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Evaluation of Various Approaches to Value at Risk

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Abstract

In the light of the current financial crisis, risk management and prediction of market losses seem to play a crucial role in finance. This thesis compares one day out-of-sample predictive performance of standard methods and conditional autoregressive VaR (CAViaR) by Engle & Manganelli (2004) for VaR (Value-at-risk) prediction of market losses. Comparison is made on US, Hong Kong, and Russian indices under tranquil period and current crisis using more than 10 years of daily returns. Performance is evaluated in terms of empirical coverage probability and predictive quantile loss on adequate models pointed out by Christoffersen test. The findings show that traditional methods such as historical simulation, normal VaR and *t*-VaR behave quite well in tranquil period if accounted for the return volatility dynamics by using GARCH volatility estimates. When unfiltered, these models fail to produce reliable results. In crisis period symmetric and asymmetric specifications of CAViaR showed good results, generally better and more stable than traditional approaches. Overall, CAViaR was found to work better on 5% than on 1% level. However, this model class is in most cases outperformed by conventional filtered models in the tranquil period. Little evidence was found that the market type has impact on the choice of ideal VaR model.

Key words: CAViaR, VaR, GARCH, market risk measuring, coverage probability

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1. Introduction

Measuring of market risk is not a new field of finance. The risk measure called VaR and its ability to forecast market losses has been object of broad previous academic research. The background of VaR, its evaluation and comparison of various classes of VaR in prior research are presented in this section. A brief review of empirical findings gives an insight into VaR and its use as a risk measure. This chapter also includes the definition of VaR, purpose, delimitations and the outline of the thesis.

1.1. Background

Effective measures of market risk have become crucial in current times of increased uncertainty in financial markets. Fragility emerging from extensive movements in market prices of financial assets as well as increased use of derivatives call for risk measures able to capture and mitigate more-than-ever growing financial risks. Not only supervisory authorities, but also management asks for a quantitative measure of market risks, in order to make sound investment decisions, allocating risk capital or fulfilling external regulations. As broadly defined by Jorion (2001) market risk is a volatility of unexpected outcomes. In other words, it is a risk, that the investment loses its value due to movements in market risk factors such as equity, exchange rate, interest rate and commodity risks. The scope of this thesis is restricted to the area of market risk management with a prominent tool called Value at Risk (VaR).

In contrast to famous Modigliani-Miller theorem postulating risk management is irrelevant in a perfect market; e.g. Bartram (2000) summarizes the benefits of effective risk management in the presence of agency costs, information asymmetries, transaction cost and taxes, i.e. in a real world. Hence, quantitative measurement of market risk is employed by the whole range of institutions such as security houses, banks, pension funds and other financial and non-financial entities. The most commonly accepted and used measure of market risk is VaR. There are several more or less equivalent definitions of VaR; however there is no general consensus on how to actually calculate it (Thompson & McCarthy, 2008). This study has the ambition to bring more light to the line of research that compares various VaR models, by evaluating predictive performance of chosen VaR classes.

1.2. Value-at-Risk – the Definition

Defined as maximum loss suffered by a given portfolio within a given time period by a given probability, VaR is a most widespread risk measure used internally as well as externally for reporting to regulatory authorities. In a statistical manner $VaR(\alpha)$ can be defined as follows:

$$P/L_t = P_t - P_{t-1}$$

$$\Pr[P/L < -VaR] = 1 - \alpha$$

$$\Pr[P/L > -VaR] = \alpha$$

- P_t : Value of Portfolio at time *t*
- P/L_t : profit/loss at time *t*
- α: confidence level (e.g. 95%, 99%)

Thus VaR is the (1-α) quantile of the return distribution, which in most cases has to be specified.[1]

The emergence of VaR reaches as far as 1952 as it is a natural evolution of Markowitz's portfolio theory (PT) in the mean-variance framework. However, there are important differences between PT and VaR. Dowd (2005, p. 11) mentions e.g.:

1. PT interprets risk in terms of standard deviation, while VaR interprets it in terms of maximum likely loss.
2. PT assumes distributions close to normal, while VaR accommodates wide range of possible distributions.
3. PT is limited to market risk, while VaR can be applied to other types of risk.
4. Some VaR approaches do not share the same variance-covariance background as PT.

As regards development of VaR, it can be dated back to the late 1980's at J.P. Morgan. Within the next couple of years, due to its many advantages VaR established itself as a prevailing risk measure, that has concerned academics ever since.

1.3. Previous Research on the topic

VaR models have been extensively discussed in literature. As the shortcomings of the traditional VaR models are well known (see section 2.1), VaR-related research aimed at more advanced approaches in order to improve the accuracy and predictive power of VaR models. Although new VaR approaches such as Conditional Autoregressive VaR (CAViaR) have been developed, there are only few studies available comparing a broader range of VaR models including both, traditional and advanced VaR models.

In an early study, Beder (1995) applied eight common VaR models to three different portfolios. The models used were the historical simulation approach as well as Monte Carlo simulation. Then variations of these VaR models were constructed by employing different assumptions with respect to the data base and/or data correlation. The three portfolios were

[1] Exception is e. g. CAViaR model that does not invert the distribution but models VaR autoregressively instead.

chosen such that the complexity in terms of optionality and/or asset class composition was increasing. Applying these models, Beder found that results varied by more than 14 times for the same portfolio. Hence, it can be seen that results are highly dependent on the input parameters, data, assumption and methodology. Therefore VaR does not provide certainty or confidence of outcomes, as the results are highly dependent on the time horizon, the underlying data as well as the assumptions and the applied methodology.

In another study Bao, Lee & Saltoğlu (2006) evaluated the predictive power of VaR models in several dimensions in emerging markets. In their study, they did not only apply traditional VaR models, but also models based on extreme value theory as well as the conditional autoregressive VaR (CAViaR). The focus of their study was on emerging markets in Asia. Their results showed that their benchmark, RiskMetrics model developed by J.P. Morgan, produced good results in tranquil periods, whereas in crisis periods VaR approaches based on extreme value theory produced better results. Furthermore Bao, Lee & Saltoğlu (2006) found that filtering is useful for EVT models, whereas it may deteriorate results for other models. In contradiction to that Danielsson & and de Vries (2000) found that common confidence levels, such as 95%, for VaR are not extreme enough and therefore VaR models using extreme value theory often produce poor results. Hence, the finding of Bao, Lee & Saltoğlu (2006) may only be coincidence. The last class of VaR models examined by the authors was CAViaR. Concluding from the results for this model class, the authors state that they produce some successful results across some periods. However they are not reliable for the whole period.

In their article Kuesters, Mittnik & Paolella (2006) state that the regulatory relevance of the VaR approach makes it necessary to develop reliable VaR estimation and prediction strategies. Therefore, the authors are comparing both conditional and unconditional VaR models with respect to their one-step-ahead prediction ability. Applying those models to NASDAQ-composite data, the authors find that most of the models are not able to produce correct estimates. The simulated VaRs do often underestimate the actual market risk. Furthermore they find that the unconditional models lead to clustered VaR violations and they are therefore not fulfilling the independency criterion of VaR estimates (see Section 2.6).

Even though conditional models of VaR estimates lead to an increased volatility in VaR estimates, approaches allowing for heteroskedasticity yield acceptable forecasts. In their conclusion Kuesters, Mittnik & Paolella (2006) state that VaR specifications of the following model classes produced the best estimates in their study: mixed normal GARCH[2], extreme value theory and filtered historical simulation. A further finding of theirs is that CAViaR models were not able to perform well overall. According to the authors, this is due to a lacking return process which is not estimated along with the quantile process.

[2] This model class links a "(…) GARCH-type structure to a discrete mixture of normal distributions, allowing for dynamic feedback between the normal components." (Keith Kuesters, Stefan Mittnik and Marc S. Paolella, 2006)

Although Giamouridis & Ntoula (2009) performed a study using different approaches for VaR and expected shortfall on hedge funds, their results are in line with aforementioned studies: Advanced models allowing for conditional mean and variance produce better VaR estimates, than standard approaches, such as historical simulation. In their study the authors use the following models: Historical simulation, Filtered Historical simulation, Gaussian model, Generalized Pareto Distribution model and the Cornish–Fisher model. Concluding from their study, they found that "parameterizations allowing asymmetry and fat tails, i.e. Cornish–Fisher and Generalized Pareto Distribution outperform the Gaussian and Historical Simulation models" (Giamouridis & Ntoula, 2009) for 1% VaR. On the 5% those three models were performing equally well in terms of average size of violations, and number of threshold violations.

Summarizing the presented literature it can be concluded, that there seems to be no ideal VaR model for any dataset. Common sense suggests that traditional VaR models in their naïve form should not produce reliable forecasts for any dataset. More advanced models, allowing for conditional parameters are usually outperforming traditional models. However, the question whether there is a VaR model producing sufficiently good estimates for different data series and regions has not been answered yet.

1.4. Purpose of the thesis

The disagreement about which model best accommodates market risk supports the relevance of further research into VaR. This thesis discusses VaR as a prominent tool in risk management and tries to provide valuable comparison between various classes of VaR in terms of their forecasting performance with regards to chosen evaluation criteria.

This thesis focuses on one-day forecast assuming that historical return data provides sufficient information necessary for forecasting. In recent years many new VaR models have been developed in order to overcome the shortcomings of traditional approaches[3]. However, only a few studies have been performed comparing a broader range of models, markets, and time periods. This thesis aims for a comparison of the traditional VaR models and the Conditional Autoregressive VaR (CAViaR) introduced by Engle & Manganelli (2004). CAViaR approaches show some promising performance properties, therefore deeper and comprehensive insight and tests of this method would surely contribute to the field of market risk measuring. Tests of chosen models are performed not only on different markets, including mature and emerging markets, but also on two different confidence levels and two different time periods representing tranquil and volatile market. In summary, this thesis seeks to answers three questions:

[3] Detailed selection of traditional approaches as well as general and specific shortcoming are presented in the theory section.

1) Do VaR forecasting models based on CAViaR have better predictive performance in one-step forecast than the traditional measures of VaR in terms of specified evaluation criteria?
2) How does the various VaR models performance change with respect to changes in return data geographical origin, time period or confidence level?
3) What is the role evaluation technique play in looking for most appropriate models?

1.5. Delimitations

Quite understandably, it is difficult to cover all aspects of VaR in the relevant quality. Exhaustive analysis would require very extensive research which is not feasible in given time horizon. Therefore, several aspects of this thesis have to be delimited.

First, it was necessary to delimit number of tested methods. Sub-section 1.3 well summarizes different approaches and results found by researchers in terms of VaR accuracy and reliability in maximum loss prediction. Methods applied vary from the simple to the most sophisticated. This study investigates Conditional Autoregressive VaR and puts it into contrast with traditional VaR models. Models based on Extreme Value theory (EVT) that model the tail of the distribution are not evaluated in this study. Theoretical description and references on EVT approach to VaR are given in the section 2.7.1.

Second, despite drawbacks embedded in it, VaR is still the main risk management tool and most discussed risk measure. VaR's exclusivity is emphasized by Basel I and Basel II, since it determines capital requirement of banks' portfolios. E.g. expected shortfall (ES) is one of the coherent improvements of VaR. Nevertheless, it has not received equal attention. Consequently, to keep the study as practically useful as possible, VaR solely remains the primary focus of this thesis.

Finally, this study will not focus on trading portfolios of banks because they do not disclose their trading positions and respective returns. It is assumed that the models would behave according to empirical findings of this thesis also for different kinds of positions or formed portfolio. Hence, estimation is conducted only on equity indices of different geographical origin.

1.6. The Plan of the Thesis

The thesis rests upon the three questions mentioned above, to which empirical analysis tries to find the answer. Organization of the thesis is then as follows. Section 2 introduces the theoretical background of various VaR models. Central theoretical aspects concern both, the traditional approaches of estimating VaR as well as the conditional autoregressive VaR. Section 2 also introduces evaluation framework used for testing VaR estimates produced by aforementioned models. In section 3 applied methods for both VaR estimation and comparison are described. Section 4 presents obtained empirical results with respect to evaluation criteria. The final section summarizes the results and concludes the most important findings.

2. Theory

In this section, the theoretical background of VaR is presented. First advantages and disadvantages of VaR are listed. Four different VaR classes with their sub-classes are described in this section. (1) Three types of Historical simulation, (2) VaR under normal distribution, (3) VaR under t-distribution, (4) Three specifications of Conditional Autoregressive VaR. Filtering of data series with volatility constitutes a striking improvement against naïve models. This requires plausible volatility estimate which is argued to be reached by GARCH(1,1). Finally, back-testing methods employed are introduced.

2.1. Attractions and Shortcomings of VaR

The reason behind the popularity of VaR is predominantly its conceptual simplicity as it aggregates all the risks in a portfolio into a single number suitable for use in the boardroom, reporting to regulators, or disclosure in an annual report (Linsmeier & Pearson, 1996). VaR can measure risk across all types of positions (almost any asset) and risk factors (not only market risk) and it provides a monetary and probabilistic expression of loss amounts.

Despite significant problems, the VaR concept can be utilized in several ways. (1) Management can set overall risk targets and from that determine the corresponding risk position. Increasing VaR means increasing risk for the firm. (2) VaR can be used to determine capital requirements. New risk-based capital adequacy framework Basel II, analogous to Basel I, approves VaR as a primary means of quantifying credit risk and thus capital adequacy. Further, according to Basel Committee, banks should keep sufficient cash to be able to cover market losses over 10 days with 99 percent probability for all their traded portfolios. This amount is to be determined by VaR. (3) VaR is useful for reporting and disclosing purposes. (4) VaR-based decision rules can guide investment, hedging, trading and portfolio management decisions. (5) VaR information can be used to provide remuneration rules for traders and managers and (6) systems based on VaR can measure other risks such as credit, liquidity and operational risks. (Dowd, 2005)

From among the critics, Taleb (1997) suggests suspension of VaR as a (1) potentially dangerous malpractice as it involves principal-agent issues and is often invalid in real world settings. (2) Over-reliance on VaR can lead to bigger losses. (3) VaR does not describe losses beyond the specific confidence level. Danielsson & Zigrand (2003) argue that VaR used for regulatory purposes may (4) distort good risk management practices. (5) Non-coherence due to non-subadditivity[4] of VaR is seen as the most serious drawback of VaR as a risk measure. It can only be made sub-additive when imposing normality restriction on return distribution, what contradicts the reality of financial time series.

VaR is used by Bank of International Settlements (BIS) for determining capital requirement to cover market risks by normal operations. This, however, requires the underlying risk to be properly estimated, otherwise it (6) may lead institutions to overestimate (underestimate) their market risks and consequently to maintain excessive high (low) capital requirements. The result is an inefficient allocation of financial resources. These facts were suggested by Engle & Manganelli (2004).

Among recent critics Whalen (2006) notes that over the past decades VaR appeared to be effective as there was little risk to measure and that relying on false assumption in regulatory framework makes VaR one of the most "(…) dangerous and widely held misconceptions in financial world." (Whalen, 2006, p. 2)

(7) Lastly, there exists a vast number of ways of VaR calculation which differ in their assumptions and have their own advantages and disadvantages and performance specifics. While bearing in mind the current popularity of VaR, we believe that addressing the problem of comparison of various classes of VaR would represent useful information for VaR users.

2.2. Traditional VaR Approaches

2.2.1. VaR using Historical Simulation

The most common non-parametric approach to VaR estimation is using historical simulation (HS). Under this approach one assumes that the historical distribution of the returns represents also the return distribution of future returns. This assumption allows forecasting future VaR directly from empirical distribution. This thesis employs three most common approaches to HS.

a) Basic Historical Simulation

Basic (naive) HS estimates VaR_{t+1} by $(1-\alpha)$-quantile of empirical distribution of returns r.

$$VaR_{t+1} = VaR_{1-\alpha}(r_t, r_{t-1}, \dots, r_1)$$

For example, using a sliding window of, say, 1000 observations, $VaR_{t+1}(0{,}95)$ is simply the negative of the 50th lowest observation in the sorted sample.

The basic (unfiltered) HS has numerous disadvantages summarized well in (Dowd, 2005). The biggest weakness is the assumption of *IID*[5] return series. In other word, basic HS would perform well only if there were no changes in volatility of returns over time.

[4] Consider two portfolio representations A and B and let ρ(.) be the risk measure for a determined period, than ρ(.) is subadditive if ρ(A+B) ≤ ρ(A)+ ρ(B). This in fact expresses diversification principle.
[5] Independently and identically distributed

b) Age-weighted Historical Simulation

To tackle this problem Boudoukh, Richardson, & Whitelaw (1998) suggested weighting the observations according to their age. Consequently instead of previous weights *1/N*, the most recent observations are assigned higher weights as follows, where w_1 is the weight for the newest observation:

$$w_1 = \frac{1-\lambda}{1-\lambda^n} \quad (1)$$

$$w_i = \lambda^{i-1} w_1 \quad (2)$$

Constant λ lies between 0 and 1 and reflects exponential rate of decay. For the special case $\lambda \to 1$ age-weighted HS converges to basic HS. Good summary of improvement of age-weighting against basic HS is given in Dowd (2005) stressing there are four major attractions of age-weighted HS:

1. It provides a nice generalization of traditional HS as mentioned earlier.
2. A suitable choice of λ can make VaR estimates more responsive to large loss observations and makes them better at handling clustering of large losses.
3. Age-weighting helps to reduce (not eliminate!) distortions caused by events that are unlikely to recur and reduces ghost effects. Older observations will probably lose their probability weights and their power to influence current VaR falls over time.
4. Unlike equally-weighted HS, age-weighting can be more effective as it gives the option of letting the sample grow with time. While under basic HS non-recurring past events would distort the current picture, age-weighting allows the importance of this events to decline. Hence, with this modification, valuable information is never thrown away, as it is necessary with basic HS, what results in jumps as sliding window puts old observations (with weight *1/N*) out of sample.

c) Volatility-weighted Historical Simulation

Finally, Hull & White (1998) suggest to apply HS to volatility weighted (filtered) series. The idea lies behind updating the return series to account for latest changes in volatility. For illustration, if the market volatility today is 5% a day, while two months ago it was only 2,5%, than the data from two months ago would clearly understate the risk tomorrow. Similar situation can also occur vice versa, resulting in overestimating tomorrow's risk if basic HS is applied. Volatility weighting updates return data to reflect volatility tomorrow and its changes from past values. Detailed description of volatility weighting method is presented in sections 2.4. and 3.6.

2.2.2. VaR under normal distribution

This approach assumes that returns are normally distributed and VaR is calculated employing quantiles of standard normal distribution. Empirical evaluation showed that return series do not follow the normal distribution. More specifically, returns are not *IID*. Kuesters, Mittnik & Paolella (2006) summarizes the three widely reported stylized facts:

(1) Volatility clustering, indicated by high autocorrelation of absolute and squared returns. Extreme values tend to occur in clusters. This implies that volatility today influences the expectation of volatility in the future period(s). Despite the widely accepted assumption of random walk (*IID*-ness) in financial returns, in what case variable shows no serial correlation, it does not follow that non-linear functions of return would also be non auto-correlated. So, while no correlation is observed in returns themselves, volatility of returns of financial time series exhibits it.

(2) Substantial kurtosis, that is, the density of the unconditional return distribution is more peaked around the center and possesses much fatter tails than the normal density. Usual values of kurtosis of financial time-series fall in the range 4 to 50, while kurtosis of normal distribution equals to 3. Sample standard deviation is, therefore, not a proper measure of variance in returns.

(3) Mild skewness of the returns, possibly of a time-varying nature.

The normal VaR is calculated as follows:

$$VaR(\alpha) = -\mu + \sigma * q_z(\alpha) \qquad (3)$$

Where $q_z(\alpha)$ is the value of the standard normal variate such that α of the probability density mass lies to its left and $(1-\alpha)$ of the probability density mass lies to its right. (Dowd, 2005, p. 58) Assuming randomly some profit/loss variable $\sim N(0;1)$, what is the 99% VaR for the next 100 days? Applying aforementioned formula yields:

$$VaR(0{,}99) = -\mu + \sigma * q_z(0{,}99) \qquad (4)$$

$$VaR(0{,}99) = -0 + 1 * 2{,}33 = 2{,}33 \qquad (5)$$

Hence in 99 trading days out of 100 trading days the loss will not exceed 2,33 *P/L*-units. A Graphical representation shows the geometrical location of the VaR(99%):

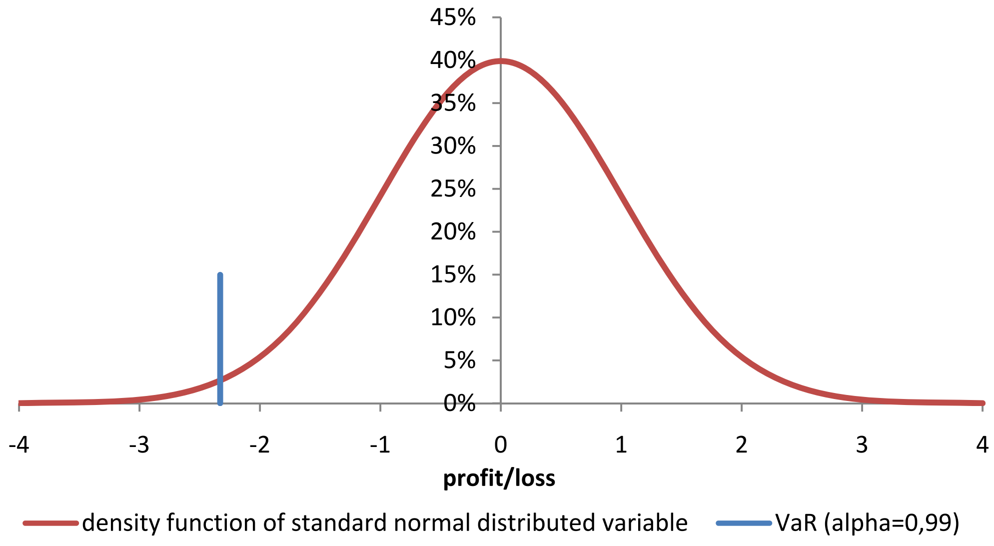

Figure 1: VaR for standard normal distributed profit/loss variable

Using a normal distribution implies a small but non-negative probability of negative asset values, which cannot occur in real world, as asset prices are limited to the interval $[0; \infty]$. Therefore a variation of the aforementioned approach is commonly used: VaR under the assumption of log-normal distribution. In this case log-returns are used instead of arithmetic returns. Hence asset prices are limited to a positive range (see Figure 2 below).

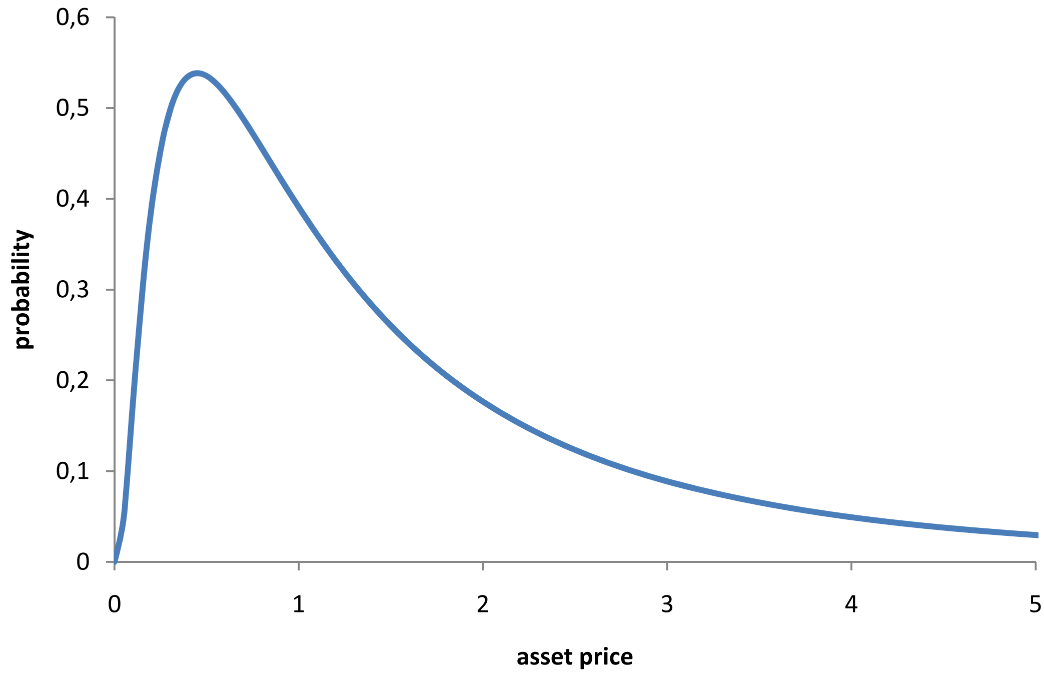

Figure 2: Log-normal distributed asset prices (following Dowd (2005), p. 61)

The use of normal distribution for VaR modeling has been heavily criticized: Empirical evidence showed that market data is more accurately described by distributions that allow occasionally for very large market movements. These movements are not covered by a normal distribution instead a heavy-tailed distribution has to be used. (Glasserman, Heidelberger, & Shahabuddin, 2000) Using the normal distribution in this case leads usually to an underestimation of the VaR. Often the normality assumption is justified by referring to the central limit theorem stating that as the sample size increases the distribution approaches normal distribution. However this applies only to "quantities and probabilities in the central mass of the density function." (Dowd, 2005, p. 157) Instead extreme value theory should be used, when dealing with extremes of the distribution.

2.2.3. VaR under t-distribution

A good description of return series is important for accurate VaR estimates. As the normality assumption of returns leads to underestimation of VaR, recent research focused often on developing advanced, complicated and calculation intensive VaR approaches. "In contrast, the Student-t distribution is a relatively simple distribution that is well suited to deal with the fat-tailed and leptokurtic features." (Chu-Hsiung & Shan-Shan, 2006, p. 292) The advantage of this approach is that the t-distribution converges to the normal distribution for an infinite number of degrees of freedom. Therefore the t-based VaR can be seen as a generalization of the VaR under assumption of normality. (Dowd, 2005, p. 159)

A crucial point of the student-t distribution is the determination of degrees of freedom. One suggestion is to set the degree of freedom according to empirical findings. Following this approach it turns out that shorter periods of time suggest degrees of freedom in the range of [4;6]. (Glasserman, Heidelberger, & Shahabuddin, 2000, p. 58) However, this thesis follows, Dowd (2005): There degrees of freedoms are obtained as follows:

$$v = \frac{4*\kappa-6}{\kappa-3} \qquad (6)$$

Where κ is the kurtosis and v are the degrees of freedom. The degrees of freedom are chosen, such that they fit the empirically observed kurtosis. The number of degrees of freedom is the closest integer satisfying aforementioned equation.

Using the α quantile of student-t distribution with v degrees of freedom, $t_{v,\alpha}$, one can calculate the value at risk under assumption of student-t distribution:

$$tVaR(\alpha) = -\mu + \sqrt{\frac{v-2}{v}} * \sigma * t_{v,\alpha} \qquad (7)$$

Where μ is the mean and σ the sample standard deviation.

However, there are also critical issues about the VaR under t-distribution that have to be considered. The Student-t distribution is criticized for its inability to capture the asymmetry of distribution of asset returns (Chu-Hsiung & Shan-Shan, 2006). Furthermore the VaR under t-distribution can produce too conservative estimates, in other words, estimates are too high. Applying VaR under t-distribution to very low or very high levels, leads to estimates that are not consistent with extreme value theory. The aforementioned problem applies also to the normal distribution. Additionally, the t-distribution is not stable, if two variables are t-distributed. The sum of these variables is not necessarily t-distributed.

2.3. Conditional autoregressive VaR (CAViaR)

One branch of recently developed VaR models focuses on extreme quantile estimation. Instead of modeling the whole distribution, this approach only focuses on the left tail of it, as this is usually the region of interest for everybody concerned about risk. The volatility in financial markets is not constant over time and shows signs of autocorrelation evident from observed volatility clustering. VaR is to a great extent determined by distribution of volatility; hence it must logically follow similar behavior.

Engle & Manganelli (2004) try to formalize this characteristic by proposing conditional autoregressive quantile specification, that they call CAViaR model (Conditional Autoregressive Value at Risk). Next, following Engle and Manganelli;

Suppose a portfolio's observed returns $\{r_t\}_{t=1}^{T}$. Let p (e.g. 0,01; 0,05) be the probability associated with VaR, let Ω_{t-1} be the information set available at time t consisting of observable variables and let β_p be a m-vector of unknown parameters. Next, let VaR_{t-i} denote the time p-quantile of the distribution of portfolio returns formed at t-i. General CAViaR specification is then given by

$$VaR_t = f(x_t, \beta_0) = \beta_0 + \sum_{i=1}^{m} \beta_i \, VaR_{t-i} + l(\beta_{m+1}, \dots, \beta_{m+q}; \Omega_{t-1}) \qquad (8)$$

where l is the function of finite number of lagged values of observables (e.g. returns).

For practical use the relation above can be in most cases reduced to

$$VaR_t = \beta_0 + \beta_1 \, VaR_{t-1} + l(\beta_2, r_{t-1}; \mathrm{VaR}_{t-1}) \qquad (9)$$

The idea behind CAViaR lies in the autoregressive term $\beta_1 VaR_{t-1}$ that ensures gradual and smooth change in VaR over time. Term $l(\beta_2, r_{t-1}, VaR_{t-1})$ links VaR to the observed variables belonging to the information set Ω_{t-1}. For the purposes of this thesis the information set consists of time lagged returns of r_{t-1}, that are thus connected to the level of VaR. All said, the VaR level responds to the level of r_{t-1} and previous periods' VaR.

In the above specification parameter β_1 measures the change based on the previous level of VaR, whilst β_2 is a measure of change in VaR level based on the latest information, i.e. the level of r_{t-1}.

Next, the three CAViaR processes that are the object of this thesis are discussed.

a) *Symmetric absolute value*

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 |r_{t-1}| \quad (10)$$

Apart from the autoregressive term, the VaR for this specification responds symmetrically to the past return. This is in line with natural behavior of the VaR level, as one can expect VaR to increase when the latest return falls below zero. Intuition suggests that the negative performance today increases probability of similar performance tomorrow. However, the model makes the same true for "good days" through the absolute value. A possible explanation suggested by the authors is that in case of volatility models we might expect VaR increases due to exceptionally good performance.

b) *Asymmetric slope*

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 (r_{t-1})^+ + \beta_3 (r_{t-1})^- \quad (11)$$

$$(r_{t-1})^- = -\min(r_{t-1}, 0)$$

$$(r_{t-1})^+ = max\,(r_{t-1}, 0)$$

This specification enables VaR level to depend asymmetrically on the sign of r_{t-1}. Thus positive and negative returns influence VaR differently.

c) *Indirect GARCH(1,1)*

$$VaR_t = \sqrt{\beta_0 + \beta_1 VaR_{t-1}^2 + \beta_2 r_{t-1}^2} \quad (12)$$

As with symmetric absolute value, the VaR level depends symmetrically on past return r_{t-1}. The authors state that indirect GARCH model would be correctly specified[6] only if the underlying distribution is really GARCH(1,1) with *IID* errors.

Further they argue that various forms of non-*IID* error distributions can be modeled with aforementioned specifications. Models are also applicable for distributions with both non-constant volatility and errors, what is a feature of many financial time series.

2.4. Filtering through volatility weighting

Various models described in the sections 2.2 and 2.3 are computed in both "unfiltered" and "filtered" form. CAViaR models will not be presented in filtered form, since filtered series is nearly *IID*, and CAViaR-specified VaRs might thus not show signs of dependence. By

[6] If a model is correctly specified, then $\Pr(r_t < VaR_t) = p$

unfiltered forms with specified conditional distribution function $F(\cdot)$ VaR estimate is, in general, given by inverting the $F(\cdot)$

$$F_t\big(q_t(\alpha)\big) = \alpha \qquad (13)$$

$$q_t(\alpha) = F_t^{-1}(\alpha) \qquad (14)$$

For filtered model this thesis uses the ideas of J. Hull and A. White, 1998 who suggested the method to account more for the recent changes in volatility of the instrument. This can also be called volatility updating as the calculations are applied to filtered series of r_t^*. In such case the model is denoted with *, e.g. VW-HS* in case of Volatility-weighted Historical Simulation. For non-parametric method (Historical Simulation) filtered VaRs are calculated using filtered series of $r_t^* = \frac{r_t}{\sigma_t}$, where the volatility, σ_t, must be estimated. Next step is to rescale r^* with the most recent forecast of volatility σ_T of asset in question. Returns in the data set are then replaced by

$$r_t^* = \frac{\sigma_t}{\sigma_t} r_t \qquad (15)$$

For parametric methods (Normal VaR and VaR under *t*-distribution) forecasted volatility enters the VaR formula directly for each observation. The task is thus to forecast conditional volatility σ_t for each day allowing for normally distributed errors (HS, Normal VaR) and *t*-distributed error terms (*t*-distribution).

2.5. Modeling volatility

Conditional variance σ_t^2 can be modeled in many ways.[7] First group is based on past standard deviations. From among these methods of modeling time varying volatility, the simplest one is the Moving Average (MA) approach described and criticized well in Jorion (2001). MA is calculated as a moving average of historical variances discarding the oldest observations. MA assigns the same weight to the past events as the most recent ones even though these past events are not likely to recur. This effect is also known as ghosting or ghost effect. From the same family, VaR estimation weighted by exponentially weighted MA (EWMA) used by RiskMetrics is usually used as a benchmark in financial industry. EWMA model, unlike MA puts greater weights on the more recent volatility estimates, what makes it more responsive to sudden movements of the market. RiskMetrics specifies EWMA model to forecasting variance σ_{t+1}^2 for $\lambda = 0{,}94$ by the following equation:

$$\sigma_{t+1}^2 = \alpha_0 + \lambda\sigma_t^2 + (1-\lambda)(r_t - \hat{\mu}_t)^2 \qquad (16)$$

[7] See Poon & Granger, 2003 for excellent coverage, reference and recommendations

Where the estimated mean of returns at time t is represented by $\hat{\mu}_t$.This model was tested by González-Rivera, Lee & Mishra (2004) founding that EWMA model proposed to calculate VaR seems to be the worst performer.

Other methods are either non-parametric or parametric. Non-parametric models are suggested by Bühlmann & McNeil (2001) who proposes simple iterative algorithm for nonparametric first-order GARCH modeling and its extensions. His procedure is often found to give better estimates of the unobserved latent volatility process than parametric modeling with the standard GARCH(1,1) model.

From among ARCH class parametric models GARCH model originally proposed by Engle (1982) and independently developed by Bollerslev (1986) and Taylor (1986) gained the largest attention by academics. In financial markets, it is common to find that the variance of returns is not constant over time. More specifically, large changes in prices tend to follow large changes and small changes tend to follow small changes. This tendency is called volatility clustering. "GARCH models are commonly used to capture the volatility clusters of returns and express the conditional variance as a linear function of past information, allowing the conditional heteroskedasticity of returns." (Curto, Pinto & Tavares, 2009, p. 313)

Throughout this thesis, GARCH(1,1) will be used as a model to capture volatility clustering since models of higher order only rarely better describe volatility, nor they are used in finance literature. Hansen & Lunde (2005) in their article *"Does anything beat a GARCH (1, 1)?"* support this by finding no evidence that simple GARCH(1,1) for σ_t^2 is outperformed by any more sophisticated model in its out-of-sample forecasting power.

2.5.1. Leverage effect

One of the biggest disadvantages of GARCH models is that it considers negative and positive error terms to have symmetric effect on volatility. In other words, negative shocks are assumed to increase volatility in the same way as positive shocks. In reality, the violations of this assumption are often observed. Bad news tend to increase volatility more than good news. This fact was first noted by Black (1976). Asymmetric response of volatility to the sign of the shock led to many parameterized extensions of GARCH. Most widely used in practice are the following extensions detailed in e.g. in Brooks (2008) or Thapar (2006): Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), GARCH in mean (GARCH-M) and Asymmetric GARCH (AGARCH). It should be stressed that although these extensions exhibit success in capturing stylized facts of volatility standard GARCH(1,1) remains the most widely used method in financial risk management and is also employed in this thesis for filtering purposes.

2.6. Back-testing

In order to evaluate the accuracy of the VaR models, which are employed in this thesis, different back-testing methodologies are employed. In this section a general theoretical description is given. An exhaustive description, including notes on the practical implementation follows in the methodology section (see section 3.8) of this thesis. Totally there are four different tests examining whether the VaR model is well specified or not.

In order to determine the accuracy of the VaR estimates produced by a model, estimates are usually back-tested. A common starting point for those procedures is the so-called hit function, or indicator function:

$$I_t = \begin{cases} 1, if\ r_t \leq VaR_t \\ 0, if\ r_t > VaR_t \end{cases} \qquad (17)$$

It takes the value one if the return at time t exceeds the VaR at time t. If the VaR is not exceeded the function takes the value 0.

In order to be accurate the indicator function has to fulfill two properties, according to Christoffersen (1998):

1. Unconditional coverage property:
   According to this criterion the probability of realizing a loss not exceeding $VaR(\alpha)$ must be exactly $\alpha * 100\%$. Hence only $(1 - \alpha) * 100\%$ of the VaR estimates should be violated.[8] In case the VaR is exceeded too few times, indicates a too conservative VaR model.
2. Independence property:
   This criterion states the manner in which violations may occur. Thus any two elements of the indicator sequence have to be independent from each other. In case this criterion is not met, this is an indicator that the VaR model is not responsive enough to consider changes in market risk. (Campbell, 2005)

The above stated properties can be combined into one single statement:

$$I_t(p) \overset{i.i.d.}{\sim} B(p)$$

The indicator function has to be an *IID* Bernoulli random variable with probability p. As the aforementioned properties turned out to be the keys to accuracy tests, many of the back-testing procedures developed in recent years focus on these features. (Campbell, 2005)

The VaR estimates presented in methodology section of this thesis are tested for the aforementioned properties, using three different evaluating criteria:

[8] In the remaining part of the thesis $(1 - \alpha)$ expresses the coverage probability denoted as p.

Firstly, the widespread unconditional Kupiec test is performed. The VaR estimates of a well specified model should not be violated more than $p * 100$. The Kupiec test examines whether the empirical number of violations $\hat{p} * 100$ exceeds the nominal number significantly. (Campbell, 2005) Hence, the test accounts only for the first property, mentioned earlier in this section. To overcome the non-consideration of the second property, Christoffersen (1998) extends the Kupiec test, in order to take the independence property into account. The Christoffersen test consists of two parts, whereof the first part coincides exactly with the Kupiec test. The second part of the test, the conditional coverage ratio test, examines whether VaR violations are clustered or independent. The advantage of the Christoffersen test is clearly that it can be seen whether the VaR model fails due to a too high or too low number of violations or if the model does not fulfill the property of independent violations.

The second test employed is a simple indicative test, the so called Basel back-testing method. This method evaluates whether the empirical coverage probability, $\hat{p}$, is lower than the significance level on which the VaR model is actually performed.

Lastly, empirical (predictive) quantile loss $\hat{Q}(\alpha)$ following Bao, Lee & Saltoğlu (2006) is employed to examine the fitness of the VaR model to decide for the most suitable VaR method.

All three methods are further detailed in the Section 3.8.

Possible evaluation criterion is Perigon's extension of the unconditional Kupiec test. This model evaluates the accuracy of the VaR models using VaR estimates for different coverage probabilities. The test proposed by Perignon & Smith (2009) combines the simplicity of the Kupiec test and additional information about the left tail of the distribution as suggested by Berkowitz (2001). The likelihood ratio test, examines whether the empirical coverage probability significantly deviates from the hypothesized one. This test is, however, not empirically employed in this paper.

2.7. Other VaR models

This thesis will not be exhaustive with respect to the range of different methodologies to estimate VaRs. There are several other VaR models, that will not be tested, but it is of great interest to introduce their attractions. The largest attention is dedicated to extreme-value VaR. Further, NIG-ACD model is briefly mentioned.

2.7.1. VaR models based on extreme value theory (EVT)

Papers dealing with VaR with the help of extreme value theory jointly share the opinion that traditional parametric models for VaR estimation are unsuitable for event with extremely low probability of occurrence. This follows from the notorious fact that financial returns distributions have heavy tails and parametric models usually assume normality or lognormally distributed returns. Fitting the distribution into the return series necessarily leads to underestimation of tails as the majority of observation lies in the centre, which is accommodated by the distribution. Hence, these models tend to fail, when they are needed most; i.e. when low-probability event occurs, what can lead to huge losses. EVT has advantage over non-parametric models as well.

Extreme value theorists handle this problem by extracting as much information as possible straight from the tail. In practice, extreme-value VaR requires first the estimation of parameters of the whole distribution with any standard estimation technique. Next, these parameters are used in one of many formulae specifications to estimate VaR. Figure 3 from Aragonés, Blanco, & Dowd (2000) shows the difference between normal VaR and extreme-value VaR. Particularly at low significance levels (high confidence levels) extreme-value VaRs are much higher than normal VaRs hence extreme-value are much better copying actual extreme return observation (dotted line). For more evidence see Gencay & Selcuk (2004).

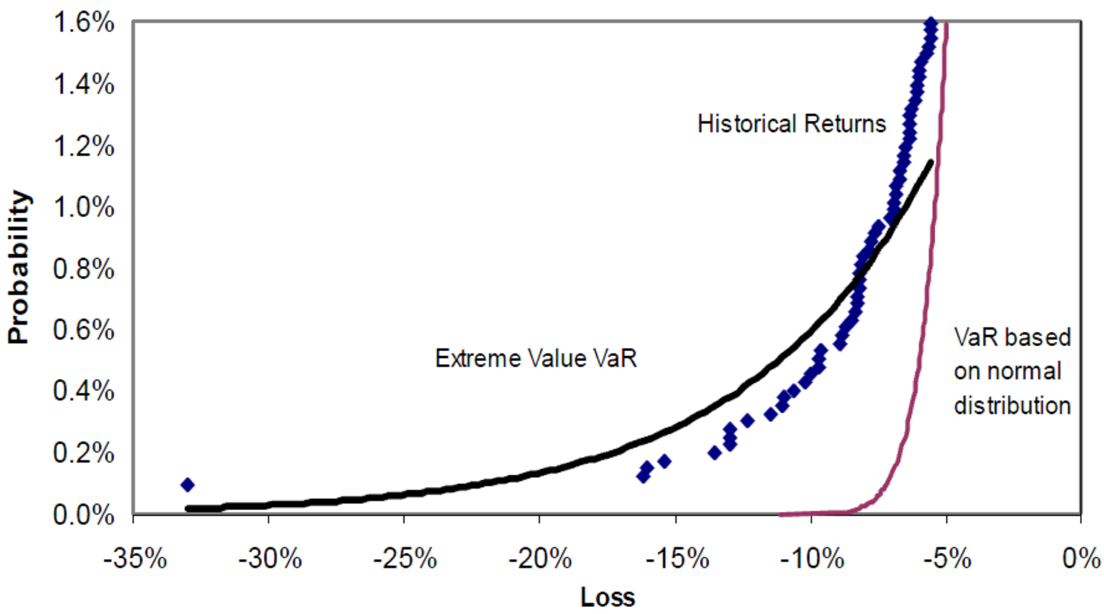

Figure 3: West Texas Intermediate (WTI) daily return distribution from 1983 to 1999

EVT is extremely promising approach to VaR estimation and deserves additional academic attention. Interested reader is encouraged to see Ahangarani (2005), Longin (1999), Aragonés, Blanco & Dowd (2000) as well as Bao, Lee & Saltoğlu (2006) for more information on extreme-value VaR.

2.7.2. VaR with time varying variance, skewness and kurtosis- NIG-ACD model

From the family of parametric VaR models, NIG-ACD by Wilhelmsson (2009) belongs to the newest. NIG-ACD allows for time varying moments based on Normal Inverse Gaussian (NIG) distribution with autoregressive conditional density (ACD). The author evaluates his models by their VaR forecast finding they provide more correct VaR forecast then extant models or Gaussion GARCH model.

3. Methodology

The section starts with specification of collected data and gives an overview of applied methods. Next, empirical employment of GARCH (1,1) approach to conditional volatility estimation is described in details. Optimization problem for all three CAViaR specifications is also presented. This section ends with an in-depth description of applied back-testing methods. The evaluation criteria are comprehensively stated.

3.1. Data Description

Price index data for three different markets was received from Datastream. Mature markets are represented by NYSE Composite, whilst Russia RTS, FTSE W Hong Kong represent emerging markets. Prices were converted into the daily log-returns.

In order to analyze the performance of the models in different time periods, two out-of-sample periods are used. The periods are denoted as Period 1/P1 (tranquil) and Period 2/P2 (crisis). Period 2 is selected to reflect the current financial crises. It is thus possible to investigate performance of chosen models under two different stress situations. Both periods are divided into two sub-samples, in-sample and out-of-sample period. The in-sample period for both periods starts on September 4, 1995. As standard practice, out-of-sample period is approximately one year long and consists of P=250 observations for both periods. Period 1 ends on July 15, 2005 with a total number of T=R+P=2575 and Period 2 ends on May 13, 2009 with a total number of T=R+P=3573 observations. To get as accurate estimates as possible, it is important to set P very low compared to R. This empirical analysis thus operates with R= 2325, 3323 versus P= 250. Table 1 summarizes analyzed periods and sample division.

Table 1: Two in-sample and out-of-sample periods for tranquil a crisis market

	Period 1 (Tranquil)	Period 2 (Crisis)
In-sample period	4/9/1995-30/7/2004	4/9/1995-28/5/2008
	R=2325	R=3323
Out-of-sample period	2/8/2004-15/7/2005	29/5/2008-13/5/2009
	P=250	P=250
Total number of observations	T=2575	T=3573

3.1.1. Descriptive statistics

Table 2 and Table 3 below provide basic descriptive statistics for both in-sample periods. Further, Jarque-Bera[9] normality test clearly rejects the null hypothesis of normal distribution of drawn data for all five return series. Histograms Figure 4 in Appendix B illustrate the distribution of returns for all three market and both periods.

Quantile-quantile plots (QQ-plot) show how the chosen distribution fits the data. If the choice of distribution explains data well enough QQ-plot forms a straight line. QQ-plots in Figure 6 in Appendix B confirm excess kurtosis (heavy tails) and thus non-normality. VaR estimates based on the *t*-distribution address this fact partly.

As common in empirical studies, squared returns serve as a proxy for volatility[10]. Figure 5 in Appendix B plots the squared log-returns for each series. Graphs clearly demonstrate how the volatility clusters over time, what justifies the use of GARCH(1,1) volatility weighting.

Table 2: Descriptive statistics: in-sample Period 1

Sample	Period1: 4/9/1995-30/7/2004		
Series	NYSE	FTSE Hong Kong	Russia RTS
Observations (T)	2325	2325	2325
Mean	0,000318161	-3,26451E-06	0,000746213
Median	0,00027264	0,000140464	0,000935061
Minimum	-0,07250271	-0,144669756	-0,220963083
Maximum	0,056554696	0,158310186	0,155559809
Std.Dev.	0,012116873	0,018031802	0,030774807
Skewness	-0,168615854	0,061452164	-0,386161051
Kurtosis	5,208233412	11,12363454	8,287122148
Jarque-Bera	480,2650	6363,0740	2751,4570
Probability	0,000000	0,000000	0,000000

Table 3: Descriptive statistics: in-sample Period 2

Sample	Period2: 4/9/1995-28/5/2008		
Series	NYSE	FTSE Hong Kong	Russia RTS
Observations (T)	3323	3323	3323
Mean	0,000261911	0,000168434	0,000895361
Median	0,000352726	0,000523784	0,001228137
Minimum	-0,07250271	-0,144669756	-0,220963083
Maximum	0,056554696	0,158310186	0,155559809
Std.Dev.	0,01149418	0,016704328	0,027211946
Skewness	-0,161950369	0,009908082	-0,443432696
Kurtosis	5,295367054	11,43844959	9,75904615
Jarque-Bera	740,6717	9825,4700	6411,8460
Probability	0,000000	0,000000	0,000000

[9] The Jarque-Bera test of normality is described in details in Appendix A, section 7.1.2
[10] See Appendix A for proof.

Table 13 in Appendix B presents Ljung-Box test statistics[11] with associated probabilities of wrongly rejecting null hypothesis. The statistics is provided for lags 1, 10, 20 and 50 for log-returns. Period 2 data are considered here to include all the available information. For series NYSE null hypothesis of no autocorrelation for all lags (1, 10, 20 and 50) cannot be rejected. For FTSE Hong Kong and Russia RTS null hypothesis can be rejected on all lags except for first lag autocorrelation for FTSE Hong Kong. The statistics is sensitive to number of chosen lags. With increasing number of tested lags, autocorrelations are becoming more significant for the examined emerging markets series. As a consequence, standard methods based on assumption of *IID* returns are prone to fail when capturing real behavior of data.

3.2. VaR models and Notation

Table 4 below defines the notation used in this thesis.

Table 4: VaR models and Notation

Distribution	Unfiltered	Filtered
Historical distribution	HS	AW-HS*
		VW-HS*
Normal Distribution	Normal	Normal*
T-Distribution	t	t*
No Distribution	$CAViaR_S$	
	$CAViaR_A$	
	$CAViaR_G$	

Legend:

1. Abbreviations stand for the following methods:

* indicates volatility- or age -filtered model

HS=Historical Simulation; VW-HS*= Volatility-weighted historical simulation

AW-HS*=Age-weighted historical simulation

Normal= VaR estimated under normal distribution

t= VaR estimated under t-distribution

$CAViaR_S$ = Symmetric CAViaR model by Engle and Manganelli (2004)

$CAViaR_A$ = Asymmetric CAViaR model by Engle and Manganelli (2004)

$CAViaR_G$ = Indirect GARCH CAViaR model by Engle and Manganelli (2004)

2. Note that quantiles of CAViaR models are not computed by inverting the distribution function. Filtered version of CAViaR is not considered as updated returns r* follow random walk. CAViaR model is based on dependent (autoregressive) quantiles; hence filtered version is not appropriate.

[11] Detailed description of The Ljung-Box test of random walk is presented in Appendix A, section 7.1.3

3.3. VaR- Historical Simulation

Historical simulation is the least complex approach from among the models in this thesis. Estimation of quantile requires the choice of appropriate historical time frame used as an empirical distribution. Precision of this method increases with the number of observations included, as more observations might fall to the tails region. One day VaR forecasts are estimated using excels percentile function for rolling window for the entire out-of-sample forecasting period.

The age-weighted HS, denoted AW-HS*, employs the same procedure for the filtered series as described in subsection 2.2.1.b., i.e. each value is assigned weight, that is exponentially decaying toward the past values.

$$w_i = \lambda^{i-1} w_1 \quad (18)$$

The weight for newest observation i=1; λ=0,995.

The volatility-weighted HS, VW-HS*, is computed using filtered series r*, equation 15. Volatility forecasts used in volatility updating are estimated using GARCH (1, 1), where GARCH (1, 1) parameters are re-estimated each 25 observations (250/25=) 10 times and volatility weighting is performed 250 times, i.e. for each out-of sample observation using the sliding window.

3.4. VaR under Normal Distribution

If the assumption about the distribution of the risk factor is correct, parametric models are precise. Standard normal distribution q_z is considered and VaR equation is given by:

$$VaR_t(\alpha) = -\mu_t + \sigma_t q_z(\alpha) \qquad (19)$$

For unfiltered Normal VaR (Normal), σ_t equals to a sample standard deviation.

For filtered Normal VaR (Normal*) σ_t is estimated by a GARCH(1, 1) model, which is re-estimated 10 times during the out-of-sample period.

3.5. VaR under *t*-distribution

VaR under *t*-distribution is used in order to address some of the stylized facts of financial return data. To implement this approach, first degrees of freedom must be determined as suggested by equation 6. Then using the α quantile of student-*t* distribution with *v* degrees of freedom, VaR is given by equation 7 in section 2.2.3.

For unfiltered *t*-VaR (t), σ_t equals to the sample standard deviation.

For filtered t-VaR (t*) σ_t is estimated by GARCH(1, 1) model, which is re-estimated 10 times during the out-of-sample period. Note, that conditional volatility parameters for t-distribution are obtained by maximizing different log-likelihood function from the one used for Normal distribution. Details are given in the next section.

3.6. GARCH(1,1)

In the GARCH framework conditional variance depends upon own lags and the past values of the squared errors, so that the conditional variance is given by

$$r_t = \mu + \varepsilon_t \quad (20)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (21)$$

As long as $\alpha_1 + \beta < 1$ the unconditional (long-term) variance of errors for GARCH(1,1) is given by

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta} \quad (22)$$

This condition corresponds to stationary of GARCH model, where forecast of conditional variance converges to the mean with increasing forecasting period. Mean reversion means that the conditional variance will arrive at its unconditional mean.

Parameter β expresses how persistent shocks are that were caused by extreme values of conditional variance. Parameter α_1 is the measure of volatility response to movements in the market. Equation 21 is a nice demonstration how GARCH model deals with volatility clustering. If the volatility of the previous period is high, next period will also be high unless the return of the portfolio in the previous period does not differ significantly from its mean.

Parameters α_0, α_1, β, μ can be found by maximizing the log-likelihood function:

$$lnL(\cdot) = \sum_{t=1}^{T} \left(-0.5 * \ln(2\pi\sigma_t^2) - \frac{\varepsilon_t^2}{2\sigma_t^2} \right) \quad (23)$$

Log-likelihood function (equation 23) implies that the errors follow the normal distribution. When calculating the filtered version of VaR under t-distribution, the GARCH model has to take into account, that the underlying distribution is a t-distribution and not normal. Therefore the following log-likelihood function is maximized:

$$lnL(\cdot) = \sum_{t=1}^{T} \left(ln \left[\frac{\Gamma[(v+1)/2]}{\Gamma[v/2]} \frac{1}{\sqrt{(v-2)\pi}} \right] - \frac{1}{2} \ln(\sigma^2) - \left(\frac{v+1}{2} \right) ln \left[1 + \frac{\varepsilon_t^2}{\sigma^2 (v-2)} \right] \right) \quad (24)$$

Where $\Gamma(\cdot)$ is the gamma function and v is the number of degrees of freedoms, which are chosen in accordance to equation 6.

To get an accurate volatility forecast, parameters of GARCH (1,1) are re-estimated each 25 observations. The estimation window for GARCH(1,1)slides down to out-of-sample period abandoning the oldest estimations.

As GARCH-effect are a common feature of financial return data, statistical testing for GARCH effects was not conducted. Empirical estimation revealed, that this assumption was correct since all the coefficients of GARCH (1,1) were significantly different from zero.

3.7. CAViaR

The paper by Koenker & Bassett (1978) introduces the new class of the robust alternatives to the least square estimators for linear models. Its discussion below presents application of quantile regression to estimation of CAViaR parameters.

3.7.1. Quantile Regression

Equations 25, 26, 27 below specify one-day CAViaR. Parameter vector β must be estimated.

Symmetric absolute value:

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 |r_{t-1}| \quad (25)$$

Asymmetric slope:

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 (r_{t-1})^+ + \beta_3 (r_{t-1})^- \quad (26)$$

$$(r_{t-1})^- = -\min (r_{t-1}, 0)$$

$$(r_{t-1})^+ = max\ (r_{t-1}, 0)$$

Indirect GARCH(1,1):

$$VaR_t = \sqrt{\beta_0 + \beta_1 VaR_{t-1}^2 + \beta_2 r_{t-1}^2} \quad (27)$$

The parameter vector β must be found so that it would be as near as possible to the true parameter vectors describing the data sample. One is therefore facing a common problem of estimating vector of unknown parameters β from the sample of observations.

As previously stated, let $\{r_t\}_{t=1}^T$ be the series of observed returns. Let $x_t' \beta_p$ be the p-quantile, where x_t is a vector of regressors. In the specification above x_t corresponds to lagged VaRs and returns. Quantile can also be expressed as

$$p = \int_{-\infty}^{x_t' \beta_p} f_r(s|x_t) ds \quad (28)$$

or, as common in the literature, the quantile of the random variable r_t is conditional on the vector of known regressors x_t.

$$r_t = x_t' \beta_p + \varepsilon_{pt} \quad (29)$$

$$\varepsilon_{pt} = r_t - x_t'\beta_p \qquad (30)$$

$$Quant_p(r_t|x_t) = x_t'\beta_p \qquad (31)$$

Koenker & Bassett (1978) showed that parameters $\hat{\beta}_p$ can be obtained by solving the minimization problem minimizing the errors ε_{pt}. Hence the p-quantile of r_t sample is defined as any solution to the following minimization problem:

$$\min_\beta \frac{1}{T}\left\{\sum_{t:r_t \geq x_t'\beta} p\,|r_t - x_t'\beta| + \sum_{t:r_t < x_t'\beta} (1-p)\,|r_t - x_t'\beta|\right\} \qquad (32)$$

From the earlier discussion of three difference autoregressive CAViaR processes, the above problem can be rewritten as follows:

$$\min_\beta \frac{1}{T}\{\sum_{t:r_t \geq -VaR_t} p\,|r_t - VaR_t| + \sum_{t:r_t < -VaR_t} (1-p)\,|r_t - VaR_t|\} \qquad (33)$$

The objective function above takes a non-linear form, due to non-linear regression quantile estimation (three specifications above). For this reason, the function is not differentiable and standard numerical optimization procedures based on differentiation might not find the global optimum. Global optimum can be theoretically found using evolutionary genetic algorithm. This algorithm is based on Price & Storn (1997) and unlike traditional routines can explore the whole target area without stopping at particular local optimum. This optimization problem (equation 33) can be solved e.g. using a built-in excel upgrade Risk Solver premium. This software is, however, not publicly available and programming this algorithm independently would far exceed the scope of this thesis. Nevertheless, trying sufficiently high number of different initial parameter values, linear excel solver provides surprisingly robust minimums yielding the parameters matching the theoretical expectations as for both magnitude and signs.

3.8. Comparing VaR models

The sample of all observations T was divided into two subsamples, in-sample R and out-of-sample P so that T=R+P. Forecasts made for out-of-sample period are evaluated in the following way.

First, VaR forecasts are tested using Christoffersen test of Conditional Coverage consisting of Kupiec test unconditional coverage and test of serial independence. Models assessed as adequate (those passing the test) are then compared in terms of their empirical coverage ratio (failure rate) and predictive quantile loss.

3.8.1. Kupiec test of unconditional coverage

One of the earliest and most widespread tests for back-testing VaR is Kupiec's unconditional coverage test. In order to be an accurate VaR model, VaR estimates should not be violated more than $p * 100$ times. If the observed number of violations $\hat{p} * 100$ significantly exceeds the nominal number of violations $p * 100$, the applied method is not appropriate to produce useful VaR estimates. (Campbell, 2005)

Kupiec (1995) uses a log-likelihood ratio test which is asymptotically $\chi^2(1)$ distributed. To apply the test, the indicator function is defined in order to count violated VaR estimates:

$$I_t = \begin{cases} 1, if\ r_t \leq VaR_t \\ 0, if\ r_t > VaR_t \end{cases} \qquad (34)$$

Where r_t is t-time observation in the return series and VaR_t is the corresponding VaR estimate. Once this is defined, the likelihood under the null hypothesis is, according to Christoffersen (1998), simply given by:

$$L(p; I_1, I_2, \dots, I_T) = (1-p)^{n_0} p^{n_1} \qquad (35)$$

The likelihood under the alternative hypothesis is then given by:

$$L(\hat{p}; I_1, I_2, \dots, I_T) = (1-\hat{p})^{n_0} \hat{p}^{n_1} \qquad (36)$$

The number of non-violated VaR estimates is given by n_0 and the number of violated VaR estimates is consequently given by n_1. Further let $\hat{p}$ be the observed coverage probability. Its maximum likelihood estimator is defined as $\frac{n_1}{n_0+n_1}$ (Christoffersen, 1998, p. 845). Testing for unconditional coverage ratio can be done by a simple log-likelihood ratio test:

$$LR_{UC} = -2 \ln \left(\frac{L(p; I_1, I_2, \dots, I_T)}{L(\hat{p}; I_1, I_2, \dots, I_T)} \right) = -2 \ln \left(\frac{(1-p)^{n_0} p^{n_1}}{(1-\hat{p})^{n_0} \hat{p}^{n_1}} \right) \sim \chi^2(1) \qquad (37)$$

Thus, under the null hypothesis of correct parameter p LR lies in the interval:

$$chiinv\left(1 - \frac{p}{2}; 1\right) \leq LR_{UC} \leq chiinv\left(\frac{p}{2}; 1\right) \qquad (38)$$

This test is only concerned with the coverage of the VaR estimates it does not account for any clustering of violated VaR estimates. As defined earlier the estimates do not only have to fulfill the coverage criterion but estimates should also be independently distributed. Hence it may happen, that a VaR model satisfies the unconditional coverage probability but they may exhibit dependent VaR estimates (Campbell, 2005). This should therefore be tested in an additional test, suggested by Christoffersen (1998).

Furthermore unconditional VaR tests suffer from the shortcoming of inability of identification estimation procedures that "systematically under report risk" (Campbell, 2005, p. 7). Moreover the size of underreporting can be quite substantial as reported by Campbell (2005). Although Kupiec's test is considered to be the standard test for VaR estimates, it lacks statistical power, when applied to usual datasets, such as one year of daily data (Perignon & Smith, 2009, p. 4).

3.8.2. Christoffersen test of Conditional Coverage

As mentioned above, the unconditional coverage (UC) by Kupiec does not suffice to explain the fact that violations are not *IID* but occur in the groups. In other words UC does not account for the alternative that ones and zeroes of I_t does not occur independently, but are clustered together over time. Christoffersen (1998) provides likelihood test of unconditional coverage (UC) and test of serial independence (ind) and conditional coverage (CC). While UC test is similar to Kupiec UC test, the LR test of independence and LR test of CC tackle the weakness of dependency-pattern in indicator function I_t. The biggest attraction of conditional coverage test is that it considers the model inadequate (=rejects the null hypothesis) if the number of clustered violations is either too high or too low.

The idea behind the test is to separate conditional coverage into two parts. Note that LR_{UC} is defined as in the previous section.

$$LR_{UC} = -2\ln\left(\frac{L(p;I_1,I_2,\dots,I_T)}{L(\hat{p};I_1,I_2,\dots,I_T)}\right) = -2\ln\left(\frac{(1-p)^{n_0}p^{n_1}}{(1-\hat{p})^{n_0}\hat{p}^{n_1}}\right)\sim\chi^2(1) \qquad (39)$$

$$LR_{ind} = -2\ln\left(\frac{L(\hat{\pi}_2;I_1,I_2,\dots,I_T)}{L(\hat{\pi}_1;I_1,I_2,\dots,I_T)}\right) = -2\ln\left(\frac{(1-\pi_2)^{n_{00}+n_{11}}\pi_2^{n_{01}+n_{11}}}{(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{11}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}}\right)\sim\chi^2(1) \qquad (40)$$

$$\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}\ \pi_{11} = \frac{n_{11}}{n_{10}+n_{11}}\ \pi_2 = \frac{n_{01}+n_{11}}{n_{00}+n_{10}+n_{01}+n_{11}}$$

Where n_{ij} is the number of observations with value *i* at time *t-1* followed by *j* at time *t*. (takes values 1 if violation occurs, 0 if loss was lower that VaR).

Christoffersen (1998) proves that when ignoring the first observation, there exists a numerical relation between the two tests. His paper also finds that the distribution of the LR test of conditional coverage is asymptotically χ^2 with 2 degrees of freedom.

$$LR_{CC} = LR_{UC} + LR_{ind}\sim\chi^2(2) \qquad (41)$$

LR_{CC} tests jointly for independence and correctness of probability parameter *p*.

Consequently, H_0 is not rejected on the significance level $p = 1-\alpha$ if

$$chiinv\left(1-\frac{p}{2};2\right) \le LR_{CC} \le chiinv\left(\frac{p}{2};2\right) \qquad (42)$$

This test is employed in this thesis as a primary indicator of model adequacy.

3.8.3. Empirical Coverage Probability

Next, relative frequency of violations (failure rate) is employed to indicate the best model from among those that pass the LR(CC). According to this indicator the best model is the one with the lowest empirical coverage probability, $\hat{p}$, i.e. ratio between number of VaR breaks and size of the out-of-sample window. The model is considered satisfactory if the actual loss is smaller than the VaR forecast at least 99 percent (or 95 percent) of the time. In other words the empirical coverage probability, $\hat{p}$, must be lower than the significance level on which the VaR model is performed.

3.8.4. Predictive quantile loss

Finally, evaluation technique is conducted in terms of quantile loss as in Bao, Lee & Saltoğlu (2006). Quantile loss can be regarded as a measure of lack of fit of the VaR model.

If the loss predicted by VaR is $q_t(p)$ for the realized return r_t, then a predictive quantile loss $Q(\alpha)$ for a given level of $p=1\text{-}\alpha$ is given by

$$Q(\alpha) = E\left[p - 1\left(r_t < q_t(\alpha)\right)\right][r_t - q_t(\alpha)] \qquad (43)$$

The function used for evaluation is the average of out-of-sample predictive quantile losses

$$\hat{Q}(\alpha) = P^{-1} \sum_{t=R+1}^{T} [p - 1(r_t < \hat{q}_t(\alpha))][r_t - \hat{q}_t(\alpha)] \qquad (44)$$

where $\hat{q}(\alpha)$ is the VaR forecast at time *t* and ***1*** is a usual indicator function with value either 1 if the argument in the brackets is true or 0 if it is false. The better the fit of the model, the lower the value of the check-function. Check-function penalizes each violation at the same time. Consequently, minimum value of $\hat{Q}(\alpha)$ indicates the best model.

4. Empirical Results

This section is split into two parts. Each part presents the results for one period in terms of three evaluation criteria: Christoffersen test of Conditional Coverage, Empirical Coverage probability (failure rate) and Empirical Quantile loss. Comments on each test interpret the results. Each part seeks to find the best model for a given market and confidence level. Different periods give ground to assessment of the models under two stress scenarios.

4.1. Period 1 (Tranquil Period)

4.1.1. Christoffersen test of Conditional Coverage

In the Tables 5, 6 below the results of Christoffersen test of Conditional Coverage are presented. The Tables show the LR(UC), LR(ind) and LR (CC) for all models, for p=0,05 and p=0,01 with adequate models highlighted in bold in the last column. As presented in Table 5, for p=0,05 all unfiltered traditional models (HS, Normal, t) are considered inadequate mainly due to the overestimation of VaR as indicated by LR(UC) or visible form Figure 7 in Appendix B. Filtering seems to improve the results letting majority of VW-HS*, Normal* and t* pass the test for all markets. All three CAViaR specifications pass the tests with exception of $CAViaR_S$ for NYSE. For p=0,01 overwhelming majority of models pass all LR tests as apparent from the Table 6, confirming that further comparison must be made to draw more precise conclusion.

These results have graphical representation in Figures 7 in Appendix B.

Table 5: Test of conditional coverage - Period 1, *p*=5%

Index	probability: p=5%	Kupiec test -LR(UC)			Independence test -LR(ind)			Christoffersen test -LR(CC)		
	Calculation Method	Confidence interval		LR(UC)	Confidence interval		LR(ind)	Confidence interval		LR(CC)
		lower bound	upper bound		lower bound	upper bound		lower bound	upper bound	
NYSE	HS	0,000982069	5,02388647	10,8123	0,00098207	5,02388647	0,0732	0,05063562	7,37775891	10,8855
	AW-HS*	0,000982069	5,02388647	8,1852	0,00098207	5,02388647	0,1306	0,05063562	7,37775891	8,3158
	VW-HS*	0,000982069	5,02388647	4,3687	0,00098207	5,02388647	2,4232	0,05063562	7,37775891	**6,7919**
	Normal	0,000982069	5,02388647	10,8123	0,00098207	5,02388647	0,0732	0,05063562	7,37775891	10,8855
	Normal*	0,000982069	5,02388647	4,3687	0,00098207	5,02388647	2,4232	0,05063562	7,37775891	**6,7919**
	t	0,000982069	5,02388647	10,8123	0,00098207	5,02388647	0,0732	0,05063562	7,37775891	10,8855
	t*	0,000982069	5,02388647	10,8123	0,00098207	5,02388647	0,0732	0,05063562	7,37775891	10,8855
	CAViaR (S)	0,000982069	5,02388647	7,5204	0,00098207	5,02388647	0,8671	0,05063562	7,37775891	8,3876
	CAViaR (A)	0,000982069	5,02388647	0,0213	0,00098207	5,02388647	1,0215	0,05063562	7,37775891	**1,0428**
	CAViaR (G)	0,000982069	5,02388647	0,4961	0,00098207	5,02388647	1,6760	0,05063562	7,37775891	**2,1720**
FTSE W Hong Kong	HS	0,000982069	5,02388647	25,6466	0,00098207	5,02388647	0,0000	0,05063562	7,37775891	25,6466
	AW-HS*	0,000982069	5,02388647	8,1852	0,00098207	5,02388647	4,1070	0,05063562	7,37775891	12,2922
	VW-HS*	0,000982069	5,02388647	1,9441	0,00098207	5,02388647	1,3809	0,05063562	7,37775891	**3,3250**
	Normal	0,000982069	5,02388647	10,8123	0,00098207	5,02388647	0,0729	0,05063562	7,37775891	10,8852
	Normal*	0,000982069	5,02388647	5,0972	0,00098207	5,02388647	0,0341	0,05063562	7,37775891	**5,1313**
	t	0,000982069	5,02388647	8,1851	0,00098207	5,02388647	0,1343	0,05063562	7,37775891	8,3194
	t*	0,000982069	5,02388647	5,0972	0,00098207	5,02388647	0,0319	0,05063562	7,37775891	**5,1291**
	CAViaR (S)	0,000982069	5,02388647	1,9441	0,00098207	5,02388647	0,4067	0,05063562	7,37775891	**2,3508**
	CAViaR (A)	0,000982069	5,02388647	0,0213	0,00098207	5,02388647	1,0215	0,05063562	7,37775891	**1,0428**
	CAViaR (G)	0,000982069	5,02388647	0,5634	0,00098207	5,02388647	0,0000	0,05063562	7,37775891	**0,5634**
Russia RTS	HS	0,000982069	5,02388647	14,1272	0,00098207	5,02388647	0,0324	0,05063562	7,37775891	14,1596
	AW-HS*	0,000982069	5,02388647	8,1852	0,00098207	5,02388647	0,1306	0,05063562	7,37775891	8,3158
	VW-HS*	0,000982069	5,02388647	4,3687	0,00098207	5,02388647	2,4232	0,05063562	7,37775891	**6,7919**
	Normal	0,000982069	5,02388647	18,4966	0,00098207	5,02388647	0,0081	0,05063562	7,37775891	18,5047
	Normal*	0,000982069	5,02388647	6,0715	0,00098207	5,02388647	0,2049	0,05063562	7,37775891	**6,2764**
	t	0,000982069	5,02388647	14,1272	0,00098207	5,02388647	0,0324	0,05063562	7,37775891	14,1596
	t*	0,000982069	5,02388647	3,0089	0,00098207	5,02388647	1,8383	0,05063562	7,37775891	**4,8472**
	CAViaR (S)	0,000982069	5,02388647	0,5634	0,00098207	5,02388647	0,5054	0,05063562	7,37775891	**1,0687**
	CAViaR (A)	0,000982069	5,02388647	0,5634	0,00098207	5,02388647	0,5054	0,05063562	7,37775891	**1,0687**
	CAViaR (G)	0,000982069	5,02388647	0,1827	0,00098207	5,02388647	0,0601	0,05063562	7,37775891	**0,2428**

Table 6: Test of conditional coverage - Period 1, *p*=1%

Index	probability: p=1%	Kupiec test -LR(UC)			Independence test -LR(ind)			Christoffersen test -LR(CC)		
	Calculation Method	Confidence interval		LR(UC)	Confidence interval		LR(ind)	Confidence interval		LR(CC)
		lower bound	upper bound		lower bound	upper bound		lower bound	upper bound	
NYSE	HS	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	AW-HS*	3,92704E-05	7,879438691	0,1084	3,92704E-05	7,879438691	0,0324	0,010025084	10,59663473	**0,1408**
	VW-HS*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0731	0,010025084	10,59663473	**0,1680**
	Normal	3,92704E-05	7,879438691	1,1765	3,92704E-05	7,879438691	0,0081	0,010025084	10,59663473	**1,1846**
	Normal*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0731	0,010025084	10,59663473	**0,1680**
	t	3,92704E-05	7,879438691	5,0250	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0250**
	t*	3,92704E-05	7,879438691	5,0250	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0250**
	CAViaR (S)	3,92704E-05	7,879438691	7,7336	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**7,7336**
	CAViaR (A)	3,92704E-05	7,879438691	3,5554	3,92704E-05	7,879438691	0,2963	0,010025084	10,59663473	**3,8517**
	CAViaR (G)	3,92704E-05	7,879438691	15,8906	3,92704E-05	7,879438691	0,8404	0,010025084	10,59663473	16,7310
FTSE W Hong Kong	HS	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	AW-HS*	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	VW-HS*	3,92704E-05	7,879438691	1,1765	3,92704E-05	7,879438691	0,0081	0,010025084	10,59663473	**1,1846**
	Normal	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	Normal*	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	t	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	t*	3,92704E-05	7,879438691	1,1765	3,92704E-05	7,879438691	0,0081	0,010025084	10,59663473	**1,1846**
	CAViaR (S)	3,92704E-05	7,879438691	12,9555	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	12,9555
	CAViaR (A)	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**0,0949**
	CAViaR (G)	3,92704E-05	7,879438691	4,3687	3,92704E-05	7,879438691	0,7645	0,010025084	10,59663473	**5,1332**
Russia RTS	HS	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	AW-HS*	3,92704E-05	7,879438691	1,1765	3,92704E-05	7,879438691	0,0081	0,010025084	10,59663473	**1,1846**
	VW-HS*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0732	0,010025084	10,59663473	**0,1681**
	Normal	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	Normal*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0732	0,010025084	10,59663473	**0,1681**
	t	3,92704E-05	7,879438691	5,0252	3,92704E-05	7,879438691	0,0000	0,010025084	10,59663473	**5,0252**
	t*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0735	0,010025084	10,59663473	**0,1684**
	CAViaR (S)	3,92704E-05	7,879438691	7,7336	3,92704E-05	7,879438691	1,1304	0,010025084	10,59663473	**8,8640**
	CAViaR (A)	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	CAViaR (G)	3,92704E-05	7,879438691	7,7336	3,92704E-05	7,879438691	1,3809	0,010025084	10,59663473	**9,1145**

4.1.2. Empirical coverage probability

Relative frequency of VaR breaks for p=0,05 and p=0,01 is presented in Table 7. Highlighted values are the models with models that previously passed LR(CC). From this group the best models in this criterion are marked with *.On 5% level most models give empirical coverage close to the nominal one-5%. Filtered VW-HS* and Normal*, however, seem to perform the best (have lowest $\hat{p}$) over the most of the markets. Exception is Hong Kong where Normal* and t* perform rather poorly, whilst VW-HS* and $CAViaR_S$ dominate. CAViaR models generally drag closely behind traditional approaches on 5% in this period, although $CAViaR_S$ does extraordinarily well for Hong Kong.

Situation changes on 1% level where unfiltered models dominate in this criterion. Figure 7 in Appendix B plots the returns against the negative of VaR. Graphs reveal the lack of fit of unfiltered methods by giving graphical overview of predictive power of all models. For trading purposes higher allocated capital can cause problems. Good results are again produced by their filtered equivalents, whilst having much better fitness to observed losses. Asymmetric CAViaR ranks among the best for 1%.

Table 7: Empirical Coverage Probability - Period 1 p=5% and p=1%

	Calculation Method	NYSE	FTSE W Hong Kong	Russia RTS
5% VaR	HS	0,012	0	0,008
	AW-HS*	0,016	0,016	0,016
	VW-HS*	**0,024***	**0,032***	**0,024**
	Normal	0,012	0,012	0,004
	Normal*	**0,024***	**0,084**	**0,02***
	t	0,012	0,016	0,008
	t*	0,012	**0,084**	**0,028**
	CAViaR (S)	0,092	**0,032***	**0,04**
	CAViaR (A)	**0,048**	**0,048**	**0,04**
	CAViaR (G)	**0,06**	**0,04**	**0,056**
1% VaR	HS	**0***	**0***	**0***
	AW-HS*	**0,008**	**0***	**0,004**
	VW-HS*	**0,012**	**0,004**	**0,012**
	Normal	**0,004**	**0***	**0***
	Normal*	**0,002**	**0,02**	**0,012**
	t	**0***	**0***	**0***
	t*	**0***	**0,004**	**0,012**
	CAViaR (S)	**0,032**	0,04	**0,032**
	CAViaR (A)	**0,024**	**0,012**	**0,02**
	CAViaR (G)	0,044	**0,024**	**0,032**

Note: Best model is marked with *

4.1.3. Empirical quantile loss

Table 8 shows the empirical quantile loss $\hat{Q}(\alpha)$ for all models. The model with lowest value of $\hat{Q}(\alpha)$ is highlighted in bold for each market and level of p. Again, it can be observed, that introducing GARCH volatility dynamics considerably improves the models for both levels of p. Normal* is indicated as best model five out of six times. This means, that VaRs produced by quantile of standard normal distribution are able to fit the actual data most precisely when accounted for volatility dynamics. In Period 1, t* and $CAViaR_A$ also show good results for NYSE and Hong Kong. The worst models in terms of $\hat{Q}(\alpha)$ on both p levels are unfiltered models. Unfiltered models on 1% are an example of models good in terms $\hat{p}$ but bad in terms of $\hat{Q}(\alpha)$.

Table 8: Empirical quantile loss - Period 1 *p*=5% and *p*=1%

	Calculation Method	NYSE	FTSE W Hong Kong	Russia RTS
5% VaR	HS	0,0010856	0,0014276	0,0024000
	AW-HS*	0,0010483	0,0012202	0,0029076
	VW-HS*	0,0009216	0,0011371	0,0018162
	Normal	0,0009823	0,0014774	0,0024500
	Normal*	**0,0007335**	**0,0009243**	0,0015071
	t	0,0181615	0,0266374	0,0441318
	t*	0,0009360	0,0009328	**0,0014067**
	CAViaR (S)	0,0009734	0,0010748	0,0018515
	CAViaR (A)	0,0009139	0,0010815	0,0017191
	CAViaR (G)	0,0009071	0,0010896	0,0017663
1% VaR	HS	0,0003226	0,0004788	0,0008780
	AW-HS*	0,0003025	0,0003304	0,0009319
	VW-HS*	0,0002992	0,0003087	0,0005404
	Normal	0,0002792	0,0004179	0,0006965
	Normal*	**0,0002075**	**0,0002614**	**0,0004263**
	t	0,0301299	0,0463579	0,0773532
	t*	0,0002962	0,0003115	0,0004698
	CAViaR (S)	0,0003182	0,0003397	0,0006038
	CAViaR (A)	0,0002824	0,0002930	0,0006372
	CAViaR (G)	0,0003437	0,0003227	0,0006443

Summarizing the results for Period 1, out of the two best models VW-HS* and Normal*, the later seems to yield superior performance by having the best empirical quantile loss results, whilst at the same time providing reasonable coverage probability on both levels of *p*. Asymmetric CAViaR fluctuates little above the target coverage probability (1%) or below it (5%) and belongs to the best models in term of quantile loss as well. Market choice in period 1 impacts models' performance only slightly, so no certain conclusion can be drawn.

4.2. Period 2 (Crisis Period)

4.2.1. Christoffersen test of Conditional Coverage

Tables 9, 10 below summarize the results of all three Christoffersen tests. Turning to the Period 2 which represents the current crisis all unfiltered models with exceptions of 1% *t, HS* for Hong Kong and *HS* for Russia fail the LR(CC). This result could be expected as with increased volatility naïve models tend to fail as described in section 2.2. Volatility weighting improves the situation almost uniformly for traditional models (exception NYSE on 5%). Adequate models in general also pass the test of independence of violations. CAViaR models pass the test with a few exceptions. The fact that only CAViaRs passes the test on 5% for NYSE is also worth mentioning.

Table 9: Test of conditional coverage - Period 2, *p*=5%

Index	probability: p=5%	Kupiec test -LR(UC)			Independence test -LR(ind)			Christoffersen test -LR(CC)		
	Calculation Method	Confidence interval		LR(UC)	Confidence interval		LR(ind)	Confidence interval		LR(CC)
		lower bound	upper bound		lower bound	upper bound		lower bound	upper bound	
NYSE	HS	0,000982069	5,02388647	86,0810	0,00098207	5,02388647	17,5156	0,05063562	7,37775891	103,5966
	AW-HS*	0,000982069	5,02388647	99,8779	0,00098207	5,02388647	11,6115	0,05063562	7,37775891	111,4894
	VW-HS*	0,000982069	5,02388647	7,5204	0,00098207	5,02388647	4,6895	0,05063562	7,37775891	12,2099
	Normal	0,000982069	5,02388647	86,0810	0,00098207	5,02388647	5,0128	0,05063562	7,37775891	91,0938
	Normal*	0,000982069	5,02388647	6,2590	0,00098207	5,02388647	4,2710	0,05063562	7,37775891	10,5300
	t	0,000982069	5,02388647	89,4617	0,00098207	5,02388647	5,7119	0,05063562	7,37775891	95,1736
	t*	0,000982069	5,02388647	7,5204	0,00098207	5,02388647	4,7104	0,05063562	7,37775891	12,2308
	CAViaR (S)	0,000982069	5,02388647	0,9514	0,00098207	5,02388647	2,1992	0,05063562	7,37775891	**3,1505**
	CAViaR (A)	0,000982069	5,02388647	1,5403	0,00098207	5,02388647	2,4936	0,05063562	7,37775891	**4,0339**
	CAViaR (G)	0,000982069	5,02388647	2,2555	0,00098207	5,02388647	2,8080	0,05063562	7,37775891	**5,0636**
FTSE W Hong Kong	HS	0,000982069	5,02388647	20,7920	0,00098207	5,02388647	2,8623	0,05063562	7,37775891	23,6543
	AW-HS*	0,000982069	5,02388647	24,8941	0,00098207	5,02388647	3,4391	0,05063562	7,37775891	28,3332
	VW-HS*	0,000982069	5,02388647	0,0208	0,00098207	5,02388647	0,1499	0,05063562	7,37775891	**0,1707**
	Normal	0,000982069	5,02388647	15,1920	0,00098207	5,02388647	1,2224	0,05063562	7,37775891	16,4144
	Normal*	0,000982069	5,02388647	0,9514	0,00098207	5,02388647	0,0009	0,05063562	7,37775891	**0,9523**
	t	0,000982069	5,02388647	20,7920	0,00098207	5,02388647	2,8296	0,05063562	7,37775891	23,6216
	t*	0,000982069	5,02388647	2,2555	0,00098207	5,02388647	1,9700	0,05063562	7,37775891	**4,2255**
	CAViaR (S)	0,000982069	5,02388647	0,5634	0,00098207	5,02388647	0,8371	0,05063562	7,37775891	**1,4004**
	CAViaR (A)	0,000982069	5,02388647	0,5634	0,00098207	5,02388647	0,7055	0,05063562	7,37775891	**1,2689**
	CAViaR (G)	0,000982069	5,02388647	0,4961	0,00098207	5,02388647	0,0113	0,05063562	7,37775891	**0,5074**
Russia RTS	HS	0,000982069	5,02388647	15,1970	0,00098207	5,02388647	10,3764	0,05063562	7,37775891	25,5734
	AW-HS*	0,000982069	5,02388647	38,8244	0,00098207	5,02388647	6,8657	0,05063562	7,37775891	45,6901
	VW-HS*	0,000982069	5,02388647	2,2555	0,00098207	5,02388647	2,8080	0,05063562	7,37775891	**5,0635**
	Normal	0,000982069	5,02388647	11,8655	0,00098207	5,02388647	12,9458	0,05063562	7,37775891	24,8113
	Normal*	0,000982069	5,02388647	1,5403	0,00098207	5,02388647	2,4936	0,05063562	7,37775891	**4,0339**
	t	0,000982069	5,02388647	15,1970	0,00098207	5,02388647	10,3175	0,05063562	7,37775891	25,5145
	t*	0,000982069	5,02388647	1,5403	0,00098207	5,02388647	2,5044	0,05063562	7,37775891	**4,0447**
	CAViaR (S)	0,000982069	5,02388647	0,9514	0,00098207	5,02388647	0,8512	0,05063562	7,37775891	**1,8026**
	CAViaR (A)	0,000982069	5,02388647	0,4961	0,00098207	5,02388647	1,2330	0,05063562	7,37775891	**1,7291**
	CAViaR (G)	0,000982069	5,02388647	3,0905	0,00098207	5,02388647	0,1806	0,05063562	7,37775891	**3,2711**

Table 10: Test of conditional coverage - Period 2, *p*=1%

Index	probability: p=1%	Kupiec test -LR(UC)			Independence test -LR(ind)			Christoffersen test -LR(CC)		
	Calculation Method	Confidence interval		LR(UC)	Confidence interval		LR(ind)	Confidence interval		LR(CC)
		lower bound	upper bound		lower bound	upper bound		lower bound	upper bound	
NYSE	HS	3,92704E-05	7,879438691	77,0794	3,92704E-05	7,879438691	0,0364	0,010025084	10,59663473	77,1158
	AW-HS*	3,92704E-05	7,879438691	135,4520	3,92704E-05	7,879438691	0,6040	0,010025084	10,59663473	136,0560
	VW-HS*	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	Normal	3,92704E-05	7,879438691	107,8293	3,92704E-05	7,879438691	0,4266	0,010025084	10,59663473	108,2559
	Normal*	3,92704E-05	7,879438691	7,7333	3,92704E-05	7,879438691	0,5312	0,010025084	10,59663473	**8,2645**
	t	3,92704E-05	7,879438691	87,0136	3,92704E-05	7,879438691	0,0106	0,010025084	10,59663473	87,0242
	t*	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2058	0,010025084	10,59663473	**2,1626**
	CAViaR (S)	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	CAViaR (A)	3,92704E-05	7,879438691	3,5554	3,92704E-05	7,879438691	0,2963	0,010025084	10,59663473	**3,8517**
	CAViaR (G)	3,92704E-05	7,879438691	12,9555	3,92704E-05	7,879438691	0,8371	0,010025084	10,59663473	13,7926
FTSE W Hong Kong	HS	3,92704E-05	7,879438691	3,5554	3,92704E-05	7,879438691	0,2963	0,010025084	10,59663473	**3,8517**
	AW-HS*	3,92704E-05	7,879438691	37,0420	3,92704E-05	7,879438691	0,5918	0,010025084	10,59663473	37,6338
	VW-HS*	3,92704E-05	7,879438691	0,7691	3,92704E-05	7,879438691	0,1306	0,010025084	10,59663473	**0,8997**
	Normal	3,92704E-05	7,879438691	22,3170	3,92704E-05	7,879438691	0,1499	0,010025084	10,59663473	22,4669
	Normal*	3,92704E-05	7,879438691	0,7691	3,92704E-05	7,879438691	0,1306	0,010025084	10,59663473	**0,8997**
	t	3,92704E-05	7,879438691	7,7336	3,92704E-05	7,879438691	0,5334	0,010025084	10,59663473	**8,2670**
	t*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0735	0,010025084	10,59663473	**0,6069**
	CAViaR (S)	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0732	0,010025084	10,59663473	**0,1681**
	CAViaR (A)	3,92704E-05	7,879438691	0,1084	3,92704E-05	7,879438691	0,0324	0,010025084	10,59663473	**0,1408**
	CAViaR (G)	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
Russia RTS	HS	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	AW-HS*	3,92704E-05	7,879438691	49,4453	3,92704E-05	7,879438691	5,9587	0,010025084	10,59663473	55,4040
	VW-HS*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0732	0,010025084	10,59663473	**0,1681**
	Normal	3,92704E-05	7,879438691	25,7803	3,92704E-05	7,879438691	13,1702	0,010025084	10,59663473	38,9505
	Normal*	3,92704E-05	7,879438691	1,9568	3,92704E-05	7,879438691	0,2049	0,010025084	10,59663473	**2,1617**
	t	3,92704E-05	7,879438691	15,8906	3,92704E-05	7,879438691	18,9258	0,010025084	10,59663473	34,8164
	t*	3,92704E-05	7,879438691	0,0949	3,92704E-05	7,879438691	0,0735	0,010025084	10,59663473	**0,1684**
	CAViaR (S)	3,92704E-05	7,879438691	5,4970	3,92704E-05	7,879438691	1,8452	0,010025084	10,59663473	**7,3422**
	CAViaR (A)	3,92704E-05	7,879438691	10,2290	3,92704E-05	7,879438691	0,6752	0,010025084	10,59663473	10,9042
	CAViaR (G)	3,92704E-05	7,879438691	12,9555	3,92704E-05	7,879438691	0,8371	0,010025084	10,59663473	13,7926

4.2.2. Empirical coverage probability

Empirical coverage probabilities are presented in Table 11 below. As previously models that passed the LR(CC) test in Tables 9, 10 are market with bold. The model with minimum value of $\hat{p}$ is marked with *. On the 5% level, considering only the LR(CC) adequate models, all three volatility weighted models understated VaR. Despite the fact that t* accounts for increased skewness and kurtosis during the crises, it fails to beat Normal* in this criterion on 5% level. Both models however perform quite poorly. Within volatility updated group, VW-HS* yields the best result for Hong-Kong. On 1% level, VW-HS* and t* are found to be best performers from the filtered models family substantially beating Normal*. It must be noted that these results are not surprising in the crisis period. In general on 5% level of *p*, the most precise models in terms of deviations from the correct coverage are with a few exceptions CAViaR models, from which $CAViaR_S$ and $CAViaR_A$ are outstanding. 1% level does not change the order substantially, ranking $CAViaR_S$, t* and VW-HS* jointly best for NYSE, and $CAViaR_A$ far best for Hong Kong. The only market where CAViaR is outperformed by t* together with VW-HS* is Russia.

Table 11: Empirical Coverage Probability - Period 2 *p*=5% and *p*=1%

	Calculation Method	NYSE	FTSE W Hong Kong	Russia RTS
5% VaR	HS	0,22	0,124	0,112
	AW-HS*	0,236	0,132	0,156
	VW-HS*	0,092	**0,052**	**0,072**
	Normal	0,22	0,112	0,104
	Normal*	0,088	**0,064**	**0,068**
	t	0,224	0,124	0,112
	t*	0,092	**0,072**	**0,068**
	CAViaR (S)	**0,064***	**0,04***	**0,064**
	CAViaR (A)	**0,068**	**0,04***	**0,06***
	CAViaR (G)	**0,072**	**0,06**	**0,076**
1% VaR	HS	0,104	**0,024**	**0,02**
	AW-HS*	0,148	0,068	0,08
	VW-HS*	**0,02***	**0,016**	**0,012***
	Normal	0,128	0,052	0,056
	Normal*	**0,032**	**0,016**	**0,02**
	t	0,112	**0,032**	0,044
	t*	**0,02***	**0,012**	**0,012***
	CAViaR (S)	**0,02***	**0,012**	**0,028**
	CAViaR (A)	**0,024**	**0,008***	0,036
	CAViaR (G)	0,04	**0,02**	0,04

Note: Best model is marked with *

4.2.3. Empirical quantile loss

Figure 8 plots VaR forecasts showing the number of violations as well as the overall goodness of fit of all models in Period 2. Note the different scale in plots for Figure 7 and Figure 8 caused by crisis. Goodness of fit expressed in numbers can be also found in the Table 12 that reports empirical quantile loss for 5% as well as 1% level of p. On 5% level results are rather dispersed. For NYSE, Hong Kong and Russia the lowest empirical quantile losses $\hat{Q}(\alpha)$ -in bold are respectively reached by $CAViaR_G$, VW-HS* and t*. On 1% level NYSE is addressed best by $CAViaR_S$, Hong Kong by $CAViaR_A$ and Russia by VW-HS*.

Table 12: Empirical quantile loss - Period 2 *p*=5% and *p*=1%

	Calculation Method	NYSE	FTSE W Hong Kong	Russia RTS
5% VaR	HS	0,0052339	0,0031687	0,0054174
	AW-HS*	0,0057662	0,0037620	0,0005088
	VW-HS*	0,0030664	**0,0024644**	0,0003120
	Normal	0,0050607	0,0030946	0,0035926
	Normal*	0,0030448	0,0024666	0,0044845
	t	0,0052219	0,0031936	0,0004050
	t*	0,0030506	0,0024771	**0,0003104**
	CAViaR (S)	0,0031656	0,0025960	0,0043118
	CAViaR (A)	0,0030793	0,0027351	0,0041432
	CAViaR (G)	**0,0029757**	0,0024719	0,0042192
1% VaR	HS	0,0023864	0,0009654	0,0018427
	AW-HS*	0,0048601	0,0014876	0,0027957
	VW-HS*	0,0008181	0,0006976	**0,0011688**
	Normal	0,0028947	0,0011381	0,0023391
	Normal*	0,0009081	0,0007097	0,0012022
	t	0,0025268	0,0009731	0,0020304
	t*	0,0008309	0,0006922	0,0011719
	CAViaR (S)	**0,0008069**	0,0007085	0,0016195
	CAViaR (A)	0,0008155	**0,0006910**	0,0013018
	CAViaR (G)	0,0009260	0,0007055	0,0015251

In the crisis period (Period 2) symmetric and asymmetric CAViaR perform better in predicting VaR than in tranquil period (Period 1). While empirical quantile loss points out VW-HS* or t* as the leading models in few cases, CAViaRs appears to provide superior results over all markets and target probabilities in Period 2. With respect to a market choice, no influences observed impacting the choice of the most appropriate models. However, it might be interesting to highlight the dominance (on 5%) of CAViaR models on the US market during the crisis. On the other hand, as e.g. for Russia, 1% CAViaR might underestimate VaR considerably.

5. Conclusion

This thesis examined one-day predictive power of traditional VaR models and a recently introduced VaR class, CAViaR, by Engle & Manganelli (2004). The comparison is made on three markets: US, Hong Kong and Russia. Following the questions in the purpose of this thesis (Section 1.4), the findings can be summarized as follows:

1) Based on the number of violations volatility-weighted historical simulation (VW-HS*) and GARCH-weighted normal VaR (Normal*) gave the best results in the tranquil period. This result suggests that for this period the assets returns do not divert largely enough from assumption of normality although different statistics suggested strong non-normality. Asymmetric CAViaR behaves also reasonably well in the calm period. Results based on empirical quantile loss correspond largely to those based on number of violations making Normal* generally dominate in this period. While conventional methods seem to outperform CAViaR models during the period of lower volatility (tranquil), CAViaR models work generally better during the crisis. Filtering with volatility estimated by GARCH(1,1) seems to be uniformly useful, while age-weighting proved to be harmful. For the data used in this thesis, one generally best model could not be pointed out.

2) Conventional models tend to overstate VaR in tranquil period at both confidence levels. Opposite is true for crisis period, in which traditional approaches generally produced higher number of violations then the nominal coverage. With few exceptions CAViaR models worked better on the lower confidence level in both periods. No conclusion can be drawn about which model is the best performer depending on the market choice.

3) Evaluation was based on three evaluation criteria. Christoffersen test of conditional coverage usually disqualified unfiltered methods on 5% level due to their over-conservatism during tranquil period and understating of VaR in the crisis period. Since Christoffersen test considers adequate too many models, deeper look at the models performance must be taken. Subsequent criteria, i.e. empirical coverage probability and empirical quantile loss report widely compatible results.

These results are in line with findings of Bao, Lee & Saltoğlu (2006) who found CAViaR working well in some periods but not reliable in the whole period. As demonstrated, the forecasting of market risk in different economy cycles might be a challenging matter. It is argued that CAViaR may be a promising crisis tool and deserves further academic attention. It would be interesting to see if the approach of this thesis applied to another type of data or different market would yield other results.

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7. Appendix

7.1. Appendix A

7.1.1. Proxy for actual volatility

Define returns by the following equations

$$r_t = \mu + \varepsilon_t \text{ and } \varepsilon_t = \sigma_t z_t ;$$

z_t is *IID*; $E[\mu|\Omega_{t-1}]$=0, where Ω_{t-1} is an information available at time *t-1*

$$E[r_t^2|\Omega_{t-1}] = E[\varepsilon_t^2|\Omega_{t-1}]$$

hence

$$E[r_t^2|\Omega_{t-1}] = E[z_t^2\sigma_t^2|\Omega_{t-1}] = \sigma_t^2 E[z_t^2|\Omega_{t-1}] = \sigma_t^2$$

Since $E[z_t^2|\Omega_{t-1}] = 1$ $z_t \sim N(0,1)$

7.1.2. Jarque-Bera normality test

Bera & Jarque (1981) test whether the coefficients of skewness and excess kurtosis are jointly zero.

Let the errors be ε and their variance be σ^2; the skewness and kurtosis coefficients are respectively given by $s = \frac{E[\varepsilon^3]}{(\sigma^2)^{\frac{3}{2}}}$ and $k = \frac{E[\varepsilon^4]}{(\sigma^2)^2}$.

Normal distribution has skewness equal to zero and kurtosis equal to 3. Excess kurtosis is thus *(k-3)* equal to zero.

The Jarque-Bera test statistics can be expressed by

$$W = T\left[\frac{s^2}{6} + \frac{(k-3)^2}{24}\right]$$

where T is the number of observations. Under the null hypothesis than the distribution is normal (symmetric and mesokurtic) the test statistics is asymptotically $\lambda^2(2)$ distributed. If residuals are significantly skewed, leptokurtic or both, null hypothesis is rejected.

7.1.3. The Ljung-Box Test of Random Walk

Ljung & Box (1978) test whether all autocorrelations up to lag m are zero and used as a general test of linear independence in time series modeling.

The statistics,

$$Q_m = T(T+2)\sum_{k=1}^{m}\frac{\rho_k^2}{(T-k)}$$

is λ^2 distributed with m degrees of freedom under null hypothesis of no autocorrelation.

T is a sample size, m is number of lags and ρ_k is k -th autocorrelation.

7.2. Appendix B

Figure 4: Histograms

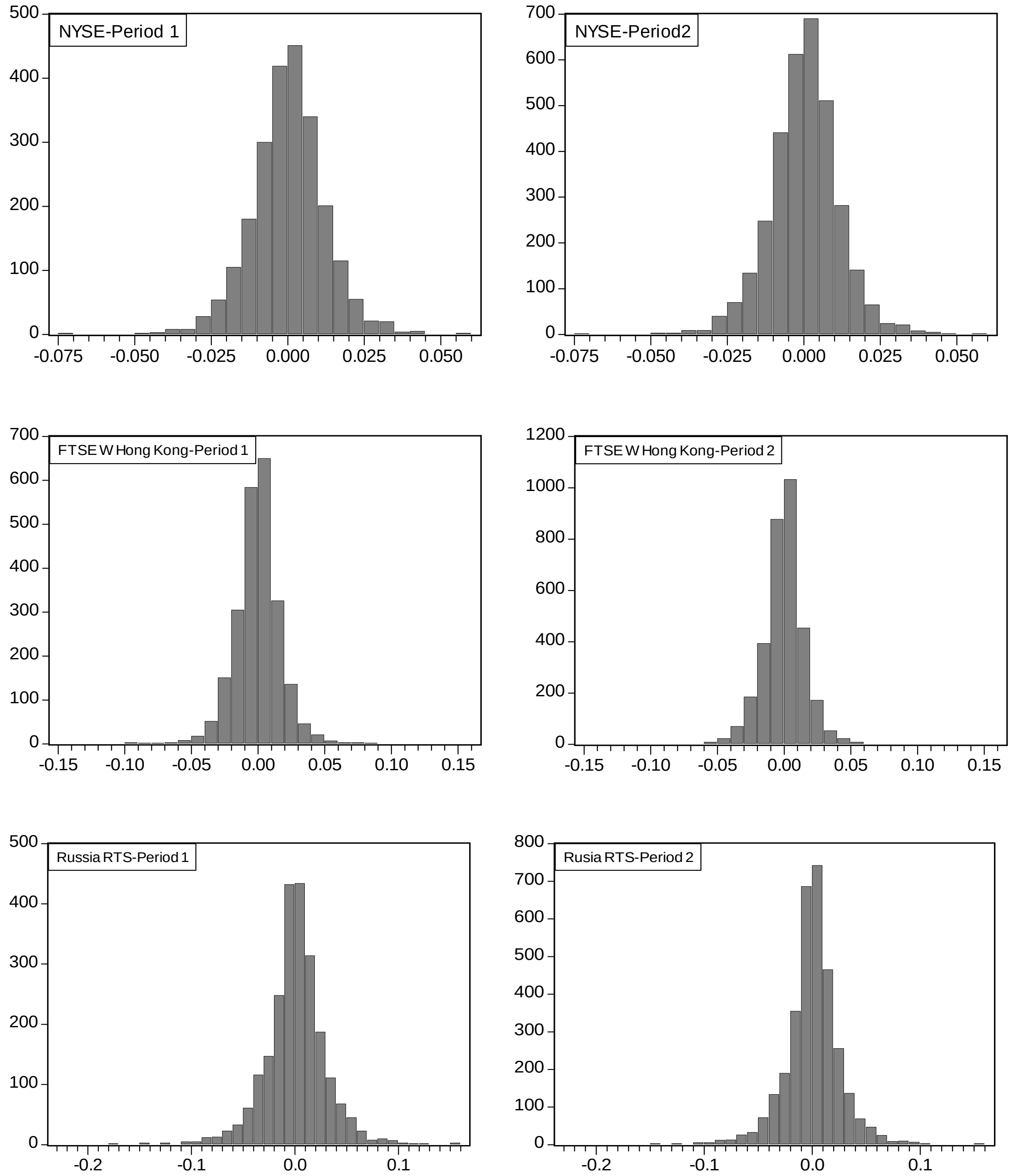


Figure 5: Squared log-returns for the in-sample Period 2

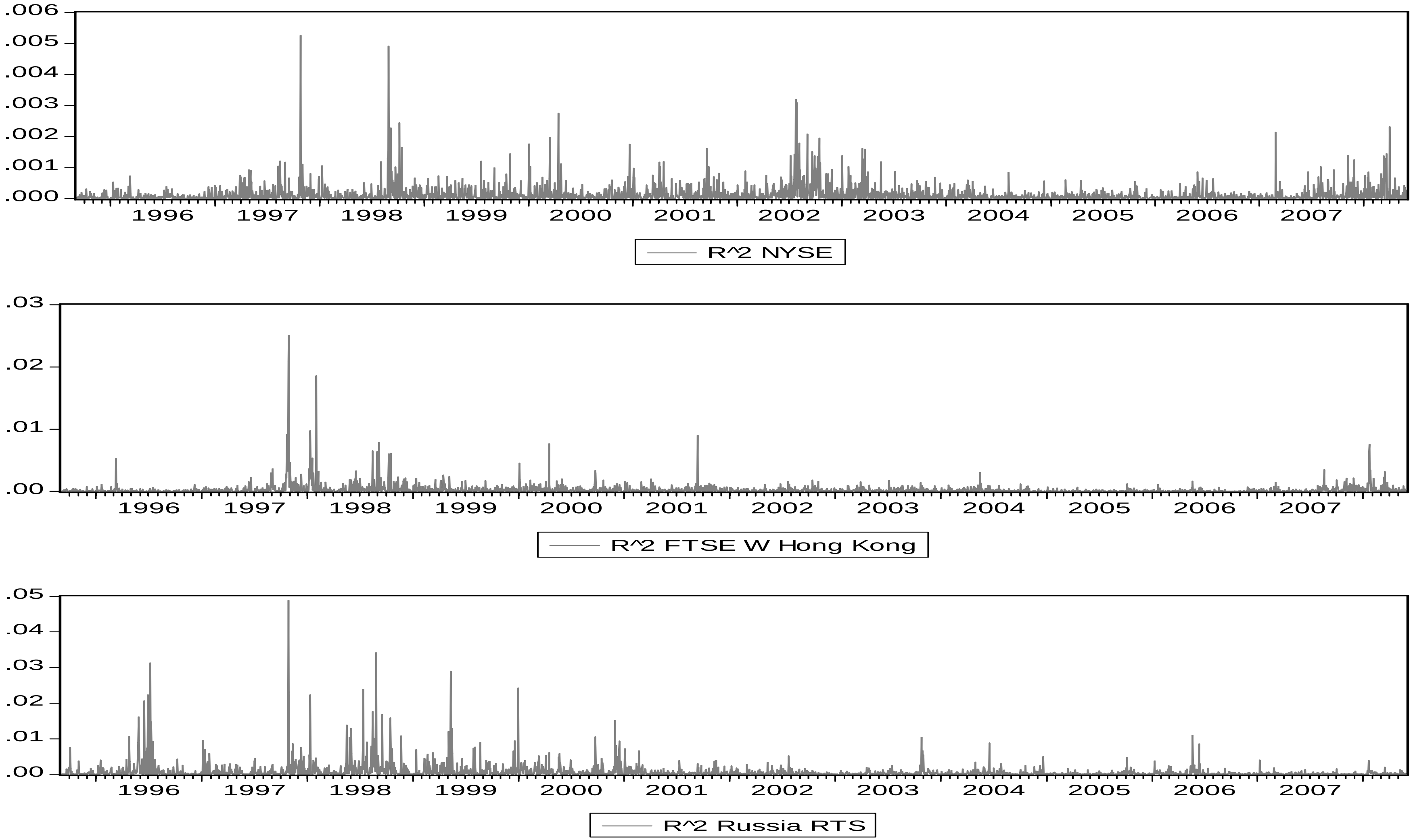


Figure 6: QQ-plots, In-sample Period 2

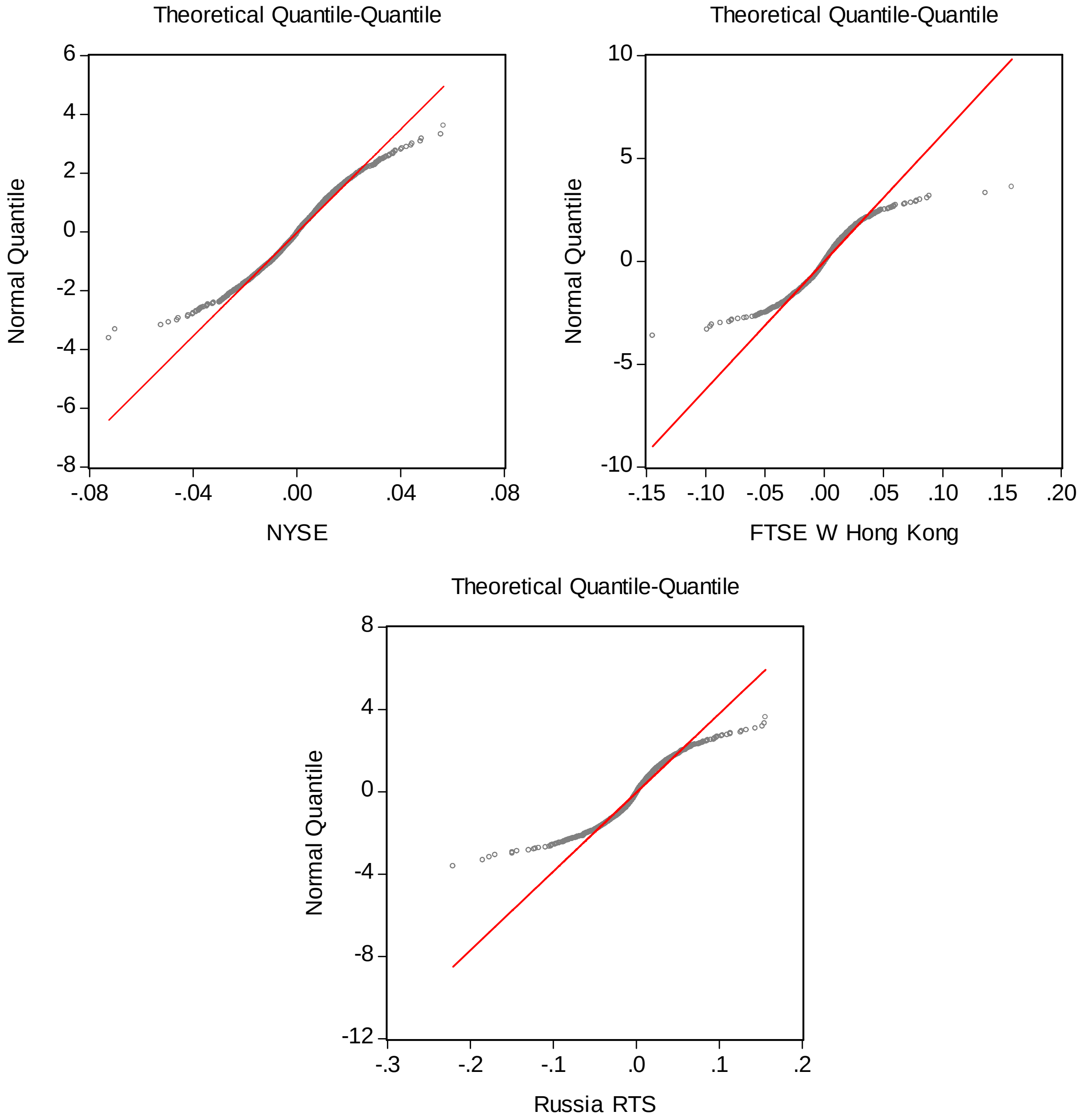
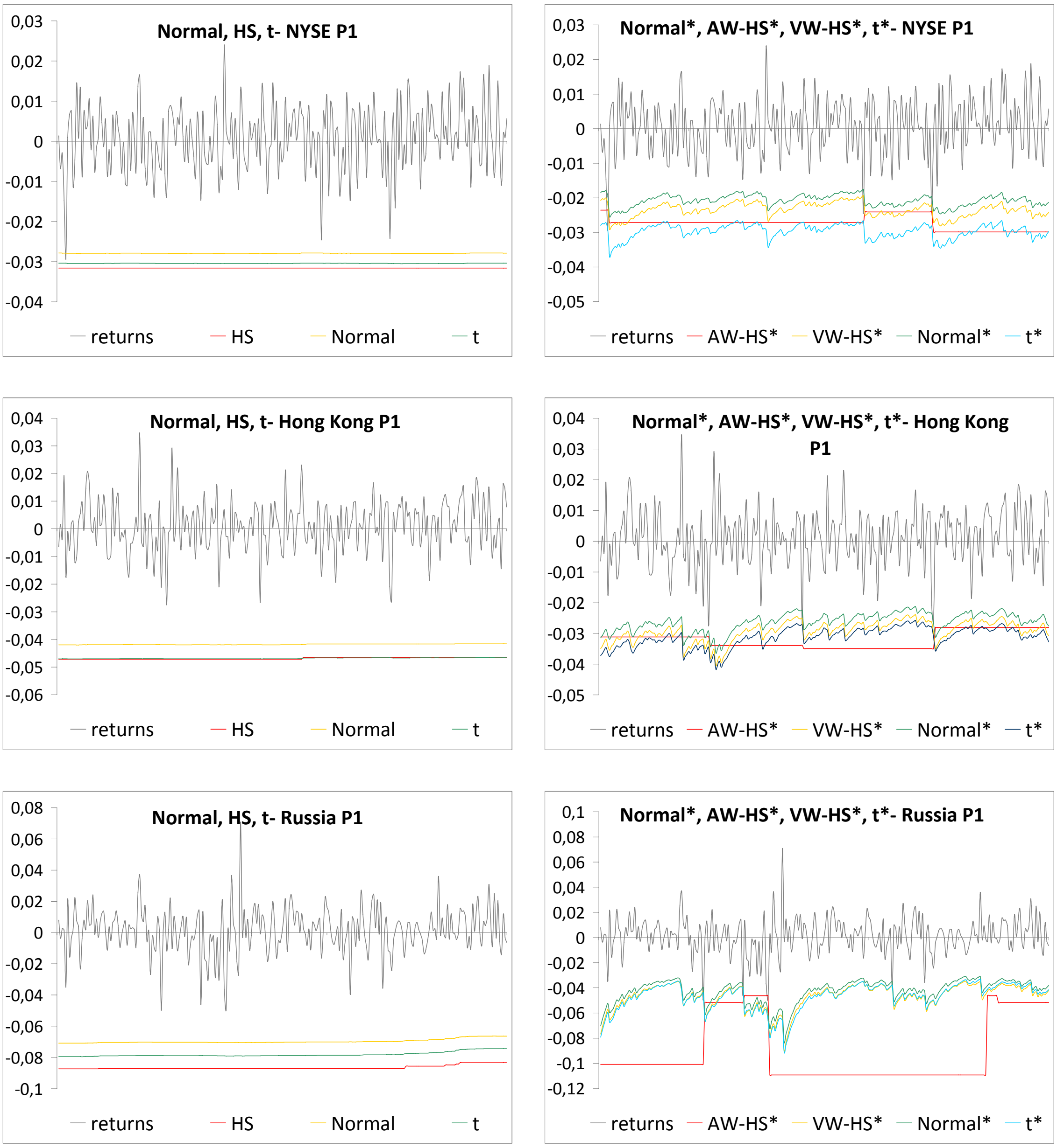


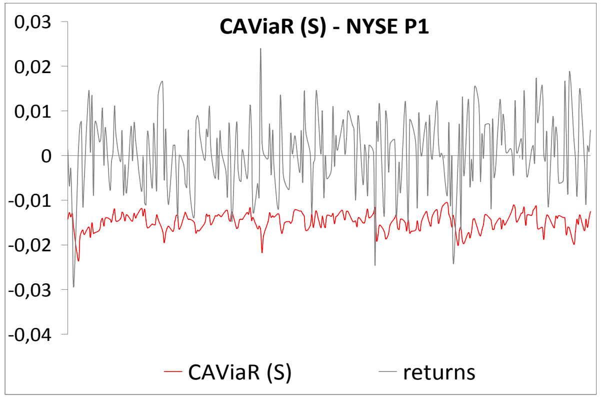
Table 13: Ljung-Box test for Random Walk

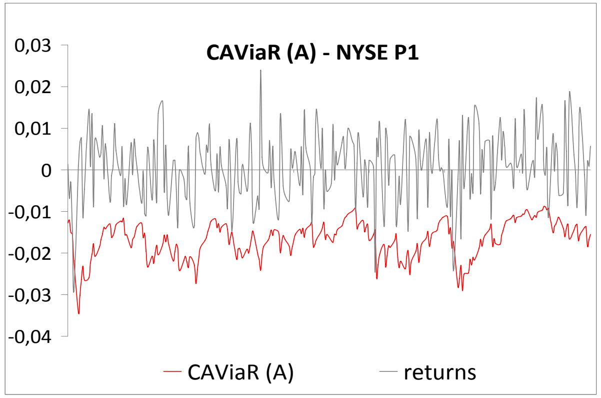
	Q(1)	p	Q(10)	p	Q(20)	p	Q(50)	p
NYSE	3.2889	0.070	11.353	0.331	23.440	0.268	60.185	0.153
FTSE W Hong Kong	3.4406	0.064	34.179	0.000	46.465	0.001	94.613	0.000
Russia RTS	32.038	0.000	56.360	0.000	78.348	0.000	145.92	0.000

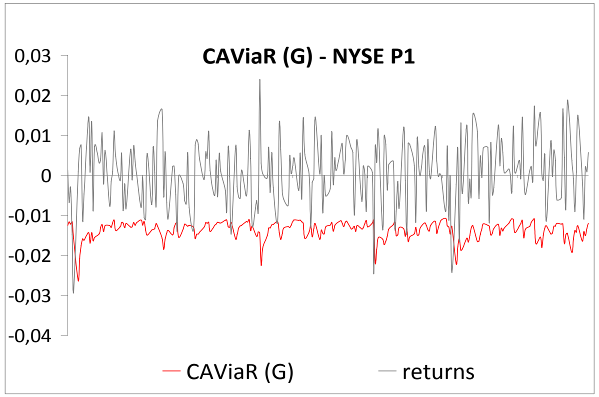
Figure 7: Out-of sample VaR forecasts Period 1

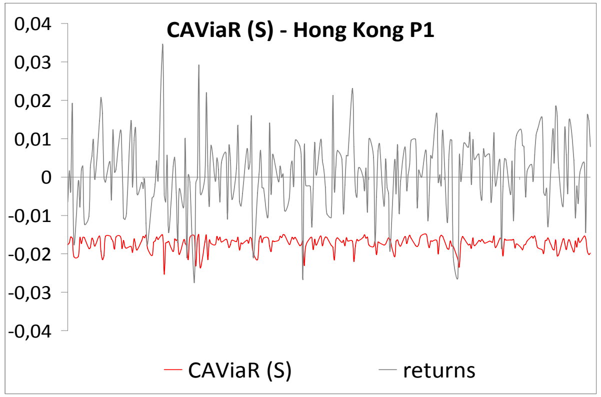
P=0,01

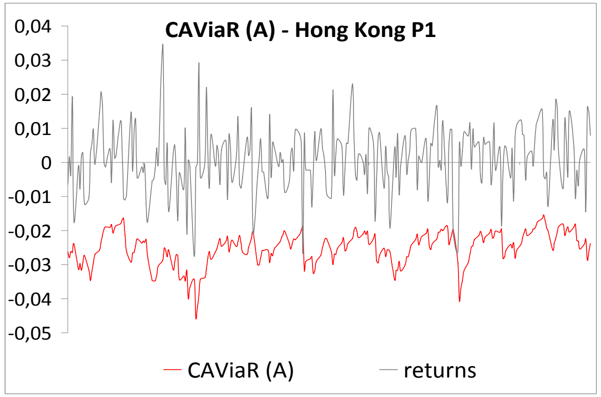


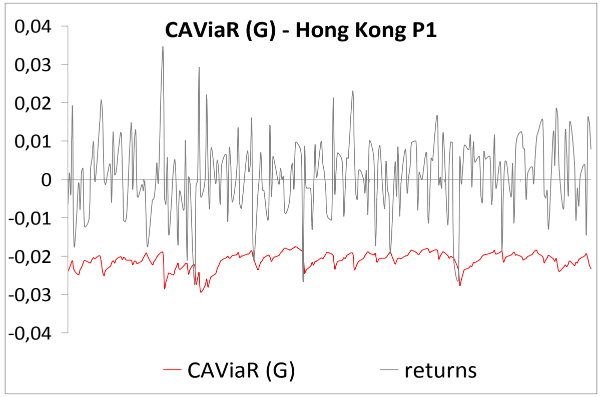

CAViaR (S) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (S)
returns

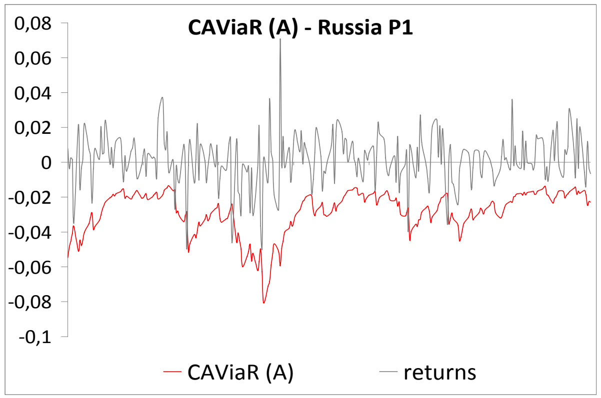

CAViaR (A) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (A)
returns

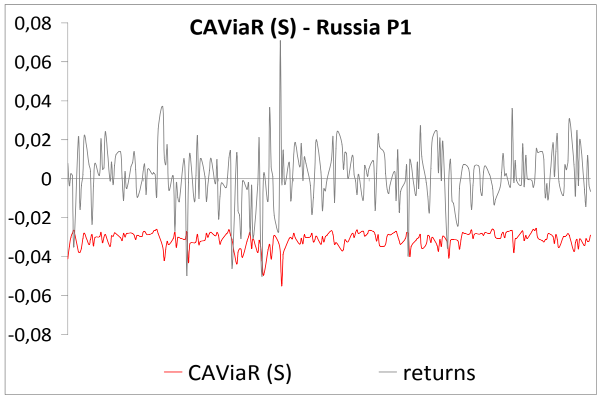

CAViaR (G) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (G)
returns

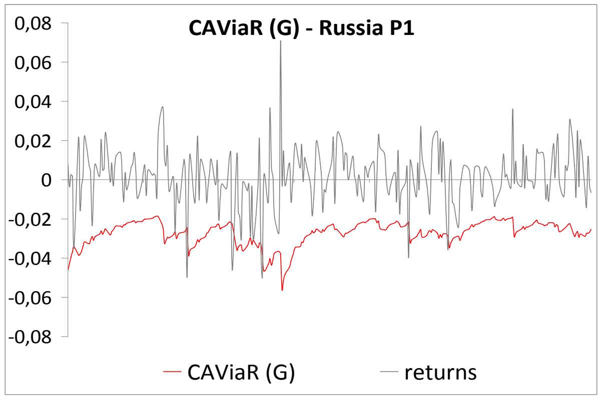

CAViaR (S) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (S)
returns


CAViaR (A) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
-0,05
CAViaR (A)
returns

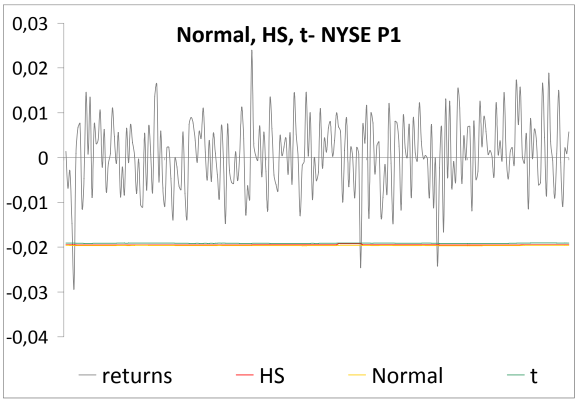

CAViaR (G) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (G)
returns

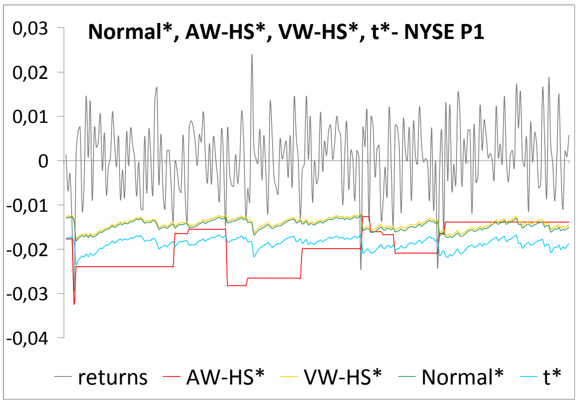

CAViaR (A) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
-0,1
CAViaR (A)
returns

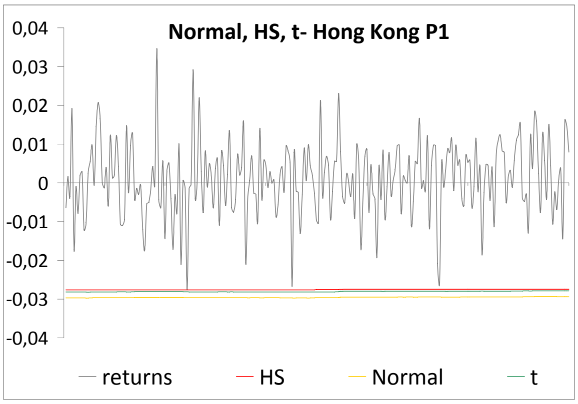

CAViaR (S) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
CAViaR (S)
returns

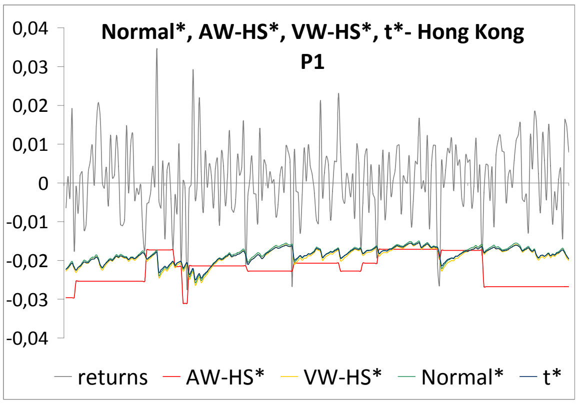

CAViaR (G) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
CAViaR (G)
returns

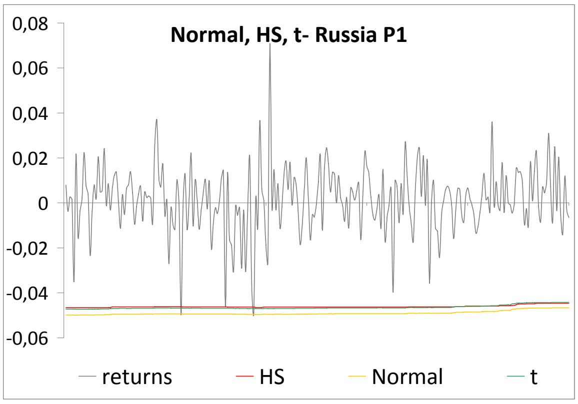
p=0,05

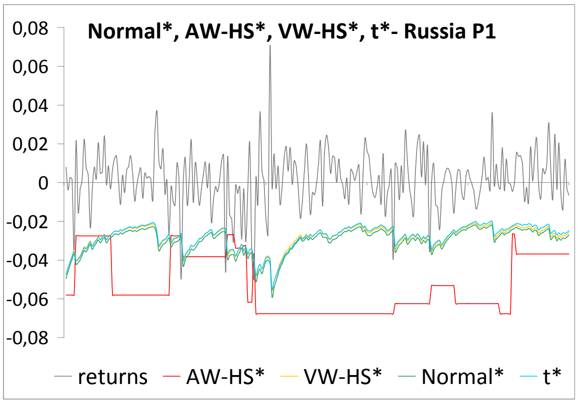

Normal, HS, t- NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
returns
HS
Normal
t


Normal*, AW-HS*, VW-HS*, t*- NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
returns
AW-HS*
VW-HS*
Normal*
t*


Normal, HS, t- Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
returns
HS
Normal
t


Normal*, AW-HS*, VW-HS*, t*- Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
returns
AW-HS*
VW-HS*
Normal*
t*


Normal, HS, t- Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
returns
HS
Normal
t


Normal*, AW-HS*, VW-HS*, t*- Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
returns
AW-HS*
VW-HS*
Normal*
t*

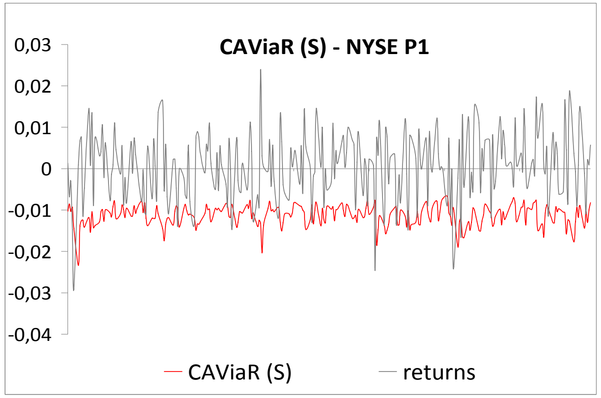
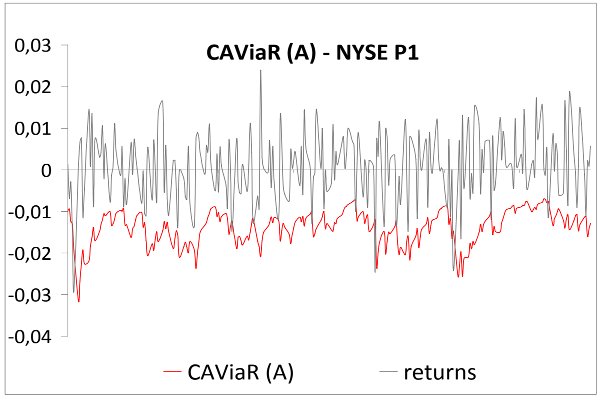
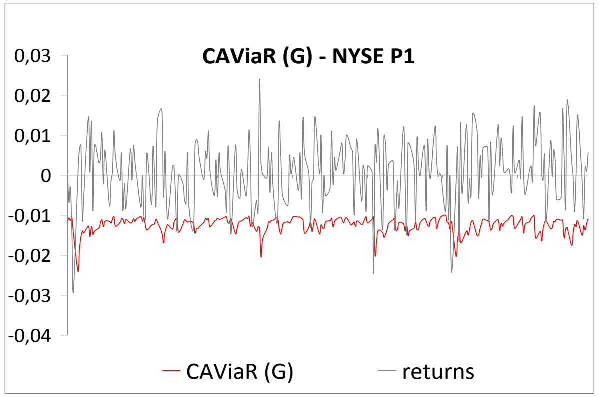
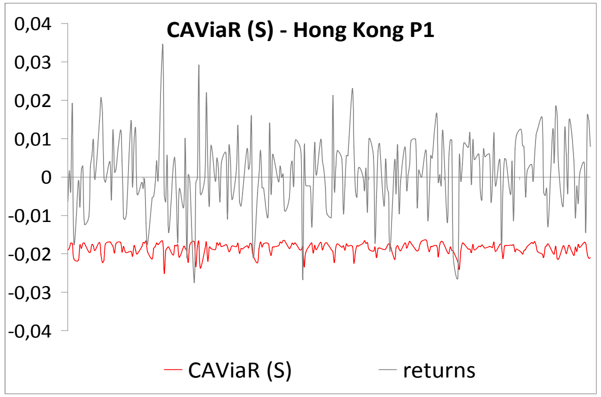
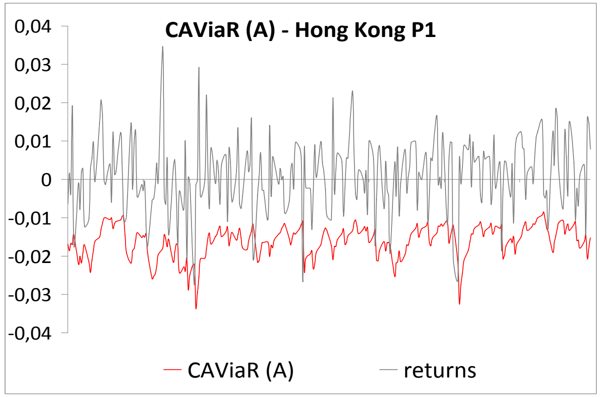
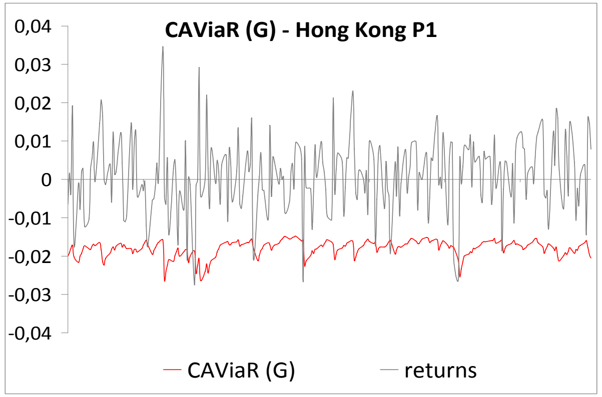
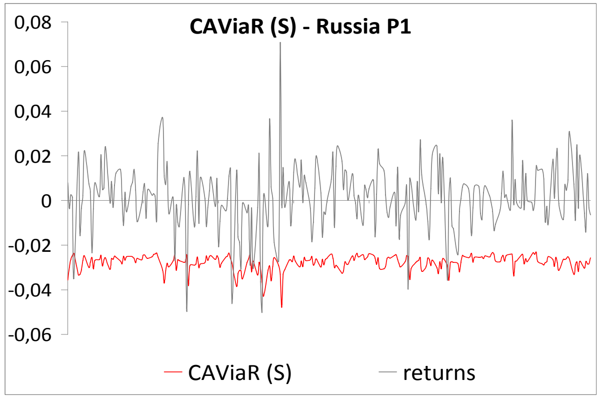
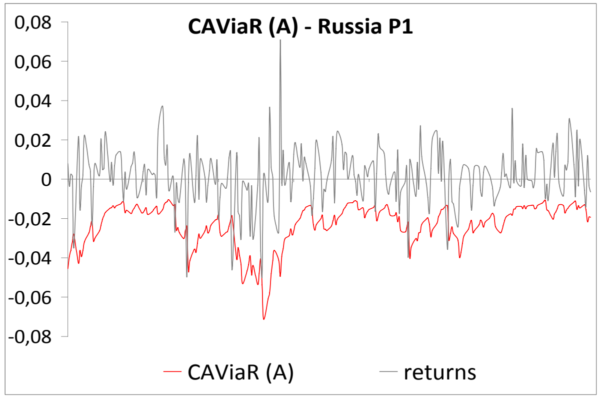
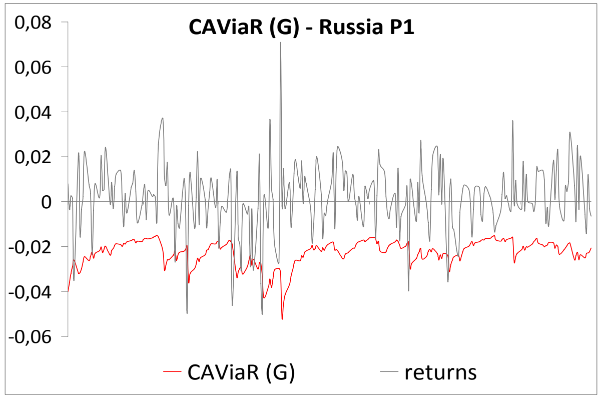
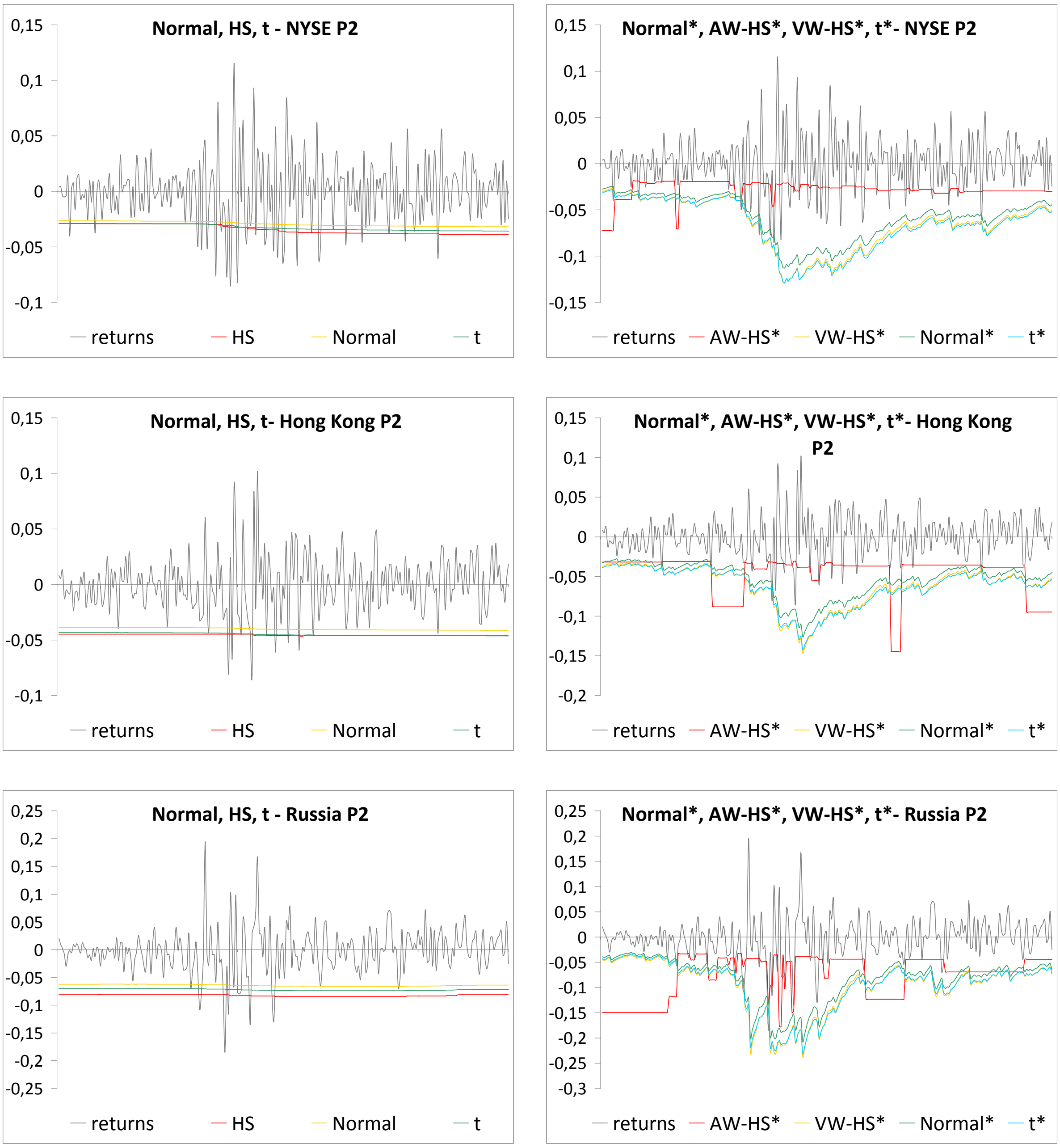
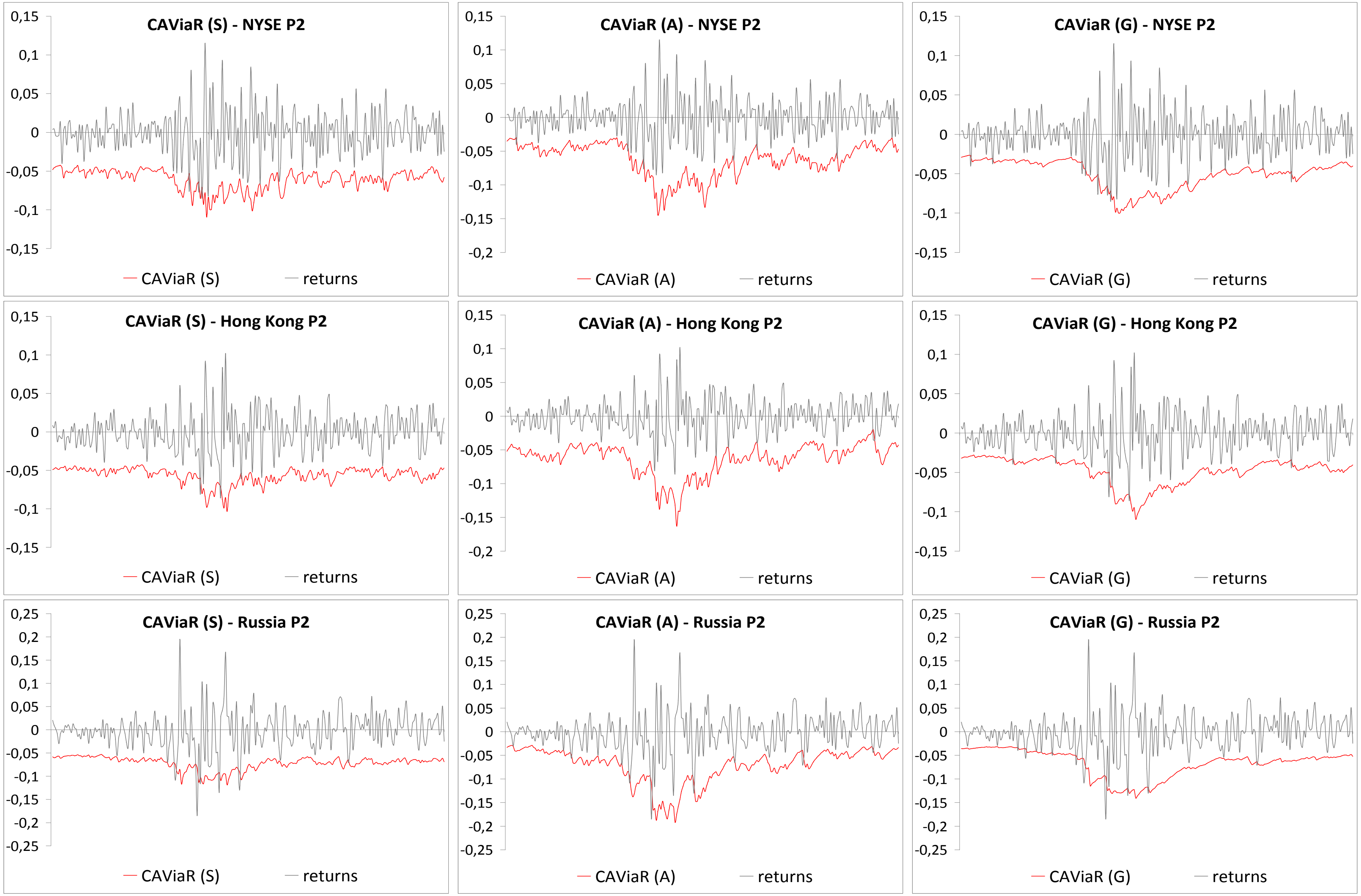

CAViaR (S) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (S)
returns

CAViaR (A) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (A)
returns

CAViaR (G) - NYSE P1
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (G)
returns

CAViaR (S) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (S)
returns

CAViaR (A) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (A)
returns

CAViaR (G) - Hong Kong P1
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
CAViaR (G)
returns

CAViaR (S) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
CAViaR (S)
returns

CAViaR (A) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
CAViaR (A)
returns

CAViaR (G) - Russia P1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
CAViaR (G)
returns

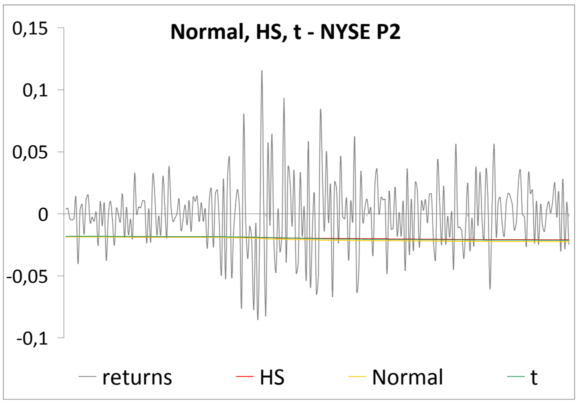
Figure 8: Out-of sample VaR forecasts Period 2

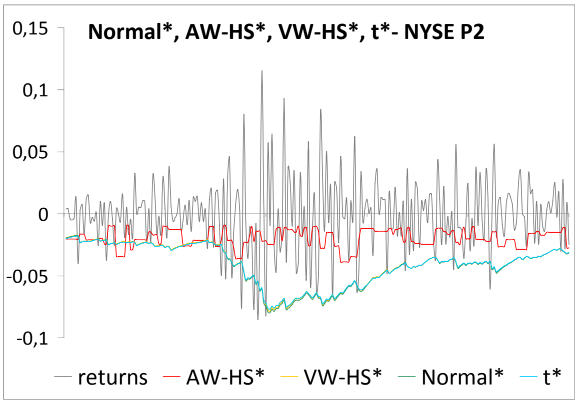
p=0,01

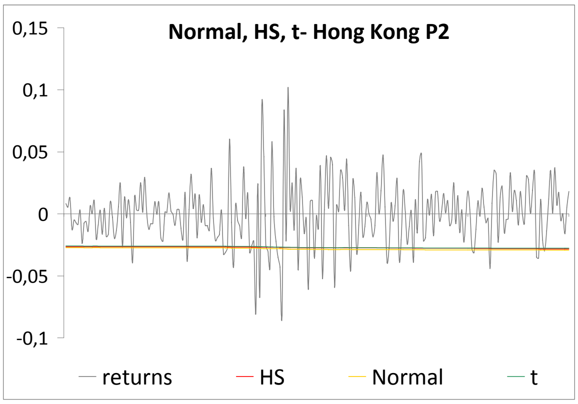


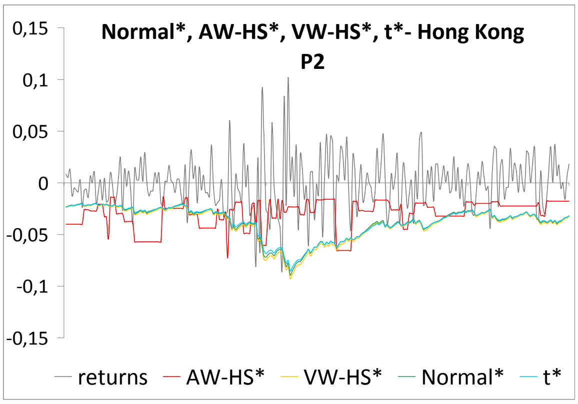

CAViaR (S) - NYSE P2
CAViaR (A) - NYSE P2
CAViaR (G) - NYSE P2
CAViaR (S) - Hong Kong P2
CAViaR (A) - Hong Kong P2
CAViaR (G) - Hong Kong P2
CAViaR (S) - Russia P2
CAViaR (A) - Russia P2
CAViaR (G) - Russia P2
CAViaR (S)
CAViaR (A)
CAViaR (G)
returns
0,25
0,2
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
-0,2
-0,25

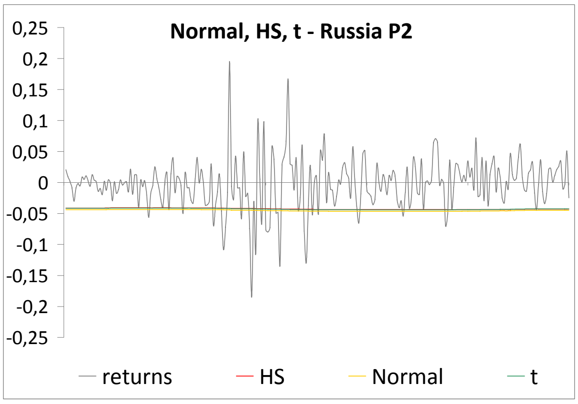
p=0,05

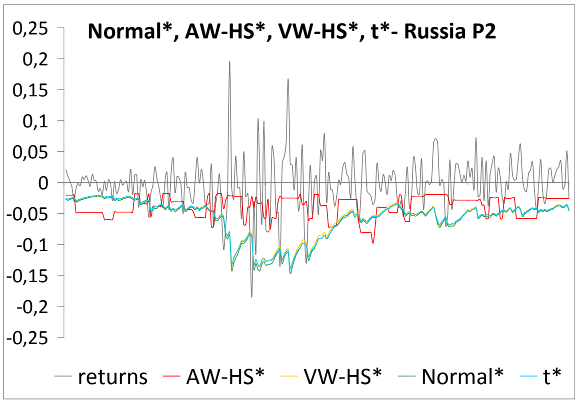

Normal, HS, t - NYSE P2
0,15
0,1
0,05
0
-0,05
-0,1
returns
HS
Normal
t

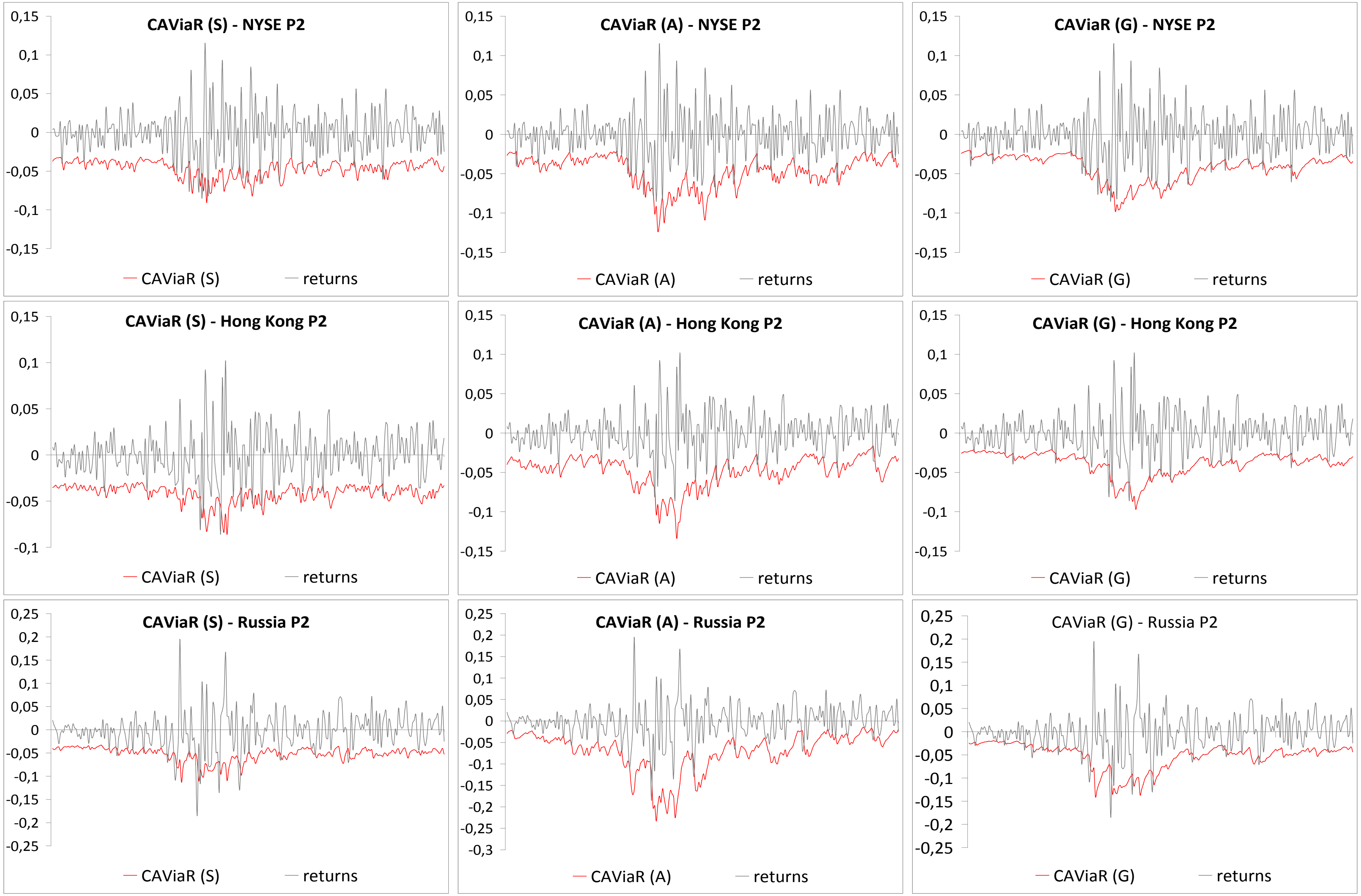

Normal*, AW-HS*, VW-HS*, t*- NYSE P2
0,15
0,1
0,05
0
-0,05
-0,1
returns
AW-HS*
VW-HS*
Normal*
t*


Normal, HS, t- Hong Kong P2
0,15
0,1
0,05
0
-0,05
-0,1
returns
HS
Normal
t


Normal*, AW-HS*, VW-HS*, t*- Hong Kong P2
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
returns
AW-HS*
VW-HS*
Normal*
t*


Normal, HS, t - Russia P2
0,25
0,2
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
-0,2
-0,25
returns
HS
Normal
t


Normal*, AW-HS*, VW-HS*, t*- Russia P2
0,25
0,2
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
-0,2
-0,25
returns
AW-HS*
VW-HS*
Normal*
t*


CAViaR (S) - NYSE P2
CAViaR (A) - NYSE P2
CAViaR (G) - NYSE P2
CAViaR (S) - Hong Kong P2
CAViaR (A) - Hong Kong P2
CAViaR (G) - Hong Kong P2
CAViaR (S) - Russia P2
CAViaR (A) - Russia P2
CAViaR (G) - Russia P2
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
0,25
0,2
-0,2
-0,25
-0,3
CAViaR (S)
CAViaR (A)
CAViaR (G)
returns