

# Trading CDS Indices vs. Equity Index Futures – A pairs trade

Daniel Alavei, Tobias Olsson

Department of Economics, Lund University

July 12, 2015



**LUND**  
UNIVERSITY

### **Abstract**

In this thesis we use a unique data set to show that there is a cointegrating relationship between the EURO STOXX 50 index and the Markit iTraxx Europe index that can be exploited through trading. As far as we know, we are the first ones to write about trading this pair in an academic paper. On our way we deal with various peculiarities in the data set before we manage to find cointegration. The cointegration is evident enough to be utilized when trading the pair. We deal with many practical issues when simulating the trading environment, like marking-to-market of CDS index positions. We successively add more layers of reality to the trading simulation, such as bid-offer-spreads and even in the last step our strategies are highly profitable. Finally, we benchmark our strategies against indices and show that they have low market correlation. This is achieved by introducing an alternative beta measure, customized for the involved assets.

**Keywords:** Pairs Trading, CDS Index, Equity Index Future, Cointegration

**JEL Classifications Numbers:** G11

## Acknowledgements

There are a few people without whom this thesis would never have happened.

First off, we would like to thank Ulf Erlandsson at the Fourth Swedish National Pension Fund (AP4) for generously sharing the unique data, on which this whole thesis is based. Without him this paper would be on an entirely different subject. We would also like to thank him for sharing the experiences he has obtained during his career. We must not forget to thank Fredrik Regland, also at AP4, for getting us in contact with Erlandsson in the first place.

A big thank you to our supervisor Rikard Green for guiding us in the process of writing this thesis. Without him the structure of this thesis would leave a lot to be desired. Thank you for being flexible, and also for giving us the chance to work so independently.

We would also like to thank Magnus Wiktorsson and Hans Byström for insightful tips that may have not only saved us a lot of work, but also made the finished product a little better.

We must not forget the value of the objective and critical eyes of Mårten Bertenstam, Viktor Börnfors, Svante Dieden Sandell, Mattias Karlsson, Halwest Mohammad, Carl Olofsson, Rakel Saldeen and Melker Samuelsson who all committed to the comprehensive task of proofreading our thesis before it became a final product.

Last but not least, we would like to send a special thank you to the people in charge of M.Sc. Industrial Engineering and Management at Lund University Faculty of Engineering; Pia Jablonsky, Karin Cherfils-Karlsson, Eva Berg and Jan Eric Larsson for making our lives a lot easier in times when we did the opposite for them.

# Contents

Abbreviations and Glossary .....	3
1 Introduction.....	4
2 Relevant Financial Markets.....	7
2.1 The CDS Market .....	7
2.1.1 Credit Default Swap .....	7
2.2 The Equity Futures Market .....	12
3 Data.....	14
3.1 Uniqueness .....	14
3.2 Statistical Properties of the Data .....	15
3.3 Details About the Data .....	18
3.3.1 The Equity Data.....	18
3.3.2 The CDS Data .....	18
3.4 Modified Time Series .....	19
3.4.1 Resampled Series .....	20
3.4.2 Synthetic Series.....	21
4 Theory and Methods .....	24
4.1 Time Series Model .....	24
4.2 Mean and Covariance .....	24
4.3 White Noise .....	24
4.4 Stationarity .....	24
4.5 Moving Average Process – MA(q).....	25
4.6 Autoregressive Process – AR(p) .....	25
4.7 First Difference Operator.....	26
4.8 Order of Integration $d - I(d)$ .....	26
4.9 Dickey-Fuller Unit Root Test.....	26
4.9.1 Case I.....	27
4.9.2 Case II.....	27
4.9.3 Case III .....	27
4.10 Augmented Dickey-Fuller Test.....	28
4.11 Cointegration .....	29
4.12 Engle-Granger Test .....	29
5 Statistical Testing .....	31
5.1 Testing for Unit Root .....	31
5.2 Testing for Cointegration .....	33

5.3	Evaluating the Cointegrating Relationship .....	36
5.4	Forecasting Volatility .....	38
5.5	Conclusions on Statistical Testing .....	40
6	Trading Strategies .....	41
6.1	Pairs Trading .....	41
6.2	CDS Pricing – Our Implementation .....	42
6.2.1	Bias .....	42
6.2.2	The ISDA Standard Model .....	42
6.3	The Trading Strategies .....	44
6.3.1	Simplifications .....	44
6.3.2	The Original Trading Model .....	45
6.3.3	Modification of the Original Strategies .....	46
6.3.4	The Three Layer Model .....	47
7	Results .....	48
7.1	Simulation on Resampled Series .....	49
7.1.1	Round 1 .....	49
7.1.2	Round 2 .....	55
7.2	Simulations on Real Data .....	57
7.3	Simulations on Real Data with Transaction Costs .....	58
7.4	Evaluation .....	59
7.4.1	An Alternative Evaluation Approach .....	60
8	Conclusion and Suggestions for Further Research .....	64
	References .....	65
	Appendix .....	69
	A.1 ISDA-model Discount Curve Specifications .....	69

## Abbreviations and Glossary<sup>1</sup>

AP4	Fourth Swedish National Pension Fund
BIC	Bayesian Information Criterion
Bid-offer-spread	Difference between buy- and sell quote
Bloomberg	Financial software
Bp	Basis point, 1bp = 0.01 %
Buy-side	Asset managers
Carry	The return or cost from holding a position
Cash settled	Cash payment instead of physical delivery
CDS	Credit Default Swap
Clearing house	Intermediary in transactions reducing counterparty risk
Credit event	Default on a significant transaction
CSA	Credit Support Annex
Free-float	Market capitalization of shares available to acquire publicly
Libor	London Interbank Offered Rate
Margin	Collateral that has to be posted to reduce counterparty risk
Mid-price	The average of the best bid and offer prices
MTM	Mark-to-market
Notional	The value which the yield is calculated on
NPV	Net Present Value (negative and positive cash flows)
Off-the-run	No longer the most recently issued contract
OLS	Ordinary Least Squares
On-the-run	Most recently issued contract
OTC	Over-the-counter, trading done directly between two parties
Par yield	Yield at which the NPV of a position is 0 (for swaps)
PnL	Profit & Loss
PV	Present Value (only positive cash flows)
Recovery rate	Percentage of bond notional remaining after default
Roll	When the on-the-run contract changes
Roll down	How much the yield changes with maturity
RPV01	Risky Present Value of 1bp
Sell-side	Investment banks
Spot price	The current price a security can be traded at

---

<sup>1</sup> Note that these explanations are valid in the context of this thesis and should not be used as a reference in general.

# 1 Introduction

Even though there has been a lot of research on the subject of pairs trading, there has been very little or none on cross-asset pairs trading. With this paper, we would like to fill that gap, and perhaps make way for further research on the subject. The purpose of this paper is to investigate the relationship between a CDS index and an equity index future and from this relationship form a profitable trading strategy. We believe that an appropriate way to model the dynamics of the series is through cointegration. Thus, in this paper we have decided to use the concept of cointegration to model this relationship that we want to exploit by forming a market neutral pairs trading strategy. We are indeed able to confirm that there is a cointegrating relationship. By using this we can form several trading strategies that after evaluation showed to be successful.

The main idea of the paper, to investigate this particular pair, was given to us by Ulf Erlandsson, Ph.D. in Economics, Senior Portfolio Manager Corporate Credit and SSA, Global Macro Trading at the Fourth Swedish National Pension Fund. He has been looking at the pair himself, but not carried out a formal quantitative analysis. He is looking to trade the spread but would need some academic research to back up his hypothesis before he feels comfortable allocating risk mandate to this trade. Besides serving its academic purpose, which would be fully enough to motivate the thesis, it looks to serve as the academic research that Erlandsson and AP4 need.

What makes this paper special is the unique data set that we are working with. Normally the spreads (levels) of CDS indices can only be observed on a daily basis. But Erlandsson gets quotes with these spreads from sell-side counterparties which combined gives him many intraday observations. He has then written code to extract several years' worth of data.

There is a vast amount of players on financial markets nowadays and they are getting more transparent, liquid and accessible. Thus it may sound surprising that there would be an unexploited opportunity, as obvious as this one, to trade on. But if it turns out to work there are several reasons to believe that this trade has not yet been discovered by the market and thus not yet disappeared.

First of all the spread is only observable for some market participants, see the Data section for an explanation. Even if the spreads were observable there would be few market participants that would care about it because most portfolio managers do not have a cross-asset mandate, i.e. being able to trade credit derivatives and equities in the same portfolio. Even if they could, it may not be a possible strategy if they are benchmarked against the market. Since trading the pair is a market neutral strategy they cannot afford the risk of underperforming their benchmark even if the alpha would be high. AP4 is one of the big players in its niche and not everyone can get as tight spreads as they get, which might be necessary in order to make the trade profitable. One might argue that retail investors have no rules to relate to and can do this trade. However, the credit derivatives market is not open to retail investors and would be too complicated anyway.

Although pairs trading has been around since the 80's the cointegration technique to asset allocation was first pioneered by Lucas (1997) and Alexander (1999). Alexander (1999) discussed its advantages on correlation analysis, which has been the standard analysis for portfolio management and risk measurement for a long time. Correlation analysis is valid only for stationary variables but financial variables are often integrated of order one. One therefore has to take the first difference in log prices. By doing so there is a risk of losing valuable information such as common trends in prices. Cointegration analysis on the other hand can be used on non-stationary series avoiding that valuable information is lost by differencing the data. She also brings up the fact that while correlation is a measure of a short term relationship, cointegration quantifies the long term relationship. In the case of a sufficiently large data sample, cointegration might therefore be a preferred tool. Alexander & Dimitriu (2002) made further research on applying the cointegration technique to constructing various trading strategies, from index tracking to long-short market neutral strategies such as pairs trading. In addition to further providing its applicability for asset allocation overall, Alexander & Dimitriu (2002), among other things, provide evidence for the mean reverting nature of cointegration-based index tracking to contribute to great over-performance of the market index during sharp market declines such as the Russian crisis or the IT-bubble.

Someone who is often quoted in discussions on cointegration as an approach for pairs trading is Vidyamurthy (2004). He, just like Alexander (1999), favors cointegration for its measure



of long-term equilibrium and mean reversion. After using cointegration for the pairs selection, Vidyamurthy (2004) suggests two different approaches to exploit its mean reverting nature. One is to model the residuals as an autoregressive-moving-average (ARMA) process. This is followed by using Rice's formula to evaluate the rate of zero crossings and level crossings, for different levels measured as deviations from the mean, to maximize the profit. The other, non-parametric, approach is to evaluate the zero and level crossings only from the estimation sample. He favors the latter approach for its simplicity and avoidance of model misspecifications.

Reason to believe our chosen pair to be a good pick was given by Byström (2005). In his paper he presents and provides early evidence for the theory that there is a link between the CDS index market and the stock market. Byström (2005) provides this evidence not only with correlation estimates but also by regressing daily CDS spread changes on yesterday's CDS spread and both today's and yesterday's stock returns. First of all, in this OLS regression model all the coefficients for the contemporaneous stocks were significant, indicating a strong negative correlation between the CDS spread changes and the stock returns. Secondly, the regression on the lagged CDS spread changes provided evidence suggesting that the CDS market might have positive first order autocorrelation, hinting that this market might not be efficient. Last but not least, half of the lagged stock return coefficients were significant, indicating that the stock market might be driving the CDS market by incorporating firm-specific information quicker. To further strengthen this theory, the previously mentioned correlation estimates that Byström (2005) made, were also made between lagged stock returns. These were found to be almost as correlated to CDS spread changes as the unlagged returns. When instead the CDS spread changes were lagged, no correlation was found.

The rest of this paper is organized as follows. In section 2 the relevant financial markets is presented. Section 3 goes into further detail on the unique data and how it was prepared to make it more manageable for our purposes. Section 4 covers the theory and methods used in this paper. In section 5 we present the results from the statistical testing on the data. In section 6 we describe the trading strategy and how it is implemented. Section 7 presents results and analysis from the trading and its evaluation. Finally, section 8 presents our conclusions and gives suggestions for further research.

## 2 Relevant Financial Markets

### 2.1 The CDS Market

In 1994, Exxon faced the potential of \$5 billion in damages for the Valdez oil spill in 1989. As a result, JP Morgan extended a \$4.8 billion credit line to their old client. Since doing so would tie up a lot of reserve cash, Blythe Masters, a member of the JP Morgan swap team got the idea of selling the credit risk to the European Bank of Reconstruction and Development, EBRD. For covering the risk of Exxon not being able to pay their debt, EBRD would receive a periodic premium and JP Morgan would untie massive amounts of reserve cash for more lucrative business. This is considered to be the first Credit Default Swap. (Newsweek LLC, 2008; The New Yorker, 2009) Today CDS are not only traded for risk protection, they are also traded by players speculating in credit events. CDS data is also often used to monitor how the market views the credit risk of the underlying entities. (Simkovic & Kaminetzky, 2010)

#### 2.1.1 Credit Default Swap

##### 2.1.1.1 Definition

A Credit Default Swap is a bilateral Over-the-Counter (OTC) agreement transferring credit risk from one party to another. The buyer of the credit protection pays a periodic premium to the seller of the credit protection. In return, the seller offers protection against a credit event on the underlying credit specified in the contract. There are two legs in the agreement (just as in the case with an interest rate swap), the premium leg and the protection leg (later also referred to as the default leg). The market convention is to refer to the parties as protection buyer and -seller respectively. (Merrill Lynch, 2006)

The premium is usually paid quarterly, either until the contract matures, is offset or if a credit event occurs in the underlying asset. In case of a credit event the protection buyer can deliver any qualifying debt instrument as stated in the agreement and in exchange receive a cash payment of the original notional amount of the delivered instrument. A CDS

can also be cash settled and would instead result in a cash payment from the protection seller to the protection buyer corresponding to net value of the transaction described above which involved physical delivery. (Merrill Lynch, 2006)

#### **2.1.1.2 Valuation**

In a paper from the Quantitative Credit Division at Lehman Brothers, O'Kane & Turnbull (2003) discuss the model which they at the time considered was the most used in the market. The following model is based on their work.

At the time their paper was written CDS, just like interest rate swaps, were entered at a par yield so that neither party had to pay the other in order to enter the contract since the net present value (NPV) of holding each side of the contract was zero. Today CDS are still quoted at par, as if they would be entered at zero NPV but the contract that is in fact obtained pays a standardized premium. Therefore the actual contract entered has non-zero NPV and has to be marked-to-market (MTM) at initiation and closure of the trade in order to compensate or get compensated by the counterparty for the non-zero NPV. (Barclays Capital, 2010)

Assume that an investor buys  $n$  years protection on a company with a par spread of  $x$  basis points (bps) but the contract entered has a fixed coupon of  $y$  bps. In case  $x > y$  the protection buyer has to compensate the protection seller with a cash amount and in case  $y > x$  it is the other way around. The reason for this is that from the protection buyer's point of view it is more attractive to hold the contract with coupon  $y$  when  $x > y$ . In order to calculate the amount that the protection buyer has to pay the protection seller we want to calculate the NPV of holding the CDS with fixed coupon  $x$  when the market is trading at a coupon of  $y$ . This can be rephrased as "what is the value of a portfolio with  $n$  years protection bought at  $y$  bps and sold at  $x$  bps?" because the "attractiveness" of holding the contract can be locked in by entering into an offsetting position at the market spread. This portfolio is both long and short the protection leg so the cash flows from these two positions will always cancel out and thus have a NPV of zero. The value of the portfolio is just the NPV of receiving  $x$  bps and paying  $y$  bps on the premium leg.

It may sound like an easy task to calculate the NPV of a net cash flow of  $x - y$  bps but we must not forget that even though the contract is on  $n$  years it is uncertain for how long this cash flow will continue. The cash flow will stop when the contract matures or when a credit event occurs, whichever happens first. A new term is introduced, Risky PV01 (RPV01) as the expected present value of 1 bp paid on the premium leg. The NPV of the portfolio then becomes  $(x - y) * RPV01$ . (O'Kane & Turnbull, 2003)

The problem of MTM the position has now been reduced to calculating the RPV01. To do this we need a model that takes into account the riskiness of each premium payment. If  $Frac_i$  denotes the fraction of a year between time  $i - 1$  and  $i$  according to used conventions, the RPV01 can be calculated as the sum of all payments multiplied by the risk neutral probability of payments being made (survival probability) and discounted.

$$RPV01 = \sum_{i=1}^n Frac_i * Notional * 1bp * (1 - p_i) * DF_i \quad (2.1)$$

$DF_i$  is the discount factor and  $p_i$  is the cumulative probability of default up to time  $i$ . In order to perform this calculation the probability of default term structure is needed. Our purpose is to MTM CDS contracts and therefore we want probabilities consistent with current market pricing and not something based on historical defaults for instance. We will now show how an arbitrage relationship can be used to calculate the market implied probabilities of default. Assuming no arbitrage on the market, the present value (PV) of the premium leg, when quoted at par, has to equal the PV of the default leg.

The PV of the premium leg can be written as the sum of the premiums, discounted by the risk-free rate and multiplied by the risk-neutral survival probabilities. We will come back to how the risk-neutral survival probabilities implied by the market are calculated.

$$PV_{premium\ leg} = \sum_{i=1}^n premium * DF_i * (1 - p_i) \quad (2.2)$$

In order to calculate the PV of the default leg we assume that defaults can occur only on a series of discrete dates. If  $R$  denotes the recovery rate the PV is calculated as

$$PV_{default\ leg} = (1 - R) * \sum_{i=1}^n (p_i - p_{i-1}) * DF_i \quad (2.3)$$

Each term in the sum is the amount received in case of default, multiplied by the marginal risk neutral probability that a default occurs from time  $i - 1$  to time  $i$ , discounted at time  $i$ . By summing the terms over all discrete time steps the full PV is obtained. Both (2.2) and (2.3) are expressed as a fraction of the notional. (Merrill Lynch, 2006)

### 2.1.1.3 Risk-Neutral Probabilities of Default

Assume that there are several CDS traded on a name with different maturities and that the spread can be observed on the market. By putting the PV of the two legs equal to each other we get one equation that has to be satisfied for each traded contract. If some value is assigned to  $R$  the only thing unknown is the default probability term structure. By using bootstrapping, the market implied term structure can be backed out.

Say that the shortest maturity contract traded has a maturity of  $m$  and is traded on a spread of  $premium_m$ . The arbitrage relationship for this contract becomes

$$premium_m * DF_m * (1 - p_m) = (1 - R) * (p_m - p_{m-1}) * DF_m \quad (2.4)$$

Since  $m$  is the shortest maturity contract, the time scale can be defined in a way that makes  $m - 1 = 0$  today. If the company has not defaulted today we have  $p_{m-1} = p_0 = 0$ , leaving us with one unknown. Solving for  $p_m$  yields

$$p_m = \frac{premium_m}{1 - R + premium_m} \quad (2.5)$$

In the same way, by observing the spread of the contract with maturity  $m + 1$  the next point on the term structure,  $p_{m+1}$  can be calculated since  $p_m$  is now known. Deriving all probabilities iteratively in this way is called bootstrapping. The different premiums used are obtained from the credit curve which is the premiums as a function of time.

Recall that the default probabilities are needed at the points in time where the premium payments are planned to occur in order to evaluate expression (2.1). The points in time at which the implied default probabilities are calculated are the maturity dates of the CDS contracts currently traded on the market. A problem that can arise is that the points in time where the default probabilities are obtained do not match the points in time where the premium payments occur. Therefore implied default probabilities are needed at points in

time where no contracts may be traded and this is solved by interpolating between the points obtained from the bootstrapping. (Merrill Lynch, 2006)

#### **2.1.1.4 Recovery Rate**

The final assumption needed in order to calculate the default probabilities is the recovery rate. From (2.5) we see that a higher recovery rate gives a higher probability of default which gives a lower RPV01 and in the same way a lower recovery rate implies a higher RPV01 so the assumption indeed affects the MTM of the CDS. Fortunately, we are only trying to MTM the CDS as the market would do it and it is therefore enough to use a recovery rate in line with the rest of the market.

#### **2.1.1.5 Discount Factor**

As we can see from (2.1) a discount factor is needed in order to MTM the CDS contract. Risk neutral probabilities are used throughout the pricing model so the risk free rate can be used to discount the cash flows. In theory this is a simple statement but in real world applications an appropriate discount curve has to be picked. O'Kane (2008) argues that the sellers of credit derivatives need to hedge their risks and thus the rate used to discount cash flows is the one at which they have to fund the purchase of hedging instruments with. Libor (London Interbank Offered Rate) fits this description well. A discount curve linked to the current and expected future level of the Libor would be desirable.

#### **2.1.1.6 Quotation Conventions and Upfront Payment**

In the theoretical valuation section above we assumed that the CDS were quoted in a certain way just in order to show the classical ideas of CDS valuation. In practice there are different ways of quoting CDS contracts resulting in the same MTM of a contract (or upfront payment) if the appropriate valuation method corresponding to the quotation convention is used.

##### **2.1.1.6.1 Par Spread**

In the preceding section we have assumed that the contracts are quoted at par spread which resulted in the following formula for calculating the MTM value (upfront payment needed).

$$\text{Upfront Payment} = (\text{Par Spread} - \text{Fixed Coupon}) * \text{RPV01}_{\text{Par Spread Curve}} \quad (2.6)$$

Note that the RPV01 in this case is calculated by using a credit curve consisting of all the par spreads on other maturity contracts observable on the market. (Barclays Capital, 2010)

#### 2.1.1.6.2. Quoted Spread

In the market there is another way of quoting CDS contracts as well, called the quoted spread. This quoted spread has been introduced in order to simplify the calculation of the upfront payment. The spread has been defined in a way that it results in the same upfront payment as the par yield approach when using certain conventions. These conventions are to assume a flat credit curve equal to the quoted spread of the contract being valued, a fixed predefined recovery rate (often 40 %) and an ISDA-standard hazard rate model. The formula instead becomes:

$$\text{Upfront Payment} = (\text{Quoted Spread} - \text{Fixed Coupon}) * \text{RPV01}_{\text{Quoted Spread Flat Curve}} \quad (2.7)$$

In contrast to (2.6) the RPV01 is calculated with the above conventions making the upfront payment in both calculations equal even if the first part of the expression, the spread, differs. (Barclays Capital, 2010)

## 2.2 The Equity Futures Market

The reader of this paper should be familiar with the concept of futures contracts but we will still introduce them briefly for formality. A futures contract is an agreement between two parties, where one sells and the other buys, an asset at a certain time in the future for a certain price. Futures contracts are normally traded on an exchange and when this is the case they have standardized features. (Hull, 2011a)

For the exchange traded futures the trading is organized in a way so that contract defaults are avoided. This is obtained through the operations of margins. In summary, the counterparties have a margin account at a clearing house where cash, or other collateral, has to be deposited so that the exchange can make sure that the counterparties can fulfil their obligations if the position moves against them. Each day the contracts are marked-to-market and the margins have to be adjusted accordingly. If a counterparty cannot post the

required margin the exchange will have to close out the position. Futures contracts can be either cash settled or involve physical delivery. However, the latter is rarely the case especially when the underlying is a financial asset. (Hull, 2011b)

In this paper the underlying asset on the futures are stock indices and the price of the future is closely related to the one of the underlying asset. The arbitrage free price of a stock index future,  $F_0$  is given by  $F_0 = S_0 * e^{(r-q)*T}$  where  $S_0$  is the spot price of the stock index,  $r$  the risk free interest rate for the maturity of the contract,  $q$  the dividend yield and  $T$  the time to maturity. (Hull, 2011c)

Calculating PnL (profit & loss) from unwinding a futures position is straight forward. Assume that a long position in a futures contract is entered at time  $t_0$  and that this position is unwound, by entering an offsetting short position at time  $t_1$ . At maturity the position will generate a cash flow of  $F_1 - F_0$ . In order to obtain the value of this cash flow at the time the position is unwound it has to be discounted to time  $t_1$ .



## 3 Data

In this paper the data is a central part due to its unique nature. The data consists of two time series, one with EURO STOXX 50 index future prices and one with Markit iTraxx Europe index (iTraxx EUR) spread levels. Let us refer to them as the equity data and CDS data respectively. First we describe why the data is so unique and then we will present some of its statistical properties. Then we have a detailed section on what exact data has been collected for the interested reader. Finally we present new time series used in the thesis that have been created from the existing ones.

### 3.1 Uniqueness

The equity data is not unique at all as it is accessible to anyone with a Bloomberg terminal or similar. There is a significant difference between how the equity index futures are traded and how the CDS index is traded. The equity index futures are exchange traded, very liquid and accessible to almost any investor. In other words this market is transparent, the transactions are documented and easy to obtain through data providers.

The CDS index is traded OTC, is not as liquid as the equity index futures market and is not accessible to everyone. Thus this market is less transparent. When a trade is done on an exchange all the market participants can see that trade, but when a trade is done OTC it is just the two counterparties involved that are aware of the trade. As a consequence two contracts could theoretically be traded at different levels at the same time with different counterparties. The CDS index provider, Markit Group Limited (2009), calculates daily levels on the index according to a methodology similar for calculating Libor. So the only information available to the market in general is daily data points of some calculated averages.

The CDS data is very unique and we are probably the only ones having access to this particular data set. AP4, that are on the buy-side, have a lot of sell-side counterparties quoting them prices on different OTC derivatives such as iTraxx EUR. The exact counterparties are of course confidential but the major investment banks in London are included so the sources are reliable. The counterparties quote prices on Bloomberg every

now and then to indicate at what level it is possible to trade. Erlandsson has automated forwarding of these quotes to his Outlook. He has also written a script that can extract the data from Outlook to a handier format which was handed over to us.

### 3.2 Statistical Properties of the Data

To get an intuition on how the two time series move in relation to each other, they are plotted in the same window as can be seen in Figure 3.1. The plots look like they do, with flat sections during the nights and a lot of movements in-between because no modification has been made to the series and they are plotted against the exact observation time on the horizontal axis. We note that the two series move inversely just as one would expect from existing theories like the Merton Model. The intuition behind it is that the equity of a company can be seen as a call option on its assets with strike price equal to its debt. The stock market moves inversely to market implied volatility (see for instance VIX against S&P 500) (CBOE, 2015). When the stock market goes down, the volatility increases and thus the probability of the value of the assets moving lower than the debt (which is the same as a default) also increases. This will of course make the CDS spread widen and hereby the inverse relationship. (Merton, 1974)

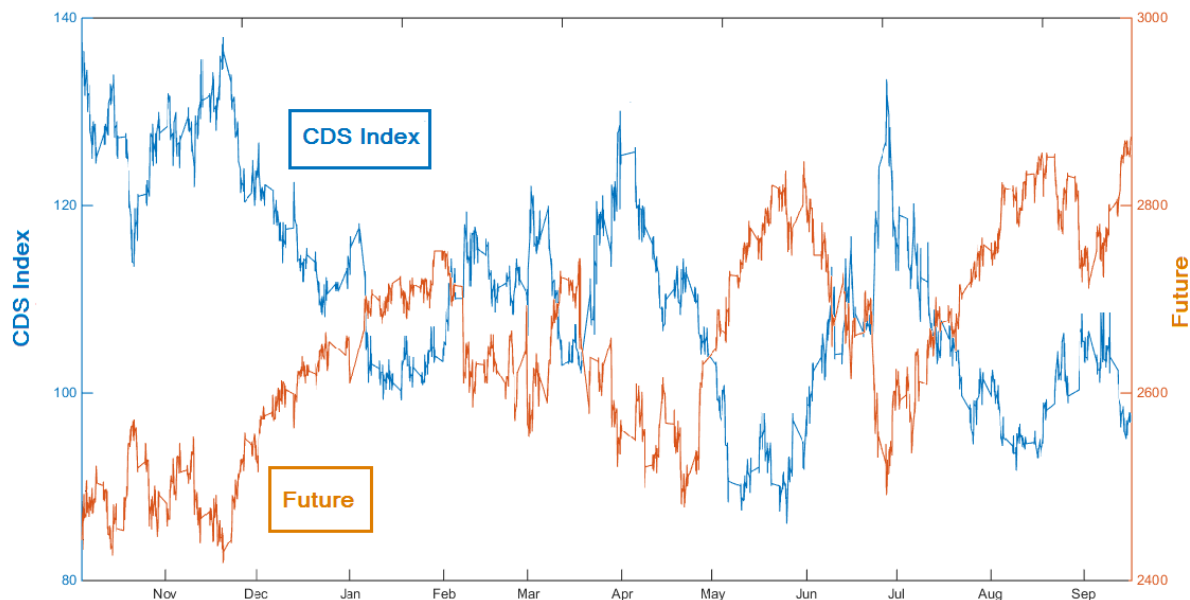


Figure 3.1: CDS Index and Future.

The statistical properties of the data set are summarized below in Table 3.1. It should be noted that the measure of kurtosis and skewness are calculated from the log differences. The fact that the mean log returns for the two series are of opposite sign but approximately equal in absolute terms makes sense looking at Figure 3.1 since they show sign of equally large percentage drifts but in opposite directions.

Table 3.1: Summary statistics.

	CDS index	Future
Mean	112.12	2653.60
Mean log return	-7.38e-06	8.28e-06
Annualized volatility <sup>2,4</sup>	60.13 %	24.67 %
Min	87.00	2425.00
Max	138.88	2872.00
Kurtosis <sup>3</sup>	221.41	113.67
Skewness <sup>3</sup>	0.61	-0.31
Number of observations	45686	19728
Start	01 October 2012	01 October 2012
End	13 September 2013	13 September 2013
Average intraday observations <sup>4</sup>	191	82

Below are the histograms of the log differences (Figure 3.2 and Figure 3.3). These show the many outliers that generate the high values for the kurtosis (kurtosis for a standard normal distribution is 3). These outliers could be explained, by changes over weekends or non-trading days, since these might be a lot bigger. A potential explanation to why the kurtosis is higher for the CDS Index is that while the price of the future is sampled every ten minutes, the spreads for the CDS index is sampled with a mixed frequency, potentially generating bigger changes. Another possible explanation for the outliers in the CDS histogram is that the skew or basis can potentially be a lot bigger for the CDS index than for the equity index, i.e. the equity index is more bound by arbitrage relations. Reasons for this is that underlying CDS trade wider than underlying stocks and the CDS index has more constituents making the arbitrage more costly. Thus the CDS index can be more receptive to supply-demand

<sup>2</sup> The CDS series has irregular sampling frequency but here it is assumed to have a regular sampling frequency so the number is not necessarily meaningful.

<sup>3</sup> Calculations made on log differences.

<sup>4</sup> Based on 252 trading days in a year of 365 days.

effects. Citigroup (2010) mentions several other reasons as “index CDS are rolled every 6m while single name CDS are quarterly, resulting in reliance on interpolation”.

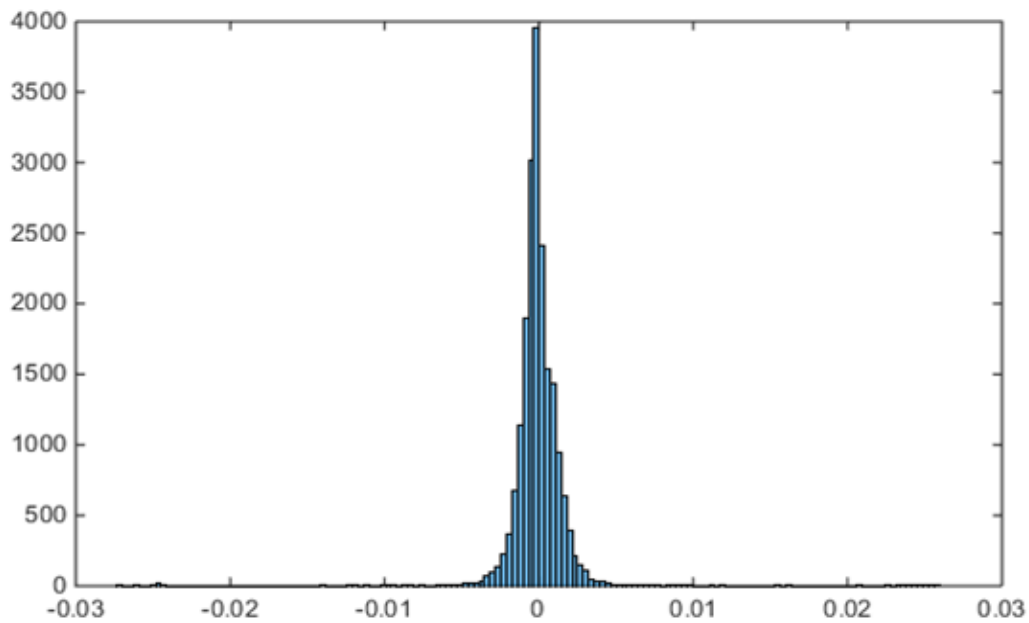


Figure 3.2: Histogram for log differences of future.

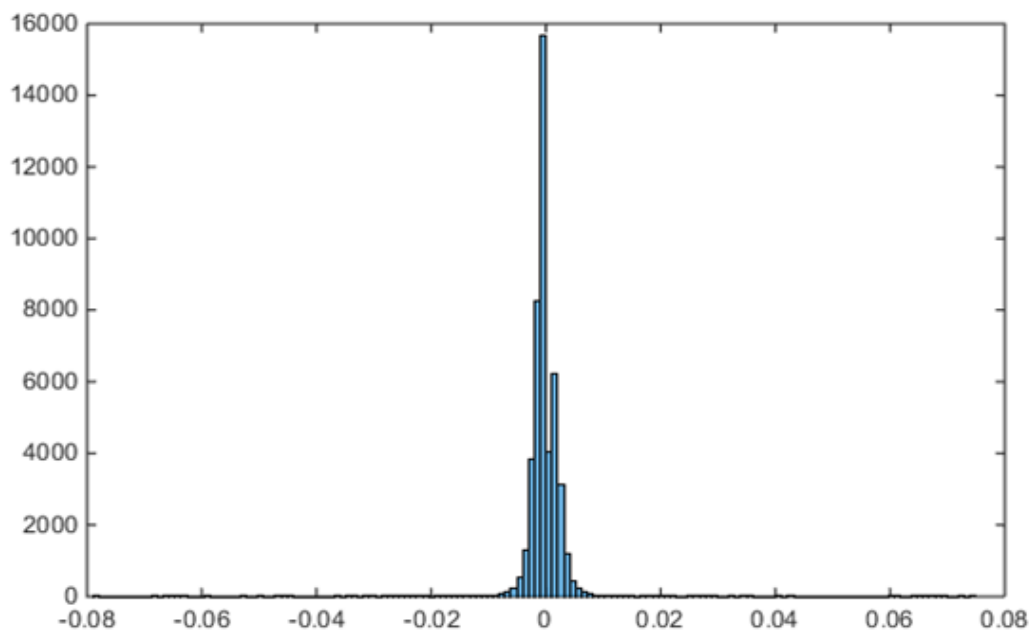


Figure 3.3: Histogram for log differences of CDS Index.

### 3.3 Details About the Data

The reader is probably already familiar with equity indices and therefore the equity section will not be as thorough as the CDS section.

#### 3.3.1 The Equity Data

EURO STOXX 50 is an equity index consisting of 50 of the largest and most liquid stocks in the Eurozone. These are weighted by the free-float market cap. The index futures on EURO STOXX 50 are among the most liquid futures in the world. (STOXX Limited, 2015) In the thesis we are looking to trade the EURO STOXX 50 index but the standard way to do this is by trading a future on the index. This has several advantages like the high liquidity mentioned above and the natural leverage they offer. The futures roll every quarter so the approximately one year worth of data consists of four smaller time series of on-the-run future contracts as can be seen in Table 3.2.

Table 3.2: Future contract dates.

Contract	VGZ2	VGH3	VGM3	VGU3
Start	01 Oct 2012	17 Dec 2012	15 Mar 2013	15 Jun 2013
End	14 Dec 2012	14 Mar 2013	14 Jun 2013	13 Sep 2013

The equity data has a regular sampling frequency and is sampled every 10 minutes. There are exceptions but most days the data stretches from 08:00am to 10:00pm. Naturally there is, in addition to the futures prices, a column with times corresponding to the observations and one with the name of the futures contract being quoted.

#### 3.3.2 The CDS Data

The Markit iTraxx Europe index is composed of 125 liquid European entities with investment grade credit ratings that trade in the credit market. The index will begin on the first business day later than or equal to September 20 and March 20 of each calendar year. The contracts are issued with maturities of 3 years, 5 years, 7 years and 10 years. When a contract of maturity  $n$  years is issued on September 20 it matures on December 20  $n$  years later and for contracts issued on March 20 they mature on June 20  $n$  years later. The index

is approximately equally weighted. (Markit Group Limited, 2013) Due to the semi-annual roll, the one year series consists of two smaller series of on-the-run contracts as specified in Table 3.3.

Table 3.3: CDS contract dates.

Contract	iTraxx Europe Series 18 Version 1 5Y	iTraxx Europe Series 19 Version 1 5Y
Start	01 Oct 2012	20 Mar 2013
End	20 Mar 2013	13 Sep 2013

The CDS sampling frequency is highly irregular ranging from several observations in a minute to no observation in ten minutes. How the data stretches over the days is also irregular but the quotes normally start coming in at around 8am and stop at around 6pm. The data set consists of four columns; time when the quote was made, bid and offer of spread using quoted spread as convention (see section 2.1.1.6.2) and the name of the contract being quoted.

As mentioned above the indices are traded and quoted in spread (not in price). A buyer of the index takes on the credit exposure, so buying the index is equivalent to selling protection. By selling the index the exposure is passed on to another party. The indices trade with a fixed coupon that is paid quarterly for the past quarter by the seller of the index (the buyer of protection on the index). This coupon is received by the buyer of the index (protection seller). At initiation and closure of the trade upfront payments are made to account for the difference between the fixed coupon and the real spread of the index. The quoted spread is clean, that is accruals are not considered. (Markit Group Limited, 2012)

### 3.4 Modified Time Series

The time series are used throughout the thesis and their original format is not always appropriate for our purposes. Naturally minor modifications are done frequently without having to be mentioned in this section. However, all the minor modifications are based on three series, one of them being the original series not modified at all. The following section will describe the two other series referred to as the resampled series and the synthetic series. A shortcoming with both of these series that is not handled is that when the contracts are rolled, there will be a big jump that is not part of the dynamics. But since it is only 1 point

for the CDS series and 3 points for equity series out of around 130 000 it should not affect the statistical tests too much.

### 3.4.1 Resampled Series

For someone used to looking at equity pairs our time series pair looks really unpleasant. There are especially three properties that have to be dealt with. These are the irregular sampling frequency of one of the time series, the mixed sampling frequencies of the two series and the different sampling periods of the two series. The motivation behind creating this new series is primarily because our cointegration tests, which will be explained in later sections, cannot handle the unpleasant original series.

Each series is first divided into daily series. Only the days of data that existed on both the equity and CDS set were kept. Assume now that one day the CDS time series stretches from  $t_0^{CDS}$  to  $t_1^{CDS}$  and that the equity one stretches from  $t_0^{Eq}$  to  $t_1^{Eq}$ , then the new series would stretch from  $\max(t_0^{CDS}, t_0^{Eq})$  to  $\min(t_1^{CDS}, t_1^{Eq})$ . This modification solves the problem with the different sampling periods. To solve the problem with the mixed and irregular sampling frequencies an easy solution would be to get the CDS series on the same sampling frequency as the equity series since this is already regularly sampled every 10 minutes. The CDS data however sometimes has more than 10 observations in 10 minutes with quite some movements in this interval so too much information would be lost by this resampling method. Instead both series are resampled to 1 observation per minute. This is made by doing a linear interpolation between all observations and then resampling this, theoretical continuous series, with 1 minute intervals. Finally all these daily observations are put together to get back the full original time series. The number of observations are after resampling 130 793 for both series.

Figure 3.4 shows the new resampled series plotted in the same window. They now look more like we are used to when it comes to financial assets. Compared to Figure 3.1 we avoid the flat areas overnight and the inverted relation becomes even more evident. Note that the inverted relation seems to disappear for a short period during January to February where they have a trend in the same direction.

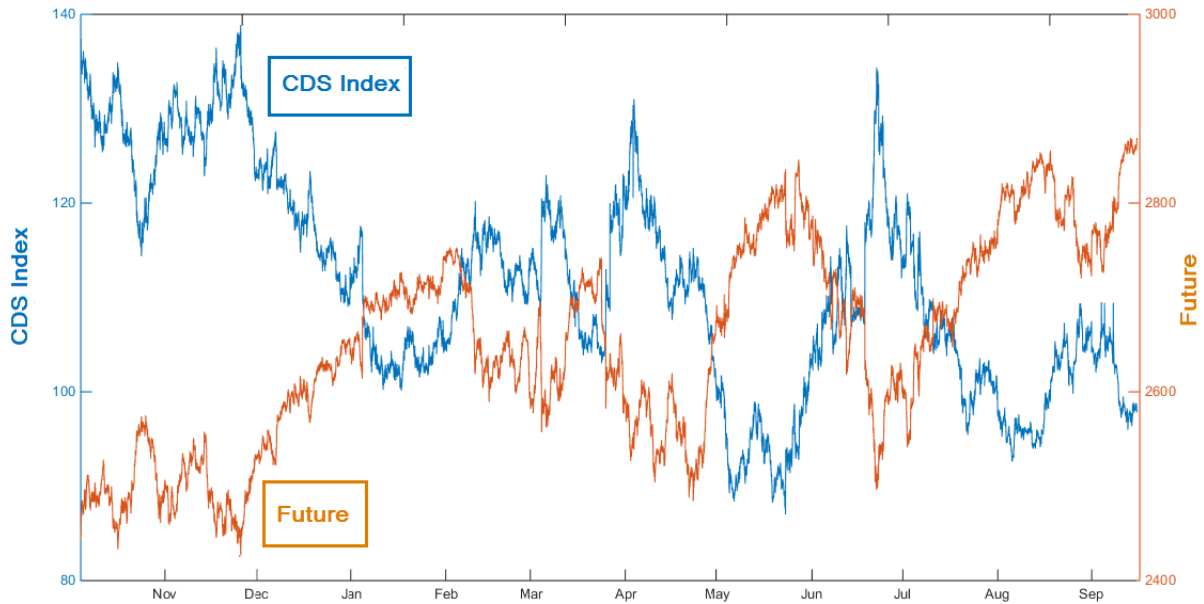


Figure 3.4: Plot of the two resampled series.

Furthermore, we are not aware of how the modifications that have been done will affect the cointegration relation between the series. There is a tradeoff between two effects, one working in favor for proving cointegration and one working against it. More data makes it easier to prove cointegration but the resampling hides some of the dynamics of the series. In the 10 minutes between two real equity observations the time series will always be monotone, due to the resampling method even if the real series probably is not, while the CDS may move both up and down. This effect can make the cointegration harder to detect.

### 3.4.2 Synthetic Series

The motivation behind creating a synthetic series is to obtain a more clean series on which the tests can be run. The synthetic series is a further development of the resampled series. All the returns in the resampled series except 233 (234 days of data) are one minute returns and the rest are overnight or even longer period returns. These 233 returns should have a significantly different distribution than those of the rest of the series. Table 3.4 shows some key figures for these returns and we can indeed see that the mean and volatility almost are of order  $10^2$  greater. It may be that these big movements make the intraday movements appear less significant and by removing them the series could be cleaner and thus the



intraday relation more evident. After all, cointegration assumes constant (time invariant) mean and covariance for the residual series, see section 4.11 and 4.4.

Table 3.4: Mean and volatility for removed data.

	CDS index	Future
Mean log return	-8.14e-04	5.02e-04
Annualized volatility <sup>5</sup>	1037.92 %	327.74 %

Consequently, the synthetic series is constructed by creating a new series without the overnight moves. An array with all the log returns in the resampled series is created. The 233 overnight moves are then removed. From the start value of the resampled series a new synthetic series is built by rebuilding the series from the log returns, but this time excluding the overnight moves. Just as with the resampled series we need to keep in mind that this is no longer real data and we need to be careful about which conclusions we draw.

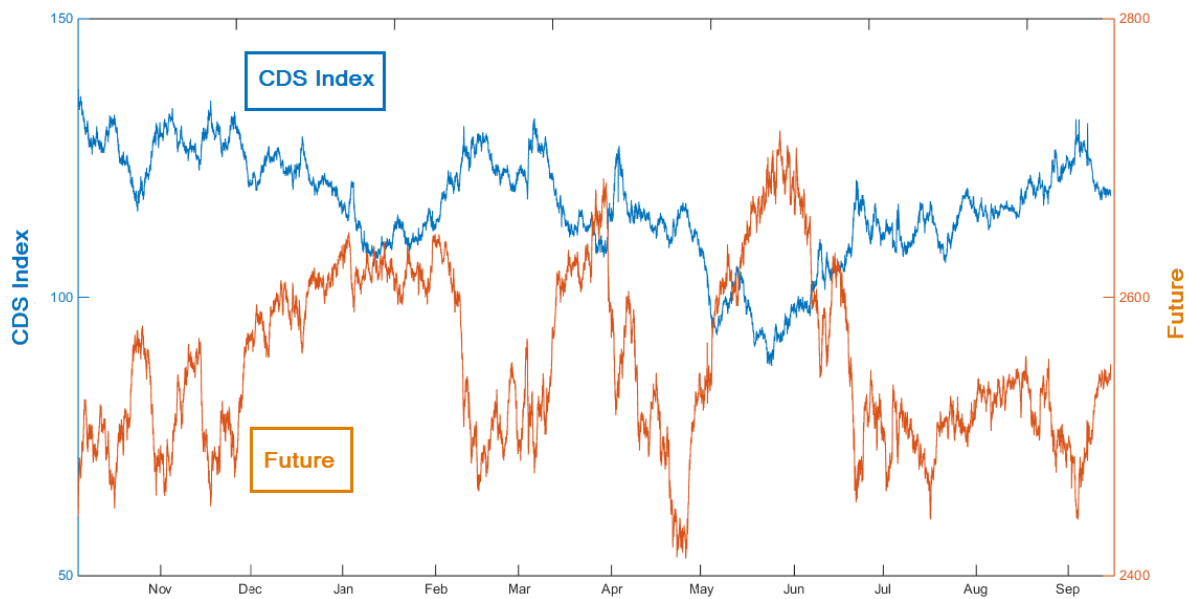


Figure 3.5: Plot of the two synthetic series.

Figure 3.5 shows the synthetic series plotted. It differs more from Figure 3.4 than one would maybe expect since only 233 out of 130 793 observations (less than 1 in 500) were removed.

<sup>5</sup> With the same calculation method as in Table 3.1.

This confirms the different nature of the overnight observations. One would especially note that a lot of the trend in both series have disappeared which confirms the strong bias that can be derived from Table 3.4. To further show this we plot the resampled against the synthetic series for each asset, see Figure 3.6 and Figure 3.7.

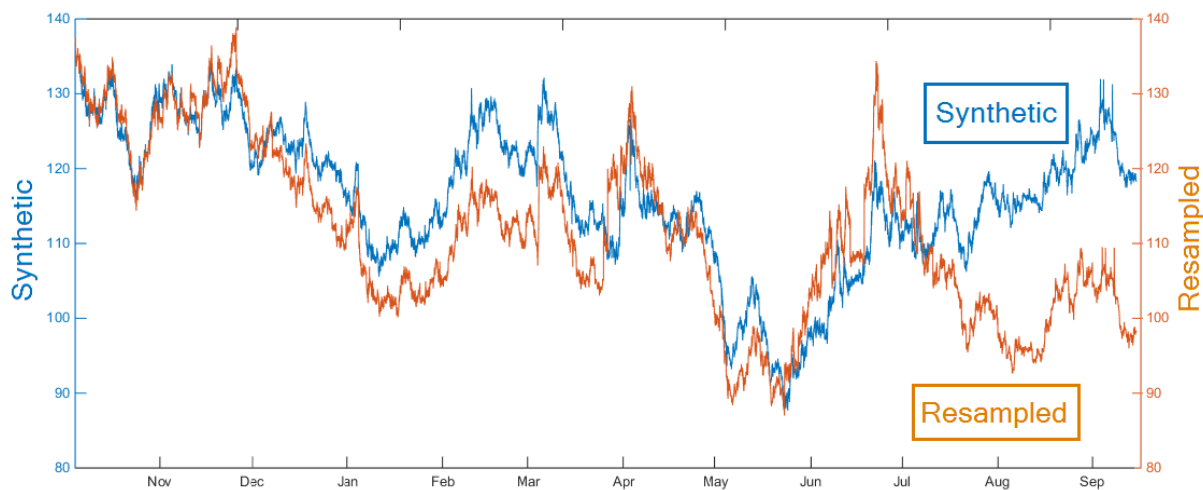


Figure 3.6: Synthetic against resampled CDS series.

One must keep in mind that there is a risk that too much of the dynamics have disappeared in the synthetic series. One explanation to why the series diverge so much is because it is not rare that the stock market differs more than a percent between closing the night before and opening the morning after and a lot of the trend can be present at these points in time

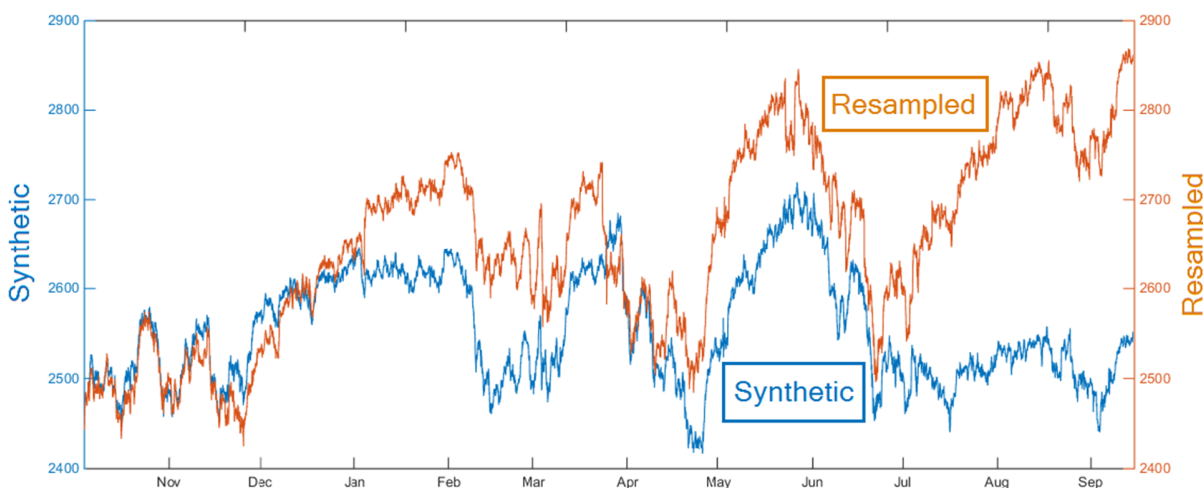


Figure 3.7: Synthetic against resampled futures series.

## 4 Theory and Methods

The following section covers the theory and assumptions that form the foundation on which the Statistical Testing section is built on. The section covers both basic theory and concepts that might be new to the reader.

### 4.1 Time Series Model

Given the observed data  $\{x_t\}$ , Brockwell & Davis (2002) define a time series model as the specifications of the joint distributions of a sequence of random variables  $\{X_t\}$  of which  $\{x_t\}$  is a realization. Said specifications may only consist of mean and covariance.

### 4.2 Mean and Covariance

The mean function of a time series  $\{X_t\}$  with  $E(X^2) < \infty$  is

$$\mu_X(t) = E(X_t) \quad (4.1)$$

and the covariance function is

$$\gamma_{j,t} = Cov(X_{t-j}, X_t) = E \left[ (X_t - \mu_X(t)) (X_{t-j} - \mu_X(t-j)) \right] \quad (4.2)$$

### 4.3 White Noise

White noise plays a big role in time series analysis. If  $\{X_t\}$  is a white noise process it can be denoted as  $X_t \sim \text{i.i.d}(0, \sigma^2)$ . Hamilton (1994) defines white noise as a sequence of uncorrelated random variables with a zero mean and variance  $\sigma^2$ . If the series also has a normal distribution, denoted  $N(0, \sigma^2)$  it is called a Gaussian white noise process.

### 4.4 Stationarity

A time series  $\{X_t\}$  is said to be weakly stationary if

- $\mu_{X_t}$  is independent of  $t$ ,  $E[X_t] = \mu \forall t$
- $\gamma_{j,t}$  is independent of  $t$ ,  $E[(X_t - \mu_{X_t})(X_{t-j} - \mu_{X_t})] = \gamma_j \forall t$  and any  $j$

This means that the mean is constant and the covariance only depends on the lag. In this thesis, weak stationarity will be shortened to just stationarity. (Hamilton, 1994)

## 4.5 Moving Average Process – MA( $q$ )

Hamilton (1994) defines  $\{X_t\}$  as a moving average process of order  $q$ ,  $MA(q)$ , if it can be characterized by

$$X_t = \mu + u_t + \theta_1 u_{t-1} + \dots + \theta_q u_{t-q} \quad (4.3)$$

$\{u_t\}$  being white noise with zero mean and variance  $\sigma^2$  and  $\mu, (\theta_1, \theta_2, \dots, \theta_q)$  being constants. We see as a consequence that

$$E[X_t] = \mu \quad (4.4)$$

$$\gamma_0 = (1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2)\sigma^2 \quad (4.5)$$

$$\gamma_j = \begin{cases} (\theta_j + \theta_{j+1}\theta_1 + \theta_{j+2}\theta_2 + \dots + \theta_q\theta_{q-j})\sigma^2 & \text{for } j = 1, 2, \dots, q \\ 0 & \text{for } j > q \end{cases} \quad (4.6)$$

With a mean and a covariance both independent of  $t$ , all  $MA(q)$ -processes are stationary.

## 4.6 Autoregressive Process – AR( $p$ )

Hamilton (1994) defines a time series as an autoregressive process of order  $p$ ,  $AR(p)$ , if it can be characterized by

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + u_t \quad (4.7)$$

with  $\{u_t\}$  still being white noise with zero mean and variance  $\sigma^2$ . If the roots of the autoregressive polynomial

$$1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p = 0 \quad (4.8)$$

all lie outside the unit circle, i.e.  $|z| > 1$ , the process is stationary and

$$\mu = c/(1 - \phi_1 - \phi_2 - \dots - \phi_p) \quad (4.9)$$

$$\gamma_j = \begin{cases} (\phi_1 \gamma_{j-1} + \phi_2 \gamma_{j-2} + \dots + \phi_p \gamma_{j-p}) & \text{for } j = 1, 2, \dots \\ (\phi_1 \gamma_1 + \phi_2 \gamma_2 + \dots + \phi_p \gamma_p + \sigma^2) & \text{for } j = 0 \end{cases} \quad (4.10)$$

## 4.7 First Difference Operator

The first difference operator  $(1 - L)$  or  $\Delta$  of a sequence  $\{x_t\}$  is defined as

$$\Delta x_t = x_t - x_{t-1} \quad (4.11)$$

## 4.8 Order of Integration $d - I(d)$

Hamilton (1994) defines a non-stationary time series  $\{X_t\}$  to be integrated of order  $d$ ,  $\{X_t\} \sim I(d)$ , if the series becomes stationary after differencing  $d$  times. A stationary time series is integrated of order 0 or  $I(0)$ . The autoregressive polynomial of an  $AR(p)$ -process that is integrated of order  $d$  has  $d$  unit roots, i.e. with  $z = 1$ .

## 4.9 Dickey-Fuller Unit Root Test

The Dickey-Fuller test was presented by Dickey & Fuller (1979) and the test works under the assumption that  $\{X_t\}$  follows an  $AR(1)$  model, that is

$$X_t = c + \rho X_{t-1} + u_t \quad (4.12)$$

The test tests the null hypothesis of a time series  $\{X_t\}$  being non-stationary against the alternative of stationarity. This can also be expressed as  $H_0: \{X_t\} \sim I(1)$  against  $H_1: \{X_t\} \sim I(0)$  which is equivalent to testing  $H_0: \rho = 1$  against  $H_1: \rho < 1$ . Rewrite (4.12) by subtracting both sides by  $X_{t-1}$

$$X_t - X_{t-1} = c + (\rho - 1)X_{t-1} + u_t \quad (4.13)$$

which from (4.11) is equivalent to

$$\Delta X_t = c + (\rho - 1)X_{t-1} + u_t \quad (4.14)$$

or

$$\Delta X_t = c + \delta X_{t-1} + u_t \quad (4.15)$$

with  $\delta = (\rho - 1)$ .

Now the null hypothesis can be expressed as  $H_0: \delta = 0$  against  $H_1: \delta \neq 0$ . This is obvious since if  $\delta = 0$  the increment  $\Delta X_t$  is just a random walk with drift  $c$

$$\Delta X_t = c + u_t \quad (4.16)$$

which is evidence for  $X_t$  being non stationary. As it turns out,  $\hat{\delta}$ , the OLS estimator of  $\delta$  does not have an ordinary  $t$ -distribution. Fortunately, the asymptotic distribution was simulated by Dickey & Fuller (1979) using a Monte Carlo Simulation. Its critical values were presented by Fuller (1996). The  $t$ -statistic of the estimator is

$$t = \frac{\hat{\delta} - \delta_0}{\hat{\sigma}_{\hat{\delta}}} = \frac{\hat{\delta}}{\hat{\sigma}_{\hat{\delta}}} \quad (4.17)$$

where the last equality follows from the fact that  $\delta_0 = 0$ .  $\hat{\sigma}_{\hat{\delta}}$  is the standard error of the estimate. To be able to obtain the correct critical value from the tables one has to make a distinction between three different cases (Dickey & Fuller, 1979; Hamilton, 1994).

#### 4.9.1 Case I

The true process is a random walk,  $X_t = X_{t-1} + u_t$ . The OLS estimation of  $\rho$  is based on an  $AR(1)$  regression

$$X_t = \rho X_{t-1} + u_t \quad (4.18)$$

#### 4.9.2 Case II

The true process is still a random walk,  $X_t = X_{t-1} + u_t$ . The OLS estimation of  $\rho$  is now based on an  $AR(1)$  regression with an intercept

$$X_t = c + \rho X_{t-1} + u_t \quad (4.19)$$

#### 4.9.3 Case III

The true process is now a random walk with or without drift,  $X_t = c + X_{t-1} + u_t$ . This time the OLS estimation of  $\rho$  is based on an  $AR(1)$  regression with an intercept and a time trend

$$X_t = c + \delta t + \rho X_{t-1} + u_t \quad (4.20)$$

When choosing which case to use for the test Hamilton (1994) only gives the suggestion of using case III for series with an obvious trend and case II for series without significant trend.

## 4.10 Augmented Dickey-Fuller Test

If the time series is an AR-process, but contains more than one lagged term, the ordinary Dickey-Fuller test does not work. To fix this problem Said & Dickey (1984) presented the Augmented Dickey-Fuller (ADF) test, that tests  $AR(p)$ -processes for stationarity. Recall the characteristic polynomial for an  $AR(p)$  process from (4.7)

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + u_t \quad (4.7)$$

The first step of the test is to choose how many lags  $p$  the model should include. Cryer & Chan (2008) suggest doing this by using an information criterion such as Akaike (AIC) or Bayesian (BIC) to find the best fitting approximation of the AR-process. Such a criterion gives a score that for each added variable decreases with its increased goodness of fit and is punished by adding a constant multiplied by the logarithm of the number of variables to avoid overfitting. This makes picking the right number of variables a tradeoff between goodness of fit and the punishing term. When an appropriate  $p$  has been determined Hamilton (1994) suggests rewriting (4.7) by defining

$$\rho = \phi_1 + \phi_2 + \dots + \phi_p \quad (4.21)$$

$$\zeta_j = -[\phi_{j+1} + \phi_{j+2} + \dots + \phi_p], \quad j = 1, 2, \dots, p-1 \quad (4.22)$$

The auto regression can now be written

$$X_t = c + \rho X_{t-1} + \zeta_1 \Delta X_{t-1} + \zeta_2 \Delta X_{t-2} \dots + \zeta_{p-1} \Delta X_{t-p+1} + u_t \quad (4.23)$$

The null hypothesis for the ADF test is the same as for the ordinary Dickey-Fuller test,  $H_0: \{X_t\} \sim I(1)$  against  $H_1: \{X_t\} \sim I(0)$ . For the process that generated  $X_t$  to contain one single unit root, to be  $I(1)$ , then one root of (4.24) has to be 1 and the others have to be outside the unit circle. The autoregressive polynomial still being

$$1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p = 0 \quad (4.24)$$

If there is a unit root then

$$1 - \phi_1 - \phi_2 - \dots - \phi_p = 0 \quad (4.25)$$

Keeping (4.21) in mind, this leads up to the same null hypothesis as in the ordinary Dickey-Fuller test, that is  $\rho = 1$ . We rewrite (4.23) in the same manner as in the ordinary Dickey-Fuller test and get

$$\Delta X_t = c + \delta X_{t-1} + \zeta_1 \Delta X_{t-1} + \zeta_2 \Delta X_{t-2} \dots + \zeta_{p-1} \Delta X_{t-p+1} + u_t \quad (4.26)$$

where as before  $\delta = (\rho - 1)$ , once again rephrasing the null hypothesis to  $H_0: \delta = 0$  against  $H_1: \delta \neq 0$ . The  $t$  statistic for  $\delta$  is calculated in the same manner and compared to the same tabulated critical values as for the ordinary Dickey-Fuller test. (Said & Dickey, 1984)

## 4.11 Cointegration

The two economists Engle and Granger observed that even though two time series are non-stationary, there might be some sort of a stationary relationship between the two. They found that some time series have a long-term equilibrium. They coined this mean reverting behavior cointegration and this was one of the ideas for which they were awarded the Nobel Prize in economics in 2003. (Vidyamurthy, 2004)

Hamilton (1994) defines an  $(n \times 1)$  vector time series  $\mathbf{y}_t$  as cointegrated if each of the  $n$  series taken individually are  $I(1)$ , nonstationary with a unit root, while some linear combination of the series  $\mathbf{a}'\mathbf{y}_t$  is stationary,  $I(0)$ , for some nonzero  $(n \times 1)$  cointegrating vector  $\mathbf{a}$ .

## 4.12 Engle-Granger Test

Although there is another commonly used cointegration test, the Johansen test, we will only apply the Engle-Granger test. Reasons for selecting this particular test are that it is intuitive and the fact that its only limitation would not become a problem in our case. The only limitation compared to the Johansen test is that it is less accurate in the case of testing for cointegration between more than two time series. Alexander (1999), one of the pioneers of the cointegration approach to pairs trading, even considered the Engle-Granger test to be more accurate than Johansen in the case of only two series and a big enough data sample.



The Engle-Granger test is a very straight forward two step test. Shortly put, all you do is regress one of the two non-stationary series on the other using a linear OLS regression and then test the residuals of the equation for stationarity with a unit root test, such as Dickey-Fuller. The regression equation is

$$X_t = \beta_0 + \beta Y_t + u_t \quad (4.27)$$

And if the residuals of the regression equation are left alone on one side

$$X_t - \beta Y_t - \beta_0 = u_t \quad (4.28)$$

where  $\beta_0$  might or might not be zero, the logic of the test is obvious. If the residuals are stationary then the linear combination of the two time series in (4.28) is also stationary, which is the definition of cointegration. Since the residuals are not observed but only estimated, the ordinary critical values for the Dickey-Fuller test do not apply and new values have to be computed as described by MacKinnon (2010).

## 5 Statistical Testing

The following section covers all the statistical testing that is based on the theory and methods presented in the previous section. The testing is performed on the three different data sets that are the original data, the resampled data and the synthetic data. This testing is done primarily to test the series for cointegration, upon which our entire trading strategy is based. When cointegration has been confirmed, further testing is done to gain more understanding of how the two series relate. We also try to find other peculiarities in the data that may be of use when building trading strategies.

The section is divided into five parts. The first part tests the series for unit roots, which is a condition for cointegration. The second part tests the different data sets for cointegration. The third part further quantifies and evaluates the cointegrating relationship. The fourth part investigates a potential way to forecast volatility in the residual series. The last part briefly concludes all the testing that has been performed.

### 5.1 Testing for Unit Root

The definition of cointegration demands the two time series to be non-stationary and integrated of order 1,  $I(1)$ . The MATLAB built-in `adftest` is applied to the series to test for a unit root and thus order of integration. To confirm the results from these tests, the same test is applied on both of the series after differencing them. When running the tests on the differenced series the results should be reversed and they should be stationary and not have a unit root.

The `adftest` tests a time series for a unit root and returns `1` if it can reject the null hypothesis of unit root ( $p$  – value  $< 0.05$ ) and `0` otherwise. There are four settings that can be altered if one does not wish to use the default settings. For our test the two settings that are not left on default are the model specification and the lag specification. The default setting is the alternative model of an  $AR(1)$ -process without drift (Case I). (The MathWorks, Inc., 2015c) But as mentioned in the theory and method section, Hamilton (1994) recommends using the alternative model of an  $AR(1)$ -process with drift and either with or without a time trend (Case II & III). The default setting for the number of lagged terms,  $p$  in (4.7),

is 0. As mentioned in the theory and method section the number of lags should be determined by using an information criterion such as BIC. To determine whether to use a model with or without a trend and how many lags to include in the model, the BIC-values for each combination are presented in the Table 5.1-6 below. The model with the lowest BIC has the best fit. In the tables only the three lags around the one with the minimum BIC are presented.

Table 5.1: CDS index, Original series, model specification.

	Drift			Drift & Trend		
Lag	2	3	4	4	5	6
BIC	18831	18768	18777	18785	18763	18768

Table 5.2: Future, Original series, model specification.

	Drift			Drift & trend		
Lag	11	12	13	11	12	13
BIC	10461	10461	10461	10462	10461	10462

Table 5.3: CDS index, Resampled series, model specification.

	Drift			Drift & trend		
Lag	1	2	3	1	2	3
BIC	-19974	-19982	-19982	-19973	-19981	-19981

Table 5.4: Future, Resampled series, model specification.

	Drift			Drift & trend		
Lag	5	6	7	5	6	7
BIC	26860	26854	26854	26861	26855	26855

Table 5.5: CDS index, Synthetic series, model specification.

	Drift			Drift & trend		
Lag	5	6	7	5	6	7
BIC	-29986	-29988	-29987	-29985	-29986	-29985

Table 5.6: Future, Synthetic series, model specification.

	Drift			Drift & trend		
Lag	11	12	13	11	12	13
BIC	-25495	-25845	-25844	-25494	-25845	-25843

From the tables above, the best model specifications can be obtained and used to test the undifferenced series for a unit root. When testing the differenced series for a unit root, the test returned the minimal p-value of 0.001 for every model specification setting.

Table 5.7: adftest on undifferenced data.

	Original series		Resampled series		Synthetic series	
	CDS index	Future	CDS index	Future	CDS index	Future
Model(lag)	Drift & trend(5)	Drift(12)	Drift(2)	Drift(6)	Drift(2)	Drift(12)
p-value	0.1052	0.3806	0.0946	0.2816	0.0685	0.0507

Table 5.8: adftest on differenced data.

	Original series		Resampled series		Synthetic series	
	CDS index	Future	CDS index	Future	CDS index	Future
p-value	0.001	0.001	0.001	0.001	0.001	0.001

The unit root tests on both of the time series in all three data sets confirmed that both of the series are non-stationary and integrated of order 1,  $I(1)$ . Since the two series have been confirmed to fulfill the requirements for cointegration, the Engle-Granger test for cointegration can be applied.

## 5.2 Testing for Cointegration

The built in `egcitest` in MATLAB tests for cointegration within a matrix, which in this case consists of two time series. Since these series must be of the same size, the test cannot be applied on the original series and the Engle-Granger test is only applied on the resampled and synthetic series. Critical values for MATLAB's `egcitest` are only tabulated for a sample size of up to 10 000. To account for this the test is run on every  $n:th$  element so that

$\frac{\text{Sample size}}{n} < 10\,000$  where  $n = 1, 2, 3 \dots$ . Since we have more than 500 data points per day we feel confident in doing so. The test returns **1** if it can reject the null hypothesis of no cointegration ( $p$  – value  $< 0.05$ ) and **0** if it cannot reject cointegration. The test estimates the residuals of the OLS regression and runs a Dickey-Fuller test on them. As mentioned in the Theory and Methods section, the critical values for estimated residuals are not the same as for an ordinary Dickey-Fuller test so critical values are computed as described by MacKinnon (2010) in MATLAB as well. (The MathWorks, Inc., 2015d)

When running the `egcitest`, the OLS estimates of  $\beta_0$  and  $\beta$  in (4.28) are obtained. From these estimates and (4.28) the residuals can be calculated and also plotted for visualization. When running the Engle-Granger test on the entire sample for both the resampled and the synthetic data we obtain very different results, see Table 5.9. The resampled data shows strong evidence for cointegration while the synthetic data does not.

Table 5.9: `egcitest` on Synthetic and Resampled data.

	Synthetic data	Resampled data
p-value	0.1935	0.0010

Looking at the plotted residuals for the two data sets (Figure 5.1 and Figure 5.2) the results make sense since the residuals from the synthetic data do not seem to oscillate around 0 to the same extent. The major difference between the series is that we in the synthetic data removed the changes that took place over nights, weekends and non-trading days. For both of the residuals series there is a distinct dip in April, and while the resampled data reverts to 0 rather quickly, the synthetic one reverts slowly. Recall that the synthetic data suddenly deviates more from the resampled data towards the end of the sample. Due to this and the fact that the synthetic does not show evidence of cointegration while the resampled data does, the rest of the testing in this section will be performed on the resampled data alone.

When running the `egcitest` it is, as mentioned, run on every  $n$ :th element to not exceed the maximum of 10 000 elements. To test how sensitive the `egcitest` is to this frequency another test is performed. The  $p$ -value from the `egcitest` is calculated for a rolling window of size 10 000 observations. The mean  $p$ -value for these windows is then calculated using  $n = 1, 2$  and  $5$ . The results can be seen in Table 5.10. It is obvious that a cointegrating relationship

becomes more evident with a decreasing sampling frequency. A possible reason for that might be that the estimated residuals become smoother with less movement between every zero crossing. This way, a bigger portion of the movements are zero crossings, which might make the residuals easier for the test to detect as stationary.

Table 5.10: egctest's sensitivity to sampling frequency.

n	1	2	5
Average $p$ -value	0.4266	0.3550	0.2921

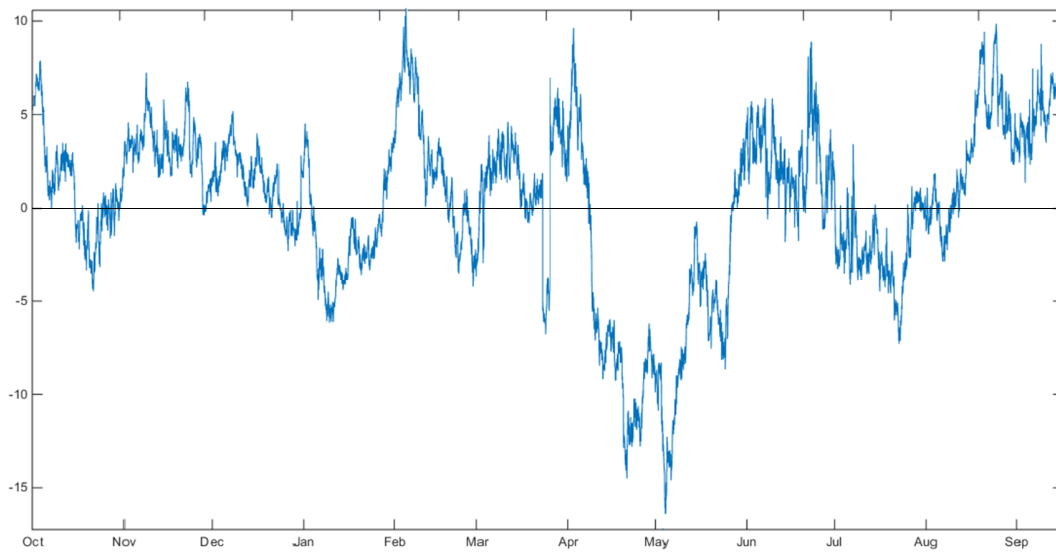


Figure 5.1: Residuals of regression on Resampled data.

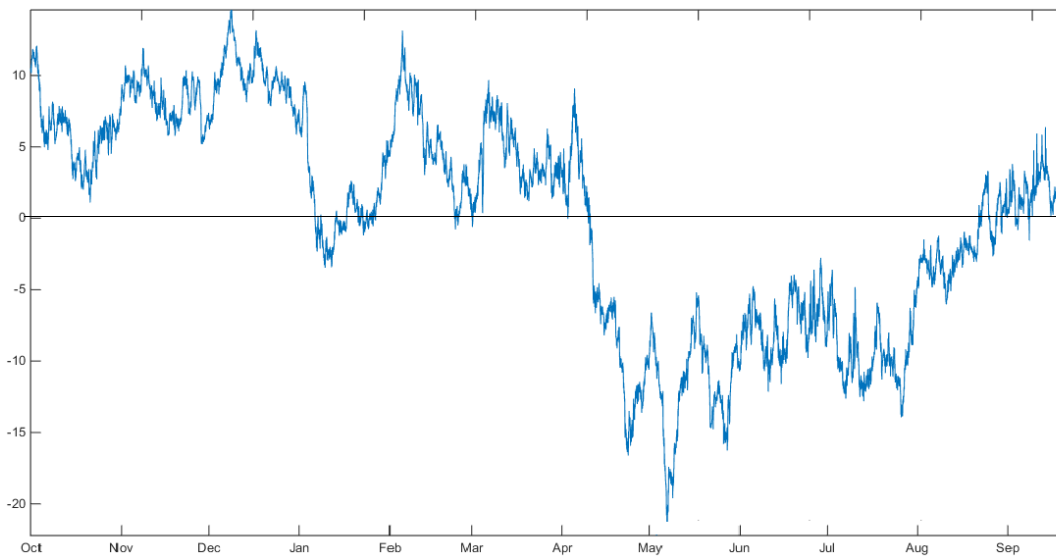


Figure 5.2: Residuals of regression on Synthetic data.

### 5.3 Evaluating the Cointegrating Relationship

Below in Figure 5.3 and Figure 5.4 we can see how the parameters  $\beta_0$  and  $\beta$  in (4.28) change over time (calculated with a rolling window of size 10 000 observations). One thing worth noting is that  $\beta$  changes sign at three times. That is because the two series divert from their inverse relation as pointed out in the Data section and show sign of a temporary trend in the same direction. This diversion from the pattern can be seen in Figure 3.1 in January where both series seem to have an upgoing trend. Even though the parameters seem to change quite a lot, it seems that changes are only temporary and corrected with time. This gives us further reason to believe that our two time series do indeed possess the mean reverting trait of cointegration. The fact that the parameters can change rather dramatically, even if only temporarily, gives us a hint that when constructing the trading strategy it might be wisest to estimate the parameters continuously as opposed to using fixed parameters estimated from one single training period.

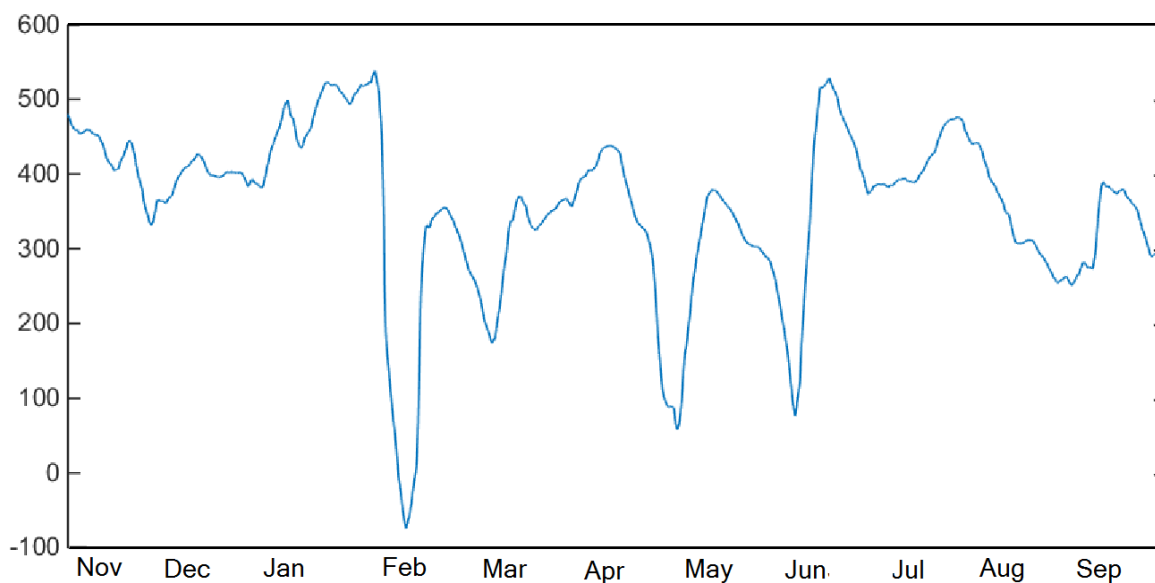


Figure 5.3: Parameter stability for  $\beta_0$ .

Further reason to believe so comes from looking at the frequency of zero crossings in the residuals in Figure 5.1. A high frequency of zero crossing is a trait that Vidyamurthy (2004) points out to be very desirable for pairs trading, since it would lead to more possible trades. Looking at the residuals in Figure 5.1 we can see that in the periods November 2012 and

mid August 2013 - mid September 2013 there is not a single zero crossing but they seem to oscillate around another value. Say that the residuals cross a threshold that triggers a trade in the beginning of November. This trade would not be unwound until the end of November when the residuals revert back down to 0. During this time, the residuals have been oscillating around another value (see Figure 5.6 and Figure 5.5) and several potentially profitable trades could have been done. If the residuals were estimated for these windows individually we would not miss out on these opportunities.

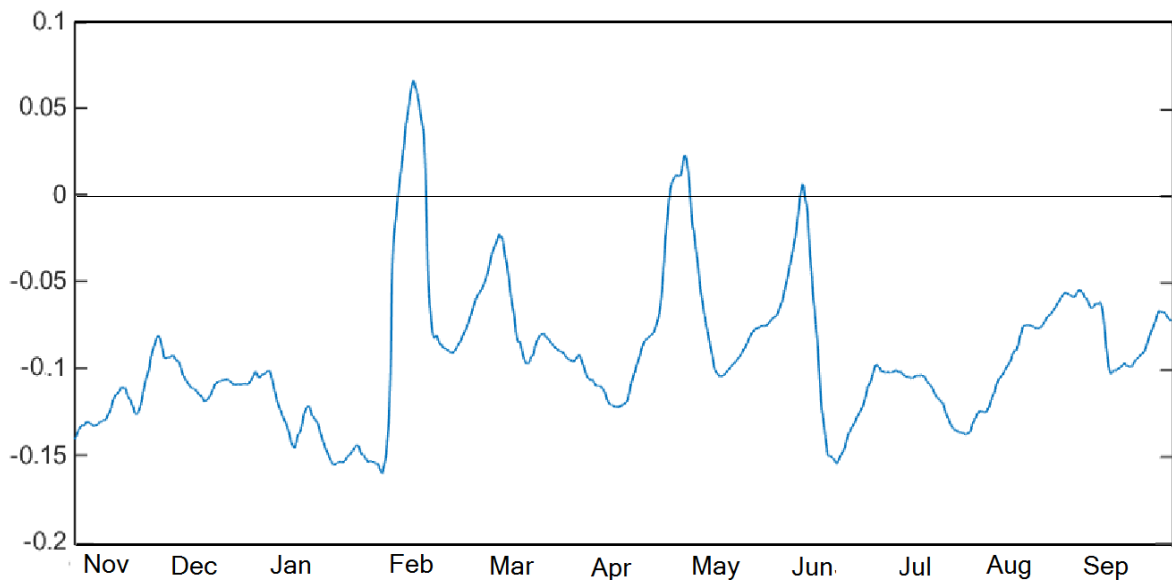


Figure 5.4: Parameter stability for  $\beta$ .

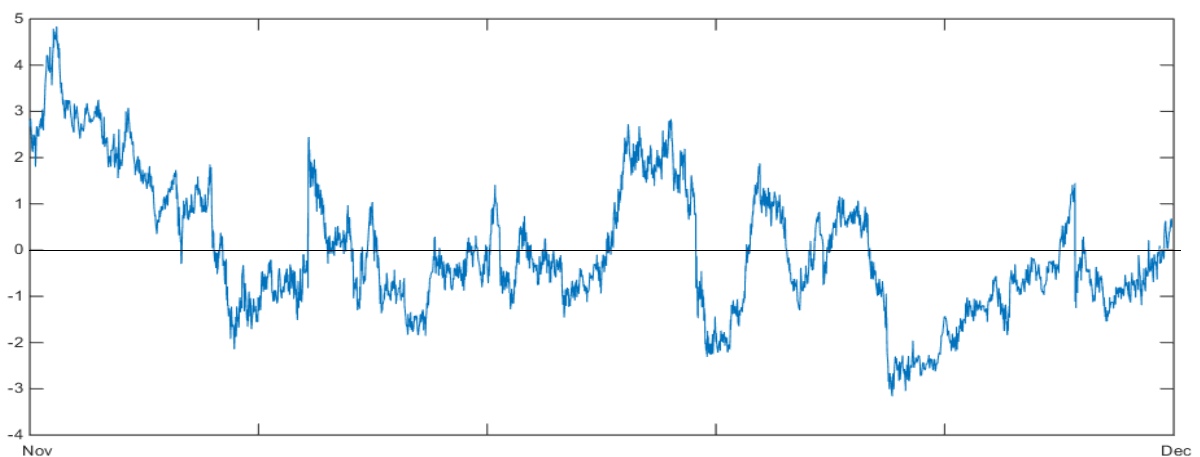


Figure 5.5 Residuals of regression on window November 2012.



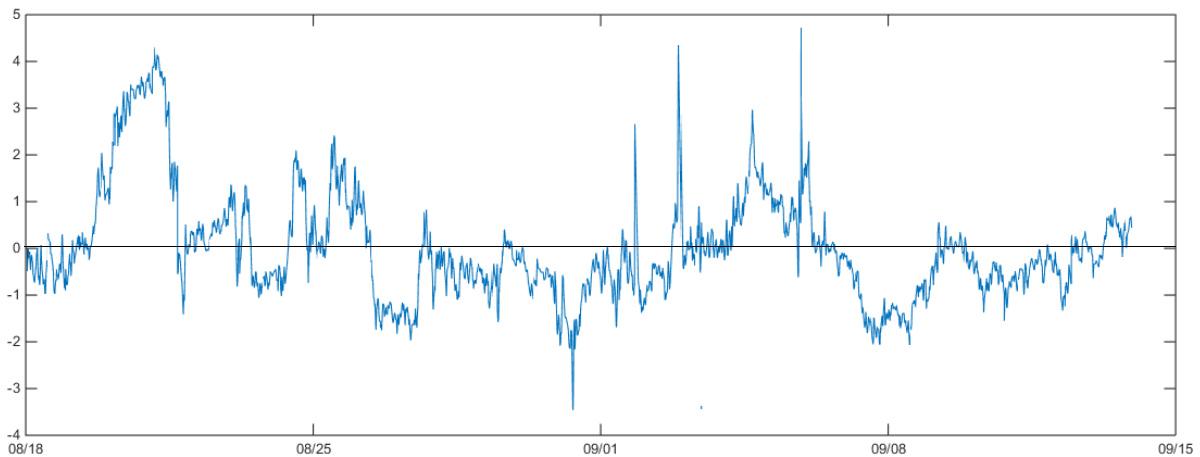


Figure 5.6: Residuals of regression on window mid-August 2013 to mid-September 2013.

## 5.4 Forecasting Volatility

The sampling frequency of the CDS series or the frequency of CDS quotes is irregular and we believe that this may be used to forecast volatility of CDS series and thus residual series. This could be used to make the trading strategies better. We perform tests on whether the volatility is higher during a period with frequent quotes. We also test the lagged hypothesis, which is that a period with frequent quotes is followed by a period of higher volatility.

Figure 5.7 shows a scatter plot of the number of quotes during a day against the hourly volatility that day. Just by observing the plot we can see that the number of quotes does not seem enough to explain the volatility. Trying to fit a simple linear regression model the results suggests that the model is better than a model with only intercept, see Table 5.11. An  $F$ -test is used when calculating the  $p$ -values. However the variance of the data against the linear model is too high to be used in practice which can be seen by the low R-squared value. One could continue to investigate more complicated models, like logistic regression and multiple regressors but as mentioned before we have decided to keep things simple and close to intuition in order to avoid overfitted strategies so we end the search here. Even if we would find a relationship with a method like logistic regression we would not be able to use it in a trading strategy unless it can be quantified.

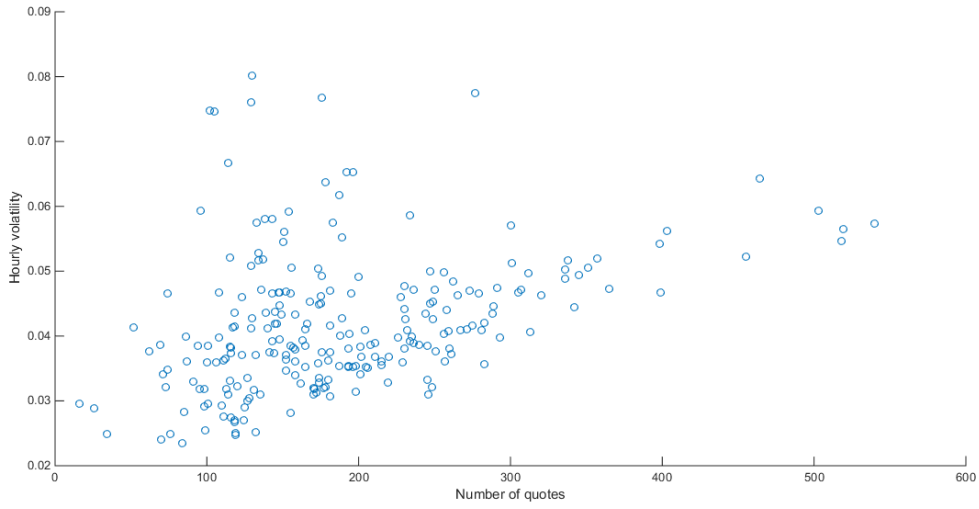


Figure 5.7: Scatter plot of daily number of quotes against hourly volatility.

Table 5.11: Summary statistics for the regression models.

	Adj. R-Sq.	p-val against only intercept
Linear Regression(Unlagged)	0.124	1.72e-08
Linear Regression(Lagged)	0.062	6.57e-05

Figure 5.8 shows a scatter plot with the number of daily quotes against next day's hourly volatility. The regression that was run on the unlagged data is also run on the lagged one and the goodness of fit was even lower in this case, see Table 5.11. The search for a better model is ended for the same reason as for the unlagged data.

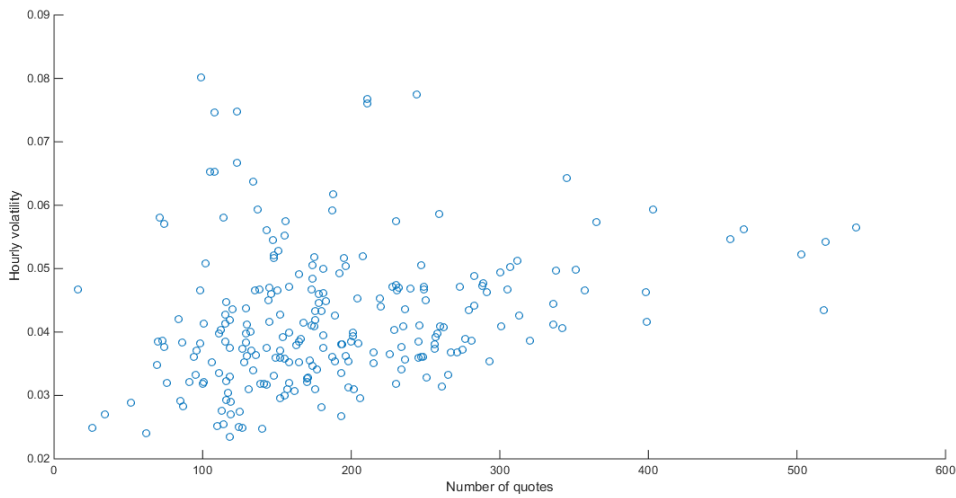


Figure 5.8: Scatter plot of daily number of quotes against hourly volatility (1 day lag).

## 5.5 Conclusions on Statistical Testing

To conclude, building the synthetic data unfortunately did not contribute by making cointegration easier to detect, and will therefore not be used for further testing. The resampled data on the other hand shows strong evidence for cointegration. The residuals seem to oscillate around a long term equilibrium over the entire sample, but in smaller windows as well. The cointegrating parameters do vary a lot throughout the sample, but tend to revert back to some equilibrium over time. The results also indicate that using as much data as possible might not be the best approach as a lower sampling frequency for the egcitest makes it easier to detect cointegration. Finally, forecasting the CDS volatility based on the frequency of the quotes turned out to be a dead end, for now.

## 6 Trading Strategies

In this section we present all the work related to the pure development of the trading strategies. The section is divided into three parts. The first part briefly discusses previous work on pairs trading and key takeaways that may be used in our trading strategies. The second section outlines our implementation of the CDS pricer used in the trading simulations due to its central part. The third part describes the development of the trading strategy itself.

### 6.1 Pairs Trading

Pairs trading was first developed in the 1980's by a team lead by Nunzio Tartaglia, while he was working for Morgan Stanley. The team consisted of mathematicians, physicists and computer scientists and they did not only develop the theory of pairs trading, they also developed automated systems to detect and exploit mispricings in the financial markets. (Vidyamurthy, 2004)

Vidyamurthy (2004) further describes pairs trading as a market neutral strategy, using only two securities with similar betas canceling each other out. If the two securities are chosen wisely the spread between these two securities will be a stationary process. The whole idea of pairs trading is based on the fact that two cointegrated series will have a long-run equilibrium that in the case of diversion will eventually be corrected. If the spread deviates from its mean at a certain magnitude,  $u$ , the strategy is to put a long position at the relatively undervalued security and a short position at the relatively overvalued security. When the spread is back to its mean the positions are unwound and profit is made. When using a cointegration approach, this spread is instead of a difference in price, the stationary residuals from a linear combination of the two time series. The weights of the two assets are decided by the cointegrating coefficient  $\beta$  in the regression model (4.7). Vidyamurthy (2004) also proves that the threshold  $u$  that maximizes profit is  $0.75\sigma$ , where  $\sigma$  is the standard deviation of the normal density around the mean. This is of course with the approximation that residual are of normal distribution.

## 6.2 CDS Pricing – Our Implementation<sup>6</sup>

In theory, CDS pricing is straight forward with not so many ambiguities but there are many ways to do a practical implementation and therefore it is worth having a section explaining our implementation. We need to keep in mind that the goal is to price i.e. MTM the CDS as closely as possible to what the market would have done at the time it was valued. Thus there is no point in building a pricer that perfectly finds the value of the CDS according to our or someone else's model, using market conventions is enough. The implementation of the CDS pricer is done in MATLAB since this is where the trading is simulated.

### 6.2.1 Bias

During the testing of the trading strategy we assume that the trading book is empty at the initiation of the trading period and that all positions are unwound at the end of the trading period. Thus we expect to be equally many times on the short side of the contracts as on the long side. This makes the pricing model less vulnerable to bias in the pricing. Ideally, the model has no bias but if it happens to systematically over- or underprice the contracts the total effect of this on the trading profit will probably cancel out to some extent. There is a vast amount of articles written on CDS pricing alone or even on how swap curves are built. The main focus of this work is not to price CDS and there will be details in the pricing that can be improved.

### 6.2.2 The ISDA Standard Model

ISDA, the International Swaps and Derivatives Association, is a global trade association for OTC derivatives and has since 1985 worked to make the OTC derivatives markets safe and efficient (International Swaps and Derivatives Association, Inc., 2015). This association has developed a standardized model, the ISDA CDS Standard Model for MTM of CDS (ISDA and Markit Group Limited, 2009a). No existing implementation in MATLAB could be found of this model and to implement the full ISDA model into MATLAB code was not feasible for several reasons. First of all the task of writing the code itself would be a task too time consuming and complicated for our purposes and capabilities, secondly the model

---

<sup>6</sup> We are happy to share the CDS pricer code upon request, please contact the authors on [daniel.alavei@gmail.com](mailto:daniel.alavei@gmail.com).

requires some input data that is not available to us. The model we implement is inspired by this model when possible but is a lot simpler.

### **6.2.2.1 Discount Curve**

In order to build the discount curve the standard model suggests using 20 points on the curve with specified maturities, 6 of them being deposit rates and the rest swap rates. Details about which rates that are used can be found in Appendix A.1.

#### **6.2.2.1.1 Interpolation**

We need a discount curve with daily points from a curve with 20 points. A curve can be built by interpolation of the 20 known points on the curve. In MATLAB there is a standard tool for building interest rate curves from market data (The MathWorks, Inc., 2015a). Some important input arguments have to be specified when using this tool and one of them is the interpolation method to use. The interpolation method used is Piecewise Cubic Hermite Interpolating Polynomial, referred to as PCHIP in MATLAB (The MathWorks, Inc., 2015b). In Zakrajsek, et al. (2005) the authors, all affiliated with the Division of Monetary Affairs, Federal Reserve Board, use this method to fit a CDS curve. They claim that it is appropriate to use since; it is interpolating, it preserves monotonicity in the data and where the data has extreme points so has the curve. Thus artificial oscillations are avoided which may not be the case when using splines for instance. Note that all 20 rates must be brought to the same compounding frequency before interpolation, preferably continuous compounding.

### **6.2.2.2 Credit Curve**

In section 2.1.1.2 about CDS pricing we mentioned that in order to calculate the RPV01 the term structure of probabilities of default is needed and in order to calculate this contracts of different maturities on the same name is needed. In the data section we described the nature of the data and that the data set consists of contracts of one maturity and thus the theoretical pricing model cannot be used. However, it happens to be that the quoting convention used on the data is quoted spread so the simplified approach assuming a flat credit curve can be used and will result in the right upfront payment.

### 6.2.2.3 Recovery Rate

For our purposes the recovery rate 40 % should be used throughout all calculations according to the ISDA standard model. (ISDA and Markit Group Limited, 2009b)

## 6.3 The Trading Strategies<sup>7</sup>

Simulating a real world trading environment is a very complex task and for this reason we chose to start with a simplified model of the reality and then successively put on more layers of reality, we call this the three layer model and it will be discussed later. When constructing the trading strategy we always keep in mind that the strategy has to make intuitive sense in order to avoid overfitting. We therefore start with a strategy that is intuitive and pleasing according to the theory we have. This strategy has several settings, some of them as simple as parameters, some of them models that can be modified. We do not strive to find the exact parameters that would optimize the profit for the particular data set but try some different settings that have different orders of magnitude. In other words we do not calibrate our trading algorithm based on historical data and as a result we do not need to divide our data into a training set and a test set. The whole sample can be used for testing since the algorithm is not based on our data.

### 6.3.1 Simplifications

As mentioned before, the markets and products we are dealing with are way too complex to fully model in a trading simulation. Several simplifications have already been made and mentioned on the way but some important ones done in the trading implementation will be mentioned here. In section 2.1.1.1, describing CDS market, we mentioned that premiums are paid quarterly and therefore one has to deal with accruals in terms of clean and dirty prices. Since we do not take the time value of money into account anywhere in the trading we might as well assume that premiums are paid every minute to avoid dealing with accruals. There is another approximation being made when it comes to rolling the contracts. The series are not overlapping when contracts are rolled so we assume that contracts are exited at the closing price of the night before the roll and entered at the opening price of the

---

<sup>7</sup> We are happy to share the code upon request, please contact the authors on [daniel.alavei@gmail.com](mailto:daniel.alavei@gmail.com).

morning after the roll. This is to avoid having to make approximations about the roll down to the next contract. Furthermore the discount curve is only updated on a daily basis since the effects of intraday movements in the discount curve are negligible.

A drawback of the data set is that we do not have access to information about the defaults in the underlying names of the CDS index. Assume that you are long protection, i.e. paying premium, then you would expect to at some point get something back due to a default in an underlying name but this will never happen in our data set. In a risk neutral world one would expect that the par yield would cancel out default payments in the long run. We do not have the par spread, only the quoted spread. They tend to be close and are therefore assumed to be the same, this is the first approximation. The second approximation is that the default leg will on average pay as much as the premium leg (with par spread or quoted spread after approximation). We illustrate the use of this in the trading algorithm with an example. Assume that we enter into a long protection position quoted at 120 bps with a fixed coupon of 100 bps. Then you have to make a payment when entering into the fixed coupon contract since it is more attractive. At that time you expect to pay 100 bps on the premium leg and receive 120 bps on the protection leg on average, a positive carry of 20 bps in other words. This is exactly what we assume that you get in our implementation. Finally we need to keep in mind that all the CDS quotes are indicative quotes of where they can be traded, not actual trades. However, we have to assume that they can be executed at the given time and price since we have no other information.

### **6.3.2 The Original Trading Model**

First we go through how the so called original model trades and which inputs are needed and which are used. The algorithm looks for cointegration in a moving window, since the cointegration parameters change dramatically over time, on which it bases its trading decisions. The first parameter that has to be specified is thus the window size, the default setting in the original model is 10 000 observations. This value is used since this is the maximum number of observations that there are tabulated critical values for in the cointegration testing. After that we specify the window sampling frequency, which is set to 1 observation/minute by default, since we want to use all our data. With these settings the algorithm first has to collect 10 000 data points before it can start trading.



When the algorithm has collected enough data to start trading (window size) it starts to look for cointegration in the moving window. In the search for cointegration one has to specify the significance level to use in the cointegration test. 5 % is our default setting since this is the MATLAB default setting. The first criterion for trading is that the algorithm cannot reject cointegration, but this is not enough to trigger a trade. From the cointegration regression the current value of the residual can be calculated. A trade is only put on if the residual has diverged more than a certain threshold from its mean (in this case 0). The threshold used is  $0.75\sigma$  based on the motivation in section 6.1. The next setting is naturally how to calculate the volatility and here the default setting is just the sample standard deviation.

Finally, one has to specify criteria for when trades are closed. We choose to close the trade when the residual, calculated using the regression coefficients from when the trade was put on, has reverted back to its mean. That is when it is back to 0 or when the trading period ends, whichever happens first. One could implement a stop loss functionality or a maximum holding period before the trade is closed as well but this is left out in the first model.

The aforementioned model has a certain unwanted behavior leading to one more parameter being added. If cointegration is found for a window and the residual has diverged enough a trade is put on. In the next step the floating window, say of size 10 000, moves one step and unless there has been any big movements in the assets, cointegration will likely be found again and almost the same trade will be put on again just one minute later. So the same trade is basically doubled in size and this can happen for several consecutive minutes which is not the idea. Therefore there is a parameter, set to 1 hour by default that restricts how often a trade can be put on.

### **6.3.3 Modification of the Original Strategies**

With a modifiable trading algorithm in place the next step is to run it with different settings but still big changes, not fine tuning, in order to avoid bias. Ideally, all the settings would be altered simultaneously but this would result in a tremendous amount of cases and each simulation is very time and resource consuming. Therefore a two-step pseudo optimization

is made. In the first step we change one setting at a time, in some cases two when it makes sense to do. In the next step we try to combine the winners into better ones but one must keep in mind that there is no guarantee that this would generate an optimal strategy.

### **6.3.4 The Three Layer Model**

All the above mentioned tests are run on the resampled series, a simplified model of the real world. This is mainly because we cannot run the cointegration tests on the original series. Another reason is that both positions can always be offset at the same time so we avoid having to take outright risk. In some sense it should thus be easier to find a profitable strategy on the resampled series.

The best strategies from simulation on the resampled series qualify for the next level of testing where we put on one more layer of the reality. The trades from the previous stage have timestamps so they are now instead made on the original series, as closely as it is possible since all the observations in the resampled data are not real. If the trades are still profitable in this stage they qualify for the final layer which is probably the hardest test, namely the original series with transaction costs.

The reader might find it unnecessary to run the simulations on the original data without transactions costs at all. Thinking that moving on straight to the original data with transaction costs would be better. But there is actually a good reason for not doing so. When the simulations are first run on the resampled data the algorithm can at any time enter into market neutral positions by trading both assets at the same point in time. When the real data is introduced it is not always, or rarely, possible to trade both assets at the exact same time. This means that when a position in one of the assets has been entered but not yet in the other there will be an unwanted market exposure for a short period of time. This exposure could hit either way depending on how the market moves and therefore it is good to be aware of how big this effect is before adding the transaction costs to avoid drawing the wrong conclusions about the effect of transaction costs.

## 7 Results

In this section we present the results from the testing. The section is divided into four parts. The first part presents the results from the different settings for the parameters. The second part presents the results from evaluating some of these settings on the original data. The third part presents results with transaction costs added. In the last part we try to evaluate how good these strategies are in absolute terms, not only compared to each other.

In all the result tables 13 key figures have been handpicked in order to interpret the results from the trading simulations. The one with the highest profit in absolute terms is definitely not always the better choice. A definition of the key figures and a short explanation, where needed, of why they are relevant to look at will be provided below.

The exposure range for each asset is defined as the minimum and maximum net exposure during the trading period. Too high exposures are not good for obvious reasons. Risk mandates can be reached and exceeded and more collateral is needed to do the trade so the return on equity becomes lower. For the CDS contracts the exposure is measured in notional and for the equity index in number of futures contracts. The average exposure in absolute terms for each asset is the next figure. It is important that it is in absolute terms so the high negative exposures do not cancel out high positive ones. This number can be used to compare the final PnL to the amount of risk taken on average.

Total number of trades is another figure presented, followed by total number of unique trading days. This is calculated by counting the number of unique days a trade was entered. The total number of trades can sometimes be misleading when there are many trades entered the same day. These trades are often basically the same and thus number of unique trading days better reflects the risk diversification of the strategy. The success rate is defined as the proportion of the trades with positive PnL. The average holding period, measured in calendar days is also presented. A figure that reminds of the gross margin of a company is introduced to measure how much profit is made in relation to the total trade volume. First we define the revenue as the sum of all trades with positive PnL and the sum of the absolute value of the trades with negative PnL. Then the gross margin is defined as the PnL divided by this. The mean and standard deviation of the PnL of each trade are also presented.

Finally, the total PnL for the period is presented, but also the more interesting figure; annualized total PnL by dividing by the fraction of a year used for trading. A summary of all the key figures is presented next to the unit and abbreviation that will be used when presenting the results, see Table 7.1.

Table 7.1: Key figures and abbreviations.

Abbreviation	Unit	Key figure
Exp. rng. fut.	Number of contracts	Exposure range for futures
Exp. rng. CDS	Notional in EUR	Exposure range for CDS
Avg. exp. fut.	Number of contracts	Average absolute exposure for futures
Avg. exp. CDS	Notional in EUR	Average absolute exposure for CDS
# trades	No unit	Total number of trades
# unique days	Days	Total number of unique trading days
Success rate	Percent	Success rate
Avg. holding	Calendar days	Average holding period
Gross margin	Percent	Gross margin
Mean PnL	EUR	Mean of all trade PnLs
Std. PnL	EUR	Standard deviation of trade PnLs
Total PnL	EUR	Total PnL
PnL p.a.	EUR / year	Annualized PnL

## 7.1 Simulation on Resampled Series

Just as mentioned in 6.3.3 a two-step pseudo optimization is performed by first changing one or two parameters at the time in the original trading model (round 1) and then recombining the best performers from this stage into new strategies (round 2). This method, known as genetic programming, was first proposed by Holland (1975).

### 7.1.1 Round 1

In the first round, 16 simulations are run. The results from the simulations are presented in groups of four preceded by a key explaining the settings used in each strategy simulation. The key only states what differs from the default settings and therefore we begin by reminding the reader of the default settings mentioned in 6.3.2, see Table 7.2.

Table 7.2: Trading strategy default settings.

Parameter	Value
Window size	10 000 data points
Window sampling frequency	60 observation / hour
Cointegration test significance level	5 %
Trading threshold	$0.75 \sigma$
Stop loss	No
Maximum trading frequency	1 / hour

In Table 7.3 the settings for the first four strategies can be seen and the results can be found in Table 7.4. The very first strategy is with default settings, this can be used as a benchmark throughout the section to quickly evaluate if a set of settings improved the algorithm or not. We also try to change the maximum allowed trading frequency and the window size. When comparing the strategies in terms of profit the risk weighted (by average exposure) profit per annum is a better measure than just comparing the total PnL. The argument is that a strategy with a shorter trading period could be run for longer and that a strategy with lower exposure could scale up risk and profit with the same factor. In terms of risk weighted profit strategy 1-2 are far better than 3-4. It looks like strategies 1 and 2 are almost doing the same trades, except 2 is doing them twice as often.

Table 7.3: Strategies 1-4 settings.

Strategy	Settings different from default
1	Default
2	Maximum trading frequency: 2 / hour
3	Maximum trading frequency: 1/ day
4	Window size: 5 000

Table 7.4: Strategies 1-4 results.

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Exp. Rng. Fut.	[-691, 1028]	[-1380, 1879]	[-115, 190]	[-771, 1147]
Exp. Rng. CDS	[-14e06, 21e06]	[-27e06, 38e06]	[-2e06, 4e06]	[-17e06, 21e06]
Avg. exp. Fut.	157.74	271.47	38.46	139.16
Avg. exp. CDS	3.67e06	6.50e06	0.96e06	3.16e06
# trades	143	252	31	116
# unique days	35	35	31	30
Success rate	98.60 %	98.81 %	96.77 %	93.97 %
Avg. holding	8.86	8.89	10.06	9.24
Gross margin	84.56 %	87.03 %	65.10 %	62.28 %
Mean PnL	799.05	833.97	634.50	408.64
Std. PnL	822.6	779.97	1162.03	679.99
Total PnL	114 264	210 160	19 669	47 402
PnL p.a.	128 641	236 601	19 670	51 328

Below we present the settings for the next four strategies (Table 7.5) followed by the results from the simulations (Table 7.6). The main theme of these strategies is the cointegration. We run two simulations with changed values of the cointegration significance level, then one where we trade regardless of cointegration but still use its parameters and finally one where we look for cointegration in the full historic data. From a risk weighted profit perspective strategy 8 is clearly the better one. But its success rate is significantly lower than the others' which can also be seen from the high standard deviation. Another drawback is the number of trades it is using which will disfavor it when putting on transaction costs.

Table 7.5: Strategies 5-8 settings.

Strategy	Settings different from default
5	Cointegration test significance level: 100 %.
	Does not look for cointegration but uses regression parameters in trading.
6	Cointegration test significance level: 10 %.
7	Cointegration test significance level: 2.5 %.
8	Window size is set to max(10 000, all historical data).

Table 7.6: Simulation 5-8 results.

	Strategy 5	Strategy 6	Strategy 7	Strategy 8
Exp. Rng. Fut.	[-5554, 5189]	[-1837, 1808]	[-581, 272]	[-10 451, 9 688]
Exp. Rng. CDS	[-0.18e09, 0.18e09]	[-47e06, 35e06]	[-11e06, 6e06]	[-0.26e09, 0.22e09]
Avg. exp. Fut.	2031.10	432.76	48.91	2142.87
Avg. exp. CDS	69.25e06	11.07e06	1.06e06	48.86e06
# trades	1401	344	61	1341
# unique days	197	69	19	179
Success rate	91.43 %	96.80 %	96.72 %	81.95 %
Avg. holding	19.52	10.44	5.57	12.12
Gross margin	63.83 %	73.35 %	60.85 %	71.56 %
Mean PnL	1056.49	819.44	520.72	1320.60
Std. PnL	1797.04	1105.07	1109.93	1832.80
Total PnL	1 480 144	281 888	31 764	1 770 895
PnL p.a.	1 666 373	317 355	35 760	1 993 705

In strategies 9-12, that can be seen in Table 7.8, the windows size and sampling frequency on the window used are altered, see Table 7.7. The holding period increases as the sampling frequency gets lower, just as one would expect. Strategy 9 is the clear winner in several ways and does also beat the default settings. It has the highest annual risk weighted profit by far, the highest success rate and top 2 in number of unique trading days. One drawback is the high maximum exposure that it has even if the average is low. It is also a bit surprising that the average holding period does not differ too much from that of the default strategy. Furthermore, all four strategies do significantly more trades on more unique days compared to the default settings indicating that it seems easier to find cointegration with lower sampling frequency, consistent with the results in the Statistical Testing section.

Table 7.7: Strategies 9-12 settings.

Strategy	Settings different from default
9	Window size: 10 000. Window sampling frequency: 30 / hour.
10	Window size: 10 000. Window sampling frequency: 12 / hour.
11	Window size: 5 000. Window sampling frequency: 30 / hour.
12	Window size: 5 000. Window sampling frequency: 12 / hour.

Table 7.8: Strategies 9-12 results.

	Strategy 9	Strategy 10	Strategy 11	Strategy 12
Exp. rng. fut.	[-2151, 3050]	[-1423, 4289]	[-1109, 1613]	[-2421, 3449]
Exp. rng. CDS	[-38e06, 61e06]	[-31e06, 0.11e09]	[-24e06, 34e06]	[-57e06, 69e06]
Avg. exp. fut.	456.75	910.93	377.69	886.51
Avg. exp. CDS	9.73e06	29.28e06	8.72e06	21.80e06
# trades	319	300	300	439
# unique days	54	48	59	68
Success rate	96.87 %	93.33 %	94.00 %	87.70 %
Avg. holding	9.73	18.35	9.73	14.24
Gross margin	90.88 %	97.07 %	52.23 %	66.20 %
Mean PnL	1255.39	1948.03	611.68	1084.57
Std. PnL	925.82	1353.66	1418.76	1489.22
Total PnL	400 468	584 409	183 503	476 126
PnL p.a.	490 081	976 791	206 591	604 977

The last four simulation results of round 1 (Table 7.10) and their settings (Table 7.9) are shown below. Here we examine different thresholds for entry and exits of trades. The two leftmost columns are stop losses for when to exit if the trade moves in the wrong direction. The stop loss is set so that the probability of triggering the stop loss is equal to the percentage value under normality assumptions, given that there is cointegration and given the volatility at the time the trade was initiated. The idea is that cointegration can be rejected if the residual diverges from its mean, but the problem is that the volatility from when the trade was initiated may now, most likely, have changed. The stop loss strategies do not do well but strategy 16 does outstanding. It has a very high risk weighted profit, 100% success rate, not too many trades but still good distribution of unique trading days.

Table 7.9: Strategies 13-16 settings.

Strategy	Settings different from default
13	Stop loss: $1.96 \sigma$ (one sided 97.5 %)
14	Stop loss: $2.58 \sigma$ (one sided 99.5 %)
15	Threshold: $0.50 \sigma$
16	Threshold: $1.50 \sigma$



Table 7.10: Strategies 13-16 results.

	Strategy 13	Strategy 14	Strategy 15	Strategy 16
Exp. rng. fut.	[-251, 379]	[-572, 775]	[-744, 1182]	[-528, 480]
Exp. rng. CDS	[-5e06, 7e06]	[-12e06, 15e06]	[-15e06, 24e06]	[-10e06, 9e06]
Avg. exp. fut.	8.31	19.83	198.01	53.83
Avg. exp. CDS	0.17e06	0.42e06	4.46e06	1.19e06
# trades	143	143	185	52
# unique days	35	35	43	17
Success rate	44.76 %	50.35 %	98.92 %	100 %
Avg. holding	0.42	1.06	8.34	8.29
Gross margin	9.78 %	-4.71 %	86.33 %	100 %
Mean PnL	41.77	-37.50	710.11	1174.02
Std. PnL	539.76	954.01	742.91	390.35
Total PnL	5 972	-5 362	131 370	61 049
PnL p.a.	6 724	-6 037	147 899	68 730

We pick five strategies to qualify for a second round of simulations and thus be combined into new strategies. These are strategies 5<sup>8</sup>, 7<sup>9</sup>, 8<sup>10</sup>, 9<sup>11</sup> and 16<sup>12</sup> and will be followed by individual motivations. 5 qualifies mainly because it trades almost every day and still had a reasonable risk weighted profit. To trade every day is a valuable property because then success of the strategy probably has little to do with the exact things that happened in this data set and more with the dynamics of the assets. 7 qualifies because of its good trades to unique days-ratio and that it brings down the number of trades and would thus be interesting to combine with 5 and 8. 8 qualifies because it, just as 5, has a lot of trading days but differs on some other points, like having higher risk weighted profit, lower success rate and shorter average holding period. Finally, 16 qualifies because it does just outstanding in terms of risk weighted profit and success rate. Just as 7 it brings down the total number of trades which could result in a very successful strategy when combined with 5 or 8.

---

<sup>8</sup> Cointegration test significance level: 100 %.

<sup>9</sup> Cointegration test significance level: 2.5 %.

<sup>10</sup> Window size is set to max(10 000, all historical data).

<sup>11</sup> Window size: 10 000. Window sampling frequency: 30 / hour.

<sup>12</sup> Threshold:  $1.50\sigma$ .

## 7.1.2 Round 2

The five strategies that qualified to the second round cannot be recombined arbitrarily, only certain settings can be used together. A total of 8 new strategies are created and they use at most settings from 3 different strategies from round 1. We strive to narrow the 8 strategies down to 4 before moving on to the testing on the real data. We group the results so that the strategies with few trades are in Table 7.13 and the strategies with many trades are in Table 7.14 with corresponding settings in Table 7.11 and Table 7.12.

Table 7.11: Strategies 17-20 settings.

Strategy	Settings different from default
17	Window size: 10 000. Window sampling frequency: 30 / hour. Threshold: $1.50 \sigma$
18	Window size: 10 000. Window sampling frequency: 30 / hour. Cointegration test significance level: 2.5 %.
19	Cointegration test significance level: 2.5 %. Threshold: $1.50 \sigma$
20	Window size: 10 000. Window sampling frequency: 30 / hour. Cointegration test significance level: 2.5 %. Threshold: $1.50 \sigma$

Table 7.12: Strategies 21-24 settings.

Strategy	Settings different from default
21	Window size: 10 000. Window sampling frequency: 30 / hour. Cointegration test significance level: 100 %.
22	Window size: 10 000. Window sampling frequency: 30 / hour. Cointegration test significance level: 100 %. Threshold: $1.50 \sigma$
23	Window size is set to $\max(10\,000, \text{all historical data})$ . Threshold: $1.50 \sigma$
24	Cointegration test significance level: 100 %. Threshold: $1.50 \sigma$

All the strategies 17-20 have really good risk weighted profits compared to strategies 21-24. Strategy 17 has slightly lower risk weighted profit than its peers. Strategy 19 is doing very few trades but is successful in all of them. Even if strategies 21-24 do not have as high risk weighted profit they do a lot of trades. This is good because it is hard to blame a strategy of being overly fitted when it trades almost every day and has a success rate of around 90%. On the other hand it can be hard for a trading strategy that does many trades to perform well after adding transaction costs. We have chosen to pick strategies 18, 19, 20 and 22 to run tests on the real data since they are appropriate according to expressed preferences.

Table 7.13: Strategies 17-20 results.

	Strategy 17	Strategy 18	Strategy 19	Strategy 20
Exp. rng. fut.	[-1579, 2074]	[-1770, 3050]	[-418, 56]	[-1198, 2074]
Exp. rng. CDS	[-28e06, 42e06]	[-31e06, 61e06]	[-7e06, 1e06]	[-21e06, 42e06]
Avg. exp. fut.	303.51	313.38	18.34	204.33
Avg. exp. CDS	6.50e06	6.44e06	0.43e06	4.20e06
# trades	158	255	20	130
# unique days	30	44	7	25
Success rate	94.94 %	98.04 %	100 %	96.92 %
Avg. holding	13.12	8.31	7.50	10.58
Gross margin	89.53 %	94.49 %	100 %	93.41 %
Mean PnL	1633.95	1196.75	1042.31	1554.74
Std. PnL	1078.42	759.45	290.16	844.59
Total PnL	258 164	305 171	20 846	202 116
PnL p.a.	315 934	373 459	23 469	247 344

Table 7.14: Strategies 21-24 results.

	Strategy 21	Strategy 22	Strategy 23	Strategy 24
Exp. rng. fut.	[-6842, 6243]	[-4095, 4954]	[-8306, 8420]	[-3069, 2758]
Exp. rng. CDS	[-.23e09, .20e09]	[-.11e09, .16e09]	[-.20e09, .19e09]	[-.10e09, 85e06]
Avg. exp. fut.	2501.66	1691.59	1501.04	1209.68
Avg. exp. CDS	73.76e06	50.27e06	34.40e06	43.24e06
# trades	1332	667	729	649
# unique days	178	101	117	117
Success rate	88.29 %	91.90 %	73.80 %	90.60 %
Avg. holding	17.75	23.67	15.59	24.10
Gross margin	82.60 %	92.76 %	73.16 %	72.49 %
Mean PnL	1572.14	2382.24	1637.21	1477.29
Std. PnL	1637.54	1667.11	2142.02	1820.83
Total PnL	2 094 088	1 588 954	1 193 525	958 759
PnL p.a.	2 562 684	1 944 516	1 343 691	1 079 388

## 7.2 Simulations on Real Data

Recall that the cointegration tests cannot be run on the original data, which was why the data was resampled in the first place. Instead, the exact trade times from the simulations on the resampled data were saved and used in the new simulation. A trade is performed with the original date at the first time possible greater than or equal to the saved trading time from the simulation on the resampled data. Note that all this is doable in reality and is not just something that can be done with access to future data. In that case the algorithm would create a resampled series in every instance of time, then run necessary tests and decide whether to trade or not and finally execute it on real data.

The results from the simulations on the real data is presented below in Table 7.15. We see that in two of the cases 18 and 20 the total PnL is lower when run on the real data and in the two other cases it is higher. This agrees with the purpose of running these tests, which is that the market exposure can hit both ways and that we need to be aware of this.

Table 7.15: Strategies 18, 19, 20 & 22 on real data.

	Strategy 18	Strategy 19	Strategy 20	Strategy 22
Exp. rng. fut.	[-1770, 3050]	[-418, 56]	[-1198, 2074]	[-4095, 4954]
Exp. rng. CDS	[-31e06, 61e06]	[-7e06, 1e06]	[-21e06, 42e06]	[-.11e09, .16e09]
Avg. exp. fut.	313.38	18.34	204.33	1691.59
Avg. exp. CDS	6.44e06	0.43e06	4.20e06	50.27e06
# trades	255	20	130	667
# unique days	44	7	25	101
Success rate	93.33 %	100 %	93.08 %	90.55 %
Avg. holding	8.31	7.50	10.58	23.66
Gross margin	81.90 %	100 %	88.72 %	91.60 %
Mean PnL	1027.20	1195.14	1429.21	2436.05
Std. PnL	953.24	779.27	959.68	1718.48
Total PnL	261 936	23 903	185 797	1 624 847
PnL p.a.	320 550	26 910	227 373	1 988 441

Since the trading times are the same several key figures do not change. The good news is that the unwanted market exposure does not seem to have too much of an impact on the PnL. Furthermore one can most likely trade even more often in reality than what is implied by the data set, meaning that the market exposure will have an even lower impact in a real world implementation.

### 7.3 Simulations on Real Data with Transaction Costs

The final stage, to be as close to reality as we get in this thesis, is to add transaction costs. The trading is performed with the same methodology as in 7.2, based on the resampled series, the only difference being the transaction costs. The transaction costs when trading these assets is the bid-offer-spread. Recall that the CDS data has real spreads quoted while the equity data only has mid prices. A symmetric spread of one index unit is artificially added to the equity data which is fairly reasonable given how liquid the contracts are. This is also the spread we observed on Eurex (2015). Another assumption made is no bid-offer-spread when the contracts were rolled. The logic behind this is that the contracts would not have to be rolled in reality due to the short holding period but in the trading simulation they have to be rolled due to lack of quotes for off-the-run contracts.

The results from the simulations with transaction costs are presented in Table 7.16. As we can see the PnL is lower for all strategies just as expected. However the results are promising since the PnL is still far from zero. By looking at the change in mean PnL we see that the transaction cost for each trade is roughly EUR 3-400. It is a big cost but still small enough for many of the strategies in earlier stages to break even. The transaction costs vary across the strategies because the spread is not constant for the CDS contracts. Strategy 19 keeps impressing with 100 % success rate so all its trades must be significantly profitable.

At this stage there is not really much more to be said. In all of section 7 we have been presenting different key figures like PnL and average exposure. Nevertheless it is still hard to get a feeling of how good the results are, especially for someone not used to working with these products and therefore does not have the intuition for them. The strategies need to be compared to more tangible strategies that we are used comparing to.

Table 7.16: Strategies 18, 19, 20 &amp; 22 on real data with transaction costs.

	Strategy 18	Strategy 19	Strategy 20	Strategy 22
Exp. rng. fut.	[-1770, 3050]	[-418, 56]	[-1198, 2074]	[-4095, 4954]
Exp. rng. CDS	[-31e06, 61e06]	[-7e06, 1e06]	[-21e06, 42e06]	[-.11e09, .16e09]
Avg. exp. fut.	313.38	18.34	204.33	1691.59
Avg. exp. CDS	6.44e06	0.43e06	4.20e06	50.27e06
# trades	255	20	130	667
# unique days	44	7	25	101
Success rate	92.55 %	100 %	93.08 %	89.36 %
Avg. holding	8.31	7.50	10.58	23.66
Gross margin	72.14 %	100 %	83.06 %	88.15 %
Mean PnL	683.10	876.92	1080.42	2089.22
Std. PnL	940.51	786.84	966.79	1679.07
Total PnL	174 190	17 538	140 455	1 393 509
PnL p.a.	213 169	19 745	171 885	1 705 337

## 7.4 Evaluation

In this section we strive to evaluate the strategies from the previous section in order to get an understanding of how profitable the strategies are in more than just absolute terms. A standard measure for investigating how good an investment is, is the return on equity or just the return. Following the CAPM framework this return is only interesting to look at when put in relation to the market risk taken. The return in excess to the one predicted by CAPM is defined as alpha, which is what asset managers try to maximize. Another popular way of evaluating strategies is to benchmark them against indices. If we were trading say cash equities we could easily use all these evaluation methods.

The assets we trade are however much more complex which makes these simple calculations impossible. There is no way to, in a generic way, calculate the amount of capital used when entering the CDS and futures contracts. Take the CDS contracts for instance, they come with a Credit Support Annex (CSA) that specifies the rules for how collateral is posted between two parties in a contract. Note that the CSA is different between every pair of parties. A fund has different CSAs with its counterparties and all the funds have different

CSAs. The process of posting margins when trading futures is also non trivial to deal with. To make things more complicated it is not always obvious how much the real cost of capital for a big fund is when posting collateral that can originate from other trades. No matter how much we would like it, we will not be able to obtain that one number, the return, in order to represent the whole strategy all by itself.

#### **7.4.1 An Alternative Evaluation Approach**

We have decided to develop some measures of their own that could be used to judge the strategies. The first one is an index to benchmark against. For each strategy it is known how big the average exposure is for both futures and CDS contracts. Therefore the strategy can easily be compared to the market with the same average amount of risk taken. Normally trades are compared to a long position in the stock market but in this case they have to be compared to both a position in the stock market and a CDS position. To follow the convention the benchmark consists of a long futures position equal to the average futures position of the strategy and a short (protection) CDS position equal to the average CDS position of the strategy (short protection will have the same direction as long future). The benchmarks and the strategies are MTM every day so the standard deviation of the absolute daily PnL is also calculated as a risk measure.

Finally, we invent our own measure of market risk which reminds of how standard equity beta is calculated. Two of these beta proxies are calculated, one against the futures and one against the CDS. We start with the daily PnL from the strategy and the daily PnL from one of the benchmarks, both in absolute terms. Now since we cannot divide by the equity to get the return we instead divide by the average exposure of CDS or futures whichever we calculate the beta against. Now we have two normalized series that can be treated just like returns and be used to calculate beta the standard way. This alternative beta should be a fairly good measure of market correlation. The calculations for these measures are performed without transaction costs mainly for two reasons. The first one being that all the players on the market get quoted different spreads at different times so these results will be of wider relevance. The second reason is that transaction costs can disturb the beta calculation since they are done on a daily basis. Assume one day that the market has been climbing and we have been flat, no positions open. Then suddenly before closing, the trading

algorithm is triggered and a trade is executed. Then immediately a loss will be recorded due to the transaction costs indicating a negative beta which may not be the case at all.

The performance measures can be seen in Table 7.17. All strategies except 22 beat their benchmark in terms of pure PnL. All except 19 have lower standard deviation on the PnL than their benchmark. The beta measures indicate that our strategies are indeed market neutral, at least all except the CDS beta for 19. Even if 22 does not beat its benchmark it does not mean that it is not a strategy worth implementing. We must not forget that this PnL has almost no market correlation. The strategies could be leveraged up several times and still have a low market risk. More impressive is that market neutral strategies like 18 and 20 crush their benchmarks both in terms of profit and standard deviation during a period when the stock market rallied more than 17 % and the CDS index spread tightened more than 28 %.

Table 7.17: Performance measures for strategies 18, 19, 20 & 22.

	Strategy 18	Strategy 19	Strategy 20	Strategy 22
PnL	305 171	20 846	202 116	1 588 954
Std. absolute returns	11 209	1 352	7 431	61 651
Benchmark PnL	269 526	15 831	170 319	2 542 586
Std. absolute returns	14 292	966	9 325	94 742
$\beta_{CDS}$	- 0.19	0.43	-0.21	0.07
$\beta_{Future}$	-0.12	-0.10	-0.15	0.13

The strategies did surprisingly well, and to get a better overview we have also plotted their performance during the year against their benchmarks'. Figure 7.1 shows how strategy 18 does and an interesting observation is that while the benchmark is very bumpy during the second part of the year the strategy barely moved. Its market neutrality becomes very evident in the plot.



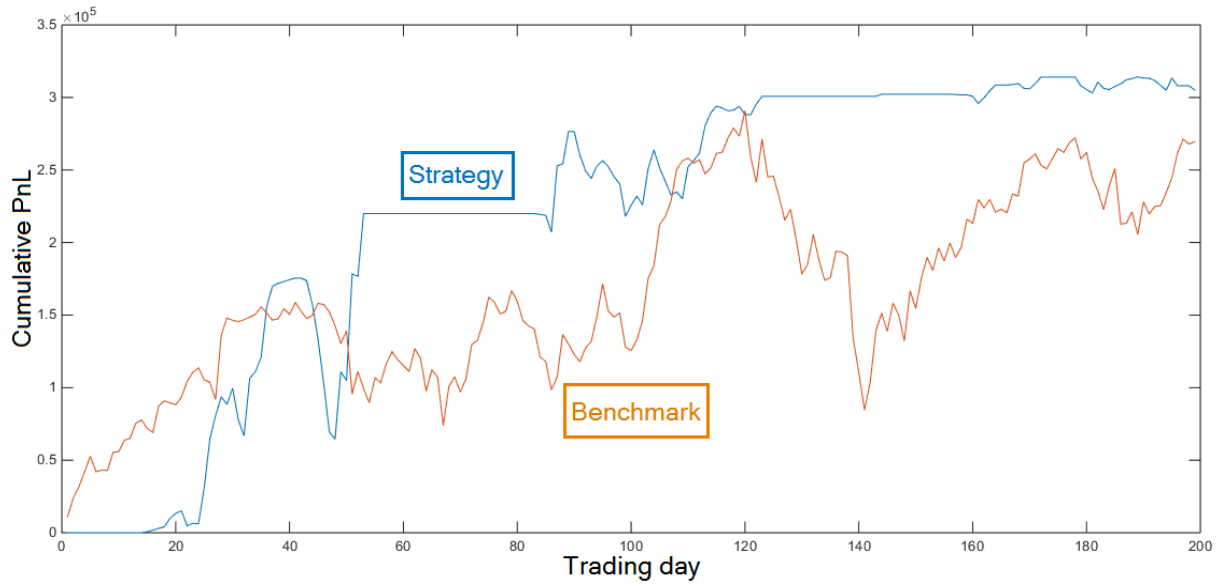


Figure 7.1: Strategy 18 against benchmark.

Figure 7.2 shows the same plot for strategy 19. Here it is evident that it does very few trades because the graph is totally flat during long periods. Another weakness is that a lot of the profit is made during a very short amount of time after day 40. On the other hand it manages to make a nice profit during the downturn around day 160 that none of the other strategies managed to make.



Figure 7.2: Strategy 19 against benchmark.

Figure 7.3 shows how strategy 20 does. As we can see it looks very much like strategy 18 which makes sense because the only thing distinguishing them is the different thresholds. It can be the low significance level of 2.5 % in the cointegration test that makes them avoid trading during the second half.

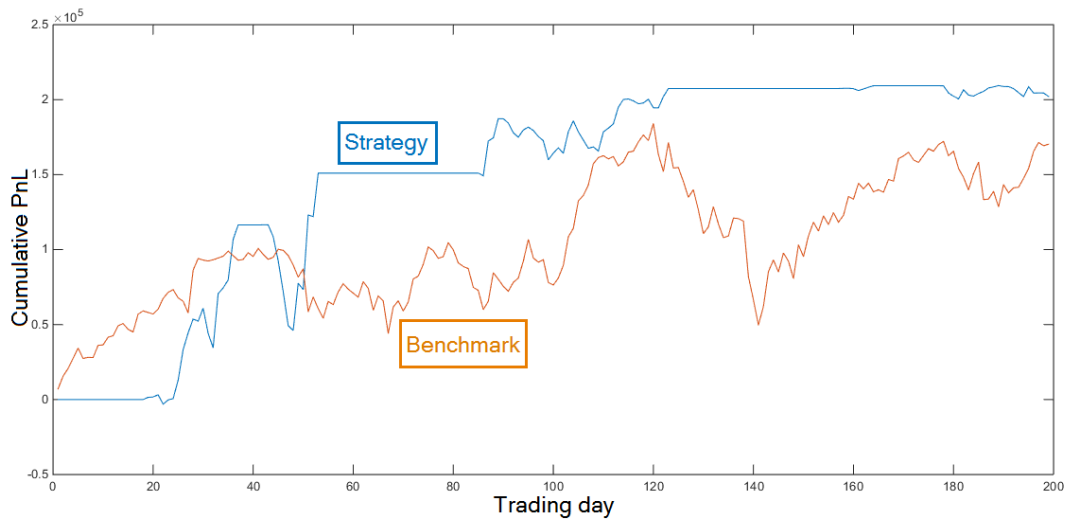


Figure 7.3: Strategy 20 against benchmark.

Figure 7.4 shows how strategy 22 does. It is evident that it is doing a lot of trades. It may not beat its benchmark but no one can blame it for being lucky. It systematically climbs the ladder of cumulative PnL during the year. It may look like the market is driving its profits but by observing the betas from Table 7.17 we see that it is indeed market neutral.



Figure 7.4: Strategy 22 against benchmark.

## 8 Conclusion and Suggestions for Further Research

In this thesis we have evaluated the relationship between the EURO STOXX 50 index and the Markit iTraxx Europe index. We could easily draw the conclusion that the two series possessed the mean reverting trait of cointegration. Although the two series seem to cointegrate over time, it is not an attribute that is present in every window of the sample. The cointegrating relationship was used to form several trading strategies. Four of these trading strategies have been tested with as many layers of reality as we have been able to add, including transaction costs. Even after this, all the strategies were profitable. After evaluation we could see that several strategies were indeed close to being market neutral, which was our goal. We could also see that the strategies did well against their benchmarks. To conclude, we managed to fulfill the purpose of the thesis, which has been to model the relationship between the series and use it to form a profitable market neutral trading strategy.

There is plenty of room for further research given that we have just started to explore this particular area of pairs trading. First of all, it would be interesting to run similar tests on a new data series, either with the same pair or on another market, like the American. A suggestion is to look at one of Markit's CDX indices (the American equivalent to iTraxx) against S&P 500. It would also be interesting to build another trading framework than the one used in this thesis.

## References

- Alexander, C. O., 1999. Optimal Hedging Using Cointegration. *Philosophical Transactions of the Royal Society*, 357(1758), pp. 2039-2058.
- Alexander, C. O. & Dimitriu, A., 2002. The Cointegration Alpha: Enhanced Index Tracking and Long-Short Equity Market Neutral Strategies. *ISMA Discussion Papers in Finance*, Volume 08.
- Barclays Capital, 2010. *Standard Corporate CDS Handbook: Ongoing Evolution of the CDS Market*, London: Barclays Capital.
- Brockwell, P. J. & Davis, R. A., 2002. Introduction. In: 2nd, ed. *Introduction to Time Series and Forecasting*. New York: Springer-Verlag New York, Inc, pp. 1-40.
- Byström, H., 2005. Credit Default Swaps and Equity Prices: The Itraxx CDS Index Market. *Working Papers*.
- CBOE, 2015. *The Relationship of the SPX and the VIX Index*. [Online]  
Available at: <https://www.cboe.com/micro/vix/part3.aspx>  
[Accessed 9 July 2015].
- Citigroup, 2010. Why is there a skew at all? . In: *Credit derivatives: under the bonnet* .  
London: Citigroup Global Markets, p. 34.
- Cryer, J. D. & Chan, K.-S., 2008. Model Specification. In: *Time Series Analysis: With Applications in R*. 2nd ed. New York: Springer Science+Business Media, LLC, pp. 109-141.
- Dickey, D. A. & Fuller, W. A., 1979. Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association*, 74(366), pp. 427-431.
- Eurex, 2015. *EURO STOXX 50 Index Futures (FESX)*. [Online]  
Available at: <http://www.eurexchange.com/exchange-en/products/idx/stx/blc/EURO-STOXX-50--Index-Futures/18954>  
[Accessed 8 July 2015].
- Fuller, W. A., 1996. Percentiles for Unit Root Distributions. In: *Introduction to Statistical Time Series*. 2nd ed. s.l.:John Wiley & Sons, Inc., pp. 641-652.

- Hamilton, J. D., 1994. *Time Series Analysis*. 1st ed. s.l.:Princeton University Press.
- Holland, J. H., 1975. *Adaptation in natural and artificial systems : an introductory analysis with applications to biology, control and artificial intelligence*. Michigan: University of Michigan Press.
- Hull, J. C., 2011. Chapter 1. Introduction. In: *OPTIONS, FUTURES, AND OTHER DERIVATIVES*. 8th ed. Boston: Prentice Hall, pp. 1-21.
- Hull, J. C., 2011. Chapter 2. Mechanics of futures markets. In: *OPTIONS, FUTURES, AND OTHER DERIVATIVES*. 8th ed. Boston: Prentice Hall, pp. 22-46.
- Hull, J. C., 2011. Chapter 5. Determination of forward and futures prices. In: *OPTIONS, FUTURES, AND OTHER DERIVATIVES*. 8th ed. Boston: Prentice Hall, pp. 101-128.
- International Swaps and Derivatives Association, Inc., 2015. *About ISDA*. [Online]  
Available at: <http://www2.isda.org/about-isda/>  
[Accessed 25 May 2015].
- ISDA and Markit Group Limited, 2009. *ISDA CDS Standard Model*. [Online]  
Available at: <http://www.cdsmodel.com/cdsmodel/>  
[Accessed 25 May 2015].
- ISDA and Markit Group Limited, 2009. *ISDA Standard Model Settings for Fee Computations*. [Online]  
Available at: <http://www.cdsmodel.com/cdsmodel/fee-computations.html?>  
[Accessed 26 May 2015].
- Lucas, A., 1997. Strategic and Tactical Asset Allocation and the Effect of Long Run Equilibrium Relations. *Serie Research Memoranda*, Volume 42.
- MacKinnon, J. G., 2010. Critical Values for Cointegration Testing. *Queen's Economics Department Working Paper*, Issue 1227.
- Markit Group Limited, 2009. *Markit Itraxx Indices: Index Level Calculations*. [Online]  
Available at: <http://www.markit.com/assets/en/docs/products/data/indices/credit-and-loan-indices/iTraxx/iTraxx%20index%20pricing%20Final.pdf>  
[Accessed 10 July 2015].

- Markit Group Limited, 2012. *Markit Credit Indices A Primer*. [Online]  
 Available at: [http://www.markit.com/assets/en/docs/products/data/indices/credit-index-annexes/Credit\\_Indices\\_Primer\\_Oct\\_2012.pdf](http://www.markit.com/assets/en/docs/products/data/indices/credit-index-annexes/Credit_Indices_Primer_Oct_2012.pdf)  
 [Accessed 10 July 2015].
- Markit Group Limited, 2013. *Markit iTraxx® Europe Index Rules*. [Online]  
 Available at: <http://www.markit.com/assets/en/docs/products/data/indices/credit-and-loan-indices/iTraxx/Markit%20iTraxx%20Europe%20Index%20Rules%20S19.pdf>  
 [Accessed 10 July 2015].
- Merrill Lynch, 2006. *Credit Derivatives Handbook - Volume 1: A Guide to Single-Name and Index CDS Products*, s.l.: s.n.
- Merton, R. C., 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance*, 29(2), pp. 449-470.
- Newsweek LLC, 2008. *How Credit Default Swaps Became a Timebomb*. [Online]  
 Available at: <http://www.newsweek.com/how-credit-default-swaps-became-timebomb-89291>  
 [Accessed 25 June 2015].
- O'Kane, D., 2008. Modelling Single-name and Multi-name Credit Derivatives. In: *Building the Libor Discount Curve*. 1st ed. Chichester: John Wiley & Sons, Ltd, pp. 9-28.
- O'Kane, D. & Turnbull, S., 2003. *Valuation of Credit Default Swaps*, s.l.: Lehman Brothers.
- Said, S. E. & Dickey, D. A., 1984. Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika*, 71(3), pp. 599-607.
- Simkovic, M. & Kaminetzky, B., 2010. Leveraged Buyout Bankruptcies, the Problem of Hindsight Bias, and the Credit Default Swap Solution. *Columbia Business Law Review*, 2011(1), pp. 118-221.
- STOXX Limited, 2015. *EURO STOXX 50®*. [Online]  
 Available at: [http://www.stoxx.com/indices/index\\_information.html?symbol=SX5P](http://www.stoxx.com/indices/index_information.html?symbol=SX5P)  
 [Accessed 25 June 2015].
- The MathWorks, Inc., 2015. *MathWorks Documentation adftest*. [Online]  
 Available at: <http://se.mathworks.com/help/econ/adftest.html>  
 [Accessed 10 June 2015].

The MathWorks, Inc., 2015. *MathWorks Documentation bootstrap*. [Online]  
Available at: <http://se.mathworks.com/help/fininst/bootstrapirdatacurve.html>  
[Accessed 26 May 2015].

The MathWorks, Inc., 2015. *MathWorks Documentation pchip*. [Online]  
Available at: <http://se.mathworks.com/help/matlab/ref/pchip.html>  
[Accessed 26 May 2015].

The MathWorks, Inc., 2015. *MatWorks Documentation Cointegration and Error Correction*.  
[Online]  
Available at: <http://se.mathworks.com/help/econ/identify-cointegration.html>  
[Accessed 10 Juni 2015].

The New Yorker, 2009. *Outsmarted*. [Online]  
Available at: <http://www.newyorker.com/magazine/2009/06/01/outsmarted>  
[Accessed 25 June 2015].

Vidyamurthy, G., 2004. *Pairs Trading : Quantitative Methods and Analysis*. 1st ed. New Jersey: John Wiley & Sons, Inc..

Zakrajsek, E., Levin, A. & Perli, R., 2005. The Determinants of Market Frictions in the Corporate Market. *Computing in Economics and Finance*, Issue 379.

# Appendix

## A.1 ISDA-model Discount Curve Specifications

The table below specifies the rates used by the CDS pricer. The data was downloaded from Thomson Reuters Datastream (or just Datastream, version 5.1). The “Name” column is the names as it appears on Datastream and the “Symbol” is the symbol used by Datastream. The source abbreviations are EBF - European Banking Federation, ACI - The Financial Markets Association and TR - Thomson Reuters.

Table A.1: ISDA-model discount curve specification.

Maturity	Type	Name	Symbol	Source
1MO	Deposit	EBF EURIBOR 1M DELAYED	EIBOR1M	EBF/ACI
2MO	Deposit	EBF EURIBOR 2M DELAYED	EIBOR2M	EBF/ACI
3MO	Deposit	EBF EURIBOR 3M DELAYED	EIBOR3M	EBF/ACI
6MO	Deposit	EBF EURIBOR 6M DELAYED	EIBOR6M	EBF/ACI
9MO	Deposit	EBF EURIBOR 9M DELAYED	EIBOR9M	EBF/ACI
1Y	Deposit	EBF EURIBOR 12M DELAYED	EIBOR1Y	EBF/ACI
2Y	Swap	TR EURO 6M IRS 2Y	TRE6S2Y	TR
3Y	Swap	TR EURO 6M IRS 3Y	TRE6S3Y	TR
4Y	Swap	TR EURO 6M IRS 4Y	TRE6S4Y	TR
5Y	Swap	TR EURO 6M IRS 5Y	TRE6S5Y	TR
6Y	Swap	TR EURO 6M IRS 6Y	TRE6S6Y	TR
7Y	Swap	TR EURO 6M IRS 7Y	TRE6S7Y	TR
8Y	Swap	TR EURO 6M IRS 8Y	TRE6S8Y	TR
9Y	Swap	TR EURO 6M IRS 9Y	TRE6S9Y	TR
10Y	Swap	TR EURO 6M IRS 10Y	TRE6S10	TR
12Y	Swap	TR EURO 6M IRS 12Y	TRE6S12	TR
15Y	Swap	TR EURO 6M IRS 15Y	TRE6S15	TR
20Y	Swap	TR EURO 6M IRS 20Y	TRE6S20	TR
25Y	Swap	TR EURO 6M IRS 25Y	TRE6S25	TR
30Y	Swap	TR EURO 6M IRS 30Y	TRE6S30	TR