New Equations for Atomic Mass, Mass per Nucleon and Mass defects of Elements in Terms of Rest Mass Based Neutron Numbers

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Abstract

Conventional neutron numbers are based on the approximations that the mass of the electron is zero and that those of the proton and neutron are each equal to unity. It was shown, by using the exact rest masses, that the neutron numbers are actually one or two less than the conventional ones for atomic masses greater than 108. Presented here are new equations for the atomic mass, mass per nucleon and mass defects for the most abundant stable isotope (or the one with the longest half life) of each of the 118 elements of the Periodic Table.

1. Introduction

Atomic masses of nuclides are usually interpreted through the mass defects (MD), $\Delta A_{Z,N}$, or the binding energies (B.E.), [1-3]

$$MD = A_{Z,N} - A_r = \Delta A_{Z,N} = B.E./c^2 \quad (1a)$$

$$A_{Z,N} = Zm_H + Nm_n \quad (1b)$$

where $A_{Z,N}$ is the expected mass sum for a nuclide with atomic number Z and neutron number N (= A – Z), A is the mass number[4a], A_r is the actual atomic mass[4b] relative to that of the carbon 12 isotope of atomic mass 12 u (u = atomic mass unit[4c], 1u = 931.494028 MeV), B.E., the binding energy, is the energy equivalent of the mass defect and c is the velocity of electromagnetic radiation. There have been several attempts to explain the exact values of the B.E., see e.g.[5-8]. However, the equations have so far been semi empirical and no exact solution has been found yet. In this work, rather than mass defects, the atomic masses as such have been interpreted based on the rest masses of the electron, proton and neutron. Simple equations have been presented for the atomic mass, mass per nucleon and mass defects, and the article has been divided into suitable sections.

2. The conventional neutron numbers

The mass number, A, of an isotope of an element is defined[1-3,4a] as the rounded up whole integer closest to the relative atomic mass, A_r, of the isotope. (The recommended symbols A for mass number[4a] and A_r for relative atomic mass[4b] used here replace M and A, respectively, used in the previous work[9,10] on rest mass based neutron numbers). In Table 1 are presented the values of Z (column 2), A_r (column 3, data from[11,12]) and the conventional neutron number N (= A –Z, column 4) for the most abundant stable isotope (or the one with the longest half life) of each of the 118 elements of the Periodic Table[13]. Also marked with asterisks in column 1 of Table 1, are the 21 elements which occur 100% in only one stable natural isotopic form. The exact atomic masses for the nine nuclides listed in the last rows of Table 1 are not yet known[13], but for the whole numbers.

3. The rest mass based neutron numbers

In the absence of the knowledge of the exact masses, the mass (m_e) of the electron was assumed[1-3] to be zero and those of the proton (m_p) and neutron (m_n) to be 1u, and the mass number, A, was considered to be equal to the nucleon number, Z + N. The conventional neutron numbers (N) are simply taken as the difference, A - Z. Note that this also implies that in equation (1b), $A_{Z,N}$ = Z + N = A and the mass defect and binding energy, $\Delta A_{Z,N}$ = A - A_r. It was pointed out in[9] that, although the exact rest masses of the electron, proton and neutron have become known with high precision[8,11,12] (m_e = 0.000548580u , m_p = 1.007276467u and m_n = 1.008664916 u), and the original idea that atomic masses of isotopes are whole numbers[14] have long changed in the light of later accurate data[8,11,12], the definition of the conventional neutron numbers as N = A - Z has continued to be in use, while the rest mass of the neutron and the conventional neutron number were used in equation (1b) to calculate $A_{Z,N}$, e.g. see[1-3].

Therefore, the exact rest mass based neutron numbers, $N_{r.m.}$ (this replaces the symbol N' used before[9]) were calculated in[9,10] for the then known 105 nuclides, from the atomic masses (A_r) in[11,12a] as,

$$A_r = Zm_H + N_{r.m.}m_n \qquad (2)$$

$$N_{r.m.} = (A_r - Zm_H)/m_n \qquad (3)$$

where $m_H = (m_e + m_p) = 1.00782505$ u is the mass of an atom of hydrogen isotope (H). Here, the values of $N_{r.m.}$ calculated using the recent atomic mass data[12b] (see column 3, Table 1) are tabulated in column 5 of Table 1. In[9] the values of $N_{r.m.}$, were rounded up and denoted by *N* (in italics, to distinguish it from N, the conventional number) such that $N > N_{r.m.} > (N - 1)$, were shown to be 1) *N* = N, for all nuclides with A_r < 108 (for Z < 48, see Table 1), 2) *N* = N -1 for those with A_r in the range, 108 < A_r > 251 (for Z = 48 to Z = 98) and 3) *N* = N – 2 for A_r > 251 (for Z = 99 and above). These can be seen by comparing the values of N (column 4) and *N* (column 6) in Table 1. Besides, it was pointed out[9] that, of the three particles, the electron, proton and neutron, since the last is the least stable (decays with a half life of about 10 mins. in the free state[15]), $N_{r.m.} < N$ and that the above equation (2) for the atomic mass gives the highest mass defect per nucleon, in accordance with the criterion for stability[2].

4. Equation for the atomic mass

This work reports the interesting finding that a graph of A_r versus the nucleon number, $Z + N_{r.m}$ (see column 7, Table 1) for all the present nuclides from Z = 1 to 118**,** is a straight line with a slope, m*, and a small intercept (see Figure 1). Also plotted in this graph are the coordinates for the nuclides of 21 elements which exist in only one stable isotopic form. The slope of this line is 1.00833917 u. This is close to that for the whole region from Z = 1 to 118, m* = 1.00834045 u. The product $(Z + N_{r.m})m^*$ reproduces A_r for all the 118 nuclides with an accuracy of ~ - 0.005 u in the third decimal place (see column 10, Table 1).

On re-writing equation (2) in terms of the nucleon number, $(Z + N_{r.m})$, it follows that the atomic mass, A_r is given (up to the sixth decimal place) by the equations,

$A_r = (Z + N_{r.m})m^* + [N_{r.m}\,\delta m_n - Z\delta m_H]$ (4a)

$= Z\,(m^* - \delta m_H) + N_{r.m}\,(m^* + \delta m_n)$ (4b)

$= Zm_H + N_{r.m.}m_n$ (2)

where $m^* = m_H + \delta m_H = m_n - \delta m_n = 1.00834045$ u, $\delta m_H + \delta m_n = (m_n - m_H) = 0.00083987$ u, $\delta m_n = 0.00032447$ u and $\delta m_H = 0.00051540$ u ($< m_e = 0.00054858$ u). The second term in the square brackets, $[N_{r.m}\,\delta m_n - Z\delta m_H] = \delta A_r$ is negative (see column 11, Table 1) and it accounts for the deviation of A_r from $(Z + N_{r.m})m^*$ from the third to the sixth decimal place. Note that the equation (4a) for the atomic mass contains the three constants, m^*, δm_H and δm_n.

Equation (4a) may be interpreted to mean that out of the mass, $(Z + N_{r.m})m^*$, the subtracted amount, δA_r is the mass shared by both Z and $N_{r.m}$, where δm_H and δm_n are distributed between H and n as shown by equation (4b) and their sum $\delta m_H + \delta m_n = (m_n - m_H)$. These values are compared with those of the quarks in the section below.

5. Neutrons, protons and quarks

If one considers the mass (m_n) of a neutron as constituted by two down quarks of mass m_{qd} each and one up quark of mass m_{qu} and the mass of a proton (m_p) as made of those of two up quarks and one down quark,[16] then, $(m_n + m_p)/3 = m_{qd} + m_{qu} = 0.671947128$ u = 625.9147369 MeV and $m_n - m_p = m_{qd} - m_{qu} = \delta m_H + \delta m_n + m_e = 0.001388449$ u = 1.2933320 MeV. From these, one gets,

$m_{qd} = 0.33666779$ u (= 313.6040349 MeV) (6a)

$m_{qu} = 0.33527934$ u (= 312.3107025 MeV) (6b)

These masses are close to those of constituent quarks[17]. The mass of the quark by itself, which is bound to the constituent quark by gluons, is termed the current quark mass, which is around 2.01 MeV (= 0.002158 u) for the up quark and 4.79 MeV (= 0.005142 u) for the down quark[17d,e]. These are about ten times higher than δm_n = 0.00032447 u (= 0.3022419 MeV), δm_H = 0.0005154 u (= 0.4800920 MeV) and m_e = 0.00054858 u (= 0.5109990 MeV).

6. Equation for the mass per nucleon

On dividing equation (4a) by $Z + N_{r.m}$, the atomic mass per nucleon, m_A is obtained as,

$$m_A = A_r/(Z + N_{r.m}) = m^* - \delta A_r/(Z + N_{r.m}) \quad (5a)$$

$$= m_H + N_{r.m}(m_n - m_H)/(Z + N_{r.m}) \quad (5b)$$

Equation (5b) shows the linear increase of m_A from the value $m_A = m_H$ at $N_{r.m} = 0$ as the ratio $N_{r.m.}/(Z + N_{r.m})$ (see column 8, Table 1) increases and reaches $m_A = m_n$ at Z = 0. The values of m_A are given in column 10 of Table 1. Figure 2 shows the linear variation of m_A with $N_{r.m}/(Z + N_{r.m})$ for all the nuclides. The slope of the line = $(m_n - m_H)$ = 0.00083987u and the intercepts, $m_A = m_n$ at $N_{r.m} = 1$ (for the neutron, Z = 0) and $m_A = m_H$ at $N_{r.m} = 0$ (for hydrogen, Z = 1).

7. Variation of the ratio $(A_r - Zm_H)/A_r$ with Z

The ratio, $(A_r - Zm_H)/A_r = N_{r.m.}m_n/A_r$ of the mass of neutrons in the atom to the atomic mass (data in column 8, Table 1) varies with Z as shown in Figure 3. The 21 nuclides which are available 100% in only one natural isotopic form are shown as filled circles. The rare gas nuclides are marked as open squares with the dotted line passing through them. The nuclides (He, C, N, O, Ne, Mg, Si, S and Ca) for which the ratio, $N_{r.m.}m_n/A_r \sim 0.5$ are marked as open squares. Note that of these, C, N, O and S, are the primary atoms in the life giving molecules, like the amino acids and DNA[18]. It is interesting that for values of Z > 75, the ratio, $N_{r.m.}m_n/A_r$ reaches a limiting value of about 0.61 (which is close to 0.618 = $1/\phi$, where ϕ is the Golden ratio[19,20]).

8. Equation for the mass defects of nuclides

The equation (1) for the mass defect written with the rest mass based whole neutron number, N, becomes

$$MD = A_{Z,N} - A_r = \Delta A_{Z,N} = (N - N_{r.m.})m_n \qquad (6)$$

where $A_{Z,N} = Zm_H + Nm_n$. Figure 4 shows a plot of MD vs Zm_H. It can be seen that there are three regions i) Z= 1 to 47, ii) Z = 48 to 98 and iii) Z = 99 to 118. The points in the first two regions are found to be close to straight lines (with the slopes a ~ 0.02 as given in the Figure 4) which can be represented by,

$$MD = \Delta A_{Z,N} = (N - N_{r.m.})m_n \sim aZm_H - b \qquad (7)$$

9. Summary

In this work, instead of the conventional semi empirical interpretations of the atomic masses through the binding energies, atomic masses as such of the most abundant stable isotopes of all the elements have been represented by the equation (4), in terms of the rest mass based neutron numbers and three constants. Also, the mass per nucleon is expressed as a linear function of the ratio $Z/(Z + N_{r.m})$ or $N_{r.m}/(Z + N_{r.m})$ by the equations (5a, b). For large values of Z, the ratio $(A_r - Zm_H)/A_r$ is found to reach a limiting value of about 0.61, which is close to the reciprocal of the Golden ratio. The mass defects are shown to vary nearly linearly with the atomic number for Z = 1 to 98. All the relevant data are presented in Table 1 and supporting Figures 1 – 4 have been provided.

References

1. Pauling L. *General Chemistry* (Freeman and Co., San Francisco, 1954).

2. Gucker, F. T. & Seifert, R. L. *Physical Chemistry* (The English Universities Press Ltd., London, 1967).

3. Housecroft, C. E. & Sharp, A. G. *Inorganic Chemistry* (Pearson Prentice Hall, N.J., 2008).

4. a) http://goldbook.iupac.org/M03726.html Last update: 2009-09-07; (last accessed: 18th May 2010)

b) http://goldbook.iupac.org/R05258.html, Last update: 2009-09-07; (last accessed: 18th May 2010)

c) http://physics.nist.gov/cgi-bin/cuu/Value?eqtukg;(last accessed: 18th May 2010)

5. Samyn, M., Goriely, S., Bender, M., Pearson, J. M. Further explorations of Skyrme-Hartree-Bogoliubov mass formulas. III. Role of particle-number projection. *Phys. Rev*. C **70,** 044309, 2004.

6. Koura, H., Tachibana, T., Uno, M., & Yamada, M., Nuclidic mass formula on a spherical basis with an improved even-odd term. *Progr. Theor. Phys*., **113,** 305-325, 2005.

7. Cook, N. D. *Models of the Atomic Nucleus: With Interactive Software.* (Springer, Berlin, New York, 2006).

8. Mohr, P. J., Tailor, B. N. & Newell, D. B. CODATA recommended values of the fundamental physical constants: 2006, *Rev. Mod. Phys.*, **80**, 633 -730, 2008; also in *J. Phys. Chem. Ref. Data*, **37,** 1187-1284, 2008; http://www.nist.gov/physlab/div842/fcdc/upload/chart.pdf

9. Heyrovska, R. On neutron numbers and atomic masses. *J. Chem. Educ*., **69**, 742-743, 1992.

10. Heyrovska, R. 216th National Meeting of the American Chemical Society, Boston, August 1998, a) short abstract nr. 9: Text of the full contribution printed by Desktop Publisher, Arjun Consultancy & Publishing Inc., Wayne NJ (USA), 1998, and b) short abstract nr. 11.

11. a) *Handbook of Chemistry and Physics*, 46th and 67th edns., CRC Press, Florida, 1965- 66 and 1986-87; b) Commission on Atomic Weights and Isotopic Abundances *Pure and Appl. Chem.*, **58**, 1677-1692, 1986.

12. a) Audi, G. & Wapstra, A. H. Nuclear and hadronic physics, *Nuclear Physics A*, **565**, Issue: 1, 1-65, 66- 157, 158-192, 1993.

b) NIST Standard Reference Database 144: http://physics.nist.gov/cgi-bin/Compositions/stand_alone.pl (last accessed: 18th May 2010).

13. http://www.webelements.com/ (last accessed: 18th May 2010).

14. http://nobelprize.org/nobel_prizes/chemistry/laureates/1922/aston-lecture.pdf, (see Table on page 16).

15. Christensen, C. J., Nielson, A., Bahnsen, A., Brown, W. K. & Rustad, B. M. Free-neutron beta-decay half-life. *Phys. Rev. D*, **5**, 1628-1640, 1972.

16. M. J. G. Veltman, http://www.youtube.com/watch?v=Xcww72_6gCI&feature=related , June 2006 (last accessed: 21st May 2010).

17. a) http://en.wikipedia.org/wiki/Constituent_quark: Ref. to A. Watson (2004). *The Quantum Quark*. Cambridge University Press. pp. 285–286, b) http://en.wikipedia.org/wiki/Quark: Ref. to C. Amsler *et al.*

"Review of Particle Physics: Quarks". *Physics Letters B* **667** (1): 1–1340, c)

http://pdg.lbl.gov/2008/tables/rpp2008-sum-quarks.pdf., d)

http://www.sciencedaily.com/releases/2010/05/100503162349.htm

e) http://news.sciencemag.org/sciencenow/2010/04/mass-of-the-common-quark-finally.html

18. Heyrovska, R. & Narayan, S. Atomic structures of molecules based on additivity of atomic and/or ionic radii: *Nature Precedings*, http://dx.doi.org/10.1038/npre.2009.3292.1, 2009.

19. Livio, M. *The Golden Ratio, the Story of Phi, the World's Most Astonishing Number* (Broadway books, New York, 2003).

20. Heyrovska, R. The Golden ratio in the creations of Nature arises in the architecture of atoms and ions. Ch. 12 in book: *Innovations in Chemical Biology*, (Ed.: Sener, B. Springer, 2009).

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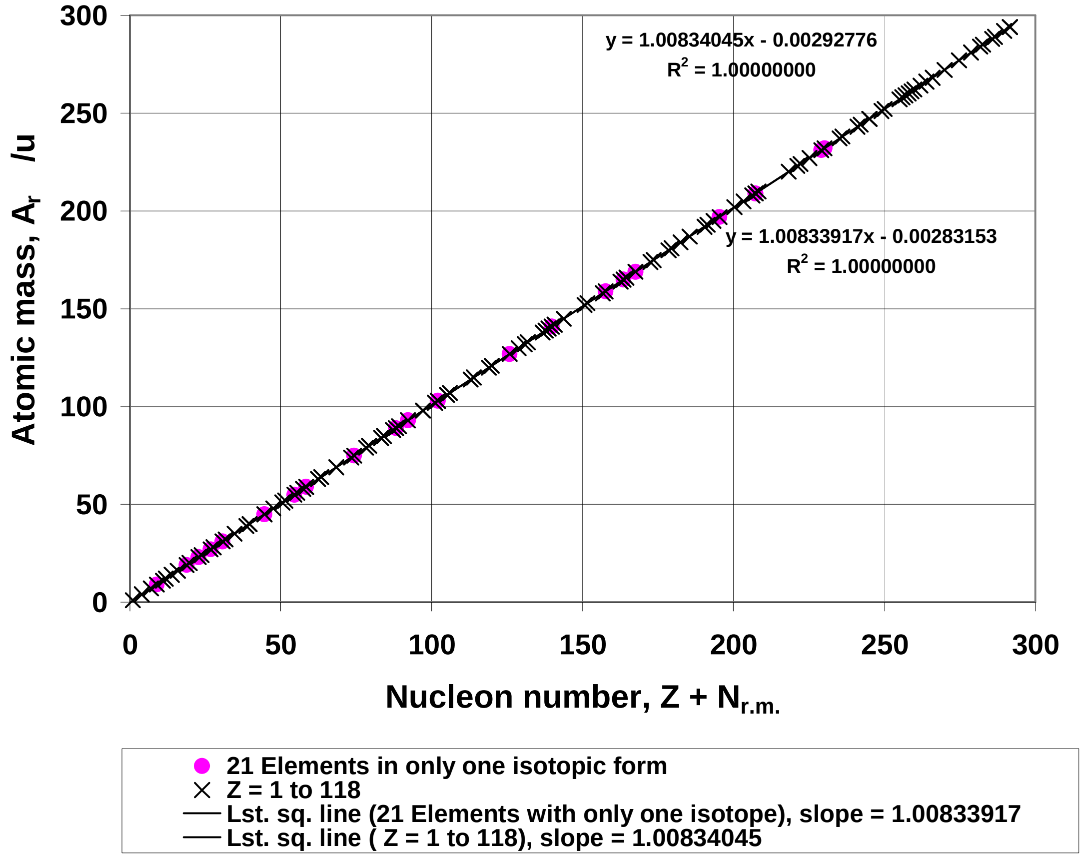

Figure 1: Dependence of relative atomic mass (A_r) on the nucleon number, $Z + N_{r.m.}$ for the most abundant stable isotope of each of the nuclides from Z = 0 to 118.

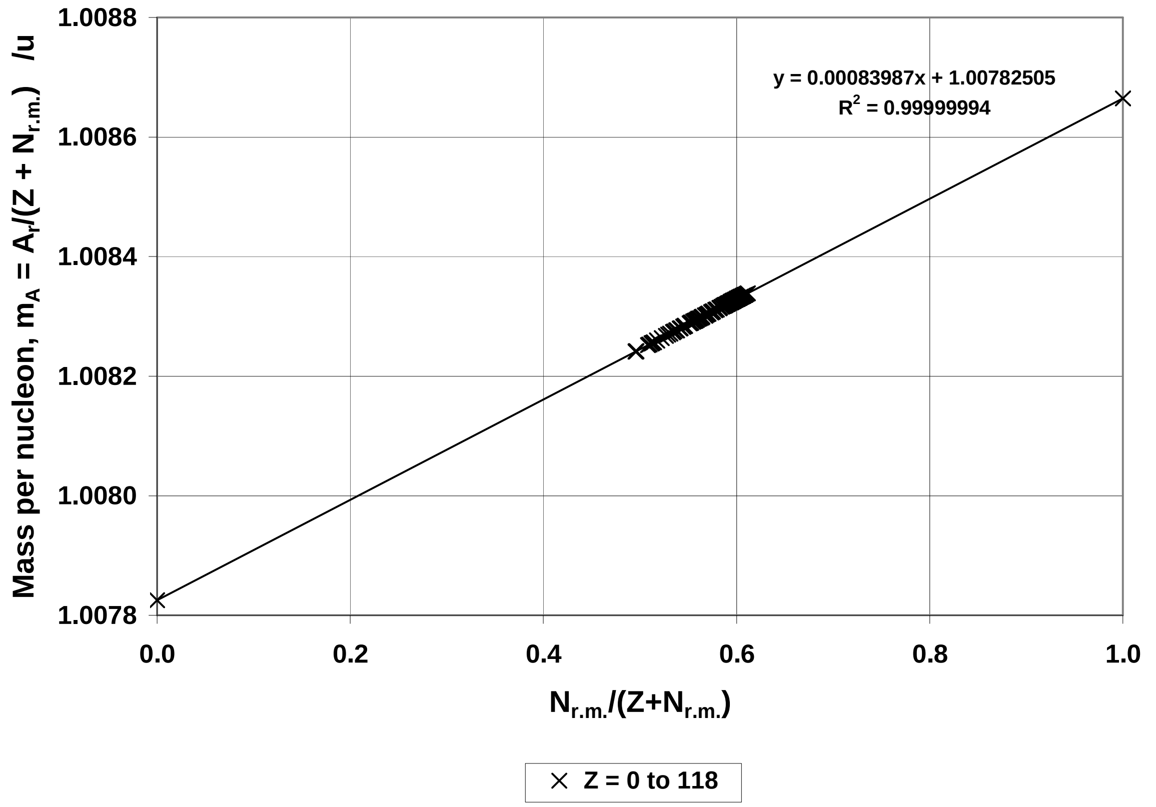

Figure 2: Linear increase of the mass per nucleon (m_A) with $N_{r.m.}/(Z + N_{r.m.})$ for the most abundant stable isotope of each of the nuclides from Z = 0 to 118.

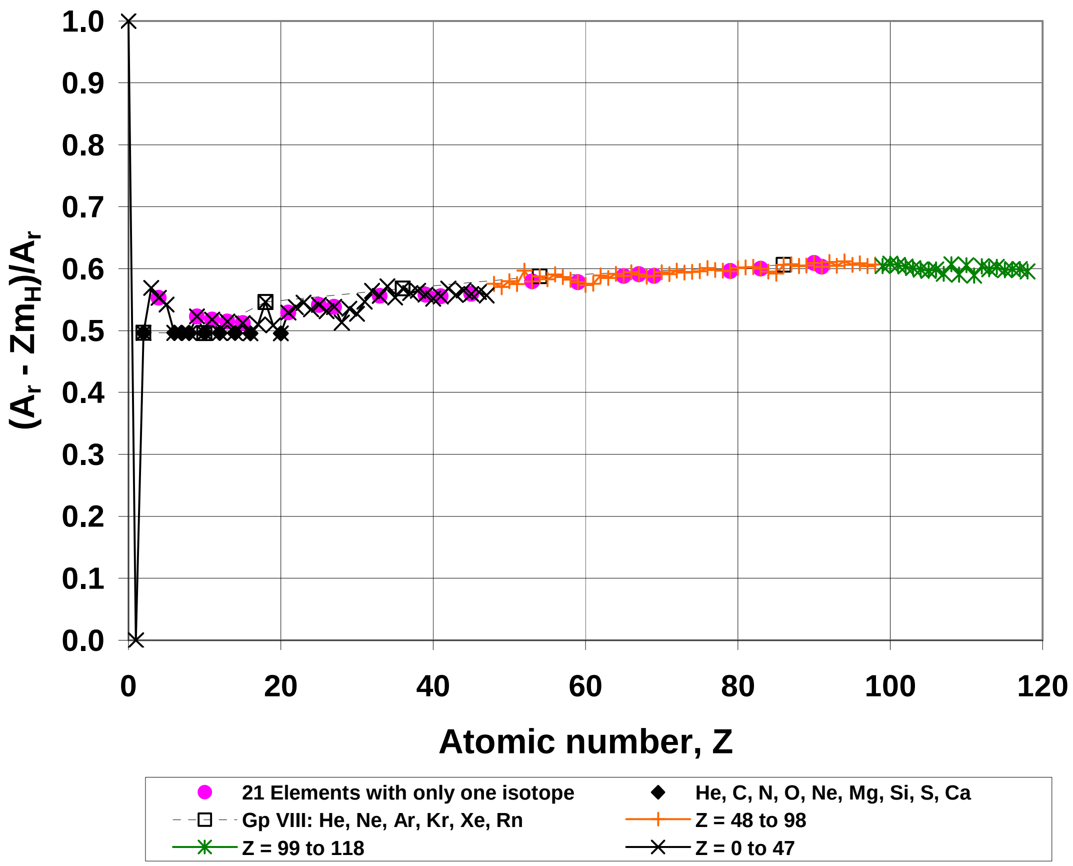

Figure 3: Dependence of the ratio, $(A_r - Zm_H)/A_r$, of the mass of neutrons to that of atomic mass, on atomic number, Z for the most abundant stable isotope of each of the nuclides from Z = 0 to 118.

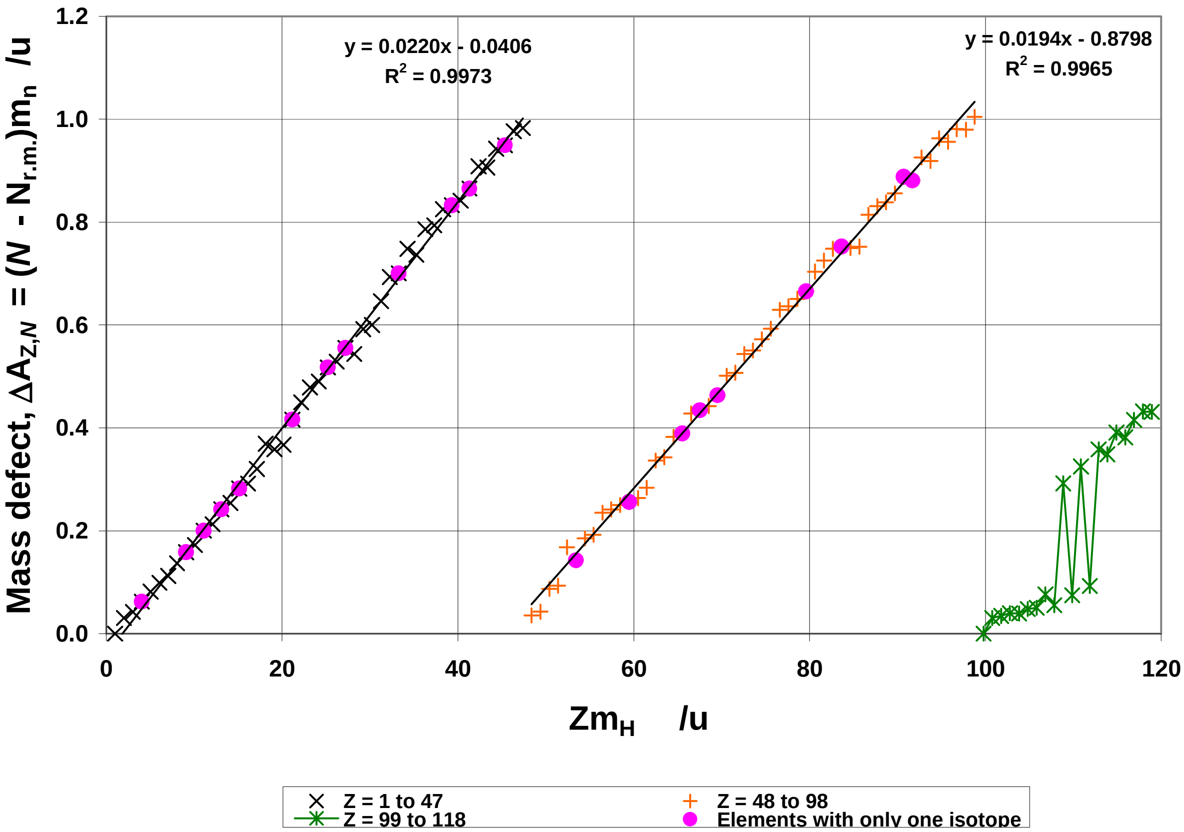

Figure 4: Dependence of mass defects, $(N - N_{r.m.})m_n$ on Zm_H for the most abundant stable isotope of each of the nuclides from Z = 0 to 118.

Table 1: Atomic numbers, Z, relative atomic masses, A_r, conventional neutron numbers, N, rest mass based neutron numbers, $N_{r.m.}$, rounded up values, *N*, of $N_{r.m.}$, and mass per nucleon, m_A, for the most abundant isotope (or one with longest half life) of each element. The 21 nuclides available in only one natural isotopic form are marked with asterisks (*). $\delta A_r = N_{r.m.}\delta m_n - Z\delta m_H$; A_r,cal = $(Z+N_{r.m.})m^*+\delta A_r$ and m^* = 1.0083392 u

1	2	3	4	5	6	7	8	9	10	11	12
X	**Z**	**A_r (in u)**	**N**	**$N_{r.m.}$**	*N*	**$Z+N_{r.m.}$**	**$N_{r.m}m_n/A_r$**	**m_A (in u)**	**$(Z+N_{r.m.})m^*$**	**δA_r**	**A_r,calc**
n	0	**1.008665**	1	1.000000	1	1.000000	1.000000	1.0086649	1.008340	0.000324	**1.008665**
H	1	**1.007825**	0	0.000000	0	1.000000	0.000000	1.0078250	1.008340	-0.000515	**1.007825**
He	2	**4.002603**	2	1.969884	2	3.969884	0.496415	1.0082418	4.002995	-0.000392	**4.002603**
Li	3	**7.016004**	4	3.958231	4	6.958231	0.569060	1.0083028	7.016266	-0.000262	**7.016004**
Be*	4	**9.012182**	5	4.938094	5	8.938094	0.552683	1.0082891	9.012641	-0.000459	**9.012182**
B	5	**11.009306**	6	5.918894	6	10.918894	0.542285	1.0082803	11.009963	-0.000656	**11.009306**
C	6	**12.000000**	6	5.901910	6	11.901910	0.496087	1.0082415	12.001177	-0.001177	**12.000000**
N	7	**14.003074**	7	6.888609	7	13.888609	0.496198	1.0082416	14.004447	-0.001373	**14.003074**
O	8	**15.994915**	8	7.864172	8	15.864172	0.495927	1.0082414	15.996487	-0.001572	**15.994915**
F*	9	**18.998403**	10	9.842691	10	18.842691	0.522569	1.0082638	18.999848	-0.001445	**18.998403**
Ne	10	**19.992440**	10	9.829022	10	19.829022	0.495897	1.0082414	19.994405	-0.001965	**19.992440**
Na*	11	**22.989770**	12	11.801436	12	22.801436	0.517782	1.0082597	22.991610	-0.001840	**22.989770**
Mg	12	**23.985042**	12	11.788991	12	23.788991	0.495773	1.0082413	23.987402	-0.002360	**23.985042**
Al*	13	**26.981538**	14	13.760578	14	26.760578	0.514419	1.0082569	26.983773	-0.002235	**26.981538**
Si	14	**27.976927**	14	13.748249	14	27.748249	0.495672	1.0082412	27.979682	-0.002755	**27.976927**
P*	15	**30.973762**	16	15.720172	16	30.720172	0.511930	1.0082548	30.976392	-0.002630	**30.973762**
S	16	**31.972071**	16	15.710738	16	31.710738	0.495647	1.0082412	31.975220	-0.003149	**31.972071**
Cl	17	**34.968853**	18	17.682609	18	34.682609	0.510049	1.0082532	34.971877	-0.003024	**34.968853**
Ar	18	**39.962383**	22	21.634075	22	39.634075	0.546052	1.0082835	39.964641	-0.002258	**39.962383**
K	19	**38.963707**	20	19.644810	20	38.644810	0.508551	1.0082520	38.967126	-0.003418	**38.963707**
Ca	20	**39.962591**	20	19.635946	20	39.635946	0.495616	1.0082411	39.966528	-0.003937	**39.962591**
Sc*	21	**44.955910**	24	23.587203	24	44.587203	0.529220	1.0082693	44.959080	-0.003170	**44.955910**
Ti	22	**47.947947**	26	25.554369	26	47.554369	0.537579	1.0082764	47.950994	-0.003047	**47.947947**
V	23	**50.943964**	28	27.525482	28	50.525482	0.544991	1.0082826	50.946887	-0.002923	**50.943964**
Cr	24	**51.940512**	28	27.514302	28	51.514302	0.534317	1.0082736	51.943954	-0.003442	**51.940512**
Mn*	25	**54.938050**	30	29.486922	30	54.486922	0.541381	1.0082796	54.941367	-0.003317	**54.938050**
Fe	26	**55.934942**	30	29.476083	30	55.476083	0.531537	1.0082713	55.938778	-0.003836	**55.934942**
Co*	27	**58.933200**	32	31.449417	32	58.449417	0.538269	1.0082770	58.936912	-0.003711	**58.933200**
Ni	28	**57.935348**	30	29.460970	30	57.460970	0.512921	1.0082557	57.940220	-0.004872	**57.935348**
Cu	29	**62.929601**	34	33.413152	34	62.413152	0.535562	1.0082747	62.933706	-0.004105	**62.929601**
Zn	30	**63.929147**	34	33.404944	34	63.404944	0.527058	1.0082675	63.933770	-0.004623	**63.929147**
Ga	31	**68.925581**	38	37.359289	38	68.359289	0.546720	1.0082840	68.929437	-0.003855	**68.925581**
Ge	32	**73.921178**	42	41.312804	42	73.312804	0.563719	1.0082983	73.924266	-0.003088	**73.921178**
As*	33	**74.921596**	42	41.305461	42	74.305461	0.556093	1.0082919	74.925202	-0.003606	**74.921596**
Se	34	**79.916522**	46	45.258311	46	79.258311	0.571227	1.0083046	79.919361	-0.002839	**79.916522**
Br	35	**78.918338**	44	43.269534	44	78.269534	0.553033	1.0082894	78.922338	-0.003999	**78.918338**
Kr	36	**83.911507**	48	47.220642	48	83.220642	0.567619	1.0083016	83.914740	-0.003233	**83.911507**
Rb	37	**84.911789**	48	47.213164	48	84.213164	0.560844	1.0082959	84.915540	-0.003751	**84.911789**
Sr	38	**87.905614**	50	49.182103	50	87.182103	0.564336	1.0082988	87.909241	-0.003627	**87.905614**
Y*	39	**88.905848**	50	49.174577	50	88.174577	0.557901	1.0082934	88.909993	-0.004145	**88.905848**
Zr	40	**89.904704**	50	49.165685	50	89.165685	0.551603	1.0082881	89.909367	-0.004663	**89.904704**
Nb*	41	**92.906376**	52	51.142404	52	92.142404	0.555242	1.0082912	92.910913	-0.004537	**92.906376**
Mo	42	**97.905408**	56	55.099325	56	97.099325	0.567658	1.0083016	97.909177	-0.003769	**97.905408**
Tc	43	**97.907216**	55	54.101950	55	97.101950	0.557372	1.0082930	97.911824	-0.004608	**97.907216**
Ru	44	**101.904350**	58	57.065579	58	101.065579	0.564844	1.0082993	101.908512	-0.004162	**101.904350**
Rh*	45	**102.905504**	58	57.058966	58	102.058966	0.559284	1.0082946	102.910183	-0.004679	**102.905504**
Pd	46	**105.903483**	60	59.032023	60	105.032023	0.562243	1.0082971	105.908038	-0.004554	**105.903483**

Ag	47	**106.905094**	60	59.025863	60	106.025863	0.556917	1.0082926	106.910166	-0.005072	**106.905094**
Cd	48	**113.903358**	66	64.964841	65	112.964841	0.575293	1.0083080	113.907018	-0.003660	**113.903358**
In	49	**114.903878**	66	64.957598	65	113.957598	0.570220	1.0083038	114.908056	-0.004178	**114.903878**
Sn	50	**119.902197**	70	68.913812	69	118.913812	0.579730	1.0083118	119.905607	-0.003410	**119.902197**
Sb	51	**120.903818**	70	68.907661	69	119.907661	0.574876	1.0083077	120.907745	-0.003927	**120.903818**
Te	52	**129.906223**	78	76.833564	77	128.833564	0.596579	1.0083259	129.908094	-0.001871	**129.906223**
I*	53	**126.904468**	74	72.858428	73	125.858428	0.579095	1.0083112	126.908144	-0.003676	**126.904468**
Xe	54	**131.904155**	78	76.815998	77	130.815998	0.587408	1.0083182	131.907062	-0.002907	**131.904155**
Cs	55	**132.905447**	78	76.809521	77	131.809521	0.582934	1.0083145	132.908872	-0.003425	**132.905447**
Ba	56	**137.905241**	82	80.767197	81	136.767197	0.590746	1.0083210	137.907897	-0.002656	**137.905241**
La	57	**138.906348**	82	80.760537	81	137.760537	0.586441	1.0083174	138.909522	-0.003173	**138.906348**
Ce	58	**139.905434**	82	80.751873	81	138.751873	0.582190	1.0083138	139.909126	-0.003692	**139.905434**
Pr*	59	**140.907648**	82	80.746310	81	139.746310	0.578010	1.0083103	140.911857	-0.004209	**140.907648**
Nd	60	**141.907719**	82	80.738623	81	140.738623	0.573882	1.0083069	141.912446	-0.004727	**141.907719**
Pm	61	**144.912744**	84	82.718666	83	143.718666	0.575763	1.0083084	144.917344	-0.004600	**144.912744**
Sm	62	**151.919728**	90	88.666289	89	150.666289	0.588696	1.0083193	151.922914	-0.003185	**151.919728**
Eu	63	**152.921226**	90	88.660016	89	151.660016	0.584799	1.0083160	152.924929	-0.003703	**152.921226**
Gd	64	**157.924101**	94	92.620747	93	156.620747	0.591571	1.0083217	157.927034	-0.002933	**157.924101**
Tb*	65	**158.925343**	94	92.614220	93	157.614220	0.587803	1.0083186	158.928794	-0.003450	**158.925343**
Dy	66	**163.929171**	98	96.575896	97	162.575896	0.594237	1.0083240	163.931852	-0.002680	**163.929171**
Ho*	67	**164.930319**	98	96.569276	97	163.569276	0.590589	1.0083209	164.933517	-0.003198	**164.930319**
Er	68	**165.930290**	98	96.561489	97	164.561489	0.586983	1.0083179	165.934006	-0.003716	**165.930290**
Tm*	69	**168.934211**	100	98.540438	99	167.540438	0.588361	1.0083190	168.937801	-0.003589	**168.934211**
Yb	70	**173.938858**	104	102.502925	103	172.502925	0.594411	1.0083241	173.941677	-0.002819	**173.938858**
Lu	71	**174.940768**	104	102.497061	103	173.497061	0.590973	1.0083212	174.944105	-0.003336	**174.940768**
Hf	72	**179.946549**	108	106.460673	107	178.460673	0.596750	1.0083261	179.949115	-0.002566	**179.946549**
Ta	73	**180.947996**	108	106.454349	107	179.454349	0.593412	1.0083233	180.951079	-0.003083	**180.947996**
W	74	**183.950933**	110	108.432322	109	182.432322	0.594571	1.0083242	183.953890	-0.002957	**183.950933**
Re	75	**186.955751**	112	110.412160	111	185.412160	0.595696	1.0083252	186.958581	-0.002830	**186.955751**
Os	76	**191.961479**	116	114.375719	115	190.375719	0.600989	1.0083296	191.963538	-0.002059	**191.961479**
Ir	77	**192.962924**	116	114.369394	115	191.369394	0.597837	1.0083270	192.965501	-0.002576	**192.962924**
Pt	78	**194.964774**	117	115.354880	116	193.354880	0.596797	1.0083261	194.967546	-0.002772	**194.964774**
Au*	79	**196.966552**	118	116.340294	117	195.340294	0.595778	1.0083253	196.969520	-0.002968	**196.966552**
Hg	80	**201.970626**	122	120.302213	121	200.302213	0.600803	1.0083295	201.972824	-0.002198	**201.970626**
Tl	81	**204.974412**	124	122.281028	123	203.281028	0.601736	1.0083303	204.976483	-0.002071	**204.974412**
Pb	82	**207.976636**	126	124.258294	125	206.258294	0.602640	1.0083310	207.978581	-0.001945	**207.976636**
Bi*	83	**208.980383**	126	124.254251	125	207.254251	0.599726	1.0083286	208.982845	-0.002461	**208.980383**
Po	84	**208.982416**	125	123.257099	124	207.257099	0.594907	1.0083245	208.985717	-0.003300	**208.982416**
At	85	**209.987131**	125	123.254016	124	208.254016	0.592046	1.0083221	209.990948	-0.003817	**209.987131**
Rn	86	**220.011384**	134	132.192989	133	218.192989	0.606052	1.0083339	220.012816	-0.001432	**220.011385**
Fr	87	**223.019731**	136	134.176325	135	221.176325	0.606847	1.0083346	223.021035	-0.001304	**223.019732**
Ra	88	**224.020202**	136	134.169034	135	222.169034	0.604104	1.0083322	224.022024	-0.001821	**224.020203**
Ac	89	**227.027747**	138	136.151576	137	225.151576	0.604910	1.0083329	227.029441	-0.001693	**227.027748**
Th*	90	**232.038050**	142	140.119670	141	230.119670	0.609097	1.0083364	232.038972	-0.000921	**232.038051**
Pa*	91	**231.035879**	140	138.126941	139	229.126941	0.603040	1.0083314	231.037963	-0.002083	**231.035880**
U	92	**238.050783**	146	144.082416	145	236.082416	0.610504	1.0083376	238.051450	-0.000666	**238.050784**
Np	93	**237.048167**	144	142.089246	143	235.089246	0.604605	1.0083327	237.049996	-0.001829	**237.048168**
Pu	94	**244.064198**	150	148.045839	149	242.045839	0.611842	1.0083387	244.064610	-0.000411	**244.064199**
Am	95	**243.061373**	148	146.052461	147	241.052461	0.606094	1.0083339	243.062947	-0.001573	**243.061374**
Cm	96	**247.070347**	151	149.027829	150	245.027829	0.608406	1.0083359	247.071471	-0.001123	**247.070348**
Bk	97	**247.070299**	150	148.028614	149	245.028614	0.604327	1.0083324	247.072263	-0.001963	**247.070300**
Cf	98	**251.079580**	153	151.004286	152	249.004286	0.606631	1.0083344	251.081093	-0.001513	**251.079581**
Es	99	**252.082970**	153	150.999889	151	249.999889	0.604199	1.0083323	252.085000	-0.002030	**252.082971**

Fm	100	**257.095100**	157	154.969795	**155**	254.969795	0.607995	1.0083355	257.096358	-0.001257	**257.095101**
Md	101	**258.098425**	157	154.965333	**155**	255.965333	0.605614	1.0083335	258.100199	-0.001774	**258.098426**
No	102	**259.101021**	157	154.960149	**155**	256.960149	0.603251	1.0083315	259.103313	-0.002291	**259.101022**
Lr	103	**260.109693**	157	154.960989	**155**	257.960989	0.600915	1.0083296	260.112500	-0.002806	**260.109694**
Rf	104	**261.108751**	157	154.952297	**155**	258.952297	0.598582	1.0083276	261.112076	-0.003324	**261.108752**
Db	105	**262.114152**	157	154.949894	**155**	259.949894	0.596277	1.0083257	262.117993	-0.003840	**262.114153**
Sg	106	**266.121933**	160	157.924079	**158**	263.924079	0.598570	1.0083276	266.125324	-0.003391	**266.121934**
Bh	107	**264.124733**	157	154.944868	**155**	261.944868	0.591718	1.0083218	264.129606	-0.004873	**264.124734**
Hs	108	**277**	169	166.710363	**167**	274.710363	0.607057	1.0083347	277.001571	-0.001571	**277.000001**
Mt	109	**268.138823**	159	156.926141	**157**	265.926141	0.590313	1.0083207	268.144084	-0.005261	**268.138824**
Ds	110	**281**	171	168.677667	**169**	278.677667	0.605478	1.0083334	281.001964	-0.001963	**281.000001**
Rg	111	**272.153484**	161	158.907979	**159**	269.907979	0.588950	1.0083195	272.159133	-0.005649	**272.153485**
Cn	112	**285**	173	170.644970	**171**	282.644970	0.603942	1.0083321	285.002356	-0.002356	**285.000001**
Uut	113	**284**	171	168.654393	**169**	281.654393	0.598999	1.0083280	284.003518	-0.003517	**284.000001**
Uuq	114	**289**	175	172.612273	**173**	286.612273	0.602450	1.0083309	289.002749	-0.002748	**289.000001**
UUp	115	**288**	173	170.621697	**171**	285.621697	0.597570	1.0083268	288.003910	-0.003909	**288.000001**
Uuh	116	**292**	176	173.588167	**174**	289.588167	0.599631	1.0083285	292.003463	-0.003462	**292.000001**
Uus	117	**294**	177	174.571819	**175**	291.571819	0.598927	1.0083279	294.003659	-0.003658	**294.000001**
Uuo	118	**294**	176	173.572652	**174**	291.572652	0.595499	1.0083250	294.004499	-0.004498	**294.000001**