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MASTER THESIS

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Comparing Return-Risk and Direct Utility Maximization Portfolio Optimization Methods by 'Certainty Equivalence Curves'

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Abstract

Mean-Risk portfolio optimization method proposes an efficient frontier that consists of portfolios not dominated by any portfolio. Consequently, this method reduces the choice set by excluding inefficient portfolios. Different risk measures offer different efficient frontiers, which can be interpreted as different optimal choice sets. The question is whether these different risk measures lead to significantly different efficient frontiers for the investors, and which risk measure should be used.

My purpose is to present a method to assess the effect of the choice set reduction from different Return-Risk models and to answer the question presented earlier. The most important contribution of the paper is the creation of a two-dimensional space "*Risk-Aversion – Certainty Equivalence* (**CE**)" as a platform for comparisons. The curves, representing different risk-averse investors and different models, on this space are called "*Certainty Equivalence Curves* (**CEC**)". The empirical analysis shows that the Mean-Variance method is very effective in ranking portfolios for exponential utility investors. Therefore, it is not recommended to use more complicated methods such as Mean-CVaR.

Key words: Portfolio Optimization Return Risk Direct Utility Maximization Certainty Equivalence CE CEC

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I. INTRODUCTION

1. Background

Return-Risk portfolio optimization methods have been used and discussed extensively by financial practitioners and scholars. One clear advantage of these methods is a reduction in the choice set that facilitates the portfolio selection and evaluation processes. Instead of choosing among all possible portfolios, investors just choose portfolios on the efficient frontier. This implies that the reduction of the choice set loses some portfolios that are optimal from the perspective of investors who follow exponential utility.

There is abundance of risk measures available to assess portfolio allocation. Some of them concern the whole return distribution, whereas others focus on only a half or a specific range of the return distribution. Different moments are also used, such as 1^{st} moment for Mean Absolute Deviation and 2^{nd} moment for Variance. It may therefore be confusing and difficult for practitioners and researchers in evaluating assets portfolios such as hedge funds. For example, if the return is normally distributed or investors have mean-variance preference, Mean-Variance portfolio optimization method is perfectly applicable. However, if the investors are supposed to follow exponential utility and the distribution is not normally distributed, then the Mean-Variance method is obviously not exactly correct.

Obviously we must analyze the appropriateness of these risk measure toward investors, since investors are the ones who value the portfolios. If the investors really care about downward movement but are minimally concerned with upward movements, then one-sided risk measures may be employed. However, common sense shows that people care much about upside as well; although, it may be less important than downside. Therefore, both one-sided and two-sided risk measures are not perfectly appropriate for all investors in every situation.

2. Problem and Purpose

Each risk measure determines one efficient frontier, but these frontiers are obviously not identical. Therefore, it raises a problem concerning whether these risk measures lead to similar or significantly different efficient frontiers. Investors would also be concerned with which risk measure they should use to assess portfolios. This paper has the purpose of analyzing and assessing the efficient frontiers gained from Return-Risk methods. Throughout the analysis, for a particular investor, one portfolio is only considered better than another portfolio when it has higher utility for this investor. There are only two risk measures under investigation: standard deviation and Conditional Value at Risk (CVaR). A specific question follows from this purpose: is standard deviation or CVaR recommended for assessing portfolios?

There are several reasons for choosing standard deviation and Conditional Value at Risk as our risk measures. Firstly, both risk measures are very well-known, and covered in thousands of papers. Secondly, standard deviation is the most representative for two-sided risk measures; while CVaR is the best candidate in the one-sided risk-measure class. Although it is not as popular as Value at Risk but it has proven to be more effective in terms of coherence. Thirdly, standard deviation takes into account the whole return distribution, whereas CVaR only focuses on the worst possible situations. In other words, standard deviation measures dispersion of returns around the mean, and CVaR measures how bad the return could be if the worst cases happen. We may expect differences that make it interesting to compare the efficient frontiers. However, we can not say whether these differences are significant or not.

3. Methodology Introduction

Exhibit 1 illustrates that an investor with risk-aversion β^* chooses a portfolio on the M-V and M-CVaR efficient frontiers, which is the tangent point to the highest indifference curve. We can compare these two portfolios (1) and (2) by comparing certainty equivalences of these two portfolios with respect to the investor with risk-aversion β^*. In order to compare the whole range of risk-aversion β, which is equivalent to study effects of choice set reduction to utility maximization problem, we need to create a platform where these methods are comparable. This paper proposes a two-dimensional space *"Risk-Aversion – Certainty Equivalence* (**RA-CE**)*"* as a platform for comparison. For a specific investor with risk-aversion β^*, we can find the most preferred portfolio by direct portfolio optimization method; this portfolio will be represented by one point on the RA-CE space. Two portfolios on the two efficient frontiers chosen by this investor are represented by other two points on the RA-CE space. There are the following benefits of using this space: (1) we can see how investors perceive optimal portfolios, which are generated by different portfolio optimization methods, in the language of 'Certainty Equivalence'; (2) we can have an overall view on investors with different risk-aversion.

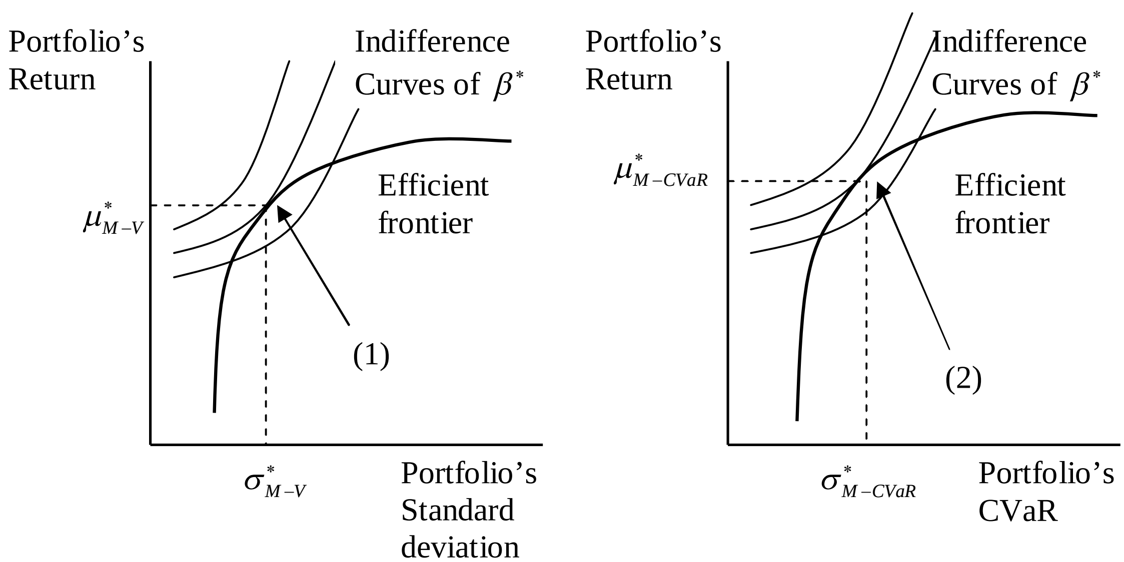


Exhibit 1: Efficient frontiers

The benchmark for the comparison is the expected utility, which is analogous to certainty equivalence. The curve representing the most preferred portfolios obtained from direct utility maximization method for different risk-aversion β is called "*Utility Certainty Equivalence Curve (***UCEC***)*". To deal with a Return-Risk portfolio optimization method, we go through 2 steps. The first step is finding a set of optimal portfolios from that

Return-Risk method, which is usually called "*Efficient Frontier*". Investors with different risk-aversion will choose different portfolios on the efficient frontier. Each of these portfolios is correspondent to certainty equivalence to the investor who chose the portfolio. The second step is sketching an UCEC on the "*risk-aversion – CE"* space.

By sketching these graphs, we visualize the efficiency of the funds and risk-measure methods relatively to direct utility-maximization method. Furthermore, the CE values allow numerical comparison among portfolios. The guidance to draw "*Utility Certainty Equivalence Curve (***UCEC***)*" and "*Efficient Certainty Equivalence Curve (***ECEC***)*" is described latter in the paper.

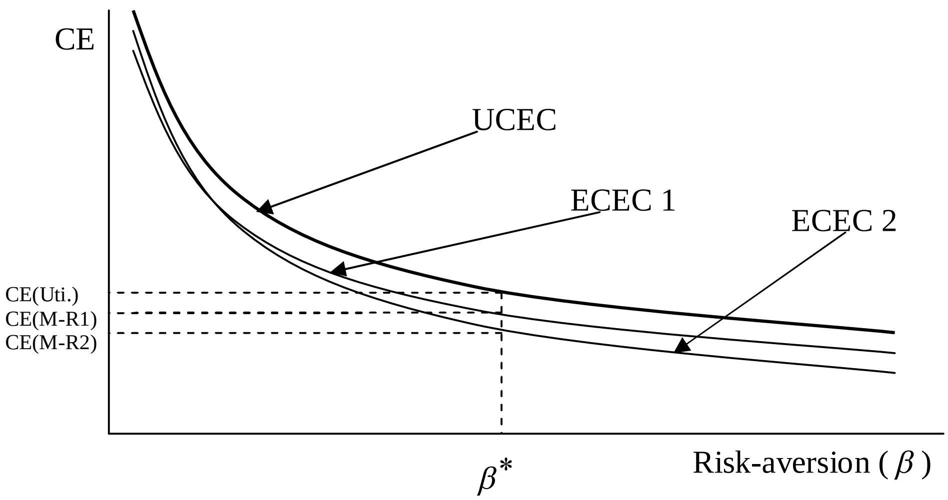


Exhibit 2: Certainty Equivalence Curves

Exhibit 2 shows UCEC and two ECEC respected to the two risk measures. When risk-aversion is low, investors tend to take more risk, which bring them higher certainty equivalence. In opposition, highly risk-averse investors prefer safer portfolios, which usually have lower returns. Therefore, certainty equivalences of highly risk-averse investors are lower than those of lowly risk-averse investors. UCEC should lie above the two ECEC since the initial choice set is a superset of the two efficient frontiers.

Under the constraint of time, the paper examines the most common portfolios, which are portfolios of stocks and portfolios of indices. Besides, the paper also studies compound portfolios of options and stocks.

II. LITERATURE REVIEW

In this part, we study *utility maximizing* method and *return-risk* methods. The scenarios generation techniques available for generating inputs for these methods are also introduced. Although scenarios generation techniques are not directly related to the methodology, inaccurate scenarios generation techniques may lead to inaccurate conclusions. Therefore, it is worthy to review this part of materials for further research purpose.

1. Utility maximizing method

Utility is intensively researched over hundreds of years as the fundamental measure of psychological preferences. This concept has been applied to many areas of economics; one of which is portfolio optimization problem. By maximizing the expected average utility of possible outcomes in a bundle of choices, the most favorite one would be selected. The biggest problem relating to this approach is the uncertainty in the utility function. In other words, a universal utility function, which can be applied for every one, does not exist. To deal with this problem, people use different utility functions such as *exponential* utility and *power* utility alternatively to observe different classes of investors. This paper uses exponential utility function as the benchmark. The second problem is that the utility of a person is considered in a whole, which means that a portfolio held by an investor should be assessed together with his remaining assets. However, this paper takes into consideration the separate portfolio with all other assets.

2. Return – Risk method

Under the assumption of risk-averse, Von Neumann-Morgenstern investors prefer certainty to uncertainty with the same expected return. Another research direction of portfolio optimization problem is based on the trade-off between expected return and risk, which was firstly proposed by Nobel Prize holder Markowitz. After over a half of a century, the optimization methods have been broadly expanded. However, these methods still keep the shape of the origin – return-risk compromising. There are many risk measures suggested as criteria for the optimization problem in this approach. Numerous papers such as Angelelli et al. (2008) and Adam et al (2008) focus on comparing these risk measures

The most popular four models which use moment-based risk measure are: *mean – variance* model (MV) (Markowitz 1952), *mean – lower semi-variance* model (MLSV) (Markowitz 1959), and *mean – absolute-deviation* model (MAD) (Konno and Yamazaki 1991). These models are consistent with *second stochastic dominance* (SDD) (Porter 1974; Konno and Yamazaki 1991), which are representing risk-averse investors. The first two models are two-sided, which present symmetric behavior towards profits and losses. Under Normal distribution, there is no difference between one-sided and two-sided methods. However Fama (1965) found that return distributions of financial instruments are more leptokurtic than normal distribution and "fat-tails". This statement was also tested by Lo and MacKinlay (1999). Many non-normal distributions were suggested for return distribution such as *Varian-Gamma* (VG) process introduced Madan and Seneta (1990). However, for either normally or non-normally distributed return, MAD is equivalent to *downside mean semi-deviation* model (Kenyon et al. 1999). Hence, MAD is excluded in the comparison. Fischer (2003) discussed the general form of one-sided moment-based risk measures.

Value-at-Risk model (VaR) (Morgan J.P., Inc. 1996) is a very popular risk measure, which can be found in thousands of documents. Despite its popularity, its shortcomings were pointed out by Artzner et al. (1997, 1999). *Worst Conditional Expectation* model

(WCE) (Mansini et al. 2003) and its similar models – *Conditional Concentration* model (Shalit and Yitzhaki 1994), *Expected Shortfall* (ES) (Embrechts et al. 1997) and *Conditional Value-at-Risk* (CVaR) (Rockafellar and Uryasev 2000) – have improved the downsides of VaR model. Dhaene et al. (2004) showed that ES dominated VaR. Chen and Wang (2006) presented a generalized model representing these models.

To assess risk measures, Artzner et al. (1999) proposed four criteria for a coherent risk measure. They are positive *homogeneity*, *translation-invariance*, *monotonicity* and *sub-additivity*. Standard deviation and VaR are not coherent measures since standard deviation violates *translation-invariance* and *monotonicity*, VaR fails *sub-additivity*. One-sided moment-based risk measures were proved to be coherent by Fischer (2003). *Expected shortfall* and *CVaR* are also coherent risk measures (Acerbi and Tasche 2002, Rockafellar and Uryasev 2002).

MAD and CVaR are implicitly compared with real features in Angelelli et al. (2008). However, the paper reveals a drawback when a minimum return of 0% is used to compare these two risk measures. With this assumption, the paper just focuses on investors who have the requirement of non-negative returns; it does not mean that these investors have the same risk-aversion represented by Arrow-Pratt measure (Arrow 1964, Pratt 1964). By using Certainty Equivalence Graph, we can overview the effect of choice set reduction from selecting portfolios on Efficient Frontier instead of the whole choice set.

3. Scenarios generation techniques

In order to obtain future scenarios, non-parametric scenario generation techniques or parametric scenario generation techniques could be utilized. In the category of Non-parametric techniques, *historical data* technique, *bootstrapping* technique and *block bootstrapping* technique are currently widely used. *Historical data* technique is the most simple scenarios generation technique, which is based on the assumption that historical data represents the future possibilities. The correlations between variable are implicitly considered in the data series. The trade-offs for its simplicity are the limitation in data and possible miss-representation of the pass data. *Bootstrapping* technique is another non-parametric scenarios generation technique suggested by Kouwenberg and Zenios (2006). This technique is the combination of historical data and bootstrapping technique discovered by Efron and Tibshirani (1993). Although this method can generate large samples of scenarios, it destroys the autocorrelation information of the series. It also may misrepresent the data by using historical data to interpret the future. To correct the drawback of breaking autocorrelation information, Buhlmann (2002) offered *block bootstrapping* technique. However, it does not eliminate the historical data problem.

Besides non-parametric techniques, parametric scenarios generation techniques play a very important role over a long history. On the one hand, Monte Carlo simulation techniques focus on simulating the distributions. Due to the complication of the advanced distribution functions, portfolio optimization area usually adopts Normal distribution. However, it does not capture the *skewness* and *fat-tails* effects (Mandelbrot 1993). The t-Student distribution, log-normal distribution and other distributions have taken this drawback into consideration. Furthermore, taking covariance between assets into account, scenarios generation from a multivariate Normal distribution with known mean and covariance matrix was proposed by Levy (2004). On the other hand, multivariate generalized ARCH process technique central attention to volatility cluster effect – heteroskedasticity of a time series (Bollerslev et al. 1992). CC-MGARCH is one valuable example. Guastaroba et al. (2009) referred to further details of these scenarios generation techniques.

This paper focuses on the analyzing portfolio optimization by directly maximizing expected utility method and mean-risk trade-off methods. Therefore, the most simple scenarios generation technique is utilized, which is historical data technique. However, other techniques can also be applied without difference in the later steps.

III. METHODOLOGY

Methodology part starts with the guidance to draw the Utility Certainty Equivalence Curves and Efficient Certainty Equivalence Curves in the Certainty Equivalence Graph.

UCEC: Optimal portfolios for different levels of risk-averse investors are found by *direct utility-maximization* method. In this paper, exponential utility function is used to represent different investors with different risk-aversion; changing in β represents changing in risk-aversion. For a specific value of β, we can find an optimal portfolio with maximal expected utility, which is equivalent to a specific value of CE. By changing β, we can sketch a graph of UCEC ($CE \sim \beta$).

ECECs: For one risk measure, we draw an ECEC through two steps. The first step is finding the subsets corresponding to different risk measures, which are very well-known under the concept of "Efficient Frontiers" with traditional way: minimizing risk measure corresponding to a specified return. Different risk returns result in different optimal portfolios, which are called "*Efficiency Frontier*". Among portfolios in the EFS, we choose the best portfolio corresponding to a risk-aversion (equivalent to β). This step has one advantage over previous works, which is the consideration of utility maximization after we have Efficient Frontier. This advantage facilitates investigation over different risk-averse levels. Different investors with different risk-aversion will choose different portfolios on the Efficient Frontier. These portfolios are easily being translated to Certainty Equivalence through the inverse function of the utility function.

To combine these curves for computational purpose, we go through three steps. Firstly, we find efficient frontiers for risk measures, which are subsets of initial choice set. Secondly, expected utility maximization method is applied for the initial choice set and subsets from different Return-Risk methods. The final step is to convert Certainty Equivalence from expected utility. These steps are the backbone of the Matlab code.

After that we get in detail statistical and mathematical theories and applications to the problem in this paper. The portfolio is assumed to contain N assets j for $j=[1,N]$. Asset j has T historical returns R_{jt} for $t=[1,T]$. With the assumption of historical data technique, we have T scenarios for the target future return. We also assume that the possibilities of scenarios are $p_t = 1/T$ for $j=[1,N]$. We denote w_j, $j=[1,N]$ as weights of N assets in the portfolio. To deal with the risk-free asset, we simply use $R_{t,r-f} = const$ for t=$[1,\mathrm{T}]$. We assume the initial total weight equivalent to one $\left(\sum_{j=1}^{N} w_j = 1\right)$, and there is no short-selling ($w_j \geq 0$). Then, we have return of portfolio in scenario t is:

$$R_t = \sum_{j=1}^{N} R_{jt} w_j \text{ for } t=[1,T];$$

Average return on asset j is:

$$R_j = E_t\left[R_{jt}\right] = \sum_{t=1}^{T} p_t R_{jt};$$

Return of portfolio with assets weights w in the scenario t is denoted as:

$$\mu_t(w) = \sum_{j=1}^{N} R_{jt} w_j \text{ for t=}[1,\mathrm{T}];$$

The portfolio's average return is:

$$\mu(w) = E\left[\mu_t(w)\right] = \sum_{t=1}^{T} p_t \mu_t(w)$$

1. Return – Risk Portfolio Optimization

The optimization method is simply the minimization of risk measures, or maximization of safe measures with a given return. The methods were discussed intensively in the references mentioned in section I. We directly present the optimization problems corresponding to different risk measures.

Mean – Variance Model (M-V)

This is the most well-known model in the financial industry, which is usually accompanied with normal distribution assumption. The reason is that the normal distribution has only two parameters: mean and variance, which allows the parametrical analysis with ease. Variance measures the dispersion of possible values around the mean. For a given expected return of the portfolio, the optimization problem is:

$$\min\left\{\bar{V}(w)\right\} = \min\left\{\sum_{t=1}^{T} p_t V_t\right\}$$

$s.t.\ \mathrm{V_t} = \left(\mu(w) - \mu_t(w)\right)^2$; $\mu(w) = \sum_{t=1}^{T} p_t \mu_t(w)$; $\mu_t(w) = \sum_{j=1}^{N} R_{jt} w_j$ for $\mathrm{t}=[1,\mathrm{T}]$;

$\sum_{j=1}^{N} w_j = 1$; And $w_j \geq 0$ for $j = [1, N]$

Mean – Conditional Value at Risk Model (M-CVaR)

This paper investigates the tail of 5%, which is most commonly used. To estimate CVaR(95%), we need to measure VaR at a 95% confidence level. For each weight w of N assets in the portfolio, we can find VaR(95%) of the portfolio as $\eta(w)$. Then CVaR is Linear Programming computable as following

$$CVaR_{95\%}(w) = \max\left\{\eta(w) - \frac{1}{5\%}\sum_{t=1}^{T} p_t d_t\right\}$$

$s.t.\ \mathrm{d_t} = \max\left\{0, \eta(w) - \mu_t\right\}$ for $\mathrm{t}=[1,\mathrm{T}]$

By choosing different $\mu(w)_k$ for $k = \overline{1, K}$, we have K optimal portfolios lying on the efficient frontier. K is taken large enough for latter utility maximization purpose.

2. Expected Utility Maximization

With the assumption of no friction such as transactional costs, final wealth is $W_t = e^{R_t}$ under the assumption of unit initial wealth. Expected utility of the final wealth is:

$$\bar{U}(w) = E\left[u(W_t)\right] = E\left[u(1+R_t)\right] = \sum_{t=1}^{T} p_t u(1+R_t)$$

This formula is applied to different utility functions, and is the target for optimization. Arrow-Pratt measure (Arrow 1964, Pratt 1964) is used to assess the degree of risk-aversion of investors.

This paper examines exponential utility (EU): $u(x) = -e^{-x\beta}$ with $\beta > 0$, u(x) presents non-satiation and risk-averse since the utility function is increasing and concave. In other words, the first derivative is positive and second derivative is negative.

$$u'(x) = \beta e^{-x\beta} > 0 \text{ and } u''(x) = -\beta^2 e^{-x\beta} < 0 \text{ for } \beta > 0$$

We can easily derive Arrow-Pratt's measure of absolute risk-aversion $(ARA(x))$:

$$ARA(x) = -\frac{u''(x)}{u'(x)} = -\frac{-\beta^2 e^{-x\beta}}{\beta e^{-x\beta}} = \beta$$

Investors are more risk-averse when β increase. In other words, we can expect the portfolio chosen by higher β investors having lower expected return. For each level of risk-aversion β, the maximization problem of expected average utility is:

$$\max_w \left\{ \sum_{t=1}^{T} p_t \left(-e^{-\beta e^{R_t}} \right) \right\}$$

s.t. $R_t = \sum_{j=1}^{N} R_{jt} w_j$ for $t = [1, T]$; $\sum_{j=1}^{N} w_j = 1$; and $w_j \geq 0$ for $j = [1, N]$

We apply the framework of expected utility maximization above into three sets of choices: initial choice set, the Efficient Frontier from Mean-Variance method (a subset of the initial choice set), and the Efficient Frontier from Mean-Variance method (another subset of the initial choice set). For direct utility maximization, an optimal weight w, which is corresponding to each β, is $w_U(\beta)$. For the subset of Efficient Frontier from Mean-Variance and Mean-CVaR methods, the weights for each β are $w_{M-V}(\beta)$ and $w_{M-CVaR}(\beta)$ respectively.

3. Converting to Risk-Aversion – CE Space

Certainty Equivalence is a risk-free amount which brings the same utility to an investor as a risky asset. The formula for Certainty Equivalence is:

$$\sum_{t=1}^{T} p_t \left(-e^{-\beta e^{R_t}}\right) = -e^{-\beta e^{CE}}$$

It is equivalent to:

$$CE = \ln\left[\frac{-\ln\left(-\sum_{t=1}^{T} p_t \left(-e^{-\beta e^{R_t}}\right)\right)}{\beta}\right]$$

For each β, we have corresponding $w_U(\beta)$, $w_{M-V}(\beta)$ and $w_{M-CVaR}(\beta)$. Therefore, we have corresponding $CE_U(\beta)$, $CE_{M-V}(\beta)$ and $CE_{M-CVaR}(\beta)$. Although this step is not crucial for the analysis, it clearly shows how much investors perceive investments in terms of return rates.

IV. DATA DESCRIPTION AND OUTPUT PRESENTATION

1. Data Description

This paper focuses on three types of data: single companies, indexes, and European options. For individual companies, we select 3 portfolios including different number of American companies. The companies are picked randomly over industries. To satisfy sufficient number of data points, we choose the companies listed in 1973. We have 435 monthly returns from Feb1973 to Apr2009, which are used to calculate possible next month return scenarios. We also take into account negative correlation, which would reduce variance of the portfolio. However, there are only a few stocks that have negative correlation with the majority, and these stocks usually have negative average returns. We take three portfolios of 6 stocks, 29 stocks and 61 stocks randomly from a list of 232 stocks with these issues taken into account.

To assess portfolios of indexes, we choose portfolios of industry indexes. One advantage of industry indexes is that industries have different natural characteristics deciding different risks and returns. Another advantage is that these indexes are mutually exclusive, which eliminates the duplication of some stocks in the indexes. Monthly returns of 10 and 30 industries are taken from the website of Kenneth R. French[*], which last from Jul 1926 to Dec 2008 with a total of 990 data points for each industry.

Due to the difficulty in collecting market data of options, we use theoretical Black-Scholes European calls and puts to estimate theoretical options prices and their returns. We assume to use current price as the strike price; the option expire in 1 year $(\tau = 1)$; volatility is simply standard deviation of past data; and risk-free is 2% $(r_f = 0.02)$. The current prices of call and put options on asset j $(j = [1, N])$ by Black-Scholes are:

$$c_{j0} = S_{j0} N(d_{1j}) - K_j e^{-r_f \tau} N(d_{2j}); \text{ and } p_{j0} = -S_{j0} N(-d_{1j}) + K_j e^{-r_f \tau} N(-d_{2j})$$

s.t.

$$K_j = S_{j0}; \sigma_j = stdev(S_{jt});\ d_{1j} = \frac{\ln(K_j / S_{0t}) + (r_f + \sigma_j^2/2)\tau}{\sigma_j \sqrt{\tau}}; d_{2j} = d_{1j} - \sigma_j \sqrt{\tau}$$

We can apply any scenarios generation techniques mentioned above to generate possible scenarios of stock returns in one month. We utilize the simplest method, which is basic historical method. We have T possible monthly returns R_t for $t = [1, T]$, which is equivalent to T possible stock prices $S_{jt} = S_{j0} e^{R_{jt}}$ for $t = [1, T]$.

Value of these options at time 1/12 (after 1 month) at scenario t $(t = [1, T])$ is:

$$c_{jt} = S_{jt} N(d_{1j}) - K_j e^{-r_f \tau^*} N(d_{2j}); \text{ and } p_{jt} = -S_{jt} N(-d_{1j}) + K_j e^{-r_f \tau^*} N(-d_{2j})$$

s.t. $\tau^* = 11/12; d_{1j} = \frac{\ln(K_j / S_{jt}) + (r_f + \sigma_j^2/2)\tau^*}{\sigma_j \sqrt{\tau^*}}; d_{2j} = d_{1j} - \sigma_j \sqrt{\tau^*}$

[*] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Monthly returns of these options at the state t are:

$$R_{c,jt} = \ln\left({c_{jt}}/{c_{j0}} \right) \text{ and } R_{p,jt} = \ln\left({p_{jt}}/{p_{j0}} \right)$$

The main issue of options returns is that time value of the options will decrease over time. For that reason, we should expect most of options to have negative expected returns. We checked both calls and put options returns over 232 stocks; the result is that all options returns are negative. Besides options have very high volatilities, which would distract investors from these securities. The intuition of investor's decision making is that they would accept high risk with adequate compensation of high return. Therefore, it is not reasonable to construct only portfolios of options. We examine portfolios of stocks and options on these assets, which investors may benefit from the correlations of these securities. Since put options have negative correlation with the underlying stocks, we are interested in portfolios of stocks and their corresponding put options. Two portfolios of 6 pairs and 30 pairs of stocks and their put options are randomly chosen and inspected. We also consider portfolios with and without risk-free asset at the same time.

2. Output Presentation

This paper visualizes the differences of direct utility maximization, Mean-Variance and Mean-CVaR methods through four graphs for each portfolio with and without the risk-free asset. The first one is Efficient Frontiers Graph (denoted (-a) in each appendix), the second one is Certainty Equivalence Graph (denoted (-b) in each appendix), the third one is Portfolio Returns Graphs (denoted (-c) in each appendix), and the last one is Weight Differences Graph (denoted (-d) in each appendix). These graphs are repeated over for three portfolios of stocks, two portfolios of indexes, and two portfolios of stocks and puts.

The Efficient Frontiers Graphs bring a visual view over the subsets after applying Return-Risk optimizations methods on risk-return space. For each minimum return requirement, there is one corresponding minimal risk which could be standard deviation or CVaR. The

Certainty Equivalence Graphs take into account utility maximization issue. With the assumption of exponential utility, we can solve the problem of utility maximization over a choice set. For direct utility maximization, the choice set covers all possible weight allocations under non-short-selling constrain. The choice sets for return-risk methods are efficient frontiers. Since efficient frontiers are subsets of the initial choice set, the maximized utility from direct utility maximization should be higher than utilities from return-risk methods for any level of risk-aversion. Consequently, Certainty Equivalence of direct utility maximization should be higher than utilities from return-risk methods for any level of risk-aversion. The third and the forth graphs get in detail of portfolio weight allocations. The thirds graph compares returns of optimal portfolios constructed by direct utility maximization method and return-risk methods for each level of risk-aversion. The forth graph visualizes the differences in weights of individual items inside the portfolio.

For the purposes of reference, we put all the graphs in the appendices. The first three appendices are for stock portfolios: appendix 1 shows graphs for portfolio of six stocks, appendix 2 is for portfolio of 29 stocks, appendix 3 is for portfolio of 61 stocks. The next two appendices are for index portfolios: appendix 4 presents 10 industries portfolio, and appendix 5 displays 30 industries portfolio. The last two appendices put on view portfolios of stocks and their put options: appendix 6 exhibits portfolios of 6 stocks and 6 theoretical B-S put options, and appendix 7 extends to 30 stocks and 30 put options.

V. DATA ANALYSIS

The analysis observes three types of portfolios: portfolios of stocks, portfolios of indices, and portfolios of stocks and corresponding put options. Although there are only three stock portfolios, two index portfolios and two combination portfolios considered; it is possible to draw conclusions to some extent. Within each type of portfolio, we study their Efficient Frontiers Graphs, Certainty Equivalence Graphs, Portfolio Returns Graphs and Weight Differences Graphs. The analysis places special attention on: effects of risk-free asset in the portfolios, differences between two risk measures, consequences of choice set reduction to CE, and differences in portfolios returns and weight allocations.

1. Stock Portfolios

Efficient frontier graphs

Although the Efficient Frontier Graphs do not show comparability between two risk measures, these graphs clearly illustrate the difference of the portfolios when a risk-free asset is included. Graphs 1-a, 2-a, and 3-a show that: for each return, optimal portfolio risks are significantly reduced when the portfolio contains risk-free assets. For example, in graph 1-a, standard deviation of 0.8% return is about 0.065 for portfolio without risk-free asset; while it is only 0.045 when risk-free asset is included. It is similar to the M-CVaR frontier. Expected tail losses for a portfolio with expected return 0.8% are 0.145 and 0.1 for portfolios without and with risk-free asset, respectively. It is also noticed that the effect of risk-free asset is stronger for portfolios with smaller expected returns. It is consistent with the intuition of highly risk-averse investors allocating more wealth to the risk-free asset.

The Efficient Frontiers of the M-V method with a risk-free asset reminds us about Tobin separation theorem. The theory states that all investors with different risk-aversion can be satisfied with the combination of a risk-free asset and one portfolio called the "market portfolio". The straight lines in M-CVaR Efficient Frontier Graphs with a risk-free asset show a similarity to the Tobin theorem in the M-V method. However, M-V method dominates M-CVaR in this aspect since the line is totally formulated in Return-Standard deviation space. The similar line on the M-CVaR space can only be drawn by numerical method.

Certainty equivalence graphs

The second graph of Certainty Equivalence versus risk-aversion is the most important issue of this paper. Certainty Equivalence Graphs 1-b, 2-b and 3-b visualize the maximized Certainty Equivalence gained by investors with different risk-aversion. The most significant point is that three methods bring very close Certainty Equivalence

Curves despite the differences in three methods. Although UCEC lies above the two ECEC, it is not significant. This finding encourages the statement that both Variance and CVaR are efficient to apply for stock portfolios under the judgment of exponential-utility investors. The choice set reductions due to the efficient frontiers minimally affect the initial choice set.

The second point in these three Certainty Equivalence Graphs is that the Certainty Equivalence of the portfolios with risk-free asset is above the one without the risk-free assets; this point is not very clear when they are drawn on separated graphs. It is clearly true since the investors will be better off when they have more choices, which extends the choice set. In addition, the difference becomes obvious with high risk-aversion. The graphs strengthen the statement that more risk-averse investors are better off when including a risk free asset. The third point worth noticing is that the equally weighted portfolios bring much worse Certainty Equivalence Curve than UCEC and ECEC. Therefore, we can conclude that equal weighting is not a good portfolio management strategy.

Returns Graphs and Weight Differences Graphs

The third and forth graphs examine further differences in returns and weight allocations of portfolios (graphs 1-c, 1-d, 2-c, 2-d, 3-c, and 3-d). In the three portfolios, the returns of these portfolios using all three methods are very close, even though there are some small discrepancies between graphs 1-b, 2-b, and 3-b. In all the cases, portfolios with the risk-free asset have smaller differences than the portfolios without the risk-free asset, especially in graph 1-c. One noticeable point, which is consistent to the theory, is that higher risk-averse investors choose lower expected returns. In addition, it seems that returns from M-CVaR are less smooth than the return curves from the other two methods when the portfolios do not include the risk-free asset.

The Weight Differences Graphs 1-d, 2-d, and 3-d clearly show the differences in weight allocations. It is obvious that the weight allocations obtained by these three methods are different. There are some remarkable points obtained from these graphs. Firstly, the usual differences are less than 10%. Sometimes the weight differences exceed 10%, for example in the case of portfolio of 6 stocks without the risk free asset (top left of graph 1-d). It could be interpreted as some significant distinctions among these three methods. Secondly, the differences in weights tend to be smaller as risk-aversion increases for the portfolios with the risk-free asset. However, it is not correct for portfolios without the risk-free asset. For portfolios without the risk-free asset, the differences between M-CVaR method and the other two methods seem to be not affected by risk-aversion of investors. The differences in weight allocations between M-V method and direct utility method seem to be increasing for risk-aversion, which is opposite to the case of portfolios with the risk-free asset.

In general, both risk measures are efficient to stock portfolios from the view of exponential-utility investors. Although the weight allocations among securities are different, the maximized utilities obtained from those methods are very close. Due to the simplicity of M-V method, we confidently recommend this method to assess the efficiency of a portfolio.

2. Index Portfolios

Due to the normality character of indices, we expect M-V method to be even more suitable to the index portfolios than stock portfolios. There is nothing special in the Efficient Frontier Graphs. The most noticeable thing in the Certainty Equivalence Graphs (graphs 4-b and 5-b) is that the almost identical Certainty Equivalence Curves, except two strange points at lowest betas in graph 4-b. At these risk-averse levels, the efficient frontiers destroy some values from the choice set reduction. However, it also could be caused by a mistake from Matlab; it is not necessary to get in detail. It is also noticed that the equal weighting strategy is not terribly worse than the optimization methods. To some extent, this strategy is acceptable in the case of index portfolios. Graphs 4-c and 5-c show that the weight allocation differences are more serious in the case of index portfolios than stock portfolios. Within differences exceeding 10%, there are some of them exceeding

20%. Generally, the conclusion for index portfolios is similar to the conclusion for stock portfolios. M-V is recommended to use with assurance.

3. Stock and Put Options Portfolios

In the case of put options included in the stock portfolios, people may expect there to be a significant difference between the M-V method and the M-CVaR method. However, the result from the examination is opposite. The differences are as small as in the stock portfolios case. Graphs 6-b and 7-b show that the Certainty Equivalence Curves from three methods are nearly identical. The weight allocation differences are similar to the case of stock portfolios. Therefore, we can conclude that the M-V can even be applied to portfolios containing non-normally distributed securities, like put options.

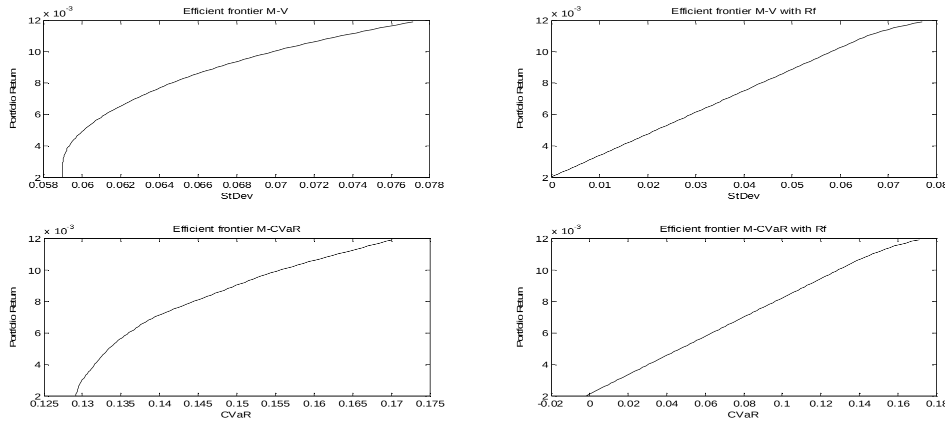
VI. CONCLUSION

The paper has compared two risk measures variance and CVaR based mainly on Certainty Equivalence Graphs. The question “whether these different risk measures lead to significantly different efficient frontiers for the investors, and which risk measure should be used?” is answered based on empirical tests with stock portfolios, indices portfolios, and stock-put compound portfolios. Both return-risk methods provide very good efficient frontiers. This means that the utility is not reduced very much after the return-risk optimization step. The traditional Mean-Variance method outperforms the Mean-CVaR in terms of simplicity and guarantee against losing utility due to choice set reduction, even in the case of non-normally distributed portfolios with options. Although the utilities of these methods are nearly the same, the weight allocations are different among these methods. In conclusion, M-V method is recommended for assessing common portfolio types such as stock portfolios, index portfolios, and even portfolios with options.

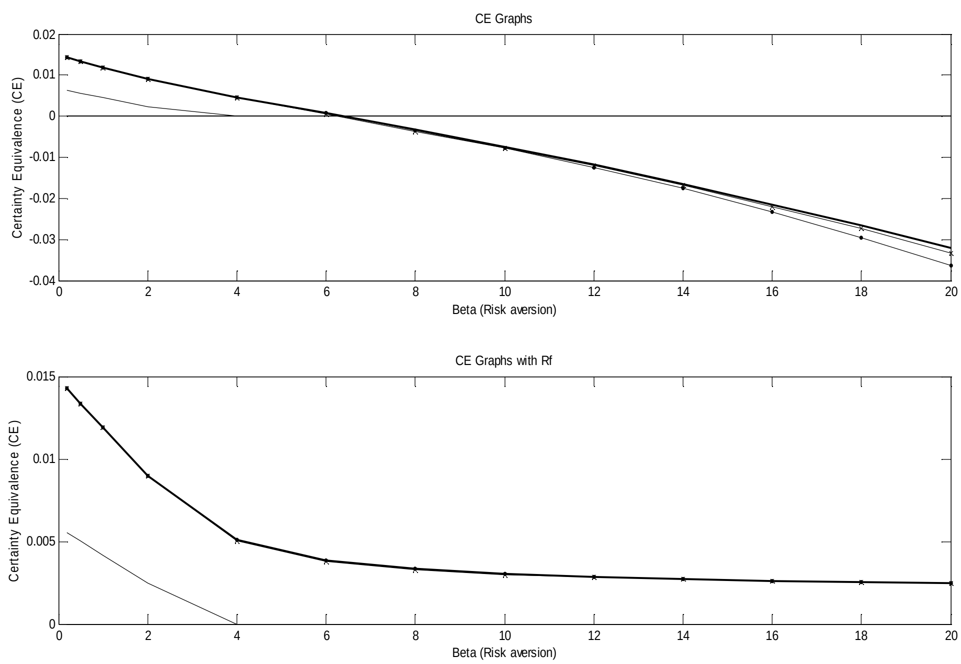
It is too early to state that the M-V method outperforms the Mean-CVaR for any portfolio and for any investors. However, we can say that application of Certainty Equivalence Curves has a high potential. They can be used to compare different risk measures, and applied to different portfolio types for the benefit of investors with different utility functions. The Certainty Equivalence Graphs are also very useful for visual presentation of the efficiency of a security selections strategy for toward different risk-averse investors.

Appendix

Appendix 1: 6 companies



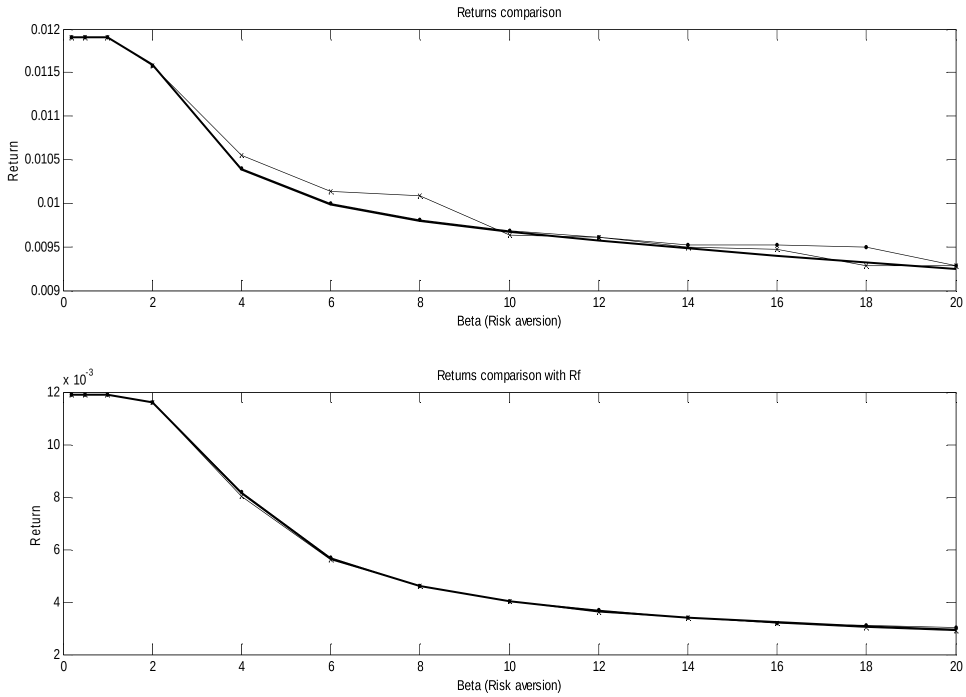
Graph 1-a



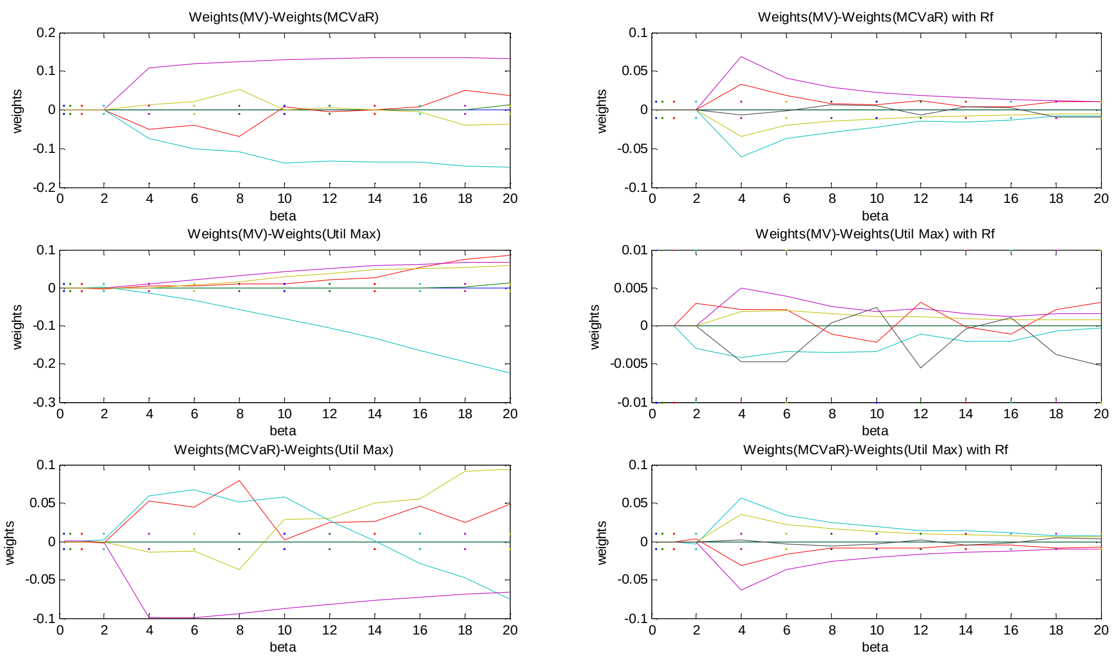
Graph 1-b*

---

* Thick solid line: for utility maximization method
Solid line with dots: for M-V method
Solid line with x: for M-CVaR method
Notice: these legends are applied for all CEG and Returns Graphs
The far below lines are representing equally weighted portfolios

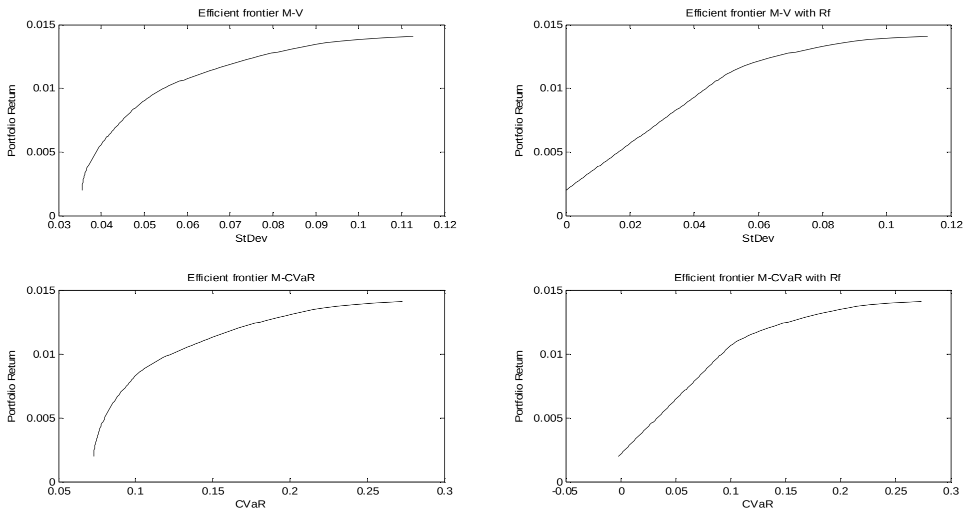


Graph 1-c

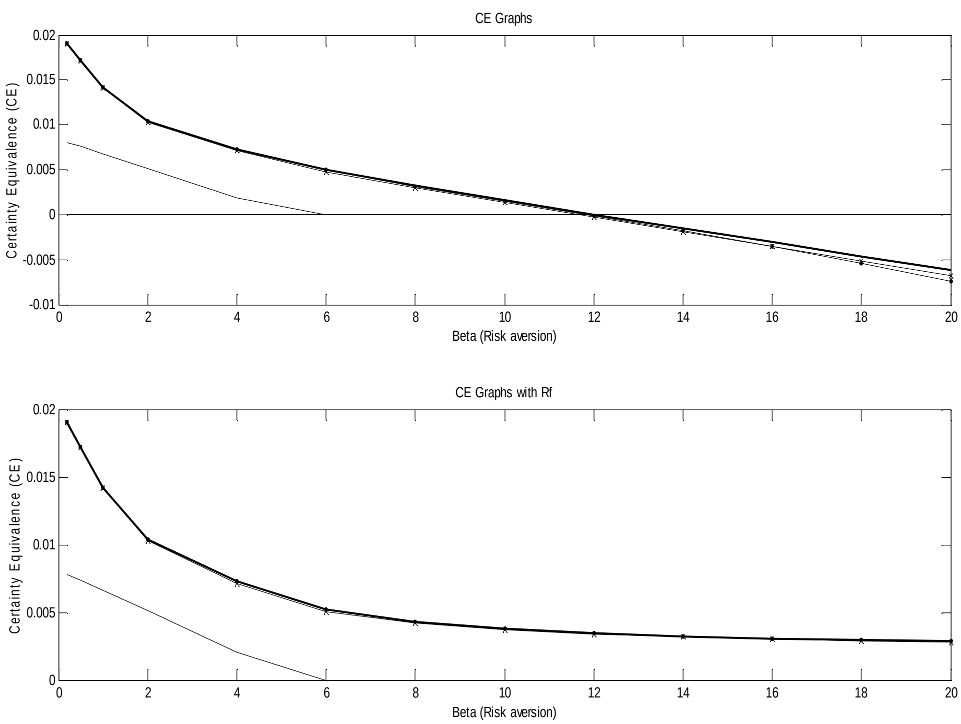


Graph 1-d

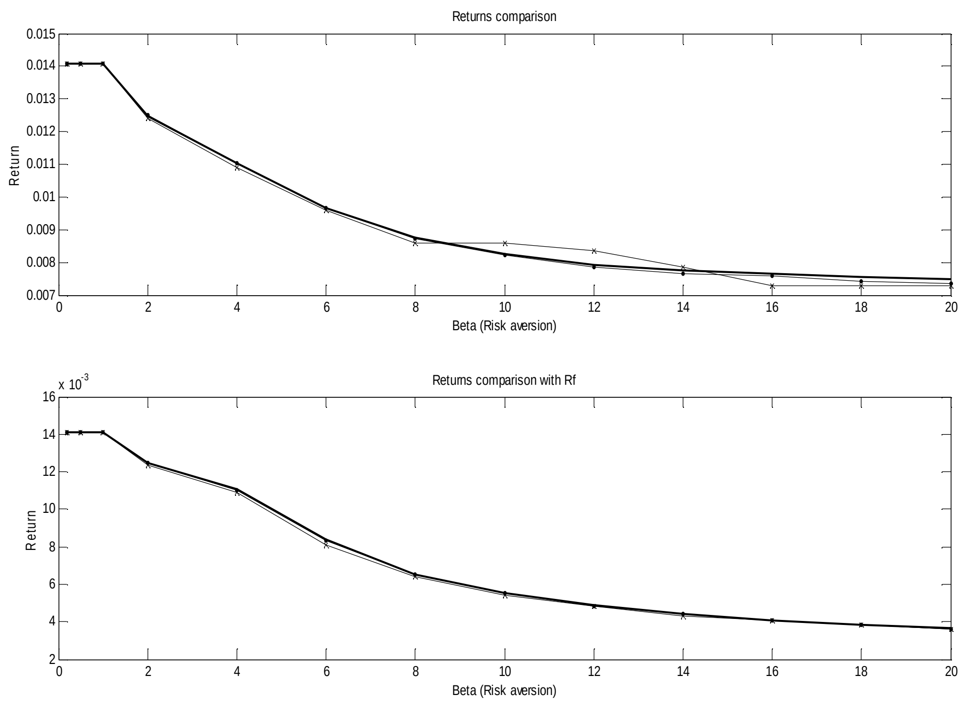
Appendix 2: 29 companies



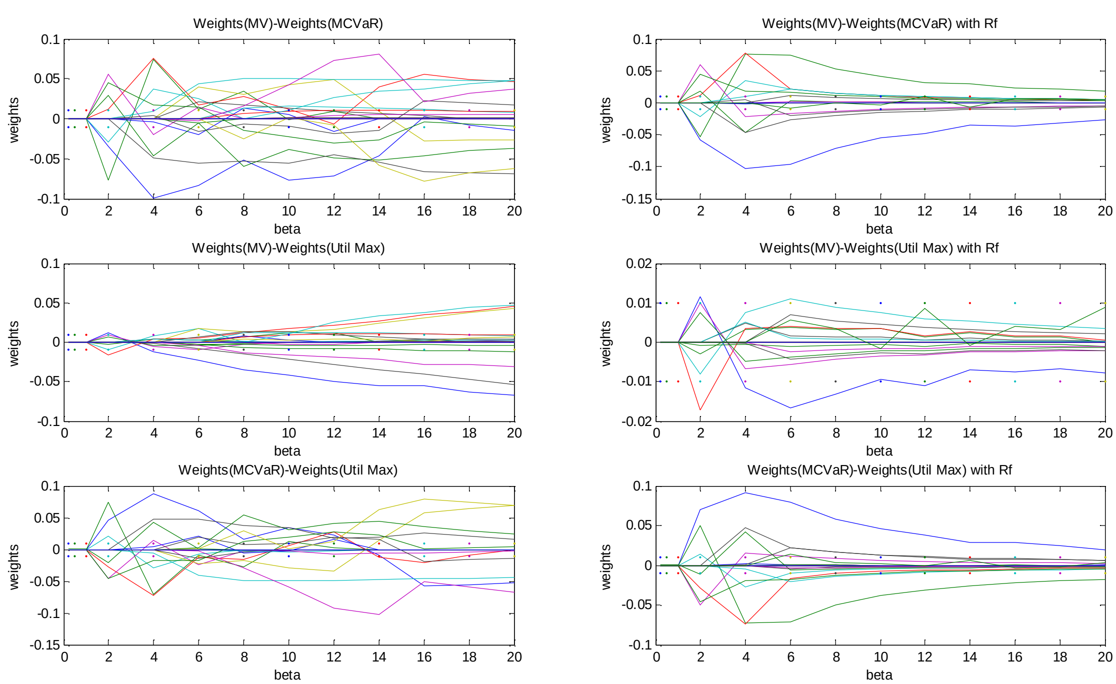
Graph 2-a



Graph 2-b

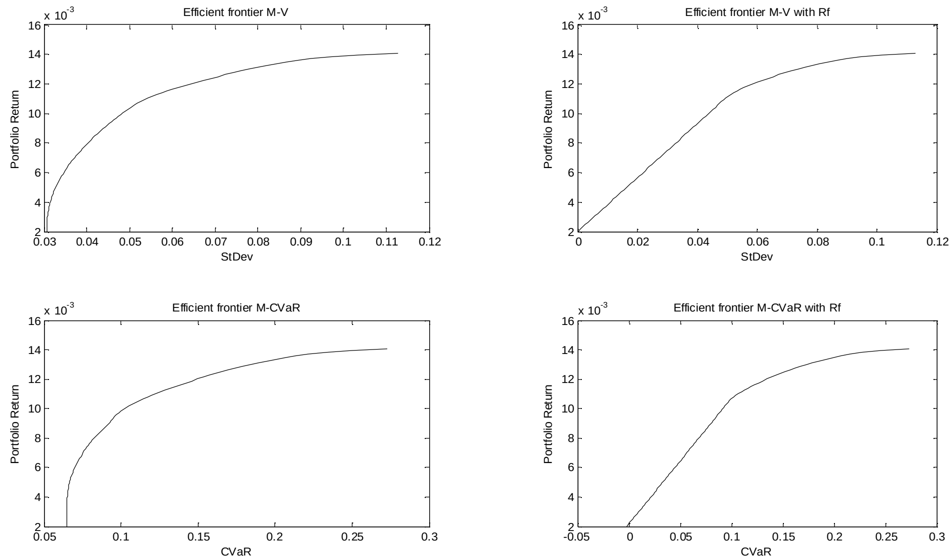


Graph 2-c

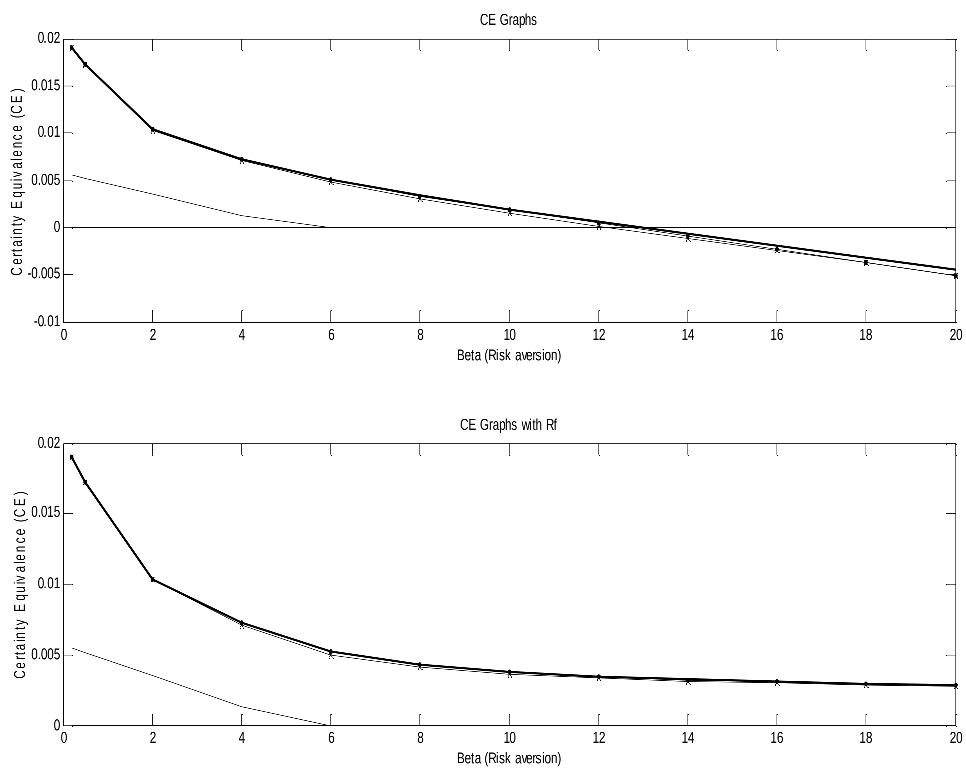


Graph 2-d

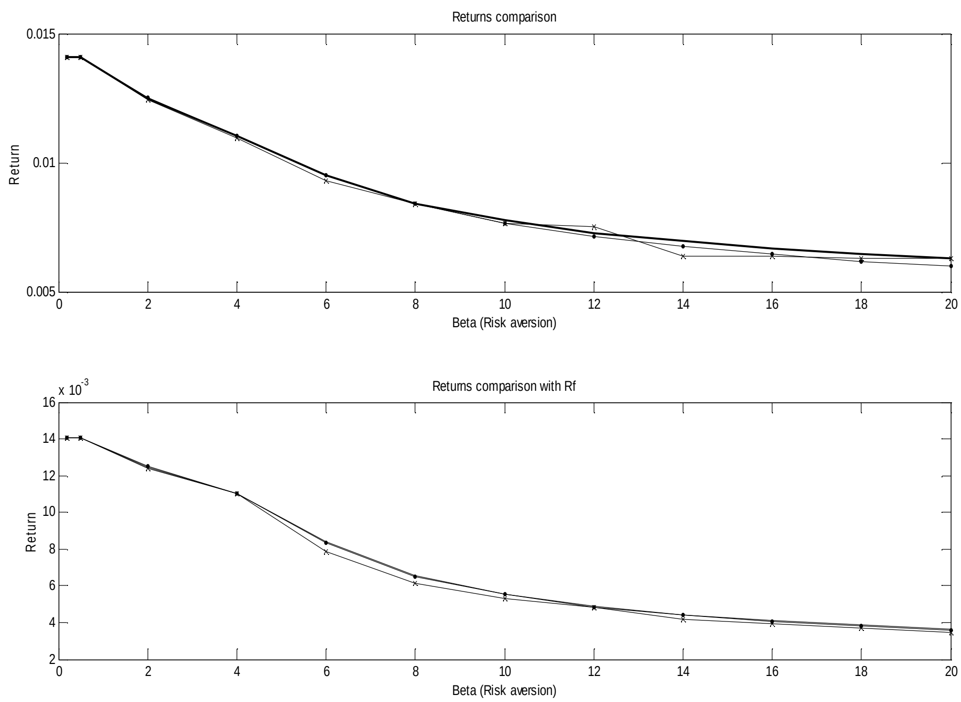
Appendix 3: 61 companies



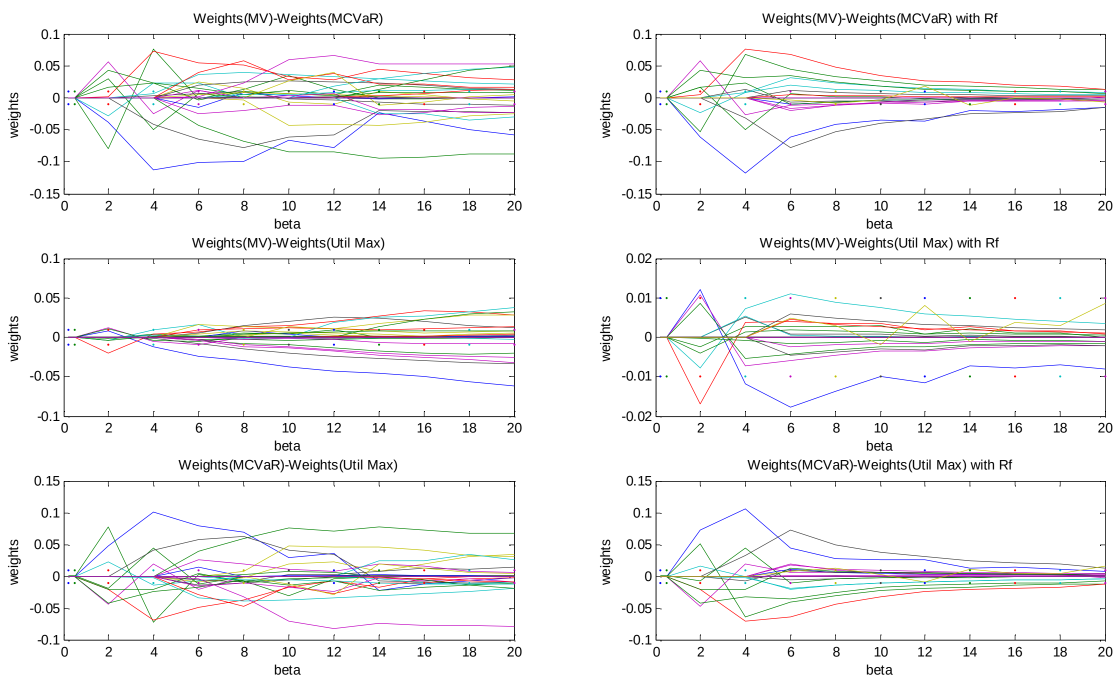
Graph 3-a



Graph 3-b

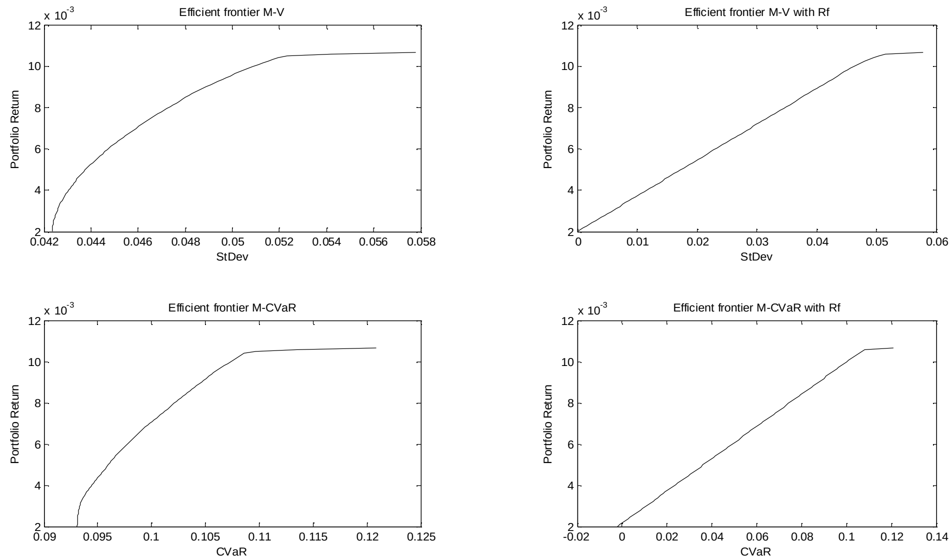


Graph 3-c

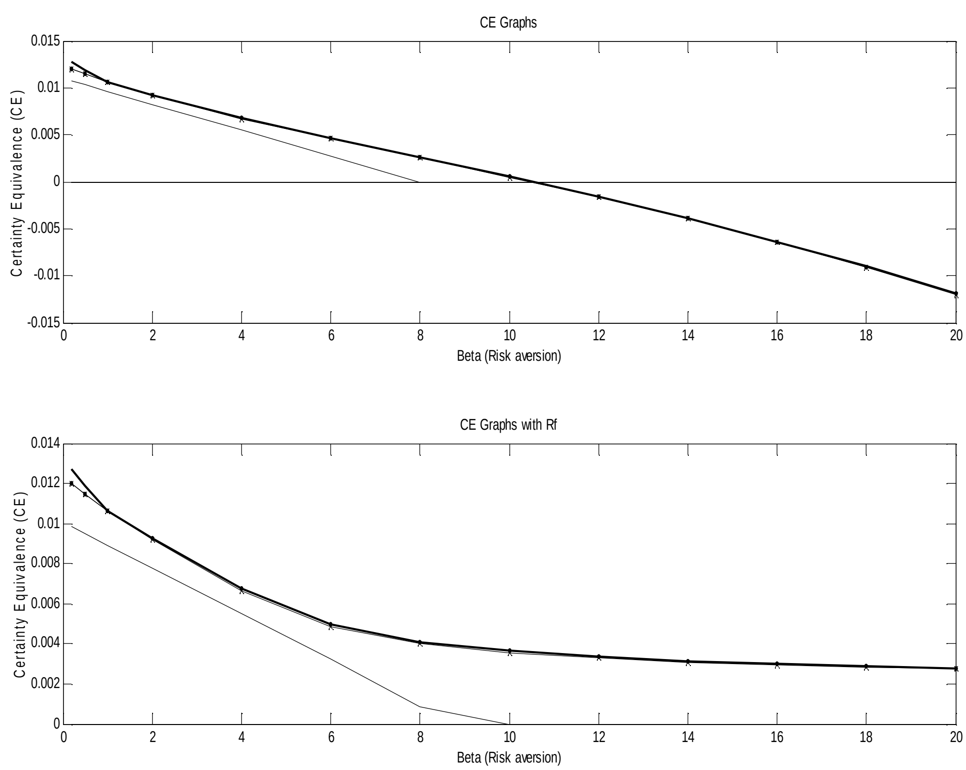


Graph 3-d

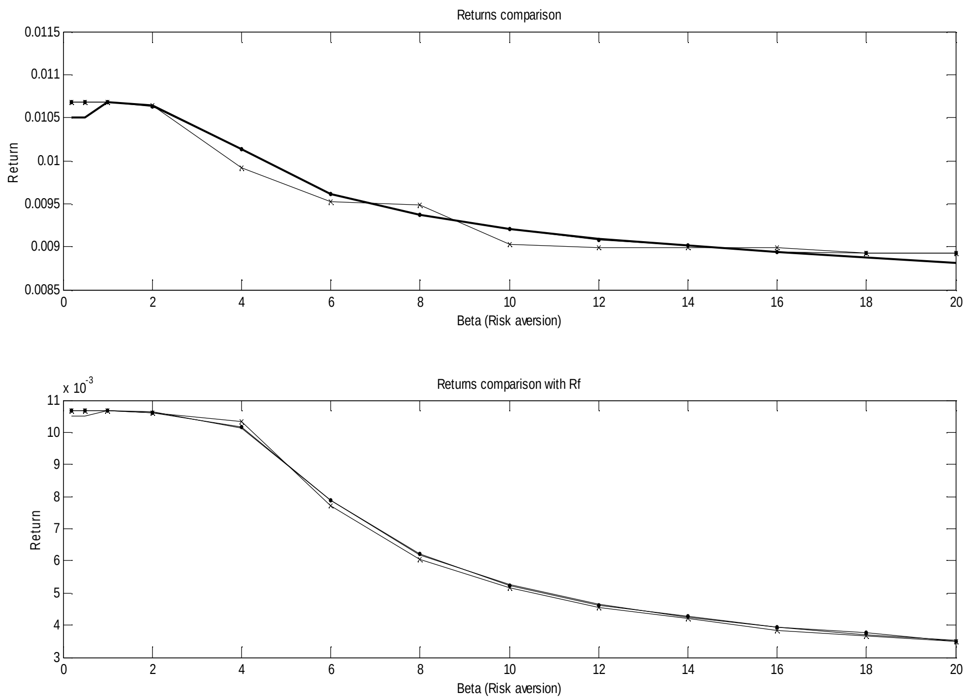
Appendix 4: 10 industries



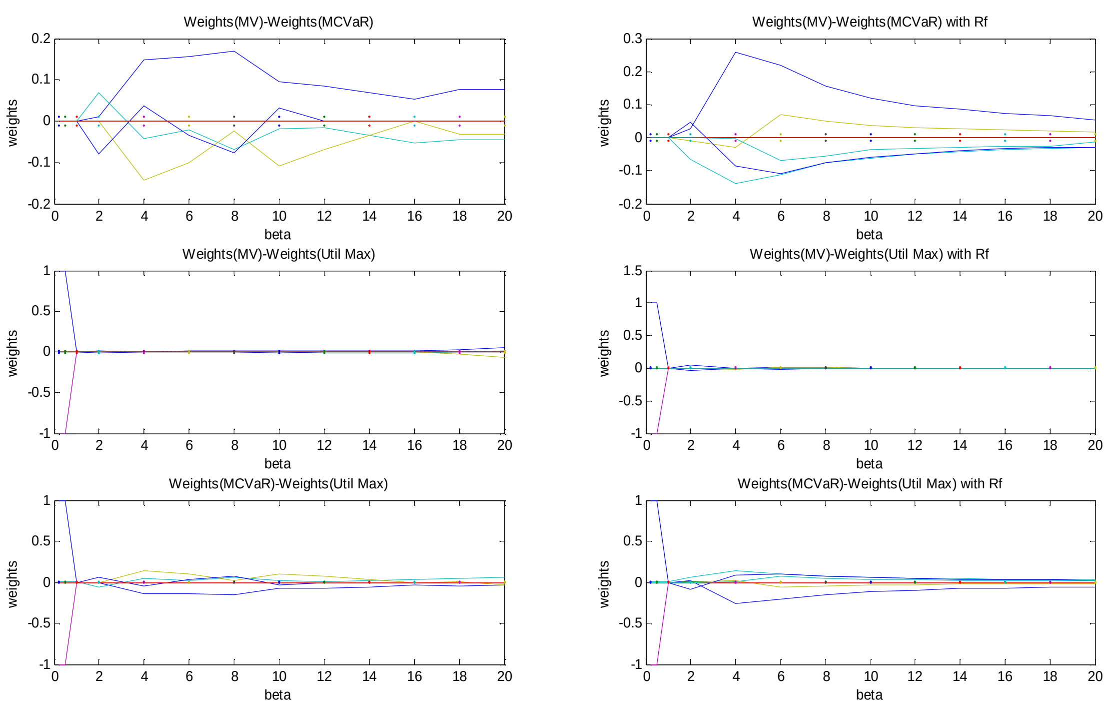
Graph 4-a



Graph 4-b

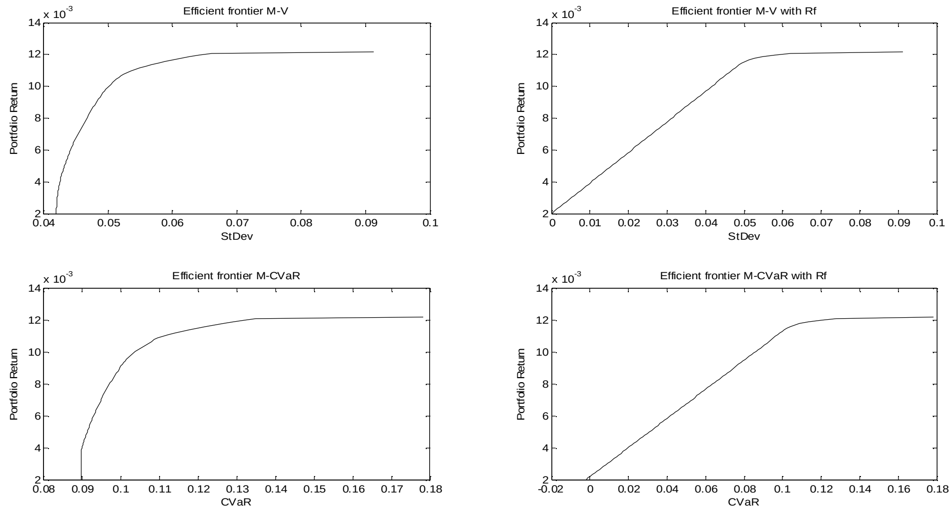


Graph 4-c

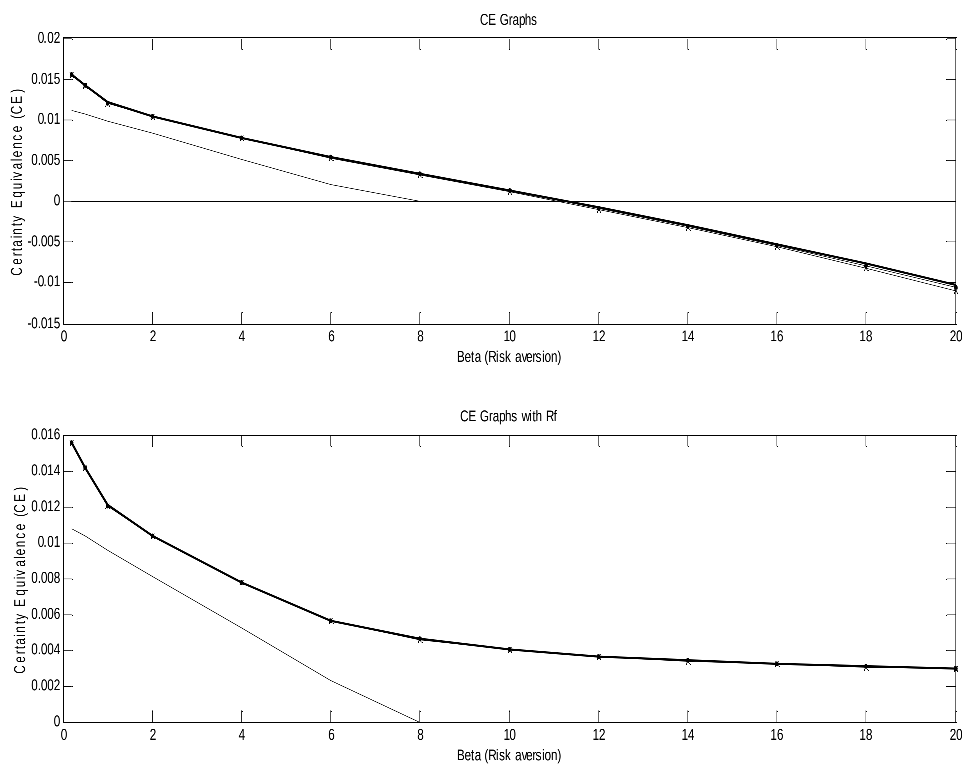


Graph 4-d

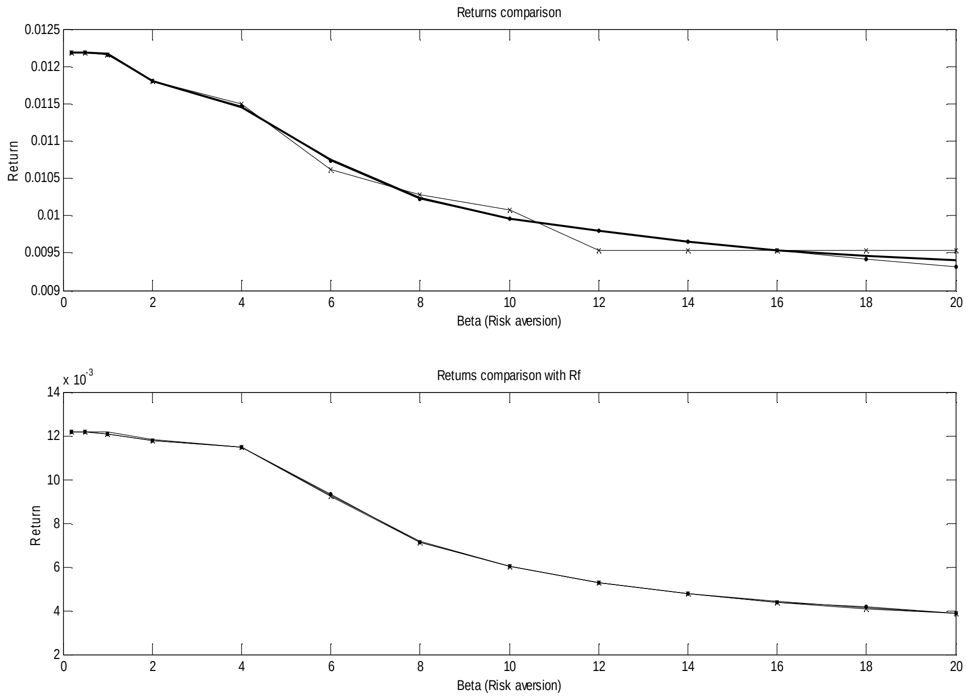
Appendix 5: 30 industries



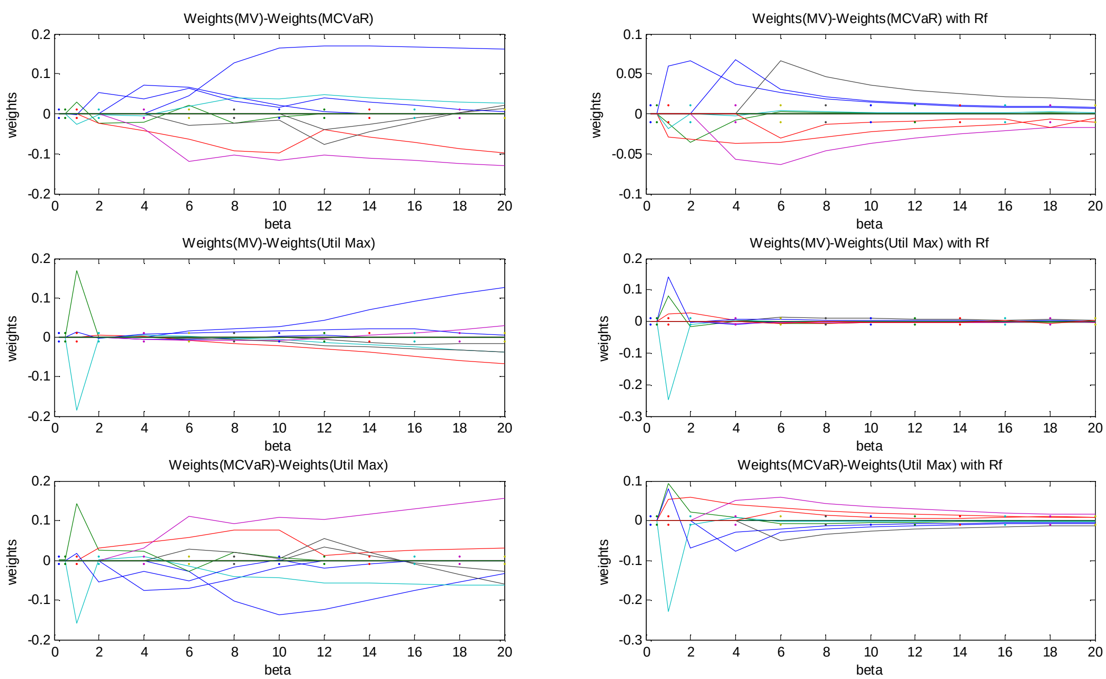
Graph 5-a



Graph 5-b

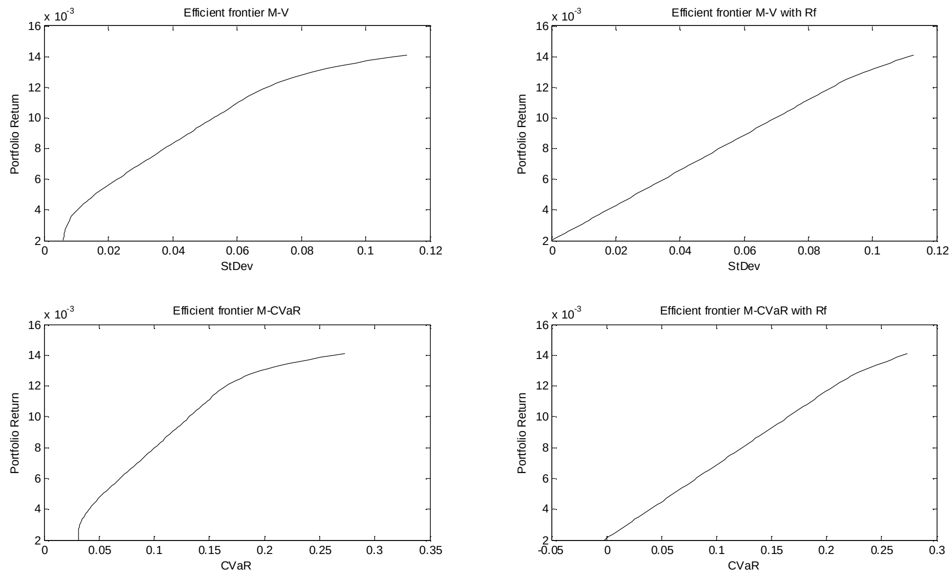


Graph 5-c

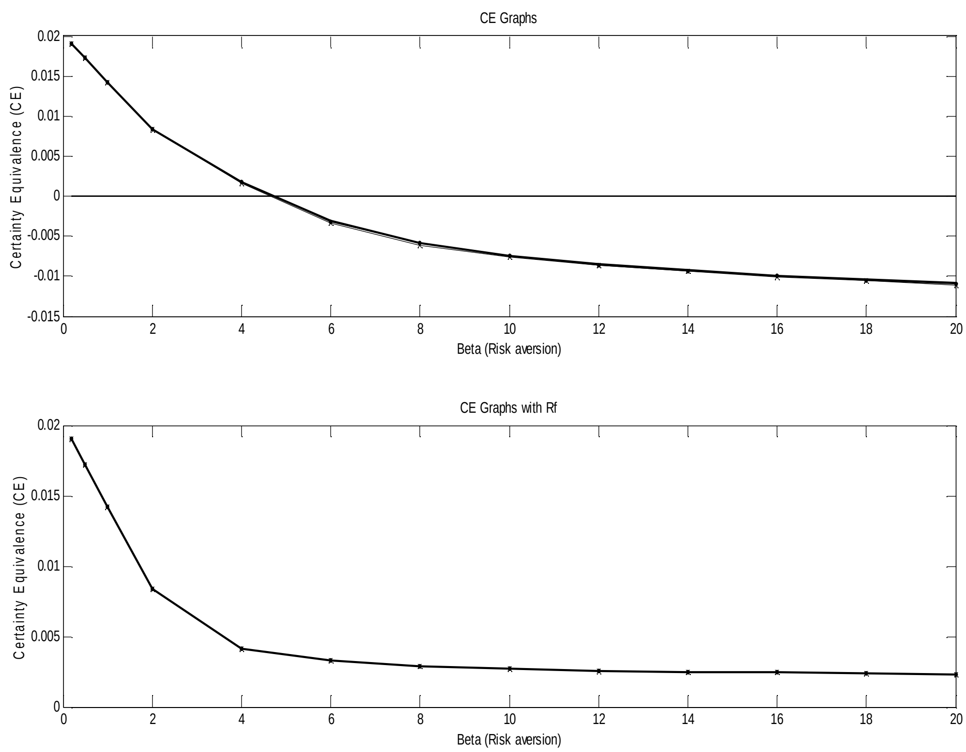


Graph 5-d

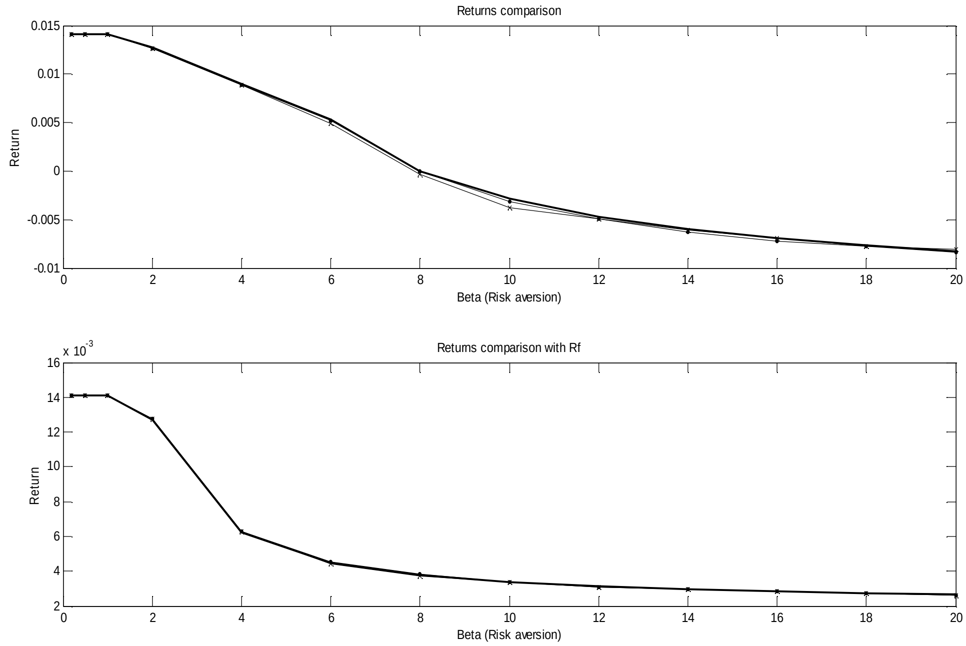
Appendix 6: 8 theoretical Black-Scholes puts and 8 stocks



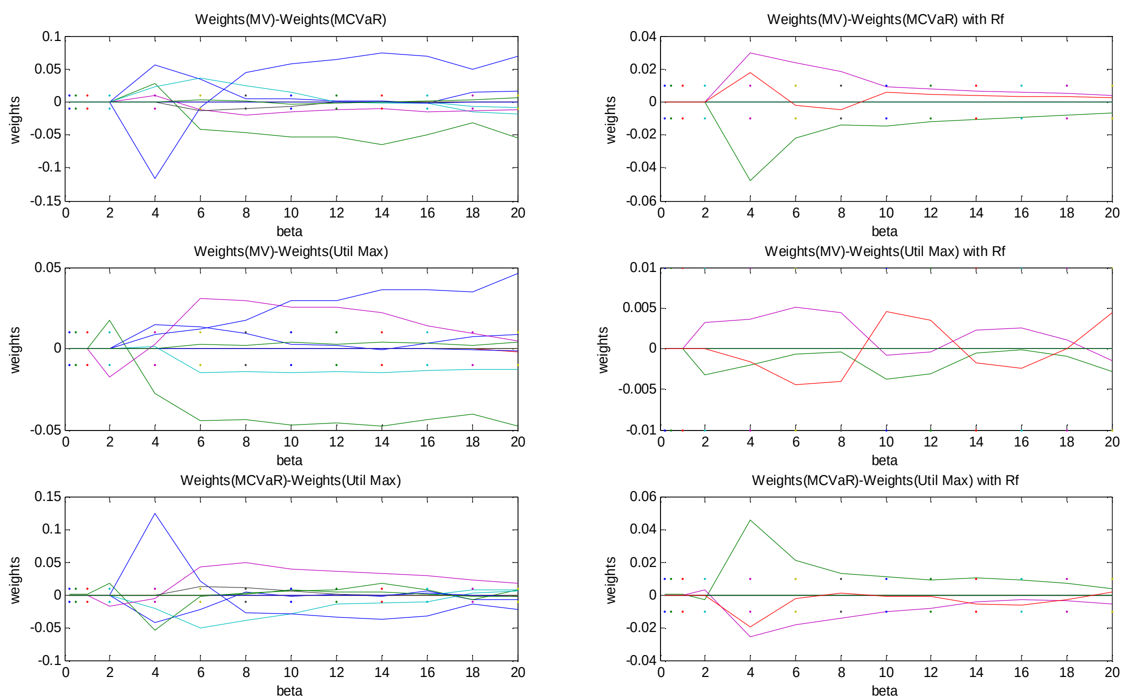
Graph 6-a



Graph 6-b

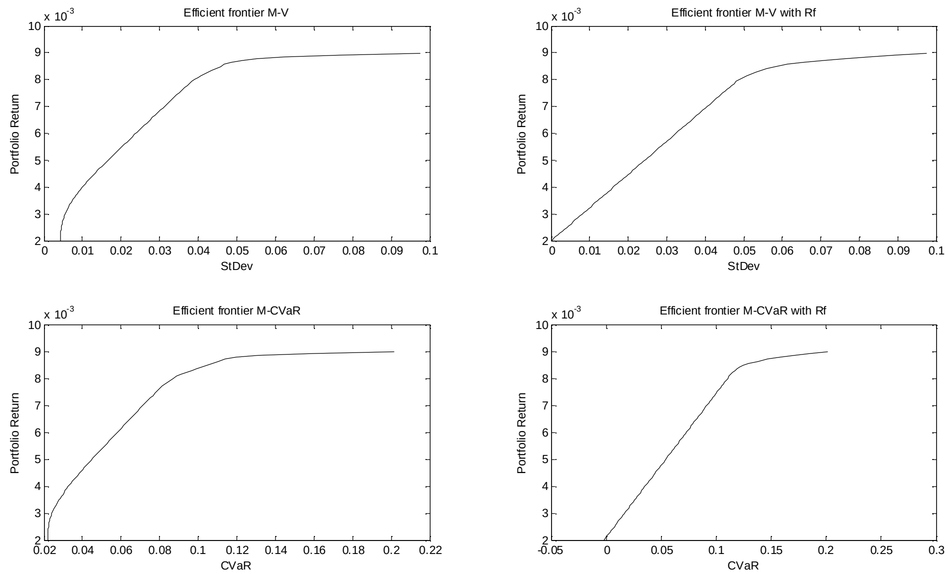


Graph 6-c

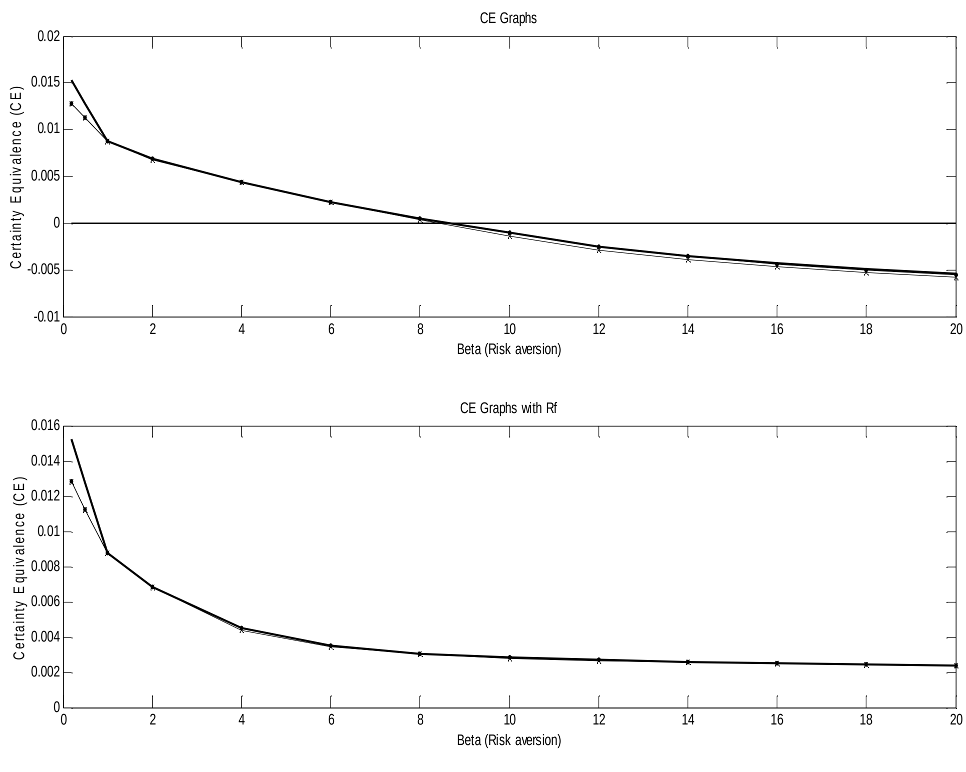


Graph 6-d

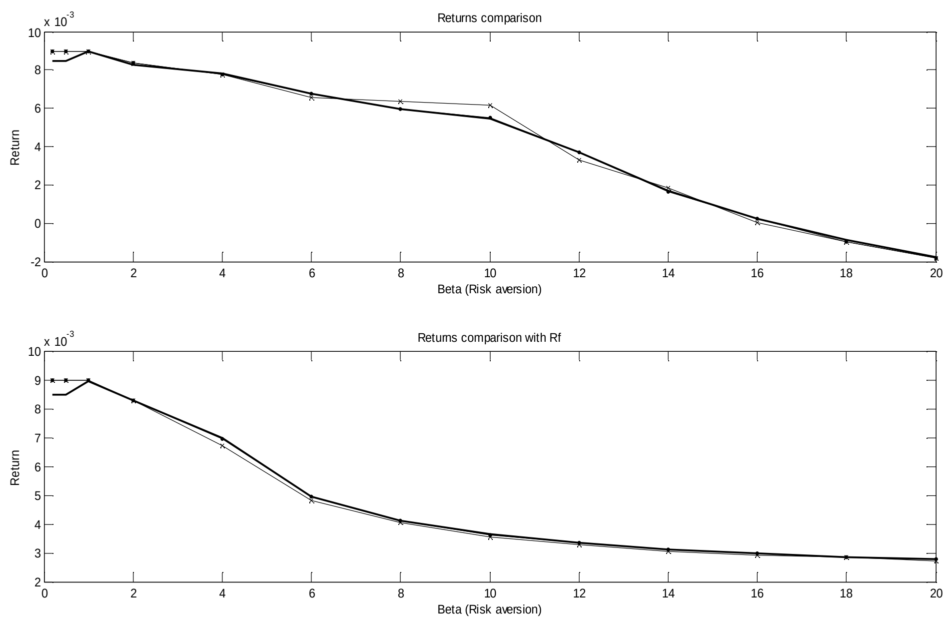
Appendix 7: 30 theoretical Black-Scholes puts and 30 stocks



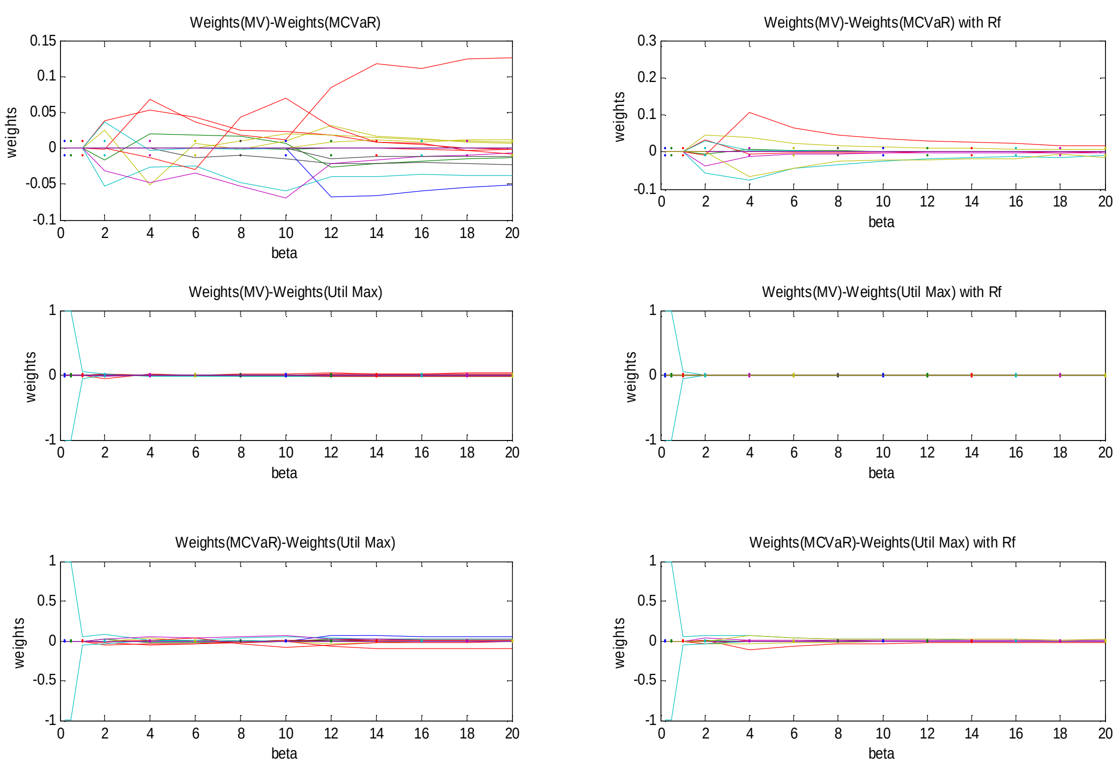
Graph 7-a



Graph 7-b



Graph 7-c



Graph 7-d

MATLAB Code

```
clear all
%I.Without Risk-free Asset
%I.1. Inputs

NumPorts=100; beta=[0.2,0.5,1,2:2:20]; alpha=0.95;
data='C:\Users\Hien Vu\Desktop\text1.txt';

   nRt=load(data); %the raw material in percentage
   [J, nAssets]=size(nRt);
   w0=[(1/nAssets)*ones(1,nAssets)];
   options=optimset('LargeScale','off');
   options=optimset(options,'TolFun',1e-40);
   options=optimset(options,'MaxFunEvals',1000000000000);
   options=optimset(options,'MaxIter',1000);
   %options=optimset(options,'TolX',1e-10);

%I.2. Utility function
% function f=exputilfun(w,nRt,bt)
% [J, nAssets]=size(nRt);
% Ut=0;
% for i=1:J
%     Ut=Ut+(-exp(-(exp(nRt(i,:)*w')*bt)))/exp(-(4/5)*bt);
% end
% f=(-Ut/J);


%I.3. Mean-Variance Criteria
ExpCovariance=cov(nRt); ExpReturn=mean(nRt);
[stdev, PortReturn, PortWtsMV] = frontcon(ExpReturn,...
ExpCovariance, NumPorts);%Matlab frontior toolbox

%I.4. Mean-CVaR Criteria
i=1:nAssets;UB=1;LB=0; R0=transpose(PortReturn); Risk=zeros(length(R0),2);
   A=[-mean(nRt) 0]; A=[A;  -eye(nAssets) zeros(nAssets,1)];
   A=[A; eye(nAssets) zeros(nAssets,1)];
   Aeq=[ ones(1,nAssets) 0]; beq=[1];
objfun=@(w) -w(nAssets+1)+(1/J)*(1/(1-alpha))...
   *sum(max(-w(i)*nRt(:,i)'+w(nAssets+1),0));
   w0=[(1/nAssets)*ones(1,nAssets)];%initial guess-equally weighted
   VaR0=quantile(nRt*w0',1-alpha); % the initial guess for VaR is the
   w0=[w0 VaR0];
for k=1:length(R0)
b=[-R0(1,k) -LB*ones(1,nAssets) UB*ones(1,nAssets)]; b=b';
[w,fval,exitflag,output]=fmincon(objfun,w0,A,b,Aeq,beq,LB,UB,[],options);
```

```
    for i=1:nAssets
    PortWtsCVaR(k,i)=w(i);
    end
Risk(k,1)=w(nAssets+1); Risk(k,2)=fval; %w(31)= portfolio VaR
clear w
end
CVaR=transpose(Risk(:,2));

%I.5. Utility Maximisation
w0=[(1/nAssets)*ones(1,nAssets)];
ub=ones(1,nAssets); lb=zeros(1,nAssets); %constrain condition
Aeq=[ones(1,nAssets)]; beq=[1]; %portfolio has unit value
for k=1:length(beta);
    bt=beta(k);
%Equally weighted
    y=exputilfun(w0,nRt,bt); [C,I]=min(y);
    CEu=max(log(-log(C*exp(-(4/5)*bt))/bt),0); CEeqwt(1,k)=CEu;
%Mean Variance
    for m=1:length(PortReturn)
    yMV(m)=exputilfun(PortWtsMV(m,:),nRt,bt);
    end
    [CMV,IMV]=min(yMV); PortWtsMVbeta(k,:)=PortWtsMV(IMV,:);
    CEMVk=log(-log(CMV*exp(-(4/5)*bt))/bt); CEMV(1,k)=CEMVk;
    RMVbeta(k)=R0(IMV);
%Mean-CVaR
    for m=1:length(Risk(:,2))
    yCVaR(m)=exputilfun(PortWtsCVaR(m,:),nRt,bt);
    end
    [CCVaR,ICVaR]=min(yCVaR); PortWtsCVaRbeta(k,:)=PortWtsCVaR(ICVaR,:);
    CECVaRk=log(-log(CCVaR*exp(-(4/5)*bt))/bt); CECVaR(1,k)=CECVaRk;
    RCVaRbeta(k)=R0(ICVaR);
%Direct Utility Maximization
    [w,fval,exitflag,output]=fmincon(@(w) exputilfun(w,nRt,bt),...
        w0,[],[],Aeq,beq,lb,ub,[],options);
    CEk=log((-log(fval*exp(-(4/5)*bt)))/bt); CEUtil(1,k)=CEk;  PortWtsUtil(k,:)=w(:);
    RUtilbeta(k)=ExpReturn*transpose(w);
end

%II. With risk-free asset
data='C:\Users\Hien Vu\Desktop\text2.txt';
nRt=load(data); [J, nAssets]=size(nRt); %the raw material in percentage

%II.1. Mean-Variance Criteria
ExpCovariance=cov(nRt); ExpReturn=mean(nRt);
[stdevRf, PortReturn, PortWtsMV] = frontcon(ExpReturn,...
ExpCovariance, NumPorts);%Matlab frontior toolbox
```

```
%II.2. Mean-CVaR Criteria
i=1:nAssets;UB=1;LB=0;
R0=transpose(PortReturn);
Risk=zeros(length(R0),2);
   A=[-mean(nRt) 0]; A=[A;  -eye(nAssets) zeros(nAssets,1)];
   A=[A; eye(nAssets) zeros(nAssets,1)]; Aeq=[ ones(1,nAssets) 0]; beq=[1];
objfun=@(w) -w(nAssets+1)+(1/J)*(1/(1-alpha))...
   *sum(max(-w(i)*nRt(:,i)'+w(nAssets+1),0));
   w0=[(1/nAssets)*ones(1,nAssets)];%initial guess-equally weighted
   VaR0=quantile(nRt*w0',1-alpha); w0=[w0 VaR0];% the initial guess for VaR is the
for k=1:length(R0)
b=[-R0(1,k) -LB*ones(1,nAssets) UB*ones(1,nAssets)]; b=b';
[w,fval,exitflag,output]=fmincon(objfun,w0,A,b,Aeq,beq,LB,UB,[],options);
   for i=1:nAssets
   PortWtsCVaR(k,i)=w(i);
   end
Risk(k,1)=w(nAssets+1); Risk(k,2)=fval; %w(31)= portfolio VaR
clear w
end
CVaRRf=transpose(Risk(:,2));

%II.3. Utility Maximization
w0=[(1/nAssets)*ones(1,nAssets)];
ub=ones(1,nAssets); lb=zeros(1,nAssets); %constrain condition
Aeq=[ones(1,nAssets)]; beq=[1]; %portfolio has unit value
for k=1:length(beta);
   bt=beta(k);
%Equally weighted
   y=exputilfun(w0,nRt,bt); [C,I]=min(y);
   CEu=max(log(-log(C*exp(-(4/5)*bt))/bt),0); CEeqwtRf(1,k)=CEu;
%Mean Variance
   for m=1:length(PortReturn)
   yMV(m)=exputilfun(PortWtsMV(m,:),nRt,bt);
   end
   [CMV,IMV]=min(yMV); PortWtsMVbetaRf(k,:)=PortWtsMV(IMV,:);
   CEMVk=log(-log(CMV*exp(-(4/5)*bt))/bt); CEMVRf(1,k)=CEMVk;
   RMVbetaRf(k)=R0(IMV);
%Mean-CVaR
   for m=1:length(Risk(:,2))
   yCVaR(m)=exputilfun(PortWtsCVaR(m,:),nRt,bt);
   end
   [CCVaR,ICVaR]=min(yCVaR); PortWtsCVaRbetaRf(k,:)=PortWtsCVaR(ICVaR,:);
   CECVaRk=log(-log(CCVaR*exp(-(4/5)*bt))/bt); CECVaRRf(1,k)=CECVaRk;
   RCVaRbetaRf(k)=R0(ICVaR);
%Direct Utility Maximization
```

```
   [w,fval,exitflag,output]=fmincon(@(w) exputilfun(w,nRt,bt),...
      w0,[],[],Aeq,beq,lb,ub,[],options);
   CEk=log((-log(fval*exp(-(4/5)*bt)))/bt); CEUtilRf(1,k)=CEk;
PortWtsUtilRf(k,:)=w(:); RUtilbetaRf(k)=ExpReturn*transpose(w);
end

%III. Plots
% %III.1. Plot Efficience Frontiers
figure
subplot(2,2,1)
plot(stdev,R0,'k-')% M-V frontier
xlabel('StDev')
ylabel('Portfolio Return')
title('Efficient frontier M-V')
subplot(2,2,2)
plot(stdevRf,R0,'k-')% M-V frontier
xlabel('StDev')
ylabel('Portfolio Return')
title('Efficient frontier M-V with Rf')

subplot(2,2,3)
plot(CVaR,R0,'k-')% M-CVaR frontier
xlabel('CVaR')
ylabel('Portfolio Return')
title('Efficient frontier M-CVaR')
subplot(2,2,4)
plot(CVaRRf,R0,'k-')% M-CVaR frontier
xlabel('CVaR')
ylabel('Portfolio Return')
title('Efficient frontier M-CVaR with Rf')


%%III.2. Plot Certainty Equivalance Graphs
%
Figure

   subplot(2,1,1)
plot(beta,zeros(length(beta)),'k-')
hold on
plot(beta,CEeqwt,'k-')
% plot CE with different level of risk averse
xlabel('Beta (Risk aversion)')
ylabel('Certainty Equivalence (CE)')
title('CE Graphs')
   subplot(2,1,1)
plot(beta,CEMV,'k.-')
   subplot(2,1,1)
plot(beta,CECVaR,'k-x')
   subplot(2,1,1)
plot(beta,CEUtil,'k-','linewidth',1.5)
% plot CE with different
   subplot(2,1,2)
plot(beta,zeros(length(beta)),'k-')
hold on
plot(beta,CEeqwtRf,'k-')
% plot CE with different level of risk averse
xlabel('Beta (Risk aversion)')
ylabel('Certainty Equivalence (CE)')
title('CE Graphs with Rf')
   subplot(2,1,2)
plot(beta,CEMVRf,'k.-')
   subplot(2,1,2)
plot(beta,CECVaRRf,'k-x')
   subplot(2,1,2)
plot(beta,CEUtilRf,'k-','linewidth',1.5)%
plot CE with different

% %III.3. Plot Comparing Returns of Optimal portfolio obtained from three methods

Figure
```

```
subplot(2,1,1)
plot(beta,RMVbeta,'k.-')
hold on
xlabel('Beta (Risk aversion)')
ylabel('Return')
title('Returns comparison')
   subplot(2,1,1)
plot(beta,RCVaRbeta,'k-x')
   subplot(2,1,1)
plot(beta,RUtilbeta,'k-','linewidth',1.5)
% plot Return with different

subplot(2,1,2)
plot(beta,RMVbetaRf,'k.-')
hold on
xlabel('Beta (Risk aversion)')
ylabel('Return')
title('Returns comparison with Rf')
   subplot(2,1,2)
plot(beta,RCVaRbetaRf,'k-x')
   subplot(2,1,2)
plot(beta,RUtilbetaRf,'k-','linewidth',1.5)
% plot Return with different

% %III.4. Plot Comparing Weights of Optimal portfolio obtained from three methods
figure
   subplot(3,2,1)
plot(beta,PortWtsMVbeta-PortWtsCVaRbeta)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MV)-Weights(MCVaR)')

   subplot(3,2,2)
plot(beta,PortWtsMVbetaRf-PortWtsCVaRbetaRf)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MV)-Weights(MCVaR) with Rf')

   subplot(3,2,3)
plot(beta,PortWtsMVbeta-PortWtsUtil)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MV)-Weights(Util Max)')

   subplot(3,2,4)
plot(beta,PortWtsMVbetaRf-PortWtsUtilRf)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MV)-Weights(Util Max) with Rf')

   subplot(3,2,5)
plot(beta,PortWtsCVaRbeta-PortWtsUtil)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MCVaR)-Weights(Util Max)')

   subplot(3,2,6)
plot(beta,PortWtsCVaRbetaRf-PortWtsUtilRf)
hold on
plot(beta,0.01)
plot(beta,-0.01)
xlabel('beta')
ylabel('weights')
title('Weights(MCVaR)-Weights(Util Max) with Rf')
```

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