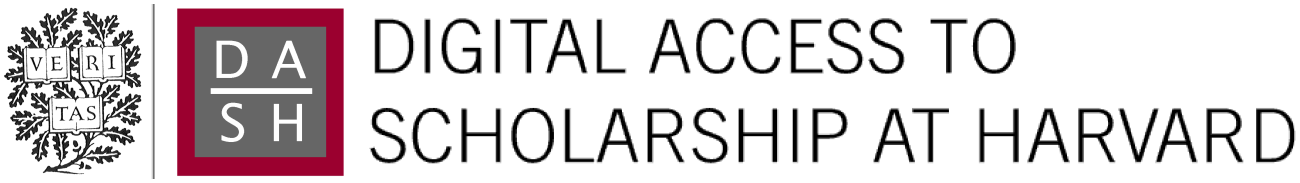

The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment

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Abstract

The long-run risks model of asset prices explains stock price variation as a response to persistent fluctuations in the mean and volatility of aggregate consumption growth, by a representative agent with a high elasticity of intertemporal substitution. This paper documents several empirical difficulties for the model as calibrated by Bansal and Yaron (BY, 2004) and Bansal, Kiku, and Yaron (BKY, 2011). US data do not show as much univariate persistence in consumption or dividend growth as implied by the model. BY's calibration counterfactually implies that long-run consumption and dividend growth should be highly predictable from stock prices. BKY's calibration does better in this respect by greatly increasing the persistence of volatility fluctuations and their impact on stock prices. This calibration fits the predictive power of stock prices for future consumption volatility, but implies much greater predictive power of stock prices for future stock return volatility than is found in the data. The long-run risks model, particularly as calibrated by BKY, implies extremely low yields and negative term premia on inflation-indexed bonds. Finally, neither calibration can explain why movements in real interest rates do not generate strong predictable movements in consumption growth.

1 Introduction

Consumption-based asset pricing models explain fluctuations in aggregate asset prices using shocks to the process driving aggregate consumption, together with assumptions about the utility function of a representative investor. A very basic question is what types of shocks are important, and different consumption-based models answer this question differently. The habit-formation model of Campbell and Cochrane (1999), for example, emphasizes shocks to the current level of consumption that move consumption in relation to a moving average of its past values, while the rare disasters model of Rietz (1988) and Barro (2006, 2009), extended by Gabaix (2010) and Wachter (2011), emphasizes changes in the probability or severity of a large drop in consumption.

Bansal and Yaron (henceforth BY, 2004) have argued that the key shocks moving aggregate stock prices are changing expectations of long-run consumption growth and its volatility. Their long-run risks model has attracted a great deal of attention, with important subsequent work by Bansal, Khatchatrian, and Yaron (2005), Bansal, Kiku, and Yaron (2007, 2010, 2011), Hansen, Heaton, and Li (2008), Bansal, Dittmar, and Kiku (2009), and Bansal and Shaliastovich (2009, 2010) among others. The purpose of this paper is to evaluate the plausibility of the claim that these shocks are the main drivers of aggregate asset prices.

The long-run risks model has four key features. First, there is a persistent predictable component of consumption growth. This component is hard to measure using univariate time-series methods, but investors perceive it directly and so it influences asset prices. In BY's original calibration of the model, this is the most important cause of stock price movements. Second, there is persistent variation in the volatility of consumption growth. A more recent calibration of the model by Bansal, Kiku, and Yaron (henceforth BKY, 2011) greatly increases the importance of changing volatility by increasing its persistence, somewhat in the spirit of Calvet and Fisher (2007, 2008). Third, consumption and dividends are not the same; the stock market is a claim to dividends, which are more volatile than consumption although correlated with consumption and sharing the same persistent predictable component and the same movements in volatility.

Finally, assets are priced by a representative investor who has Epstein-Zin-Weil preferences (Epstein and Zin 1989, Weil 1989). These preferences generalize power utility by treating the elasticity of intertemporal substitution (EIS) and the coefficient

of relative risk aversion (RRA) as separate free parameters. In the long-run risks model, EIS is greater than one and RRA is many times greater than one. The level of EIS ensures that stock prices rise with expected future consumption growth and fall with volatility of consumption growth, while the level of RRA delivers high risk premia. Because EIS is greater than the reciprocal of RRA, asset risk premia are driven not only by covariances of asset returns with current consumption, as in the classic power-utility models of Hansen and Singleton (1983) and Mehra and Prescott (1985), but also by the covariances of asset returns with expected future consumption growth (Restoy and Weil 1998, 2011).[2]

In this paper we assess the consistency of the BY and BKY calibrations of the long-run risks model with stylized facts about macroeconomic dynamics and the pricing of stocks and bonds. Some of our points have been made in recent papers by Bui (2007) and Garcia, Meddahi, and Tédongap (2008), but our examination of the long-run risks model is more comprehensive.[3]

We make five main points. First, there is evidence of mean-reversion rather than persistence in US consumption and dividend series in the period since 1930 that is emphasized by BY and BKY. Even in data from the period since World War II, the persistence of consumption and dividend growth is considerably smaller than in the BY and BKY calibrations of the long-run risk model. Thus the simplest univariate time-series analysis casts doubt on the existence of the predictable variations in long-run growth that drive the long-run risks model.

Of course, predictable variations in long-run growth might exist in the data but might be masked by temporary fluctuations that are omitted from the long-run risks model. If this is the case, however, economic agents must perceive those variations in

[2]The Epstein-Zin-Weil model can alternatively be used to derive an augmented version of the classic Capital Asset Pricing Model, in which asset risk premia are driven not only by covariances of asset returns with the current return on the aggregate wealth portfolio, but also by covariances with news about future returns on wealth (Campbell 1993, 1996, Campbell and Vuolteenaho 2004). We do not explore this approach to the model here. Campbell (2003) gives a textbook treatment of the Epstein-Zin-Weil model under homoskedasticity.

[3]A similar empirical analysis by BKY analyzes only the calibration with extremely persistent consumption volatility and ignores the original BY calibration. We evaluate both calibrations for two reasons. Many contributions to the long-run risks literature continue to use a half life of volatility shocks closer to that in BY (Bansal, Kiku and Yaron (2007) Bansal and Shaliastovich (2009) Bansal and Shaliastovich (2010)), so it remains important to understand the properties of the long-run risks model with lower volatility persistence. In addition, including both calibrations builds understanding of the sensitivity of model properties to parameters.

order for them to influence asset prices. This means that asset prices should predict consumption and dividend growth if the long-run risks model is true. Our second main point is that the level of the stock market (as measured by the log price-dividend ratio) is a poor predictor of dividend growth and particularly of consumption growth, but a strong predictor of future excess stock returns. These patterns are inconsistent with both the BY and BKY calibrations of the long-run risks model.

BKY recognize that the original calibration of the long-run risks model by BY overstates the ability of stock prices to predict consumption and dividend growth. Accordingly they calibrate a version of the model in which predictable movements in consumption volatility are a more important influence on stock prices, thereby weakening the correlation between stock prices and subsequent consumption growth. Our third main point is that while the BKY calibration does capture an important empirical regularity—that stock prices strongly predict future consumption volatility—it creates a new puzzle by overstating the ability of stock prices to predict stock return volatility.

Fourth, we show that the long-run risks model predicts a downward-sloping term structure of interest rates on real (inflation-indexed) bonds. This is because both negative shocks to consumption growth and positive shocks to volatility lower real interest rates and raise bond prices, while at the same time driving up the marginal utility of consumption, implying that real bonds hedge against such shocks. In the model, investors are willing to accept low yields on long-term real bonds for the sake of their hedging properties. While data on real bond yields are quite limited, the implied downward slope seems too large to be consistent with the data, particularly in the BKY calibration of the model.

Our final main point is that the long-run risks model cannot explain why there is predictable variation in short-term real interest rates that is unaccompanied by predictable variation in consumption growth. The model requires that the representative investor's EIS is greater than one, but this implies a strong tendency for consumption growth to move predictably with short-term real interest rates. The data show no such pattern, which has led earlier authors such as Hall (1988) and Campbell (2003) to argue that the EIS is much smaller than one. It is true that changing volatility can weaken the relation between predictable consumption growth and the short-term real interest rate, but the magnitude of this effect is too small to reconcile the long-run risks model with the data.

The paper is organized as follows. Section 2 lays out the long-run risks model

and its solution, discusses the alternative calibrations of BY and BKY, and explains our simulation methodology. We use analytical solutions to a loglinear approximate model of the sort proposed by Campbell and Shiller (1988) and Campbell (1993). This method is highly accurate for reasonable values of the intertemporal elasticity of substitution, provided that one solves numerically for the parameter of loglinearization (Campbell 1993, Campbell and Koo 1997). The empirical evaluation by BKY also uses the loglinear approximation of the model.[4]

Section 3 explains the data that we use to evaluate the performance of the model, annual US data over the period 1930-2008 and quarterly data over the period 1947.2-2008.4. This section presents basic moments from the data and the model as calibrated by BY and BKY, highlighting the ability of the model to fit some basic facts about asset prices and the crucial importance of volatility shocks in the BKY calibration. Finally, this section shows that higher-order univariate autocorrelations of consumption and dividend growth, particularly in the 1930-2008 sample, contrast with the predictions of the long-run risks model.

The key testable prediction of the long-run risks model is that stock prices reflect investors' rational expectations of long-run consumption growth and volatility. Section 4 examines the ability of stock prices to predict consumption growth, dividend growth, excess stock returns, and the volatility of these series. This section also discusses the temporal pattern of correlation between stock prices and consumption growth, and multivariate predictions of growth rates and excess returns that use not only the log dividend-price ratio, but also lagged dependent variables and real interest rates.

Section 5 studies the implications of the long-run risks model for the term structure of real interest rates. Section 6 examines instrumental variables (IV) estimates of the elasticity of intertemporal substitution, which are less than one in aggregate data, and asks whether the long-run risks model can explain this fact. The long-run risks model implies downward bias in the IV estimates, but this bias is not large enough to explain the very low estimates in the data. Section 7 concludes. An appendix available online (Beeler and Campbell 2011) provides further technical details.

[4]It is also possible to derive analytical solutions to a discrete-state approximation of the model (Garcia, Meddahi, and Tédongap 2008).

2 The Long-Run Risks Model

2.1 Model statement

Bansal and Yaron (BY, 2004) and Bansal, Kiku, and Yaron (BKY, 2011) propose the following processes for consumption and dividend growth, denoted by Δc_{t+1} and Δd_{t+1} respectively:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\ \sigma^2_{t+1} &= \overline{\sigma}^2 + \nu(\sigma^2_t - \overline{\sigma}^2) + \sigma_w w_{t+1} \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \varphi \sigma_t u_{t+1} + \pi \sigma_t \eta_{t+1} \\ & \quad w_{t+1}, e_{t+1}, u_{t+1}, \eta_{t+1} \sim i.i.d.\, \mathcal{N}(0,1). \end{aligned} \tag{1}$$

Here x_t is a persistently varying component of the expected consumption growth rate. σ^2_t is the conditional variance of consumption, also persistently time-varying, with unconditional mean $\overline{\sigma}^2$. The variance process can take negative values, but this will happen only with small probability if the mean is high enough relative to the volatility of variance. Dividends are imperfectly correlated with consumption, but their growth rate Δd_{t+1} shares the same persistent and predictable component x_t scaled by a parameter ϕ, and the conditional volatility of dividend growth is proportional to the conditional volatility of consumption growth.[5]

2.2 Solution

BY solve the long-run risks model using analytical approximations. They assume that the log price-consumption ratio for a consumption claim, z_t, is linear in the conditional mean and variance of consumption growth, the two state variables of the model:

$$z_t = A_0 + A_1 x_t + A_2 \sigma^2_t, \tag{2}$$

and that the log price-dividend ratio for a dividend claim, $z_{m,t}$, is similarly linear:

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma^2_t. \tag{3}$$

[5] This process does not impose cointegration between consumption and dividends. Some more recent research, notably Bansal, Dittmar, and Kiku (2008) and Hansen, Heaton, and Li (2008), emphasizes such cointegration.

Under the assumption that a representative agent has Epstein-Zin utility with time discount factor δ, coefficient of relative risk aversion γ, and elasticity of intertemporal substitution ψ, the log stochastic discount factor for the economy is given by

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{a,t+1}, \tag{4}$$

where $\theta = (1-\gamma)/(1-1/\psi)$ and $r_{a,t+1}$ is the return on the consumption claim, or equivalently the return on aggregate wealth.

BY use the Campbell-Shiller (1988) approximation for the return on the consumption claim in relation to consumption growth and the log price-consumption ratio:

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{5}$$

where κ_0 and κ_1 are parameters of linearization. Substituting equations (1) and (5) into equation (4), the innovation in the log SDF can be written as

$$m_{t+1} - \mathrm{E}_t\left(m_{t+1}\right) = -\lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \sigma_t e_{t+1} - \lambda_w \sigma_w w_{t+1}, \tag{6}$$

where $\lambda_\eta = \gamma$, $\lambda_e = (1-\theta)\kappa_1 A_1 \varphi_e$, and $\lambda_w = (1-\theta) A_2 \kappa_1$. The $\lambda' s$ represent the market prices of risk for consumption shocks η_{t+1}, expected consumption growth shocks e_{t+1}, and volatility shocks w_{t+1} respectively.

In order to solve the model, one must find the unknown parameters A_0, A_1, A_2, A_{0m}, A_{1m}, A_{2m}, κ_0, and κ_1. Conditional on the linearization parameters κ_0 and κ_1, the A parameters can be found analytically. The parameters A_0 and A_2 determine the mean of the price-consumption ratio, $\overline{z}$, and the parameters κ_0 and κ_1 are simple nonlinear functions of $\overline{z}$. It is straightforward to iterate numerically until a fixed point for $\overline{z}$ is found. Campbell (1993) and Campbell and Koo (1997) study a somewhat simpler model, without volatility shocks, and find that the loglinear approximation method is highly accurate provided that numerical iteration is used to find a fixed point for $\overline{z}$, but approximation accuracy deteriorates noticeably if $\overline{z}$ is prespecified. Details of the approximate solution method for the long-run risks model are given in the online appendix (Beeler and Campbell 2011).

2.3 Calibration

Table I reports parameter values from the calibrations of BY and BKY. All parameters are given in monthly terms; thus mean consumption growth of 0.0015, or 15

basis points per month, corresponds to annualized growth of 1.8%. The monthly persistence of the predictable component of consumption growth is 0.979 in BY and 0.975 in BKY, implying half-lives of between two and three years (33 and 27 months respectively).

Dividends are more variable than consumption, and this is captured in the BY calibration by the parameters ϕ and φ. The first measures the sensitivity of predictable dividend growth to predictable consumption growth, while the second measures the ratio of the standard deviations of dividend shocks and consumption shocks. The first parameter is 3 in BY and 2.5 in BKY, while the second is 4.5 in BY and 5.96 in BKY. Both calibrations imply that dividend growth is less predictable than consumption growth, but the difference between the two processes is accentuated in BKY. In addition, BKY introduces a contemporaneous correlation of consumption shocks and dividend shocks, using the parameter π, that is absent (zero) in BY.

Both calibrations of the model imply that persistent growth shocks cause extremely volatile changes in the expected long-run level of consumption and dividends, even though the conditional volatilities of expected next-period consumption and dividends are low. The average absolute magnitude of the change in the expected long-run level of consumption is 1.3% per month (not annualized) in the BY calibration and 0.9% per month in the BKY calibration. These numbers are even larger for dividends, which have a leveraged exposure to long-run risks. The average magnitude of a monthly shock to expected long-run dividends is 3.9% in the BY calibration and 2.2% in the BKY calibration.

The persistence of volatility, ν, is 0.987 in BY, implying a half-life slightly over four years, and 0.999 in BKY, implying an essentially infinite (58-year) half-life. Volatility shocks have similar standard deviations in the two calibrations (0.0000023 in BY and 0.0000028 in BKY), but the greater persistence of volatility in BKY implies that volatility shocks are very much more important for asset prices in that calibration. The original BY calibration is driven by long-run consumption growth risk, whereas in the BKY calibration long-run volatility risk is more important.

The asset pricing properties of the long-run risk model depend on the preference parameters of the representative agent. Table I reports the parameters used in BY and BKY. BY consider relative risk aversion coefficients γ of 7.5 and 10, and assume an elasticity of intertemporal substitution ψ of 1.5 and a time discount factor δ of 0.998 per month, equivalent to a pure rate of time preference of 2.4% per year. In our empirical work for BY we will use risk aversion of 10. BKY set risk aversion at

10 and the EIS at 1.5, and use a higher time discount factor of 0.9989 per month, implying a pure rate of time preference of 1.3% per year.

2.4 Simulation

In the remainder of the paper, we compare the model with quarterly and annual data by simulating the model at a monthly frequency and then time-aggregating the data to a quarterly or annual frequency. First, we generate four sets of i.i.d. standard normal random variables and use these to construct the monthly series for consumption, dividends, and state variables using equation (1). Next, we construct quarterly (annual) consumption and dividend growth by adding three (twelve) monthly consumption and dividend levels, and then taking the growth rate of the sum. Low-frequency log market returns and risk free rates are the sum of monthly values, while log price-dividend ratios use prices measured from the last month of the quarter or year. Because the price-dividend ratio in the data divides by the previous year's dividends, we multiply the price-dividend ratio in the model by the dividend in that month and divide by the dividends over the previous year.

Following BY and BKY, we censor negative realizations of conditional variance, replacing them with a small positive number, but retaining sample paths along which the volatility process goes negative and is censored. Because volatility is so persistent in the BKY calibration, it is much more likely to go negative than in the BY calibration. In our simulations, we find negative realizations of volatility 1.3% of the time for the BKY process, but less than 0.001% of the time for the BY process. When we simulate 79-year paths of volatility using the BKY calibration, over half of them go negative at some point, whereas this happens less than 0.2% of the time for the BY process.

To initialize each simulation, we set state variables to their steady-state values but run each simulation for a "burn-in" period of ten years before using the output.

In subsequent tables, we report the median moments from 100,000 simulations run over sample periods equal in length to our empirical samples. We also report the fraction of the 100,000 moments that are smaller than the ones in the empirical data, that is, the percentile of the simulated moment distribution that corresponds to the empirical moment. This number (or one minus this number) can be interpreted as a p-value for a one-sided test of the long-run risks model using the particular moment

under consideration. To highlight this fact, we use bold font for percentiles that are smaller than 0.05 (5%) or larger than 0.95 (95%).

3 Basic Moments

3.1 Data and sample periods

In order to evaluate the performance of the long-run risks model, we follow BY and use data on US real nondurables and services consumption per capita from the Bureau of Economic Analysis. We take stock return and dividend data from CRSP and convert to real terms using the CPI. We create a proxy for the ex-ante risk free rate by forecasting the ex-post quarterly real return on three-month Treasury bills with past one-year inflation and the most recent available three-month nominal bill yield. This procedure, which is equivalent to forecasting inflation and subtracting the inflation forecast from the nominal bill yield, is described in detail in the online appendix (Beeler and Campbell 2011).

We use the longest available annual (1930-2008) and quarterly (1947.2-2008.4) datasets that include all necessary data. The empirical work in BKY uses an identical annual dataset. BKY argue that the primary focus should be on the annual dataset because it covers the longest time period. However, we believe it is important to examine how well the long-run risks model works in the postwar era, just as any empirical analysis examines whether results are sensitive to the inclusion of a few outlier observations.

BKY argue that the Great Depression provides an extremely important few observations to include when studying asset prices. It should be noted that the Great Depression itself represents a period in history that the model is very unlikely to generate. Over the four-year period 1930-1933, consumption declined by a cumulative 18% (26% relative to trend). Using finite-sample simulations equal in length to the long-run annual dataset, we find that four-year cumulative consumption declines larger than this occur in 6.9% of samples for the BY model and 6.6% of samples for the BKY model. Over the four-year period 1934-1937, consumption increased by a cumulative 19% (12% above trend). A four-year decline and subsequent four-year increase in consumption of equal or greater size happens in only 0.1% of samples for

the BY model and 0.3% of samples for the BKY model. A model with rare disasters in consumption is much more likely to generate historical data consistent with the Great Depression (Barro 2006, 2009, Barro et al. 2010).

Table II reports basic moments for the annual and quarterly US datasets, and the corresponding median moments implied by simulations of the BY and BKY calibrations of the long-run risks model with relative risk aversion $\gamma = 10$. The median simulated moments are calculated from 100,000 simulations of finite samples equal in length to the historical samples. We look at five variables: the changes in log consumption and dividends, log stock return, log risk free interest rate, and log price-dividend ratio. All variables are measured in real terms. For each variable, we report the mean, standard deviation, and first-order autocorrelation.

It is apparent from the left panel of Table II that the long-run risks model does a good job of matching many basic properties of the long-run annual data, including the means, standard deviations, and first-order autocorrelations of consumption growth, dividend growth, and stock returns. However, some problems are worth noting.

The model understates the volatility of the riskless interest rate at about 1.2% in the BY calibration and 1.0% in the BKY calibration, compared to 2.9% in the data. This is despite the fact that the real interest rate does not include the volatile inflation surprises usually associated with an ex-post real interest rate. Our use of a forecasting equation for the real interest rate reduces volatility, but movements in our proxy for the ex-ante real interest rate, especially during the Great Depression, are still much more volatile than in the model.

A more serious discrepancy is that the long-run risks model greatly understates the volatility of the log price-dividend ratio. In the model, the standard deviation of the log price-dividend ratio is 0.18 for both the BY and BKY calibrations, as compared with 0.45 in the annual data. Historical stock prices display low-frequency variation relative to cash flows, which is not captured by the model.[6]

The same issues arise in postwar quarterly data in the right panel of Table III. At first glance, the behavior of quarterly dividend growth is an additional problem. The model implies a modest positive autocorrelation of dividend growth, but in quarterly data dividend growth has a first-order autocorrelation of -0.58. However this results

[6]The historical standard deviation of the log price-dividend ratio is this high in part because stock prices were persistently high at the end of our sample period. If we end the sample in 1998, as in BY, we obtain a lower standard deviation of 0.36, still somewhat higher than in the model.

merely from seasonality in dividend payments, a phenomenon that is commonly ignored in stylized asset pricing models. Dividend seasonality should not be regarded as an important omission of the long-run risks model.

A more serious difficulty is that postwar quarterly consumption growth has a much lower volatility than either calibration of the model. Without the Great Depression included in the sample, the annualized standard deviation of consumption growth is only around 1%, much smaller than the 2.4% BY and 2.3% BKY calibrated values.

3.2 The relative importance of consumption and volatility shocks

The BY and BKY calibrations of the long-run risks model assign very different roles to movements in consumption growth and volatility. Movements in volatility have very little effect in the BY calibration, whereas they are primary in the BKY calibration. To establish this, we have calculated the moments shown in Table II for two simpler models, one with constant volatility and time-varying expected consumption growth, and one with iid consumption growth. Other parameters of the BY and BKY calibrations remain unchanged.

In the BY calibration, the equity premium is zero with iid consumption growth, 5.4% with constant volatility and time-varying expected consumption growth, and 5.5% with time-varying volatility. The standard deviation of the log price-dividend ratio is 0.07 with iid consumption growth, 0.18 with constant volatility and time-varying expected consumption growth, and 0.18 with time-varying volatility. (It is positive with iid consumption growth because one divides by the previous year's dividends which adds noise.) It is apparent that time-variation in volatility is of little consequence for the results reported by BY.

In the BKY calibration, the results are very different. The equity premium is 1.6% with iid consumption growth, 3.8% with constant volatility and time-varying expected consumption growth, and 6.6% with time-varying volatility. (The positive equity premium with iid consumption growth results from the positive correlation of dividend and consumption growth assumed in the BKY calibration.) The standard deviation of the log price-dividend ratio is 0.09 with iid consumption growth, 0.13 with constant volatility and time-varying expected consumption growth, and 0.18 with time-varying volatility. A large proportion of the equity premium, and a large

proportion of the variability in stock prices relative to dividends, result from time-varying volatility in the BKY calibration of the long-run risks model.

3.3 Variance ratios

Variance ratio statistics are commonly used to summarize the persistence of growth rates. Table III sheds more light on the dynamic behavior of consumption and dividend growth by reporting variance ratios for these series. For the postwar sample, we aggregate quarterly observations to an annual frequency to remove the effects of seasonality.

The variance ratio statistic at horizon K is the variance of K-period growth rates divided by K times the variance of one-period growth rates. For consumption growth, the K-period variance ratio is

$$\widehat{V}(K) = \frac{\widehat{\text{Var}}(\Delta c_{t+1} + ... + \Delta c_{t+K})}{K\widehat{\text{Var}}(\Delta c_{t+1})}. \tag{7}$$

As the sample size increases, the variance ratio converges to a triangular weighted average of the first $K-1$ population autocorrelations:

$$V(K) = \frac{\text{Var}(\Delta c_{t+1} + ... + \Delta c_{t+K})}{K\text{Var}(\Delta c_{t+1})} = 1 + 2\sum_{j=1}^{K-1}\left(1 - \frac{j}{K}\right)\rho_j, \tag{8}$$

while in finite samples, the variance ratio can differ from the corresponding average of sample autocorrelations (Lo and MacKinlay, 1989). Table III reports variance ratios for consumption and dividend growth for horizons K of two, four, and six years, corresponding to one, three, and five autocorrelations.

In both the long-run and postwar data, both consumption and dividends have positive first-order annual autocorrelations, so two-year variance ratios are between 1.2 and 1.4 in Table III. These values are not far from the 1.25 that would be implied by time-aggregation of a continuous-time random walk (Working 1960). When we look beyond the two-year horizon there is a preponderance of negative autocorrelations in the long-run annual data, and these negative autocorrelations generate variance ratios below one at a horizon of six years. In the postwar quarterly data, higher-order

autocorrelations are very close to zero, implying that the six-year variance ratios are almost exactly equal to the two-year variance ratios.[7]

The patterns in the long-run data are quite different from the behavior implied by the BY and BKY calibrations of the long-run risk model. The median six-year variance ratios generated by the BY calibration are about 2.3 for consumption and 1.9 for dividends, and these numbers are only slightly lower in the BKY calibration, 2.0 for consumption and 1.4 for dividends. Fewer than 1% of the simulated samples have variance ratios as low as those in the long-run data, implying that six-year variance ratios reject both the BY and BKY calibrations at the 1% level in this dataset. The postwar quarterly data contrast less strongly with the long-run risk model, failing to reject either calibration of the model at the 5% significance level.

4 What Do Stock Prices Predict?

4.1 Predicting stock returns, consumption, and dividends

In the long-run risks model, the main cause of stock price variability relative to dividends is predictable and persistent variation in consumption growth, which creates similar variation in dividend growth. Thus, it is natural to test the model by evaluating the ability of the log price-dividend ratio to predict long-run consumption and dividend growth. At the same time, a large empirical literature has argued that the log price-dividend ratio predicts excess stock returns and not dividend growth or real interest rates (Campbell and Shiller 1988, Fama and French 1988, Hodrick 1992). This suggests that one should compare the predictability of excess returns with the predictability of consumption and dividend growth, in the data and in simulations from the long-run risks model.

We undertake this exercise in Table IV for both the BY and BKY calibrations of the long-run risks model. We regress excess stock returns, consumption growth, and dividend growth, measured over horizons of 1, 3, and 5 years, onto the log price-dividend ratio at the start of the measurement period. We report results both

[7] Campbell and Mankiw (1987) and Cochrane (1988) noted a similar difference in estimated persistence between pre-war and post-war data on GNP growth. The appendix reports the first five sample autocorrelations, together with simulated values from the long-run risks model.

for annual data over the period 1930-2008 and for quarterly data over the period 1947.2-2008.4. Model results and not just empirical results differ across the two data sets, both because the sample periods differ in length which affects the finite-sample properties of the model, and because time-aggregation has different effects in quarterly and annual data. Time-aggregation increases measured cash-flow predictability, an effect which is larger in the annual model.

In the data, one must adopt a convention about the timing of measured consumption. Measured consumption is a flow that takes place over a discrete time interval, but in a discrete-time asset pricing model, consumption takes place at a point of time and consumption growth is measured over a discrete interval from one point of time to the next. To match the data to the model, one must decide whether measured consumption should be thought of as taking place at the beginning of each period, or the end. The former assumption gives a higher contemporaneous correlation of consumption growth and asset returns, and is advocated by Campbell (2003). The latter assumption generates a higher correlation between consumption growth and lagged financial market data, and is used by BY and Parker and Julliard (2005) among others. Here we report results using the end-of-period timing convention. Results using the beginning-of-period timing convention are qualitatively similar. We time-aggregate the model using the same timing assumption as in the data so that the comparison of data and model is legitimate.

The top part of Table IV shows the results for predicting excess stock returns. At the left, we report regression coefficients, t statistics, and R^2 statistics in annual and then quarterly postwar data. Then we report the median simulated R^2 statistics implied by the BY and BKY calibrations, followed by the percentiles of the simulated R^2 statistics corresponding to the statistics in the data. The appendix reports a similar analysis comparing the regression coefficients in the model to the data, with similar results. The remaining parts of Table IV repeat these exercises for consumption growth and for dividend growth.

Table IV shows a striking contrast between the patterns in the data and in the BY calibration of the long-run risks model. In the data, the log price-dividend ratio predicts excess stock returns negatively, with a coefficient whose absolute value increases strongly with the horizon. At a 5-year horizon, the R^2 statistic is 27% in long-run annual data and 26% in postwar quarterly data. However there is relatively little predictability of consumption growth in the data. At a one-year horizon there is some predictability in annual data but this predictability dies out rapidly. There is

no predictability of consumption growth in postwar quarterly data. Dividend growth predictability also appears to be short-term in the annual data and absent in postwar quarterly data.

These empirical patterns are the reverse of the predictions of the BY calibration of the long-run risks model. According to the model, regressions of excess returns on log price-dividend ratios have median R^2 statistics that never rise above 3% at any horizon, while regressions of consumption and dividend growth on log price-dividend ratios have high explanatory power even at long horizons. At five years, the median explanatory power of the log price-dividend ratio is 29% for consumption growth and 26% for dividend growth.

For excess returns, median finite-sample R^2 statistics are very small under the null that the long–run risks model describes the data. However, the finite-sample distribution of these statistics has a fat right tail because of the well-known Stambaugh (1999) bias in predictive regressions with persistent regressors whose innovations are correlated with innovations in the dependent variable. The bias affects not only the coefficients, but also the t statistics and R^2 statistics of predictive regressions (Cavanagh, Elliott, and Stock 1995). Because of this problem, the predictability of excess returns can only be used to reject the model statistically at horizons greater than one year in annual data. For consumption and dividend growth, Stambaugh bias is a much less serious concern, and both the regression coefficients and R^2 statistics deliver strong statistical rejections of the long-run risk model at almost all horizons.

BY conducted a similar exercise to this with qualitatively similar but quantitatively less extreme results. However, their work used a numerical technique to solve the model that has since been abandoned in the literature. The authors have since argued that analytical solutions are more reliable for assessing the model's empirical properties (Bansal, Kiku and Yaron (2007)), and they use similar analytical solutions to ours in BKY. Bui (2007) and Garcia, Meddahi, and Tédongap (2008) also report results similar to ours.

We repeat this analysis for the BKY calibration of the long-run risks model. Recall that this calibration greatly increases the persistence of volatility; it therefore increases the effect of volatility on asset prices and the predictive power of the log price-dividend ratio for excess stock returns, and reduces the predictive power for consumption and dividend growth. At a five year horizon, the median explanatory power of the log price-dividend ratio is 4.3% for excess stock returns, 8.5% for consumption growth and 6.1% for dividend growth. In finite samples there are enough

simulations in which stock prices have spuriously increased predictive power for stock returns, and decreased predictive power for consumption and dividend growth, that statistical rejections of the model are less extreme for this calibration. In annual data only 5-year return and consumption predictability reject the BKY calibration at the 5% one-tailed significance level, while in postwar quarterly data the model is rejected only at a one-year horizon.

In summary, the long-run risks model, as its name suggests, tends to generate stock prices that reveal the long-run prospects for consumption and dividend growth. This does not seem to be the case in the data, so extreme movements in volatility, as assumed by BKY, are needed to bring the model into even rough concordance with the data.

4.2 The timing of consumption and stock price variation

The contrast between the long-run risks model and the data can be better understood by studying the timing of the relationship between stock prices and consumption growth. Consider a regression of K-period time-aggregated consumption growth onto the log price-dividend ratio, with a lead of j periods:

$$\Delta c_{t+j} + ... + \Delta c_{t+j+K} = \alpha_{jK} + \beta_{jK}(p_t - d_t) + \varepsilon_{jKt}. \tag{9}$$

When $j \geq 1$, this is a predictive regression of the sort we have reported in Table VI. The long-run risks model implies that both the regression coefficient and the R^2 statistic of the regression should be highest when j is around $1 - K/2$, declining slowly as j moves away from this value. The timing convention for consumption implies that the log price-dividend ratio in period t is most highly correlated with consumption in period $t + 1$. As one moves away from period $t + 1$, in either direction, predictability declines as the distance from the state variable x_t increases. Predictability is maximized when the K-period growth rates are centered around the time $t + 1$.

Figures 1 and 2 plot the regression coefficient β_{jK} and R^2 statistic R^2_{jK} against j, for several alternative horizons K. Figure 1 plots the β_{jK} and R^2 statistics for annual data and Figure 2 is based on quarterly data. The top panel of each figure shows a one-year horizon, the middle panel shows a three-year horizon, and the bottom panel shows a five-year horizon. The graphs on the left side of each figure show the regression coefficients and the graphs on the right side of the figure show the R^2. Each

graph contains three curves, one for the BY calibration, one for the BKY calibration, and one for the historical data. The BKY curves are lower than the BY curves, but the historical curves are much lower again.

The long-run risks model is sometimes described as a "forward looking" asset pricing model, with asset prices more correlated with future consumption growth. These figures demonstrate that in fact the price-dividend ratio in the long-run risks model is just as correlated with past consumption growth as it is with forward looking consumption growth. This is because the persistent unobserved state variable x_t is correlated with past consumption growth as well as future consumption growth.

It is noteworthy that in Figure 1 the curves for regression coefficients are particularly shifted down in the predictive region $j \geq 1$ at the right of the figures, while in Figure 2 both the regression coefficient and R^2 curves appear shifted down in this region. In this sense the empirical curves are shifted to the left relative to the theoretical curves. To the extent that stock prices are related to consumption growth, they appear relatively more responsive to lagged consumption growth, and relatively less predictive of future consumption growth, than the long-run risks model implies. Responsiveness of stock prices to lagged consumption growth is a phenomenon that is captured by habit formation models such as Campbell and Cochrane (1999), although of course the Campbell-Cochrane model shares with the long-run risks model a counterfactually strong relation between consumption growth and stock prices.

4.3 Predicting volatility

Movements of consumption volatility are also important drivers of stock prices in the long-run risks model, and particularly so in the BKY calibration of that model. Thus it is appropriate to evaluate the model by asking whether it matches the ability of stock prices to predict future realized volatility of consumption, dividends, or excess stock returns. In Table V, we do this using a measure of realized volatility suggested by Bansal, Khatchatrian and Yaron (2005).

We begin by fitting an AR(1) process for each variable y_t that we are interested in:

$$y_{t+1} = b_0 + b_1 y_t + u_{t+1}. \tag{10}$$

Then we calculate K-period realized volatility as the sum of the absolute values of

the residuals over K periods:

$$Vol_{t,t+K-1} = \sum_{k=0}^{K-1} |u_{t+k}|. \tag{11}$$

Finally, we regress the log of K-period realized volatility onto the log price-dividend ratio:

$$\ln [Vol_{t+1,t+K}] = \alpha_c + \beta_c(p_t - d_t) + \xi_t. \tag{12}$$

Table V shows that the log price-dividend ratio predicts consumption volatility, with a negative sign, at horizons from 1 to 5 years. The effect is highly statistically significant, and the explanatory power of the regressions at a 5-year horizon is 20% in long-run annual data and 46% in postwar quarterly data. The evidence that the log price-dividend ratio predicts dividend or return volatility is considerably weaker.

The BY calibration of the long-run risks model generates a relation between stock prices and consumption or dividend volatility with the same negative sign that we observe in the data. However, the effect in the model is far weaker than in the data; the explanatory power of the regressions is trivially small. The BY calibration of the long-run risks model is strongly rejected statistically on the basis of its lack of explanatory power for consumption volatility. It is, however, consistent with the weak relation between stock prices and the future volatility of stock returns and dividend growth observed in the data.

The BKY calibration of the model has a much more persistent volatility process. This increases the predictive power of stock prices for consumption and dividend volatility to levels that match the data quite well. Unfortunately, the model also predicts that stock prices should be good predictors of the future volatility of stock returns. The median R^2 at a 5-year horizon is 11% for the annual model and 18% for the quarterly model. In contrast the R^2 in the data are 0.1% for the long-run annual data and 6% for the postwar quarterly data (where the regression coefficient has the opposite sign to that predicted by the model). The discrepancies between the model-implied and empirical R^2 statistics are significant at the 5% level for 3- and 5-year horizons in the annual data, and almost significant for the 1-year horizon in the postwar quarterly data. Thus the BKY calibration creates a new puzzle: if stock prices are driven by persistent changes in the volatility of consumption, which in turn moves the stock market, why don't they forecast the future volatility of the stock market itself?

To ensure the robustness of these results, we have also considered a measure of realized volatility used by Campbell (2003). We start by regressing each variable of interest y_{t+1} onto the log price-dividend ratio:

$$y_{t+1} = b_0 + b_1(p_t - d_t) + u_{t+1}. \tag{13}$$

In the second stage, we regress the K-period average of squared residuals onto the log price-dividend ratio:

$$\frac{\sum_{k=0}^{K-1} u_{t+1+k}^2}{K} = \alpha_c + \beta_c(p_t - d_t) + \xi_t. \tag{14}$$

The general pattern of results using this method is very similar to those using the Bansal, Khatchatrian and Yaron (2005) method.

The message of this section is that the BY calibration of the long-run risks model greatly understates the effect of consumption volatility on stock prices. The BKY calibration does much better in this respect, in effect changing the driving force of the model from consumption growth to consumption volatility. However, this leads to a new difficulty, which is that there has only been a weak historical relation between stock prices and the volatility of stock returns themselves. An interesting challenge for future research will be to build a model that matches the strong relation between stock prices and consumption volatility without generating a counterfactually strong relation between stock prices and stock return volatility.

4.4 Multivariate predictability

BKY argue that a vector autoregression (VAR) provides evidence for the predictability of cash flows in annual data. Their methodology differs from ours in two important respects. First, they include a larger information set with three predictor variables: lagged cash-flow (consumption or dividend) growth, the riskfree rate, and the log price-dividend ratio. Second, instead of directly regressing long-horizon cash-flow growth onto the predictor variables, BKY estimate a first-order VAR and use it to infer long-run cash-flow predictability under the assumption that the model is correctly specified. This procedure does not give the same results as direct long-horizon regression if the model is misspecified. For example, if annual consumption is the time aggregate of a random walk, it is an MA(1) process with short-term predictability

but no longer-term predictability beyond one year ahead. In this case the VAR(1) will exaggerate the long-run predictability of consumption growth.

In order to separate the impact of a larger information set from the impact of the VAR methodology, we perform predictive regressions of cash-flow growth over horizons of 1,3 and 5 years onto BKY's information variables. These regressions expand the information set while maintaining our direct long-horizon regression methodology. We report the results in Table VI for the 1930-2008 annual data set, the same dataset used in the BKY VAR. The table also shows multivariate long-horizon regression results for excess stock returns and riskless interest rates.

Table VI reports regression R^2 statistics. The R^2 statistic from the data is on the left, followed by the median R^2 statistics from the two calibrations, and finally the percentiles of the model distribution for the R^2 statistic in the data.

When we regress excess stock returns onto a multivariate information set including lagged excess stock returns, the real interest rate, and the log price-dividend ratio, the R^2 statistics in the data are extremely close to the R^2 statistics earlier reported for univariate regressions in Table IV. The inclusion of lagged excess stock returns and the real interest rate barely increases the predictability of stock returns in the data.[8] However, the predictability of excess returns increases slightly in both calibrations of the model. As a result, the model becomes slightly more likely to generate as much excess return predictability as we see in the data.

The multivariate regression predicting consumption growth from lagged consumption growth, the real interest rate, and the log price-dividend ratio has much higher short-run predictability than the univariate regression reported in Table IV, 27% rather than 6%, reflecting short-run positive autocorrelation of consumption growth. However, the explanatory power of the multivariate regression dies out rapidly and is only 4% at a 5-year horizon. BKY report a 22% R^2 statistic at the 5-year horizon using their VAR procedure. If their VAR were correctly specified, our approach would deliver the same R^2 statistic in a large enough sample since both procedures use the same information. Thus the primary reason for the high long-horizon predictability reported by BKY is likely VAR misspecification. The long-horizon R^2 statistics implied by the BY and BKY calibrations are far larger than those we measure in the data, providing further evidence against the long-run risks model.

[8] For this case, the empirical results are similar if we use a VAR methodology as in Campbell and Shiller (1988) and Campbell (1991).

For dividend growth, there is more multivariate predictability even at long horizons, with an R^2 of 10% at the 5-year horizon. This is still much smaller than the 28% VAR implied R^2 that BKY report at the 5-year horizon, with the difference again attributable to misspecification. The BY calibration implies more multivariate predictability than found in the data, but the BKY calibration is fairly successful at matching this moment of the data.

Finally, real interest rates are substantially predictable from past real interest rates, consumption growth, and the log price-dividend ratio. This is true in the data but much more so in the BY and BKY calibrations of the long-run risks model. At a 1-year horizon, the difference between empirical and model-implied predictability is large enough to reject the BY and BKY calibrations statistically.

A limitation of Table VI is that it looks only at the overall explanatory power of multivariate regressions and not at individual regression coefficients and t statistics. Tables reported in the appendix examine the multivariate regressions more closely and find further evidence against the BKY calibration of the long-run risks model. In the regression for consumption growth, the BKY calibration implies that the real interest rate should predict next period's consumption growth positively. In the data, we find the opposite sign.

Similarly, in the multivariate regression for dividend growth, the only statistically significant variable predicting 5-year dividend growth is lagged past dividend growth, but with a negative sign. This result reflects the negative dividend autocorrelations that were reported in Table III. The implied process for dividend growth is mean-reverting, and therefore it does not generate a large equity premium. In this important sense the long-horizon multivariate predictability of dividend growth is evidence against the long-run risks model rather than evidence for it.

5 The Term Structure of Real Interest Rates

The long-run risks literature has focused primarily on stock prices, but the model has important implications for the term structure of real interest rates as well. In a consumption-based model with power utility, the risk premia on long-term real bonds relative to short-term real bonds (real term premia) depend on the covariance between innovations to consumption and innovations to real interest rates. If con-

sumption follows a mean-reverting process such that positive shocks to consumption are expected to reverse themselves through slower subsequent consumption growth, then a positive consumption shock causes real interest rates to fall, and bond prices to rise. In this case real bonds are risky and there is a positive real term premium. On the other hand, if consumption growth follows a persistent process such that positive shocks cause upward revisions in expected future consumption growth, then a positive consumption shock causes real interest rates to increase and bond prices to fall. In this case real bonds hedge consumption risk and have a negative real term premium (Campbell 1986).

With Epstein-Zin utility as assumed by the long-run risks model, revisions in expected future consumption growth command a risk premium even if they are uncorrelated with shocks to contemporaneous consumption growth. Since increases in expected consumption growth drive up real interest rates and drive down bond prices, the long-run risks model with a positive risk premium for consumption growth implies a negative real term premium. This fact has been pointed out by Piazzesi and Schneider (2006).

Shocks to consumption volatility also lower the term premium in the model. An increase in consumption volatility lowers real interest rates and increases real bond prices by stimulating precautionary savings. The increase in bond prices hedges the unfavorable shock to volatility, increasing average bond prices and reducing term premia. This effect is more powerful the more persistent is volatility.

In Table VII we report the moments of yields and returns on real zero coupon bonds for the BY calibration in the top panel and the BKY calibration in the bottom panel. The results are the medians from 100,000 finite sample simulations. In each panel we report the mean real yield, mean yield spread relative to the short-term riskless interest rate, the real term premium, and the standard deviation of excess real bond returns. We report results for a range of maturities from 3 months to 30 years.

In the top panel, we find that the BY calibration generates an average yield spread of -1.8% for the 30-year zero-coupon bond relative to a short-term riskless bond and an average spread of -1.0% between a 30-year zero-coupon bond and a 5-year zero-coupon bond. The behavior of the real term structure is another troubling difficulty for the long-run risks model of asset prices.

In the bottom panel, the problem is even more serious for the BKY calibration.

The persistence of volatility in that calibration makes long-run consumption extremely uncertain, lowering the safe long-term discount rate in the manner described by Martin (2009, 2010) and Weitzman (2007, 2009). Now the average yield spread between a 30-year zero-coupon bond and the short-term riskless bond is -2.9% and the average yield spread between 30-year and 5-year zero-coupon bonds is -2.2%. In addition, the average 30-year real interest rate implied by the BKY calibration is far below zero, at -1.7%.

Although there are less than fifteen years of inflation-indexed bond data in the United States, the observed term structure of Treasury inflation-protected securities (TIPS) has never had a quantitatively significant negative slope. The current real term structure is steeply upward sloping. A positively sloped term structure during the late 1990's and 2000's could be consistent with the BKY model if this period was a time of historically low expected short-term growth or historically high short-term volatility. However, the high price-dividend ratio of the stock market during this period suggests exactly the opposite, that this period was one of historically high expected growth or historically low volatility. Possibly even more troubling from the perspective of the data is the strongly negative real yield. Campbell, Shiller, and Viceira (2009) report that the real yield on long-term TIPS has always been positive and is usually above 2%, in contrast with the negative values implied by the BKY calibration of the long-run risks model.

6 The Elasticity of Intertemporal Substitution

6.1 The need for a high EIS

Both the BY and BKY calibrations of the long-run risks model make the assumption that the elasticity of intertemporal substitution (EIS) is greater than one. This assumption is critical for the asset pricing properties of the model to match the data.

In the appendix, we present a table showing the effects of setting the EIS to 0.5 rather than the high value of 1.5 assumed by BY and BKY. When $\psi < 1$, the riskless interest rate is high and volatile because the representative agent dislikes increasing consumption over time and would like to borrow from the future to flatten

the upward-sloping and time-varying expected consumption growth path. Also, the equity premium is trivially small and the volatility of the price-dividend ratio is low, implying that the volatility of stock returns is close to the volatility of dividend growth.

The low volatility of the price-dividend ratio results from offsetting effects of expected consumption growth on stock prices. Rapid consumption growth raises stock prices by increasing expected future dividends, but lowers them by increasing real interest rates. The leverage parameter ϕ measures the strength of the effect on future dividends, while the reciprocal of the EIS, $1/\psi$, measures the strength of the effect on interest rates. If ϕ is close to $1/\psi$, then the two effects roughly cancel and stock prices respond only weakly to long-run growth shocks (Campbell 2003). The BY calibration assumes $\phi = 3$, so setting $\psi = 0.5$ produces a relatively stable price-dividend ratio and a volatility of stock returns close to the volatility of dividend growth.

The low equity premium with $\psi < 1$ is closely related. In the BY calibration, there is no contemporaneous correlation between consumption growth and dividend growth, so in a power utility model with $\psi = 1/\gamma$, the equity premium would be zero.[9] With Epstein-Zin utility, the equity premium depends not only on the covariance of the stock return with contemporaneous consumption growth, but also on its covariances with shocks to expected future consumption growth and consumption volatility. If $\psi > 1/\gamma$ as assumed in the long-run risks literature, an asset that pays off when there is an upward revision in expected consumption growth is risky and commands a premium. However, this premium is small when the response of stock prices to expected future consumption growth is weak. In addition, the sign of the premium for consumption volatility risk is ambiguous. When $\psi < 1$, stock prices increase when volatility increases and the risk premium for volatility shocks is negative (Lettau, Ludvigson, and Wachter 2008).

In the BKY calibration, an extreme problem arises in the case where $\psi = 0.5$. Because the rate of time preference is relatively low, and long-run uncertainty about consumption is high as a result of persistently time-varying volatility, consumers have a strong desire to save in the BKY calibration. If they are sufficiently risk averse, precautionary savings can make the equilibrium real interest rate negative and an infinite-lived consumption or dividend claim can have an infinite price. This

[9] The BKY calibration does allow for a positive contemporaneous correlation of consumption and dividend shocks, producing a small positive equity premium even in the power utility case.

happens with $\psi = 0.5$ and $\gamma = 10$. Since the consumption claim appears in the stochastic discount factor for Epstein-Zin utility, this problem prevents us from even calculating the equilibrium riskless interest rate for these cases. Meaningful results can be restored only by lowering the coefficient of risk aversion substantially.

This discussion makes it clear that the long-run risks model can only match the level of the equity premium, and the volatility of stock prices in relation to dividends, if the EIS is greater than one. This observation is not new; it has been emphasized by BY and other papers in the long-run risks literature.

6.2 Estimating the EIS

In a model with constant variance, a high EIS implies that the real interest rate should be perfectly correlated with, but less volatile than, predictable consumption growth. Hansen and Singleton (1983), followed by Hall (1988), Campbell and Mankiw (1989), and others, have used an instrumental variables (IV) regression approach to estimate the elasticity of intertemporal substitution from the homoskedastic Euler equation. One way to run the regression is as

$$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right) \Delta c_{t+1} + \eta_{i,t+1}. \tag{15}$$

In general the error term $\eta_{i,t+1}$ will be correlated with realized consumption growth so OLS is not an appropriate estimation method. However $\eta_{i,t+1}$ is uncorrelated with any variables in the information set at time t. Hence any lagged variables correlated with asset returns can be used as instruments in an IV regression to estimate $1/\psi$.

Alternatively, one can reverse the regression and estimate

$$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}. \tag{16}$$

If the orthogonality conditions hold, then the estimate of ψ in (16) will asymptotically be the reciprocal of the estimate of $1/\psi$ in (15). In a finite sample, however, Staiger and Stock (1997) have shown that the IV estimator is poorly behaved if the right hand side variable is difficult to predict. This means that if the Euler equation holds and ψ is small, it is better to estimate (16); however, if ψ is large, it is better to estimate (15).

Hall (1988) estimated an extremely small value of ψ using this approach. Campbell and Mankiw (1989) found some predictability of consumption growth associated with predictable income growth, but little predictable variation associated with interest rates, again implying a low ψ. Campbell (2003) summarizes these results and finds similar patterns in international data.

BY have criticized this literature on the grounds that time-varying volatility causes time-variation in the intercept of the Euler equation and biases the estimate of the elasticity of intertemporal substitution. While this criticism is correct in principle, it is an empirical question whether there is a large downward bias. In Table VIII we simulate the long-run risks model to see whether IV estimates of ψ are importantly downward biased.

Table VIII reports two-stage least squares estimates of equations (16) and (15), in both annual and quarterly data for both the BY and BKY calibrations of the model. The first two rows of each part of Table VIII report results using the log short-term real interest rate as the asset return, while the second two rows use the realized log stock return. We use the end-of-period timing convention for consumption. The instruments are the asset return, the consumption growth rate, and the log price-dividend ratio, lagged twice to avoid difficulties caused by time-aggregation of the consumption data (Wheatley 1988, Campbell and Mankiw 1989). The columns report empirical estimates, median finite-sample estimates implied by the BY and BKY calibrations of the long-run risks model, and finally the percentiles of the finite-sample distribution of model estimates corresponding to the empirical estimates.

The original BY calibration implies no downward bias in IV estimates of ψ when the asset is the real interest rate. The finite-sample median values of the regression coefficients are always at or above the true EIS of 1.5. The discrepancy between the model estimates and the data estimates provides strong statistical evidence against the model. There is however a serious finite-sample problem with IV estimates of ψ using the stock return as the asset return. If the long-run risks model is true, these estimates are strongly downward-biased and extremely noisy, so they cannot be used to reject the model. Presumably the poor finite-sample performance of IV regressions with stock returns reflects a weak instrument problem.

In the BKY calibration we do find that IV estimates of the elasticity of intertemporal substitution are biased downwards in several cases, but the median EIS estimates are always above 1.3. This reflects the greater importance of time-varying volatility in this calibration of the model. Even in the BKY calibration, however, the low IV

estimates of ψ using the risk free rate as an asset provide statistical evidence against the model in both annual and quarterly data.

These results highlight an empirical difficulty for the long-run risks model, that the real interest rate is so volatile relative to predictable variation in consumption growth. It is hard to reconcile this with the assumption of the model that assets are priced by a representative agent with an EIS greater than one.

Some authors have looked at disaggregated data and have found greater predictable variation in consumption growth than appears in aggregate data. Attanasio and Weber (1993) and Beaudry and van Wincoop (1996) have found higher values for ψ using disaggregated cohort-level and state-level consumption data. Vissing-Jorgensen (2002) points out that many consumers do not participate actively in asset markets; using household data she finds a higher value for ψ among asset market participants. But these results do not confirm the long-run risks model because that model is calibrated to aggregate consumption data. A general equilibrium model with limited asset market participation and long-run risk in the consumption of stock market participants is a different model that remains to be explored.

7 Conclusion

The long-run risks model of asset prices (Bansal and Yaron 2004) is an important advance in that it allows economists to understand asset price variation in an economy with persistent shocks to both consumption growth and volatility, while making a realistic distinction between aggregate consumption and dividends.

However, the model has several important difficulties as a quantitative description of US financial history. In US data there is little evidence either for long-run persistent fluctuations in consumption and dividend growth rates, or for the ability of stock market participants to predict these growth rates. Results are similar whether we look only at stock prices as growth rate predictors, or whether we also consider lagged growth rates and real interest rates. This finding implies that the long-run risks model cannot use persistent variations in consumption growth as the main force driving stock market variation.

Bansal, Kiku, and Yaron (2011) recognize this problem and recalibrate the model to emphasize persistent variations in consumption volatility. However this creates a

new difficulty, which is that although stock prices strongly predict future consumption volatility, they have little predictive power for the future volatility of stock returns. The discrepancy between these two types of volatility movements is an interesting issue for future research.

The long-run risks model generates extremely low yields and negative term premia on long-term inflation-indexed bonds. In the calibration of Bansal, Kiku, and Yaron (2011), this implies real yields far below zero. While US inflation-indexed bond yields were extremely low in the early stages of the financial crisis of 2007-08, they have been high enough in most other periods to create yet another empirical challenge for the long-run risks model of asset prices.

A final difficulty for the long-run risk model is that aggregate consumption growth does not respond to variations in the short-term real interest rate in the manner required by the model's assumption that the elasticity of intertemporal substitution is greater than one. Although Bansal and Yaron (2004) correctly point out that time-varying consumption volatility can bias traditional estimates of the elasticity of intertemporal substitution that assume homoskedastic consumption growth, this bias is not large. Another challenge for future research is to resolve the apparent contradiction between the smoothness of consumption in the face of real interest variation, which suggests a low elasticity of intertemporal substitution, and the negative response of stock prices to consumption volatility, which suggests a high elasticity.

It is of course possible that other parameter choices within the long-run risks framework, or extensions of the framework to include features such as habit formation, rare disasters, or cointegration of consumption and dividends, will match the data more closely than these calibrations do. It would be natural to estimate such modifications of the model using a generalized method of moments (GMM) estimator, asking the model to fit both the volatilities and means of asset returns and the predictive moments we have emphasized in this paper. This is challenging given the number of parameters in the model, the size of available datasets, and the persistent processes followed by some state variables. Despite this technical challenge, we hope that future research will pay close attention to the forecastability of real interest rates, stock returns, consumption growth, and dividend growth, as well as the shape of the term structure of real interest rates, in judging the merits of proposed consumption-based asset pricing models.

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Table I

Long Run Risks Parameters

Endowment Process

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma^2_{t+1} &= \overline{\sigma}^2 + \nu(\sigma^2_t - \overline{\sigma}^2) + \sigma_w w_{t+1} \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \varphi \sigma_t u_{t+1} + \pi \sigma_t \eta_{t+1}
\end{aligned}
$$

Parameter	Symbol	BY Calibration	BKY Calibration
Mean Consumption Growth	μ_c	0.0015	0.0015
LRR Persistence	ρ	0.979	0.975
LRR Volatility Multiple	φ_e	0.044	0.038
Mean Dividend Growth	μ_d	0.0015	0.0015
Dividend Leverage	ϕ	3.0	2.5
Dividend Volatility Multiple	φ	4.50	5.96
Dividend Consumption Exposure	π	0.0	2.6
Baseline Volatility	$\overline{\sigma}$	0.0078	0.0072
Volatility of Volatility	σ_w	0.0000023	0.0000028
Persistence of Volatility	ν	0.987	0.999
Preference Parameters			
Risk Aversion	γ	7.5-10	10
EIS	ψ	1.5	1.5
Time Discount Factor	δ	0.9980	0.9989

Table I displays the model parameters for Bansal and Yaron (2004) (BY) and Bansal, Kiku and Yaron (2009) (BKY). The endowment process for the model is displayed above the table. All parameters are given in monthly terms. The standard deviation of the long run innovations is equal to the volatlity of consumption growth times the long run volatility multiple (LRR Volatility multiple) and the standard deviation of dividend growth innovations is equal to the volatility of consumption growth times the volatility multiple for dividend growth (Dividend Volatility Multiple). Dividend Consumption Exposure is the magnitude of the impact of the one period consumption shock on dividend growth. Dividend Leverage is the exposure of dividend growth to long-run risks.

Table II

Long Run Risks Moments

	Yearly Time Interval 1930-2008			Quarterly Time Interval 1947.2-2008.4		
Moment	**Data**	**Model BY**	**Model BKY**	**Data**	**Model BY**	**Model BKY**
$E(\Delta c)$	1.93	1.80	1.80	2.01	1.80	1.80
$\sigma(\Delta c)$	2.16	2.81	2.50	1.02	2.44	2.26
$AC1(\Delta c)$	0.45	0.47	0.40	0.26	0.31	0.27
$E(\Delta d)$	1.15	1.80	1.81	2.29	1.82	1.80
$\sigma(\Delta d)$	11.05	11.20	14.04	27.61	10.53	14.08
$AC1(\Delta d)$	0.21	0.37	0.26	-0.58	0.26	0.22
$E(r_e)$	5.47	6.75	6.07	6.36	6.75	6.02
$\sigma(r_e)$	20.17	16.70	18.68	16.52	16.54	18.51
$AC1(r_e)$	0.02	0.02	0.00	0.08	0.01	0.00
$E(r_f)$	0.56	2.59	1.23	0.89	2.59	1.25
$\sigma(r_f)$	2.89	1.21	0.97	1.82	1.23	0.97
$AC1(r_f)$	0.65	0.81	0.80	0.84	0.94	0.94
$E(p-d)$	3.36	3.01	3.13	3.46	3.01	3.14
$\sigma(p-d)$	0.45	0.18	0.18	0.43	0.18	0.17
$AC1(p-d)$	0.87	0.66	0.63	0.98	0.89	0.86

The BY and BKY models do a good job matching basic data moments with the primary exception being the low volatility of log price-dividend ratios in the models. Table II displays moments for the model and data from the annual and quarterly datasets. Columns 2-4 display the results for a yearly time interval and columns 5-7 display the results for the quarterly time interval. For each model, the moment displayed is the median from 100,000 finite sample simulations of equivalent length to the dataset. The consumption and dividend growth rates are calculated by first aggregating monthly consumption to yearly or quarterly levels, then computing the growth rate, then taking logs. The returns on equity and the risk free rate are aggregated to a yearly or quarterly level by adding log returns within a year or quarter. For the yearly data, the growth rates and returns are in annualized percentage points. For quarterly data, the means are mulitplied by four and standard deviation multiplied by two to annualize. For risk free rates, the annualized moments are the mean and standard deviation of annualized risk free rates (multiplied by four). For the log price-dividend ratio the yearly or quarterly value is taken from the last month of the year or quarter, with the price-dividend ratio divided by the previous year's dividend to match the construction in the data.

Table III

Variance Ratios for Consumption and Dividends

Consumption Variance Ratio

Moment	$\widehat{V}$	$V(50\%)$	$V(50\%)$	$\%\left(\widehat{V}\right)$	$\%\left(\widehat{V}\right)$
	data	**BY**	**BKY**	**BY**	**BKY**
$V\,(2)$ 1930-2008	1.40	1.47	1.40	0.271	0.505
$V\,(4)$ 1930-2008	1.38	2.01	1.78	**0.039**	0.126
$V\,(6)$ 1930-2008	0.84	2.32	1.97	**0.001**	**0.007**
$V\,(2)$ 1948-2008	1.27	1.46	1.39	0.087	0.202
$V\,(4)$ 1948-2008	1.29	1.96	1.74	**0.047**	0.121
$V\,(6)$ 1948-2008	1.29	2.22	1.88	0.065	0.151

Dividend Variance Ratio

Moment	$\widehat{V}$	$V(50\%)$	$V(50\%)$	$\%\left(\widehat{V}\right)$	$\%\left(\widehat{V}\right)$
	data	**BY**	**BKY**	**BY**	**BKY**
$V\,(2)$ 1930-2008	1.23	1.37	1.26	0.111	0.377
$V\,(4)$ 1930-2008	0.98	1.71	1.39	**0.008**	0.069
$V\,(6)$ 1930-2008	0.59	1.87	1.41	**0.000**	**0.006**
$V\,(2)$ 1948-2008	1.32	1.36	1.26	0.388	0.696
$V\,(4)$ 1948-2008	1.45	1.66	1.36	0.290	0.609
$V\,(6)$ 1948-2008	1.32	1.79	1.35	0.191	0.469

The longest sample of annual data shows enough evidence of mean reversion in consumption and dividend growth at long-horizons to statistically reject the persistent cash flow growth of the long-run risks model. Table III displays consumption and dividend variance ratios in the data and for the BY and BKY calibrations. For the yearly model, the consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to yearly levels, then computing the growth rate, then taking logs. For the quarterly sample period spanning the years 1948-2008, the quarterly consumption and dividend data are first added to aggregate to the yearly level. Then we calculate the growth rates and take logs. This removes the issue of dividend seasonality which has a large impact on quarterly dividend variance ratios. In the model, consumption and dividend growth rates for comparison to the annualized quarterly data are calculated using the same procedure as for the annual model. The second column displays the moment in the data, the next two display the medians for the BY and BKY calibrations, followed by the percentile of the data moment in both calibrations. The medians are from 100,000 samples of equivalent length to the data (948 or 732 months) and the percentile is the proportion of those samples with an estimate at or below that of the data. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Table IV
Predictability of Excess Returns, Consumption and Dividends

$$\sum_{j=1}^{J}\left(r_{m,t+j}-r_{f,t+j}\right)=\alpha+\beta\left(p_t-d_t\right)+\varepsilon_{t+j}$$

	$\widehat{\beta}$	t	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	-0.093	-1.803	0.044	0.007	0.011	0.918	0.841
3 Y	-0.264	-3.231	0.170	0.017	0.028	**0.980**	0.940
5 Y	-0.413	-3.781	0.269	0.025	0.043	**0.990**	**0.956**
4 Q	-0.119	-2.625	0.090	0.008	0.012	**0.980**	**0.952**
12 Q	-0.274	-3.191	0.187	0.022	0.033	**0.970**	0.933
20 Q	-0.424	-3.365	0.257	0.033	0.050	**0.969**	0.926

$$\sum_{j=1}^{J}\left(\Delta c_{t+j}\right)=\alpha+\beta\left(p_t-d_t\right)+\varepsilon_{t+j}$$

	$\widehat{\beta}$	t	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.011	1.586	0.060	0.324	0.145	**0.006**	0.202
3 Y	0.010	0.588	0.013	0.350	0.109	**0.002**	0.132
5 Y	-0.001	-0.060	0.000	0.285	0.085	**0.001**	**0.015**
4 Q	0.000	0.140	0.000	0.237	0.063	**0.000**	**0.023**
12 Q	-0.002	-0.296	0.001	0.269	0.068	**0.003**	0.069
20 Q	-0.003	-0.296	0.002	0.213	0.060	**0.014**	0.089

$$\sum_{j=1}^{J}\left(\Delta d_{t+j}\right)=\alpha+\beta\left(p_t-d_t\right)+\varepsilon_{t+j}$$

	$\widehat{\beta}$	t	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.074	1.977	0.092	0.404	0.194	**0.001**	0.165
3 Y	0.107	1.330	0.059	0.320	0.084	**0.015**	0.399
5 Y	0.089	1.214	0.039	0.255	0.061	**0.039**	0.401
4 Q	0.003	0.112	0.000	0.159	0.026	**0.001**	**0.043**
12 Q	0.012	0.193	0.001	0.180	0.029	**0.011**	0.115
20 Q	0.044	0.482	0.011	0.147	0.033	0.084	0.302

The long-run risks model, especially the BY calibration, has much more cash flow predictability and much less excess return predictability than the data. Columns 2-4 of Table IV display coefficients, t-statistics, and R-squared statistics from predictive regressions of excess returns, consumption growth and dividend growth on log price-dividend ratios in the 1930-2008 annual and 1947.2-2008.4 quarterly datasets. Throughout the table, the first part of each panel dsiplays annual results and the second quarterly. The next two columns following the data moments display the median R-squared statistics from finite sample simulations of the two calibrations. The last two columns report the percentile of the data moment for the model in both calibrations. Standard errors are Newey-West with 2*(horizon-1) lags. The medians from 100,000 samples of equivalent length to the data (948 or 741 months) and the percentile is the proportion of those samples with an estimate at or below that of the data.The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Table V
Predictability of Volatility: Excess Returns, Consumption and Dividends

Excess Return Volatility

	$\widehat{\beta}$	t	$\widehat{R}^2$	R^2 (50%)	R^2 (50%)	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	-0.084	-0.354	0.001	0.006	0.026	0.251	0.118
3 Y	-0.027	-0.166	0.001	0.015	0.082	0.114	**0.042**
5 Y	0.017	0.148	0.001	0.022	0.107	0.085	**0.032**
4 Q	0.056	0.337	0.002	0.009	0.106	0.262	0.052
12 Q	0.149	0.939	0.033	0.022	0.177	0.596	0.159
20 Q	0.161	1.732	0.061	0.030	0.183	0.666	0.238

Consumption Volatility

	$\widehat{\beta}$	t	$\widehat{R}^2$	R^2 (50%)	R^2 (50%)	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	-0.697	-2.193	0.059	0.006	0.031	**0.974**	0.687
3 Y	-0.583	-2.375	0.148	0.013	0.103	**0.982**	0.623
5 Y	-0.560	-2.999	0.204	0.020	0.132	**0.980**	0.653
4 Q	-0.619	-4.170	0.226	0.008	0.126	**1.000**	0.728
12 Q	-0.641	-4.667	0.437	0.021	0.197	**1.000**	0.891
20 Q	-0.552	-5.342	0.462	0.030	0.200	**0.999**	0.894

Dividend Volatility

	$\widehat{\beta}$	t	$\widehat{R}^2$	R^2 (50%)	R^2 (50%)	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*data*	*data*	*BY*	*BKY*	*BY*	*BKY*
1 Y	-0.501	-1.503	0.032	0.005	0.034	0.906	0.482
3 Y	-0.248	-0.681	0.017	0.013	0.105	0.554	0.172
5 Y	-0.148	-0.375	0.007	0.020	0.133	0.316	0.096
4 Q	-0.370	-1.366	0.049	0.009	0.128	0.899	0.274
12 Q	-0.247	-0.587	0.027	0.021	0.197	0.557	0.129
20 Q	-0.189	-0.412	0.019	0.030	0.199	0.409	0.109

The BKY calibration of the long-run risks model matches the high predictability of consumption volatility in the data but overstates the predictability of excess return volatility relative to the data. Columns 2-4 of Table VIII display coefficients, t-statistics, and R-squared statistics from predictive regressions of excess return, consumption or dividend volatility on the log price-dividend ratio for the 1930-2008 annual and 1947.2-2008.4 quarterly datasets. Volatility is measured as the sum of absolute residulas from an AR(1) model of consumption growth, dividend growth or excess returns. Throughout the table, the first part of each panel dsiplays annual results and the second quarterly. The next two columns following the data moments display the median R-squared statistics from finite sample simulations of the two calibrations. The last two columns report the percentile of the data moment for the model in both calibrations. Standard errors are Newey-West with 2*(horizon-1) lags. The medians from 100,000 samples of equivalent length to the data (948 or 741 months) and the percentile is the proportion of those samples with an estimate at or below that of the data. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Table VI
Multivariate Predictability

$\sum_{j=1}^{J}(r_{m,t+j}-r_{f,t+j})=\alpha+\beta_1(r_{m,t}-r_{f,t})++\beta_2(p_t-d_t)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.052	0.033	0.043	0.704	0.584
3 Y	0.171	0.050	0.086	0.947	0.812
5 Y	0.271	0.061	0.118	**0.973**	0.868

$\sum_{j=1}^{J}(\Delta c_{t+j})=\alpha+\beta_1\Delta c_t++\beta_2(p_t-d_t)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.269	0.405	0.298	0.125	0.401
3 Y	0.080	0.392	0.268	**0.010**	0.067
5 Y	0.039	0.321	0.227	**0.010**	**0.044**

$\sum_{j=1}^{J}(\Delta d_{t+j})=\alpha+\beta_1\Delta d_t++\beta_2(p_t-d_t)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.205	0.467	0.258	**0.004**	0.299
3 Y	0.129	0.341	0.138	**0.036**	0.465
5 Y	0.104	0.280	0.130	0.072	0.406

$\sum_{j=1}^{J}(r_{f,t+j})=\alpha+\beta_1\Delta c_t+\beta_2(p_t-d_t)+\beta_3 r_{f,t}+\varepsilon_{t+j}$

	$\widehat{R}^2$	$R^2\,(50\%)$	$R^2\,(50\%)$	$\%\left(\widehat{R}^2\right)$	$\%\left(\widehat{R}^2\right)$
	data	*BY*	*BKY*	*BY*	*BKY*
1 Y	0.483	0.772	0.698	**0.001**	**0.027**
3 Y	0.333	0.539	0.500	0.056	0.132
5 Y	0.325	0.384	0.379	0.337	0.372

Regressions with a larger information set have a greater ability to predict short-term cash flows but not necessarily long-term cash flows. Column 2 of Table VI displays R-squared statistics for predictive regressions of excess returns, consumption growth, dividend growth or the risk free rate on predictor variables in the 1930-2008 annual dataset. For consumption growth, dividend growth and excess returns the predictor variables are the risk free rate, the log price-dividend ratio and lagged consumption growth, dividend growth or excess returns. For the risk free rate, the predictor variables are the log price-dividend ratio, consumption growth and the lagged risk free rate. The next two columns following the data moment display the median R-squared statistics from finite sample simulations of the two calibrations. The last two columns report the percentile of the data moment for the model in both calibrations. The medians from 100,000 samples of equivalent length to the data (948 months) and the percentile is the proportion of those samples with an estimate at or below that of the data.The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Table VII
Term Structure Moments

BY Calibration									
Maturity	1m	3m	6m	1y	2y	5y	10y	20y	30y
Zero Coupon Term Structure									
Yield	2.58	2.55	2.49	2.38	2.19	1.74	1.30	0.91	0.75
Yield Spread		-0.04	-0.10	-0.20	-0.40	-0.84	-1.28	-1.68	-1.84
Excess Return		-0.08	-0.19	-0.39	-0.74	-1.39	-1.84	-2.05	-2.08
Ex Ret. Volatility		0.16	0.38	0.79	1.48	2.80	3.77	4.31	4.41

BKY Calibration									
Maturity	1m	3m	6m	1y	2y	5y	10y	20y	30y
Zero Coupon Term Structure									
Yield	1.22	1.19	1.15	1.07	0.91	0.51	-0.02	-0.88	-1.65
Yield Spread		-0.03	-0.07	-0.15	-0.31	-0.70	-1.22	-2.09	-2.85
Excess Return		-0.06	-0.14	-0.29	-0.57	-1.24	-2.09	-3.47	-4.68
Ex Ret. Volatility		0.13	0.32	0.66	1.21	2.28	3.37	5.40	7.40

The long-run risks model has a downward sloping term structure with very low long-term yields. Table VII displays moments for the real zero coupon term structure for the BY and BKY calibrations. The median moments from 100,000 simulations of equivalent length to the annual dataset are diplayed. The zero coupon yield is the average monthly log yield on a zero coupon bond of a given maturity multiplied by 12 to annualize. The zero coupon yield spread is the difference between the yield on the long maturity zero coupon bond and that on the one month bond, also annualized. Excess returns are calculated on the strategy of buying a long bond each month and selling it the following month to buy a new bond of the same maturity. Then yearly excess returns are calculated by adding monthly log excess returns. The excess return on long term bonds is the average of these yearly excess returns plus one half the variance of yearly excess returns. The standard deviation of yearly excess returns is displayed in the final row of the first panel.

Table VIII
Long Run Risks and the EIS

$$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}$$

Asset	Sample	$\widehat{\psi}$ *data*	$\psi(50\%)$ *BY*	$\psi(50\%)$ *BKY*	$\%(\widehat{\psi})$ *BY*	$\%(\widehat{\psi})$ *BKY*
$r_{f,t+1}$	1930-2008	-0.123	1.654	1.324	**0.000**	**0.020**
	1947.2-2008.4	0.306	1.495	1.343	**0.001**	**0.017**
$r_{m,t+1}$	1930-2008	-0.041	0.041	0.018	0.395	0.310
	1947.2-2008.4	0.004	0.030	0.008	0.471	0.484

$$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right)\Delta c_{t+1} + \eta_{i,t+1}$$

Asset	Sample	$\frac{1}{\widehat{1/\psi}}$ *data*	$\frac{1}{(1/\psi)}(50\%)$ *BY*	$\frac{1}{(1/\psi)}(50\%)$ *BKY*	$\%\left(\frac{1}{\widehat{1/\psi}}\right)$ *BY*	$\%\left(\frac{1}{\widehat{1/\psi}}\right)$ *BKY*
$r_{f,t+1}$	1930-2008	-0.679	1.816	1.871	**0.001**	**0.025**
	1947.2-2008.4	0.504	1.613	1.694	**0.000**	**0.007**
$r_{m,t+1}$	1930-2008	-0.137	0.428	0.160	0.445	0.399
	1947.2-2008.4	1.970	0.428	0.115	0.845	0.928

Regression estimates of the EIS in the model are close to the true value of 1.5 while estimates in the data are close to 0. Table VIII displays the EIS estimates using both the risk free rate and the market return as the asset for the BY and BKY calibrations. Medians are from a series of 100,000 samples of equivalent length to the data (948 and 741 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The instruments are consumption growth, the log price-dividend ratio and returns for the asset, all lagged twice. In the model, the EIS is 1.5. The percentile is in bold when the data moment is rejected by a 5 percent one-sided test or a 10 percent two-sided test.

Figure I Predictability of Yearly Consumption with Leads or Lags

$$\mathbf{\Delta c}_{t+j}+...+\mathbf{\Delta c}_{t+j+K}=\boldsymbol{\alpha}_{jK}+\boldsymbol{\beta}_{jK}(\mathbf{p}_t-\mathbf{d}_t)+\boldsymbol{\varepsilon}_{jKt}$$

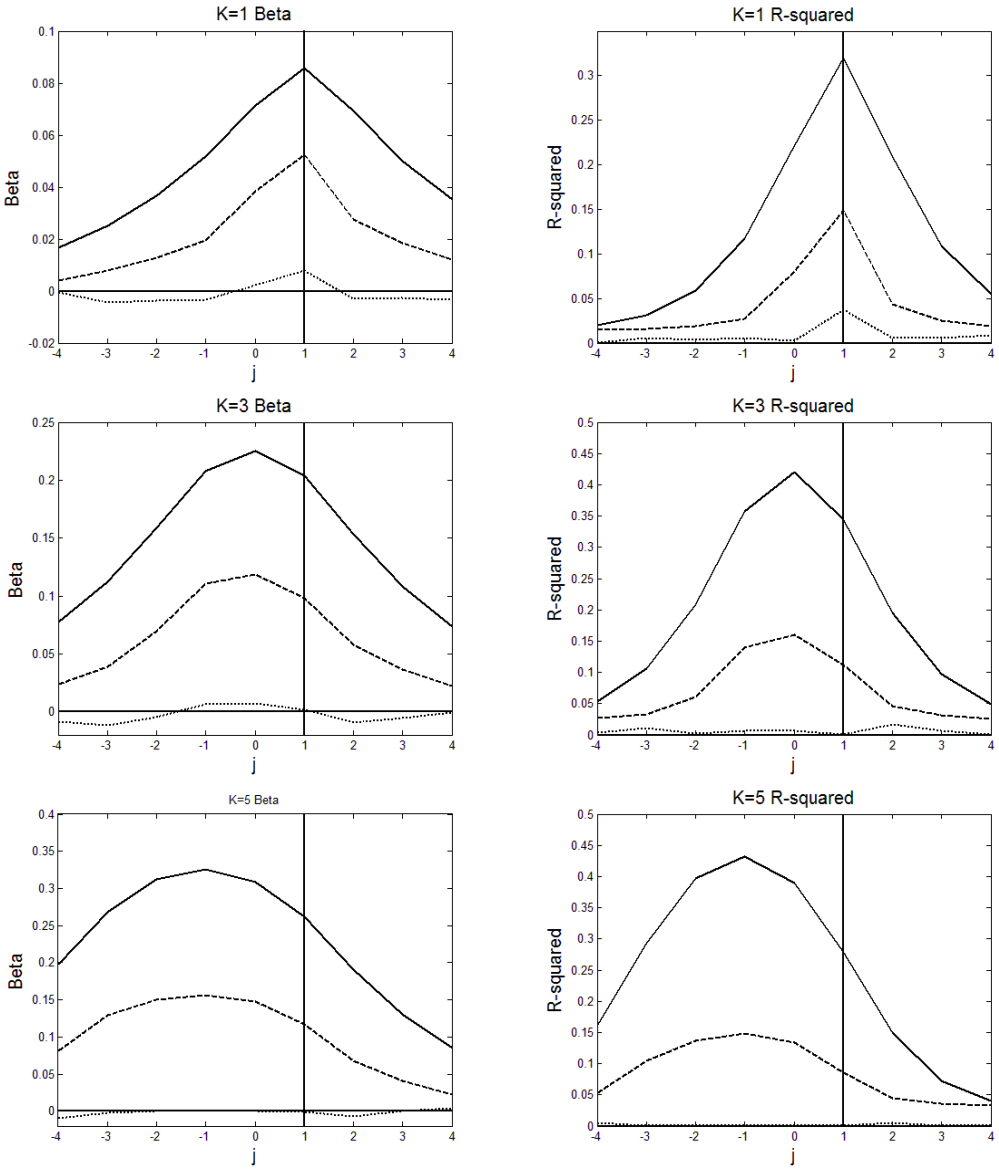

Figure 1 displays the coefficients and R-squareds from a regression of consumption growth over a 1, 3 or 5 year horizon on the log price-dividend ratio at different leads and lags. Solid lines are the BY model, dashed lines the BKY model and dotted lines are the data. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. Model regressions are the medians from 100,000 simulations of equivalent length to the data. The data is the 1930-2008 yearly dataset.

Figure II Predictability of Quarterly Consumption with Leads or Lags

$$\Delta c_{t+j} + \ldots + \Delta c_{t+j+K} = \alpha_{jK} + \beta_{jK}(p_t - d_t) + \varepsilon_{jKt}$$

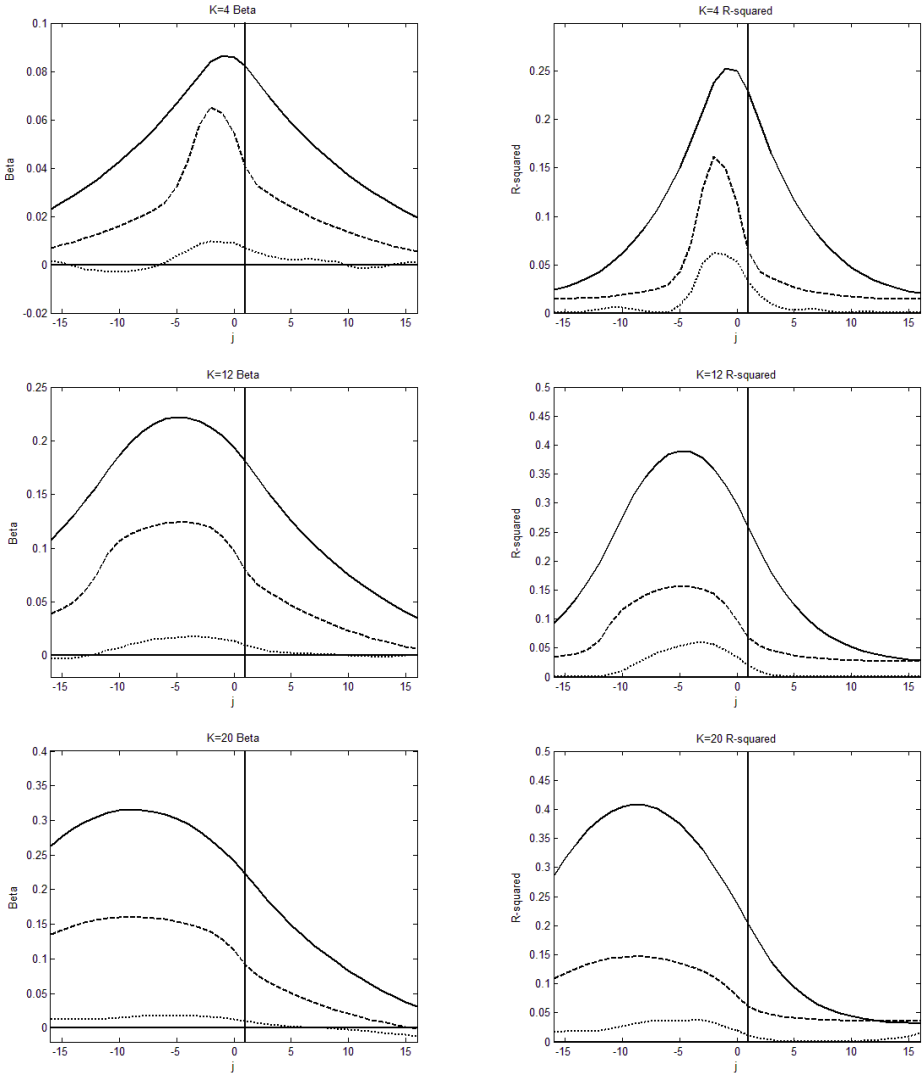

Figure 2 displays the coefficients and R-squareds from a regression of consumption growth over a 4, 12 or 20 quarter horizon on the log price-dividend ratio at different leads and lags. Solid lines are the BY model, dashed lines the BKY model and dotted lines are the data. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. Model regressions are the medians from 100,000 simulations of equivalent length to the data. The data is the 1947.2-2008.3 quarterly dataset.