

# DECOMPOSING CO<sub>2</sub> EMISSIONS DATA INTO A PAIR OF LOGISTIC GROWTH PULSES

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## ABSTRACT

Numerous scenarios of emissions growth presume that it can not grow infinitely: there are certain limits that may not or must not be exceeded. Applying the theory of pulsing logistic growth, I detected two pulses of logistic growth. This led me to the conclusion that constructing a scenario of emissions growth one should pay attention to the fact that limits to their growth do not remain constant.

## INTRODUCTION

Carbon dioxide emissions is a heavily discussed topic since 1970s. It is well-recognized that their growth, associated with industrial development, may have negative consequences, and should be put under control [Heimann and Reichstein, 2008]. Numerous scenarios of emissions growth presume that it can not grow infinitely: there are certain limits that may not or must not be exceeded. The purpose of my work is to analyze the limits to CO<sub>2</sub> emissions growth during 1850-2004 and to consider how they may change in the future.

## METHODS

Applying the theory of pulsing logistic growth [Ausubel, 1996; Meyer, 1994], I assume that CO<sub>2</sub> emissions grow in agreement with the equation

$$(1) \quad \frac{dy}{dt} = m y \left(1 - \frac{y}{K(t)}\right); \quad y(t_0) = y_0$$

where  $K(t)$  is carrying capacity (or limit to growth) at the moment  $t$ ,  $m$  is relative increment of  $y$  when  $y(t) \ll K(t)$ .

In order to find changing limits to growth, I find the general solution of the Equation 1

$$y = \frac{1}{C e^{-mt} + e^{-mt} \int \frac{m e^{mt}}{K(t)} dt}$$

approximate  $K(t)$  as follows

$$K(t) = \frac{1}{a_0 + \sum_{n=1}^N (A_n(t) + B_n(t))}$$

$$A_n(t) = a_n \cos\left(\frac{n\pi}{L}\right) (t - t_0)$$

$$B_n(t) = b_n \sin\left(\frac{n\pi}{L}\right) (t - t_0)$$

and set the constant to be

$$C = \frac{1}{y_0} - \frac{1}{K(t_0)}$$

The graphics displayed at the Fig 1 corresponds to the case of  $L=155$ ,  $m=0.045$ ,  $y_0=60$ ,  $n=2$ ,  $a_0 = -0.000258168$ ,  $a_1 = -0.0000440057$ ,  $a_2 = 0.000302174$ ,  $b_1 = 0.000951037$ ,  $b_2 = 0.000375412$

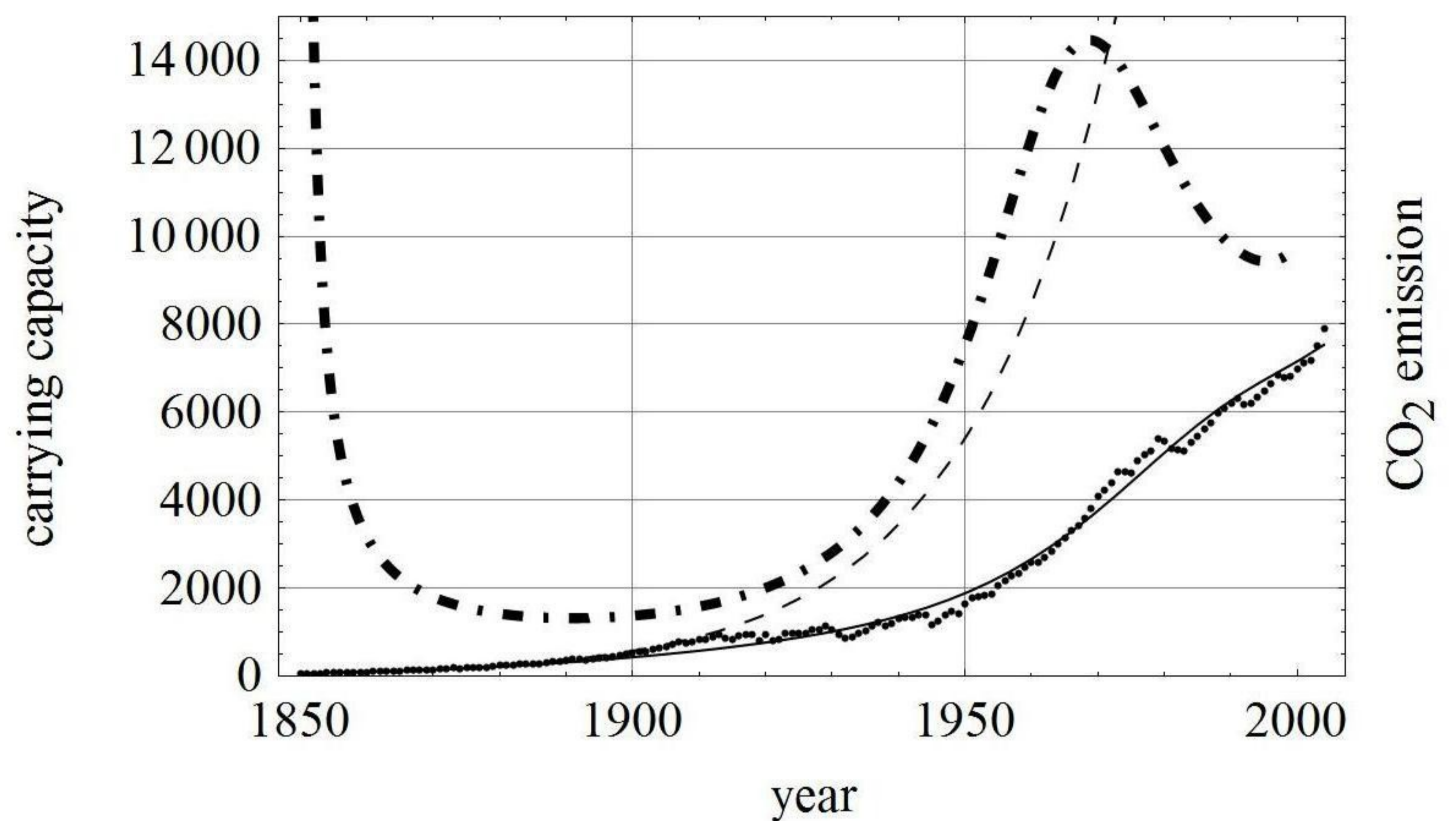


Fig. 1 CO<sub>2</sub> emissions and their limits to growth (carrying capacity). Units: MtC/y. Legend: thick dot-dashed line - carrying capacity, dots - CO<sub>2</sub> emissions data [Marland et al., 2008], solid line - solution of the Equation (1), dashed line - exponential growth ( $m=0.045$ ).

## RESULTS

Results, shown on Fig. 1, allow us to detect two pulses of logistic growth. The first pulse, with the carrying capacity of  $2 \pm 0.8$  GtC/y, culminated in 1930s. The second pulse, with the carrying capacity of  $12 \pm 2$  GtC/y, recently entered the culmination phase. Unexpected acceleration of emissions growth observed in 2000-2004 [Raupach et al., 2007] may indicate to the start of the third pulse, which carrying capacity is even higher.

## CONCLUSIONS

Obviously, constructing a scenario of emissions growth one should pay attention to the fact that limits to their growth do not remain constant. They were and would be changing because of technological advances. The numerical method, developed as a result of this study, expands opportunities for use of the theory of pulsing growth in constructing emissions scenarios.

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