Learning a Bayesian prior in interval timing

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Introduction

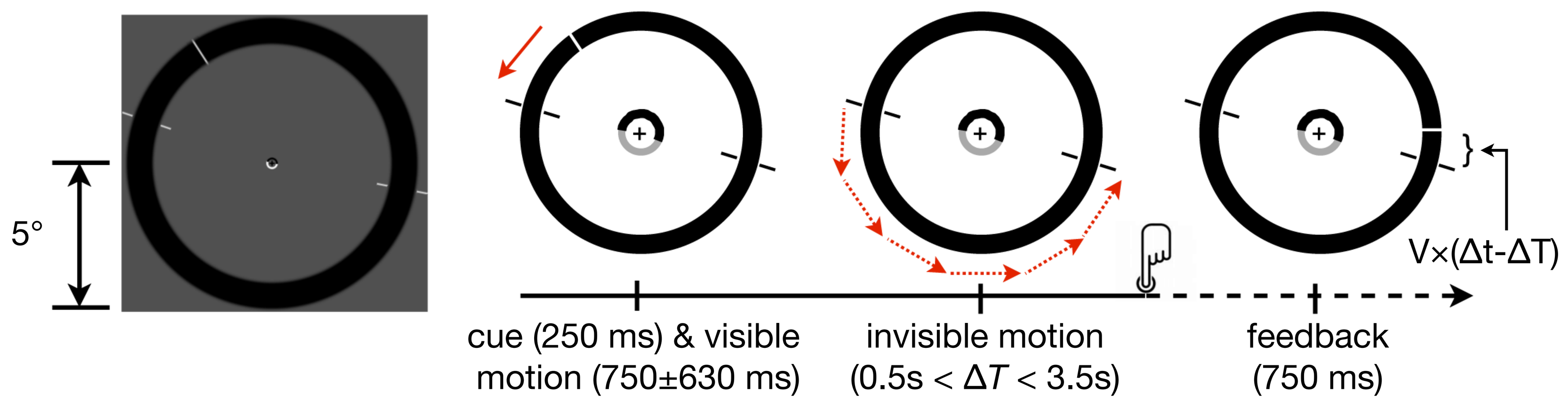
Two potential sources of ***Perceptual Learning*** (PL; improvement in task performance through repetition) within the framework of Bayesian probabilistic inference [1-5] are

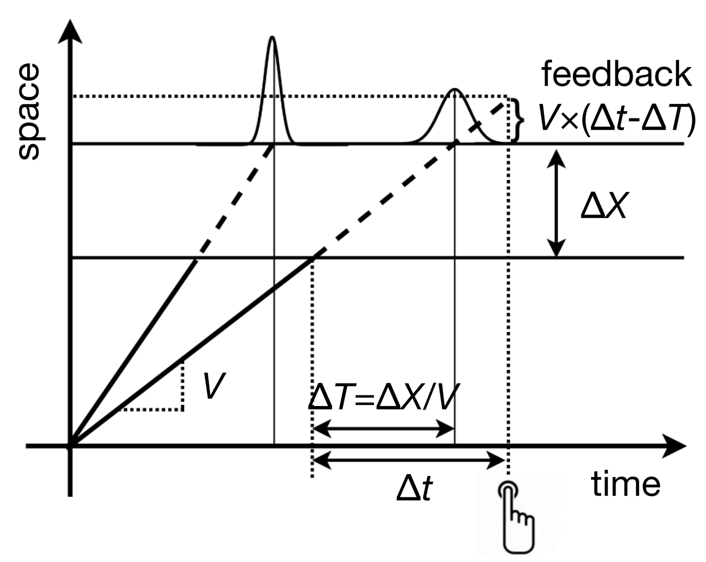
(1) ***Likelihood***: enhancement in sensory processing of stimuli

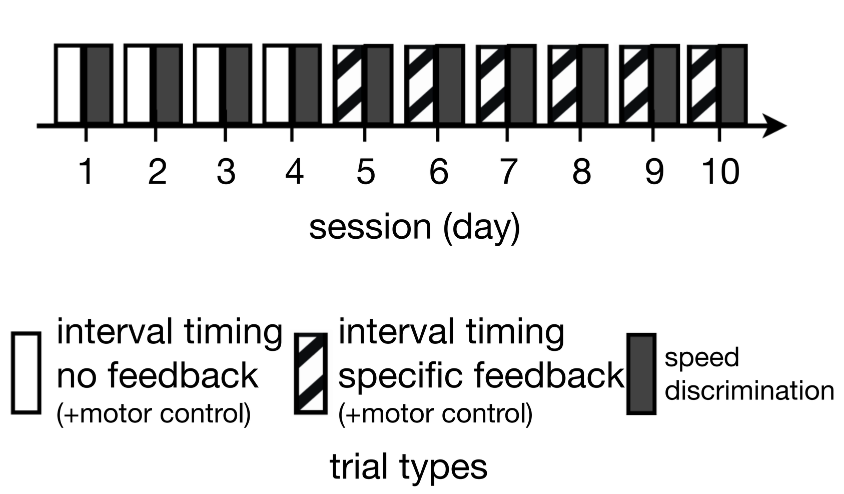
(2) ***Prior***: learning-induced changes in prior expectation

To assess relative contributions of the prior and likelihood to PL of ***Interval timing*** (IT)[5-7], we fitted ***Bayesian observer models*** to the distributions of timing latency data while human subjects were learning a novel timing task for an extended period of time (10 days).

Task and Experimental Design

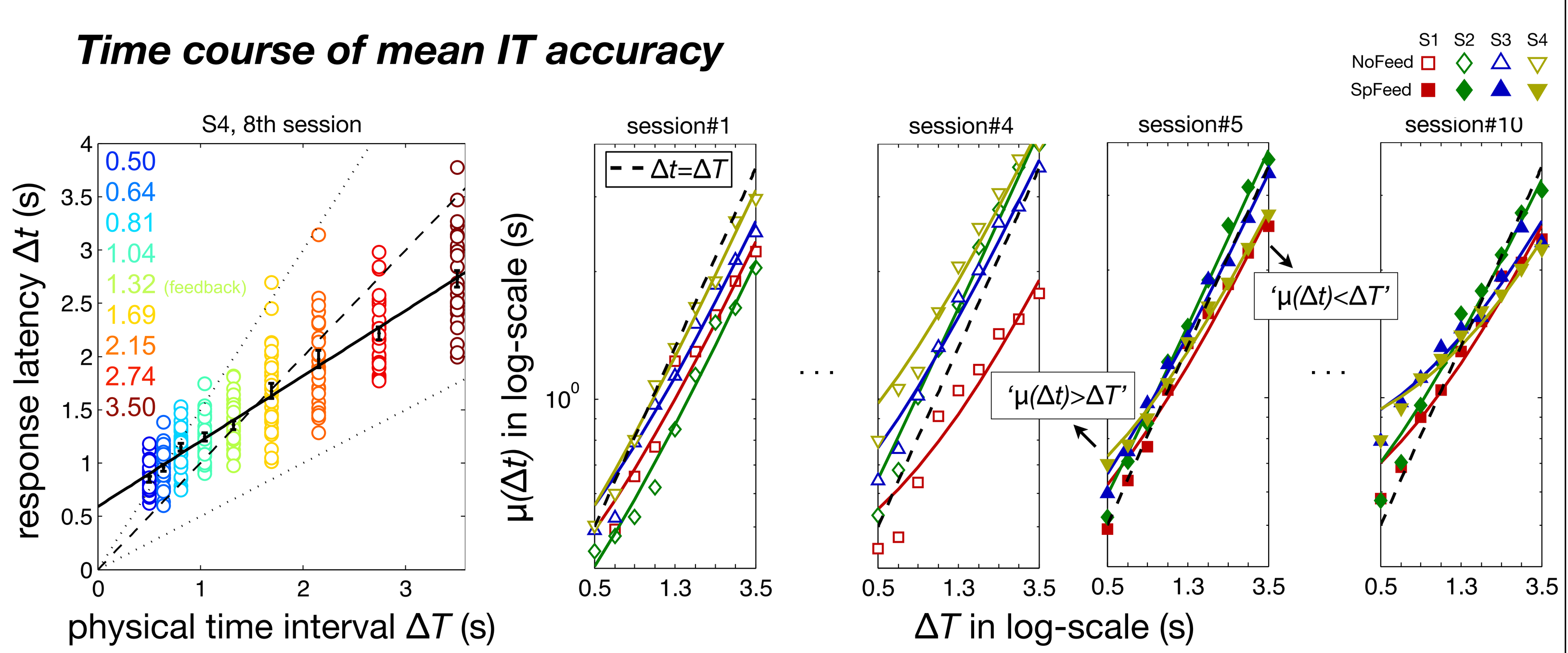




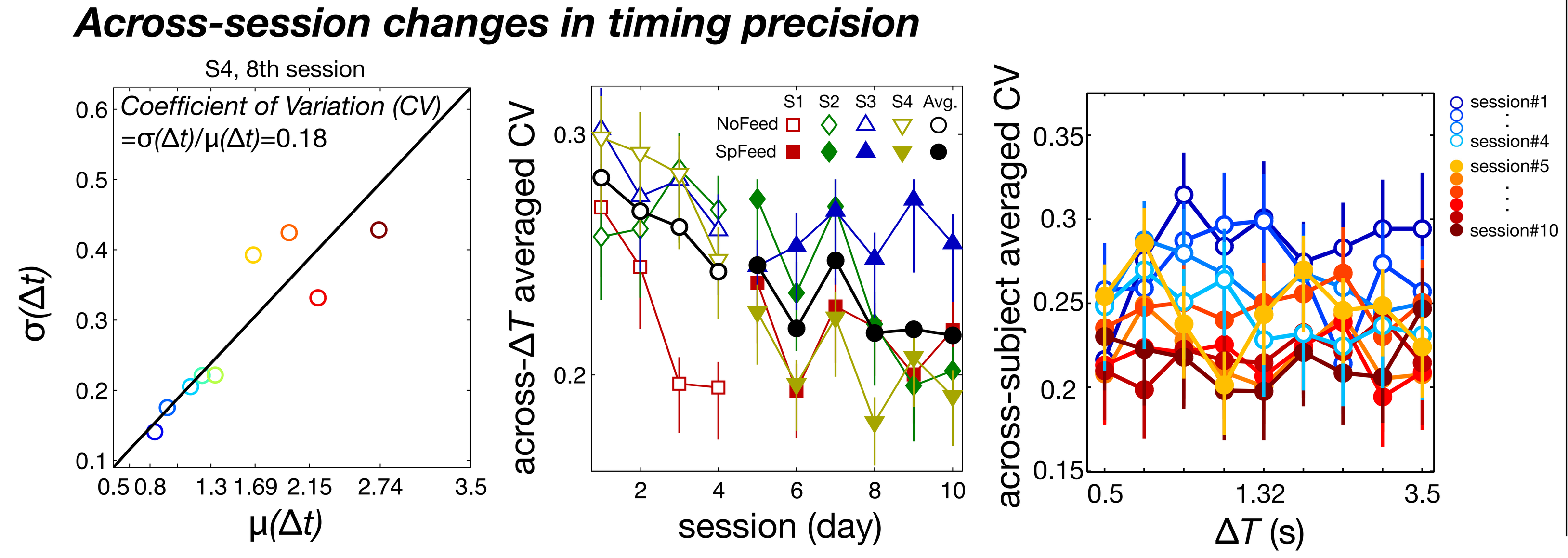


Behavioral Results

Time course of mean IT accuracy



Across-session changes in timing precision

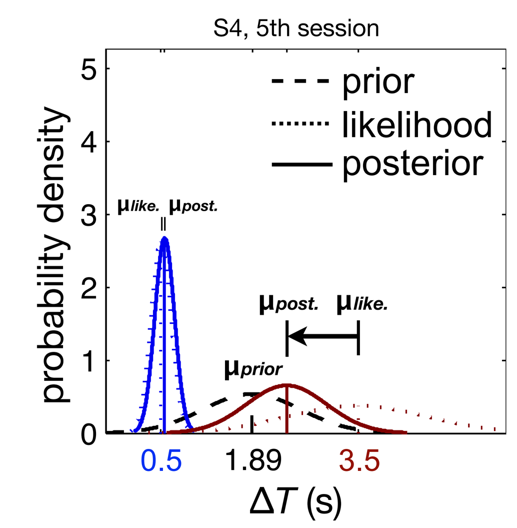


- **Mean accuracy**: bias toward center of sampled intervals (a.k.a. 'Vierordt's law')
- **Precision**: long-term, slow reduction in variability for all sampled ΔT

Bayesian Observer Model

$$\underbrace{P(\Delta T \mid \Delta t)}_{\text{posterior}} \propto \underbrace{P(\Delta t \mid \Delta T)}_{\text{likelihood}} \times \underbrace{P(\Delta T)}_{\text{prior}}$$

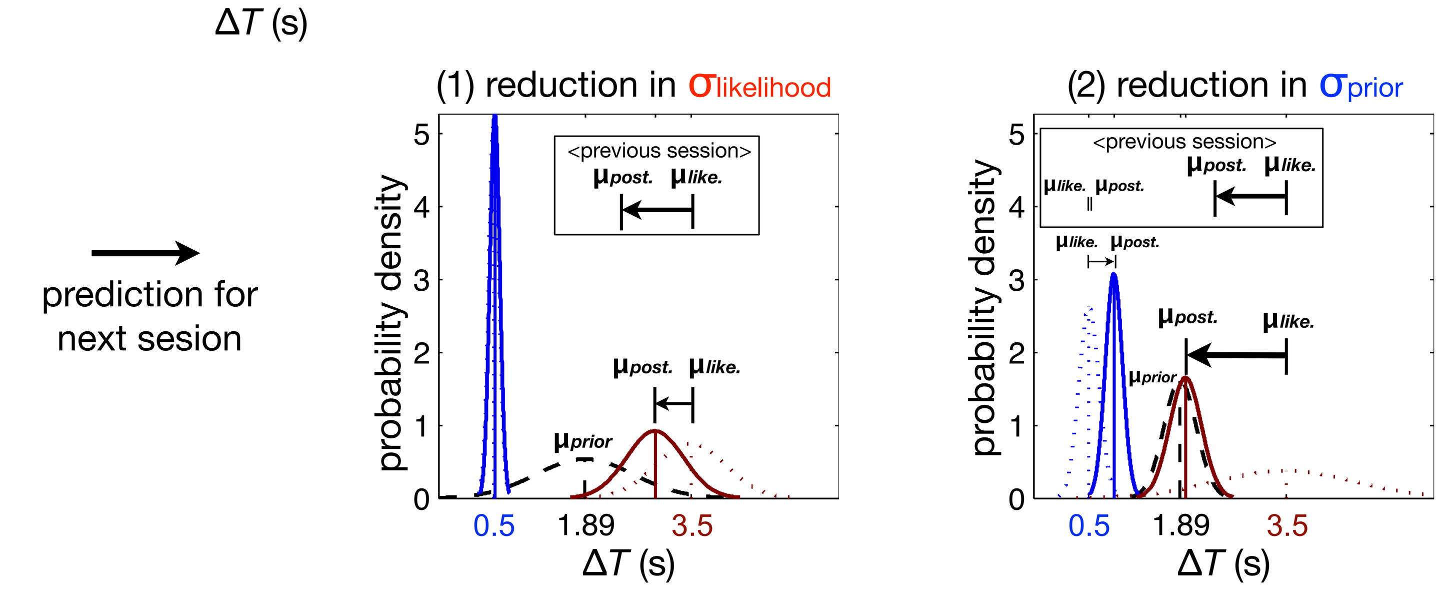
(i) Normal distribution for likelihood and prior: $P(\Delta t \mid \Delta T) \sim N(\mu_{likelihood}, \sigma_{likelihood})$, $P(\Delta T) \sim N(\mu_{prior}, \sigma_{prior})$



⇒ $P(\Delta T \mid \Delta t) \sim N(\mu_{posterior}, \sigma_{posterior})$ [1] where

$$\mu_{posterior} = \frac{\sigma^2_{prior}}{\sigma^2_{prior} + \sigma^2_{likelihood}} \cdot \mu_{likelihood} + \frac{\sigma^2_{likelihood}}{\sigma^2_{prior} + \sigma^2_{likelihood}} \cdot \mu_{prior}$$

$$\sigma^2_{posterior} = \frac{\sigma^2_{prior}}{\sigma^2_{prior} + \sigma^2_{likelihood}} \cdot \sigma^2_{likelihood}$$



(ii) Veridical mean of likelihood for 9 sampled time intervals: $\mu_{likelihood} = \Delta T$

(iii) Scalar property (constant CV for 9 intervals): $\sigma_{likelihood} = CV \cdot \mu_{likelihood}$

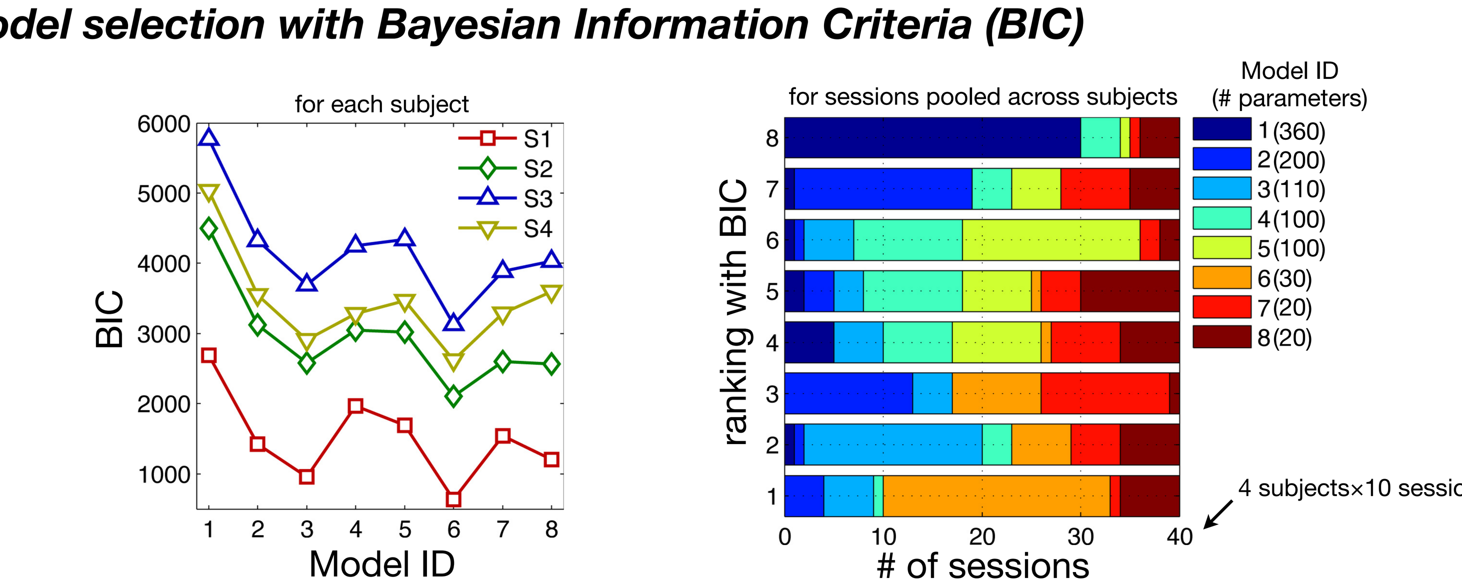
Model Comparison - *Validation of model assumption (ii) & (iii)*

(model ID) nested model variants	parameter assumptions: μ_{prior}	σ_{prior}	$\mu_{likelihood}$	$\sigma_{likelihood}$	# of free parameters for whole data†
(1) Exaggerated model	*multiple for each ΔT*	*multiple for each ΔT*	*free for 9 ΔTs*	*free for 9 ΔTs*	360=(9+9+9+9) ×10 sessions
(2) Full model	*one common prior to 9 ΔTs*	***one common prior to 9 ΔTs***	*free for 9 ΔTs*	*free for 9 ΔTs*	200=(1+1+9+9) ×10 sessions
(3) Reduced model with veridical $\mu_{likelihood}$	*one common prior to 9 ΔTs*	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	*free for 9 ΔTs*	110=(1+1+0+9) ×10 sessions
(4) Reduced model with veridical $\mu_{likelihood}$ & mean (ΔT) as μ_{prior}	*mean(ΔT)**	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	*free for 9 ΔTs*	100=(0+1+0+9) ×10 sessions
(5) Reduced model with veridical $\mu_{likelihood}$ & median (ΔT) as μ_{prior}	*median (ΔT)**	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	*free for 9 ΔTs*	100=(0+1+0+9) ×10 sessions
(6) Reduced model with veridical $\mu_{likelihood}$ & free CV, μ_{prior}, σ_{prior}	***one common prior to 9 ΔTs***	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	***scalar property (= CV × $\mu_{likelihood}$)***	**30=(1+1+0+1) ×10 sessions**
(7) Minimal model with mean (ΔT) as μ_{prior}	*mean(ΔT)**	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	***scalar property (= CV × $\mu_{likelihood}$)***	20=(0+1+0+1) ×10 sessions
(8) Minimal model with median(ΔT) as μ_{prior}	*median (ΔT)**	***one common prior to 9 ΔTs***	***veridical (= ΔT)****	***scalar property (= CV × $\mu_{likelihood}$)***	20=(0+1+0+1) ×10 sessions

*constant across sessions

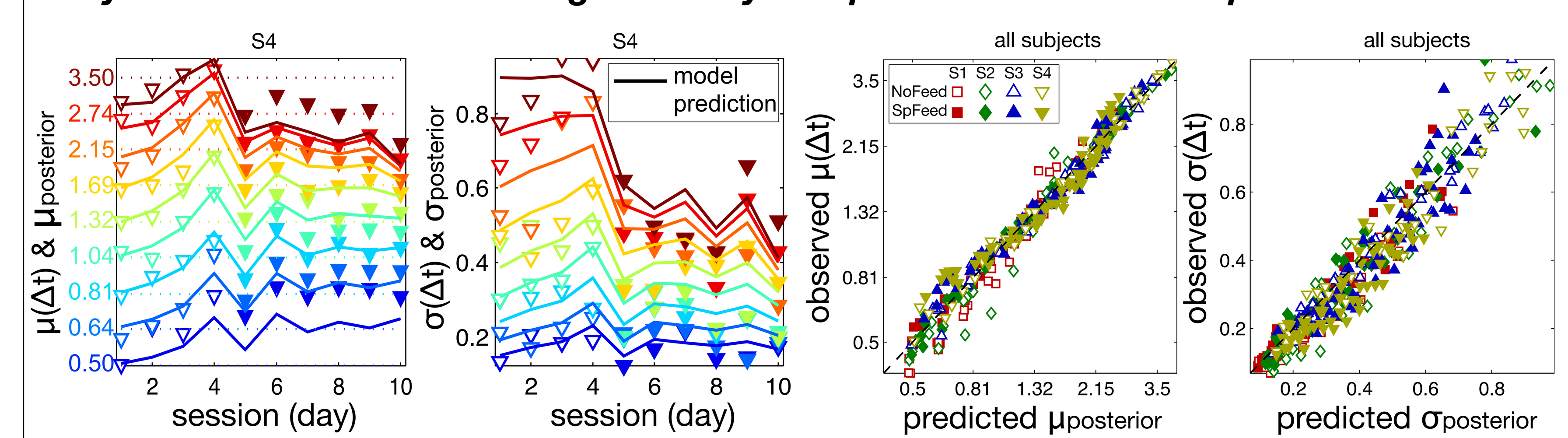
†# of whole data points for each subject: 2700 (=9 ΔTs × 30 trials × 10 sessions)

Model selection with Bayesian Information Criteria (BIC)

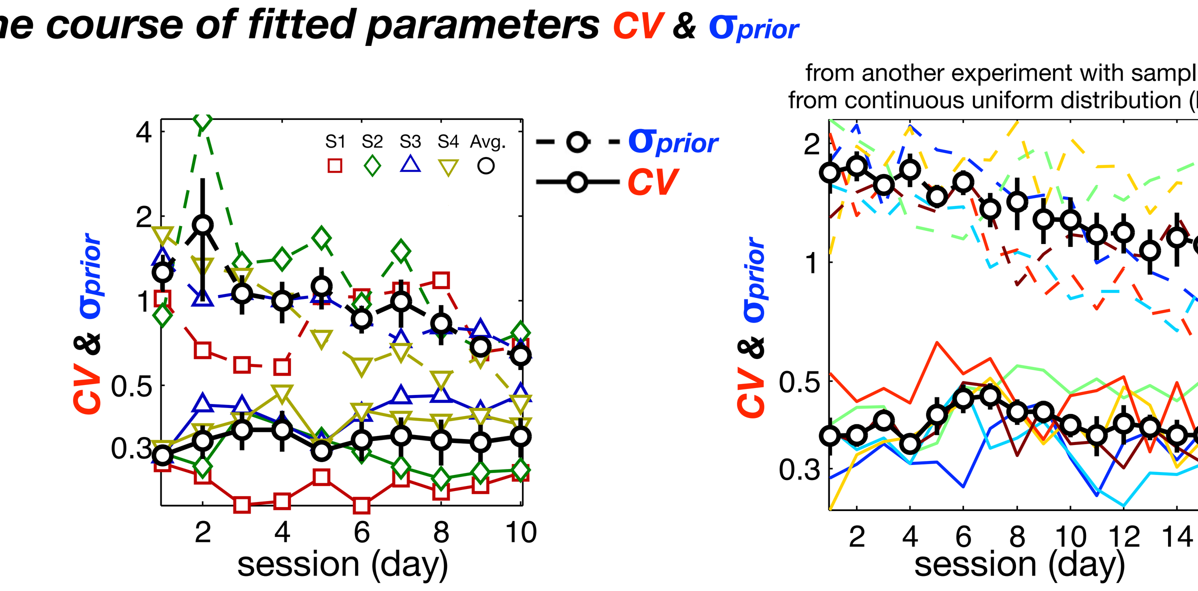


⇒ (6) reduced model with veridical $\mu_{likelihood}$ & free CV, μ_{prior}, σ_{prior} is the best-fitting yet the most parsimonious model in terms of BIC.

Dynamics of observed timing accuracy and precision with model prediction



Time course of fitted parameters CV & σprior



⇒ σ_{prior}, not CV, gradually reduces over sessions.

Discussion

- Bayesian observer model can capture the dynamics of timing accuracy and precision, in particular, biased mean perceived intervals toward the center of intervals and nonspecific reduction of timing variability.
- What underlies PL of IT is a slow long-term decrease in the prior width, not in the likelihood width.
- Open questions: What is a neural correlate of the internalized prior for IT?[1,5] How is the prior for IT formed and adjusted by experience?[7] To what extent do human observers learn or internalize various time interval distribution in the environment? [5,6]

References & Acknowledgement

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