
minus
OUTPUTS

58
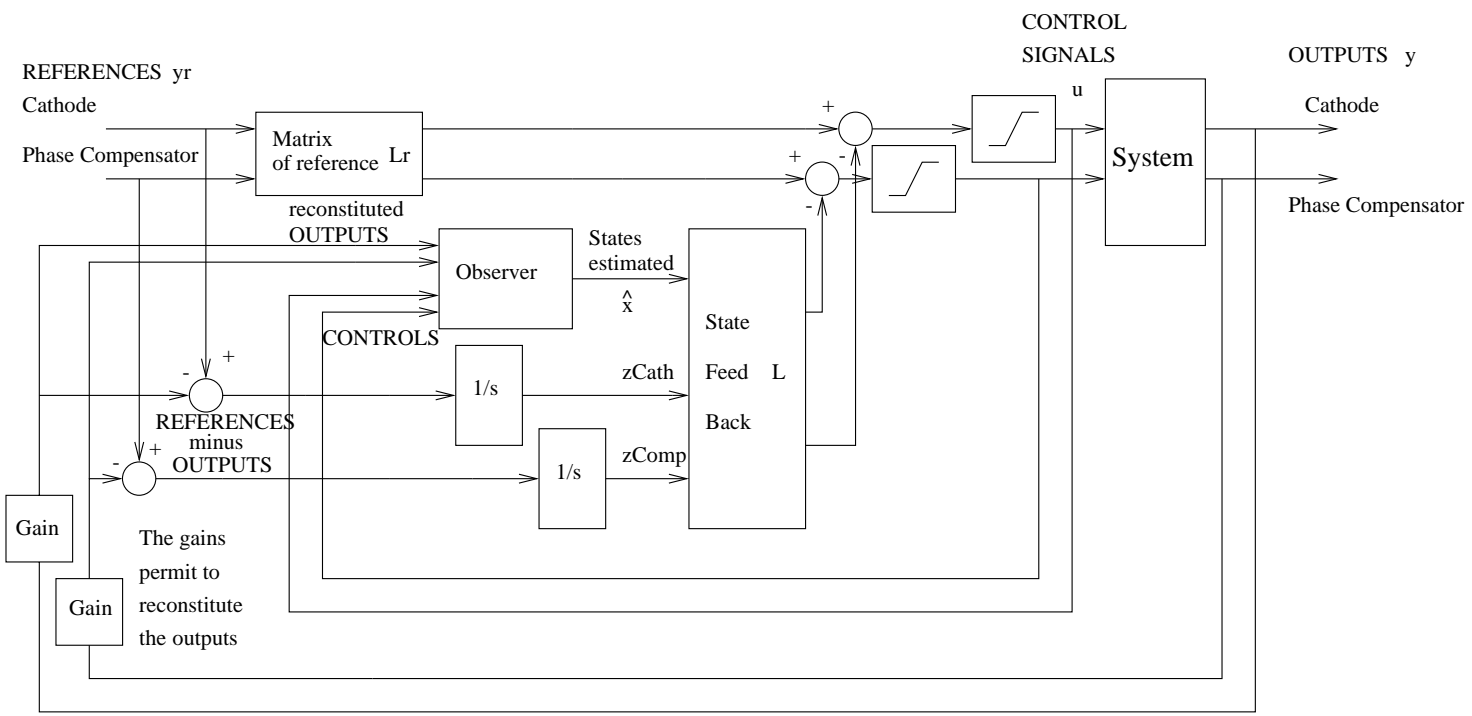
Figure 4.11: The two states for the Integral action, z_{Cath} and z_{Comp}, permit via the new state feedback to stabilize the outputs of the system.

4.8   Antiwindup

Extract from Computer Controlled Systems, Theory and Design, Karl J. Åström and Bjorn Wittenmark, page 331.
"The system [with a limitation on the control signal] can be described linearly when [the control signal] does not saturate. The nonlinearity is thus important when large changes are made. There may be difficulties with the control system during startup and shutdown, as well as during large changes, if the nonlinearities are not considered. A typical example is *integrator windup*."

Antiwindup for the estimated states

A first antiwindup system is designed so that the control signals used by the observer to compute the states are saturated by a model of the saturation that really exists in the system.
Hence the observer takes the good information to compute the state of the control signal.
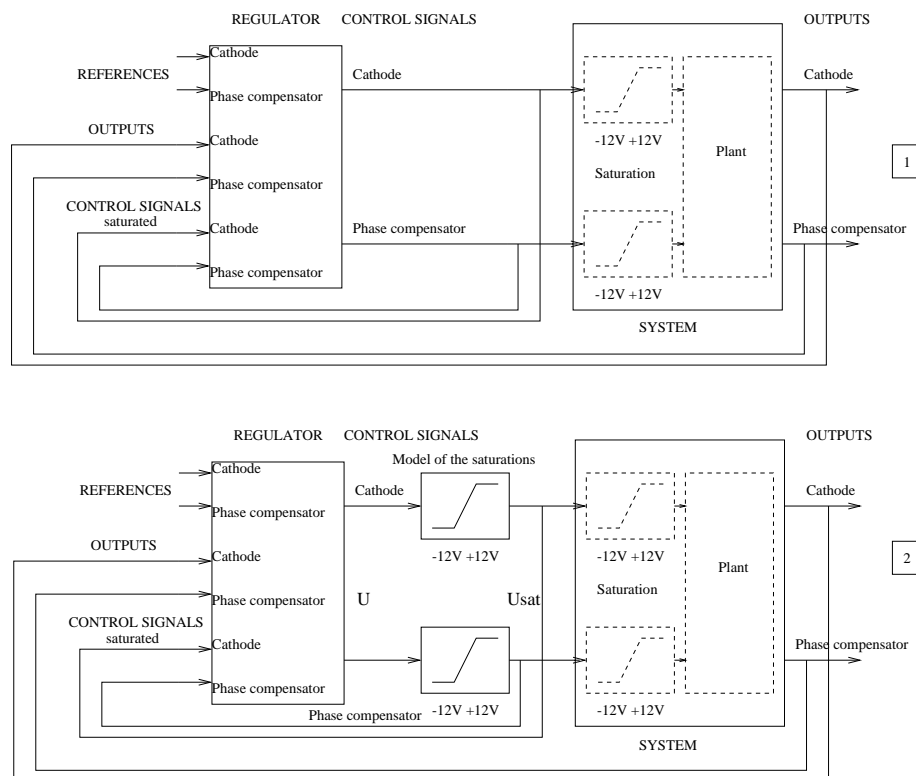
Figure 4.12: In the design $\boxed{1}$, there is no antiwindup. The design $\boxed{2}$ permits to take into account the nonlinearity of the system and hence to design an antiwindup on the controller.

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Antiwindup on the integral action states

Extract from Åström, K. J. and Hägglund Tore (1995) : "PID Controllers: Theory, Design and Tuning"

"The combination of a saturating actuator and a controller with integral action gives rise to a phenomena called *integrator windup*. If the control error is so large that the integrator saturates the feedback path will be broken because the actuator will remain saturated even if the process output changes. The integrator, being an instable system, may then integrate up to a very large value. When the error changes sign the integral may be so large that it takes considerable time until the integral assumes a normal value again."

"The so called *back-calculation* or tracking method works as follows : When the output saturates, the integral is recomputed so that its new value gives an output at the saturation limit. It is advantageous not to reset the integrator instantaneously but dynamically with a time constant T_t.

[...] The system has an extra feedback path that is generated by measuring the actual actuator output and forming an error signal (e_s) as the difference between the output of the controller and the actuator output. Signal e_s is fed to the input of the integrator through gain $1/T_t$. The signal is zero when there is no saturation. Thus , it will not have any effect on the normal operation when the actuator does not saturate. When the actuator saturates, the signal e_s is different from zero. The normal feedback path around the process is broken because the process input remains constant. There is, however, a feedback path around the integrator. Because of this, the integrator output is driven towards a value such that the integrator input becomes zero."
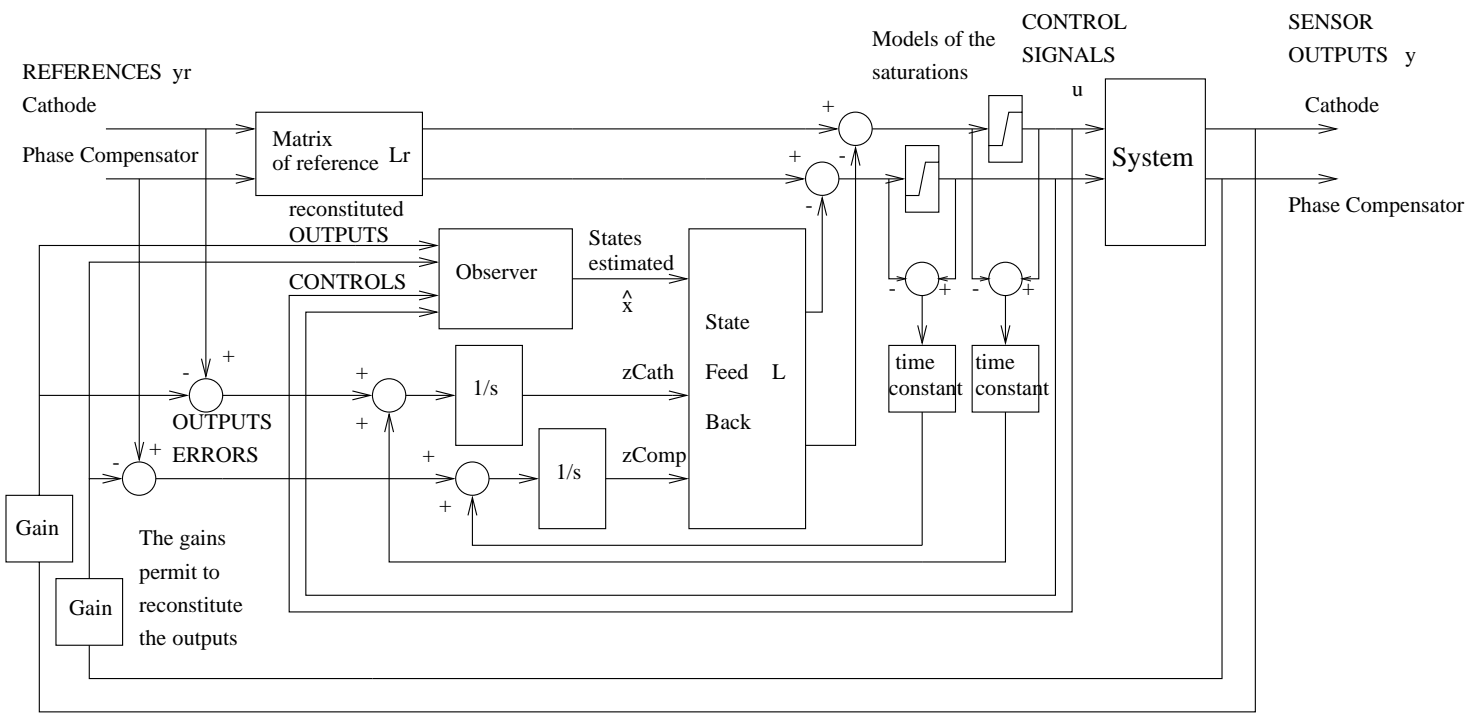
Figure 4.13: The computation of the signal error permits to design via a time constant an antiwindup for the integral action states.

Chapter 5

Control results in Simulation

Since the tune of controller based on the first reduced model of the plant is very long and difficult due to the number of variables, no real results have been found. Only results found with the second reduced model will be presented.

5.1 Simulation parameters in Matlab/Simulink

The method of integration used is a variable-step method. Its name is $ode23t$ (modified stiff / trapezoidal method). The maximal step size during the simulation is $0.1\mu s$. The initial step size, relative tolerance and absolute tolerance are automatically fixed by the method. Even if we lose accuracy during the simulation with this integral routine, the simulation is still reliable and it is the only way to keep the simulation working and to keep the time of the simulation in a reasonable range (some minutes).

5.2 How to judge the performances ?

The criterion C_r is defined by the following formula :

$$Cr = \sqrt{e_1^2 + e_2^2 + \cdots + e_9^2 + e_{10}^2}$$

The e_n, the errors, are computed as following:
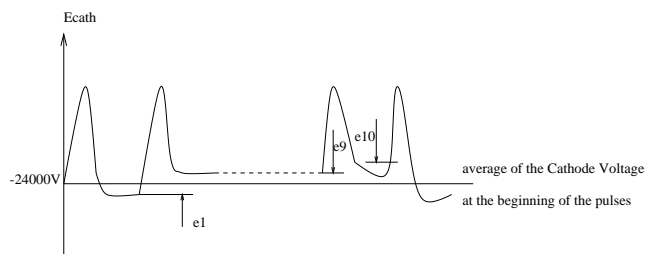
Figure 5.1: The computation of the criterion is performed from the error of the Cathode Voltage at each pulse. The error is computed from the average of the Cathode Voltage at the beginning of the 10 pulses.

For the criterion the pulses are coming alternatively every $200\,\mu s$ and $600\,\mu s$, for example, so that to have a duty cycle of 2.5%.

5.3 Controller 1 : Optimal observer and linear quadratic regulator without modification of the Cathode Voltage sensor

In this section, no modification of the sensor is performed. The same sensor is kept. So an estimation of the Cathode Voltage is done. The state space representation used is presented in section 4.3.5 on page 50.
The weights of the optimal observer and controller are the following :
For the optimal controller, the γ_n are :

$$\gamma_1 = \frac{500\ 000}{24\ 000^2} \qquad \gamma_2 = \frac{0.07}{300^2} \qquad \gamma_3 = \frac{0.03}{12^2} \qquad \gamma_4 = \frac{0.3}{12^2}$$

For the optimal observer, the γ_{1n} are :

$$\gamma_{101} = 10\ 000\ 000 \qquad \gamma_{102} = 100\ 000 \qquad \gamma_{103} = 1\ 000 \qquad \gamma_{104} = 100$$
$$\gamma_{105} = 10\ 000\ 000 \qquad \gamma_{106} = 100\ 000 \qquad \gamma_{107} = 10\ 000\ 000 \qquad \gamma_{108} = 100\ 000$$
$$\gamma_{109} = 1 \qquad \gamma_{110} = 10$$

And the γ_{2n} are :

$$\gamma_{21} = \frac{1}{1\ 000} \qquad \gamma_{22} = \frac{1}{10}$$

The reference of the Phase Compensator is 300 V and the reference of the Cathode Voltage is -24000 V.

5.3.1 General view of the results of the controller 1

Cathode Voltage

Figure 5.2: The Cathode Voltage converges to its reference quite quickly. On the zoom, the disturbances are well rejected.

Phase Compensator Voltage

Phase Compensator Voltage   Zoom during the disturbances

in Volts

Time in seconds

Figure 5.3: The Phase Compensator needs some time reach its value because of the time necessary to the Cathode Voltage to reach its nominal value and also because of the limitation at its output. There is a static error on the Compensator voltage but it is not important for the specification. On the zoom, we can see how the Phase Compensator helps very well the Cathode Power Supply reject the disturbances.

Poles placement

Eigen Values of A (with x), A-B×L (with +) and A-K×C (with o)

General view   Zoom

Imaginary
axis

Real axis

Figure 5.4: The location of the poles is quite typical for the optimal observer and controller.

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5.3.2 Detailed results of the controller 1

Ouput error

Error on the Cathode Voltage
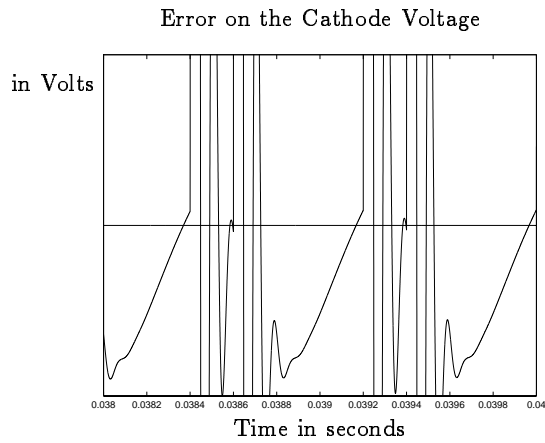
Figure 5.5: No precision about the scale is done because of confidentiality. The values of the Cathode Voltage are very close to each other when the pulses are coming. But there is some "ringing".

Control signals

Control Signals

Solid : Cathode P.S. ; Dashed : Phase Compensator

Figure 5.6: The Phase Compensator control signal saturates a bit to compensate for the disturbances.

Observer error

Observer error

Solid : Cathode Voltage ; Dashed : Phase Compensator Voltage

Time in seconds

Figure 5.7: The estimation error of the Phase Compensator is bigger than the Cathode Voltage because during the weight of optimal problem, we put much more penalty on this signal.

RMS criterion

The RMS criterion is around 50% less than the limit fixed by EMW with pulses at alternatively $200\mu s$ and $600\mu s$. The results are quite good.
When the pulses are coming closer, the performances are degraded.

Drift

If we take a close look at the Phase Compensator voltage, we can see a drift.

Zoom on the inferior limit of the Phase Compensator Voltage

Time in seconds
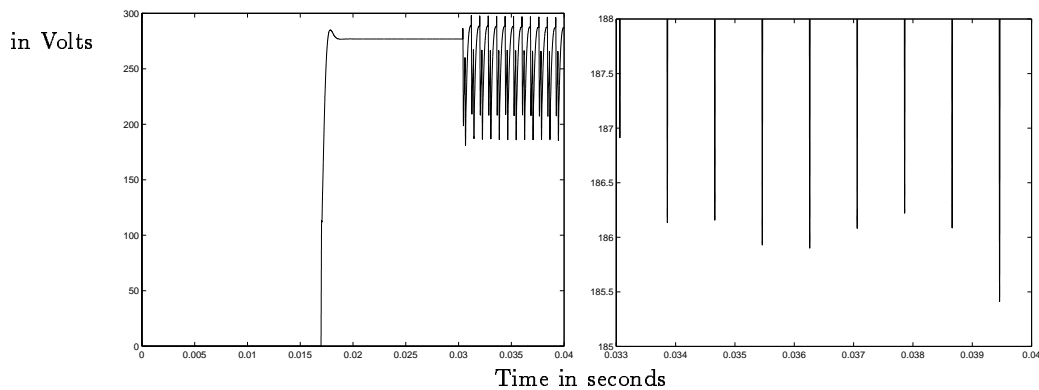
Figure 5.8: A drift appears on the Phase Compensator Voltage.

5.3.3   Conclusion on Controller 1

The specifications are met with pulses alternatively at $200\mu s$ and $600\mu s$. But there is a drift on the phase Compensator Voltage.

5.4 Controller 2 : Optimal observer and linear quadratic regulator with modification of the Cathode Voltage sensor

In this section, a modification of the sensor is performed. We assume that there is an ideal sensor which gives a perfect fraction of the Cathode Voltage. So there is no estimation of the Cathode Voltage. The state space representation used is presented in section 4.3.4 on page 49.

The weights of the optimal observer and controller are the following :

For the optimal controller, the γ_n are :

$$\gamma_1 = \frac{500\ 000}{24\ 000^2} \qquad \gamma_2 = \frac{0.07}{300^2} \qquad \gamma_3 = \frac{1}{12^2} \qquad \gamma_4 = \frac{1}{12^2}$$

For the optimal observer, the γ_{1n} are :

$$\gamma_{11} = 10\ 000\ 000 \qquad \gamma_{12} = 100\ 000 \qquad \gamma_{13} = 1\ 000 \qquad \gamma_{14} = 100$$
$$\gamma_{15} = 10\ 000\ 000 \qquad \gamma_{16} = 100\ 000 \qquad \gamma_{17} = 10\ 000 \qquad \gamma_{18} = 10\ 000$$

And the γ_{2n} are :

$$\gamma_{21} = \frac{1}{1\ 000} \qquad \gamma_{22} = \frac{1}{10}$$

The reference of the Phase Compensator is 300 V and the reference of the Cathode Voltage is -24 kV.

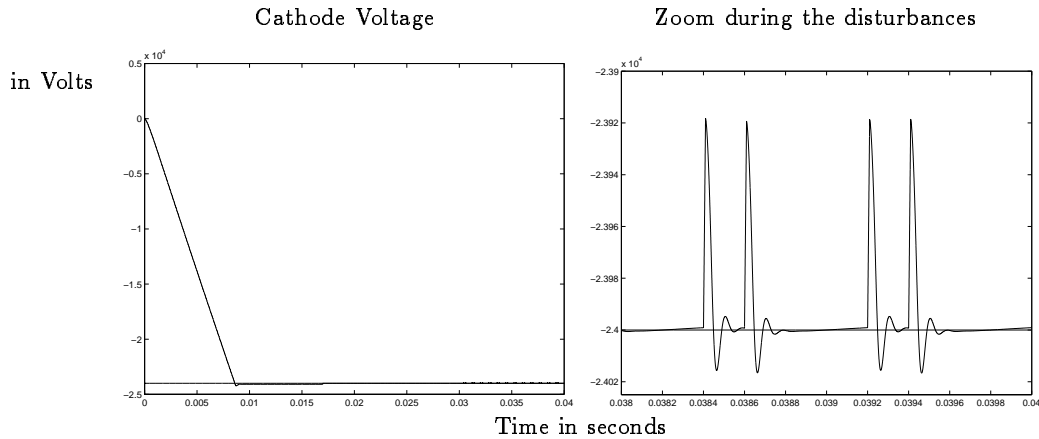### 5.4.1 General view of the results of the controller 2

Cathode Voltage

Figure 5.9: The Cathode Voltage converges to its reference quite quickly. On the zoom, the disturbances are also well rejected.

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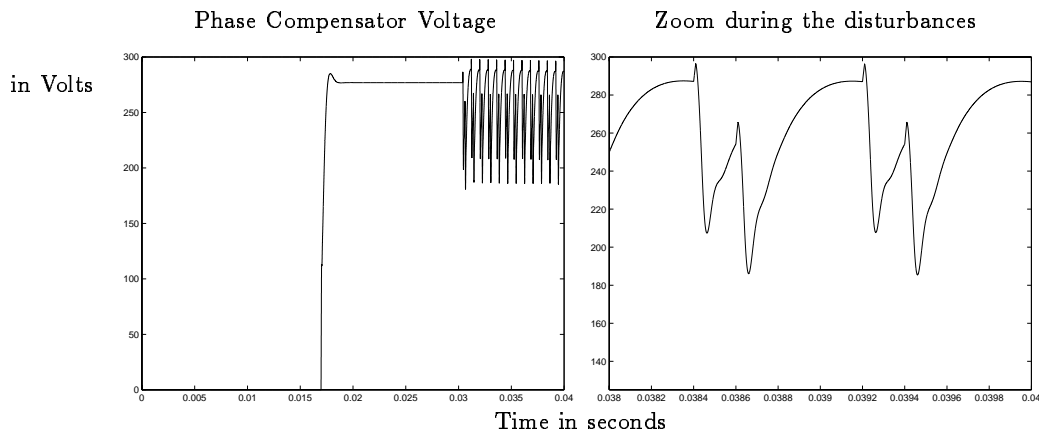**Phase Compensator Voltage**
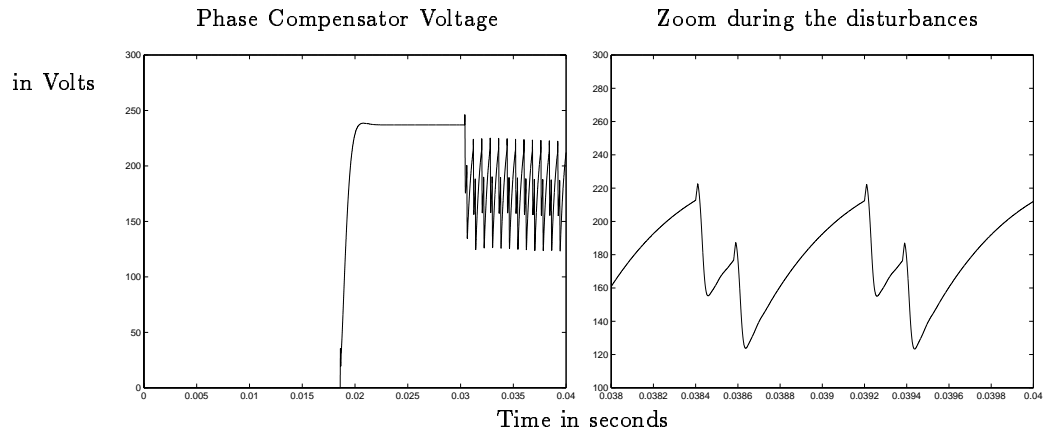
Figure 5.10: The Phase Compensator needs some time to reach its value because of the time necessary to the Cathode Voltage to reach its nominal value and also because of the limitation at its output. There is a static error on the Compensator voltage but it is not important for the specification. On the zoom, we can also see how the Phase Compensator helps very well the Cathode Power Supply to reject the disturbances even better than for the controller 1.
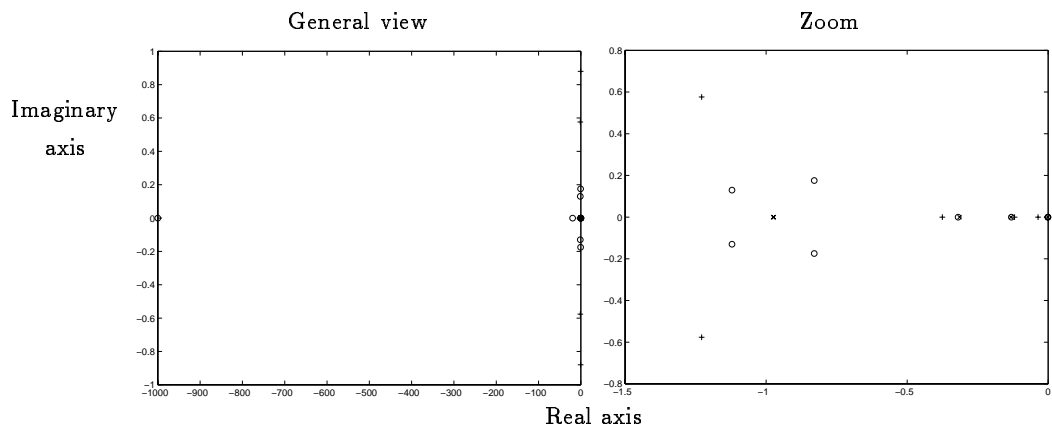
Poles placement

Figure 5.11: The location of the poles is quite typical for the optimal observer and controller.

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## 5.4.2 Detailed results of the controller 2

Ouput error

Error on the Cathode Voltage
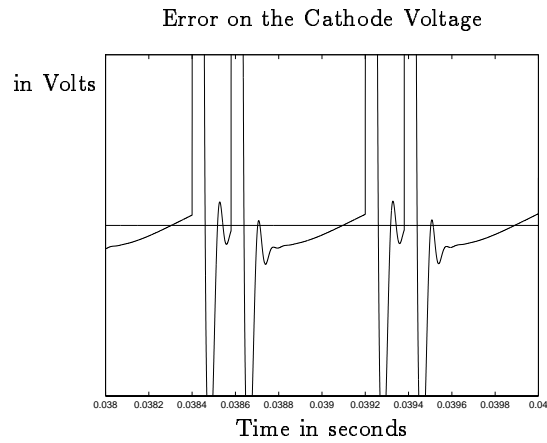
Figure 5.12: The values of the Cathode Voltage are even closer to each other when the pulses are coming. Comparing to the controller 1, there is much less "ringing".

Control signals

Control Signals

Solid : Cathode P.S. ; Dashed : Phase Compensator
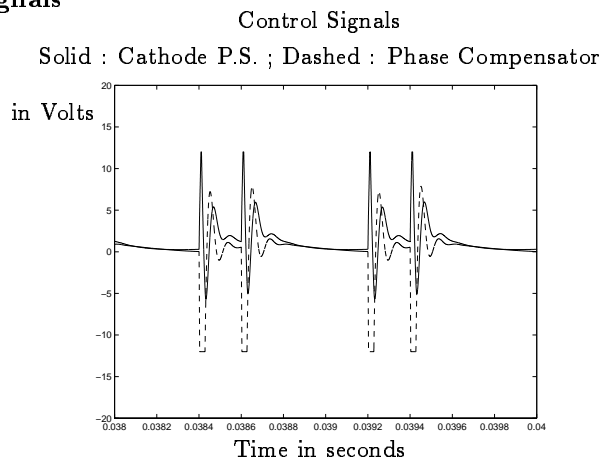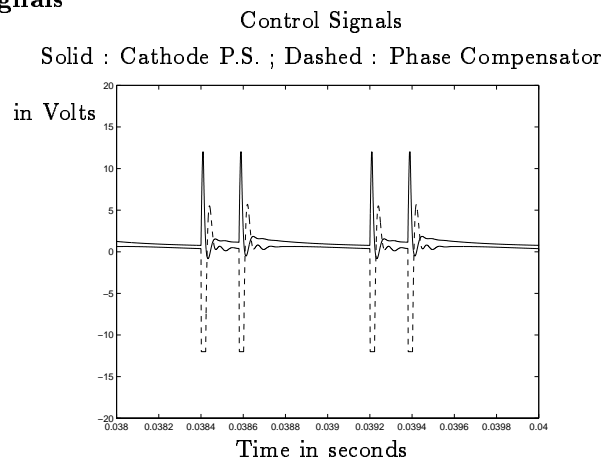

Figure 5.13: Both control signals saturate a bit to compensate the disturbance.

Observer error

Observer error

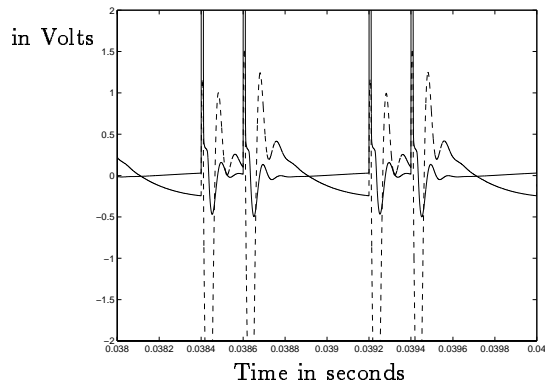Solid : Cathode Voltage ; Dashed : Phase Compensator

in Volts

Time in seconds
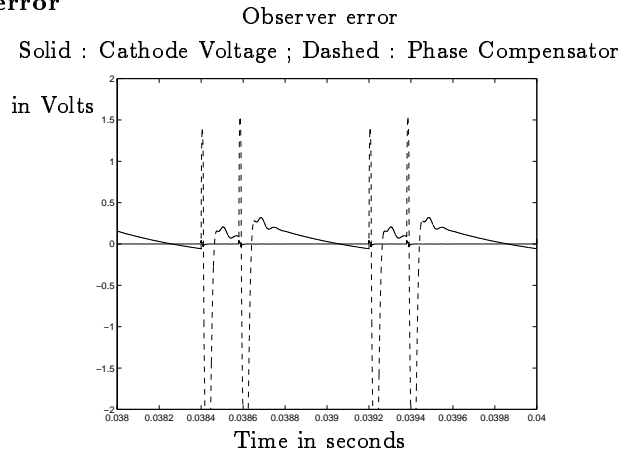
Figure 5.14: The difference between the two estimations is even bigger than for the controller 1.

RMS criterion

The RMS criterion is around 55% less than the limit fixed by EMW with pulses at alternatively every $180\mu s$ and $620\mu s$. The results are therefore very good. When the pulses are coming closer, the performances are also degraded.

Drift

If we take a close look at the Phase Compensator voltage, we can see a drift.

Zoom on the inferior limit of the Phase Compensator Voltage

in Volts

Time in seconds
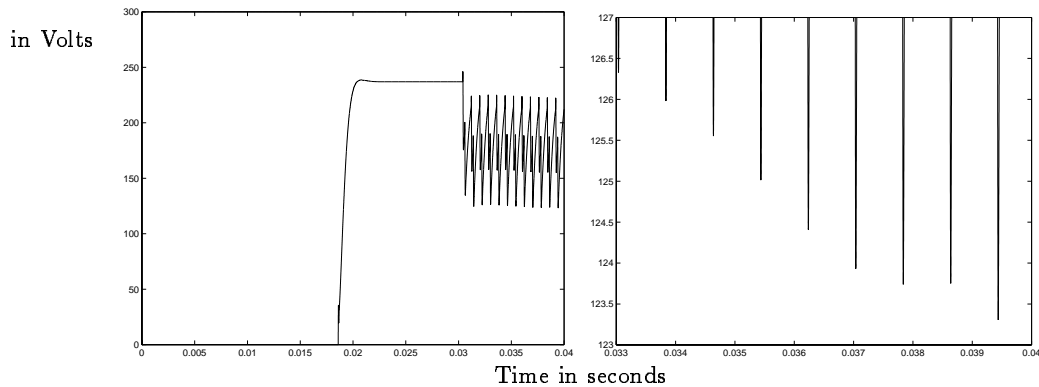
Figure 5.15: A drift appears clearly on the picture.

5.4.3   Conclusion on the controller 2

The controller 2 includes 8 states. The specifications are met with pulses alternatively at $180\mu s$ and $620\mu s$. The results are slightly better. But there is a drift on the phase Compensator Voltage. To try to solve the drift problem, we will introduce integral action in the controller.

5.5 Controller 3 : Addition of integral action

In this section, a modification of the plant is performed. We assume that there is an ideal sensor which gives a perfect fraction of the Cathode Voltage. So there is no estimation of the Cathode Voltage. The state space representation used is presented in section 4.3.4 on page 49 with the addition of the integral action on each output presented in section 4.7 on page 57.

The weights of the optimal observer and controller are the following :

For the linear, quadratic optimal regulator, the γ_n and the ϵ_n are :

$$\gamma_1 = \frac{2\ 500\ 000}{24000^2} \qquad \gamma_2 = \frac{10}{200^2} \qquad \gamma_3 = \frac{1}{12^2} \qquad \gamma_4 = \frac{1}{12^2};$$
$$\epsilon_1 = 0.000\ 001 \qquad \epsilon_2 = 0.000\ 1$$

For the optimal observer, the γ_{1n} are :

$$\gamma_{11} = 10\ 000\ 000 \qquad \gamma_{12} = 100\ 000 \qquad \gamma_{13} = 1\ 000 \qquad \gamma_{14} = 100$$
$$\gamma_{15} = 10\ 000\ 000 \qquad \gamma_{16} = 100\ 000 \qquad \gamma_{17} = 10\ 000 \qquad \gamma_{18} = 10\ 000$$

And the γ_{2n} are :

$$\gamma_{21} = \frac{1}{1\ 000} \qquad \gamma_{22} = \frac{1}{10}$$

The reference of the Phase Compensator is 300 V and the reference of the Cathode Voltage is -24000 V.

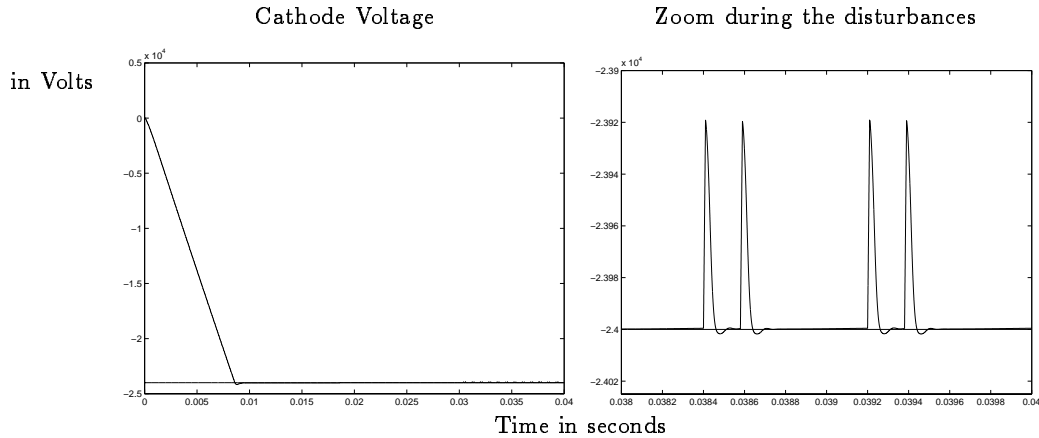### 5.5.1 General view of the results of the controller 3

Cathode Voltage
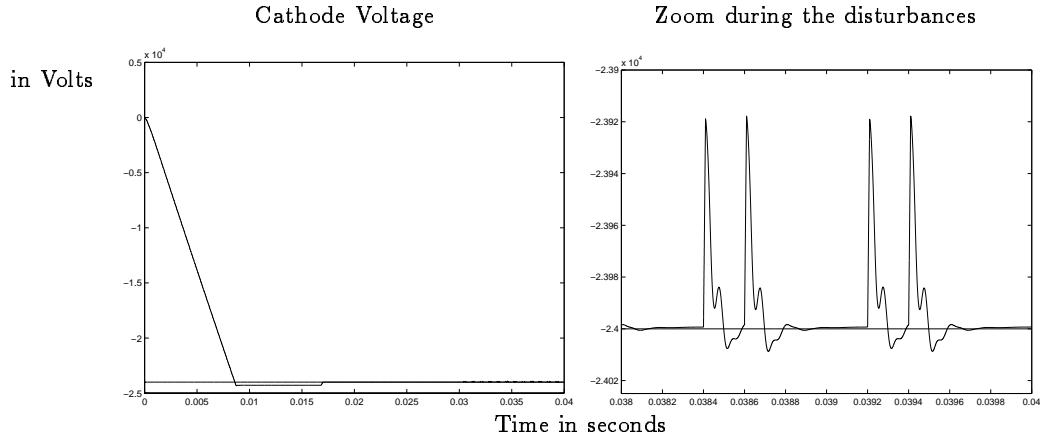
Figure 5.16: The Cathode Voltage converges to its reference also quite quickly. On the zoom, the disturbances are not rejected so quickly compared to the previous controllers.

Phase Compensator Voltage
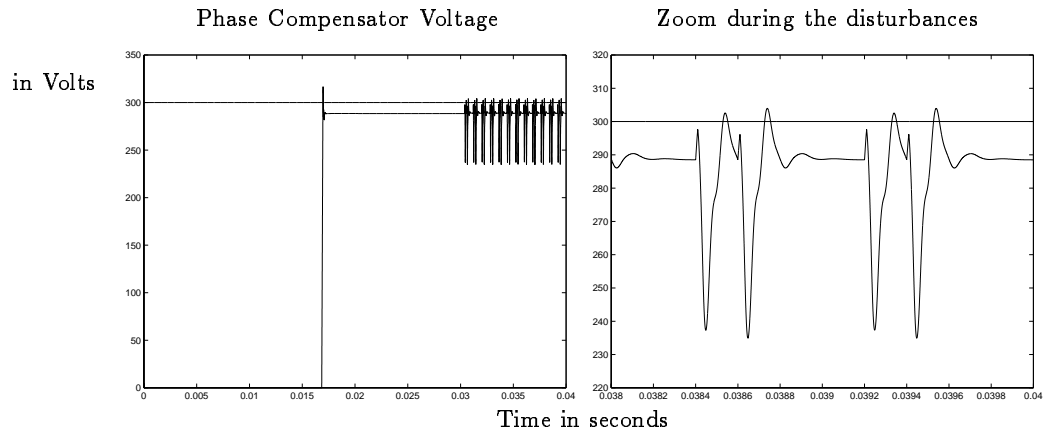
Figure 5.17: The Phase Compensator is much more stable but we can observe an overshot on the zoom when the disturbances are coming. This is the integral action effect that slows a bit the controller in general.
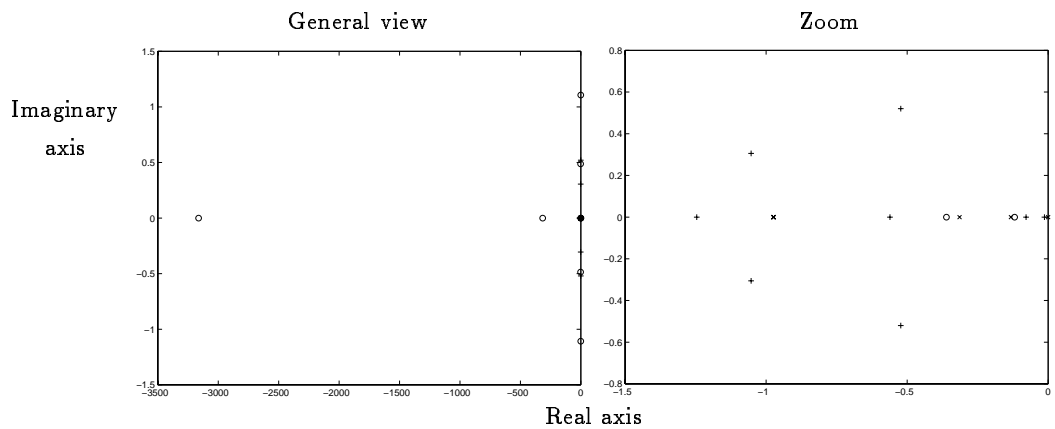
Poles placement
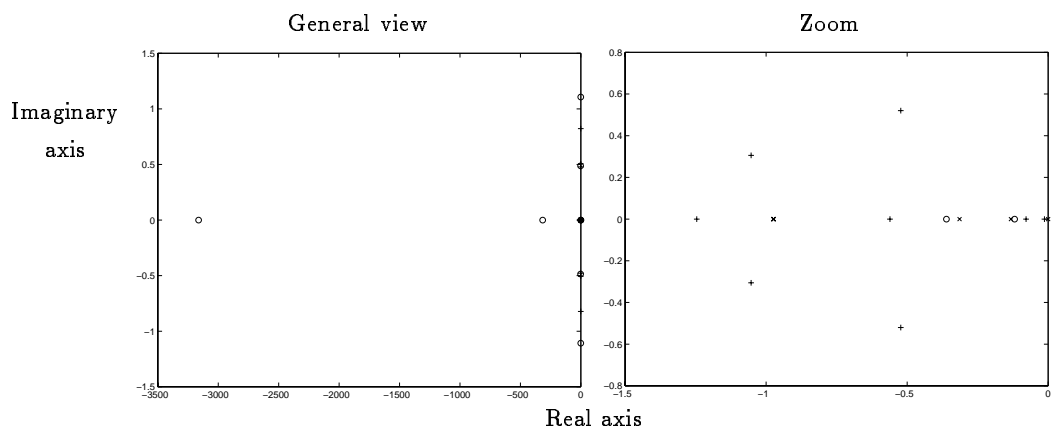
Figure 5.18: The location of the poles is also quite typical for the optimal observer and controller.

5.5.2 Detailed results of the controller 3

Ouput error

Error on the Cathode Voltage
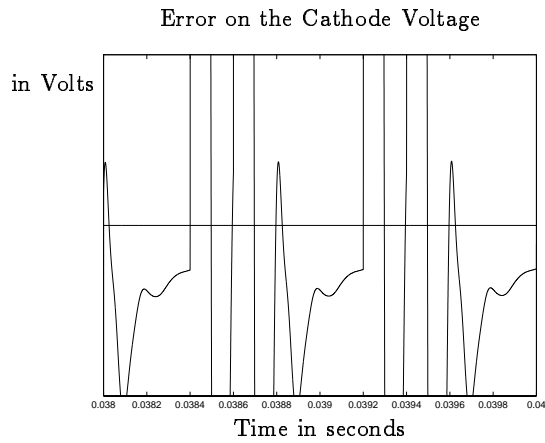
Figure 5.19: There is not any real "ringing". But the disturbances do not have the time to be rejected by the system. The second pulse comes a bit too early.

Control signals

Control Signals
Solid : Cathode P.S. ; Dashed : Phase Compensator
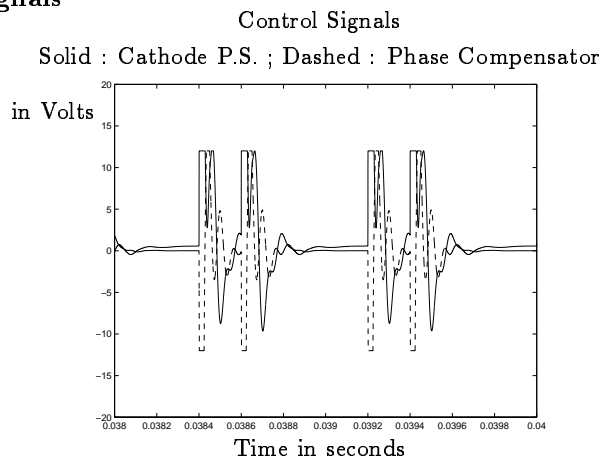
Figure 5.20: Both control signals saturate quite a lot to compensate the disturbance and for the drift.

Observer error

Observer error
Solid : Cathode Voltage ; Dashed : Phase Compensator
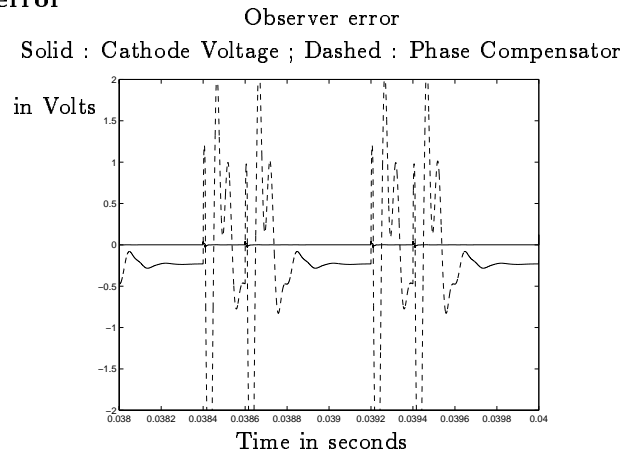
Figure 5.21: The estimation of the Phase Compensator Voltage is quite bad probably because of the saturation of the control signal.

RMS criterion

With pulses coming alternatively every $200\mu s$ and $600\mu s$, the criterion is 260% times bigger than the limit fixed by EMW.

5.5.3   Conclusion on the controller 3

The controller 3 includes 10 states.
There is not anymore a problem of drift on the Phase Compensator Voltage but the specifications are not met.

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5.6 Conclusion on all the results

The controller succeeds to reject the disturbances as specified and quite quickly in the case of an ideal sensor of the Cathode Voltage (Controller 2). But there remains the drift problem. Hence, if the duty cycle was continuously 2.5%, it would not be possible to implement the regulator. But since the duty cycle is an entire part of the specifications, it does not seem realistic to implement this controller because the Phase Compensator voltage will hit the limitation and all the performance will be lost. However, the controller 2 seems the best controller

because it includes only 8 states :

$$\dot{\hat{x}} = (\mathbf{A} - \mathbf{KC})\hat{x} + \mathbf{B}u_{sat} + \mathbf{K}y$$
$$u = -\mathbf{L}\hat{x} + \mathbf{L}rr$$

With u_{sat} : the control signal after the model of the saturations
in the antiwindup mechanism

$$\mathbf{A} - \mathbf{KC} = \begin{bmatrix} -3.8956\ e - 05 & -7.1135\ e - 06 & -2.8866\ e - 07 & -4.3925\ e - 09 \\ 8.0000\ e - 05 & 0 & 0 & 0 \\ 0 & 1.6000\ e - 04 & 0 & 0 \\ 0 & 0 & 1.6000\ e - 04 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8.2360\ e - 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -6.6712\ e - 02 & -1.7820\ e - 05 \\ 0 & 0 & -4.1634\ e - 01 & 7.4310\ e - 04 \\ 0 & 0 & 1.0489\ e + 01 & 5.7299\ e - 05 \\ 0 & 0 & 3.5896\ e + 02 & -8.6653\ e - 01 \\ -4.4611\ e - 06 & -5.1815\ e - 08 & -6.5342\ e - 02 & -1.7954\ e - 02 \\ 8.0000\ e - 05 & 0 & 1.0614\ e - 01 & -2.2982\ e - 01 \\ 5.2713\ e - 08 & 4.5541\ e - 10 & -3.1632\ e - 02 & -1.0672\ e - 08 \\ 5.2713\ e - 08 & 4.5541\ e - 10 & -1.0672\ e - 06 & -3.1626\ e - 03 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 8.0000\ e - 05 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2.5600\ e - 03 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 6.6712e - 02 & 1.7820e - 05 \\ 4.1634\ e - 01 & -7.4310\ e - 04 \\ -1.0489\ e + 01 & -5.7299\ e - 05 \\ -3.5896\ e + 02 & 8.6653\ e - 01 \\ 6.5342\ e - 02 & 1.7954\ e - 02 \\ -1.0614\ e - 01 & 2.2982\ e - 01 \\ 3.1632\ e - 02 & 1.0672\ e - 08 \\ 1.0672\ e - 06 & 3.1626\ e - 03 \end{bmatrix} \quad \mathbf{L^T} = \begin{bmatrix} 0.0074072 & -0.013614 \\ 0.003727 & -0.0078443 \\ 0.00035456 & -0.00092716 \\ 1.5036e - 05 & -5.7281\ e - 05 \\ -0.00042543 & 0.0025355 \\ -1.1004e - 06 & 1.5149\ e - 05 \\ -0.044374 & 0.35076 \\ 0.010499 & 0.0013283 \end{bmatrix}$$

$$\mathbf{L_r} = \begin{bmatrix} -0.044374 & 0.010499 \\ 0.35076 & 0.0013283 \end{bmatrix}$$

y_1 is directly the Cathode Voltage in this case without any voltage divider or sensor. Since during the simulation we do the computation in a different time domain we need to come back to the good one.

$$\frac{1}{t'} = 10^{-5}\frac{1}{t} \quad \Rightarrow t' = 10^5 t$$
$$\frac{dx}{dt'} = \frac{dx}{10^5 dt} = \mathbf{A}\,x + \mathbf{B}\,u$$
$$y = \mathbf{C}\,x + \mathbf{D}\,u$$

$$\Rightarrow \quad \frac{dx}{dt} = 10^{-5}\left(\mathbf{A}\,x + \mathbf{B}\,u\right)$$
$$y = \mathbf{C}\,x + \mathbf{D}\,u$$

This computation is done to get the matrix listed above.

Chapter 6

Robustness analysis of a Multivariable (MIMO) system

Since the system is Multi-Inputs Multi-Outputs (MIMO), it is difficult to have a classical approach (Gain Margin, Phase Margin) of the robustness problem.
It is also difficult to say if the robust control approach is sufficient and to show a unique diagram which we have to look at.
That is why a more pragmatic approach typically by varying some parameters in the simulation will be also performed.

6.1   Singular values, robust control approach

To perform a global approach of the problem, some tools will be introduced.
A study of the singular values of the complementary sensitivity equation will be performed. The multiplicative uncertainty will be chosen.

Introduction to the singular values

Extracts from Zhou K. and J. Doyle (1998) : "Essentials of Robust Control", p. 19.
"A very useful tool in matrix analysis is *singular value decomposition (SVD)*. it will be seen that singular values of a matrix are good measures of the "size" of the matrix and that the corresponding singular vectors are good indications of strong/weak input or output directions."

Let $A \in \mathbb{F}^{m \times n}$.   There exist unitary matrices

$$\mathbf{U} = [u_1, u_2, \ldots, u_m] \in \mathbb{F}^{m \times n}$$
$$\mathbf{V} = [v_1, v_2, \ldots, v_m] \in \mathbb{F}^{m \times n}$$

such that

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^{\mathbf{T}}, \qquad \Sigma = \left[\begin{array}{cc} \Sigma_1 & 0 \\ 0 & 0 \end{array} \right],$$

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where

$$\Sigma_1 = \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 \\ 0 & \sigma_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_p \end{bmatrix}$$

and

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p \geq 0, \quad p = min\{m, n\}.$$

The following notations for the singular values are often adopted:

$$\bar{\sigma}(\mathbf{A}) = \sigma_{max}(\mathbf{A}) = \sigma_1 = \text{the largest singular value of } \mathbf{A};$$

and

$$\underline{\sigma}(\mathbf{A}) = \sigma_{min}(\mathbf{A}) = \sigma_p = \text{the smallest singular value of } \mathbf{A};$$

Justification of this approach

See also section 8.2 of Zhou [9].

On the following diagram, $\mathbf{P}(s)$ is the Transfer Function matrix corresponding to the plant (plant+sensors) and $\mathbf{C}(s)$ is the controller Transfer Function matrix corresponding to the controller.

Δ is a non-linearity added to the system to symbolize the uncertainties that we have on the system and $\mathbf{W}(s)$ is the weight matrix.

r is the reference-vector and y is the outputs-vector.

The computation of the equivalent system between the points A and B will
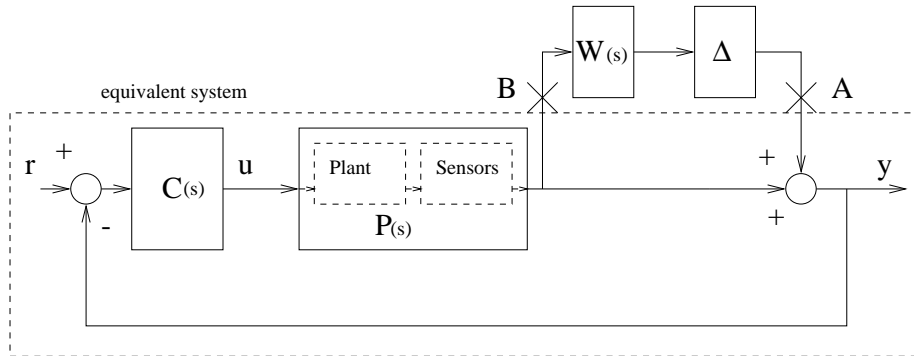
Figure 6.1: The output multiplicative uncertainty perturbes the system and hence permits to simulate the uncertainties of the total system (controller + plant).

give the complementary sensitivity function of the system :

$$\mathbf{T}(\mathbf{s}) = \mathbf{P}(\mathbf{s})\mathbf{C}(\mathbf{s})[\mathbf{I} + \mathbf{P}(\mathbf{s})\mathbf{C}(\mathbf{s})]^{-1}$$

Computation of $\mathbf{C}(s)$:
Transfer Function matrix of the controller

$$\begin{aligned} \mathbf{C}(\mathbf{s}) &= \mathbf{C_C}(s\mathbf{I} - \mathbf{A_C})^{-1}\mathbf{B_C} \\ &= -\mathbf{L}(s\mathbf{I} - (\mathbf{A} - \mathbf{KC} - \mathbf{BL}))^{-1}\mathbf{K} \end{aligned} \quad \text{with} \quad \begin{aligned} \mathbf{A_C} &= \mathbf{A} - \mathbf{KC} - \mathbf{BL} \\ \mathbf{B_C} &= \mathbf{K} \\ \mathbf{C_C} &= -\mathbf{L} \end{aligned}$$

The Outputs (control signal vector: u) - Inputs (output vector of the system : y) relationship of the controller is described in the state space representation by the following equation :

$$\begin{cases} \dot{\hat{x}} = (\mathbf{A} - \mathbf{KC} - \mathbf{BL})\hat{x} + \mathbf{K}y \\ u = -\mathbf{L}\hat{x} \end{cases}$$

Where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are the state space representation of model of the system used for the observer.

Computation of $\mathbf{P}(s)$:
Transfer Function matrix of the plant

$\mathbf{T_{Cath}}$ is computed in section 3.2.1 on page 21 and $\mathbf{T_{Comp}}$ is computed in section 3.2.2 on page 23 during the modeling of the plant.
The following equations define the relationship between $\mathbf{A_{Cath}}$, $\mathbf{B_{Cath}}$, $\mathbf{C_{Cath}}$, $\mathbf{A_{Comp}}$, $\mathbf{B_{Comp}}$, $\mathbf{C_{Comp}}$ and $\mathbf{T_{Cath}}$ $\mathbf{T_{Comp}}$:

$$\mathbf{T_{Cath}} = \mathbf{C_{Cath}}(s\mathbf{I} - \mathbf{A_{Cath}})^{-1}\mathbf{B_{Cath}}$$
$$\mathbf{T_{Comp}} = \mathbf{C_{Comp}}(s\mathbf{I} - \mathbf{A_{Comp}})^{-1}\mathbf{B_{Comp}}$$

With $\mathbf{A_{Cath}}$, $\mathbf{B_{Cath}}$, $\mathbf{C_{Cath}}$ the state space representation of the complete model of the Cathode Power Supply and with $\mathbf{A_{Comp}}$, $\mathbf{B_{Comp}}$, $\mathbf{C_{Comp}}$ the state space representation of the complete model of the Phase Compensator introduced previously in figure 3.1 on page 20, we can construct the state space representation $(\mathbf{A_P}, \mathbf{B_P}, \mathbf{C_P})$ of the plant :

$$\mathbf{A_P} = \begin{bmatrix} \mathbf{A_{Cath}} & 0 \\ 0 & \mathbf{A_{Comp}} \end{bmatrix} \qquad \mathbf{B_P} = \begin{bmatrix} \mathbf{B_{Cath}} & 0 \\ 0 & \mathbf{B_{Comp}} \end{bmatrix}$$

$$\mathbf{C_P} = \begin{bmatrix} \mathbf{C_{Cath}} & \mathbf{C_{Comp}} \\ 0 & \mathbf{C_{Comp}} \end{bmatrix}$$

$\mathbf{T_{Sensor_{Cath}}}$ computed in section 3.4.2 on page 34 and $\mathbf{T_{Sensor_{Comp}}}$ computed in section 3.4.1 on page 32 are used to take care of the sensors dynamics in the following formula to define the $\mathbf{P}(s)$:

$$\mathbf{P}(s) = \begin{bmatrix} \mathbf{T_{Sensor_{Cath}}} \\ \mathbf{T_{Sensor_{Comp}}} \end{bmatrix} \mathbf{C_P}(s\mathbf{I} - (\mathbf{A_P}))^{-1}\mathbf{B_P}$$

$$= \begin{bmatrix} \mathbf{T_{CathCath}}(s) & \mathbf{0} \\ \mathbf{T_{CompCath}}(s) & \mathbf{T_{CompComp}}(s) \end{bmatrix}$$

The formula above has a $\mathbf{0}$ instead of $\mathbf{T_{CathComp}}(s)$ because there is no relationship between the input of the Cathode Power Supply and the Phase Compensator Voltage Sensor output, see figure 3.1 on page 20.

$\mathbf{T_{CathCath}}(s)$ is the transfer function integrating the sensor between the input of the Cathode Power Supply and the Cathode Voltage Sensor output.
$\mathbf{T_{CompComp}}(s)$ is the transfer function integrating the sensor between the input of the Phase Compensator and the Phase Compensator Voltage Sensor output.
$\mathbf{T_{CompCath}}(s)$ is the transfer function integrating the sensor between the input of the Phase Compensator and the Cathode Voltage Sensor output.

Robustness criterion

By looking at the largest singular values of frequency response of the complementary sensitivity function in all the frequency domain, a robustness criterion can be deduced.

Results

The computation of the robustness criterion is done on the controller 2.

Maximum singular value
of the complementary sensitivity function

Frequency in Herz
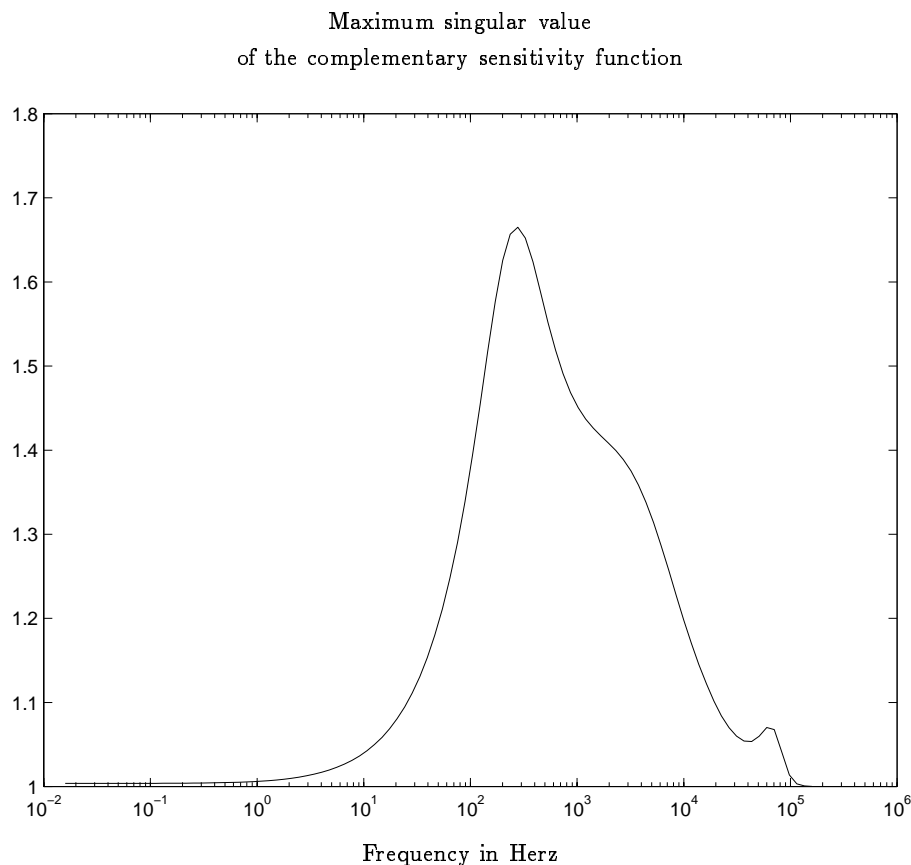
Figure 6.2: The maximum point of the curve is 1.7 so it means that 60% ($\frac{1}{1.7}$) of uncertainty is tolerated for stability in all the frequency domain. The disturbances are mainly around 1 000 Hz and the uncertainty on the plant is mainly at high frequency. At those frequencies, the percentage of uncertainty tolerated is increasing to 100% ($P(I + W\Delta) \simeq 2P$).

A good robustness stability seems to be guaranteed. But it is difficult to conclude on the robustness of the performance. A study of the performance when the controller and the plant change should be performed.

6.2 System faced with the variations of the plant and the controller

In this study, we will modify the system. We will simulate a modification of the system in order to look at the behavior of the system when there is a variation of the plant due to the modeling for an example or when the implementation is different from the ideal computed controller.

The study will be done about the performance when the gains will be modified of - 30% and + 30%, and the frequencies also of - 30% and + 30%.

Variations on the Plant

	$\times$ 1.3	$\times$ 0.7
Gain (variation of $G_{0_{Comp}}$ and $G_{0_{Cath}}$)	+ 240% (+ 10%)	+ 25% (- 45%)
Main frequency (variation of f_2 and f_{13})	+ 20% (- 50%)	+ 340% (+ 50%)

Variations of the Controller

	$\times$ 1.3	$\times$ 0.7
Gain (variation of measurement of the outputs)	+ 10% (- 50%)	+ 30% (- 40%)

The first percentage is the difference with the RMS criterion found in the initial result (result of the controller 2 descibed in section 5.4.2.). The second one is the difference compared to the EMW criterion limit.

At the view of this table, we can conclude that the controller is also quite robust in performance for quite small variation (30%) of the parameters of the system.

Chapter 7

Preliminary studies about the implementation of controller based on state space representation

There are several possibilities to implement a controller based on the state space representation. The one presented below permits not to be redundant in terms of states and follows precisely the state space approach.
The example of controller presented below has no relationship with the previous controllers.

7.1 By which kind of analog electronics should this kind of regulator be implanted ?

The two or three basic elements used will be the summator, the integrator and the invertor.

Summator

The summator will permit to do the basic additive operation neccessary to translate linear algebra operation.
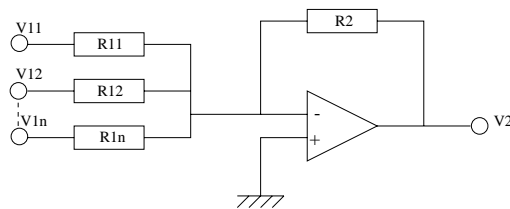
Figure 7.1: A summator permits to accomplish the basic additive operation of the tensions V_{1n}.

The following equation defines the behavior of the summator :

$$V_2 = -\frac{R_2}{R_{11}}V_{11} - \frac{R_2}{R_{12}} - \ldots - \frac{R_2}{R_{1n}}V_{1n}$$

Integrator

The integrator will permit to do the basic integration operation neccessary to translate state space representation.
The following schematic defines the integrator. The following equation defines
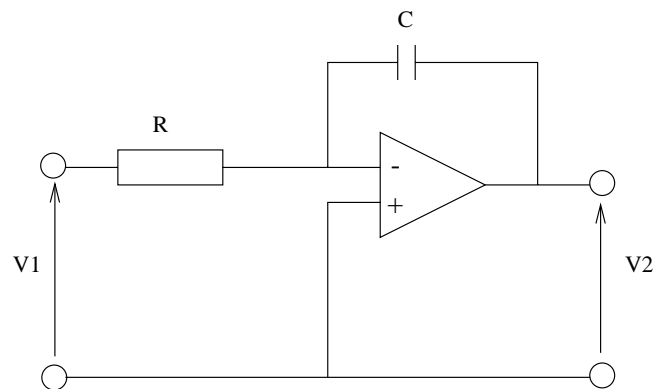
Figure 7.2: An integrator permits to accomplish the basic integration routine necessary to update a state.

the behavior of the integrator :

$$V_2 = -\frac{1}{RC}\int_0^t V_1 \quad ds \qquad \Rightarrow \qquad \dot{V_2} = -\frac{1}{RC}V_1$$

7.2 An example of how to design the analog controller (from the state space representation to the schematic)

The following state space representation defines a controller from the inputs of controller, vector u (outputs of the system to control in our case) to the outputs of the controller, vector y (control signals in our case). The state space representation should be computed in order to have a **A** matrix as diagonal as possible to prevent the saturation of the operational amplifier used in the implementation.

From the following state space representation, an implementation will be performed:

$$\dot{x} = \begin{bmatrix} -2\zeta\omega & \omega^2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{pmatrix} 0 & \omega^2 \end{pmatrix} x$$

$$x_1 = -\int \left(2\zeta\omega x_1 + \omega^2 x_2 - u\right) \, dt$$
$$x_2 = -\int (-x_1) \, dt$$
$$y = \omega^2 x_2$$

So one integrator summator, one integrator and one inversor will be used as shown on figure 7.3 on page 86.

Now, we should choose the resistors R_n so that :

$$R_1 C_1 = 1$$

$$R_2 C_1 = \tfrac{1}{2\zeta\omega}$$

$$R_6 C_1 = 1$$

$$R_3 C_2 = 1$$

$$\tfrac{R_5}{R_4 + R_5} = \omega^2$$

The numerical values will be for $\omega = 1$ and $\zeta = 0.7$:

$$R_1 = R_6 = R_3 = 470 k\Omega$$
$$C_1 = C_2 = 2\mu F$$
$$R_2 = 330 k\Omega$$
$$R_4 = 0\Omega$$
$$R_5 = 47 k\Omega$$

7.3 Remarks

A very important point in this kind of implementation is to have a **A** matrix as diagonal as possible.

The nonlinearities (typically anti-windup) can be quite easily added to the system because the state space structure is respected.
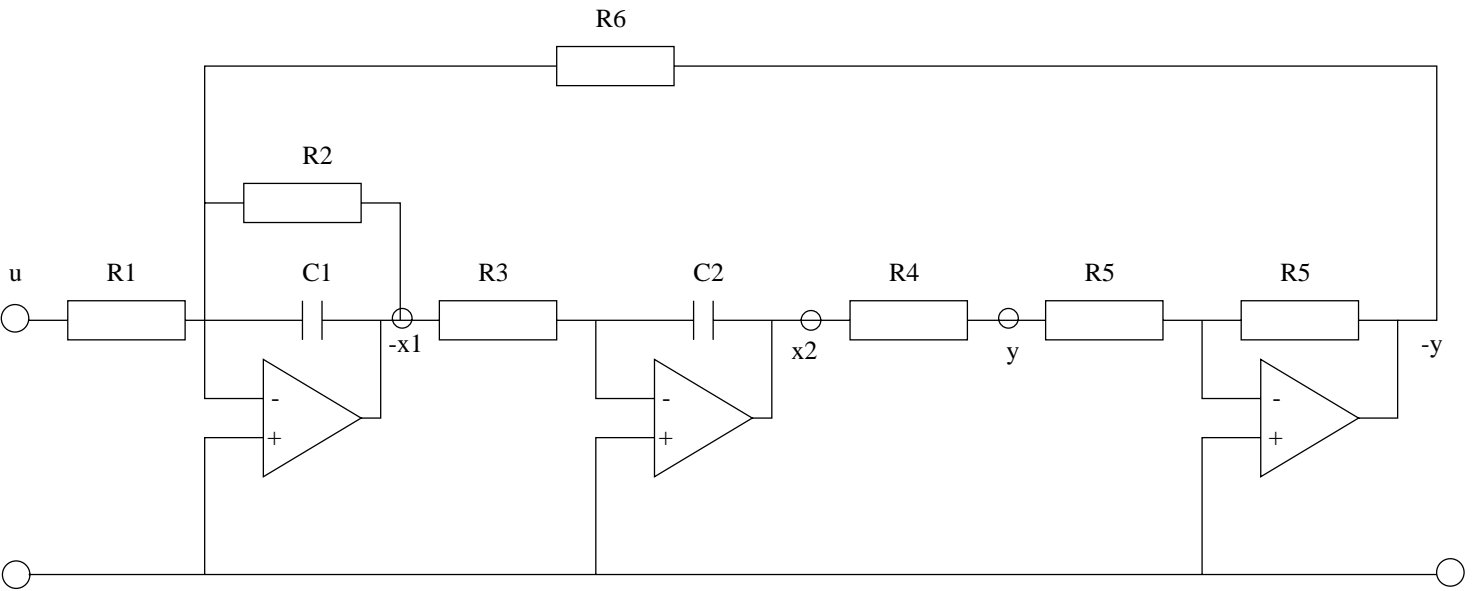
Figure 7.3: This kind of implementation permits to construct each state of the space space representation of the controller.

Chapter 8

Last control results in simulation

Since the last meeting with EMW (July 2000, the 12th), a question has appeared in the modeling of the disturbances. If we take a look at the disturbances, presented in section 3.3.5, we can see that the total of disturbances is increasing all the time.
This does not seem to be correct and to provoke the problem of the drift on the Phase Compensator Voltage.

The Controller 2 is used in the results with the following modification on the modeling of the system.

8.1 Modification of the current pulses

The modification of the modeling of the pulses is the following. The idea is to keep only the AC component of the last modeling of the pulses by removing the DC Component of 0.025 A (2.5% of duty cycle).
In this case the pulses will go from -0.025 A to 0.975 A.
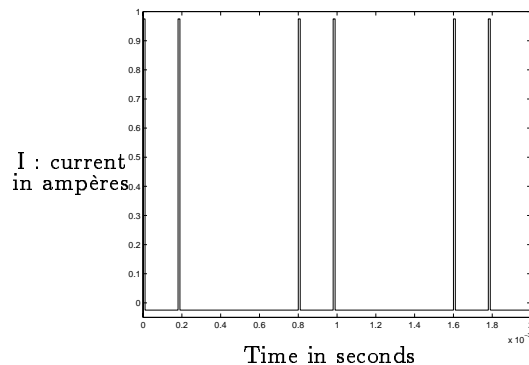
Figure 8.1: The amplitude of current pulses are now considered to be of 1 A from -0.025 A to 0.975 A.
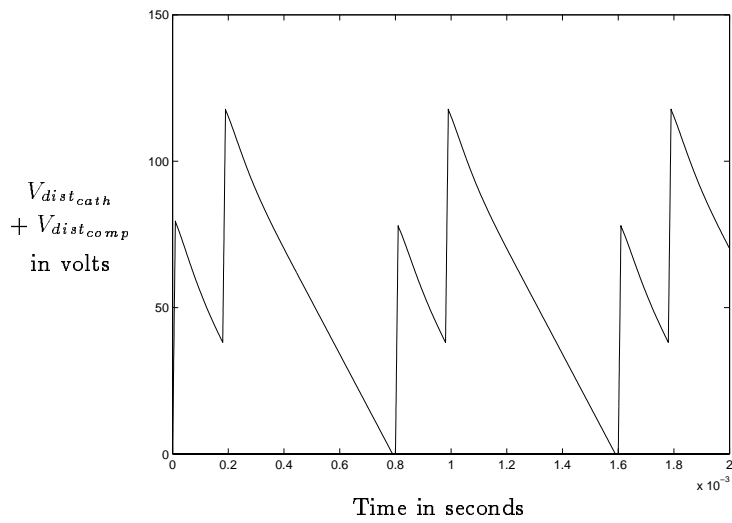
Figure 8.2: The resulting disturbance does not increase for ever. It seems to be a more correct modeling of the disturbances.

8.2 General view of the results of controller 2 with modification of the current pulses

Cathode Voltage
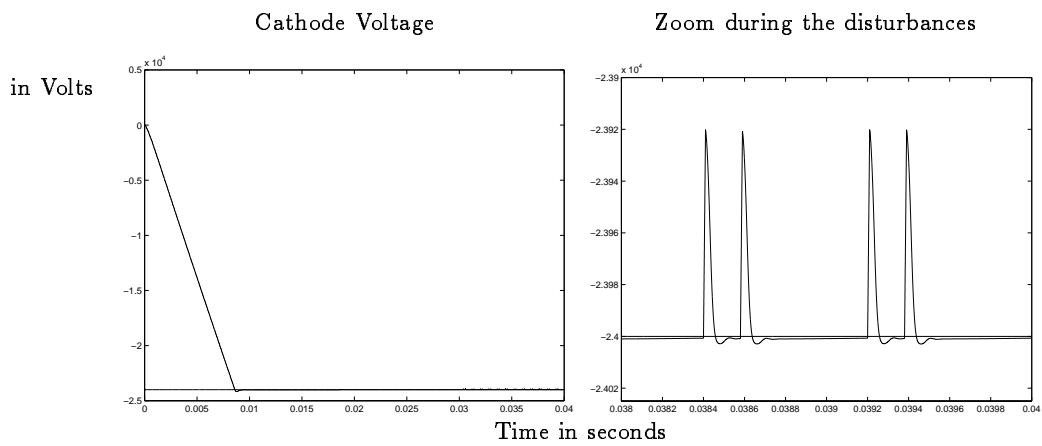
Figure 8.3: The Cathode Voltage converges to its reference quite quickly. On the zoom, the disturbances are also well rejected. It seems more easy for the controller to reject the disturbance because there is less overshot than for the previous modeling of the disturbances with the same Controller.

Phase Compensator Voltage
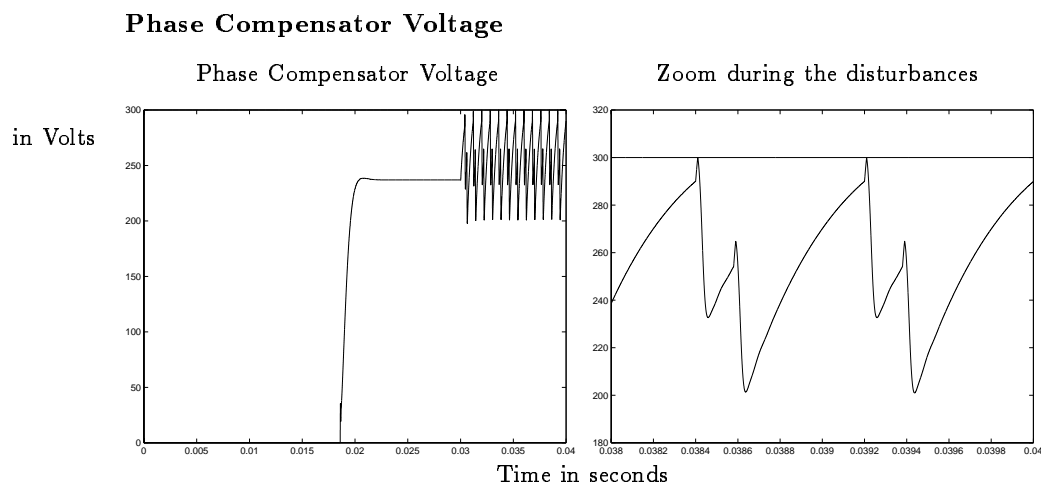
Figure 8.4: The Phase Compensator needs some time to reach its value because of the time necessary to the Cathode Voltage to reach its nominal value and also because of the limitation at its output. There is no more too much static error on the Compensator voltage. On the zoom, we can also see how the Phase Compensator helps very well the Cathode Power Supply to reject the disturbances.

8.3 Detailed results of controller 2 with modification of the current pulses

Ouput error
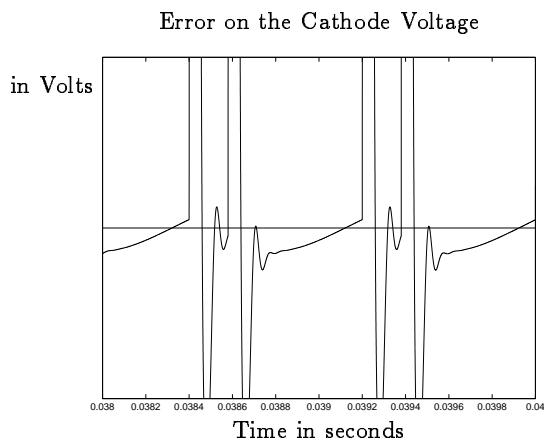
Figure 8.5: The values of the Cathode Voltage are even closer to each other when the pulses are coming. There is no "ringing".

Control signals

Control Signals

Solid : Cathode P.S. ; Dashed : Phase Compensator

in Volts

Time in seconds
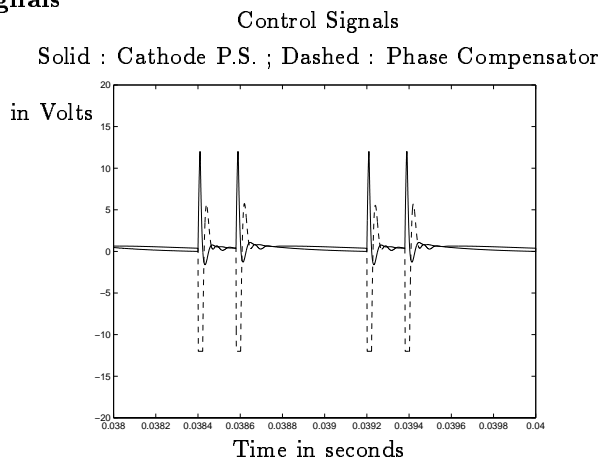
Figure 8.6: Both control signals do not saturate so much to compensate the disturbance. It seems very good.

Observer error

Observer error

Solid : Cathode Voltage ; Dashed : Phase Compensator

in Volts

Time in seconds
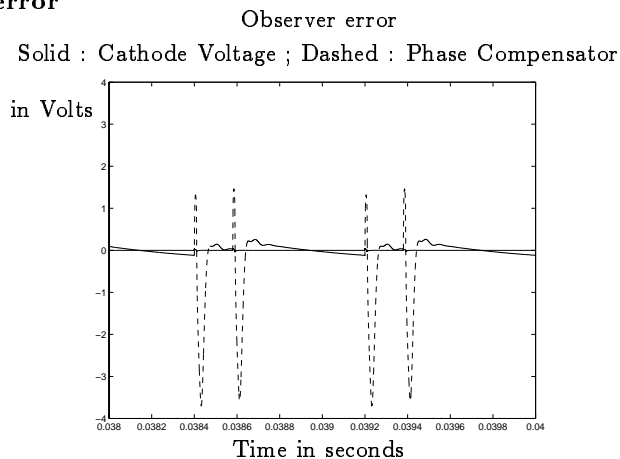
Figure 8.7: The estimation of the Phase Compensator Voltage seems to be degraded. But anyway it is not very important. The most important thing is that the Cathode Voltage is very well observed.

RMS criterion

The RMS criterion is around 55% less than the limit fixed by EMW with pulses at alternatively every 180μs and 620μs. The results are therefore very good.

Drift

If we take a close look at the Phase Compensator voltage, we can not see a drift anymore.

Zoom on the inferior limit of the Phase Compensator Voltage
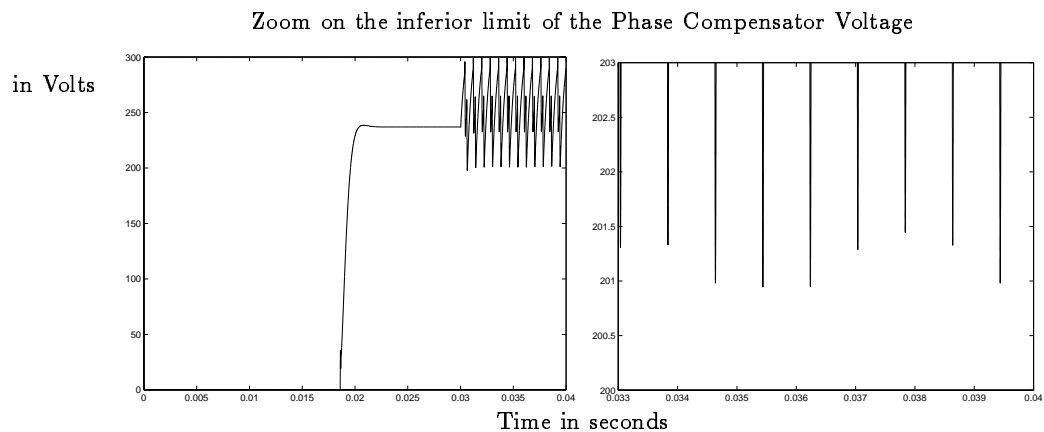
Figure 8.8: No more drift appears on the figure.

8.4   Conclusion on the controller 2 with modification of the current pulses

The controller 2 includes 8 states. The specifications are met with pulses alternatively every $180 \mu s$ and $620 \mu s$. The results are better because there is no more drift on the Phase Compensator Voltage. The controller with the new modeling of the current pulses meets all the specification with pulses every $180 \mu s$ and $620 \mu s$ which is quite close.

Chapter 9

Conclusions

The first conclusion is : since the last control results in simulation with the modification of the modeling of the disturbances, the controller meets all the specifications.

The different problems will be reviewed. The results will be presented and also possibilities of future work will be suggested.

9.1 Problems with no modification of the modeling of the pulses

If we **do not modify** the modeling of the pulses by only taking their AC component, there is no controller designed that completely meets the specifications for different reasons :

- Drift problem.
  A slow drift appears on the Phase Compensator Voltage when there is no integral action. When the Phase Compensator voltage hits the limitation, the performance will be degraded hence the specification will not be met. See also section 8.3 on page 91.

- Performance when there is use of integral action.
  The addition of integral action permits not to have the drift problem but the performance is degraded.
  See also section 5.5 on page 71.

9.2 Project results

By modifying the modeling of the pulses, the following results appear at the end of this project :

Sensor of the Cathode Voltage

The sensor of the Cathode Voltage slows down the slow loop, because, in time domain, the Cathode Voltage sensor output does not have the same shape as the

Cathode Voltage by itself. In order to be able to control the Cathode Voltage, it is necessary to include it in the model used for the observer.
See also section 4.3.5 on page 50.

Modeling of the system

A model of the system based on the linear model of the plants has been performed that can be used for later investigations. This model includes the plants, the disturbances, the known nonlinearities and the sensors.
The identification of the transfer function from the EMW data was quite conservative in order to be sure that the results correspond to the reality.

Performance and Specification

A controller has been designed which can meet the specification with disturbances alternatively every $180\,\mu s$ and $620\,\mu s$ (duty cycle of 2.5%). With the modification of the modeling of the pulses, no more drift appears on the Phase Compensator voltage. So the met performances are very interesting for EMW.

Knowledge brought to EMW

A state space approach to design a controller has been presented to EMW. New tools like optimal control approach and robust control approach have been performed in this Master Thesis. More simple development like antiwindup has also been presented.
Finally, a preliminary study of an implementation of a controller based on state space representation has been explained.

9.3 Future works suggested

Implementation of the controller

Since the specifications are met, an implementation with analog device can be done. An implementation in discrete time (more convenient from a control point of view) with a microprocessor can also be considered even if a computer controlled system should be set up.

Modeling of the system

The system studied is a quite complex electrical network. A linear model of such system does not reflect all its characteristics.
For instance, it is difficult to be sure that the Bode diagrams of the two systems are enough to build a model. Maybe, it is necessary to split further the system. Several objects could result. The connection between the several objects will be better identified and the nonlinearities of the different objects respected.
This could be performed with tools like Modelica, maybe more adapted to the problem. The new language Modelica (www.modelica.org) will permit to respect all the nonlinearities of the electrical network. There is some electrical and power system library in Modelica that permits to simulate electrical circuit easily. The model is then usable in Matlab/Simulink to design the regulator.

Such modeling will be much less conservative than the one performed in this master thesis.

Erik Möllerstedt, PhD student at the department, did some modeling of power system using Modelica. A master thesis on the modeling of the complete electrical network with the actual controller seems realistic by him.

More modest modification by keeping the same general design as before, described in section 2.2 on page 12.

- **By upgrading just a bit the slow loop**
  Some work can be done by changing the PI controller on the slow loop into a PID or better into a PID with anti-windup mechanism.

- **By trying to upgrade the fast loop**
  A study can also be performed on the way to upgrade the fast loop. But first in this case, a very good understanding of its current mechanism should be accomplished.

- **By upgrading the Cathode Voltage sensor**
  A study of the possibility to upgrade the Cathode Voltage sensor which is a limitation of the performance of the system could also be done.

$\mathcal{H}_\infty$ optimization

Another design such as $\mathcal{H}_\infty$ optimization could also be performed in this kind of problem because we mostly know in which frequency domain the uncertainties are. So we can compute a controller by adjusting, for example, the weights of the optimal problems so that the robust properties will be located in the frequency domain of the uncertainties. This approach permits to keep the general approach of Multivariable system developed in this Master Thesis.

But this kind of tool is quite complicated and requires a theoretical background in Robust Control.

See also the robustness analysis with the robust control approach in section 6.1 on page 78.

Use of the information of the moment of the arrival of the pulses

This knowledge can improve the system very much. A rejection of the disturbances in $100 \mu s$ can be considered as a good objective.

Such information would transform completely the control problem. A feedforward mechanism can be added inside the regulator.

Such information requires some study on the system by EMW. Informations of the moment of the arrival of the pulses and the kind of current pulses (amplitude, length) seem to be known by the system.

9.4   Conclusion on the project in general

A study about the possibilities to upgrade the current controller by a multivariable system approach has been done. The results do fully meet the specification.

It has been shown during this Master Thesis that there is a way to upgrade the existing system.

The project permits to bring some knowledge to EMW.

It was a good first approach of modern automatic control that will permit, I hope, further developments.

Chapter 10

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[2] Åström, K. J. and Hägglund Tore (1995) : "PID Controllers: Theory, Design and Tuning", Instrument Society of America.

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[8] Stimson, Georges W. (1980) : "Introduction to Airborne Radar", Hughes Aircraft company.

[9] Zhou K. and J. Doyle (1998) : "Essentials of Robust Control", Prentice Hall, Inc. ISBN 0-13-525833-2

- Compendium of the presentation of Ericsson Microwave System

- EEV User's Guide to the coupled cavity TWT.

- Integrated circuits unitrode Application note.

- Lecture notes of "Nonlinear control and Servo Systems", Bo Bernhardsson (LTH), 2000.

- Lecture notes of "Systèmes linéaires sous formes de représentation d'état", Didier Georges (ESISAR), 1998.

Softwares used

- Microsim Pspice Evaluation 8.0 on PC

- Matlab 5.3 Simulink 3.0 on SUN

- SIMPLIS (special simulation program for switch mode power supply) on SUN
  Simplis requires another software called Analog Workbench from Cadence as a base.

Internet sites

- http://www.control.lth.se : Web site of the department of Automatic Control of Lund.

- http://www.ericsson.se/microwave/ : Web site of the Ericsson Microwave Systems.

- http://www.esisar.inpg.fr : Web site of the ESISAR/INPG Grenoble engineering school.

- http://www.inpg.fr : Web site of INPG Grenoble (Grenoble University of Technology).

Chapter 11

Summary in French / Résumé

Introduction

Ce projet de fin d'étude porte sur le contrôle d'une alimentation haute tension de l'amplificateur d'onde radio d'un système radar. Ericsson Microwave Systems situé à Mölndahl près de Göteborg en Suède a voulu étudier les possibilités d'améliorer le système existant. Le travail a été réalisé à 90% au sein du laboratoire d'automatique de Lund et à 10% au sein de l'équipe FX/NT d'Ericsson Microwave Systems (EMW) à Mölndahl.

Formulation du problème d'automatique

Le problème d'Ericsson Microwave Systems est de pouvoir rejeter les perturbations en tension qui s'ajoutent sur les sorties du systèmes lors de chaque
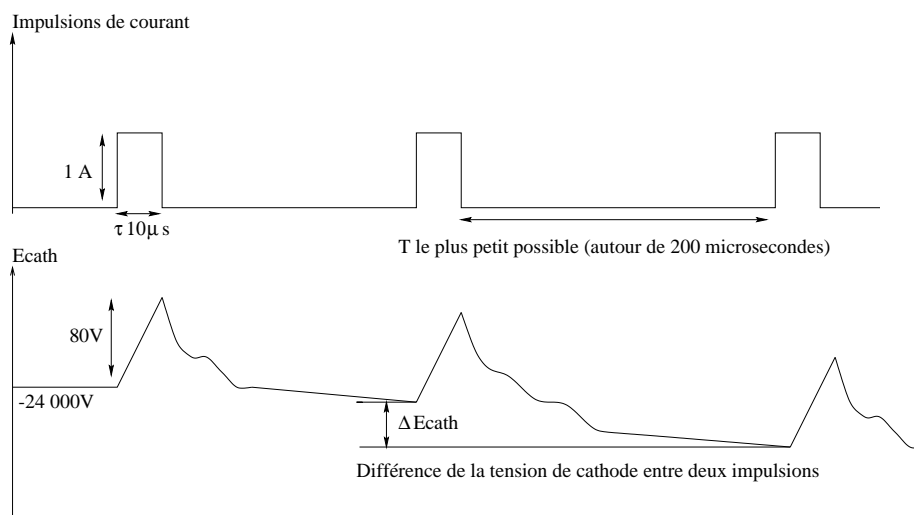
Figure 11.1: Le problème d'automatique est de concevoir un contrôleur capable de rejeter tout particulièrement ce type de perturbation avec une grande précision avant la prochaine impulsion.

amplification d'onde. De la stabilité de la tension de cathode dépend la qualité des ondes transmises et donc la qualité et la précision du système radar.

Le système étudié comporte deux principaux sous-systèmes, l'alimentation de la cathode et le compensateur de phase. Ce système est donc 2 entrées (signaux de contrôle de l'alimentation de la cathode et du compensateur de phase), 2 sorties (tension de cathode et tension de compensateur de phase).

On peut approximer le type des perturbations ajoutées au système sur la sortie tension Cathode à un échelon.

La référence de la sortie tension de cathode est constante et de -24KV et la référence de la sortie tension de compensateur de phase est également constante et de 200 V en sachant qu'elle est limitée entre 0 V et 400 V.

Modélisation du système à contrôler

Le système à contrôler comporte différentes entités : les deux sous-systèmes d'alimentation, les saturations, les perturbations et les capteurs.Le but de ce projet de fin d'étude est donc de contrôler le système ci-dessus.
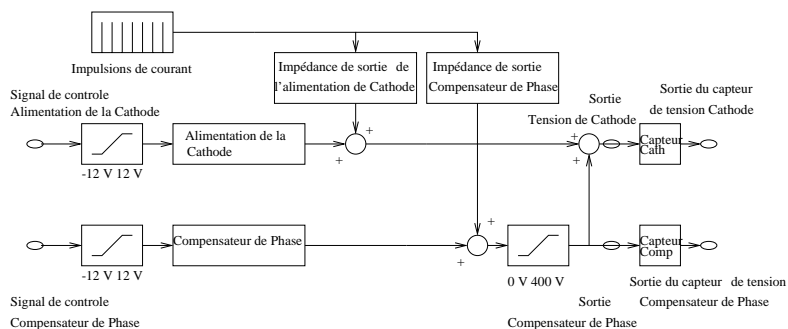
Figure 11.2: Tous les éléments du système doivent être modélisés.

Modélisation des perturbations
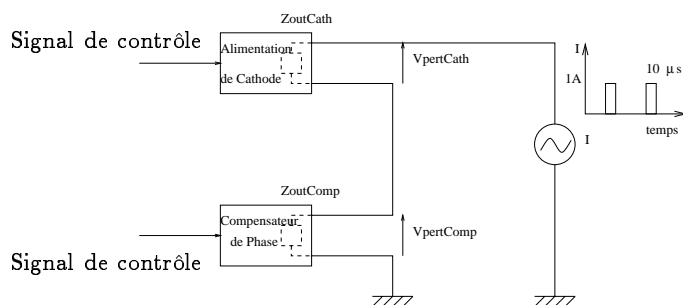
Figure 11.3: Les impulsions de courant viennent charger les impédances de sortie des deux sous-systèmes et créent donc une perturbation en tension sur la tension de Cathode.

La modélisation des perturbations doit permettre de les simuler sous MAT-LAB / SIMULINK et donc de pouvoir essayer de les rejeter par la suite.
Les impulsions de courant venant du TWT (système d'amplification des ondes radio) chargent les impédances de sortie des deux sous-systèmes.
Le calcul des impédances a été réalisé après étude des schémas électroniques du système.
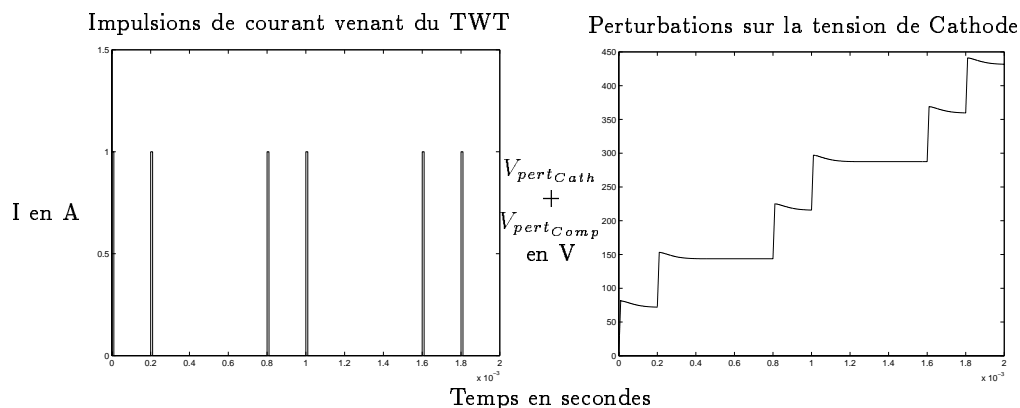
Figure 11.4: Les perturbations sur la tension de Cathode sont essentiellement proportionnelles à l'intégrale des impulsions de courant.

Modélisation des alimentations
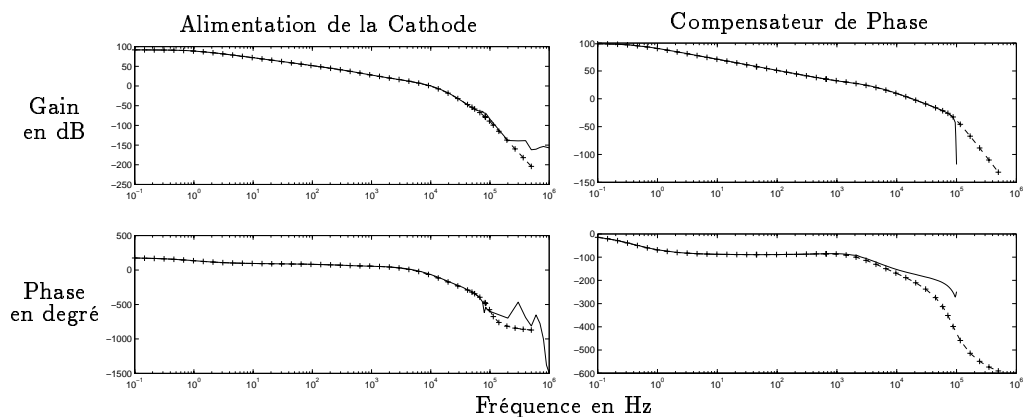
Figure 11.5: Les données fournies par EMW (la courbe) ne permettent pas de connaître le comportement des systèmes après 100 kHz pour le compensateur de phase et 80 kHz pour l'alimentation de cathode. Le présence d'un PWM (Pulse Width Modulator) sur chacun des sous-systèmes en est la raison. Les croix montrent les modèles utilisés lors des simulations sous MATLAB/SIMULINK.

Les diagrammes de Bode ont été fournis par EMW par un logiciel de simulation appelé Simplis fonctionnant sous UNIX. Donc aucun processus d'identification de mesures sur le procédé lui-même n'a été réalisé pendant ce projet.
Par contre, des fonctions de transfert ont été déduites des diagrammes de Bode fournis.

Nonlinéarités identifiées et incertitudes sur les modèles

Il y a deux limitateurs -12V +12V sur les signaux de contrôle et il y a également un limitateur en sortie de compensateur de phase.

Mise à part l'absence de connaissance sur le comportement des sous-systèmes au-delà de la fréquence d'horloge des PWMs, les modèles fournis sont des modèles linéaires d'un système rendu très non-linéaire par la présence à l'intérieur de sous-systèmes non-linéaires et l'existance de plusieurs boucles de contrôle.

La séparation du système au complet en différentes sous-parties ne faisait pas partie du projet de fin d'étude car un tel travail aurait été trop long.

Modélisation des capteurs

La modélisation des capteurs permet également de les inclure dans la simulation et de conclure sur leur influence sur le système.
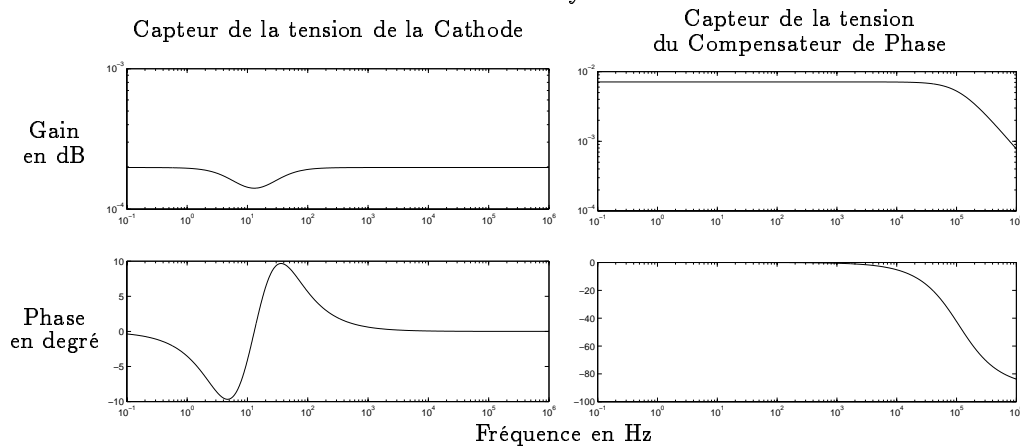
Figure 11.6: Sur les graphiques ci-dessus, on constate que les dynamiques sont très faibles donc que l'on peut considérer les capteurs comme de simples diviseurs de tension dans la conception du contrôleur.

La commande

La commande est une commande basée sur la théorie du principe de séparation. Il s'agit d'un observateur-contrôleur.

L'observateur va permettre d'estimer les états du système grâce à un modèle du système, aux informations d'entrée et de sortie de ce système .

D'après l'estimation des états du système, une commande par retour d'état permettra de contrôler le système et donc de rejeter les perturbations.

Placement optimal des pôles

Le placement optimal des pôles de l'observateur par la commande Matlab *lqe* et des pôles du contrôleur par la commande Matlab *lqr* permet de pénaliser plus ou moins certains signaux de commande et de sortie afin de mieux rejeter les perturbations. Un filtre de Kalman et un linéaire quadratique régulateur sont de fait utilisés.

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Modèles utilisés pour observer les états

Les modèles sont assez simples et d'un faible ordre pour limiter le nombre d'états afin de faciliter une éventuelle mise en œuvre.
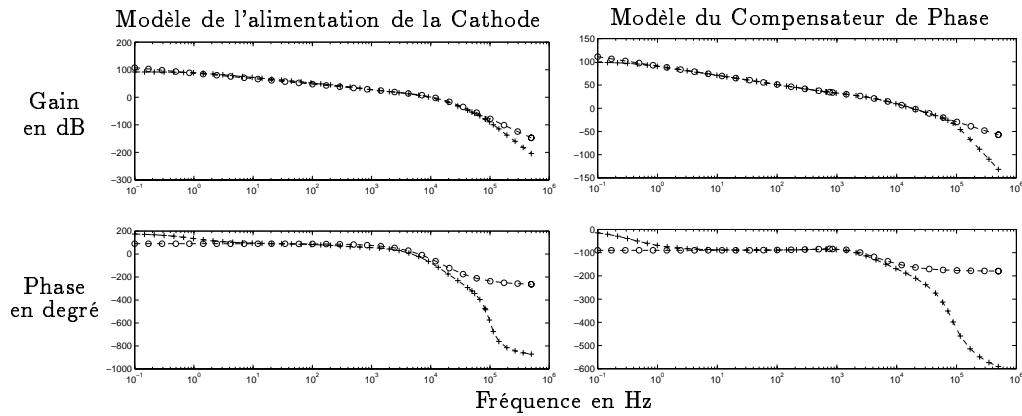
Figure 11.7: Après 20 000 Hz, on peut voir une nette différence entre le modèle utilisé pour simuler le système (croix) et le modèle utilisé dans l'observateur (cercle) principalement sur la courbe de phase. Le modèle utilisé dans l'observateur est en effet plus simple pour limiter le degré du modèle et donc le nombre d'états de l'observateur et donc la complexité du contrôleur.

Résultats de simulation
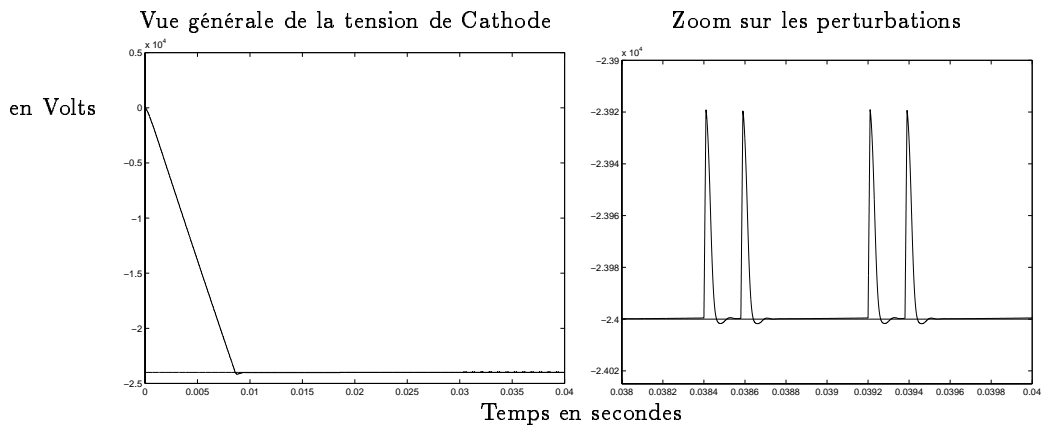
Figure 11.8: La tension de Cathode rejoint sa consigne assez rapidement. Sur le zoom pendant les perturbations, on constate qu'elles sont efficacement rejetées.
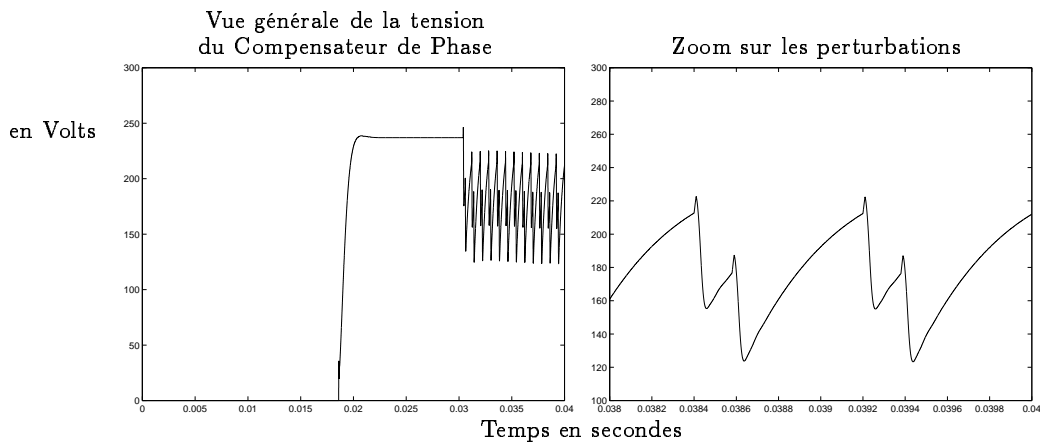
Figure 11.9: Le compensateur de Phase a besoin de plus de temps pour rejoindre la consigne compte tenu des non-linéarité et des temps nécessaire à la tension de Cathode d'atteindre sa valeur nominale. L'erreur statique n'est pas importante pour les spécifications. Sur le zoom, on voit bien comment la tension du compensateur de phase aide l'alimentation de Cathode à rejeter les perturbations.
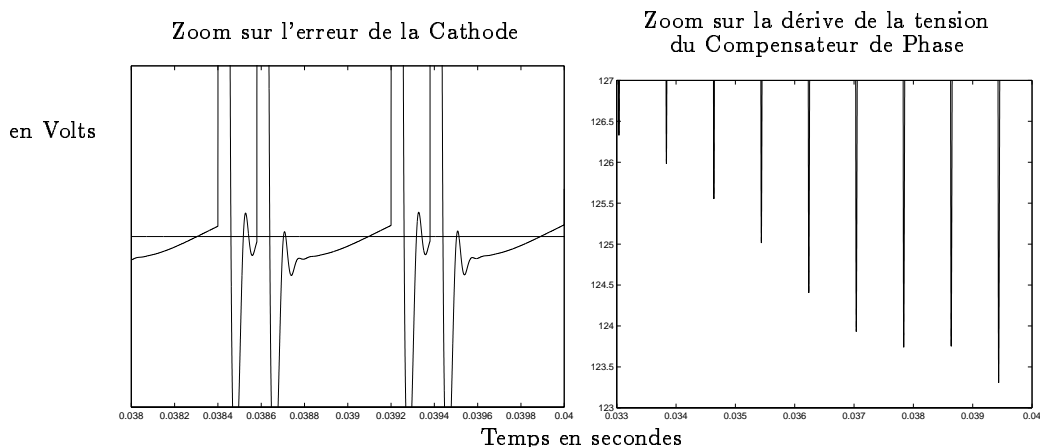
Figure 11.10: Sur ces diagrammes, les perturbations arrivent alternativement toutes les 180μs et 620μs mieux que ce que ne requiert EMW. Le critère de performance est également atteint (valeur confidentielle). Mais une dérive apparaît sur la tension du compensateur de phase qui ne permet pas d'atteindre les spécifications édictées par EMW.

Lorsque que l'on ajoute une action intégrale, le contrôle du système est ralenti et donc les spécification des performances ne sont pas atteintes.

Derniers développements

Après une modification de la modélisation des perturbations (en ne conservant que la composante alternative des impulsions de courant), le problème de dérive des sorties n'apparaît plus. Ainsi l'ensemble des spécifications et des performances demandées ont été atteintes.

Conclusion

Ce projet a permis de réaliser une étude sur les possibilités d'améliorer le système existant. Les résultats prouvent que ces possibilités existent. Une mise en œuvre peut maintenant être envisager par EMW. EMW a pu par ce projet construire un modèle linéaire du système qui pourrait être utilisé par la suite. De plus, beaucoup de connaissances ont été apportées à EMW par présentation des concepts de l'automatique moderne.

Coût du projet

Le coût du projet a été partagé par EMW (déplacements et indemnités) et le département d'automatique de Lund (en utilisant ses moyens informatiques et techniques) à hauteur de 70 000FF.

Appendix A

Data from Ericsson Microwave Systems

Phase Compensator

Frequency in Herz	Gain in dB	Phase in degree
1.0000 e-01	9.8758 e+01	-1.4030 e+01
2.0000 e-01	9.8053 e+01	-2.6550 e+01
3.0000 e-01	9.7044 e+01	-3.6848 e+01
4.0000 e-01	9.6013 e+01	-4.4968 e+01
5.0000 e-01	9.4378 e+01	-5.1310 e+01
6.0000 e-01	9.3907 e+01	-5.6273 e+01
7.0000 e-01	9.2938 e+01	-6.0222 e+01
8.0000 e-01	9.2037 e+01	-6.3401 e+01
9.0000 e-01	9.1201 e+01	-6.6000 e+01
1.0000 e+00	9.0421 e+01	-6.8181 e+01
2.0000 e+00	8.4876 e+01	-7.8665 e+01
3.0000 e+00	8.1448 e+01	-8.2371 e+01
4.0000 e+00	7.8982 e+01	-8.4247 e+01
5.0000 e+00	7.7060 e+01	-8.5375 e+01
6.0000 e+00	7.5485 e+01	-8.6126 e+01
7.0000 e+00	7.4152 e+01	-8.6659 e+01
8.0000 e+00	7.2995 e+01	-8.7058 e+01
9.0000 e+00	7.1975 e+01	-8.7366 e+01
1.0000 e+01	7.1060 e+01	-8.7612 e+01
1.0000 e+01	7.1060 e+01	-8.7610 e+01
2.0000 e+01	6.5046 e+01	-8.8657 e+01
3.0000 e+01	6.1526 e+01	-8.8940 e+01
4.0000 e+01	5.9029 e+01	-8.9033 e+01
5.0000 e+01	5.7092 e+01	-8.9050 e+01
6.0000 e+01	5.5510 e+01	-8.9028 e+01
7.0000 e+01	5.4175 e+01	-8.8985 e+01
8.0000 e+01	5.3017 e+01	-8.8929 e+01
9.0000 e+01	5.1997 e+01	-8.8863 e+01
1.0000 e+02	5.1083 e+01	-8.8792 e+01
2.0000 e+02	4.5112 e+01	-8.7968 e+01

Frequency in Herz	Gain in dB	Phase in degree
3.0000 e+02	4.1670 e+01	-8.7142 e+01
4.0000 e+02	3.9277 e+01	-8.6405 e+01
5.0000 e+02	3.7468 e+01	-8.5793 e+01
6.0000 e+02	3.6033 e+01	-8.5327 e+01
7.0000 e+02	3.4858 e+01	-8.5020 e+01
8.0000 e+02	3.3869 e+01	-8.4876 e+01
9.0000 e+02	3.3025 e+01	-8.4894 e+01
1.0000 e+03	3.2287 e+01	-8.5066 e+01
1.1000 e+03	3.1638 e+01	-8.5389 e+01
1.2000 e+03	3.1056 e+01	-8.5848 e+01
1.3000 e+03	3.0531 e+01	-8.6434 e+01
1.4000 e+03	3.0047 e+01	-8.7128 e+01
1.5000 e+03	2.9599 e+01	-8.7920 e+01
1.6000 e+03	2.9180 e+01	-8.8798 e+01
1.7000 e+03	2.8785 e+01	-8.9751 e+01
1.8000 e+03	2.8409 e+01	-9.0764 e+01
2.0000 e+03	2.8048 e+01	-9.1828 e+01
2.2000 e+03	2.7701 e+01	-9.2938 e+01
2.5000 e+03	2.6108 e+01	-9.8980 e+01
3.0000 e+03	2.4640 e+01	-1.0507 e+02
3.2000 e+03	2.4076 e+01	-1.0745 e+02
3.5000 e+03	2.3249 e+01	-1.1091 e+02
4.0000 e+03	2.1917 e+01	-1.1632 e+02
4.5000 e+03	2.0642 e+01	-1.2130 e+02
5.0000 e+03	1.9406 e+01	-1.2581 e+02
5.5000 e+03	1.8246 e+01	-1.2983 e+02
6.0000 e+03	1.7125 e+01	-1.3346 e+02
6.5000 e+03	1.6055 e+01	-1.3674 e+02
7.0000 e+03	1.5032 e+01	-1.3978 e+02
7.5000 e+03	1.4052 e+01	-1.4249 e+02
8.0000 e+03	1.3114 e+01	-1.4497 e+02
8.5000 e+03	1.2218 e+01	-1.4723 e+02
9.0000 e+03	1.1357 e+01	-1.4936 e+02
9.5000 e+03	1.0530 e+01	-1.5131 e+02
1.0000 e+04	9.7354 e+00	-1.5310 e+02
1.1000 e+04	8.2382 e+00	-1.5631 e+02
1.2000 e+04	6.8474 e+00	-1.5918 e+02
1.3000 e+04	5.5519 e+00	-1.6168 e+02
1.4000 e+04	4.3376 e+00	-1.6398 e+02
1.5000 e+04	3.1945 e+00	-1.6603 e+02
1.6000 e+04	2.1073 e+00	-1.6745 e+02
1.7000 e+04	1.0892 e+00	-1.6914 e+02
1.8000 e+04	1.2250 e-01	-1.7079 e+02
1.9000 e+04	-7.9870 e-01	-1.7229 e+02
2.0000 e+04	-1.6758 e+00	-1.7368 e+02
2.1000 e+04	-2.5112 e+00	-1.7507 e+02
2.2000 e+04	-3.3133 e+00	-1.7632 e+02
2.3000 e+04	-4.0821 e+00	-1.7758 e+02
2.4000 e+04	-4.8231 e+00	-1.7873 e+02
2.5000 e+04	-5.5326 e+00	-1.7989 e+02
2.7000 e+04	-6.8790 e+00	-1.8203 e+02
3.0000 e+04	-8.7432 e+00	-1.8505 e+02

Frequency in Herz	Gain in dB	Phase in degree
3.5000 e+04	-1.1519 e+01	-1.8918 e+02
4.0000 e+04	-1.3960 e+01	-1.9356 e+02
4.5000 e+04	-1.6156 e+01	-1.9792 e+02
5.0000 e+04	-1.8198 e+01	-2.0179 e+02
5.5000 e+04	-2.0095 e+01	-2.0634 e+02
6.0000 e+04	-2.1910 e+01	-2.1132 e+02
6.5000 e+04	-2.3700 e+01	-2.1675 e+02
7.0000 e+04	-2.5547 e+01	-2.2233 e+02
7.5000 e+04	-2.7500 e+01	-2.2939 e+02
8.0000 e+04	-2.9730 e+01	-2.3747 e+02
8.5000 e+04	-3.2541 e+01	-2.4665 e+02
9.0000 e+04	-3.6287 e+01	-2.5792 e+02
9.5000 e+04	-4.2425 e+01	-2.7200 e+02
9.7000 e+04	-6.8025 e+01	-2.6522 e+02
1.0000 e+05	-1.1785 e+02	-2.4873 e+02

Cathode Power Supply

Frequency in Herz	Gain in dB	Phase in degree
1.0000 e-01	9.1763 e+01	1.7436 e+02
2.0000 e-01	9.1640 e+01	1.6884 e+02
3.0000 e-01	9.1441 e+01	1.6351 e+02
4.0000 e-01	9.1177 e+01	1.5846 e+02
5.0000 e-01	9.0861 e+01	1.5373 e+02
6.0000 e-01	9.0502 e+01	1.4937 e+02
7.0000 e-01	9.0114 e+01	1.4536 e+02
8.0000 e-01	8.9705 e+01	1.4167 e+02
9.0000 e-01	8.9284 e+01	1.3838 e+02
1.0000 e+00	8.8856 e+01	1.3534 e+02
2.0000 e+00	8.4918 e+01	1.1677 e+02
3.0000 e+00	8.1922 e+01	1.0847 e+02
4.0000 e+00	7.9624 e+01	1.0395 e+02
5.0000 e+00	7.7780 e+01	1.0110 e+02
6.0000 e+00	7.6251 e+01	9.9156 e+01
7.0000 e+00	7.4929 e+01	9.7727 e+01
8.0000 e+00	7.3804 e+01	9.6631 e+01
9.0000 e+00	7.2796 e+01	9.5760 e+01
1.0000 e+01	7.1886 e+01	9.5039 e+01
1.0000 e+01	7.1886 e+01	9.5039 e+01
2.0000 e+01	6.5895 e+01	9.1394 e+01
3.0000 e+01	6.2374 e+01	8.9674 e+01
4.0000 e+01	5.9867 e+01	8.8439 e+01
5.0000 e+01	5.7918 e+01	8.7401 e+01
6.0000 e+01	5.6321 e+01	8.6464 e+01
7.0000 e+01	5.4965 e+01	8.5588 e+01
8.0000 e+01	5.3785 e+01	8.4751 e+01
9.0000 e+01	5.2740 e+01	8.3944 e+01
1.0000 e+02	5.1798 e+01	8.3160 e+01
2.0000 e+02	4.5415 e+01	7.6194 e+01
3.0000 e+02	4.1386 e+01	7.0601 e+01
4.0000 e+02	3.8323 e+01	6.6285 e+01
5.0000 e+02	3.5828 e+01	6.3036 e+01
6.0000 e+02	3.3731 e+01	6.0595 e+01
7.0000 e+02	3.1936 e+01	5.8719 e+01
8.0000 e+02	3.0377 e+01	5.7216 e+01
9.0000 e+02	2.9010 e+01	5.5955 e+01
1.0000 e+03	2.7796 e+01	5.4835 e+01
1.1000 e+03	2.6717 e+01	5.3798 e+01
1.2000 e+03	2.5739 e+01	5.2794 e+01
1.3000 e+03	2.4853 e+01	5.1800 e+01
1.4000 e+03	2.4039 e+01	5.0798 e+01
1.5000 e+03	2.3289 e+01	4.9777 e+01
1.6000 e+03	2.2597 e+01	4.8730 e+01
1.7000 e+03	2.1952 e+01	4.7655 e+01
1.8000 e+03	2.1348 e+01	4.6551 e+01
2.0000 e+03	2.0239 e+01	4.4261 e+01
2.2000 e+03	1.9249 e+01	4.1861 e+01
2.5000 e+03	1.7919 e+01	3.8099 e+01
3.0000 e+03	1.6015 e+01	3.1484 e+01

Frequency in Herz	Gain in dB	Phase in degree
3.2000 e+03	1.5334 e+01	2.8749 e+01
3.5000 e+03	1.4377 e+01	2.4583 e+01
4.0000 e+03	1.2914 e+01	1.7514 e+01
4.5000 e+03	1.1579 e+01	1.0355 e+01
5.0000 e+03	1.0336 e+01	3.1563 e+00
5.5000 e+03	9.1645 e+00	-4.0432 e+00
6.0000 e+03	8.0420 e+00	-1.1220 e+01
6.5000 e+03	6.9623 e+00	-1.8350 e+01
7.0000 e+03	5.9115 e+00	-2.5424 e+01
7.5000 e+03	4.8842 e+00	-3.2426 e+01
8.0000 e+03	3.8767 e+00	-3.9334 e+01
8.5000 e+03	2.8825 e+00	-4.6145 e+01
9.0000 e+03	1.8986 e+00	-5.2849 e+01
9.5000 e+03	9.2260 e-01	-5.9436 e+01
1.0000 e+04	4.7000 e-02	-6.5898 e+01
1.1000 e+04	-1.9681 e+00	-7.8357 e+01
1.2000 e+04	-3.8679 e+00	-9.0245 e+01
1.3000 e+04	-5.7380 e+00	-1.0150 e+02
1.4000 e+04	-7.5755 e+00	-1.1223 e+02
1.5000 e+04	-9.3770 e+00	-1.2248 e+02
2.0000 e+04	-1.8101 e+01	-1.6767 e+02
2.5000 e+04	-2.6264 e+01	-2.0214 e+02
3.0000 e+04	-3.3539 e+01	-2.2943 e+02
3.5000 e+04	-4.0002 e+01	-2.5291 e+02
4.0000 e+04	-4.6656 e+01	-2.7965 e+02
4.5000 e+04	-5.0774 e+01	-2.9508 e+02
5.0000 e+04	-5.5223 e+01	-3.1700 e+02
5.5000 e+04	-5.8851 e+01	-3.4049 e+02
6.0000 e+04	-6.1619 e+01	-3.6588 e+02
6.5000 e+04	-6.3323 e+01	-3.9383 e+02
7.0000 e+04	-6.4199 e+01	-4.2513 e+02
7.5000 e+04	-6.4520 e+01	-4.6208 e+02
8.0000 e+04	-6.8579 e+01	-6.1733 e+02
8.5000 e+04	-6.9470 e+01	-5.4127 e+02
9.0000 e+04	-7.4542 e+01	-5.7092 e+02
9.5000 e+04	-7.9398 e+01	-5.9021 e+02
1.0000 e+05	-8.3987 e+01	-6.0205 e+02
1.0000 e+05	-8.4330 e+01	-6.0087 e+02
2.0000 e+05	-1.3825 e+02	-6.9854 e+02
3.0000 e+05	-1.3936 e+02	-4.6552 e+02
4.0000 e+05	-1.3871 e+02	-6.8855 e+02
5.0000 e+05	-1.6192 e+02	-8.1204 e+02
6.0000 e+05	-1.6057 e+02	-6.5237 e+02
7.0000 e+05	-1.5590 e+02	-7.7493 e+02
8.0000 e+05	-1.5338 e+02	-1.0033 e+03
9.0000 e+05	-1.5537 e+02	-1.3787 e+03
1.0000 e+06	-1.5702 e+02	-1.4939 e+03

Appendix B

Matlab code used

MATLAB file : controller2script.m

```
clear all;
close all;
init;
rescaleFactor=1e-5;
bodeOfSensors;
close all;
UpperLimit=12;
LowerLimit=-12;

%%%%%% CATH %%%%%%%%%%%%%%%%%

G1o =-3.9e+04;
f1 =1.0e+00;
f2 =1.55e+04;
f7 =8.4e+02;
f8 =4.4e+02;
fx =3.2e+04;
xx =1.2e+04;
xz=2.2;

zeros_Cath=(-[-xx+i*xz*fx;-xx-i*xz*fx;f7]*(2*pi));
poles_Cath=(-[xx+i*xz*fx;xx-i*xz*fx;fx;f1;f2;f2;f2;f2;f8]*(2*pi));
G1=G1o*(prod(-poles_Cath)/prod(-zeros_Cath));
NumCath=G1*poly(zeros_Cath);
DenCath=poly(poles_Cath);
SysCath=tf(NumCath,DenCath);
set(SysCath,'InputDelay',0.0000); %tau1
[MagCath,PhaseCath,WCath]=bode(SysCath,{(0.1*(2*pi)),(500000*(2*pi))});
NumCath=[ 0 0 0 0 0 0 NumCath];

NCath=size(DenCath);
NCath=NCath(1,2)
for j=1:NCath
  NumCathrs(j)=NumCath(j)/(rescaleFactor^(NCath-j-1));
  DenCathrs(j)=DenCath(j)/(rescaleFactor^(NCath-j-1));
end
SysCathrs=tf(NumCathrs,DenCathrs);

%%%%%% COMP %%%%%%%%%%%%%%%%%

G2o =8.95e+04;
f4 =6.4e+03;
f5 =1.1e+03;
f6 =2.1e+03;
f9 =4.2e+4;
f10 =6.3e+04;
f11 =3.9e-01;
f12 =1.6e+05;
f13=1.2e+4;
f14=1.55e+4;
```

```
f15=1.8e+4;
zeros_Comp=[f5;f10;f14;f14]*(-2*pi);
poles_Comp=[f11;f4;f6;f9;f12;f13;f15]*(-2*pi);

G2=G2o*(prod(-poles_Comp)/prod(-zeros_Comp));
NumComp=G2*poly(zeros_Comp);
DenComp=poly(poles_Comp);
SysComp=tf(NumComp,DenComp);
set(SysComp,'InputDelay',0.0000);%tau2
[MagComp,PhaseComp,WComp]=bode(SysComp,{(0.1*(2*pi)),(500000*(2*pi))});

NumComp=[ 0 0 0 NumComp];

NComp=size(DenComp);
NComp=NComp(1,2)
for j=1:NComp
  NumComprs(j)=NumComp(j)/(rescaleFactor^(NComp-j-1));
  DenComprs(j)=DenComp(j)/(rescaleFactor^(NComp-j-1));
end
SysComprs=tf(NumComprs,DenComprs);


%%%%%% CATH taking into account the high frequency assumption %%%%%%%%%%%%%%%%%

G1o =-3.9e+04;
f1 =1.0e+00;
f2 =1.55e+04;
f7 =8.4e+02;
f8 =4.4e+02;


fx =1.2*3.8e+04;
xx =1.2*1.8e+04;
xz=2.2;


f50=2e+4;
f51=5e+4;
f52=8e+4;

wCath=80000*2*pi;
psy1Cath=0.38;
psy2Cath=0.92;


NumButterCathHf=wCath^4;
Den1ButterCath=[1 2*psy1Cath*wCath wCath^2];
Den2ButterCath=[1 2*psy2Cath*wCath wCath^2];
DenButterCathHf=conv(Den1ButterCath,Den2ButterCath);

zeros_CathHf=(-[-xx+i*xz*fx;-xx-i*xz*fx;f7;f51;f52]*(2*pi));
poles_CathHf=(-[xx+i*xz*fx;xx-i*xz*fx;f1;f2;f2;f2;f2;f8;f50]*(2*pi));


G1=G1o*(prod(-poles_CathHf)/prod(-zeros_CathHf));
NumCathHf=G1*poly(zeros_CathHf);
DenCathHf=poly(poles_CathHf);

NumCathHf=conv(NumCathHf,NumButterCathHf);
DenCathHf=conv(DenCathHf,DenButterCathHf);

NumCathHf=[ 0 0 0 0 0 0 0 0 NumCathHf];

NCathHf=size(DenCathHf);
NCathHf=NCathHf(1,2)
for j=1:NCathHf
  NumCathHfrs(j)=NumCathHf(j)/(rescaleFactor^(NCathHf-j-1));
  DenCathHfrs(j)=DenCathHf(j)/(rescaleFactor^(NCathHf-j-1));
end
SysCathHfrs=tf(NumCathHfrs,DenCathHfrs);
SysCathHfrs=ss(SysCathHfrs);

%%%%%% COMP taking into account  the high frequency assumption %%%%%%%%%%%%%%%%%
```

```
G2o =(8.95e+04);

f5 =1.1e+03;
f6 =2.1e+03;
f11 =3.9e-01;

%%%%%%%%%%%%%%
fx =0.05e+05;
xx =1e+05;
xz=1;

fx2 =0.11e+05;
xx2 =1e+05;
xz2=27;
%%%%%%%%%%%%

wComp=85000*2*pi;
psy1Comp=0.38;
psy2Comp=0.92;

f42=5e+03;
f43=1.2e+05;


zeros_CompHf=[f5]*(-2*pi);
poles_CompHf=[f11;f6;f42;f43]*(-2*pi);

NumButterCompHf=wComp^4;
Den1ButterComp=[1 2*psy1Comp*wComp wComp^2];
Den2ButterComp=[1 2*psy2Comp*wComp wComp^2];
DenButterCompHf=conv(Den1ButterComp,Den2ButterComp);

G2=G2o*(prod(-poles_CompHf)/prod(-zeros_CompHf));
NumCompHf=G2*poly(zeros_CompHf);
DenCompHf=poly(poles_CompHf);

NumCompHf=conv(NumCompHf,NumButterCompHf);
DenCompHf=conv(DenCompHf,DenButterCompHf);


NumCompHf=[ 0 0 0 0 0 0 0 NumCompHf];

NCompHf=size(DenCompHf);
NCompHf=NCompHf(1,2)
for j=1:NCompHf
  NumCompHfrs(j)=NumCompHf(j)/(rescaleFactor^(NCompHf-j-1));
  DenCompHfrs(j)=DenCompHf(j)/(rescaleFactor^(NCompHf-j-1));
end
SysCompHfrs=tf(NumCompHfrs,DenCompHfrs);
SysCompHfrs=ss(SysCompHfrs);

%%%%%%%%%%%%%% MODELISATION OF DISTURBANCES %%%%%%%%%%%%%%%%%%%%

%% Outputs IMPEDANCES %%
L=0.002;
C1=1e-6;
C2=20e-6;
C3=140e-9;
r=0.1;
R=80;

%% LOAD Cath %%
R3=100e+6;
C4=100e-12;
R4=1e+6;
C5=10e-9;
R5=20e+3;
C6=500e-9;
C7=10e-9;
R6=1.5e+6;

%% LOAD Comp %%
C12=94e-12;
```

```
R7=0.1;
R13=232e+3;
C13=470e-9;
R14=46.4;


NumZoutComp1=[(C2*L*r*C1) ((L*C2)+(C2*R*r*C1)) ((C2*R)+(C1*r)) 1];
DenZoutComp1=[(L*C2*C1) ((C2*R*C1)+(C2*C1*r)) (C1+C2) 0];
NumZoutCath1=[(C3*r) 1];
DenZoutCath1=[(C3) 0];

NumZout1=(conv(NumZoutComp1,DenZoutCath1)+conv(NumZoutCath1,DenZoutComp1))
DenZout1=(conv(DenZoutCath1,DenZoutComp1))
N=size(NumZout1);
N=N(1,2);

for j=1:N
  NumZout(j)=NumZout1(j)/(rescaleFactor^(N-j-1))
  DenZout(j)=DenZout1(j)/(rescaleFactor^(N-j-1))
end
N=size(NumZoutComp1);
N=N(1,2);
for j=1:N
  NumZoutComp(j)=NumZoutComp1(j)/(rescaleFactor^(N-j-1))
  DenZoutComp(j)=DenZoutComp1(j)/(rescaleFactor^(N-j-1))
end
N=size(NumZoutCath1);
N=N(1,2);
for j=1:N
  NumZoutCath(j)=NumZoutCath1(j)/(rescaleFactor^(N-j-1))
  DenZoutCath(j)=DenZoutCath1(j)/(rescaleFactor^(N-j-1))
end




SysZoutCath=tf(NumZoutCath1,DenZoutCath1);
SysZoutComp=tf(NumZoutComp1,DenZoutComp1);


%%%%%% CATH MODEL FOR THE OBSERVERS %%%%%%%%%%%%%%%%
G1obs =-15e+04;
f2 =1.55e+04;
fx =3.2e+04;
xx =1.2e+04;
xz=2.2;

zeros_CathObs=(-[-xx+i*xz*fx;-xx-i*xz*fx]*(2*pi));
poles_CathObs=(-[xx+i*xz*fx;xx-i*xz*fx;fx;f2;f2;f2;f2]*(2*pi));
%poles_CathObs=(-[f2;f2;f2;f2;fx;xx+i*xz*fx;xx-i*xz*fx]*(2*pi));
GCathObs=G1obs*(prod(-poles_CathObs)/prod(-zeros_CathObs));
NumCathObs1=GCathObs*poly(zeros_CathObs);
DenCathObs2=poly([0; poles_CathObs])
DenCathObsR1=poly([poles_CathObs]);
NumCathObs1=[0 0 0 0 0 NumCathObs1];
NumCathObs2=[0 NumCathObs1]
NR=size(DenCathObsR1);
NR=NR(1,2)
for j=1:NR
  NumCathObs(j)=NumCathObs1(j)/(rescaleFactor^(NR-j-1));
  DenCathObsR(j)=DenCathObsR1(j)/(rescaleFactor^(NR-j-1));
end
for j=1:(NR+1)
  NumCathObs3(j)=NumCathObs2(j)/(rescaleFactor^(NR-j))
  DenCathObs3(j)=DenCathObs2(j)/(rescaleFactor^(NR-j))
end

SysCathObsR=tf(NumCathObs,DenCathObsR);
SysCathObsR=ss(SysCathObsR,'inv');
ACathObsR=SysCathObsR.a;
BCathObsR=SysCathObsR.b;
CCathObsR=SysCathObsR.c;
DCathObsR=SysCathObsR.d;
SysCathObs3=tf(NumCathObs3,DenCathObs3);
```

```
%%%%%% COMP MODEL FOR THE OBSERVERS %%%%%%%%%%%%%%%%%


G2obs =22.5e+04;
f6 =2.1e+03;
f5 =1.1e+03;
f20=5e+03;
f21=1.2e+05;

zeros_CompObs=[f5]*(-2*pi);
poles_CompObs=[f6;f20;f21]*(-2*pi);
GCompObs=G2obs*(prod(-poles_CompObs)/prod(-zeros_CompObs));
NumCompObs1=GCompObs*poly(zeros_CompObs);
DenCompObs2=poly([0; poles_CompObs]);
DenCompObsR1=poly([poles_CompObs])
NumCompObs1=[0 0 NumCompObs1]
NumCompObs2=[0 NumCompObs1]
NR=size(DenCompObsR1);
NR=NR(1,2)
for j=1:NR
  NumCompObs(j)=NumCompObs1(j)/(rescaleFactor^(NR-j-1));
  DenCompObsR(j)=DenCompObsR1(j)/(rescaleFactor^(NR-j-1));
end
for j=1:(NR+1)
  NumCompObs3(j)=NumCompObs2(j)/(rescaleFactor^(NR-j))
  DenCompObs3(j)=DenCompObs2(j)/(rescaleFactor^(NR-j))
end
SysCompObsR=tf(NumCompObs,DenCompObsR)
SysCompObsR=ss(SysCompObsR,'inv');
ACompObsR=SysCompObsR.a;
BCompObsR=SysCompObsR.b;
CCompObsR=SysCompObsR.c;
DCompObsR=SysCompObsR.d;
SysCompObs3=tf(NumCompObs3,DenCompObs3);


%%%%%% CATH For OBSERVER REDUCED %%%%%%%%%%%%%%%%%
G1o =-15e+04;
f2 =1.55e+04;

poles_CathObsRed=(-[f2;f2;f2;f2]*(2*pi));
G1=G1o*(prod(-poles_CathObsRed));
NumCathObsRed=G1;
DenCathObsRedR=poly([poles_CathObsRed]);
NumCathObsRed=[ 0 0 0 0 NumCathObsRed];
NCathObsRed=size(DenCathObsRedR);
NCathObsRed=NCathObsRed(1,2)
for j=1:NCathObsRed
  NumCathObsRedrs(j)=NumCathObsRed(j)/(rescaleFactor^(NCathObsRed-j-1));
  DenCathObsRedrsR(j)=DenCathObsRedR(j)/(rescaleFactor^(NCathObsRed-j-1));
end
SysCathObsRedrsR=tf(NumCathObsRedrs,DenCathObsRedrsR);
SysCathObsRedrsR=ss(SysCathObsRedrsR,'inv');
ACathObsRedrsR=SysCathObsRedrsR.a;
BCathObsRedrsR=SysCathObsRedrsR.b;
CCathObsRedrsR=SysCathObsRedrsR.c;
DCathObsRedrsR=SysCathObsRedrsR.d;


%%%%%% COMP For OBSERVER REDUCED %%%%%%%%%%
G2o =22.5e+04;
f5 =1.1e+03;
f6 =2.1e+03;
f42=5e+03;

zeros_CompObsRed=[f5]*(-2*pi);
poles_CompObsRedR=[f6;f42]*(-2*pi);

G2=G2o*(prod(-poles_CompObsRedR)/prod(-zeros_CompObsRed));
NumCompObsRed=G2*poly(zeros_CompObsRed);
DenCompObsRedR=poly(poles_CompObsRedR);
```

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```
NumCompObsRed=[ 0 NumCompObsRed];
NCompObsRed=size(DenCompObsRedR);
NCompObsRed=NCompObsRed(1,2)
for j=1:NCompObsRed
    NumCompObsRedrs(j)=NumCompObsRed(j)/(rescaleFactor^(NCompObsRed-j-1));
    DenCompObsRedrsR(j)=DenCompObsRedR(j)/(rescaleFactor^(NCompObsRed-j-1));
end
SysCompObsRedrsR=tf(NumCompObsRedrs,DenCompObsRedrsR);
SysCompObsRedrsR=ss(SysCompObsRedrsR,'inv');
ACompObsRedrsR=SysCompObsRedrsR.a;
BCompObsRedrsR=SysCompObsRedrsR.b;
CCompObsRedrsR=SysCompObsRedrsR.c;
DCompObsRedrsR=SysCompObsRedrsR.d;


%%%%%%% CONSTRUCTION OF THE COMPLETE SYSTEM   (OBSERVER)%%%%%%%%%%%%%%%%%%%%%%
NCathR=size(ACathObsR);NCathR=NCathR(1,1);
NCompR=size(ACompObsR);NCompR=NCompR(1,1);

SysObsGene=ss([ACathObsR             zeros(NCathR,NCompR)
zeros(NCathR,1) zeros(NCathR,1);zeros(NCompR,NCathR) ACompObsR
zeros(NCompR,1) zeros(NCompR,1);CCathObsR*rescaleFactor
CCompObsR*rescaleFactor           0 0;zeros(1,NCathR)
CCompObsR*rescaleFactor           0 0],[BCathObsR
zeros(NCathR,1);zeros(NCompR,1)     BCompObsR;0            0;0
0],[zeros(1,NCathR) zeros(1,NCompR)                1 0;zeros(1,NCathR)
zeros(1,NCompR)      0 1],[0 0;0 0]);


AObsGene=SysObsGene.a;
BObsGene=SysObsGene.b;
CObsGene=SysObsGene.c;
DObsGene=SysObsGene.d;

%%%%%%% CONSTRUCTION OF THE COMPLETE SYSTEM   (REDUCED OBSERVER)%%%%%%%%%%%%%%%%%%%%%
NCathRedrsR=size(ACathObsRedrsR);NCathRedrsR=NCathRedrsR(1,1);
NCompRedrsR=size(ACompObsRedrsR);NCompRedrsR=NCompRedrsR(1,1);

SysObsGeneRedrs=ss([ACathObsRedrsR
zeros(NCathRedrsR,NCompRedrsR) zeros(NCathRedrsR,1)
zeros(NCathRedrsR,1);zeros(NCompRedrsR,NCathRedrsR) ACompObsRedrsR
zeros(NCompRedrsR,1) zeros(NCompRedrsR,1);CCathObsRedrsR*rescaleFactor
CCompObsRedrsR*rescaleFactor           0 0;zeros(1,NCathRedrsR)
CCompObsRedrsR*rescaleFactor           0 0],[BCathObsRedrsR
zeros(NCathRedrsR,1);zeros(NCompRedrsR,1)     BCompObsRedrsR;0
0;0                  0],[zeros(1,NCathRedrsR) zeros(1,NCompRedrsR)
1 0;zeros(1,NCathRedrsR) zeros(1,NCompRedrsR)      0 1],[0 0;0 0]);


AObsGeneRedrs=SysObsGeneRedrs.a;
BObsGeneRedrs=SysObsGeneRedrs.b;
CObsGeneRedrs=SysObsGeneRedrs.c;
DObsGeneRedrs=SysObsGeneRedrs.d;


%%%%%%% CONSTRUCTION OF THE COMPLETE SYSTEM   (REDUCED OBSERVER
%%%%%%% + SENSOR )%%%%%%%%%%%%%%%%%%%%%
NCathRedrsR=size(ACathObsRedrsR);NCathRedrsR=NCathRedrsR(1,1);
NCompRedrsR=size(ACompObsRedrsR);NCompRedrsR=NCompRedrsR(1,1);

SysSensorCathrs=ss(SysSensorCathrs);
SysSensorCathrs=minreal(SysSensorCathrs);
NSensorCath=2;
ASysObsGeneRedSensorrs=[ACathObsRedrsR zeros(NCathRedrsR,NCompRedrsR)
zeros(NCathRedrsR,1) zeros(NCathRedrsR,1)
zeros(NCathRedrsR,NSensorCath);zeros(NCompRedrsR,NCathRedrsR)
ACompObsRedrsR    zeros(NCompRedrsR,1) zeros(NCompRedrsR,1)
zeros(NCompRedrsR,NSensorCath);CCathObsRedrsR*rescaleFactor
CCompObsRedrsR*rescaleFactor          0  0 0 0;zeros(1,NCathRedrsR)
CCompObsRedrsR*rescaleFactor          0 0 0
0;zeros(NSensorCath,NCathRedrsR) zeros(NSensorCath,NCompRedrsR)
SysSensorCathrs.b  zeros(NSensorCath,1) SysSensorCathrs.a];
```

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```
BSysObsGeneRedSensorrs=[BCathObsRedrsR
zeros(NCathRedrsR,1);zeros(NCompRedrsR,1)      BCompObsRedrsR;0
0;0                    0;0                    0;0                    0];

CSysObsGeneRedSensorrs=[zeros(1,NCathRedrsR) zeros(1,NCompRedrsR)
SysSensorCathrs.d    0 SysSensorCathrs.c;zeros(1,NCathRedrsR)
zeros(1,NCompRedrsR)    0 1 0 0];

DSysObsGeneRedSensorrs=[0 0;0 0];

SysObsGeneRedSensorrs=ss(ASysObsGeneRedSensorrs,BSysObsGeneRedSensorrs,CSysObsGeneRedSensorrs,DSysObsGeneRedSensorrs);


AObsGeneRedSensorrs=SysObsGeneRedSensorrs.a;
BObsGeneRedSensorrs=SysObsGeneRedSensorrs.b;
CObsGeneRedSensorrs=SysObsGeneRedSensorrs.c;
DObsGeneRedSensorrs=SysObsGeneRedSensorrs.d;

%%%%%% KALMAN FILTER FOR THE REDUCED OBSERVER %%%%%
Gamma11=1/1000;
Gamma12=1/10;

Gamma101=10000000;
Gamma102=100000;
Gamma103=1000;
Gamma104=100;
Gamma105=10000000;
Gamma106=100000;
Gamma107=10000;
Gamma108=10000;

Qe=diag([ Gamma101 Gamma102 Gamma103 Gamma104 Gamma105 Gamma106 Gamma107 Gamma108]);
Ge=eye(8);
Re=1*[Gamma11 0;0 Gamma12];
Ne=zeros(8,2);
[KObsGeneRedrs,P,E] = lqe(AObsGeneRedrs,Ge,CObsGeneRedrs,Qe,Re,Ne);

%%%%%% KALMAN FILTER FOR THE REDUCED OBSERVER + SENSOR%%%%%
Gamma11=1/1000;
Gamma12=1/10;

Gamma101=10000000;
Gamma102=100000;
Gamma103=1000;
Gamma104=100;
Gamma105=10000000;
Gamma106=100000;
Gamma107=10000;
Gamma108=1000000;
Gamma109=1;
Gamma110=10;


Qe=diag([ Gamma101 Gamma102 Gamma103 Gamma104 Gamma105 Gamma106 Gamma107 Gamma108 Gamma109 Gamma110]);
Ge=eye(10);
Re=1*[Gamma11 0;0 Gamma12];
Ne=zeros(10,2);
[KObsGeneRedSensorrs,P,E] = lqe(AObsGeneRedSensorrs,Ge,CObsGeneRedSensorrs,Qe,Re,Ne);

%%%%% LINEAR QUADRATIC REGULATOR FOR THE REDUCED OBSERVER %%%%%

Gamma1=500000*1/(24000^2);
Gamma2=0.07/(300^2);
Gamma3=1/(12^2);
Gamma4=1/(12^2);
Gamma1=500000*1/(24000^2);
Gamma2=0.07/(300^2);

Qr=CObsGeneRedrs'*[Gamma1 0;0 Gamma2]*CObsGeneRedrs;
Rr=[Gamma3 0;0 Gamma4];
Nr=zeros(8,2);

[LsfbackGeneRedrs,S,E]=lqr(AObsGeneRedrs,BObsGeneRedrs,Qr ,Rr,Nr)
```

```
%%%%% LINEAR QUADRATIC REGULATOR FOR THE REDUCED OBSERVER + SENSOR %%%%%


Gamma3=1/(12^2);
Gamma4=1/(12^2);
Gamma1=1000000000000*1/(24000^2);
Gamma2=1/(2.4^2);
Qr=CObsGeneRedSensorrs'*[Gamma1 0;0 Gamma2]*CObsGeneRedSensorrs;
Rr=[Gamma3 0;0 Gamma4];
Nr=zeros(10,2);

[LsfbackGeneRedSensorrs,S,E]=lqr(AObsGeneRedSensorrs,BObsGeneRedSensorrs,Qr ,Rr,Nr)



%%%% LINEAR QUADRATIC REGULATOR FOR THE REDUCED OBSERVER WITH
%%%% 2 INTEGRAL ACTION %%%%%
epsilon2=0.0001;
epsilon1=0.000001;
Gamma1=4*500000*1/(24000^2);
Gamma2=10/(200^2);
Gamma3=1/(12^2);
Gamma4=1/(12^2);

QrInt=[CObsGeneRedrs'*[Gamma1 0;0 Gamma2]*CObsGeneRedrs zeros(8,2);zeros(1,8) epsilon1 0;zeros(1,8) 0 epsilon2];
RrInt=[Gamma3 0;0 Gamma4];
NrInt=zeros(10,2);

AObsGeneRedrsInt2=[AObsGeneRedrs zeros(8,2) ; CObsGeneRedrs(1,:) 0 0; CObsGeneRedrs(2,:) 0 0];
BObsGeneRedrsInt2=[BObsGeneRedrs;0 0;0 0];
[LsfbackGeneRedrsInt2,S,E]=lqr(AObsGeneRedrsInt2,BObsGeneRedrsInt2,QrInt ,RrInt,NrInt)

%%%%%%% CONSTRUCTION OF THE REDUCED OBSERVER %%%%%%%%%%%%%%%%%%%%

AAObsGeneRedrs=AObsGeneRedrs-KObsGeneRedrs*CObsGeneRedrs;
BBObsGeneRedrs=[BObsGeneRedrs KObsGeneRedrs];
CCObsGeneRedrs=[eye(NCathRedrsR+NCompRedrsR+2)];
DDObsGeneRedrs=[zeros(NCathRedrsR+NCompRedrsR+2,4)];

%%%%%%%% COMPUTATION OF THE MATRIX FOR THE REFERENCE  FOR THE
%%%%%%%% REDUCED OBSERVER %%%%%
AAAObsGeneRedrs=[AObsGeneRedrs
-BObsGeneRedrs*LsfbackGeneRedrs;KObsGeneRedrs*CObsGeneRedrs
AObsGeneRedrs-KObsGeneRedrs*CObsGeneRedrs-BObsGeneRedrs*LsfbackGeneRedrs];

BBBObsGeneRedrs=[BObsGeneRedrs; BObsGeneRedrs];
CCCObsGeneRedrs=[CObsGeneRedrs zeros(2,8)];
c=-CCCObsGeneRedrs*inv(AAAObsGeneRedrs)*BBBObsGeneRedrs;
LrGeneRedrs=inv(c);



%%%%%%%% COMPUTATION OF THE MATRIX FOR THE REFERENCE  FOR THE
%%%%%%%% REDUCED OBSERVER + SENSOR %%%%%
AAAObsGeneRedSensorrs=[AObsGeneRedrs
-BObsGeneRedrs*LsfbackGeneRedrs;KObsGeneRedSensorrs(1:8,:)*CObsGeneRedrs
AObsGeneRedrs-KObsGeneRedSensorrs(1:8,:)*CObsGeneRedrs-BObsGeneRedrs*LsfbackGeneRedrs];

BBBObsGeneRedSensorrs=[BObsGeneRedrs; BObsGeneRedrs];
CCCObsGeneRedSensorrs=[CObsGeneRedrs zeros(2,8)];
c=-CCCObsGeneRedSensorrs*inv(AAAObsGeneRedSensorrs)*BBBObsGeneRedSensorrs;
LrGeneRedSensorrs=[1 0 ;0 1]*inv(c);



%%%%%%% COMPUTATION OF THE MATRIX FOR THE REFERENCE  FOR THE
%%%%%%% REDUCED OBSERVER WITH 2 INTEGRAL ACTION%%%%%
%AAAObsGeneRedrs=[AObsGeneRedrs
 %-BObsGeneRedrs*LsfbackGeneRedrsInt2(:,1:8);KObsGeneRedrs*CObsGeneRedrs
 %AObsGeneRedrs-KObsGeneRedrs*CObsGeneRedrs-BObsGeneRedrs*LsfbackGeneRedrsInt2(:,1:8)];

%BBBObsGeneRedrs=[BObsGeneRedrs; BObsGeneRedrs];
%CCCObsGeneRedrs=[CObsGeneRedrs zeros(2,8)];
%c=-CCCObsGeneRedrs*inv(AAAObsGeneRedrs)*BBBObsGeneRedrs;
```

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```
%LrGeneRedrs=inv(c);


%%%%%%% CURRENT PULSES %%%%%%%%%%%%
%Pulses each 300 us - 500 us
%pulsesTime=([0 10 10  300 300 310 310 800 ]*1e-6)/rescaleFactor;

%Pulses each 250 us - 550 us
%pulsesTime=([0 10 10  250 250 260 260 800 ]*1e-6)/rescaleFactor;

%Pulses each 200 us - 600 us
%pulsesTime=([0 10 10  200 200 210 210 800 ]*1e-6)/rescaleFactor;

%Pulses each 180 us - 620 us
pulsesTime=([0 10 10  180 180 190 190 800 ]*1e-6)/rescaleFactor;

%Pulses each 150 us - 650 us
%pulsesTime=([0 10 10  150 150 160 160 800 ]*1e-6)/rescaleFactor;

%Pulses each 100 us - 700 us
%pulsesTime=([0 10 10  100 100 110 110 800 ]*1e-6)/rescaleFactor;

%Pulses each 70 us - 730 us
%pulsesTime=([0 10 10  70 70 80 80 800 ]*1e-6)/rescaleFactor;

%Pulses each 60 us - 740 us
%pulsesTime=([0 10 10  60 60 70 70 800 ]*1e-6)/rescaleFactor;

%Pulses each 50 us - 750 us
%pulsesTime=([0 10 10  50 50 60 60 800 ]*1e-6)/rescaleFactor;

%([0 10 10  200 200 210 210 800 ]*1e-6)/rescaleFactor
pulsesSize=[1 1 0 0 1 1 0 0];
%[1 1 0 0 1 1 0 0]

TstartPulses=3000;

%%%%%%% SIMULATION  %%%%%%%%%%%%

ObsStates=0;
CathStatesInit =0;
CompStatesInit=0;
simEnd=4000;

TtCath=1;
TtComp=1;

CathReference=-24000;
CompReference=300;

sim('controller2',simEnd)


%%%%%%% COMPUTATION OF THE CRITERION %%%%%%%%%%%

%computation of the indexes of the Time vector

%Pulses each 300 us - 500 us
%index(1)=searchValueBinary(Time,3360,0.1)
%index(2)=searchValueBinary(Time,3390,0.1)
%index(3)=searchValueBinary(Time,3440,0.1)
%index(4)=searchValueBinary(Time,3470,0.1)
%index(5)=searchValueBinary(Time,3520,0.1)
%index(6)=searchValueBinary(Time,3550,0.1)
%index(7)=searchValueBinary(Time,3600,0.1)
%index(8)=searchValueBinary(Time,3630,0.1)
%index(9)=searchValueBinary(Time,3680,0.1)
%index(10)=searchValueBinary(Time,3730,0.1)

%Pulses each 250 us - 550 us
%index(1)=searchValueBinary(Time,3360,0.1)
%index(2)=searchValueBinary(Time,3385,0.1)
%index(3)=searchValueBinary(Time,3440,0.1)
%index(4)=searchValueBinary(Time,3465,0.1)
```

```
%index(5)=searchValueBinary(Time,3520,0.1)
%index(6)=searchValueBinary(Time,3545,0.1)
%index(7)=searchValueBinary(Time,3600,0.1)
%index(8)=searchValueBinary(Time,3625,0.1)
%index(9)=searchValueBinary(Time,3680,0.1)
%index(10)=searchValueBinary(Time,3705,0.1)

%Pulses each 200 us - 600 us
%index(1)=searchValueBinary(Time,3360,0.1)
%index(2)=searchValueBinary(Time,3380,0.1)
%index(3)=searchValueBinary(Time,3440,0.1)
%index(4)=searchValueBinary(Time,3460,0.1)
%index(5)=searchValueBinary(Time,3520,0.1)
%index(6)=searchValueBinary(Time,3540,0.1)
%index(7)=searchValueBinary(Time,3600,0.1)
%index(8)=searchValueBinary(Time,3620,0.1)
%index(9)=searchValueBinary(Time,3680,0.1)
%index(10)=searchValueBinary(Time,3700,0.1)

%Pulses each 180 us - 620 us
index(1)=searchValueBinary(Time,3360,0.1)
index(2)=searchValueBinary(Time,3378,0.1)
index(3)=searchValueBinary(Time,3440,0.1)
index(4)=searchValueBinary(Time,3458,0.1)
index(5)=searchValueBinary(Time,3520,0.1)
index(6)=searchValueBinary(Time,3538,0.1)
index(7)=searchValueBinary(Time,3600,0.1)
index(8)=searchValueBinary(Time,3618,0.1)
index(9)=searchValueBinary(Time,3680,0.1)
index(10)=searchValueBinary(Time,3698,0.1)

%Pulses each 150 us - 650 us
%index(1)=searchValueBinary(Time,3360,0.1)
%index(2)=searchValueBinary(Time,3375,0.1)
%index(3)=searchValueBinary(Time,3440,0.1)
%index(4)=searchValueBinary(Time,3455,0.1)
%index(5)=searchValueBinary(Time,3520,0.1)
%index(6)=searchValueBinary(Time,3535,0.1)
%index(7)=searchValueBinary(Time,3600,0.1)
%index(8)=searchValueBinary(Time,3615,0.1)
%index(9)=searchValueBinary(Time,3680,0.1)
%index(10)=searchValueBinary(Time,3695,0.1)

%Pulses each 100 us - 700 us
%index(1)=searchValueBinary(Time,2320,0.1)
%index(2)=searchValueBinary(Time,2330,0.1)
%index(3)=searchValueBinary(Time,2400,0.1)
%index(4)=searchValueBinary(Time,2410,0.1)
%index(5)=searchValueBinary(Time,2480,0.1)
%index(6)=searchValueBinary(Time,2490,0.1)
%index(7)=searchValueBinary(Time,2560,0.1)
%index(8)=searchValueBinary(Time,2570,0.1)
%index(9)=searchValueBinary(Time,2640,0.1)
%index(10)=searchValueBinary(Time,2650,0.1)

%Pulses each 70 us - 730 us
%index(1)=searchValueBinary(Time,2320,0.1)
%index(2)=searchValueBinary(Time,2327,0.1)
%index(3)=searchValueBinary(Time,2400,0.1)
%index(4)=searchValueBinary(Time,2407,0.1)
%index(5)=searchValueBinary(Time,2480,0.1)
%index(6)=searchValueBinary(Time,2487,0.1)
%index(7)=searchValueBinary(Time,2560,0.1)
%index(8)=searchValueBinary(Time,2567,0.1)
%index(9)=searchValueBinary(Time,2640,0.1)
%index(10)=searchValueBinary(Time,2647,0.1)

%Pulses each 60 us - 740 us
%index(1)=searchValueBinary(Time,2320,0.1)
%index(2)=searchValueBinary(Time,2326,0.1)
%index(3)=searchValueBinary(Time,2400,0.1)
%index(4)=searchValueBinary(Time,2406,0.1)
%index(5)=searchValueBinary(Time,2480,0.1)
%index(6)=searchValueBinary(Time,2486,0.1)
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```
%index(7)=searchValueBinary(Time,2560,0.1)
%index(8)=searchValueBinary(Time,2566,0.1)
%index(9)=searchValueBinary(Time,2640,0.1)
%index(10)=searchValueBinary(Time,2646,0.1)

%Pulses each 50 us - 750 us
%index(1)=searchValueBinary(Time,2320,0.1)
%index(2)=searchValueBinary(Time,2325,0.1)
%index(3)=searchValueBinary(Time,2400,0.1)
%index(4)=searchValueBinary(Time,2405,0.1)
%index(5)=searchValueBinary(Time,2480,0.1)
%index(6)=searchValueBinary(Time,2485,0.1)
%index(7)=searchValueBinary(Time,2560,0.1)
%index(8)=searchValueBinary(Time,2565,0.1)
%index(9)=searchValueBinary(Time,2640,0.1)
%index(10)=searchValueBinary(Time,2645,0.1)

%computation of the average of the cathode voltage
averageCathode=0;
for i=1:10
averageCathode=averageCathode+Cathode(index(i),2);
end
averageCathode=averageCathode/10


%computation of the criterion
criterion=0;
for i=1:10
criterion=criterion+((Cathode(index(i),2)-averageCathode)^2);
end
criterion=sqrt(criterion)
```

Appendix C

Complete electrical schematics