Measuring Uncertainty of Factors Extracted Using Principal Components

by

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Abstract

In the context of Dynamic Factor Models (DFMs), one of the most popular procedures for factor extraction is Principal Components (PC). Measuring the uncertainty associated to PC factor estimates should be part of interpreting them. However, in this thesis, we show that the asymptotic distribution of PC factors could not be an appropriate approximation to the finite sample distribution for the sample sizes and cross-sectional dimensions usually encountered in practice. The main problem is that parameter uncertainty is not taken into account.

In the second chapter of this thesis, we show that neither the asymptotic distribution nor several bootstrap procedures with goals related to inference proposed in the context of DFMs are appropriate to measure the uncertainty of PC factor estimates. Therefore, we propose a subsampling procedure designed for this purpose. The finite sample properties of the proposed procedure are analyzed and compared with those of the asymptotic and alternative extant bootstrap procedures. The results are empirically illustrated obtaining confidence intervals of the underlying factor in a system of Spanish macroeconomic variables.

In chapter 3, the GiS (Growth-in-Stress), a new macroeconomic risk index, is proposed. The methodology for constructing the GiS is based on predictive quantile factor regressions. The factors are extracted using principal components (PC) and their joint probability density is obtained using the subsampling method proposed in the second chapter. To construct the risk index, we follow the Value-in-Stress (ViS) risk measure proposed by González-Rivera (2003). The GiS calculates the risk exposure to

stressed scenarios and the country’s ability to withstand them.

Finally, chapter 4 concludes and presents the lines of research currently being undertaken.

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Chapter 1

Introduction

Currently, large systems of variables are easily accessible, and the consequent extraction of the underlying common factors is an important issue for econometricians and policy decision makers. The latent factors are useful instruments for a wide range of applications: i) to represent cycles, trends, structural shocks and latent Engel curves; see Arouba et al. (2009), Camacho et al. (2015), Barigozzi and Moneta (2016) and Breitung and Eickmeier (2016) for some references; ii) to serve as instrumental variables; see Favero et al. (2005), Bai and Ng (2010) and Kapetanios and Marcellino (2010); iii) as regressors for the construction of Factor-Augmented Vector Autorregresive models (FAVAR) or Factor-Augmented Error Correction models (FECM); see, for example, Bernanke et al. (2005), Banerjee et al. (2014), Abbate et al. (2016) and Bai et al. (2016) or iv) in the context of factor-augmented predictive regressions; see, for example, Stock and Watson (2006), Ludvigson and Ng (2007, 2009), Ando and Tsay (2014), Bräuning and Koopman (2014) and Neely et al. (2014).

In this context, Dynamic Factor Models (DFMs), originally introduced by Geweke (1977) and Sargent and Sims (1977), have received a great deal of attention; see Breitung and Eickmeier (2006), Bai and Ng (2008), Stock and Watson (2011), Breitung and Choi (2013) and Bai and Wang (2016) for excellent surveys. The main goal of DFMs is to explain the dynamics of the system using a reduced number of unobservable common factors. Although several factor extraction methods have been proposed in the DFM literature, the most popular procedures for large data sets are still based on Principal Components (PC) techniques; see, for example, Ludvigson and Ng (2007, 2009, 2010), Wang (2009), Foester et al. (2011), Ando and Tsay (2014), Gonçalves and Perron (2014), Neely et al. (2014), Djogbenov et al. (2015), Fossati (2016) and Jackson et al. (2016) just to name a few recent references. The popularity of PC factor extraction relies on its good theoretical properties and on its computational simplicity which allows dealing with very large systems of economic or financial variables. However, in practice, it is crucial to obtain not only accurate point estimates of the latent factors, but also of their associated uncertainty. For example, Bai (2003) remarks the importance of constructing confidence intervals of the extracted factors in empirical applications in which they represent economic indices. Boivin and Ng (2006) also pay attention to the uncertainty of factor estimates in the context of predictive regressions while Bai and Ng (2006) argue about the importance of measuring correctly the uncertainty of factors in FAVAR models. More recently, Jackson et al. (2016) argue that measures of factor uncertainty should always accompany applied work in order to establish the statistical legitimacy of the results.

The asymptotic distribution of the factors extracted using PC is derived by Bai (2003) assuming weak dependence in the idiosyncratic term while Bai and Ng (2006) propose estimators of the asymptotic covariance matrix of the factors. More recently, Bai and Ng (2013) derive the limiting distribution of the factors and its corresponding covariance matrix estimation for different identification restrictions. However, results on the performance of the asymptotic distribution to approximate the finite sample distribution of the estimated factors are scarce. Ouysse (2006) shows that, if the factor

[1] In the context of inference for the OLS estimator of the parameters of factor-augmented predictive regression models, Gonçalves and Perron (2014) show that the finite sample properties of the asymptotic approach proposed by Bai and Ng (2006) can be poor, especially if the cross-sectional dimension is not sufficiently large relative to the temporal dimension.

is static, the asymptotic variance is underestimated while Poncela and Ruiz (2016) show that PC intervals based on the asymptotic distribution could underestimate the uncertainty of the extracted factors [1]. The poor performance of the asymptotic distribution could be attributed to the fact that parameter uncertainty is not considered. Alternatively, the finite sample distribution of the estimated factors can be obtained using resampling procedures which could incorporate parameter uncertainty. Several authors propose using bootstrap in the context of DFMs with other objectives than obtaining the distribution of the underlying factors. For example, Yamamoto (2016) obtains bootstrap bands for impulse response functions (IRF) in the context of FAVAR models; see also Barigozzi et al. (2016) and Forni et al. (2014) for empirical applications. Ludvigson and Ng (2007, 2009 and 2010), Gospodinov and Ng (2013), Gonçalves and Perron (2014), Djogbenou et al. (2015), Jackson et al. (2016) and Gonçalves et al. (2017) implement bootstrap procedures in the context of the parameters of factor-augemented predictive regression models; see also Alonso et al. (2008) and Alonso et al. (2011) who use bootstrap procedures for constructing forecasting intervals for population projections and electricity prices, respectively. Finally, Shintani and Guo (2015) also propose using bootstrap to test about the autoregressive parameter governing the dependence of the latent factor. However, the procedures proposed in these papers obtain either the marginal Mean Squared Errors (MSEs) of the underlying estimated factors and/or do not incorporate parameter uncertainty. Furthermore, none of these papers analyze the performance of the bootstrap procedures when they are used to obtain confidence bands for the extracted factors.

In the second chapter of this thesis we show, through an extensive Monte Carlo experiment, the conditions under which the asymptotic and the bootstrap distribution of the factors extracted using PC are a good approximation of the finite sample distribution. We see that the asymptotic and most of the bootstrap approaches only take into account the model uncertainty. Also in this chapter, we propose a new subsampling procedure for constructing conditional confidence bands for PC factors. The proposed procedure takes into account parameter and model uncertainty simultaneously.

In the third chapter, using the algorithm described in the second chapter, we propose a new global risk index, Growth-in-Stress (GiS), that measures the expected fall in a country GDP as the global factors, which drive world growth, are subject to stressful conditions. Stress is measured as the 95% contours of the joint probability distribution of the factors. With GDP growth rates of a sample of 87 countries from 1985 to 2015, we extract three global factors: a first world growth factor driven mainly by all industrial and emerging countries; a second factor driven by other developing countries in Africa and America; and a third factor that is mostly related to East Asian economies. We find that the average GiS across industrialized, emerging and other developing countries has been going down from 1987. Post 2008 financial crisis, mainly from 2011 on, the world overall has fallen in a state-of-complacency with the cross-sectional average GiS falling quite dramatically; in 2015 the average worst outcome seems to be no growth at the 95% probability factor stress. However, the cross-sectional dispersion within groups is quite wide. It is the smallest among industrialized countries and the largest among emerging and developing countries. We also measure the factor stress on different quantiles of the GDP growth distribution of each country. We calculate an Armageddon-type event as we seek to find the average 5% GiS quantile under the extreme 95% probability events of the factors and find that it can be as large as an annual 20% fall in GDP.

Finally, the fourth chapter concludes and explains some further research projects in this field. Firstly, the subsampling procedure proposed in the second chapter is extended to obtain forecast intervals in the context of factor augmented predictive regressions. Secondly, we disentangle the best strategy to model and forecast log-mortality rates in different countries using Factor Models. Finally, we propose a new method for measuring gender discrimination in the allocation of intrahousehold resources based on an approximate factor model in which the factors are extracted via Maximum Likelihood.

It should be noted that, with the aim of making the reading easier, each chapter is self-contained so certain equations might be occasionally repeated throughout the thesis.

Chapter 2

Accurate Subsampling Intervals of Principal Components Factors

2.1. Introduction

In this chapter we provide extensive Monte Carlo simulations in order to asses the conditions under which the asymptotic distribution of the factors extracted using PC is a good approximation of the finite sample distribution. In concordance with the results in Poncela and Ruiz (2016), we show that the asymptotic confidence intervals of the estimated factors are unrealistically tiny when the time series size is not large relative to the cross-sectional size. However, if the temporal dimension is large relative to the cross-sectional dimension with the latter being large enough, the asymptotic distribution is appropriate to approximate the finite sample distribution of PC factors. Note that, in this latter case, parameter uncertainty is not relevant while a large cross-sectional dimension minimizes the disturbance noise. The presence of serial dependence or heteroscedasticity of the idiosyncratic noises only have mild effects on the properties of asymptotic intervals. However, when the idiosyncratic noises are cross-sectionally correlated, the undercoverage of asymptotic intervals could be very severe if the signal to noise ratio is small. We also analyze the performance of the main available bootstrap methods mentioned above when implemented to obtain confidence bands of PC factors. We show that, if they obtain the marginal distribution of the factors, the corresponding intervals are too wide as to be informative. On the other hand, if they do not incorporate parameter uncertainty, their performance is similar to that of asymptotic intervals.

The second and main contribution of this chapter is to propose a subsampling procedure designed to construct conditional confidence bands for PC factors. The proposed procedure takes into account parameter uncertainty incorporating simultaneously the uncertainty attributed to the fact that the factors are unobserved. The finite sample performance of the proposed procedure is analyzed and compared with that of the asymptotic approach and alternative bootstrap procedures. We show that the converages of the intervals based on the proposed procedure are very close to the nominal coverages.

Finally, the last contribution of this chapter is an empirical illustration of the implications of using different procedures to construct confidence intervals for the Spanish economic cycle extracted using PC implemented to a system of macroeconomic variables.

The rest of the chapter is organized as follows. Section 2 describes the PC factor extraction procedure and its asymptotic distribution. Monte Carlo experiments are carried out to asses the adequacy of the asymptotic distribution to approximate the finite sample distribution of the factors. Section 3 describes available bootstrap procedures proposed for DFM and analyzes their finite sample performance. In Section 4, the new resampling procedure is proposed and its finite sample performance are analyzed. Section 5 illustrates the results with an empirical illustration to compute the uncertainty associated with the Spanish economic cycle. Finally, Section 6 concludes.

2.2. Factor extraction

In this section, we describe the DFM considered in this chapter and introduce notation. We also describe the asymptotic properties of PC factor estimates. Finally, we carry out Monte Carlo experiments to analyze the finite sample performance of asymptotic confidence intervals for the extracted factors.

2.2.1. The Dynamic Factor Model

We consider the following stationary DFM in which the latent factors and the idiosyncratic components are VAR(1) processes

$$Y_{\cdot t} = PF_t + \varepsilon_{\cdot t}, \tag{2.1}$$

$$F_t = \Phi F_{t-1} + \eta_t, \tag{2.2}$$

$$\varepsilon_{\cdot t} = \Theta \varepsilon_{\cdot t-1} + a_{\cdot t} \tag{2.3}$$

where $Y_{\cdot t} = (y_{1t}, ..., y_{Nt})'$ is the $N \times 1$ vector of observed variables at time t for $t = 1, ..., T$, P is the $N \times r$ matrix of factor loadings, $F_t = (f_{1t}, ..., f_{rt})'$ is the $r \times 1$ vector of unobservable factors and $\varepsilon_{\cdot t} = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the $N \times 1$ vector of idiosyncratic noises. To uniquely fix the $T \times r$ matrix of factors, $F = (F_1, ..., F_T)'$, and P (up to a column sign change), we assume that $\frac{1}{T}F'F = I_r$ and $P'P$ is a diagonal matrix with its main diagonal values ordered in decreasing order; see Bai and Ng (2013) for an extensive discussion on identification issues. The disturbances $\eta_t = (\eta_{1t}, ..., \eta_{rt})'$ and $a_{\cdot t} = (a_{1t}, ..., a_{Nt})'$ are mutually independent Gaussian white noise vectors with finite covariance matrices Σ_η and Σ_a, respectively. The matrices Φ and Γ are diagonal with their parameters restricted so that $Y_{\cdot t}$ is stationary. The number of factors, r, is assumed to be known and fixed as the cross-sectional and temporal dimensions, N and T, respectively, grow. The DFM in equations (2.1) to (2.3) has been frequently considered in the related literature; see, for example, Jungbacker and Koopman (2015), Alvarez et al. (2016) and Jackson et al. (2016) for some recent references.

Note that, according to (2.2) and assuming that $E\left(F_t F_t'\right) = I_r$, the point-wise marginal (unconditional) distribution of the factors is given by

$$F_t \sim N(0, I_r), \tag{2.4}$$

and, consequently, one can always construct confidence intervals for the unobserved factors using this distribution. However, the corresponding confidence intervals will be uninformative. Confidence intervals with less uncertainty can be constructed conditional on $Y_{\cdot t}$. Also, it is obvious that the marginal MSE in (2.4) is not appropriate when the intervals are not centered at the marginal mean (zero) but in a estimation of the factor based on $Y_{\cdot t}$.

2.2.2. Principal Components factor extraction

In the context of iid data, PC is justified because it is optimal in the sense that is the best linear MSE dimension reduction from N to r generating mutually orthogonal factors. However, in a time series context, PC fails to exploit the information contained in the leads and lags of $Y_{\cdot t}$. It still provides the best static r-dimensional approximation but has not minimum MSE as alternative linear procedures involving the past will have smaller MSE. Furthermore, in a dynamic context, PC factors will still be mutually orthogonal at lag zero but correlated at other lags. Consequently, the resulting PC factors cannot be analysed component-wise but need to be considered as vector time series, which are less easy to handle and interpret; see Brillinger (1981). However, PC is still among the most popular factor extraction procedures due to its simplicity and low computational burden when dealing with very large systems of macroeconomic or financial variables. The method of PC minimizes the following sum of squares:

$$V(r) = (NT)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{it} - P_i' F_t \right)^2, \tag{2.5}$$

where P_i' is the $i'th$ row of P. Mechanically speaking, the factor estimates can be obtained in one of two ways. The first solution is obtained concentrating out the matrix of weights P. Using the normalization $F'F/T = I_r$, the estimated factors, $\widetilde{f}$, are $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of YY' and $\widetilde{P}' = \frac{1}{T}\widetilde{f}'Y'$, with $\widetilde{P}'\widetilde{P}$ being diagonal and Y being the $N \times T$ matrix of observations. The second solution is obtained after concentrating out the factors, F. Then, $\overline{P}$ is $\sqrt{N}$ times the eigenvectors corresponding to the r largest eigenvalues of $Y'Y$. Using the normalization $\frac{1}{N}\overline{P}'\overline{P} = I_r$, yields

$$\overline{f} = \frac{1}{N} Y'\overline{P}. \tag{2.6}$$

Note that the matrices YY' and $Y'Y$ have identical nonzero eigenvalues and, consequently,

$$\frac{1}{T}\overline{f}'\overline{f} = \frac{1}{N}\widetilde{P}'\widetilde{P} = \widetilde{V}, \tag{2.7}$$

where $\widetilde{V}$ is the $r \times r$ diagonal matrix consisting of the first r eigenvalues of the matrix $\frac{1}{TN}YY'$ arranged in decreasing order. Then, $\overline{f} = \widetilde{f}\widetilde{V}^{1/2}$ and $\widetilde{P} = \overline{P}\widetilde{V}^{1/2}$; see Bai and Ng (2008). Let $\widehat{f} = \overline{f}\left(\frac{1}{T}\overline{f}'\overline{f}\right)^{1/2} = \overline{f}\widetilde{V}^{1/2}$. From the results above, we can see that $\widehat{f} = \frac{1}{N}Y'\overline{P}\widetilde{V}^{1/2} = \frac{1}{N}Y'\widetilde{P}$, and, consequently,

$$\widehat{f}_t = \frac{1}{N}\widetilde{P}'Y_{\cdot t}. \tag{2.8}$$

The interest in expression (2.8) relies on the fact that the factor estimates are expressed as a linear filter of the original observations as in (2.6) while, simultaneously, they satisfy the restriction $\frac{1}{T}\widehat{f}'\widehat{f} = I_r$.

It is well known that the extracted factors, $\widehat{f}_t$, estimate only a rotation of the true factors, HF_t, where $H = \left(\frac{P'P}{N}\right)$. Given that the filter used to estimate the factors at time t is based on $Y_{\cdot t}$, the MSE should also be computed conditional on this information. The MSE of the estimated factors can be obtained as follows:

$$\underset{t}{E}\left[\left(\widehat{f}_t - HF_t\right)\left(\widehat{f}_t - HF_t\right)'\right] =$$

$$\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)'\right] + \underset{t}{E}\left[(f_t - HF_t)(f_t - HF_t)'\right] + 2\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)(f_t - HF_t)'\right], \tag{2.9}$$

where f_t is the factor extracted if the loadings were known, i.e.

$$f_t = \frac{1}{N}P'Y_{\cdot t}, \tag{2.10}$$

and the t bellow the expectation means that it is conditional on $Y_{\cdot t}$. Note that the total MSE of $\widehat{f}_t$ in expresion (2.9) is decomposed into the uncertainty due to parameter estimation which represents the difference between the estimates PC factors obtained with known and unknown parameters, the disturbance uncertainty which is due to the process of separating signal and noise and it is inherent to the factor extraction and the cross-product between both. First, using (2.8) and (2.10), we can obtain the following expression of the MSE attributed to parameter uncertainty

$$\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)'\right] = \frac{1}{N^2}\underset{t}{E}\left[\left(\widetilde{P} - P\right)' Y_{\cdot t}Y_{\cdot t}'\left(\widetilde{P} - P\right)\right] =$$

$$\frac{1}{N^2}E[(\tilde{P}'Y_t - P'Y_t)(\tilde{P}'Y_t - P'Y_t)'] = \frac{1}{N^2}E[(\tilde{P}' - P')Y_t y_T^*(\tilde{P} - P)]. \tag{2.11}$$

On the other hand, from equation (2.1) we can obtain the following expression for the rotated true factors

$$HF_t = \frac{1}{N} P' Y_{\cdot t} - \frac{1}{N} P' \varepsilon_{\cdot t} \tag{2.12}$$

and, consequently, the disturbance uncertainty is given by

$$\underset{t}{E}\left[\left(f_t - HF_t \right) \left(f_t - HF_t \right)' \right] = \frac{1}{N^2} \underset{t}{E}\left[P' \varepsilon_{\cdot t} \varepsilon_{\cdot t}' P \right]. \tag{2.13}$$

Finally, the expectation of the cross-product in (2.9) is zero under the assumption of conditional Normality; see Rodriguez and Ruiz (2012) [1].

2.2.3. Asymptotic distribution of PC factors

The first asymptotic result on PC factor estimates in the context of strict DFM, is due to Connor and Korajczyk (1986) who prove consistency of PC factors when N goes to infinity and T is fixed. Bai (2003) shows that, in this case, consistency requires to assume asymptotic orthogonality and homoscedasticity of the idiosyncratic components. Only under large N and T, Bai (2003) establishes consistency in the presence of serial correlation and heteroscedasticity; see also Stock and Watson (2002) who show that the space spanned by the estimated factors is consistent when both N and T tend simultaneously to infinity if the serial and cross-sectional correlations of the idiosyncratic noises are weak and the factors are pervasive. Furthermore, if $\frac{\sqrt{N}}{T} \rightarrow 0$, Bai (2003) derives the limiting distribution of the factors. Under the restrictions $\frac{1}{T} F' F = I_r$ and the diagonal elements of $P'P$ being distinct and positive and arranged in decreasing order, Bai and Ng (2013) show that

$$\sqrt{N} \left(\widetilde{f}_t - F_t \right) \xrightarrow{d} N\left(0, \Sigma_p^{-1} \Gamma_t \Sigma_p^{-1} \right), \tag{2.14}$$

[1] Full details of the derivation of (2.11) and (2.13) can be found in the appendix.

where $\Sigma_p = \lim_{N\to\infty} \frac{1}{N} P'P$ and $\frac{1}{\sqrt{N}} \sum_{i=1}^{N} P_i \varepsilon_{it} \xrightarrow{d} N(0, \Gamma_t)$. Furthermore, Bai (2003) shows that, if the idyosincratic noises are serially uncorrelated, the limiting distributions are asymptotically independent across t. From (4.5), the asymptotic MSE can be estimated as follows

$$MSE_t = \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \frac{\tilde{\Gamma}_t}{N} \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}, \tag{2.15}$$

where, according to Bai and Ng (2006), $\tilde{\Gamma}_t$ can be estimated assuming that the idyiosincratic errors are cross-sectionaly uncorrelated, as follows [2],

$$\widetilde{\Gamma}_t = \frac{1}{N} \sum_{i=1}^{N} \tilde{P}_i \tilde{P}_i' \tilde{\varepsilon}_{it}^2 \tag{2.16}$$

where, $\tilde{P}_i$ is the $i-th$ row of the estimated factor loading matrix $\tilde{P}$ and $\tilde{\varepsilon}_{it} = y_{it} - \tilde{P}_i' \tilde{F}_t$.

In the single factor model, when $r = 1$, approximated $(1-\alpha)\%$ asymptotic confidence bands for F_t can be constructed as follows

$$[L_t, U_t] = \left[\tilde{f}_t - z_{\alpha/2} MSE_t^{1/2}, \tilde{f}_t + z_{\alpha/2} MSE_t^{1/2}\right] \tag{2.17}$$

where $z_{\alpha/2}$ is the $\alpha/2$ quantile of the standard normal distribution. Given that $\hat{f} = \tilde{f}\tilde{V} = \tilde{f}\frac{1}{N}\tilde{P}'\tilde{P}$, $(1-\alpha)\,\%$ confidence bands can also be written in terms of $\hat{f}$ as follows

$$[L_t, U_t] = \left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t - z_{\alpha/2} MSE_t^{1/2}, \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t + z_{\alpha/2} MSE_t^{1/2}\right]. \tag{2.18}$$

On the other hand, if $r \geq 2$ the asymptotic $(1-\alpha)\%$ ellipsoids are given by

[2] Bai and Ng (2006) propose this estimator of the asymptotic covariance matrix arguing that, if the cross-correlation in the errors is small, assuming that they are zero could be convenient because the sampling variability from their estimation could cause an efficiency loss.

$$\left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}\hat{f}_t\right] MSE_t^{-1}\left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}\hat{f}_t\right]' \leq \chi^2_\alpha(r), \tag{2.19}$$

where $\chi^2_\alpha(r)$ is the α quantile of a Chi-squared distribution with r degrees of freedom.

As an illustration, we have generated a system of $N = 100$ variables of size $T = 50$ by the DFM in equations (2.1) to (2.3) with idiosyncratic errors being serial and cross-sectionally uncorrelated, i.e. $\Gamma = 0$ and $\Sigma_a = \sigma^2_a I$ with $\sigma^2_a = 1$. The number of factors is $r = 1$ with $\phi = 0.7$ and $\sigma^2_\eta = (1 - \phi^2)$. Finally, the weights, P, have being generated by an $U(0,1)$ distribution with $\sum_{i=1}^N p^2_{i1} = 31.27$. The top left panel of Figure 1 plots the simulated factor, F_t, together with the factor extracted by PC, $\hat{f}_t$, and the corresponding point-wise 95% asymptotic confidence bands computed as in (2.18). We can observe that, in this particular realization, the asymptotic bands are rather thin with the true factor being outside the intervals more often than expected. Additionally, a system of variables with the same structure than that described above but with $r = 2$ factors, $\phi_{11} = \phi_{22} = 0.7$ and $T = 25$ has also been generated with $\sum_{i=1}^N p^2_{i1} = 29.39$ and $\sum_{i=1}^N p^2_{i2} = 4.92$. Figure 2 plots the simulated factor, and the corresponding 95% confidence contours constructed as in (2.19) for $t = 1, ..., 25$. We can observe that the asymptotic contours are too narrow, and leave the factors outside more often than they should.

Note that the estimated finite sample approximation of the asymptotic covariance matrix of $\tilde{f}_t$ (and, consequently, of $\hat{f}_t$) in expression (4.6) is asymptotically equivalent to that of a least squares (LS) estimator in which P is treated as if it were known explanantory variables. This asymptotic approximation underestimates the covariance of $\tilde{f}_t$ as it does not take into account the MSE attributed to parameter uncertainty in (2.11). Consequently, unless T is very large relative to N, the asymptotic MSE will underestimate the finite sample MSE and the corresponding coverage of the confidence regions of F_t will be bellow the nominal; see, for example, Poncela and Ruiz (2016).

2.2.4. Finite sample performance

We carry out Monte Carlo experiments in order to assess the finite sample adequacy of the asymptotic distribution when constructing confidence regions for the latent unobserved factors. These experiments complement those carried out by Poncela and Ruiz (2016) and are carried out for the shake of completeness [3]. The Monte Carlo experiments are performed using DFM of increasing complexity. The first model considered is the ubiquitous single factor model with temporal and cross-sectionally independent idiosyncratic components. Then, we consider the single factor model with the idiosyncratic components being either cross-correlated, temporally dependent or heteroscedastic. Finally, we generate simulated systems by a DFM with $r = 2$. We consider $N, T = 20, 50$ and 100 and the number of Monte Carlo replicates is $R = 1000$.

The first data generating process considered (DGP1) is the DFM in equations (2.1)-(2.3) with $r = 1$, and the idiosyncratic noises being homoscedastic and cross-sectionally uncorrelated white noises. The matrix of factor loadings, P, is generated once from a uniform distribution in [0,1] with $\sum_{i=1}^{N} p_i^2 = 6.62, 15.87$ and 33.91 for $N = 20, 50$ and 100, respectively. In order to analyze the effect of the temporal dependence of the factor, we consider several values of the autorregresive parameter, $\phi = 0.3, 0.5$ and 0.7. In each case, the noise in equation (2.2), η_t, has variance such that $Var(F_t) = 1$. Finally, the covariance matrix of the idiosyncratic noises is given by $\Sigma_a = q^{-1}I$. Note that, given $Var(F_t) = 1$, the signal to noise ratio is given by $qN^{-1}\sum_{i=1}^{N} p_i^2$. We consider $q = 2, 1$ and 0.5 and, consequently, regardless of N, the signal to noise ratios are approximately given by $0.66, 0.33$ and 0.16, respectively; see Breitung and Eickmeier (2016) who point out that the accuracy of factor estimates can depend on the signal to noise ratio. For each replicate, $i = 1, ..., R$, and moment of time, $t = 1, ..., T$, we construct asymptotic point-wise intervals, $\left(L_t^{(i)}, U_t^{(i)}\right)$ as in

[3] Note that our Monte Carlo design is different from that in Ouysee (2006) as she considers F_t as fixed.

(2.18) with nominal coverages 70% and 95% [4]. Then, at each moment of time, the empirical coverage is computed by counting how many true factors, $F_t^{(i)}, i = 1, ..., R$, lie inside the corresponding interval through the Monte Carlo replicates as $C_t = \frac{1}{R}\sum_{i=1}^{R} I\left(F_t^{(i)} \in \left[L_t^{(i)}, U_t^{(i)}\right]\right)$ where $I(\cdot)$ is the indicator function. We should mention that, in our Monte Carlo experiments, regardless of N and T, the coverages are rather constant over time. Finally, we also compute the length of each interval at each moment of time and for each replicate. Table 1 reports the average coverage across time and the average length across time and Monte Carlo replicates for different temporal and cross-sectional dimensions when $\phi = 0.7$ and $q = 1$[5]. We also report the Monte Carlo average of the scoring rule proposed by Gneiting and Raftery (2007) to measure interval accuracy which is given by

$$SR_t^{(i)} = (U_t^{(i)} - L_t^{(i)}) + \frac{2}{\alpha}(L_t^{(i)} - F_t^{(i)})I(F_t^{(i)} < L_t^{(i)}) + \frac{2}{\alpha}(F_t^{(i)} - U_t^{(i)})I(F_t^{(i)} > U_t^{(i)}). \quad (2.20)$$

Table 1 shows that, regardless the cross-sectional and temporal dimensions, N and T respectively, the coverages of the asymptotic bands are always well bellow the nominal coverages. Furthermore, we can observe that, for fixed T, the undercoverage is larger as N increases. On the other hand, for fixed N, increasing T reduces the undercoverage.

In order to analyze the role of q in the performance of the asymptotic bands, Table 2 reports the coverages, lengths and SRs when the systems are generated by DGP1 with $\phi = 0.7$ and $q = 2, 1$ and 0.5 when $N = T = 50$. Note that, although the coverages are approximately constant (around 0.6 and 0.85 when the nominals are 0.7 and 0.95, respectively), the length and SRs of the asymptotic intervals increase when q decreases. This result could be expected given that, when q is small, the uncertainty around the

[4]Forni et al. (2014) and Barigozzi et al. (2016) construct 64% confidence bands for IRFs. Forni et al. (2014) also consider a nominal coverage of 90% while Bai (2003) considers 95%.

[5]Results for other values of ϕ and q are similar. They are available upon request.

estimated factors is larger.

Finally, to have a better understanding of the finite sample properties of the asymptotic PC confidence bands with more realistic structures of the idiosincratyc components, we also simulate systems with the same parameters as DGP1 but with serially dependent idiosyncratic components generated by equation (2.3) with $\Gamma = \gamma I_N$ and $\gamma = 0.5$ and 0.7 (DGP2) [6,7], cross-sectionally heteroscedastic idiosyncratic components with $\Sigma_a = diag\left[q^{-1}U\left(0.1, 2\right)\right]$ (DGP3) and cross-correlated idiosyncratic components with Σ_a being a Toeplitz matrix with parameter 0.5 (DGP4). Table 2 reports the Monte Carlo coverages, the average lengths and SRs for DGP2 with $\gamma = 0.7$, DGP3 and DGP4. We can observe that the results when the idiosyncratic terms are heteroscedastic[8] are quite similar to the results when the systems were generated by DGP1. When the idiosyncratic component is serially correlated, the coverages reported in Table 2 are slightly smaller than those reported for iid idiosyncratic components. Note that this further undercoverage is more pronounced when q is small. Finally, when the idiosyncratic components are cross-sectionally correlated, the asymptotic coverages are extremely low when q is small. Recall that the asymptotic covariance matrix of the factors is computed as recommended by Bai and Ng (2006) assuming that the idiosyncratic noises are cross-sectionally uncorrelated. According to the results in Table 2, this wrong simplifying assumption may badly affect the construction of confidence intervals for the factors when q is small.

Jackson et al. (2016) show that the conclusions for $r = 1$ could not always be generalized to cases with $r > 1$. Consequently, we also perform Monte Carlo experiments in a DFM in equations (2.1)-(2.3) with $r = 2$ where $\Phi = diag\left(0.7, 0.7\right)$

[6]According to Bai (2003), the limiting distributions are only asymptotically independent if the idiosyncratic noises are serially uncorrelated. However, we still analyze the performance of point-wise intervals as an approximation.

[7]The signal to noise ratio is given by $q\left(1-\gamma^2\right)N^{-1}\sum_{i=1}^{N} p_i^2$.

[8]Results for other sample sizes and idiosyncratic structures are available from the authors upon request.

and Σ_η is diagonal and such that $E(F_tF_t') = I$. The idiosyncratic noises are defined as in DGP1 being homoscedastic and serial and cross-sectionally uncorrelated. Finally, the matrix of factor loadings, P, is generated once from a uniform distribution in [0,1] with $P'P$ being diagonal. The sums of squared loadings of the first factor are $11.40, 28.54$ and 58.53 when $N = 20, 50$ and 100, respectively, while the sums corresponding to the second factor are $2.41, 4.24$ and 8.65. Consequently, regardless of N, the signal to noise ratios of the first factor are approximately $1.14, 0.57$ and 0.29 when $q = 2, 1$ and 0.5 while for the second factor, the corresponding signal to noise ratios are $0.2, 0.1$ and 0.05. For each Monte Carlo replicate and moment of time, the asymptotic ellipsoid is computed as in (2.19). Then, at each moment of time, we compute the coverage of the ellipsiods by counting how many realizations $\left(F_{1,t}^{(i)}, F_{2,t}^{(i)}\right)$ ly within the corresponding ellipsoids. Table 3, which reports the average across time of these coverages, shows that the coverages of the asymptotic ellipsoids can be extremelly low. If $q = 1$, even when $T = 100$, the average coverages are around 0.34 and 0.65 when the corresponding nominals are 0.7 and 0.95, respectivelly. The undercoverage when $q = 0.5$ is even more severe.

Finally, note that, according to our experience in simulations with DFMs with $r = 1$, only when both T and N are larger than 100 and the ratio T/N is larger than 2.5, the asymptotic coverages are close to the nominal. We expect that for $r > 1$ the sample sizes should be even larger for the asymptotic distribution of the factors to be appropriate to approximate their finite sample distribution.

[9]Several authors propose implementing resampling techniques in the context of PC for iid observations; see, for example, Beran and Srivastra (1985), Stauffer et al. (1985), Timmerman et al. (2007), Babamoradi et al. (2013), Van Aelst et al. (2013) and Fisher et al. (2015).

2.3. Extant bootstrap procedures for PC factors

Several alternative bootstrap procedures have been proposed in the context of DFMs with other objectives than constructing confidence bands for extracted factors [9]. In this section, we describe these extant bootstrap algorithms and carry out Monte Carlo experiments to assess their adequacy when implemented to construct confidence bands for extracted PC factors. The extant algorithms can be classified into two main groups: i) Block bootstrap and ii) residual bootstrap.

2.3.1. Block bootstrap

Gospodinov and Ng (2013) propose a moving block bootstrap of the original vector of observations. Denoting by $B_{t,m} = (Y_{\cdot t}, Y_{\cdot t+1}, ..., Y_{\cdot t+m-1})$ a block of m $(1 \leq m < T)$ consecutive observations of $Y_{\cdot t}$, bootstrap replicates $Y_{\cdot t}^{*(b)}$ are obtained by drawing with replacement $K = T/m$ blocks from $(B_{1,m}, B_{2,m}, ..., B_{T-m+1,m})$, for $b = 1, ..., B$, and m growing at a slower rate than T. PC estimates $\tilde{f}_t^{*(b)}$ are obtained as $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of $Y^{*(b)}Y^{*(b)\prime}$. Denote by $\tilde{G}_t^*(x)$ the empirical distribution of $\tilde{f}_t^{*(b)}$ given by

$$\tilde{G}_t^*(x) = \#\left(\tilde{f}_t^{*(b)} \leq x\right)/B. \tag{2.21}$$

For each $t = 1, ..., T$ and $r = 1$ [10], $(1-\alpha)\,\%$ block bootstrap confidence bands for the extracted factors can be constructed as follows

$$[L_t, U_t] = \left[\tilde{q}^*_{(\alpha/2)t}, \tilde{q}^*_{(1-\alpha/2)t}\right], \tag{2.22}$$

[10] Scenarios with $r > 1$ are not considered since we will see that even when $r = 1$, this procedure is not appropriate to construct confidence intervals for the estimated factors.

where $\tilde{q}^*_{it}$ is the ith empirical quantile of $\tilde{G}^*_t(x)$. Alternatively, it is possible to compute the bootstrap MSE at time t, as follows

$$MSE_t = \frac{1}{B}\sum_{b=1}^{B}\left(\tilde{f}^{*(b)}_t - \frac{1}{B}\sum_{b=1}^{B}\tilde{f}^{*(b)}_t\right)^2. \tag{2.23}$$

Assuming normality of the factors, $(1-\alpha)\%$ block bootstrap confidence intervals are constructed by

$$[L_t, U_t] = \left[\tilde{f}_t - z_{\alpha/2}MSE_t^{1/2}, \tilde{f}_t + z_{\alpha/2}MSE_t^{1/2}\right]. \tag{2.24}$$

It is important to note that, when bootstrapping $Y^{*(b)}_{\cdot t}$ as proposed by Gospodinov and Ng (2013), one obtains replicates of the marginal distribution of $\{Y_{\cdot t}\}$ and, consequently, of the marginal distribution of F_t in (2.4). If confidence bands are constructed as in (2.22), they will be centered at zero with MSE given by (2.23). Although, they will have the correct coverages, they are uninformative. On the other hand, when the intervals are computed as in (2.24), the MSE is marginal while the intervals are centered at $\tilde{f}_t$. Note that these intervals will be too wide with coverages expected to be above the nominal. As an illustration, we consider again the same simulated system described above and construct confidence bands for the factor using (2.22) and (2.24) with, as suggested by Gospodinov and Ng (2013), $m = 4$ and $B = 1000$ bootstrap replicates; see Ludvigson and Ng (2007, 2009, 2010) for $B = 1000$. Figure 1 plots the true and PC estimated factors together with 95% confidence bands. We can observe that, when the bands are constructed as in (2.22), they are approximately constant around ± 2 as expected given that the factor is normally distributed with zero mean and variance 1. As mentioned above, these bands have the assumed coverage but they are not informative about the evolution of the factor. On the other hand, when the bands are constructed as in (2.24), they are much wider than those based on the asymptotic approximation and the true factor is always within the bands. Obviously, these bands are too wide.

The finite sample performance of the block bootstrap bands are analyzed by Monte Carlo experiments using DGP1 described above. Even this idealized setting is sufficient to demonstrate that the block bootstrap has a poor performance when implemented to obtain confidence intervals for the factors. Consequently, we do not consider any of the other DGPs considered in the previous section. Table 1 reports the coverages through Monte Carlo experiments, average lengths and SRs for 70% and 95% block bootstrap confidence intervals constructed as in (2.22) and (2.24) and denoted by block bootstrap 1 and block bootstrap 2, respectively. Consider first the intervals constructed as in (2.22). Regardless of N and T, the coverages are close to the nominal but the lengths and SRs are extremely large. The intervals are conservative to the point of being non-informative. On the other hand, when the intervals are constructed as in (2.24), they are not appropiate with coverages close to 1 even when the nominal coverage is 0.7. Observe that the length is similar to that observed for the confidence intervals in (2.22). Furthermore, the average SR measure of the block bootstrap intervals for the factors is larger than those of the asymptotic intervals except when $N = T = 20$ and 95% confidence intervals are considered.

Regardless of whether they are based on (2.22) or (2.24), the block bootstrap intervals are not appropriate to obtain a measure of the uncertainty of the estimated PC factors.

[11] Gonçalves and Perron (2014) and Djogbenou et al. (2015) propose a wild bootstrap algorithm to obtain replicates of ${\varepsilon^*_{\cdot t}}^{(b)}$ that take into account potential heteroscedasticity while Breitung and Eickmeier (2016) propose a block bootstrap scheme to account for the serial correlation of the idiosyncratic noises.

2.3.2. Residual bootstrap

Bootstrapping DFM using residual bootstrap schemes is very popular. Ludvigson and Ng (2007, 2009 and 2010) obtain bootstrap replicates of $Y_{\cdot t}$ as follows

$$Y_{\cdot t}^{*(b)} = \tilde{P}\tilde{f}_t + \tilde{\varepsilon}_{\cdot t}^{*(b)} \tag{2.25}$$

where $\tilde{\varepsilon}_{\cdot t}^{*(b)}$ are random extractions with replacement from $\tilde{G}_{\varepsilon}$[11], the empirical distribution of $\tilde{\varepsilon}_{\cdot t} = Y_{\cdot t} - \tilde{P}\tilde{f}_t$. PC estimates of the factors, $\tilde{f}_t^{*(b)}$, are obtained as $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of $Y^{*(b)}Y^{*(b)\prime}$. The residual bootstrap confidence intervals can be constructed as in (2.22) or as in (2.24) based on the corresponding empirical bootstrap density or MSE, respectively. When the intervals are constructed as in (2.24), they are called time-residual bootstrap intervals. It is important to note that all bootstrap replicates of $Y_{\cdot t}$ in equation (2.25) are centered in the estimated common factor $\tilde{P}\tilde{f}_t$ and incorporate uncertainty about the idiosyncratic noises but not about the parameters. Consequently, although the corresponding intervals are adequately centered, they are expected to have coverages bellow the nominal. As an illustration, we consider again the same simulated factor described when constructing asymptotic and block bootstrap intervals. Figure 1, which plots the factor together with 95% point-wise time-residual bootstrap intervals[12], shows that they are very similar to the asymptotic intervals with the true factor lying very often outside their limits.

Table 1, which reports the Monte Carlo results of the time-residual bootstrap confidence intervals for the same designs described above, shows that the average coverages are even lower than those of the asymptotic intervals. Furthermore, they decrease when T increases. This is due to the fact that, as T increases, the PC factor estimate is consistent and therefore the bootstrap factors are very similar in all bootstrap replicates.

[12]The results when the residual bootstrap intervals are constructed as in (2.22) are almost identical.

Shintani and Guo (2015) propose two alternative residual bootstrap procedures. For $i = 1, ..., N$, $Y_{i\cdot} = (Y_{i1}, Y_{i2}, ..., Y_{iT})$ is the ith row of Y and $\tilde{\varepsilon}_{i\cdot} = (\tilde{\varepsilon}_{i1}, \tilde{\varepsilon}_{i2}, ..., \tilde{\varepsilon}_{iT})$ the corresponding vector of residuals. The first algorithm proposed by Shintani and Guo (2015) is based on generating bootstrap replicates of $Y_{i\cdot}$, for $i = 1, ..., N$, as follows

$$Y_{i\cdot}^{*(b)} = \tilde{P}_i^{*(b)} \tilde{f} + \tilde{\varepsilon}_{i\cdot}^{*(b)} \tag{2.26}$$

where $\left(\tilde{P}_i^{*(b)}, \tilde{\varepsilon}_{i\cdot}^{*(b)}\right)$ are joint random extractions with replacement from pairs $\left(\tilde{P}_i, \tilde{\varepsilon}_{i\cdot}\right)$. PC estimates of the factors, $\tilde{f}_t^{*(b)}$, are obtained as $\sqrt{T}$ time the eigenvectors corresponding to the r largest eigenvalues of $Y_{\cdot t}^{*(b)} Y_{\cdot t}^{*(b)\prime}$. Note that the bootstrap replicates in (2.26) are based on random draws obtained from the cross-sample pairs of weights and residuals instead of bootstrapping in the time dimension as in (2.25). This procedure is called cross-residual bootstrap. Given that the estimated weights are also bootstrapped, the corresponding bands for the factors are expected to be larger than those obtained using the time residual bootstrap in (2.25). However, all replicates of $Y_{i\cdot}$ are constructed based on the same estimated factors. Therefore, given that they do not incorporate the uncertainty associated with the estimation of the factors, it is expected that the coverages of cross-residual bootstrap intervals will be bellow the nominal. As an illustration, we consider again the same simulated factor described above. Figure 1 plots the factor together with its PC estimation and 95% cross-residual bootstrap intervals constructed as in (2.24) [13]. We can observe that, as explained before, the confidence bands are slightly larger than those obtained using the asymptotic approach and the time-residual bootstrap. However, there are still too many moments of time in which the true factors are outside the bands. Table 1 reports the Monte Carlo results of the cross-residual bootstrap confidence intervals. We can observe that the coverages are better than when the time-residual bootstrap is implemented but still well bellow

[13] The results when the residual bootstrap intervals are constructed as in (2.22) are almost identical.

the nominal. Table 1 also reports the average SR interval accuracy measures when the intervals are constructed using the cross-residual bootstrap. We can observe that the average values of the SR statistic are even larger than those observed for the asymptotic intervals.

The second bootstrap algorithm proposed by Shintani and Guo (2015). Consider that $r = 1$. In this case, bootstrap replicates are obtained as follows

$$\tilde{F}_t^{*(b)} = \hat{\phi}\tilde{F}_{t-1}^{*(b)} + \tilde{\eta}_t^{*(b)} \tag{2.27}$$

$$Y_{\cdot t}^{*(b)} = \tilde{P}^{*(b)}\tilde{F}_t^{*(b)} + \hat{\varepsilon}_{\cdot t}^{*(b)} \tag{2.28}$$

where $\hat{\phi}$ is the OLS estimator of the autorregresive parameter of an AR(1) model fitted to $\tilde{f}_t$ and $\tilde{\eta}_t^*$ are random extractions with replacement from the empirical distribution function of the centered residuals, $\hat{\eta}_t = \tilde{f}_t - \hat{\Phi}\tilde{f}_{t-1}$ and $\tilde{P}^{*(b)}$ and $\hat{\varepsilon}_{\cdot t}^{*(b)}$ are defined as in (2.26). The bands, based on the factors extracted using $Y_{\cdot t}^{*(b)}$ defined as in (2.28), are marginal given that they are based on bootstrap replicates of the factors in (2.27) which are not based on the available information set. Therefore, we expect a similar behaviour as that of the bands constructed using the block bootstrap procedure [14].

Finally, Yamamoto (2016) considers two further residual-based bootstrap procedures. The first one is based on factor estimation based on bootstrap replicates generated as in (2.25) while the second one treats the original factor as in (2.27) [15]. The performance of the first procedure is the same as that of the residual bootstrap procedure proposed by Ludvigson and Ng (2007, 2009 and 2010) and, therefore, we do not consider it further in this chapter. The second one obtain marginal bands. Therefore, we expect a similar behaviour as that of the bands constructed using the block bootstrap algorithm [16].

[14] Monte Carlo results are available upon request.
[15] Alonso et al. (2008 and 2011) propose a bootstrap with the same structure for forecasting purposes.
[16] Monte Carlo results are available upon request.

Summarizing the results in this section, we can conclude that none of the residual bootstrap procedures available in the context of PC factor extraction in DFM, are adequate to construct confidence bands of the factors with coverages close to the nominal ones.

2.4. Conditional subsampling for factors

Given that the asymptotic and bootstrap procedures described in previous sections are not adequate when the aim is to construct confidence intervals of the underlying factors, in this section, we propose a resampling strategy designed for this purpose. Its finite sample performance is analyzed through extensive Monte Carlo experiments.

2.4.1. Subsampling procedure

In previous sections, we have seen that neither the asymptotic approximation nor the available bootstrap procedures are adequate to measure the uncertainty associated with factors extracted using PC when the temporal and cross-sectional sizes are not large enough. The main problem associated with asymptotic intervals and regions is that they do not incorporate the parameter estimation uncertainty. On the other hand, there are two main problems associated with the failure of boostrap procedures. First, as explained above, these procedures either compute the marginal MSE of the factors and/or do not incorporate paramater uncertainty. Second, there is evidence about the bootstrap being fraught with problems when implemented in models with high dimensions. For example, El Karoui and Purdom (2015) show that both residual bootstrap and pairs bootstrap give poor inference on the LS estimator of the parameters of a regression model when the number of regressors is large relative to the sample size. They show that the residual bootstrap tend to give anti-conservative estimates while the pairs bootstrap gives very conservative estimates as the ratio between the number of regressors and the sample size grows. Recall that the PC estimator is related to the LS estimator and, as such, we expect the same problems observed in regression models to affect PC factor extraction.

First, to solve the problem of incorporating parameter uncertainty, in this chapter, we propose to compute the uncertainty associated with PC factor extraction by considering the decomposition of the total MSE into the noise uncertainty and the parameter estimation uncertainty as in (2.9). While the noise uncertainty can be computed using the asymptotic covariance in (4.7), we propose computing the parameter estimation uncertainty in (2.11) by first computing the MSE of $\hat{f}_t$ conditional on the parameter estimates and then computing the average of this conditional MSE over the distribution of the parameter estimator; see Hamilton (1986), Pfeffermann and Tiller (2005) and Rodriguez and Ruiz (2012) for the same strategy in the context of state space models. Consequently, the MSE attributed to parameter uncertainty is given by

$$\underset{\tilde{P}}{E}\left[\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)'|\tilde{P}\right]\right] = \frac{1}{N^2}\underset{\tilde{P}}{E}\left[\left(\widetilde{P} - P\right)' Y_{\cdot t}Y_{\cdot t}'\left(\tilde{P} - P\right)\right], \tag{2.29}$$

where the expected value $\underset{\tilde{P}}{E}$ is taken over the sampling distribution of $\tilde{P}$. Note that because the MSE in (2.29) depends on $Y_{\cdot t}$, it is conditional on the particular observed sample and it is not marginal with respect to all possible realizations of $Y_{\cdot t}$.

Second, in order to deal with the lack of adequacy of the bootstrap in the context of high-dimensional LS problems, we propose to estimate the sampling distribution of $\tilde{P}$ using subsampling as proposed by Politis and Romano (1994); see Politis (2003) for the advantages of subsampling. The basic idea is to approximate the sampling distribution of $\tilde{P}$ based on estimates of the loadings computed over subsets of data of cross-sectional size $N^* < N$. Under weak hypothesis, the sample distribution of the PC estimator of the loadings based on N^* and that based on N should be close. Consequently, using subsampling, it is possible to accurately estimate the sampling distribution of the PC estimator of the loadings.

The subsampling algorithm is given next. For $b = 1, ..., B$, obtain the $N^* \times T$ matrix $Y^{*(b)}$ by drawing N^* vectors randomly without replacement from $Y_{i\cdot} = (y_{i1}, y_{i2}, ..., y_{iT})$, where $N^* = p \times N$ with $0 \leq p \leq 1$. Using $Y^{*(b)}$, obtain PC estimates $\widetilde{P}^{*(b)}$ and compute

$$\hat{f}_t^{*(b)} = \frac{1}{N^*}\widetilde{P}^{*(b)\prime}Y_{\cdot t}^{*(b)}. \tag{2.30}$$

The subsampling analog of the MSE due to parameter uncertainty conditional on the parameter estimates given by $\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)' |\tilde{P}\right]$ is given by $\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)'$ and, consequently,

$$\underset{\tilde{P}}{E}\left[\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)' |\tilde{P}\right]\right] = \frac{1}{B}\sum_{b=1}^{B}\left(\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)'\right). \tag{2.31}$$

Finally, the subsampling analog of the MSE_t^* of $\widehat{f}_t$ is given by

$$MSE_t^* = \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}\left(\frac{1}{B}\sum_{b=1}^{B}\left(\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)\left(\widehat{f}_t^{*(b)} - \widehat{f}_t\right)'\right) + \tilde{\Gamma}_t\right)\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}, \tag{2.32}$$

where $\tilde{\Gamma}_t$ is defined as in (4.7).

Subsampling works well under weak assumptions because each subset of size N^* (taken without replacement from the original data $(Y_{1\cdot}, ..., Y_{N\cdot})$ is indeed a sample of size N^* from the true DGP and, consequently, the sampling distributions of $\tilde{P}$ based on samples of size N^* and N should be close. As a result, choosing an adequate subsampling size, N^*, is very important for both the asymptotic and finite sample validity of the procedure. Note that if N^* is too large, there is not enough variability in the estimated loadings as all of them will be obtained using similar samples. On the other hand, if N^* is too small, the variability could be too large and not similar to that corresponding to the estimator based on the cross-sectional dimension N. For the asymptotic validity of the subsampling estimator, $\tilde{P}^*$, $\frac{N^*}{N} \rightarrow 0$ and $N^* \rightarrow \infty$ when $N \rightarrow \infty$; see Politis et al. (2001). However, the optimal block size is unknown in practice. In finite samples, we carry out extensive simulation experiments and conclude that if $q = 1$ and $p = 0.8 + 0.09 \log_{10}\left(\frac{T}{N}\right)$, the coverages of the estimated factors are optimal. On the other hand if $q < 1$, i.e. the signal to noise ratio decreases, then p should be smaller while if $q > 1$ then p should be larger for the subsampling coverages to be optimal.

The proposed procedure is computationally very simple [17].

In the context of Gaussian DFM, as that considered in this chapter, the PC extracted factors are normally distributed. Consequently, when $r = 1$, the corresponding subsampling $(1-\alpha)\,\%$ point-wise confidence interval for the true factor, F_t, is given by

$$[L_t, U_t] = \left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t - z_{\alpha/2} MSE_t^{*1/2}, \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t + z_{\alpha/2} MSE_t^{*1/2}\right], \tag{2.33}$$

where MSE_t^* is defined as in (2.32). When $r > 1$, the subsampling regions are given by

$$\left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t\right] MSE_t^{*-1} \left[\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1} \hat{f}_t\right]' \leq \chi^2_\alpha(r). \tag{2.34}$$

Bai (2003) shows that normality of the factors holds even without assuming normality. Consequently, we guess that the coverages of the intervals and regions given by (2.33) and (2.34) can be close to the nominals even in the context of non-Gaussian DFMs.

[17] For $B = R = 500$ and $N = T = 50$, it takes 6 minutes and 52 seconds to compute the subsampling MSE on Intel i7-6700 (4 cores - 2.6 GHz).

As an illustration of the behaviour of the subsampling intervals in (2.33), Figure 1 plots the same simulated and PC estimated factor considered above together with the corresponding 95% point-wise subsampling confidence bands.[18] When compared with the asymptotic or the (time) (cross) residual bootstrap bands, we can observe that the subsampling bands are wider. On the other hand, the subsampling bands are more informative than the marginal bootstrap bands and than the wrong bands constructed using the block bootstrap MSEs and centered in the estimated factors.

We also illustrate the new proposed procedure to construct regions for estimated PC factors when $r = 2$. With this purpose, we consider the factors simulated by the same DFM with $r = 2$ described in subsection 2.3 when dealing with the asymptotic confidence regions. Figure 2 plots the point-wise subsampling and asymptotic 95% contours obtained for $t = 1, ..., 25$. It can be observed that the subsampling regions are considerably wider than the asymptotic ones and contain the true factors in a larger proportion of times.

It is important to note that subsampling is carried out in the cross-sectional dimension. Consequently, the information on the temporal dependence is kept. This is why this procedure can be valid even if the factors are non-stationary as far as the idiosyncratic noises are stationary; see Bai (2004) for the consistency of PC non-stationary factors.

Finally, note that the subsampling procedure proposed in this chapter could also be easily extended to compute the parameter uncertainty of the common component if it were of interest.

[18]The subsampling has been carried out with $B = 500$. The results based on $B = 500$ are already very reliable.

2.4.2. Finite sample performance

We carry out Monte Carlo experiments in order to asses the adequacy of the propose subsampling procedure to approximate the MSEs of PC factors and, consequently, to construct confidence intervals and regions. The Monte Carlo experiments are performed using the same DGPs considered above. The number of subsampling replicates is $B = 1000$. For each Monte Carlo replicate, $i = 1, ..., R$, we construct point-wise intervals as in equation (2.33). Table 1 reports, for nominal coverages of 70% and 95%, the average coverage across time and the average length across time and Monte Carlo replicates for different temporal and cross-sectional dimensions when $\phi = 0.7$ and $q = 1$[19]. Observe that, regardless N and T, the proposed subsampling procedure estimates correctly the uncertainty of PC factors with coverages always close to the nominal. Furthermore, the SR measure of the subsampling intervals is also considerably smaller than those of the asymptotic intervals and of the extant bootstrap procedures.

Table 2 reports the Monte Carlo results when $T = N = 50$ for DGP2, DGP3 and DGP4 described before, with serial ($\gamma = 0.7$), cross-sectional heteroscedasticity and cross-sectional dependence in the idiosyncratic term, respectively. It can be observed that, the presence of serial correlation and heteroscedasticity of the idiosyncratic term does not affect the finite sample performance of the proposed subsampling procedure. Regardless of the signal to noise ratio, the coverages are rather close to the nominal and much larger than those obtained when using the asymptotic approximation. The same is true when there is cross-dependence and the signal to noise ratio is large enough, $q = 1, 2$. However, when the signal to noise ratio is small ($q = 0.5$), the proposed procedure has smaller coverages than the nominal. This could be due to the way the covariance matrix is computed.

[19] For brevity, we give only brief descriptions of the simulations in what follows. Detailed descriptions are available upon request.

DFMs with $r = 2$ have also been considered. In particular, we consider the same DGP1 described when dealing with asymptotic regions. For each Monte Carlo replicate, we construct the point-wise subsampling regions as in equation (2.34). Table 3, which reports the average coverages across time, shows that, regardless the system dimensions and the value of q, the subsampling coverages are very close to the nominal ones. Therefore, the new procedure estimates correctly the uncertainty for more than one factor.

2.5. Empirical illustration

In this section, we illustrate the importance of a proper measurement of the uncertainty associated with PC estimates of the factors by analyzing a system of $N = 60$ seasonally adjusted macroeconomic Spanish variables observed quarterly from the first quarter of 1980 to the last of 2015 with $T = 144$ [20]. The variables are converted to stationary. The list of all variables and their stationary transformations are reported in the Appendix. After centering and standardizing each of the variables in the system, the number of common factors is determined using the criteria by Ahn and Horenstein (2013) as one. The factor is extracted by PC and confidence intervals are constructed using the asymptotic approximation and the subsampling procedure proposed in this chapter. The sum of squared weights is $\sum_{i=1}^{N} \tilde{p}_i^2 = 9.71$ with estimated weights larger than 0.8 in absolute value corresponding to: gross capital formation, capital stock, imports, unemployment rate, rest of the word clients' GDP and total resources of public administrations. The estimated autorregressive parameter is $\hat{\phi} = 0.6$, $\hat{\sigma}_a^2 = [0.29, 0.99]$ with the mode close to 0, and serial dependence with $\hat{\gamma} = [-0.75, 0.96]$ distributed uniformly in this interval. Figure 3 plots the estimated PC factor together with 95% confidence bands constructed using the asymptotic approach and the subsampling

[20] The Database considered is built by the Ministry of Treasure and Public Administration, "Base de datos trimestrales de la economía española".

procedure proposed in this chapter. Additionally, a line representing a scenario of zero-growth has been included in order to facilitate the interpretation of the cycles. As expected, the asymptotic confidence intervals are narrower than those constructed following the procedure proposed in this chapter. Figure 4 plots the asymptotic and the subsampling MSEs. It can be seen how the uncertainty estimated under both methods is fairly similar, with the exception of periods of economic crisis (1993, 2001 and 2008). It can also be observed that the new procedure detects phases of high macroeconomic uncertainty several periods in advance. If practitioners and policy decision makers use the asymptotic approximation for constructing confidence bands for the latent factors, it could lead to a wrong interpretation of the economic reality -cycles and recessions-. The conclusion of a favourable economic situation could be drawn when both the extracted factor and its confidence intervals have positive values. However, when the intervals are constructed using subsampling, they include negative values. Consequently, it is not possible to confirm a period of economic growth at the established level of confidence.

Tables and figures

Table 2.1: Monte Carlo coverages (C), lengths (L) and Scoring Rule (SR) of asymptotic, extant bootstrap procedures and new resampling bands when the idiosyncratic component is homoscedastic and serial and cross-sectionally uncorrelated with $r = 1$, $\phi = 0.7$ and $q = 1$.

		T=20			T=50			T=100		
		N=20	N=50	N=100	N=20	N=50	N=100	N=20	N=50	N=100
Asymptotic										
70%	C	0.50	0.50	0.47	0.59	0.59	0.54	0.61	0.62	0.62
	L	0.73	0.50	0.35	0.77	0.52	0.36	0.77	0.51	0.37
	SR	1.87	1.27	1.05	1.49	1.03	0.84	1.41	0.91	0.7
95%	C	0.77	0.78	0.74	0.86	0.87	0.83	0.88	0.90	0.90
	L	1.38	0.94	0.67	1.45	0.98	0.67	1.46	0.96	0.64
	SR	4.02	2.77	2.55	2.67	1.86	1.72	2.41	1.48	1.21
Block Bootstrap 1										
70%	C	0.77	0.77	0.79	0.76	0.76	0.76	0.74	0.74	0.75
	L	2.01	2.01	2.01	2.04	2.02	2.04	2.06	2.05	2.03
	SR	2.59	2.61	2.58	2.74	2.72	2.8	2.89	2.82	2.78
95%	C	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	L	3.55	3.52	3.58	3.74	3.68	3.73	3.8	3.74	3.76
	SR	3.67	3.65	3.69	3.85	3.79	3.86	3.93	3.86	3.85
Block Bootstrap 2										
70%	C	0.93	0.97	0.98	0.98	0.99	0.99	0.98	0.99	0.99
	L	1.97	1.96	1.97	2.03	2.00	2.03	2.05	2.04	2.04
	SR	1.97	1.99	1.99	2.05	2.01	2.03	2.07	2.04	2.04
95%	C	0.99	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00
	L	3.73	3.71	3.74	3.85	3.79	3.85	3.89	3.86	3.85
	SR	3.79	3.71	3.74	3.85	3.79	3.85	3.89	3.86	3.85
Time-Residual Bootstrap										
70%	C	0.48	0.5	0.59	0.37	0.39	0.39	0.35	0.37	0.33
	L	0.66	0.51	0.50	0.39	0.3	0.23	0.33	0.27	0.16
	SR	1.66	1.18	0.98	1.44	1.06	0.84	1.35	1.01	0.75
95%	C	0.76	0.78	0.85	0.64	0.66	0.66	0.61	0.64	0.58
	L	1.24	0.97	0.95	0.75	0.57	0.44	0.63	0.52	0.31
	SR	3.57	2.5	1.98	3.77	2.76	2.24	3.79	2.63	2.27
Cross-Residual Bootstrap										
70%	C	0.63	0.61	0.54	0.65	0.66	0.62	0.67	0.67	0.64
	L	1.61	1.48	1.26	1.67	1.28	1.06	1.43	1.18	0.83
	SR	2.64	2.57	2.36	2.66	1.99	1.98	2.27	1.69	1.43
95%	C	0.86	0.85	0.82	0.83	0.87	0.87	0.91	0.91	0.88
	L	2.84	2.68	2.38	2.84	2.45	2.03	2.89	2.24	1.50
	SR	3.14	3.00	2.72	3.05	2.56	2.24	2.91	2.35	1.69
Subsampling										
70%	C	0.68	0.71	0.71	0.72	0.71	0.70	0.73	0.73	0.71
	L	1.12	0.80	0.62	1.02	0.67	0.50	100	0.64	0.46
	SR	1.71	1.18	0.95	1.47	1.00	0.79	1.39	0.90	0.69
95%	C	0.93	0.94	0.93	0.94	0.94	0.94	0.95	0.96	0.95
	L	2.12	1.51	1.17	1.93	1.26	0.95	1.89	1.22	0.87
	SR	2.73	1.92	1.54	2.35	1.62	1.34	2.23	1.4	1.07

Table 2.2: Monte Carlo coverages (C), lengths (L) and Scoring Rule (SR) of intervals based on the asymptotic approximation and on subsampling for different idiosyncratic structures with $r = 1$, $T = N = 50$.

	q	Nominal		Independence	Serial Dependence	Cross-Dependence	Heteroscedasticity
Asymptotic	2	70	C	0.57	0.55	0.58	0.58
			L	0.38	0.37	0.38	0.53
			SR	0.81	0.83	0.81	0.83
		95	C	0.84	0.83	0.85	0.85
			L	0.72	0.70	0.72	0.73
			SR	1.58	1.49	1.39	1.47
	1	70	C	0.58	0.55	0.56	0.58
			L	0.51	0.48	0.5	0.52
			SR	1.03	1.04	1.01	1.05
		95	C	0.86	0.83	0.85	0.86
			L	0.96	0.91	0.95	1.01
			SR	1.86	1.85	1.89	1.84
	0.5	70	C	0.58	0.52	0.39	0.57
			L	0.67	0.61	0.58	0.71
			SR	1.38	1.44	1.87	1.35
		95	C	0.86	0.81	0.66	0.85
			L	1.26	1.16	1.10	1.34
			SR	2.45	2.65	4.91	2.53
Subsampling	2	70	C	0.69	0.68	0.69	0.69
			L	0.5	0.49	0.49	0.5
			SR	0.81	0.78	0.75	0.79
		95	C	0.93	0.93	0.94	0.94
			L	0.94	0.92	0.93	0.95
			SR	1.27	1.15	1.09	1.47
	1	70	C	0.7 0	0.69	0.70	0.70
			L	0.65	0.64	0.66	0.69
			SR	1.03	0.99	0.96	1.00
		95	C	0.93	0.92	0.93	0.94
			L	1.24	1.21	1.25	1.31
			SR	1.62	1.57	1.62	1.55
	0.5	70	C	0.71	0.67	0.58	0.71
			L	0.88	0.88	0.98	0.97
			SR	1.38	1.35	1.59	1.30
		95	C	0.95	0.93	0.86	0.94
			L	1.66	1.66	1.84	1.83
			SR	2.1 0	2.16	2.69	2.21

Table 2.3: Monte Carlo averages of coverages of asymptotic and subsampling ellipsoids when the idiosyncratic component is homoscedastic and serial and cross-sectionally uncorrelated.

			T=20			T=50			T=100		
	q	Nominal	N=20	N=50	N=100	N=20	N=50	N=100	N=20	N=50	N=100
Asymptotic	2	70	0.22	0.19	0.15	0.33	0.32	0.30	0.40	0.40	0.41
		95	0.45	0.37	0.28	0.59	0.58	0.55	0.69	0.71	0.70
	1	70	0.17	0.16	0.10	0.31	0.27	0.25	0.34	0.34	0.35
		95	0.45	0.33	0.23	0.54	0.51	0.48	0.65	0.66	0.68
	0.5	70	0.17	0.11	0.08	0.19	0.15	0.16	0.20	0.21	0.24
		95	0.44	0.23	0.17	0.47	0.31	0.29	0.47	0.46	0.48
Subsampling	2	70	0.72	0.71	0.70	0.71	0.70	0.70	0.70	0.70	0.70
		95	0.91	0.91	0.90	0.89	0.90	0.90	0.91	0.90	0.91
	1	70	0.70	0.70	0.71	0.70	0.7	0.71	0.70	0.71	0.71
		95	0.94	0.95	0.92	0.94	0.93	0.93	0.95	0.93	0.91
	0.5	70	0.70	0.70	0.73	0.71	0.71	0.71	0.70	0.70	0.70
		95	0.92	0.89	0.89	0.91	0.91	0.93	0.92	0.92	0.92

Figure 2.1: Factor generated by a DFM (black continous lines) together with its PC estimates (blue discontinous lines) and 95% confidence bands (red continous lines) constructed using the asymptotic approximation (first row, first column), the subsampling procedure (first row, second column), block bootstrap based on bootstrap quantiles (second row, first column) and on Gaussian densities with bootstrap MSEs (second row, second column), time-residual bootstrap (third row, first column) and cross-residual bootstrap (third row, second column).

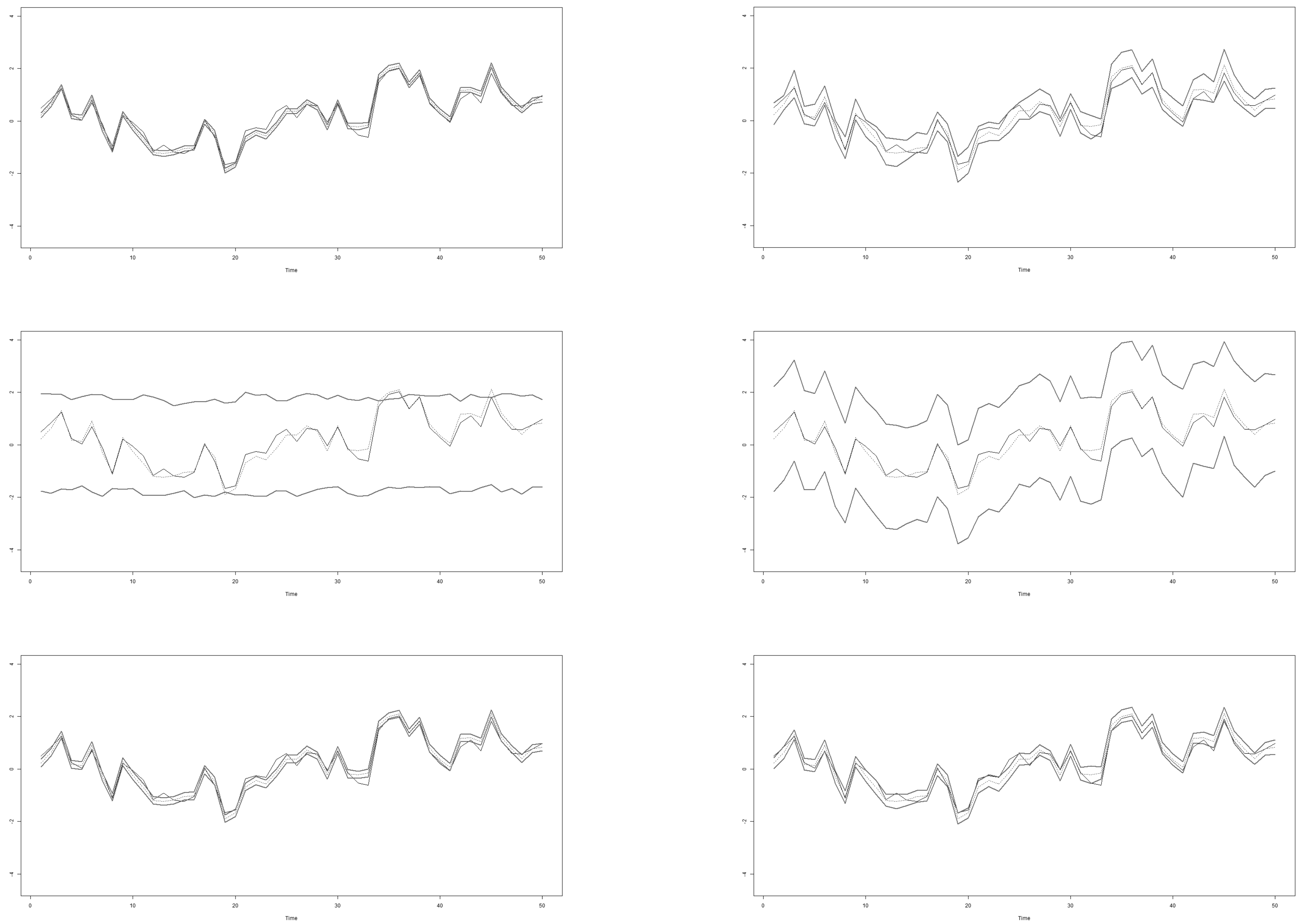


Figure 2.2: Factor generated by a DFM (black points) together with 95% confidence contours constructed using the asymptotic approximation (blue lines) and the subsampling procedure (red line) for every $t = 1, ..., 25$.

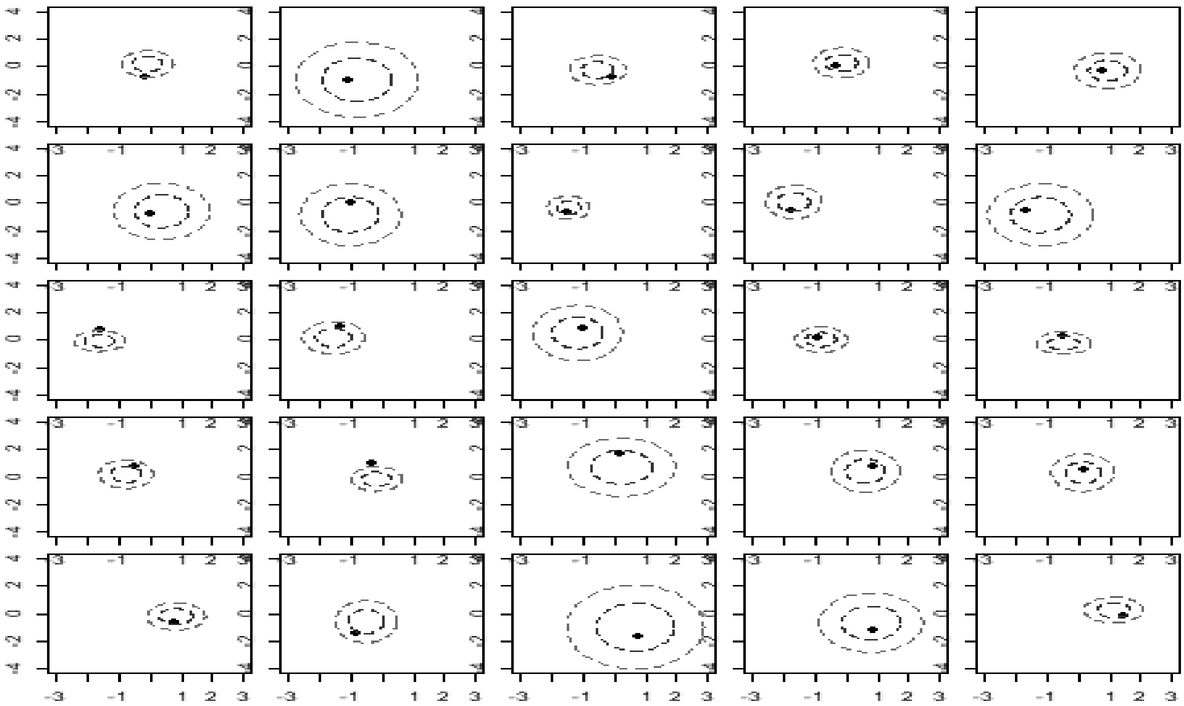


Figure 2.3: Asymptotic (blue continous lines) and resampling 95% intervals (red discontinous lines) for estimated economic cycle in Spain (black line).

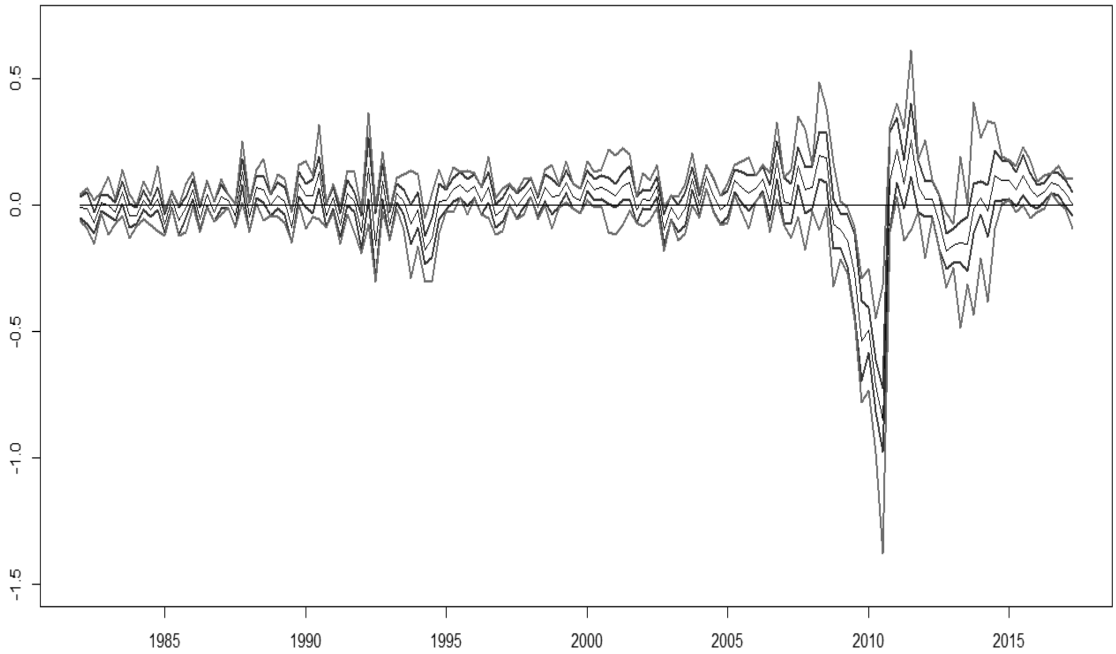


Figure 2.4: Asymptotic (blue lines) and resampling MSE (red lines) for estimated economic cycle in Spain.

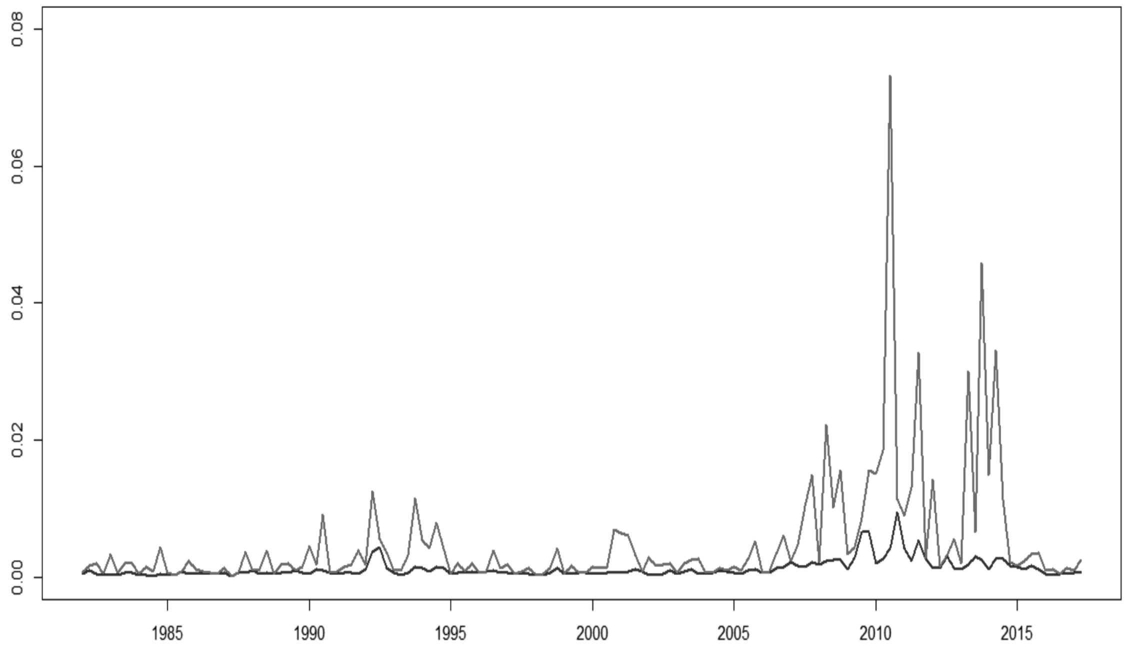


Table 2.4: List of the macroeconomic Spanish variables and their stationary transformations.

Variable	Stationarity
Gross Domestic Product mp	I(2)
Non-Market Servive Sector GAV	I(2)
Private GDP	I(2)
Households and non-profit institutions serving households final consumption expenditure	I(1)
Final consumption expenditure of Public Administrations	I(2)
Gross Capital Formation	I(1)
Gross Fixed Capital Formation	I(2)
Stock variations	I(0)
Exports of goods and services	I(1)
Imports of goods and services	I(1)
Imports of goods	I(1)
Imports of consumer goods	I(1)
Imports of capital goods	I(1)
Imports of intermediate goods	I(1)
Capital stock	I(2)
GDPmp deflator	I(2)
Labour market	I(2)
Workers in employment	I(2)
Workers in employment: full-time job equivalents	I(2)
Ratio full-time job equivalents/workers in employment	I(1)
Employees	I(2)
Employees: full-time job equivalents	I(2)
Workers in non-market services	I(2)
Workers in non-market services: full-time job equivalents	I(2)
Unemployment rate	I(2)
Number of hours worked	I(2)
Total compensation of employees (cp)	I(2)
Net taxes on products	I(2)
Private GDP at basic prices	I(2)
GDP deflator at basic prices (2010=1)	I(1)
Energy prices index	I(1)
Spain's Monetary Supply (M1)	I(1)
Spain's Monetary Supply (M3)	I(2)
US 3-month interest rates	I(1)
Vacancies	I(2)
Nominal exchange rate	I(1)
Debt of public administrations	I(2)
Net financial assets national economy	I(2)
Total resources of the public administrations	I(1)
Market production (P. 11)	I(1)
Non-market payments	I(1)
Taxes on production and imports	I(1)
Property income	I(1)
Current taxes on income, wealth, etc.	I(1)
Social contributions	I(2)
Other current transfers	I(1)
Capital transfers	I(1)
Overall employement in public administrations	I(2)
Intermediate consumption	I(2)
Other taxes on production	I(1)
Subsidies	I(1)
Social benefits other than transfers in kind	I(2)
Social transfers in kind related to the expenses in products supplied to households by market producers	I(1)
Purchases minus Transfers of non-financial Assests	I(0)
Lending (+)/Borrowing (-) capacity	I(1)
Unemployment benefits	I(2)
Stock of public capital	I(2)
Current taxes onincome	I(1)
3-month yields	I(1)

Chapter 3

Growth in Stress

3.1. Introduction

There is a large evidence on the presence of cross-country links in macroeconomic fluctuations with world and regional business cycles having different effects on developing and developed economies. For example, Kose et al. (2003), Imbs (2010), Kose et al. (2012) and Bjornland et al. (2017) conclude that the world factor is more important in explaining fluctuations in developed stable economies, whereas country-specific factors are more important in developing, volatile economies. Similarly, Ozturk and Sheng (in press) show that some regional recession episodes are associated with higher uncertainty than global recession episodes. For instance, the peaks of uncertainty in Indonesia and South Korea are higher around the 1997 Asian financial crisis than around the recent global recession. The presence of world and regional business cycles leads to the possibility of exploring a macroeconomic global risk when these common cycles are subject to extreme negative scenarios. The related literature considering common factors and macroeconomic risk has considered the factors fixed at their (estimated) expected values. However, if factors are drivers of economic growth, the potential growth risk must naturally be a function of factor risk.

Thus, we need to consider factors beyond their expected values and to explore their lower quantiles where stress is measured.

The proposed methodology is based on using predictive quantile regressions of output growth augmented with common factors as predictors. The factors are extracted using principal components (PC) from a large set of macroeconomic aggregates modeled using Dynamic Factor Models (DFMs), and their joint probability density is computed by the subsampling method proposed in the second chapter. To construct the risk index for each country, we consider the Value-in-Stress (ViS) risk measure proposed by González-Rivera (2003) in the context of monitoring capital requirements to control market risk. Adapted to a macroeconometric context, the ViS, denoted as GiS for Growth-in-Stress, is defined as (minus) the lowest expected Gross Domestic Product (GDP) growth (or quantile of growth) in a given country when there is extreme stress in the macroeconomic common factors. We calculate the risk exposure of each country to extreme changes in the macroeconomic factors and the country's ability to withstand stressful scenarios, which may eventually generate economic crises. One important advantage of our approach is that, together with the calculation of GiS, we are able to concurrently learn the magnitude of the factor stress; in other words, the stressful scenarios are endogenously determined, which is very different from the standard practice in stress testing where the stressful scenarios are chosen *a priori*. We also analyze whether the risk exposure is different across industrialized, emerging and other developing countries. We calculate the GiS of 87 countries using the annual data on macroeconomic growth from 1985 to 2015, obtained from the World Bank's World Development Indicators and supplemented with the International Monetary Fund's World Economic Outlook (WEO) data base.

The most recent literature in macroeconomic risk analyzes two different but related dimensions of risk. Some works focus on uncertainty indexes and some others on downside risk to economic growth. The main difference between uncertainty and risk indexes is that the former measure variances (uncertainty) while the latter measure the lower tail (risk) of growth. Though variances take into account deviations from the mean in both directions, a policy maker, who wishes to monitor downside risk, would be more interested in the lower quantiles of growth. Our work measures the effect of stressed factors not only on the average growth but also on different quantiles of growth.

The proposed macroeconomic risk index is related to the macroeconomic uncertainty indexes proposed by Jurado et al. (2015) who use augmented predictive regressions based on PC factors, and by Henzel and Rengel (2017) who implement two step Kalman filter factors. However, there are two main differences with our work. First, Jurado et al. (2015) and Henzel and Rengel (2017) construct uncertainty indexes based on weighted combinations of the uncertainty of the idiosyncratic components while we are concerned with the common factors instead of the idiosyncratic noises. Second, instead of focusing on conditional variances, we measure the risk in the tails of the factors' joint distribution, i.e. we consider multivariate quantiles instead of variances. Other uncertainty indexes are proposed by Rossi and Sekhposyan (2015) and Ozturk and Sheng (in press), which are based on survey data from the European Central Bank Survey of Professional Forecasters and the Consensus Forecasts, respectively; see Ozturk and Sheng (in press) for a detailed survey of the literature on economic uncertainty indexes.

More closely related to our proposal is the risk index proposed by Adrian et al. (in press) who model the full distribution of future real GDP growth as a function of current financial and economic conditions. They estimate a semi-parametric distribution of growth using quantile regressions. Risk is computed either as the expected shortfall of this distribution or using an entropy measure with respect to the unconditional distribution of growth that is time invariant and based on quantile regressions in which only the constant term is included. In this latter case, they quantify upside and downside vulnerability of future GDP growth as the "extra" probability mass that the conditional density assigns to extreme right and left tail outcomes relative to the probability of these outcomes under the unconditional density. There are three main differences between our proposed GiS index and that of Adrian et al. (in press). First, the GiS is based on stressed conditions of the common factors and their effects on growth while Adrian et al. (in press) consider that factors fixed at their estimated mean values. Second, the factors considered in this chapter are world and regional factors while Adrian et al. (in press) focus on financial local factors. Finally, Adrian et al. (in press) focus their analysis on growth risk in USA, while we extend our analysis to 87 countries around the world. Our methodology is also related to that proposed by Giglio et al. (2016) who also fit factor augmented quantile regressions to evaluate the ability of various measures of systemic financial risk to predict real activity outcomes. In this case, there are also important differences with our procedure. As in Adrian et al. (in press), they consider the effect of financial common factors, which are treated as observable. However, they are not proposing a proper risk measure for growth but just predicting it. In their empirical application, they consider US and European countries but not developing or emerging ones.

The rest of the chapter is organized as follows. In section 3.2, we describe the GiS index. In section 3.3, we estimate the common factors and GiS index for a large number of industrialized, emerging and other developing countries. In section **??**, we conclude. The appendix provides detailed results of the estimation of the predictive and quantile regressions.

3.2. Growth-in-Stress Index

The choice of key macroeconomic variable(s) is crucial to describe the state of the economy. Following the standard choice in the related macroeconomic literature, we focus on GDP growth as representative of the business cycle. Let GDP_{it} be the GDP of country i at time t, and define the corresponding growth as $y_{it} \equiv \triangle log(GDP_{it})$. For each country, we forecast growth by the following single equation autoregressive model augmented with factors

$$y_{it+1} = \mu_i + \phi_i y_{it} + \sum_{k=1}^{r} \beta_{ik} F_{kt} + u_{it+1}, \tag{3.1}$$

where F_{kt}, for $k = 1, ..., r$ are the r unobserved common factors, also known as diffusion indexes, that summarize the variations of the large cross-section of growths and u_{it} is a white noise process; see Stock and Watson (1999) and Forni et al. (2000) for the introduction of factor-augmented predictive regressions. Factor augmented regressions as that in (4.1) have been considered by Jurado et al. (2015) to construct their uncertainty index.

If the interest is not only the center of the probability distribution of growth but also its lower or upper tails, we can consider a factor-augmented quantile regression model that estimates the τ quantile of y_{it+1} conditional on y_{it} and F_t; see Ando and Tsay (2011) for factor-augmented quantile regressions. In particular, we consider the following model

$$q_\tau(y_{it+1}|y_t, F_t) = \mu_i(\tau) + \phi_i(\tau) y_{it} + \sum_{k=1}^{r} \beta_{ik}(\tau) F_{kt} + v_{it+1}, \tag{3.2}$$

where $q_\tau(y_{it+1}|y_t, F_t)$ is the τth quantile of y_{it+1} conditional on y_{it} and $F_t = (F_{1t}, ..., F_{rt})'$, and v_{it} is an uncorrelated sequence such that $q_\tau(v_{it+1}|y_t, F_t) = 0$. Quantile regressions with factors as explanatory variables have also been considered by Adrian et al. (in press) to compute their risk index and by Giglio et al. (2016) to evaluate the ability of various measures of systemic risk to predict real activity outcomes. The quantile approach is appropriate for evaluating the potentially asymmetric and nonlinear association between global and regional factors and economic growth.

The GiS index for country i at time $t+1$ is defined as the minimum expected growth (or quantile of growth) of the country when the underlying factors are subject to α-probability extreme scenarios, that is

$$GiS_{t+1}^{(i)} = -\min h(y_{i,t+1}) \tag{3.3}$$

$$s.t. \quad g(F_t, \alpha) = 0$$

and depending on whether the interest is in the average growth or in a quantile of growth, $h(y_{it+1})$ is given by the predicted y_{it+1}, as defined in equation (4.1), or by the predicted $q_\tau(y_{it+1}|y_t, F_t)$, as defined in equation (3.2), respectively. Note that to easy the interpretation, we multiply the sign of $h(y_{it+1})$ by -1 so that larger values of GiS mean larger risk. The constraint in (3.3) requires to know the multivariate probability density of the factors, from which the function $g(F_t, \alpha) = 0$ is a contour. The function $g(F_t, \alpha) = 0$ is the ellipsoid that contains the true factor vector, F_t with probability α. For instance, if $\alpha = 95\%$, the ellipsoid will contain 95% of the factor events. Those values of F_t on the boundary of the ellipsoid $g(F_t, \alpha) = 0$ are considered the extreme events. Therefore, if $\alpha = 0.95$, the GiS measures the minimum expected growth (or quantile of growth) at time $t+1$ when the factors are on the boundary of the ellipsoid $g(F_t, 0.95) = 0$. In Figure 1, we illustrate graphically how to obtain the GiS for two different probability contours, $\alpha_1 < \alpha_2$, when the number of factors is two, i.e. $r = 2$. First, we plot the two ellipsoids, $g(F_t, \alpha_1) = 0$ and $g(F_t, \alpha_2) = 0$. Second, we plot the so called iso-growth curves. These are the combinations of F_1 and F_2 that produce the same predicted value of growth (or quantile of growth) $h(y_{i,t+1})$. For α_1, the GiS is given by the predicted value of growth corresponding to the iso-growth curve that is tangent to $g(F_t, \alpha_1) = 0$, while for α_2, the GiS is given by the predicted value of the iso-growth curve tangent to $g(F_t, \alpha_2) = 0$. Observe that, as result of the minimization exercise, we will obtain not only the GiS but also the combination of factors giving rise to this GiS. This combination corresponds to the point where the ellipsoid and the iso-growth curve are tangent. In Figure 1, GiS_1 is generated by the combination (F_{11}, F_{21}) while GiS_2 is generated by (F_{12}, F_{22}). This is an important advantage of our approach; once the α-probability level is chosen, the stressful scenarios are endogenously determined, which is very different from the standard practice in stress testing exercises where the stressful scenarios are chosen *a priori*.

The factors to calculate the GiS in (3.3) are modeled using a dynamic factor model (DFM). The specification of the DFM follows common practice in the literature; see Jurado et al. (2015), Giglio et al. (2016) and Henzel and Rengel (2017), among others.[1] We consider the following DFM

$$Y_t = PF_t + \varepsilon_t, \tag{3.4}$$

where $Y_t = (y_{1t}, ..., y_{Nt})'$ is the $N \times 1$ vector of growth rates observed at time t for $t = 1, ..., T$; P is the $N \times r$ matrix of factor loadings such that $P'P$ is a diagonal matrix with distinct entries arranged in decreasing order; F_t is the vector of unobserved common factors; and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the $N \times 1$ vector of idiosyncratic noises, which are assumed to be potentially weakly cross-correlated and heteroscedastic; see Bai (2003) for the assumptions on model (4.30) to guarantee the asymptotic validity of the Principal Components (PC) factor extraction procedure. The disturbances $\eta_t = (\eta_{1t}, ..., \eta_{rt})'$ and $a_t = (a_{1t}, ..., a_{Nt})'$ are mutually independent Gaussian white noise vectors with positive covariance matrices Σ_η and Σ_a, respectively. The matrices Φ and Γ are diagonal with their parameters restricted so that Y_t is stationary. The number of factors r is assumed to be known.

We extract the factors using PC due to its well known computational simplicity

[1] Note that our approach is different from other related DFM models as we do not specify *a priori* global and specific factors for industrialized, emerging and other developing countries as in Kose et al. (2012) or global and regional factors as in Aastveit et al. (2016) and Bjornland et al. (2017).

and popularity; see Bai and Ng (2008a) for a review of PC factor extraction. For a unique identification of the factors, we assume $\frac{F'F}{T} = I_r$; see Bai and Ng (2013) for a discussion on identification issues in the context of PC factor extraction. The $r \times T$ matrix of extracted factors $\hat{F} = (\hat{F}_1, ..., \hat{F}_T)$ is given by $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $Y'Y$ where $Y = (Y_1, ..., Y_T)$. The matrix of estimated factor loadings, $\hat{P}$, is computed by $\hat{P} = \frac{Y\hat{F}'}{T}$. Bai (2003) shows that, if $\frac{\sqrt{N}}{T} \to 0$ when $N, T \to \infty$, then $\hat{F}$ is a consistent estimator of the space spanned by the true factors. Finally, to obtain the joint probability density of the factors to compute $g(F_t, \alpha)$ in (3.3), we follow the second chapter of this thesis in which we propose constructing ellipsoids based on the point-wise asymptotic normality of the PC estimated factors (Bai, 2003) with a covariance matrix computed by using a subsampling procedure, which is designed to measure parameter uncertainty associated with the factor estimation.[2]

The estimated factors are substituted either in equation (4.1) or in equation (3.2) depending on whether the interest is on the macroeconomic global risk affecting the center or one particular quantile of the growth distribution. In the former case, the estimated factors are substituted in equation (4.1) and the predictive regression parameters are estimated by Least Squares (LS) as in Stock and Watson (1999). When the interest is on a particular quantile of the growth distribution, the parameters of the quantile regressions in equation (3.2) can be estimated as in Koenker and Bassett (1978); see Ando and Tsay (2011).[3] Recently, Ohno and Ando (2018) propose a shrinkage procedure to estimate the parameters of factor augmented predictive regressions,

[2] Note that the bootstrap procedure implemented by Aastveit et al. (2016) to compute prediction intervals of the factors underestimates the uncertainty as they do not consider parameter uncertainty; see the second chapter of this thesis in which we show that the subsampling correction of the covariance asymptotic matrix provides point-wise prediction regions for the factors with coverage very close to the nominal.

[3] Stock and Watson (2002a) show the consistency of the LS estimator while Bai and Ng (2006) derive its asymptotic normality. Bai and Ng (2006, 2008b) show that when the generated regressors are the estimated factors, they can be plugged in as if they were observed as far as $\frac{\sqrt{T}}{N} \to 0$ for $N, T \to \infty$ in regression models or $\frac{T^{5/8}}{N} \to 0$ for $N, T \to \infty$ in quantile regressions, respectively.

which can be implemented in both (4.1) and (3.2).

Finally, with the estimated ellipsoids containing the true factors, $g(F_t, \alpha) = 0$, and the estimated predictive regression or quantile regression augmented with the factors, $h(y_{t+1})$, it is possible to solve numerically the minimization problem in (3.3) by evaluating (4.1) or (3.2) in *all* points of the ellipsoid[4].

3.3. GiS indexes in industrialized, emerging and other developing countries

We compute the GiS of 87 countries.[5] The data consists of GDP measured at constant national prices and observed annually from 1985 to 2015 for $N = 87$ countries, obtained from the World Bank's World Development Indicators and supplemented with the International Monetary Fund's WEO data base. The same data base has been considered by Kose et al. (2012) for a larger number of countries (106) and variables (GDP, real private consumption and real fixed asset investment) over the period 1960-2008. Given the dramatic shift of the global landscape since the mid-1980s, we only consider the period starting in 1985, which is defined by Kose et al. (2012) as the wave of globalization. On the other hand, we extend the sample period with data observed after the 2008 global financial crisis. GDP is transformed to growth rates by taking the first differences of log of GDP. Consequently, the time series length is $T = 30$.

3.3.1. Estimating the factors

Previous to factor extraction, the growth series are demeaned and standardized. Notice that the demeaning procedure eliminates differences in mean growth rates

[4]Note that this "brute force" approach of minimizing growth is only feasible when the number of factors is relatively small. When the number of factors is large, one needs to use optimization techniques, for example, second-order cone programming (SOCP); see Bertsimas et al. (2013) and the references there in. Alternatively, Chassein and Goerigk (2017) proposed using regret combinatorial optimization.

[5]The software to estimate the GiS has been developed by the third author in R programming language. It is available upon request.

among countries. To identify the number of common factors, we implement the procedure proposed by Alessi et al. (2010), which selects $r = 3$. After extracting the factors using PC, we obtain the idiosyncratic residuals and identify outliers as those residuals exceeding six times the interquartile range[6]; see Marcellino et al. (2003), Artis et al. (2005), Stock and Watson (2002b) and Breitung and Eickmeier (2011) who also use the interquartile range to identify outliers in the context of DFM. We identify the following outliers due to exceptional events: i) the consumer response to the Mexican Peso crisis in 1994, which caused a fall in Mexican growth in 1995, see McKenzie (2006); ii) in 1994, Rwanda's growth fell due to the genocide against the Tutsi, see Lopez and Wodon (2005); iii) the political crisis of 2002 in Madagascar that seriously hampered economic growth, see Vaillant et al. (2014). As in Breitung and Eickmeier (2011), we substitute each outlying original growth by the median of the last previous five observations. From now on, the growth rates considered in the analysis, denoted by y_{it}, are the corresponding growth rates corrected by outliers.

After demeaning and standardizing the outlier-corrected growth series, y_{it}, Alessi et al. (2010) still selects $r = 3$ common factors explaining 42% of the total growth variability with the first factor accounting for 20%. These percentages are comparable to those found by other authors in related research. For example, Aastveit et al. (2016) find that global and regional factors explain around 30% and 20% respectively of the business cycle variation in four small open economies (Canada, New Zealand, Norway and United Kingdom). Kose et al. (2003) attribute up to 35% of the variance in GDP across G7 countries to one common international business cycle. Finally, Bjornland et al. (2017), who analyze quarterly real GDP growth from 1978 to 2011 for 33 countries covering four geographical regions and both developed and emerging economies, report that the common business cycle accounts for 5% to 45% of the total variability

[6]Kristensen (2014) analyzes the effects of outliers on PC factor extraction and predictive regressions. He proposes a robust factor extraction procedure based on Least Absolute Deviations (LAD). However, this robust procedure cannot be implemented in our context because of the lack of an asymptotic distribution, which is needed to obtain the probability ellipsoids containing the factors.

of growth depending on the particular region of the world and the period of time considered. Consequently, we extract three factors by PC and compute their confidence bounds as well as those for the corresponding weights, $\hat{P}$, using the subsampling procedure proposed in the second chapter.[7] After visual inspection, the idiosyncratic components are considered approximately stationary.[8]

In Figures 2 to 4, we plot the estimated factors and weights corresponding to the DFM in equation (4.30) together with their 95% bounds. Following Kose et al. (2012), the countries are classified into three groups: i) Industrial whose weights are represented by red bars; ii) Emerging markets represented by blue bars; and iii) Other developing countries represented by gray bars. In Table 1, we report the classification of each country and we list the countries in the same order as their weights plotted in Figures 2 to 4. Consider the first factor plotted in Figure 2 together with its weights and corresponding 95% confidence intervals. This factor can be interpreted as a world growth factor with all industrial and emerging countries but Morocco, Peru and China having positive weights. In the case of Morocco, the weight is not significant while in Peru and China, the weights are negative although relatively small in magnitude. We also observe that the weights are negative and relatively small or non-significant in several "other developing countries", mainly in Africa. It is also remarkable that the weights of India and Indonesia, although positive, are relatively small. The dynamic profile of the estimated global factor is very similar to that found in Kose et al. (2012), Aasveit et al. (2016) and Bjornland et al. (2016), with declines in the early 1990s, in 2000/2001 during the bursting of the dot-com bubble, and in 2008-2009 during the

[7]Kose et al. (2003) and Kose et al. (2012) extract common factors of macroeconomic variables by implementing a data augmentation Bayesian procedure based on the spectral density matrix. Alternatively, Bjornland et al. (2017) implement Bayesian estimation of the corresponding state space model using Gibbs simulation. These procedures also provide predictive densities for the factors.

[8]We do not formally test for non-stationarity of the idiosyncratic noises because the temporal dimension is rather small and the lack of power of most popular nonstationarity tests is well known in this case; see, for example, Kwiatkowski et al. (1992). Banerjee et al. (2008) also point out related problems associated with cointegration tests in the context of non-stationary panels.

Great Recession, which is by far the most severe.

In Figure 3, we plot the second factor together with its weights. We observe that this factor is negative until the mid-1990s and then is positive with a relatively weak drop during the Great Recession. This factor has positive weights in most "other developing" countries in Africa and America. Furthermore, China's weight is not significant while India's is positive and large. As far as we know, this factor has not been identified before. Other related works, as in Aastveit et al. (2016), have not included African countries or developing countries in South America. Only Kose et al. (2012) extract factors using data from a similar set of countries as those considered in this thesis. However, they specified *a priori* common factors associated with industrialized, emerging and other developing countries. According to our results, the factors are not exactly associated with these groups of countries but with a mixture of these groups and geographic regions.

Finally, the third factor, plotted in Figure 4 together with its weights, is not affected by the 2008 global crisis. Furthermore, its weights are negative for all industrialized countries but Japan (non-significant) and Germany (rather small positive weight). In America and Asia, the weights are positive for all emerging and other developing countries. In particular, China's weight is rather large. This factor is related to an East Asian common factor; compare with the factor estimated by Moneta and Ruffer (2009) for the period 1993-2005 based on quarterly growth from ten East Asian countries, and by Bjornland et al. (2017) for the period 1978-2011. This factor clearly reflects the Asian financial crisis, which affected output in 1998; see, for example, Radelet and Sachs (1998) and Cabalu (1999).

According to the interpretation of the factors above, the impressive growth performance of emerging market economies, such as China and India, seems not to be affected by the growth slowdown observed in the world factor. This conclusion is in agreement with Kose et al. (2012) who conclude that emerging markets have "decoupled" from industrial economies, meaning that their business cycle dynamics were no longer tightly linked to the business cycles of industrial countries.

As an illustration of the joint ellipsoids of the factors obtained by the subsampling procedure, we plot the 95% ellipsoids for 1998 and 2004 for USA (Figure 8) and China (Figure 9). The ellipsoid corresponding to 1998 has larger volume, meaning that the uncertainty of the underlying factors in 1998 is larger just around and after the Asian financial crisis. Furthermore, we observe that the increase in uncertainty is mainly due to the first and second factors.

3.3.2. Predictive regressions

For each country growth, we estimate the predictive regression (4.1) by LS using the estimated factors $\hat{F}_{1t}, \hat{F}_{2t}$ and $\hat{F}_{3t}$ as regressors. Note that the predictive regressions are estimated using the original growth rates without demeaning and standardizing so that we can recover information about the average growth. In Figure 5, we summarize the estimated parameters, $\hat{\beta}_{i1}, \hat{\beta}_{i2}$ and $\hat{\beta}_{i3}$, by plotting a histogram of their values across all countries (first row) and across countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row). Across all countries (first row), there are not clear patterns either in the signs or magnitudes of the estimates. Their histograms are roughly centered around zero and have similar ranges going from -2.5 to 2.5 approximately. The marginal effect of the first factor (first column), $\hat{\beta}_{i1}$, to forecast growth is similar across Africa, America, and Asia with values roughly centered around zero but it tends to be mainly positive in the Europe/Oceania group. The marginal

[9] Note that the results in Bai and Ng (2006) require $\frac{\sqrt{T}}{N} \to 0$ for the asymptotic normality of the LS estimator. In our application, $\frac{\sqrt{30}}{87} = 0.06$. However, Gonçalves and Perron (2014) show that the LS estimator of the parameters of the predictive regressions may be affected by negative biases. In addition, the contemporaneous correlation between growth and the estimated factors is rather large for some countries and multicolinearity could be severe. Therefore, we should be cautious about inference on the parameters of the predictive regressions. The estimated parameters together with their p-values and the Box-Ljung statistic for the joint significance of the first four autocorrelations of the residuals, $Q(4)$, of each predictive regression are reported in an online appendix.

effect of the second factor (second column), $\hat{\beta}_{i2}$, tends to be positive in Africa and negative in Asia and virtually zero in Europe/Oceania, and the marginal effect of the third factor (third column), $\hat{\beta}_{i3}$ is mainly positive in America. It is interesting to observe the link of the American continent with the third factor, which is loading mostly in East Asian countries. We should mention that the factors are mildly significant and the estimated magnitudes are rather small.[9]

In Table 2, we report the coefficient of determination, R^2, for each factor augmented predictive regression. Overall, we observe that half of the predictive regressions have R^2 larger than 30% and only 10% of the regressions have R^2 larger than 50%. The results above show that the effects of the factors on one-step-ahead average growth are very mild.

Next, we analyze the effect of the factors on different quantiles of growth by estimating the factor augmented quantile predictive regressions (3.2) with $\tau = 0.05, 0.5$ and 0.95.[10] Note that when $\tau = 0.5$, the quantile regression reduces to the conditional median regression, which is more robust to outliers than the conditional mean regression (4.1); see Ando and Tsay (2011). In Figure 6, we plot the cross-sectional histograms of the estimated parameters $\hat{\beta}_{i1}(\tau), \hat{\beta}_{i2}(\tau)$ and $\hat{\beta}_{i3}(\tau)$ for the lower quantile $\tau = 0.05$.[11] The main difference with the results of the predicitive regression for expected growth is that the magnitude of the parameter estimates is much larger for all countries. Across all countries (first row), the histograms are roughly centered around zero with an approximate range from -5 to 5. The marginal effect of the first factor (first column), $\hat{\beta}_{i1}(\tau)$, to forecast the 0.05 quantile of growth tends to be mainly positive in the America and the Europe/Oceania group and negative in Asia. The marginal effect of the second factor (second column), $\hat{\beta}_{i2}(\tau)$, tends to be positive in Africa, and the

[10]The estimator of the parameters is based on the algorithm by Koenker and d'Orey (1987). Results based on the shrinkage estimator proposed by Ohno and Ando (2018) are similar. They are available upon request.

[11]Histograms for $\tau = 0.5$ and 0.95 are available in the on-line appendix.

marginal effect of the third factor (third column), $\hat{\beta}_{i3}(\tau)$ is mainly positive in America and negative in Europe/Oceania. In general, the joint effect of the three factors is more relevant to forecast the 0.05 quantile of growth than to forecast expected growth.

In Table 2, we report the goodness of fit measure proposed by Koenker and Machado (1999), denoted as R^1, which is the analogous counterpart to the coefficient of determination in regression models.[12] We observe that the fit of the median regression is in general lower than that of the average growth regression. However, the fit improves dramatically in the tail quantiles. For the lower tail, the 5% quantile, we find that about 30% of the regressions have R^1 coefficients larger than 50%. Therefore, it seems that the factors are more relevant to explain future tails than the center of the growth distribution. This conclusion is in agreement with the main findings of Giglio et al. (2016) and Adrian et al. (in press) who conclude that the estimated lower quantile of growth depends on financial conditions, while the upper quantiles are stable over time.[13]

Finally, following Adrian et al. (in press), we use the factor augmented quantile predictive regressions for different values of τ to compute the growth densities for each country and for each year.[14] In Figure 7, we plot these densities for two countries, namely China and USA. We observe that, in both countries, the densities are skewed to the left with the densities in China having the concentration of mass in values of growth larger than those in USA (less risk). Furthermore, the dispersion (uncertainty) of the densities in China is also smaller than that of the densities for USA. Finally, note that the effect of the global crisis in the USA densities is very obvious while there is not any clear effect on the densities in China.

[12] The estimated parameters and their corresponding p-values are reported in the online appendix.

[13] Adrian et al. (in press) show that current economic conditions forecast the median of the distribution of growth, but do not contain information about the other quantiles of the distribution.

[14] Adrian et al. (in press) fit the skewed t-distribution proposed by Azzalini and Capitanio (2003) to obtain a density by smoothing the quantile function.

3.3.3. Forecasting recession risk under stressed factors

To obtain the GiS corresponding for each country, we solve the optimization problem in (3.3) with $h(y_{it+1}) = \hat{y}_{it+1}$ being the predicted expected mean growth, which is calculated by plugging in the LS estimates of the parameters in (4.1). The ellipsoid $g(F_t, \alpha) = 0$ is estimated using the resampling procedure in the second chapter. In Figures 8 and 9, we illustrate this optimization problem by plotting the 95%-probability ellipsoids $g(F_t, 95\%) = 0$ corresponding to 1998 and 2004 for USA and China, respectively. In the top left panel figure, we also plot the iso-growth surfaces corresponding to the predictive regressions for 1999 and 2005 that are tangent to the ellipsoids. We observe that the surfaces of the predictive regressions are rather different in shape and orientation in the two countries considered.

After estimating the GiS for each country and year, we observe that, in Africa, the country with the lowest GiS over time is Cameroon while the country with the largest GiS and, consequently, the highest risk of recession is Uganda.[15] These two countries also have the smallest and the largest risk among the developing countries. In America, the country with the lowest risk of recession is Guatemala while the country with the largest risk is Venezuela. For Asian countries, Syria and China have the largest and the smallest risk of recession, respectively. It is also important to note that among the countries classified as emerging, China has the lowest risk while Venezuela has the largest. Finally, in Europe/Oceania, the largest risk of recession corresponds to Iceland while Norway has the lowest. These two countries also have the largest and the lowest risks among the industrialized countries.

In Figure 10, we summarize the GiS results by plotting year-by-year the cross-sectional average GiS together with the cross-sectional bounds constructed as ± 2 cross-sectional standard deviations of the GiS when countries are grouped by continent.

[15] Time series plots of the GiS estimated in each country appear in the online appendix.

In Figure 11, we plot the same quantities when countries are grouped by type. Several conclusions emerge from these figures. We observe that in all continents, the average risk has been slightly decreasing over time, with the Asian continent enjoying the smallest average GiS. The African and American continents offer very similar average risk profiles. Note that the decrease in average GiS is more pronounced among countries in Africa, America and Asia than among countries in Europe/Oceania. In this latter case, the GiS is more stable over time. This result is in contrast with other macroeconomic uncertainty indexes which conclude that risk has been increasing over time. There are two potential explanations for this apparent contradiction. First, note that most uncertainty indexes focus on industrialized countries while we consider growth in countries all over the world. As explained above, the decrease of the GiS is more pronounced in emerging and developing countries than in industrialized countries. Second, our index measures growth risk when the global and regional common factors are stressed while most alternative indexes focus on uncertainty. Even if the variance (uncertainty) of the distribution of growth increases, the expected growth under stressed factors can also increase and, consequently, the GiS decreases. The ± 2 standard deviation bounds are also becoming narrower over time and have very similar profiles in the African, Asian, and American continents with a sharp jump in 1999 coinciding with the Asian financial crisis. The lower bound is rather stable when compared with the upper bound that is more volatile over time. This is because the standard deviations during the years with high recession risk are larger than the standard deviations when the risk is low. The plot for the European/Oceania continents is rather different from the other plots as the bounds are much narrower indicating that these countries are very similar in risk profile. We observe that post 2008 financial crisis, mainly from 2011 on, the world has fallen in a state-of-complacency with the average GiS falling quite dramatically to reach the lowest levels of risk, between 1 and 0% in 2015. In Figure 8, we summarize risk among developing, emerging and industrialized countries. We observe that the GiS plots of industrialized and emerging countries coincide with those of Europe/Oceania and Asia, respectively, and the plot corresponding to developing countries is very similar to that of African countries.

In addition to analyzing the effects of stressed factors on the average of growth, we also predict the GiS of each country for the $\tau = 0.05$, 0.5 and 0.95 quantiles of the country growth distribution. We solve the minimization problem in (3.3) with $h(y_{it+1}) = \hat{q}_\tau(y_{it+1}|y_t, F_t)$ and compute $\hat{q}_\tau(y_{it+1}|y_t, F_t)$ as in equation (3.2) by plugging in the parameter estimates. As an illustration, in Figures 8 and 9, we plot the 95% ellipsoids for the factors in 1998 and 2004 together with the tangent iso-growth surfaces for one-step-ahead (1999 and 2005) growth quantiles ($\tau = 0.05, 0.5$ and 0.95) obtained from the estimated factor augmented predictive regressions for USA and China, respectively. We observe that the tangent surfaces based on the mean and those based on the median growth are rather similar. However, the tangent surfaces for the 5% and/or 95% growth quantiles can be very different in shape and orientation from the mean and median surfaces as we show in the case of China. In summary, the effect of stressed factors can be rather different depending on the specific quantile of the growth distribution being considered.

In Figure 12, we plot a summary of the τ-quantile GiS. As before, we plot the cross-country average and ± 2 times the standard deviations of the predicted τ-quantile GiS for all industrialized, emerging and other developing countries. First, compare the GiS results for $\tau = 0.5$ with those plotted in Figure 11 where GiS is predicted for the mean growth. The plots in both cases are almost identical for industrialized and emerging countries. However, for developing countries, the bounds become narrower mainly because the upper bound has coming down substantially. For the $\tau = 0.05$ quantile of growth, we are looking at catastrophic outcomes. For the three groups, the cross-country average of the predicted 5% quantile GiS is rather high at 20% (or slightly below 20%) and it does not decrease much over time. Obviously, these are the worst outcomes. Extreme events in the three world factors could wipe out one-fifth of GDP in those countries that are already going through deep recessions. On the contrary, when a country is in its 95% growth quantile, it could withstand extreme events in the world factors as the predicted average GiS for this quantile is close to 0%, that is, no growth on average, and with bounds becoming narrower over time.

Tables and Figures

Table 3.1: List of Countries

Conuntry	Group	Code
Algeria	Other	DZA
Benin	Other	BEN
Botswana	Other	BWA
Burkina Faso	Other	BFA
Cameroon	Other	CMR
Congo, Rep.	Other	COG
Egypt, Arab Rep.	Emerging	EGY
Gabon	Other	GAB
Gambia, The	Other	GMB
Ghana	Other	GHA
Kenya	Other	KEN
Lesotho	Other	LSO
Madagascar	Other	MDG
Mali	Other	MLI
Mauritania	Other	MRT
Mauritius	Other	MUS
Morocco	Emerging	MAR
Mozambique	Other	MOZ
Nigeria	Other	NGA
Rwanda	Other	RWA
Senegal	Other	SEN
Seychelles	Other	SYC
South Africa	Emerging	ZAF
Tanzania	Other	TZA
Togo	Other	TGO
Tunisia	Other	TUN
Uganda	Other	UGA
Zimbabwe	Other	ZWE
Argentina	Emerging	ARG
Bolivia	Other	BOL
Brazil	Emerging	BRA
Canada	Industrialized	CAN
Chile	Emerging	CHL
Colombia	Emerging	COL
Costa Rica	Other	CRI
Dominican Republic	Other	DOM
Ecuador	Other	ECU
El Salvador	Other	SLV
Guatemala	Other	GTM
Honduras	Other	HND
Mexico	Emerging	MEX
Nicaragua	Other	NIC
Panama	Other	PAN
Paraguay	Other	PRY
Peru	Emerging	PER
Trinidad and Tobago	Other	TTO
United States	Industrialized	USA
Uruguay	Other	URY
Venezuela, RB	Emerging	VEN
Bangladesh	Other	BGD
China	Emerging	CHN
Hong Kong SAR, China	Emerging	HKG
India	Emerging	IND
Indonesia	Emerging	IDN
Iran, Islamic Rep.	Other	IRN
Israel	Emerging	ISR
Japan	Industrialized	JPN
Korea, Rep.	Emerging	KOR
Malaysia	Emerging	MYS
Nepal	Other	NPL
Pakistan	Emerging	PAK
Philippines	Emerging	PHL
Singapore	Emerging	SGP
Sri Lanka	Other	LKA
Syrian Arab Republic	Other	SYR
Thailand	Emerging	THA
Turkey	Emerging	TUR
Austria	Industrialized	AUT
Belgium	Industrialized	BEL
Denmark	Industrialized	DNK
Finland	Industrialized	FIN
France	Industrialized	FRA
Germany	Industrialized	DEU
Greece	Industrialized	GRC
Iceland	Industrialized	ISL
Ireland	Industrialized	IRL
Italy	Industrialized	ITA
Luxembourg	Industrialized	LUX
Netherlands	Industrialized	NLD
Norway	Industrialized	NOR
Portugal	Industrialized	PRT
Spain	Industrialized	ESP
Sweden	Industrialized	SWE
Switzerland	Industrialized	CHE
United Kingdom	Industrialized	GBR
Australia	Industrialized	AUS
New Zealand	Industrialized	NZL

Table 3.2: Goodness of fit: R^2 of factor-augmented predictive regressions and R^1_τ of factor-augmented predictive quantile regression

Africa

	DZA	BEN	BWA	BFA	CMR	COG	EGY	GAB	GMB	GHA	KEN	LSO	MDG	MLI	MRT	MUS	MAR	MOZ	NGA	RWA	SEN	SYC	ZAF	TZA	TGO	TUN	UGA	ZWE
R^2	0.39	0.14	0.11	0.43	0.64	0.13	0.23	0.08	0.13	0.29	0.32	0.34	0.40	0.29	0.11	0.14	0.49	0.14	0.17	0.46	0.39	0.19	0.31	0.55	0.06	0.06	0.10	0.26
$R^1_{0.95}$	0.42	0.36	0.34	0.15	0.41	0.40	0.40	0.47	0.23	0.52	0.47	0.37	0.36	0.62	0.29	0.24	0.63	0.21	0.39	0.22	0.24	0.18	0.42	0.28	0.13	0.30	0.24	0.31
$R^1_{0.50}$	0.20	0.15	0.06	0.31	0.37	0.20	0.17	0.17	0.16	0.20	0.29	0.26	0.29	0.23	0.16	0.19	0.35	0.14	0.18	0.29	0.31	0.22	0.18	0.38	0.12	0.18	0.21	0.25
$R^1_{0.05}$	0.54	0.49	0.33	0.49	0.66	0.43	0.28	0.37	0.35	0.21	0.21	0.25	0.55	0.29	0.22	0.37	0.51	0.56	0.49	0.50	0.45	0.24	0.29	0.58	0.47	0.22	0.45	0.36

America

	ARG	BOL	BRA	CAN	CHL	COL	CRI	DOM	ECU	SLV	GTM	HND	MEX	NIC	PAN	PRY	PER	TTO	USA	URY	VEN
R^2	0.20	0.35	0.07	0.24	0.41	0.11	0.08	0.09	0.16	0.38	0.12	0.07	0.09	0.34	0.27	0.21	0.45	0.53	0.44	0.41	0.20
$R^1_{0.95}$	0.21	0.49	0.43	0.27	0.48	0.23	0.44	0.15	0.31	0.55	0.49	0.15	0.25	0.3	0.34	0.54	0.40	0.49	0.27	0.20	0.20
$R^1_{0.50}$	0.21	0.44	0.1	0.12	0.23	0.09	0.08	0.11	0.11	0.24	0.21	0.09	0.23	0.36	0.23	0.09	0.26	0.41	0.28	0.27	0.16
$R^1_{0.05}$	0.43	0.67	0.39	0.36	0.43	0.46	0.29	0.37	0.38	0.29	0.41	0.4	0.33	0.38	0.50	0.33	0.67	0.39	0.58	0.53	0.44

Asia

	BGD	CHN	HKG	IND	IDN	IRN	ISR	JPN	KOR	MYS	NPL	PAK	PHL	SGP	LKA	SYR	THA	TUR
R^2	0.53	0.38	0.22	0.19	0.14	0.35	0.10	0.37	0.33	0.17	0.16	0.21	0.28	0.27	0.22	0.36	0.47	0.03
$R^1_{0.95}$	0.39	0.36	0.57	0.18	0.39	0.53	0.27	0.50	0.48	0.23	0.43	0.40	0.26	0.47	0.35	0.54	0.48	0.1
$R^1_{0.50}$	0.35	0.37	0.22	0.21	0.27	0.28	0.1	0.18	0.34	0.27	0.06	0.27	0.24	0.19	0.17	0.24	0.33	0.08
$R^1_{0.05}$	0.56	0.46	0.34	0.3	0.12	0.47	0.23	0.44	0.23	0.24	0.43	0.17	0.27	0.37	0.39	0.63	0.51	0.12

Europe and Oceania

	AUT	BEL	DNK	FIN	FRA	DEU	GRC	ISL	IRL	ITA	LUX	NLD	NOR	PRT	ESP	SWE	CHE	GBR	AUS	NZL
R^2	0.33	0.37	0.09	0.25	0.42	0.16	0.55	0.37	0.38	0.44	0.24	0.38	0.31	0.54	0.62	0.27	0.16	0.41	0.19	0.39
$R^1_{0.95}$	0.35	0.4	0.22	0.22	0.47	0.27	0.24	0.31	0.43	0.37	0.30	0.46	0.32	0.59	0.39	0.33	0.29	0.48	0.34	0.21
$R^1_{0.50}$	0.21	0.22	0.21	0.22	0.26	0.12	0.34	0.36	0.32	0.25	0.19	0.28	0.23	0.33	0.39	0.13	0.12	0.15	0.14	0.38
$R^1_{0.05}$	0.43	0.44	0.34	0.43	0.49	0.44	0.62	0.36	0.46	0.48	0.43	0.54	0.44	0.55	0.54	0.47	0.42	0.51	0.50	0.39

Figure 3.1: Graphical illustration of computation of GiS when the number of common factors is two.

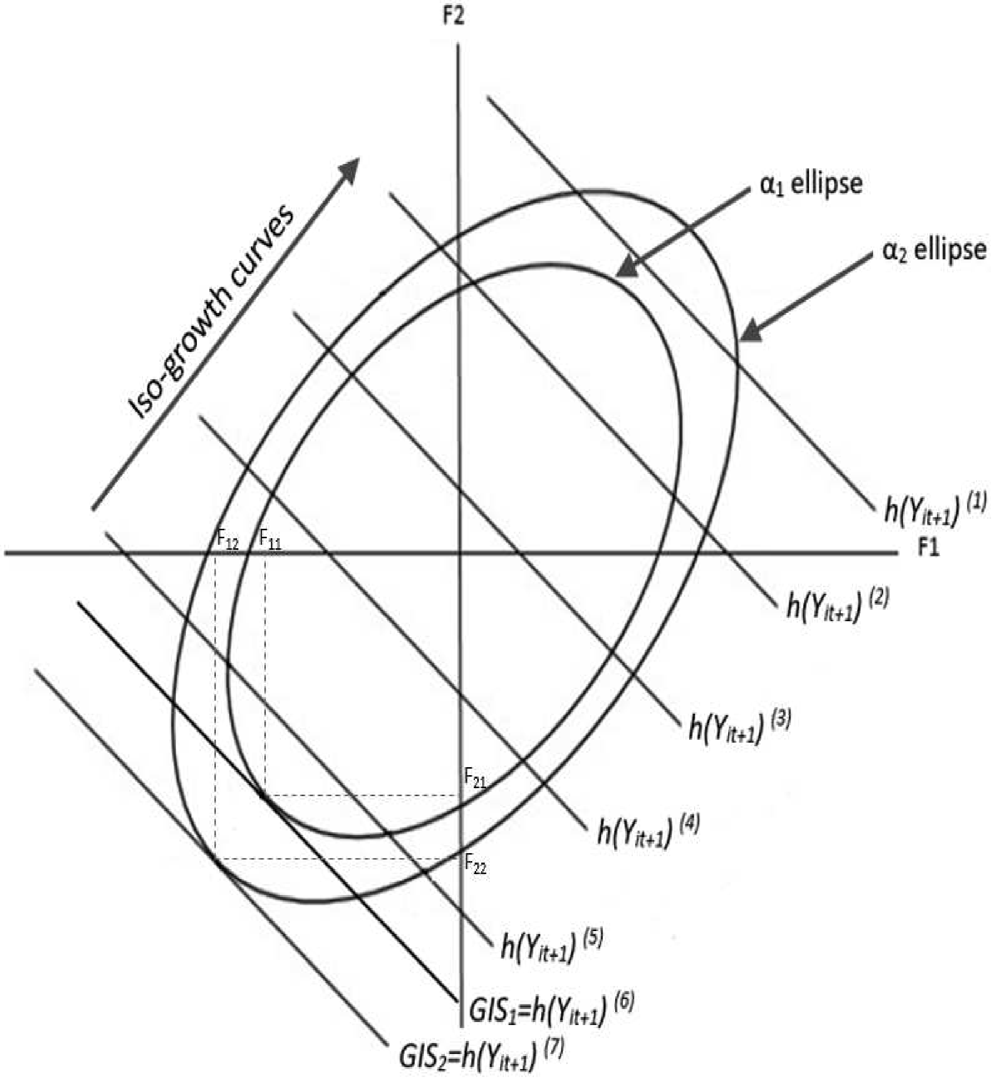


Figure 3.2: Top panel: First factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the first factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.

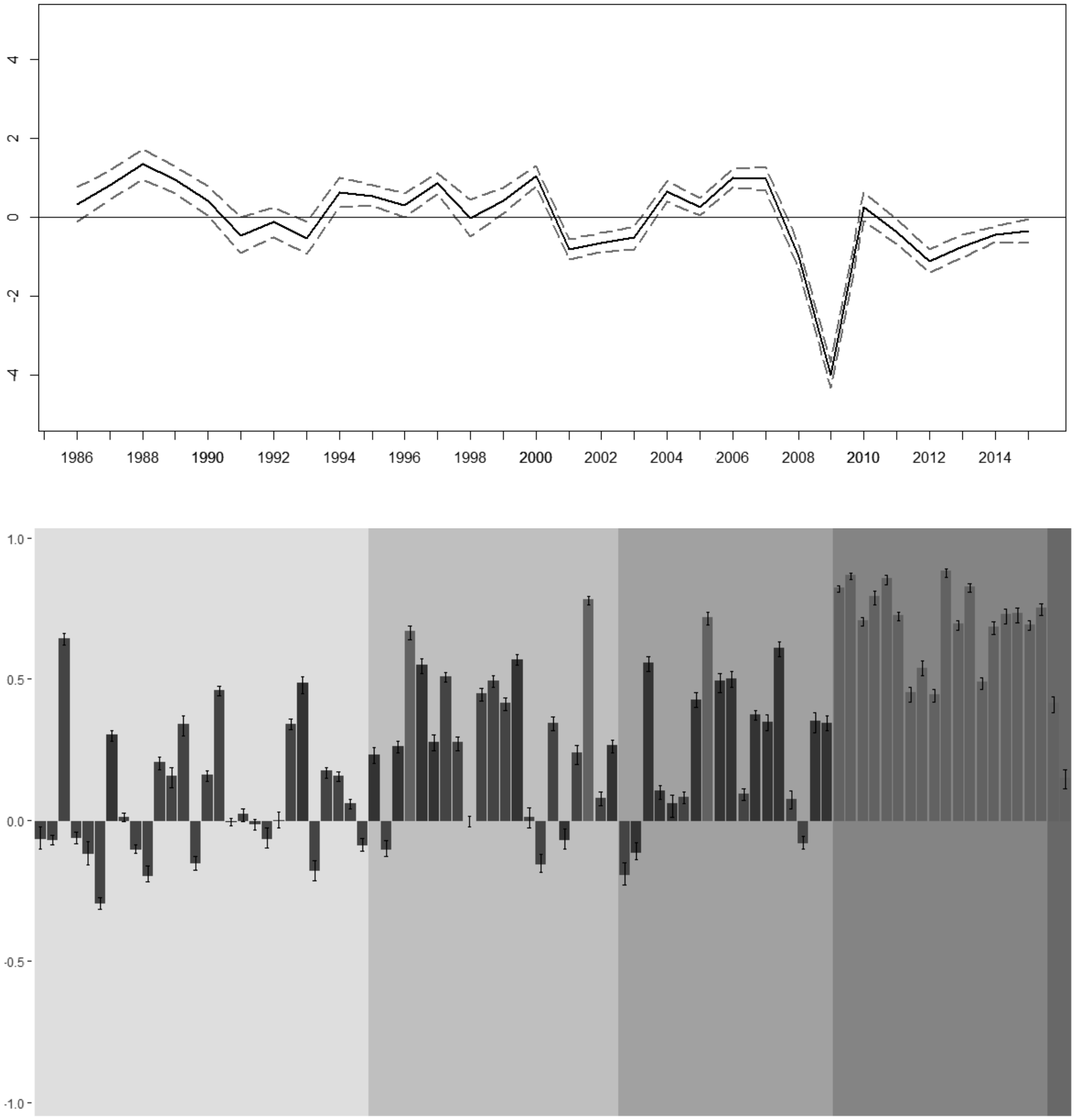
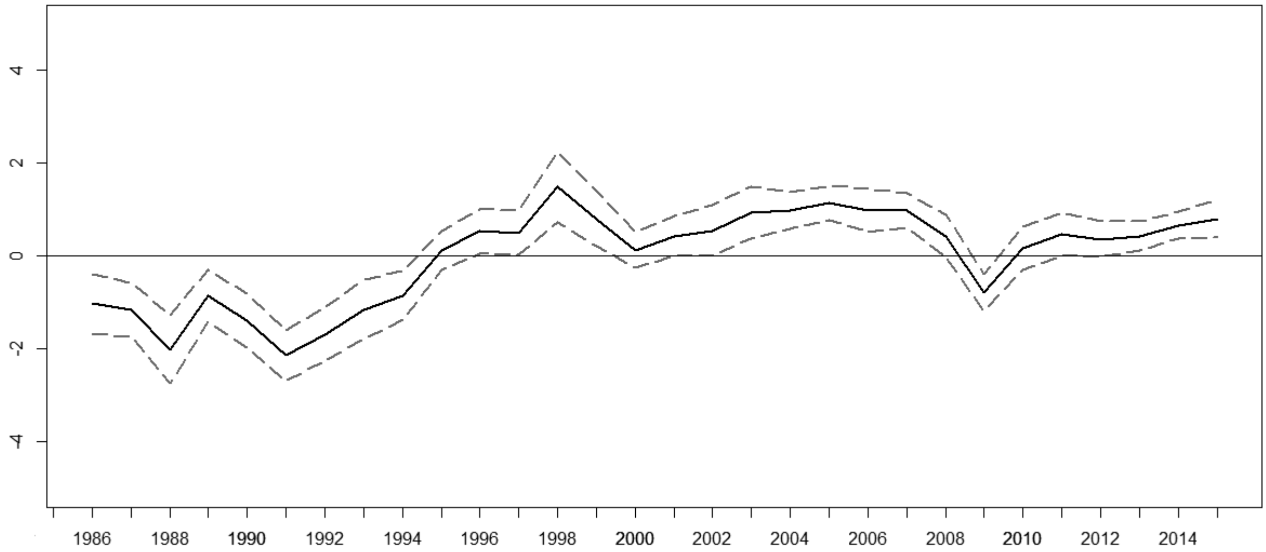


Figure 3.3: Top panel: Second factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the second factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.



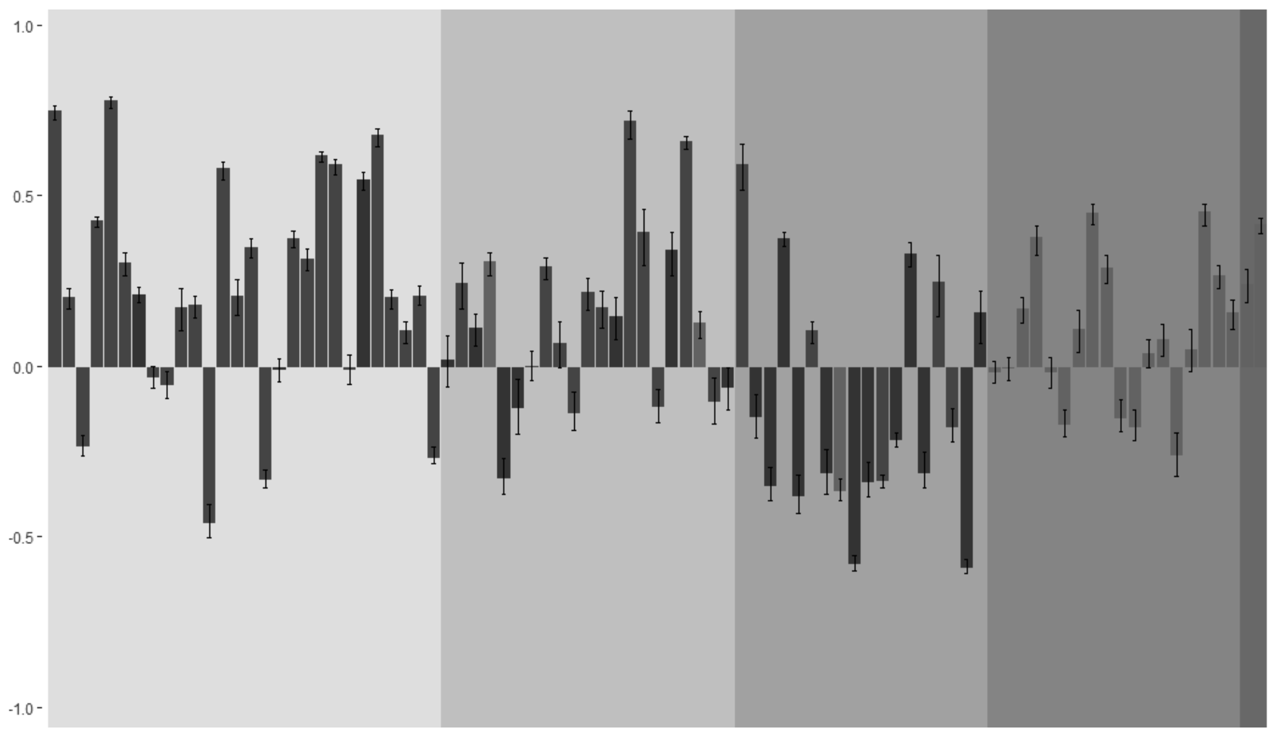
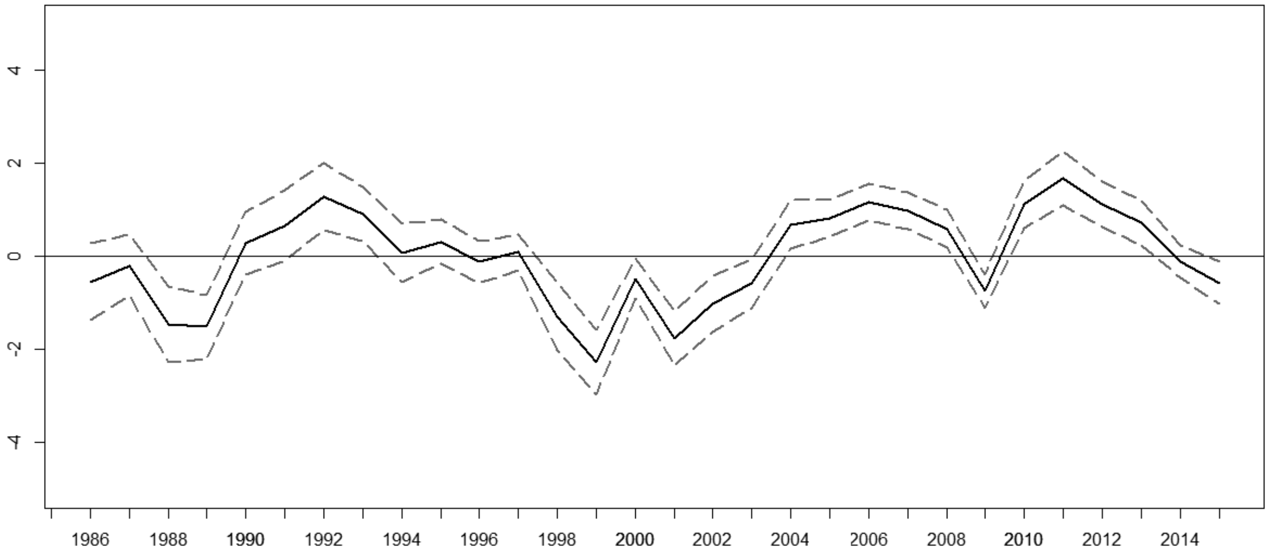


Figure 3.4: Top panel: Third factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the third factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.



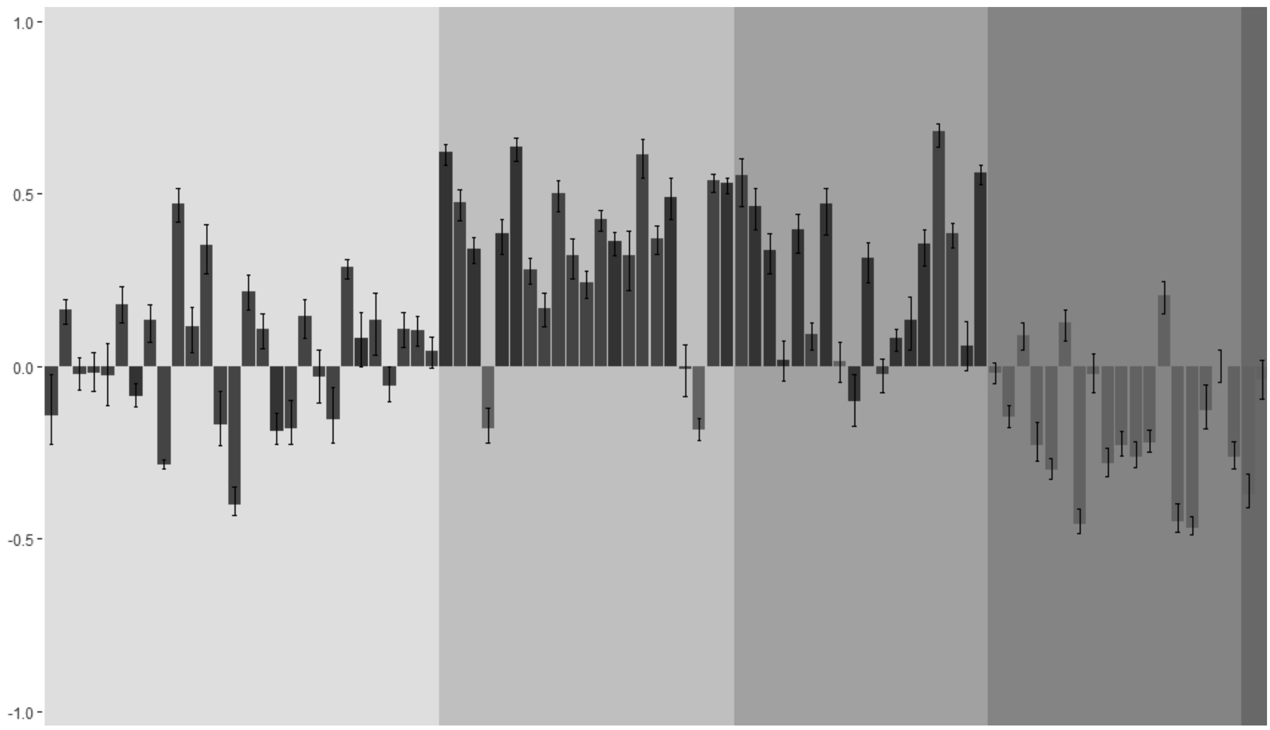


Figure 3.5: Cross-sectional histograms of estimated parameters of the factor augmented predictive regressions corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).

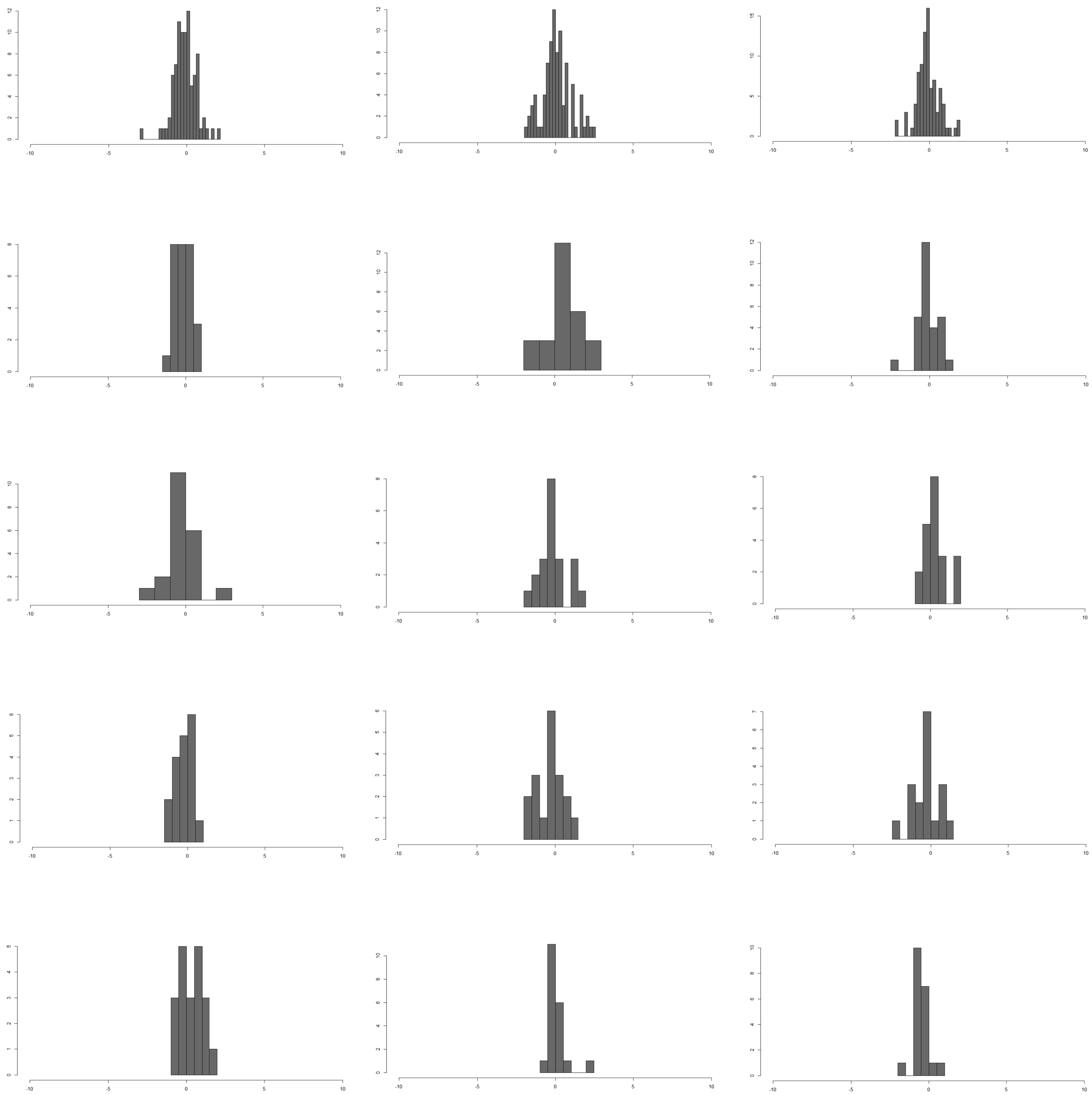


Figure 3.6: Cross-sectional histograms of estimated parameters of the factor augmented quantile predictive regressions for $\tau = 0.05$ corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).

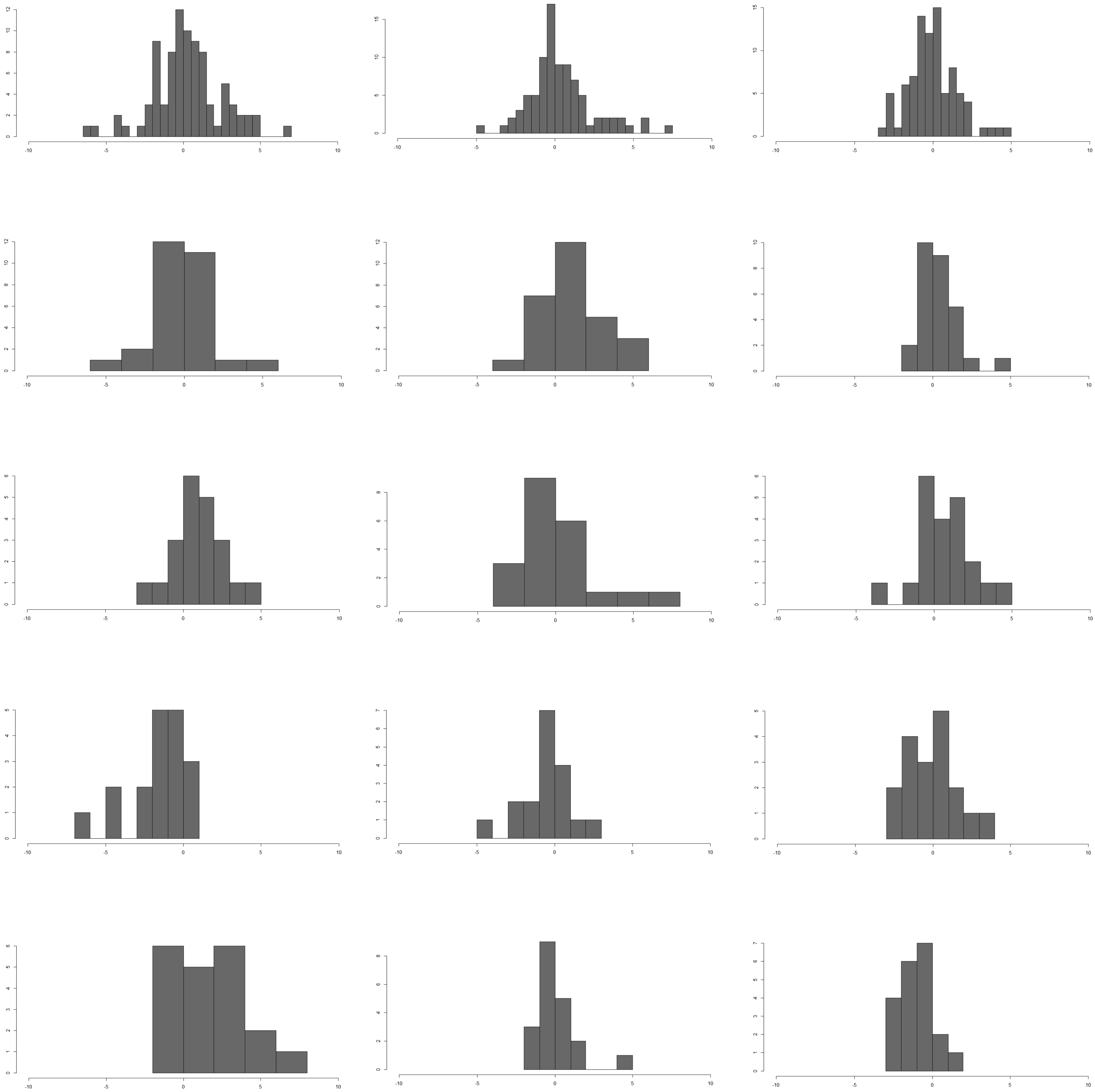


Figure 3.7: Estimated densities of growth for USA (top panel) and China (bottom panel) based on factor augmented quantile regressions.

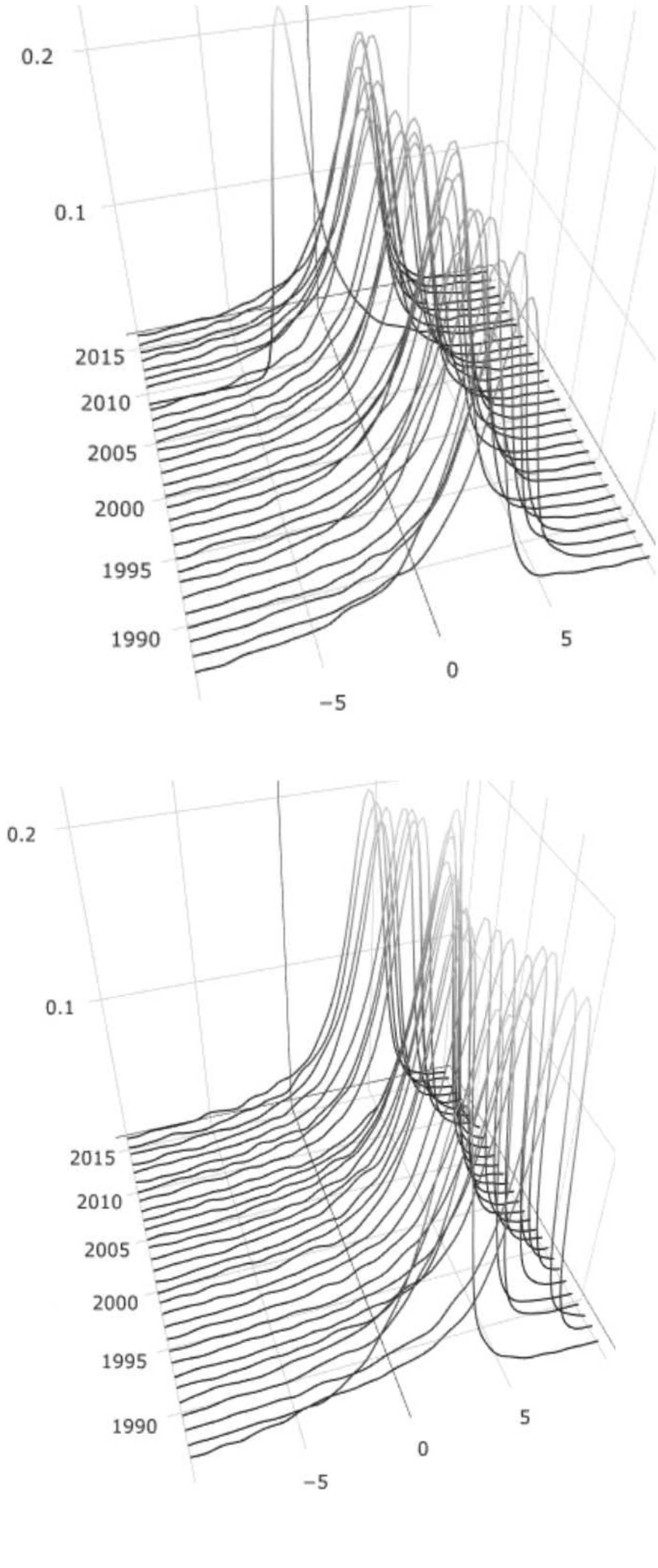


Figure 3.8: Resampling ellipsoids for the three factors in 1998 (blue) and 2004 (red). Predicted iso-growth surfaces in USA for 1999 and 2005 based on predictive regression (top left panel) and quantile regressions with $\tau = 0.05$ (bottom left panel), $\tau = 0.5$ (top right panel) and $\tau = 0.95$ (bottom right panel). For each year, the GiS is the tangency point between the ellipsoid and the corresponding surface.

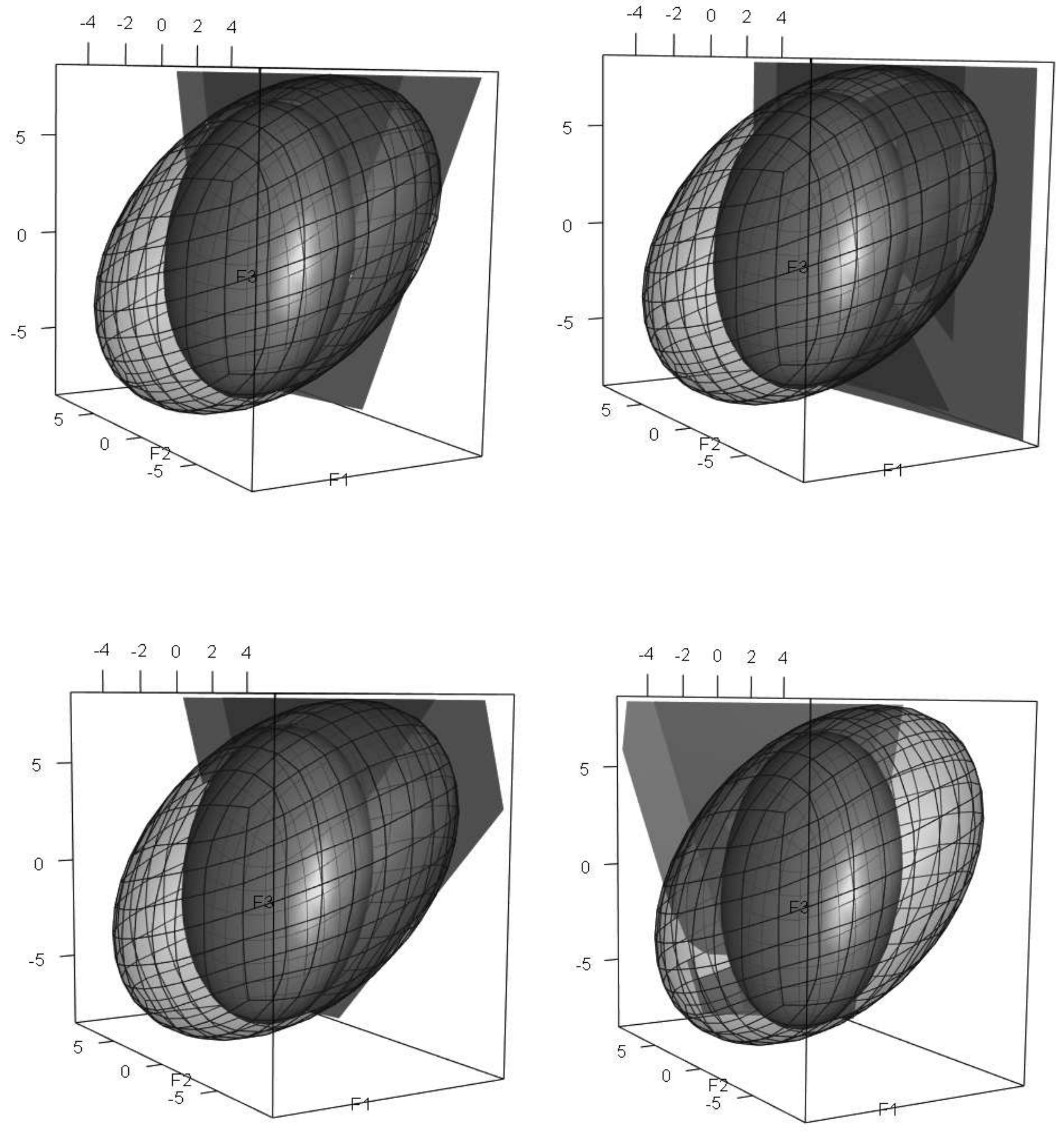


Figure 3.9: Resampling ellipsoids for the three factors in 1998 (blue) and 2004 (red). Predicted iso-growth surfaces in China for 1999 and 2005 based on predictive regression (top left panel) and quantile regressions with $\tau = 0.05$ (bottom left panel), $\tau = 0.5$ (top right panel) and $\tau = 0.95$ (bottom right panel). For each year, the GiS is the tangency point between the ellipsoid and the corresponding surface.

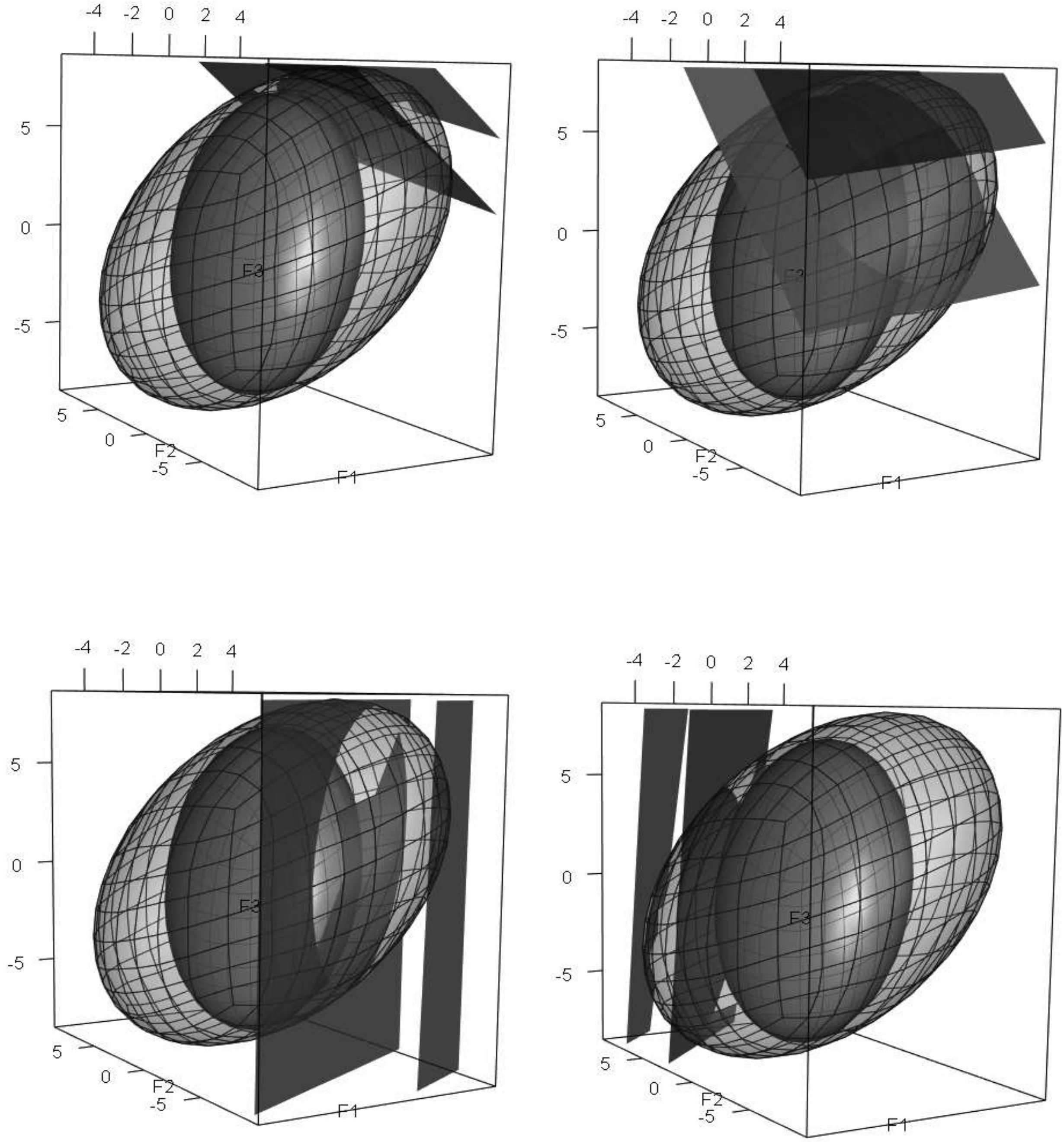


Figure 3.10: Cross-sectional average GiS (black line) and ±2 standard deviations (red lines) among countries in Africa (top left panel), America (top right panel), Asia (bottom left panel) and Europe and Oceania (bottom right panel).

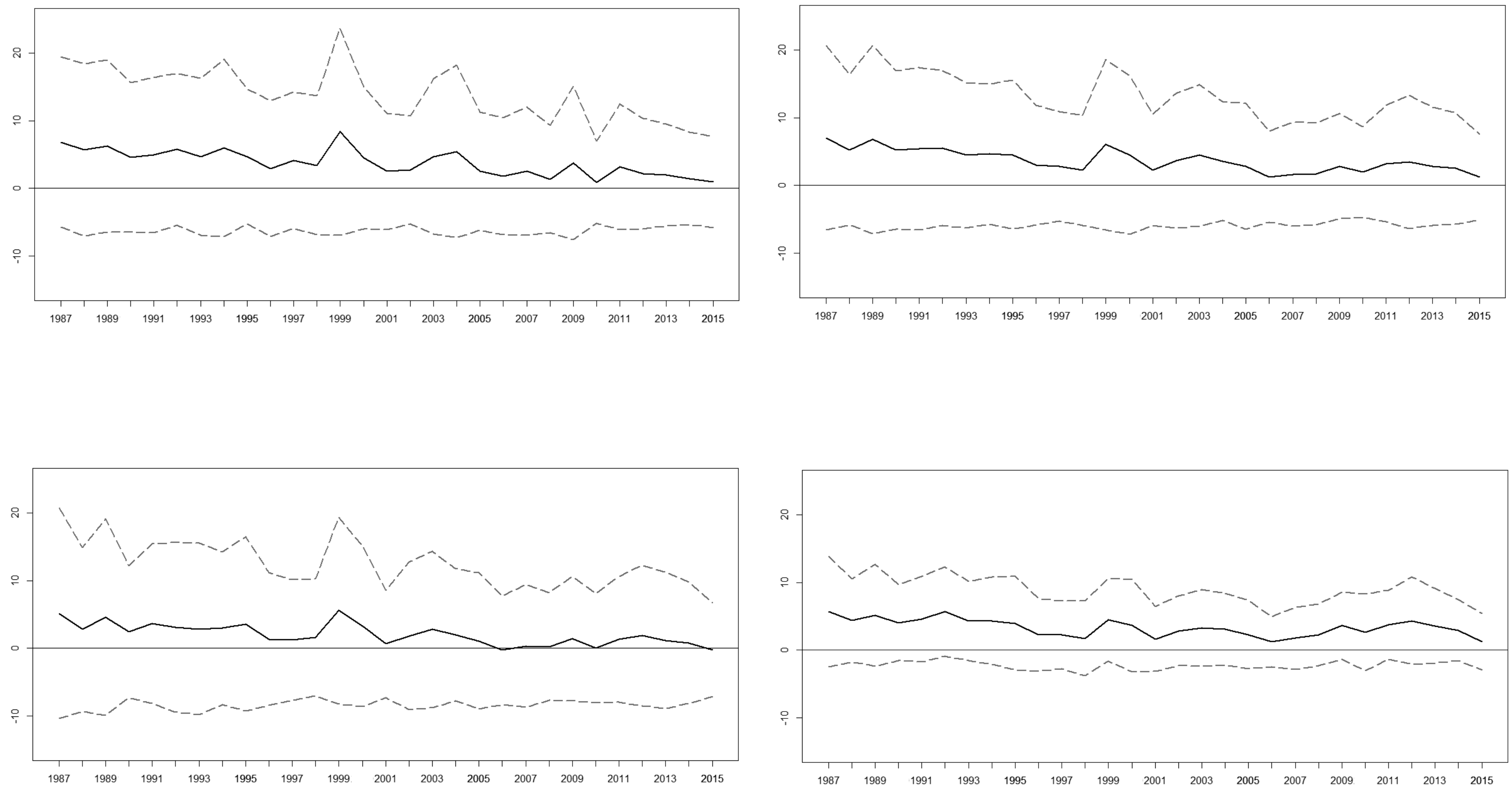


Figure 3.11: Cross-sectional average GiS (black line) and ±2 standard deviations (red lines) among other developing (top panel), emerging (middle panel) and industrialized (bottom panel) countries.

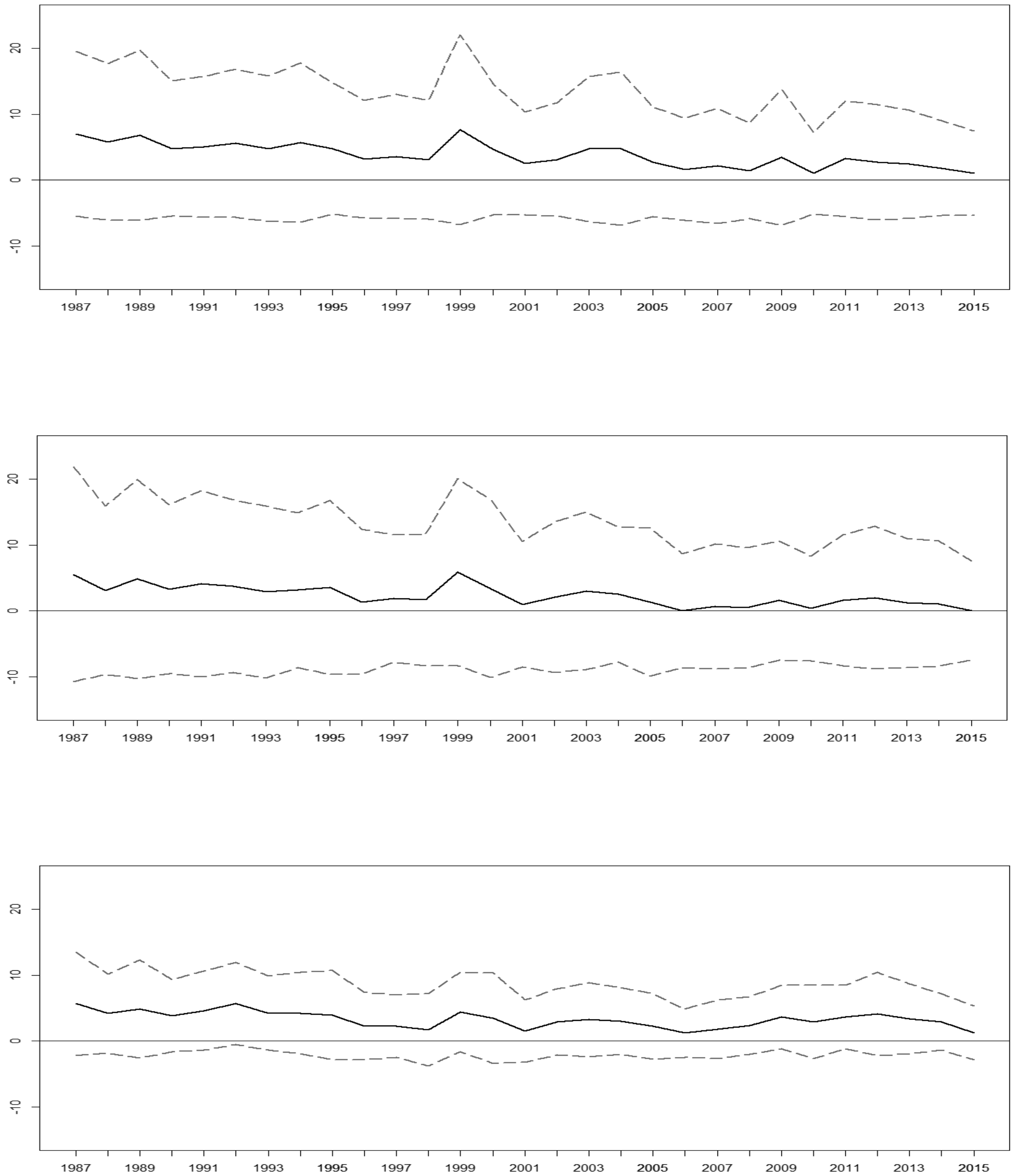
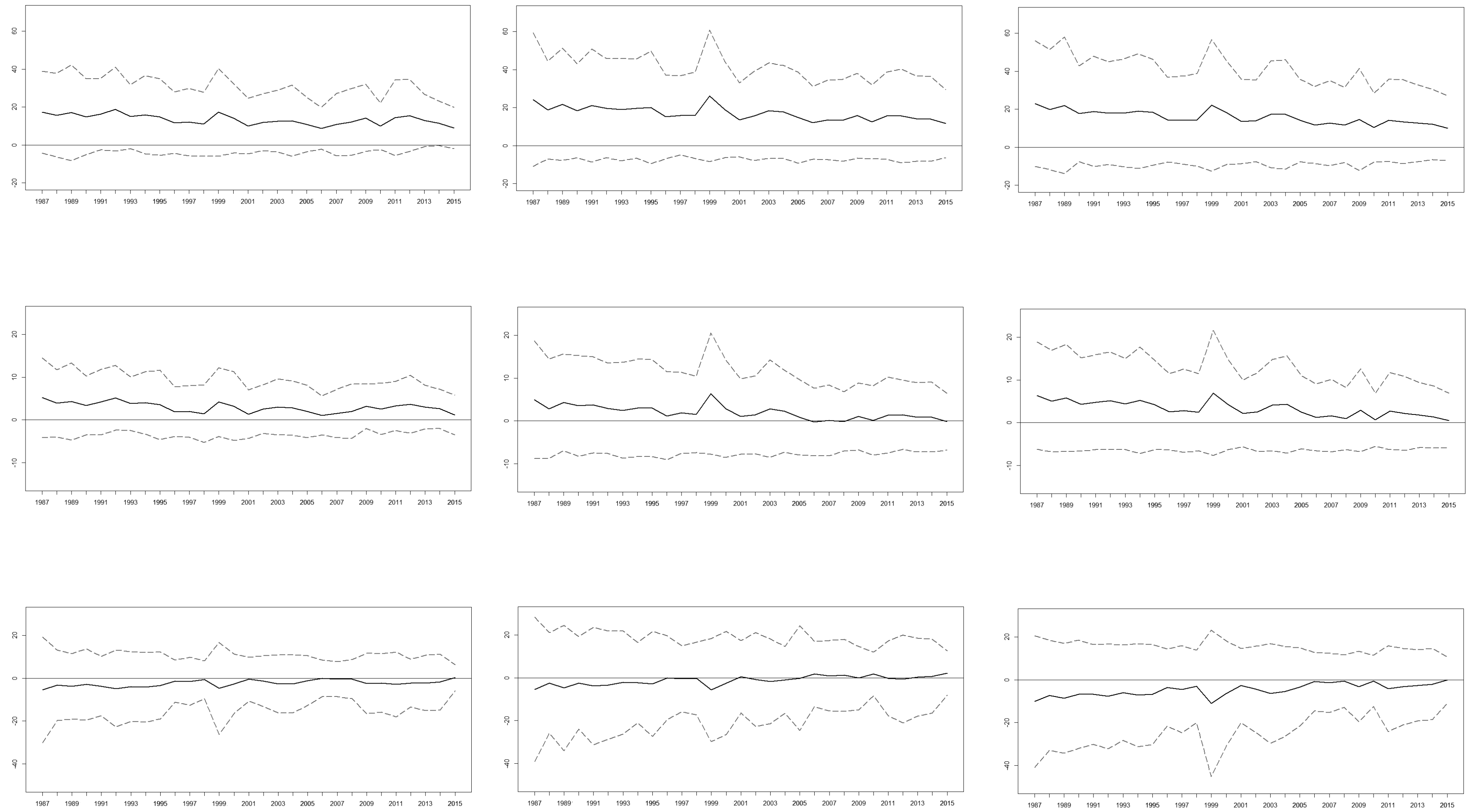


Figure 3.12: Cross-sectional average GiS (black line) and ± 2 standard deviations (red lines) for $\tau = 0.05$ (first row), 0.5 (second row) and 0.95 quantiles of the growth distribution among industrialized (first column), emerging (second column) and other developing (third column) countries.



Chapter 4

Summary and Future Research

4.1. Conclusions

In this thesis, we have investigated how to properly measure the uncertainty of the Principal Component Factor extraction and how this correct measurement could lead us to a much better understanding on the vulnerability of individual country economies when facing extreme scenarios in those factors that drive world growth.

The second chapter explores different methods for computing the uncertainty associated to factors extracted using PC in DFMs. By means of extensive Monte Carlo experiments, the finite sample performance of the asymptotic approximation is investigated. We show that it does not incorporate parameter uncertainty and, consequently, underestimates the uncertainty of PC factors, causing narrower confidence intervals and regions than desired. Moreover, we show that the extant bootstrap procedures proposed in the context of PC extraction in DFM are not capable of measuring correctly the uncertainty associated to the factors. Some of them compute the marginal MSE instead of the conditional one, while others do not take into account the parameter uncertainty.Thus, we propose a subsampling algorithm to compute the uncertainty of PC factors and to construct confidence intervals. The subsampling intervals and regions are computationally very simple and asymptotically valid. Furthermore, they have better finite sample coverages than those constructed using the asymptotic approximation or the bootstrap procedures available in the literature. Finally, we construct confidence intervals for the factor extracted from a system of Spanish macroeconomic variables and show the importance of having adequate intervals when interpreting whether the growth is truly positive.

In the third chapter, we propose a new global risk index, Growth-in-Stress (GiS), that measures the expected fall in the GDP of a country when the global factors are subject to stressful conditions. There are three components to this measure: the existence of global factors, the definition of stress, and the choice of the objective function.

We have extracted three global factors out of a sample of GDP growth of 87 countries, classified as industrialized, emerging, and other developing, over the period 1985-2015. The first factor, which accounts for 20% of the total variability of growth, is driven by all industrial and emerging countries, and is considered a world growth factor; the second factor is driven by other developing countries in Africa and America; and the third factor is mainly related to East Asian economies. All three factors account for 42% of the total growth variability. To our knowledge, the African/American factor has not been reported in the literature yet. We have defined stressful events in the factors by considering the extreme multivariate quantiles of the joint distribution of the three factors. We have constructed 95% probability ellipsoids that contain the true factors so that the extreme events are those seating on the boundary of the ellipsoid. Obviously, it is up to the researcher to choose the level of risk or stress desired. It is this approach of considering stress directly on the factors that makes our index a risk index instead of an uncertainty index. Finally, we have estimated country-specific predictive regressions augmented with the three factors to predict (i) the one-step-ahead average growth, and (ii) the one-step-ahead τ-quantile growth in each country. With these three elements in place (factors, stress, and objective function), we proceed to compute GiS as the predicted minimum growth and minimum τ-quantile generated by the point of tangency between the 95% probability ellipsoid and the properly oriented surfaces based on the predictive regressions.

Our results confirm that global risk has been decreasing over time. Not only the cross-sectional average GiS has been going down but also the ± 2 standard deviation bounds have become narrower over time. The cross-sectional average GiS was about 5% in 1987 and between 0-1% in 2015 considering the 87 countries in Africa, America, Asia and Europe/Oceania. However, there is a lot of heterogeneity across countries and continents. Several countries in Africa and America are exposed to very high risks with GiS larger than 10%. The countries in the Europe/Oceania group are more homogeneous as the bounds around the cross-sectional average GiS are the tightest of all continents. From 2011 on, all continents have entered in a state-of-complacency and by 2015 the average worst outcome seem to be no growth at the 95% factor stress. We also measure the factor stress on different quantiles ($\tau = 0.05, 0.5$ and 0.95) of the GDP growth distribution of each country. Overall, the 50% quantile GiS and the average GiS are quite similar. For those countries that are already or approaching recession i.e., those in the 5% quantile of the growth distribution, an extreme event in the factors has catastrophic consequences as we have calculated that GDP may experience a 20% drop.

The exercise that we have described is predictive but it has been conducted in-sample. The time series is too short to implement an out-of-sample exercise though it would be possible to increase the frequency of the series to obtain a larger sample size. The methodology that we propose is general enough to be applicable to any other macroeconomic aggregates beyond GDP growth. Moreover, the factors could also be extracted from systems of macroeconomic/financial variables instead of extracting them from the system of growths.

4.2. Further Research

4.2.1. Forecast uncertainty in factor augmented predictive regressions

Forecasting macroeconomic variables is of great interest for policy makers. In this context, factor augmented regressions are a very popular forecasting models when there are a large amount of predictors. Just to mention a few applications, Hofman (2009) and Cicarelli and Mojon (2010) use factor augmented regressions to predict inflation while Ludvigson and Ng (2007, 2009), Cheng and Hansen (2015), Cakmakli and van Dijk (2016) and Ohno and Ando (2018) predict financial returns using macroeconomic factor-based forecasts.

Following Bai and Ng (2006) and Gonçalves, Perron and Djogbenou (2017), we consider the following autoregressive model augmented with factors

$$y_{t+h} = \beta' W_t + \alpha' F_t + u_{t+h}, \tag{4.1}$$

where, for $t = 1, ..., T-h$, y_{t+h} denotes the observation at time $t+h$ of the variable to be forecast, with h being the forecast horizon, W_t is a vector of observed regressors which could include a column of ones and lags of y_t, and $F_t = (F_{1t}, ..., F_{rt})$ is the $r \times 1$ vector of unobserved unobserved common factors which summarize the variations of a large cross-section of variables, $X_t = (x_{1t}, ..., x_{Nt})'$ of dimension $N \times 1$ which are modelled according to the following approximated DFM

$$X_t = PF_t + \varepsilon_t, \tag{4.2}$$

where P is the $N \times r$ matrix of factor loadings such that $P'P$ is a diagonal matrix with distinct entries arranged in decreasing order and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the $N \times 1$ vector of idiosyncratic noises which can be potentially temporal and cross-sectionally correlated and heteroscedastic; see Gonçalves, Perron and Djogbenou (2017) for the assumptions about model (4.1) and (4.30). For a unique identification of the factors, we assume that $\frac{F'F}{T} = I_r$; see Bai and Ng (2013) for a discussion on identification issues in the context of PC factor extraction. Finally, u_t is the forecast error which is assumed to be an independent white noise process with common distribution F_u and variance σ_u^2; see Stock and Watson (1999) and Forni et al. (2000) for the introduction of factor-augmented predictive regressions.

If the loss function is quadratic and the factors, F_t, were observable for $t = 1, ..., T-h$, then h-step-ahead forecasts of y_{T+h} with minimum Mean Square Forecast Error (MSFE) are given by the following conditional mean

$$y_{T+h|T} = E\left(y_{T+h}|\Omega_T\right) = \beta' W_T + \alpha' F_T, \tag{4.3}$$

where Ω_T is the information set available at time T. However, the forecasts in (4.3) are not feasible because the parameters in β and $\alpha' = (\alpha_1, ..., \alpha_r)$ and the factors are all unobserved. Feasible forecasts are obtained by replacing the unknown parameters and factors by their corresponding estimates as follows

$$\hat{y}_{T+h|T} = \hat{\beta}' W_T + \hat{\alpha}' \tilde{F}_T. \tag{4.4}$$

In practice, the forecasts in equation (4.4) are obtained in two steps. First, in order to obtain $\tilde{F}_t$, the factors are often extracted using PC in the context of approximated DFMs; see, for example, Jiang et al. (2017) for empirical implementations of DFM in the context of factor augmented predictive regressions. Assuming that the number of factors, r, is known, the $r \times T$ matrix of extracted factors $\tilde{F} = (\tilde{F}_1, ..., \tilde{F}_T)$ is given by $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $X'X$ where $X = (X_1, ..., X_T)$. The matrix of estimated factor loadings, $\tilde{P}$, is computed by $\tilde{P} = \frac{X\tilde{F}'}{T}$; see Bai and Ng (2008) for a review of PC factor extraction.

Under some regularity conditions, Bai (2003) shows that, if $\frac{\sqrt{N}}{T} \to 0$ when $N, T \to \infty$, $\tilde{F}$ is a consistent estimator of the space spanned by the true factors. Furthermore, for each t,

$$\sqrt{N}\left(\widetilde{F}_t - HF_t\right) \xrightarrow{d} N\left(0, \Sigma_p^{-1}\Gamma_t\Sigma_p^{-1}\right), \tag{4.5}$$

where $H = \tilde{V}^{-1}\frac{\tilde{F}'F}{T}\frac{P'P}{N}$ with $\tilde{V}$ being the $r \times r$ diagonal matrix containing on the main diagonal the r largest eigenvalues of $\frac{XX'}{NT}$ in decreasing order, $\Sigma_p = \lim\limits_{N\to\infty}\frac{1}{N}P'P$ and $\frac{1}{\sqrt{N}}\sum\limits_{i=1}^{N} P_i\varepsilon_{it} \xrightarrow{d} N(0, \Gamma_t)$. Furthermore, Bai (2003) shows that, if the idyosincratic noises are serially uncorrelated, the limiting distributions in (4.5) are asymptotically independent across t. From (4.5), the asymptotic MSE of the extracted factors can be estimated as follows

$$MSE_t = \left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}\frac{\tilde{\Gamma}_t}{N}\left(\frac{\tilde{P}'\tilde{P}}{N}\right)^{-1}, \tag{4.6}$$

where, according to Bai and Ng (2006), $\tilde{\Gamma}_t$ can be estimated assuming that the idyiosincratic errors are cross-sectionaly uncorrelated, as follows [1],

$$\widetilde{\Gamma}_t = \frac{1}{N}\sum_{i=1}^{N}\tilde{P}_i\tilde{P}_i'\tilde{\varepsilon}_{it}^2 \tag{4.7}$$

where, $\tilde{P}_i$ is the $i-th$ row of the estimated factor loading matrix $\tilde{P}$ and $\tilde{\varepsilon}_{it} = y_{it} - \tilde{P}_i'\tilde{F}_t$.

In the second step, the estimated factors are substituted in equation (4.1) and the predictive regression parameters are estimated by Least Squares (LS) to obtain $\hat{\beta}'$ and $\hat{\alpha}'$; see Stock and Watson (1999) who introduce factor-augmented regressions estimated by LS. Consequently, the LS estimator of the parameters and the point forecasts, $\hat{y}_{T+h|T}$, are functions of the estimated factors, $\tilde{F}_T$. Therefore, the statistical properties of $\hat{y}_{T+h|T}$ depend on those of the forecast errors, u_t, of the estimated parameters and of the estimated factors. Feldstein (1971) argues that even if the errors in the factor

[1] Bai and Ng (2006) propose this estimator of the asymptotic covariance matrix arguing that, if the cross-correlation in the errors is small, assuming that they are zero could be convenient because the sampling variability from their estimation could cause an efficiency loss.

augmented predictive regression and in the DFM are normal, the forecasts $\hat{y}_{T+h|T}$ will be non-normal in finite samples as it involves the sum of products of normal variables. However, having estimated factors as regressors does not affect consistency of the regression parameters and of $\hat{y}_{T+h|T}$; see Stock and Watson (2002a). Furthermore, Bai and Ng (2006) show that it is only when N is large relative to T that the effect of estimated factors can be completely ignore. The precise condition on N and T depends on whether interest is in inference of the regression parameters, the conditional mean in equation (4.3) or the forecast in equation (4.1).

Moreover, there is also an increasing interest on the construction of probabilistic forecasts of macroeconomic and financial variables; see, for example, the recent work by Proietti et al. (2016) forecasting growth in the euro area. Therefore, we focus on the construction of probabilistic forecasts based on factor augmented regressions which should take into account two additional sources of uncertainty on top of that associated with the stochastic errors of the model. First, forecasts are based on estimated parameters and, consequently, the estimation uncertainty should be taken into account. Second, in factor augmented regressions, estimated factors are included as predictors instead of the true unobserved latent factors; see Feldstein (1971) for the sources of uncertainty when forecasting using regression models with stochastic regressors. When the factors are estimated by Principal Components (PC) in the context of Dynamic Factor Models (DFM), as it is often the case, having estimated factors as regressors does not affect consistency of the regression parameters and of forecasts; see Stock and Watson (2002a). Furthermore, Bai and Ng (2006) show that it is only when the cross-correlation dimension, N, is large relative to the temporal dimension, T, that the effect of estimated factors can be completely ignore. The precise condition on N and T depends on whether interest is on inference on the predictive regression parameters or on the forecasts of the conditional mean or of future values of the variable of interest. Furthermore, assuming normality of the forecast errors, Bai and Ng (2006) derive forecast intervals for these three quantities. When the conditional mean in equation (4.3) is forecast using (4.4), the forecast error is given by

$$\hat{y}_{T+h|T} - y_{T+h|T} = \left(\hat{\delta} - \delta\right)' \hat{z}_T + \alpha' H^{-1} \left(\tilde{F}_T - HF_T\right). \tag{4.8}$$

where $\hat{\delta} = (\hat{\beta}', \hat{\alpha}')'$ and $\delta = (\beta', \alpha' H^{-1})$ is the probability limit of $\hat{\delta}$, $\hat{z}_T = (W_T, \tilde{F}_T)$ and H is defined as in (4.5). Using the asymptotic distribution of the PC factor, Bai and Ng (2006) show that, if $\frac{\sqrt{N}}{T} \to 0$, then

$$\frac{\hat{y}_{T+h|T} - y_{T+h|T}}{\sqrt{var\left(\hat{y}_{T+h|T}\right)}} \xrightarrow{d} N(0,1), \tag{4.9}$$

where

$$var\left(\hat{y}_{T+h|T}\right) = \frac{1}{T}\hat{z}_t' Avar(\hat{\delta})\hat{z}_t + \frac{1}{N}\hat{\alpha}_t' Avar(\tilde{F}_T)\hat{\alpha}_t \tag{4.10}$$

with $Avar(\hat{\delta})$ and $Avar(\tilde{F}_T)$ being the asymptotic covariance matrices of $\sqrt{T}(\hat{\delta} - \delta)$ and $\sqrt{N}(\tilde{F}_T - HF_T)$, respectively. Bai and Ng (2006) propose the following consistent estimator of $Avar(\hat{\delta})$

$$\widehat{Avar}(\hat{\delta}) = \left(\frac{1}{T}\sum_{t=1}^{T-h} \hat{z}_t \hat{z}_t'\right)^{-1} \left(\frac{1}{T}\sum_{t=1}^{T-h} \hat{u}_{t+h}^2 \hat{z}_t \hat{z}_t'\right) \left(\frac{1}{T}\sum_{t=1}^{T-h} \hat{z}_t \hat{z}_t'\right)^{-1} \tag{4.11}$$

while $Avar(\tilde{F}_T)$ can be estimated as in (4.6). The overall convergence rate for $\hat{y}_{T+h|T}$ is $min\,[N, T]$.

When the objective is forecasting y_{T+h}, we should consider the distribution of the following forecast error

$$\hat{y}_{T+h|T} - y_{T+h} = \left(\hat{y}_{T+h|T} - y_{T+h|T}\right) - u_{T+h} \tag{4.12}$$

Bai and Ng (2006) show that if the forecast errors, u_t are normally distributed, then the forecast error in (4.12) are asymptotically

$$\frac{\hat{y}_{T+h|T} - y_{T+h}}{\sqrt{\sigma_u^2 + var\left(\hat{y}_{T+h|T}\right)}} \xrightarrow{d} N\left(0,1\right), \tag{4.13}$$

where $var(\hat{y}_{T+h})$ is defined as in equation (4.10) and σ_u^2 can be estimated from the corresponding residuals. However, these forecast intervals may be not appropriate in finite samples due to biases on the estimation of the parameters of the predictive regression model and/or non-normality of the forecast errors. Consequently, Gonçalves, Perron and Djogbenou (2017) propose constructing forecast intervals based on bootstrapping both the estimated factors and the regression parameters so that parameter bias is corrected and there is not need to assume any particular distribution of the forecast errors. In the second chapter of this thesis, we show that the bootstrap procedure implemented by Gonçalves, Perron and Djogbenou (2017) underestimates the uncertainty associated with the unobserved factors and propose a subsampling procedure which delivers accurate forecast intervals of the factors.

In this future line of research, we extend the subsampling procedure of the second chapter to obtain forecast intervals of future conditional means and observations in the context of factor augmented predictive regressions. The properties of the proposed method are analyzed in finite samples and compared to those of the bootstrap procedure by Gonçalves, Perron and Djogbenou (2017).

4.2.2. Forecasting mortality rates: does non-stationarity matters?

Since the 20th Century, mortality rates have been dramatically dropping for all age cohorts in countries all over the world. This decrease in mortality has caused a number of social and economic problems. In particular, in the social security systems and in the insurance industry, the fast unexpected reduction in death probabilities has been the main cause of solvency becoming vulnerable.

The adverse consequences of longevity risk will be mitigated if changes in longevity could be anticipated by obtaining accurate forecasts of mortality rates.[2] Consequently, there is a large number of alternative methods proposed in the literature to forecasting mortality rates with the most popular being based on time series models; see Booth (2006), Booth and Tickle (2008) and Pitacco et al. (2009) for excellent reviews. Time series models represent mortality rates as a multivariate time series system with each component of the system corresponding to observations of the time series of mortality rates of a given group age.[3] The most popular time series forecast model is due to Lee and Carter (1992), denoted by LC, and originally proposed for U.S. data. The LC model is now the dominant method for mortality forecasting and it is used as benchmark by the U.S. Consensus Bureau for its long-term forecasts of life-expectancy. The LC model is specified as follows:

$$x_t^{(i)} = \alpha_i + \beta_i k_t + \varepsilon_t^{(i)}, \tag{4.14}$$

where $x_t^{(i)}$ is the log-mortality rate observed for group of age i at time t, k_t is the common factor governing mortality rates and $\varepsilon_t^{(i)}$ is the specific component of each age group which is assumed to be an uncorrelated white noise vector. In order to identify the model, Lee and Carter (1992) impose the restrictions $\sum_{t=1}^{T} k_t = 0$ and $\sum_{i=1}^{N} \beta_i = 1$, where N is the number of age groups and T is the temporal dimension. Under these assumptions, the factor extracted using PC is given by Lee and Carter (1992, Appendix A) as follows

$$\hat{k}_t = \sum_{i=1}^{N} x_t^{(i)}. \tag{4.15}$$

[2]Accurate forecasts of longevity are crucial not only for Actuarial Sciences but also for many other disciplines, such as Demography, Biomedical and Epidemiological Studies.

[3]Alternatively, several authors propose forecasting mortality rates using functional time series models which are based on assuming that mortality rates in a time period can be considered together as a finite realization of an underlying continuous function; see, for example, Chiou and Muller (2009), Hyndman and Ullah (2007), Hyndman and Booth (2008), Hyndman et al. (2013) and Shang and Hyndman (2017) among others.

The estimates of α_i and β_i are given by

$$\hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T} x_t^{(i)} \tag{4.16}$$

$$\hat{\beta}_i = \frac{\sum_{t=1}^{T}(x_t^{(i)} - \hat{\alpha}_i)}{\sum_{i=1}^{T}\hat{k}_t^2}. \tag{4.17}$$

Finally, the variance of each of the specific noises can be estimated using the corresponding residuals as follows

$$\hat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T}(x_t^{(i)} - \hat{\alpha}_i - \hat{\beta}_i\hat{k}_t)^2. \tag{4.18}$$

After estimating the underlying factor, Lee and Carter (1992) propose modelling the dynamic dependence of k_t by an ARIMA model. For the U.S. data considered in their paper, they conclude that the best fit is obtained by the following random walk plus drift

$$k_t = \mu + k_{t-1} + u_t \tag{4.19}$$

where u_t is a white noise process with variance σ_u^2. The random walk plus drift model seems to be a good fit when modelling log-mortality rates in different countries; see the references in Shang et al. (2011). Note that (4.19) implies that the common component of log-mortality rates changes at a constant rate, μ. Note that, in tis case, log-mortality rates are I(1).

The parameters of model (4.19) can be estimated by

$$\hat{\mu} = \frac{1}{T}\sum_{t=2}^{T}(\hat{k}_t - \hat{k}_{t-1}) = \frac{1}{T}\left(\hat{k}_T - \hat{k}_1\right) \tag{4.20}$$

$$\hat{\sigma}_u^2 = \frac{1}{T}\sum_{t=1}^{T}(\hat{k}_t - \hat{k}_{t-1} - \hat{\mu})^2. \tag{4.21}$$

The h-step ahead forecast of the log-mortality rate given the information available at time T is obtained as follows

$$\hat{x}^{(i)}_{T+h|T} = \hat{\alpha}_i + \hat{\beta}_i \hat{k}_{T+h|T} \tag{4.22}$$

The main strengths of the LC method is its simplicity and robustness in situations where log-mortality rates have trends with constant slopes; see Lee and Miller (2001) for the main advantages of LC. However, LC also has two main weaknesses. First, it attemps to capture patterns in mortality rates using only one principal component. If more factors were present, then the specific noises may have strong cross-sectional and temporal correlations; see Lee and Carter (1992) who point out that the correlations between specific noises can be large when modeling mortality rates in USA. In order to deal with this limitation, several authors have proposed including more factors; see, for example, Bell (1997), Booth et al. (2002), Yang et al. (2010), Mitchell et al. (2013) and Hunt. The second main limitation of the LC method is that it assumes that the common trend in mortality rates, k_t, has a constant slope, μ. However, in practice, several authors have pointed out that there could be changes in the slope of k_t. In order to cope with changes in the slope, Sweeting (2011) and Borger and Schupp (2018) propose modeling k_t using a piecewise linear trend. Alternative, Carter (1996) consider that the drift itself follows a random walk as follows

$$\hat{k}_t = \mu_t + \hat{k}_{t-1} + u_t \tag{4.23}$$

$$\mu = \mu_{t-1} + \eta_t. \tag{4.24}$$

Note that if k_t is given by the random walk plus noise model in equations (4.23) and (4.24), the log-mortality rates are I(2) while when it is given by the random walk plus drift model, they are I(1).

The parameters involved in the random walk plus noise model for $\triangle\hat{k}_t$ in equations (4.23) and (4.24) can be obtained by Quasi Maximum Likelihood (QML) based on using the Kalman filter to obtain the one-step-ahead prediction error decomposition of the Gaussian log-likelihood as explained in, for example, Harvey (1989).[4]

LC fit the model directly to log-mortality rates. However, several authors, consider fitting DFM to stationary first differenced log-mortality rates. Mitchell et al. (2013) implement the LC model to first differences of log-mortality rates, denoted by $y_t^{(i)} = \Delta x_t^{(i)}$. They model the underlying factors, k_t using three alternative models. Two of these models for k_t are stationary while the third one is I(1) with the latest having the best fit in their empirical application to several developed countries.

Forecasts of future log-mortality rates are obtained by

$$\hat{x}_{T+h|T}^{(i)} = x_T^{(i)} + \hat{y}_{T+1|T}^{(i)} + ... + \hat{y}_{T+h|T}^{(i)} \tag{4.25}$$

where $\hat{y}_{T+j|T}^{(i)}$ is the j-step ahead forecast of the variations in log-mortality rates obtained from the DFM.

Note that, when assuming stationary models for k_t in the context of the model in first differences proposed by Mitchell et al. (2013), one is implicitly assuming that log-mortality rates are I(1) while if k_t is I(1), then log-mortality rates are I(2). Therefore, this latter alternative is equivalent to assuming the LC model for log-mortality rates with the common trend specified as in equations (4.23) and (4.24) with this latter alternative being much simpler from a computational point of view.

Finally, French and O'Hare (2013) also consider a DFM with a single factor which is estimated using the Generalized DFM procedure proposed by Forni et al. (2005).

The variant adopted now as the standard LC model is due to Lee and Miller (2001) and denoted by LM; see Booth and Tickle (2008) and Shang et al. (2011) who

[4]Details can be found in Appendix A.

compare several extensions of the LC method and show the good performance of the LM method.

The LC model and all their variants are fitted to log-mortality rates which, as mentioned above, have been declining over time and, consequently, are not stationary. It is well known that the stationarity properties of the series under analysis are crucial for inference on the extracted factors. In the context of systems of variables with non-stationary integrated of order 1 factors, if the specific noises are stationary, extracting the factors directly from non-stationary log-mortality rates is a valid procedure; see Bai (2004). Most authors extract the factors from the original log-mortality rates; see Lee and Carter (1992), Lee and Miller (2001), Li and Lee (2005) and Heberman and Renshaw (2011) among many others. However, if the specific noises are not stationary, then the extracted PC factors are not consistent. The statistical properties of PC factors extracted when the log-mortality rates are I(1) and not cointegrated can be affected by the presence of spurious correlations; see Mitchell et al. (2013). In order to deal with this problem, Mitchell et al. (2013) propose implementing the LC methodology to first differences of log-mortality rates; see also French and O'Hare (2013). Bai and Ng (2004) show that this approach delivers consistent estimators of the factors although they are not efficient as neither the dynamic nature of the factors nor the serial and cross-correlation dependence or heteroscedasticity of the specific noises are exploited. After forecasting the increments of log-mortality rates, forecasts of the levels are obtained by cumulation. In a first difference approach, one is implicitly assuming that all common factors have a unit root and the specific noises are non-stationary. This could not always be a valid assumption. Finally, it is important to point out that, as far as we know, the properties of PC factors extracted from I(2) systems have not been previously analyzed in the literature; see Haldrup (1998) for a survey of the literature dealing with I(2) variables in economic time series.

Consequently, the first objective of this future line of research is to disentangle which is the best strategy to model and forecast log-mortality rates in different countries. We consider industrialized, emerging and developing countries. For each country considered, we analyze the stationarity properties of the factors and specific noises in log-mortality rates by using the procedures proposed by Peña and Poncela (2006) to determine the number of stationary and non-stationary factors and by Bai and Ng (2004) to test for the non-stationarity of the specific noises in the context of I(1) systems.

When forecasting future mortality rates, it is important to obtain not only point forecasts but also measures of their associated uncertainty; see, for example, the discussion by Lee and Carter (1992), Keilman (2001), Cairns et al. (2011), Li and Chan (2011) and Shang at al. (2011), among others. Lee and Carter (1992) consider uncertainty of the innovations. Forecast intervals with nominal coverage $(1-\delta)\%$ are given by the inverse transformation of the extreme of the forecast intervals obtained for log-mortality rates as follows

$$\left[exp\left(\hat{x}^{(i)}_{T+h|T} - z_{\delta/2}\hat{\beta}_i se_{ht}\right), exp\left(\hat{x}^{(i)}_{T+h|T} + z_{\delta/2}\hat{\beta}_i se_{ht}\right)\right] \tag{4.26}$$

where $z_{\delta/2}$ is the $\delta/2$ quantile of the normal distribution.

Several limitations of forecast intervals constructed as in (4.26): i) They are based on the normal assumption of log-mortality rates; ii) They do not incorporate parameter uncertainty which can be important if T is not very large as it is often the case when forecasting mortality; iii) They are based on the inverse transformation of the extremes of the forecast intervals for log-mortality rates and, consequently, they are subject to biases; see, for example, Pascual et al. (2005).

This future line of research also contributes to the literature by considering not only point predictions but also density predictions which take into account the uncertainty associated with the errors in the model and with parameter uncertainty. The comparison of the procedure proposed in this paper to forecast log-mortality rates with those available in the literature is carried out not only in terms of point forecasts but also for forecast intervals.

4.2.3. Looking for Intrahousehold Resource Allocation Bias

Extensive literature shows that individual wealth not only depends on the total wealth of the household, but also on how that wealth is distributed among its members. Therefore, the intrahousehold distribution of expenditure in the individual welfare is a topic of great relevance nowadays; see, among many others, Deaton et al. (1989), Bargain and Donni (2012), Dunbar et al. (2013), Rodriguez (2016). Particularly, measuring the proportion of resources allocated by parents to their children is a topic of great interest, mainly due to the fact that they are the weakest group in the household because they do not belong to it by choice, and they do not have decision-making power or freedom to consume. Moreover, the existence of gender discrimination in the intrahousehold resource allocation has been empirically proven in many societies - generally against girls -. For example, Das Gupta (1987), Sen (1990), Klasen (1996), Das Gupta et al. (2003) show a bias against women in terms of mortality and morbidity rates. Sen and Das Gupta (1983) find nutrition discrimination against girls in India, Hazarika (2000) points out that, in South Asia, boys have better access to health services although girls are better fed, Rose (2000) finds gender bias in time allocation in rural India, Song (2000) finds discrimination against very young girls in China, Gibson and Rozelle (2004) find bias in favor of boys aged 7 to 14 years in Papua New Guinea, Gong et al.(2005) find that there is bias in favor of boys in the expenditure on education in rural China, Kingdon (2005) finds lower educational allocation to girls than to boys in rural India, Choi and Lee (2006) find gender bias in child immunization in rural areas in India, Kebede (2008) finds gender discrimination against girls in rural

Ethiopia, Himaz (2010) finds pro-female bias in rural Sri Lanka in the allocation of education expenditure, Zimmermann (2012) finds evidence of gender bias in favor of girls in education expenditure in India, Azam and Kingdon (2013) find out the existence of pro-male gender bias in the intrahousehold educational expenditure allocation, Barcellos et al. (2014) show that boys receive more investment from parents than girls in rural India. However, most of the studies on gender discrimination focus on external observable factors such as school enrollment, nutrition indicators or mortality rates. This fact is a consequence of the difficulty of measuring the possible bias in the allocation of intrahousehold resources, mainly explained by expenditure data being available usually at household level instead of by its individual members, and by the fact that, even if it could be measured, some additional types of sample biases would be incurred. Furthermore, there are goods that are consumed jointly and it is therefore difficult to differentiate them among the various members of a household. Finally, gender bias cannot be directly measured because of the impossibility to find two families that are exactly alike in real life.

Because of these data limitations, researchers should use an indirect method commonly known as the Engel curve approach; see, among many others, Haddad and Reardon (1993), Horrel and Oxley (1999), Song (2000), Gibson and Rozelle (2004), Gong et al. (2005) Fuwa et al. (2006), Kebede (2008) and Lee (2008) who apply this method to find gender bias in different societies. The Engel curve approach seeks to find demographic separability of goods by means of a system of regressions in which the dependent variable is the level or the share of expenses in each good or service, and the regressors are some demographic variables and dummies. According to Deaton (1989) and following Rothbarth (1943) approach, boy-girl discrimination can be found by looking at the adult goods -those goods which are not typically consumed by children-. In this way, the inclusion of a child in the family will cause a negative income effect on the demand for those goods. If the difference in the income effect between both genders is statistically significant, it means that there is gender discrimination in the allocation of resources. The analysis begins by specifying an Engel curve that relates the expenditure in each individual good with the total expenditure and with other demographic and socioeconomic variables. The specification of the Engel curves can be different. The most commonly used one is the Engel curve proposed by Deaton (1989) extending the work of Working (1943) and Leser (1963) [5]

$$W_g/pq = \alpha_g + \beta_g ln(x/N) + \eta_g lnN + \sum_{1}^{J-1} \gamma_{gj}(n_j/N) + \delta_g z + \varepsilon_g \tag{4.27}$$

where W_g represents the expenditure share on some commodity or group of commodities, x is the log of total consumption expenditure per household, N is the total number of household members, n_j is the number of people in the household in the jth demographic category (girls, boys, women, men), z is a vector of demographic characteristics and dummy variables that allow for possible effects of other household characteristics, such as location, region, nationality or education and ε is the error term.

Once, the Engel curves have been estimated, the most direct way of checking the effects of gender in the allocation of resources by the parents would be to compare the coefficients γ_{gj} for boys and girls using a $t - test$ or an $F - test$. However, Deaton (1989, 1997) suggests a better way to measure these effects. Concretely, Deaton (1989) introduces the "outlay-equivalent ratio" defined as the derivatives of expenditures on each adult good with respect to an additional child divided by the corresponding derivatives with respect to total expenditure.

$$\pi_{gj} = \frac{\partial W_g/\partial n_j}{\partial Y_g/\partial X} \frac{N}{X} \tag{4.28}$$

The ratio shows the change in expenditure when a new member of one of

[5] Some evidences suggest that a linear Engel curve could not be appropriate for many commodities. See, for example, Bhalotra and Attfield (1998) and Blundell et al. (1998). A possible alternative to the linear approach could be to use a semiparametric model; see Gong et al. (2005)

the categories is added to the household, expressed as a ratio of total household expenditure per person. In this way, ceteris paribus, the effect of an additional person of type j to the expenditure in a particular good is given by $\partial W_g/\partial n_j$. The ratio $\partial W_g/\partial n_j$ to $\partial W_g/\partial X$ shows the increase in the total expenses necessary to generate the same additional expense in the good g that is generated by increasing the household with an additional member of category j. Deaton (1997) explains that the convenience of this expression comes from the fact that if g is an adult good and there is no substitution effect, then the "outlay-equivalence ratio" must be identical for all adult goods. In this way, once a series of adult goods have been defined, the equality of the ratios for each age and gender group can be tested.

Estimates of the $\pi-$ratios are obtained by replacing the parameters with their Ordinary Least Squares (OLS)'s estimates and replacing Q_g and (n_j/N) with their values at the sample mean of the data.

$$\pi_{gj} = \frac{\eta_g - \beta_g + \gamma_{gj} - \sum_1^{J-1}(n_j/N)}{\beta_g + Q_g} \tag{4.29}$$

Once the $\pi-$ratios are calculated, it is necessary to test the null hypothesis that the ratios for each child and adult age are equal across the list of hypothetical adult goods. Full details of the inferential procedure can be found in Deaton et al. (1989). When a proper group of adult goods is found, it is necessary to test the null hypothesis that the ratios for different demographic categories (boys, girls) are equal for the same adult good. The $\pi-$ratios in this case will be negative so, if the null hypothesis is rejected, the $\pi-$ratio will be more negative for one gender than for the other meaning that we are facing a case of gender discrimination in the allocation of resources [6].

Despite the popularity and methodological intelligence of the Engel curve approach, the results have not been as good as one might expect. Zimmermann

[6]The inferential process can be found again in Deaton et al. (1989)

(2012) notes that bias at the individual level seems to disappear when the Engel curve approach is used at a household level, and Kingdon (2005) shows that the Engel curve procedure does not detect gender bias even when there is significant discrimination between children at an individual level. Examples of this can be found in Deaton (1989) in Thailand and Ivory Coast, Haddad and Reardon (1993) in Burkina Faso, Subramanian (1994) in India, Deaton (1997) in Pakistan and Taiwan, Horrel and Oxley (1999) in late Victorian Britain, Fuwa et al. (2006) in rural India, and Gong et al. (2005) and Lee (2008) in rural China, among many others.

There can be several reasons behind the apparent failure of Engel curves when detecting gender bias. Case and Deaton (2003,11) point out that ”it is not clear if there is no discrimination or if, for some reason that is unclear, the method simply does not work.”

One of these reasons may be that the concepts of adult goods and demographic separability are difficult to determine; see Lee (2008). Additionally, and this is perhaps the most important reason, it should be noted that, in order to identify adult goods, the Engel curve approach only considers the income effect, i.e. it does not take into account neither the substitution nor the direct effects; see, Deaton (1997) and Kebede (2008), respectively.

The solution to these problems can be found by testing the demographic separability in preferences rather than in goods. From Engel (1857), the latent causes that explain the form of the Engel curves have been an object of study. For example, Engel (1857) identifies several categories of goods according to their final purpose: ”nourishment”, ”clothing”, ”recreation”, etc. Thus, and by way of example, the consumption of food may have as motives both the fundamental caloric gain to survive -nourishment- and when nourishment is satiated, a recreational activity. Therefore, the consumption of food would be determined by two fundamental reasons: i) to satisfy a basic need, and ii) to serve as a leisure activity. In this way, and returning to the concept of demographic separability and to the terminology proposed by Deaton et al. (1989), a proportion of food expenditure could, hypothetically, be considered as adult good.

Our objective is to capture those fundamental reasons by representing the budget shares system as a model of latent factors where each factor can be interpreted as one of the fundamental forces driving consumption patterns; see Lewbel (1991). These latent factors are a linear combination of expenditure in the different categories of goods. Therefore, we can write each observable Engel curve as a linear combination of $R < G$ independent basic Engel curves where R represents the number of latent factors [7].

In this way, the factors are modelled using a factor model (FM). The specification of the FM presented here follows common practice in the literature; see Bai (2003), among others. In particular, we consider the following FM

$$W_h = PF_h + \varepsilon_h, \quad h = 1, ..., H \tag{4.30}$$

where W_h is the G-dimensional vector of the expenditure of a household h in each goods' category; P is the $G \times R$ matrix of factor loadings, F_h is the R-dimensional vector of latent factors and ε_h is the G-dimensional vector of idiosyncratic noises. R represents the number of latent factors in the model which is assumed to be known.

Considering G categories of goods and a sample of H households, the model can be expressed in matrix notation as follows:

$$W = PF + \varepsilon, \tag{4.31}$$

where $W = (W_1, ..., W_H)$ is the $G \times H$ matrix of household expenditure, $F = (F_1', ..., F_H')$ is the $R \times H$ matrix of latent factors and ε is the $G \times H$ matrix of disturbances. The errors are assumed to be potentially cross-correlated among good categories and heteroscedastic, while the factors are assumed to have diagonal covariance matrix. Both

[7] For full details of the concept of latent Engel curves, see Barigozzi and Moneta (2016).

assumptions have an economic interpretation. The idiosyncratic components are likely to be correlated since they capture good-specific reasons behind their consumptions and not just income. On the other hand, this specification of the covariance matrix of the factors implies that they are orthogonal, i.e each factor determines an underlying motive for consumption. Further details on Factor Models and their assumptions can be found in Bai (2003), among others.

The estimation of the Factor Models depends on the structure of the covariance matrix of the errors and on the sample size. Commonly, standard expenditure national surveys provide a limited amount of expenditure categories or, if they do, there is usually a large number of households whose corresponding expenditure amount is zero or missing. Nevertheless, there are some exceptions for which there are balanced datasets with a large amount of expenditure categories. In any case, it is possible to construct a large data set pooling together different waves of the same survey; see, Kneip (1994) and Barigozzi and Moneta (2016) for full details.

In such cases, when dealing with very large systems of variables, Principal Components (PC) is still among the most popular factor extraction procedures due to its simplicity and low computational burden. The $R \times H$ matrix of extracted factors is given by $\sqrt{H}$ times the eigenvectors corresponding to the R largest eigenvalues of the $H \times H$ matrix $W'W$. The matrix of estimated factor loadings, $\hat{P}$, is computed by $\hat{P} = \frac{W\hat{F}'}{H}$; see Bai and Ng (2008) for a review of PC factor extraction. Bai (2003) shows that, if $\frac{\sqrt{G}}{H} \rightarrow 0$ when $G, H \rightarrow \infty$ and the errors are weakly correlated, then $\hat{F}$ is a consistent estimator of the space spanned by the true factors.

One of the drawbacks of the PC estimator is that it is only efficient if $\Sigma_\epsilon = cI_g$, for a constant $c > 0$. Since the residuals are likely to be correlated, it will provide a non-efficient estimation of the latent factors. However, the efficient estimates can be obtained by solving the generalized least squares (GLS) objective function

$$\min_{F,P} \text{tr}[(W - FP)'\Sigma_\epsilon^{-1}(W - FP)] \tag{4.32}$$

where Σ_ϵ is the $G \times G$ covariance matrix of the idiosyncratic noises. Choi (2012) shows that the factor estimation is given by $\sqrt{T}$ times the first R eigenvectors of the matrix $W\Sigma_\epsilon^{-1}W$. Of course, in practice, Σ_ϵ is unknown. Bai and Liao (2013) show that a thresholding method based on the residuals from the previous PC procedure will provide a consistent estimator of Σ_ϵ. In finite samples, the GLS estimator is more efficient than PC.

However, when working with standard expenditure surveys, G is commonly small. Zimmermann (2012) shows that the approach of the Engel curves fails especially in small samples, probably due to problems in the aggregation of the data. Under fixed G, the PC procedure is inconsistent unless Σ_ϵ is proportional to an identity matrix. Moreover, when G is fixed, $\hat{\varepsilon}_h = W_h - \hat{P}\hat{F}_h$ is not a consistent estimator of ε_h since $\hat{F}_h$ is not a consistent estimator of F_h, and so, the GLS estimator ceases to be more efficient.

Therefore, an alternative procedure for estimating the factors in small samples is required. Anderson and Rubin (1956), Lawley and Maxwell (1971), Anderson (2013) and Bai and Wang (2016) show that, under fixed G, the Maximum Likelihood Estimator (MLE) of the factors is consistent. Bai and Liao (2016) consider the consistent MLE estimation of a non-diagonal Σ_ϵ as follows.

The quasi-likelihood function (under non-normality in the disturbances) is

$$L(P, \Sigma_\varepsilon, S_F) = \frac{1}{N}\log|PS_FP' + \Sigma_\varepsilon| + \frac{1}{N}\,\text{tr}\left(S_W\left(PS_FP' + \Sigma_\varepsilon\right)^{-1}\right) \tag{4.33}$$

being $S_f = \sum_{h=1}^{H}\left(F_h - \bar{F}\right)\left(F_h - \bar{F}\right)'$ and $S_y = \sum_{h=1}^{H}\left(W_h - \bar{W}\right)\left(W_h - \bar{W}\right)'$ the sample variance of the latent factors and of the observed data, respectively, and $\bar{F} = \frac{1}{H}\sum_{h=1}^{H} F_h$ and $\bar{W} = \frac{1}{H}\sum_{h=1}^{H} W_h$ their corresponding sample means.

Numerically minimizing the loss function with respect to Σ_ε is difficult since it implies a concave and convex optimization. In order to do so, Bai and Liao (2016) propose the "Majorize-minimize EM algorithm" which firstly approximates the concave component through a maximization of Σ_ε, and thereafter optimizes the objective function by a convex function. Defining $(\hat{P}_i, \hat{\Sigma}_{\varepsilon,i})$ as the loadings and the covariance matrix of the errors at iteration i respectively, the algorithm can be computed as follows:

- Step 1: Set $i = 0$. Initialize $\hat{P}_0$ and $\hat{\Sigma}_{\varepsilon,0}$. We use the Principal Components estimator as the initial value; see Bai and Li (2012).
- Step 2: At iteration $i+1$, $\hat{\Sigma}_{y,i} = \hat{P}_i\hat{P}_i' + \Sigma_{\varepsilon,0}$, $\hat{P}_{i+1} = AM^{-1}$, where $M = \hat{P}_i'\hat{\Sigma}_{y,i}^{-1}S_y\hat{\Sigma}_{y,i}^{-1}\hat{P}_i + I_r - \hat{P}_i'\hat{\Sigma}_{y,i}^{-1}\hat{P}_i$ and $A = S_y\hat{\Sigma}_{y,i}^{-1}\hat{P}_i$
- Step 3: Also at iteration $i+1$, $\hat{\Sigma}_{\varepsilon,i+1} = \hat{\Sigma}_{\varepsilon,i} - k\left(\hat{\Sigma}_{y,i}^{-1}S_y\hat{\Sigma}_{y,i}^{-1}\right)$ where $k > 0$[8]

Once $\hat{P}$ and $\hat{\Sigma}_\varepsilon$ have been obtained, the factors can be estimated as:
(Projection formula)

$$\tilde{F}_h = \left(\hat{S}_F + \hat{P}'\hat{\Sigma}_\varepsilon\hat{P}\right)^{-1}\hat{P}'\hat{\Sigma}_\varepsilon^{-1}\left(W_h - \bar{W}\right) \tag{4.34}$$

(GLS)

$$\hat{F}_h = \left(\hat{P}'\hat{\Sigma}_\varepsilon\hat{P}\right)^{-1}\hat{P}'\hat{\Sigma}_\varepsilon^{-1}\left(W_h - \bar{W}\right) \tag{4.35}$$

being $\hat{S}_F$ the MLE of S_F. When G is large, there are not big differences between (4.34) and (4.35). However, when N is fixed, the differences can be remarkable; see Bai and Li (2012).

As a conclusion, in large sample settings and when dealing with homoscedastic errors, it is preferable to extract the factors using PC since it is computationally easier. If the errors are heteroscedastic and the data set is large, it would be advisable to compute

[8]In our empirical studies $k = .1$; see, Bai and Liao (2016) who also fix k as 0.1.

the GLS estimator because it is efficient. However, under fixed G, which actually is the most common situation in practice, the PC estimator will be inconsistent and the GLS will not be efficient any more. Thus, when G is not large, it is advisable to use the MLE estimator as it remains consistent under fixed G[9].

However, it is well known that $\hat{F}$ is only a consistent estimate of the space spanned by the true factors. Therefore, for a unique identification of the factors, it is necessary to impose some identification conditions to avoid the rotational indeterminacy. There are many ways to impose restrictions. In this further line of research we consider three commonly applied identification conditions in Factor Models:

- IC1: $\frac{1}{T}F'F = I_r$, $P'P$ is diagonal with distinct entries.
- IC2: $\frac{1}{T}F'F = I_r$, the upper $r \times r$ block of P is lower triangular with nonzero diagonal entries.
- IC3: The upper $r \times r$ block of P is given by I_r.

Bai and Li (2012) explain how to obtain estimators that satisfy any of these restrictions. For a full discussion on identification issues in the context of PC factor extraction, readers are referred to Bai and Ng (2013).

However, even restricting the model with these identification conditions, there is not a straightforward economic interpretation of the factors. Therefore, in order to identify the factors and to make them easier to understand, we use independent component analysis (ICA); see, Comon (1994) and Barigozzi and Moneta (2016) among others.

ICA minimizes all statistical dependencies between the extracted factors so that the rotated factors are unique up to a permutation, a sign and a scaling factor. This identification procedure is particularly convenient because it is data-driven and

[9]A possible alternative would be to compute the two-step estimation procedure proposed by Breitung and Tenhofen (2011). This estimator allows for heteroscedastic and serially correlated errors.

does not require the use of microeconomic models of consumption behaviour. The most popular algorithm to compute ICA is the Joint Approximate Diagonalization of Eigen-matrices (JADE) by Cardoso and Souloumiac (1993). Once the factors are extracted following the procedure explained in the previous section, JADE determines the final orthogonal transformation maximizing the non-Gaussianity of the extracted factors.

In order to apply ICA, we need to impose two assumptions that are easy to comply with: i) the factors are mutually independent, and ii) their marginal densities are non-Gaussian. This means that the factors and their corresponding Engel curves, will reflect the basic needs that drive consumption behaviour and that express an independent consumption pattern of different nature.

Once the latent factors are extracted and identified, the Latent Outlay Equivalence Ratio (LOER) can be estimated in an analogous way as was proposed by Deaton (1989). The first step is to obtain the latent Engel curves as:

$$L_g = F_R/pq = \alpha_g + \beta_g ln(x/N) + \eta_g lnN + \sum_{1}^{J-1} \gamma_{gj}(n_j/N) + \delta_g z + u_g \tag{4.36}$$

Therefore, the Latent Outlay Equivalence Ratio can be computed as follows: :

$$\Pi_{gj} = \frac{\eta_g - \beta_g + \gamma_{gj} - \sum_1^{J-1}(n_j/N)}{\beta_g + L_g} \tag{4.37}$$

Estimates of the ratios are obtained by replacing the parameters with their Ordinary Least Squares' estimates and replacing L_g with the values of their sample means. The LOERs are the analogous to the OERs but in the context of the basic Engel curves. In this way, the LOERs can be interpreted as the increase in the total expenses necessary to generate the same additional expense in the fundamental purpose f that is generated by increasing the household with an additional member of the category j. Therefore, following the same reasoning that we explained in the previous section for OERs, if f is a purpose that has nothing to do with children, the ratio will be negative. In this way, once the factor that corresponds to adult preferences has been identified, the ratios for each age and gender group can be analysed. For that purpose, it is necessary to test the null hypothesis that the ratios for different population categories are equal for the same adult good. It should be noted that it is necessary to incorporate both sources of uncertainty: the error derived from the parameter estimation, and the one made in estimating the factors; see, Barigozzi and Moneta (2016). In this open line of research we explore how to do this inferential process and try to obtain and functional form of the variance of the Latent Outlay Equivalent Ratio.

By using this new methodology, we will be automatically separating the goods according to their purpose, and we will test if there is gender discrimination in those purposes associated with superfluous consumption, i.e. those that are not associated with basic needs.

In this way, we solve the fundamental problems currently observed in the Engel curves methodology. That is, pre-lists on possible adult goods are no longer made, and it is not necessary to make many combinations of goods until the OERs coincide. The proposed methodology is completely data driven and groups together the goods that are associated with a similar purpose. Additionally, there used to be a naive misconception that a good was either adult or not. However, as we have seen previously, it might be the case that, from a certain level of expenditure, goods that are not initially considered as adult goods -such as food- can also be superfluous.

Finally, the Engel Curve approach only considers the Income Effect, thus assuming that there is no substitution effect. This results in counter-intuitive results. When looking for the fundamental purposes that govern consumer behaviour, there is no need to assume such a strong restriction, since two goods with a fundamental purpose (substitutable) can be explained by the same latent factor that will later be used to measure gender discrimination.

Bibliography

[1] Aastveit, K.A., Bjornland, H.C. and Thorsrud, L.A. (2016). The world is not enough! Small open economies and regional dependence. *Scandinavian Journal of Economics*, 118(1), 168-195.

[2] Abbate, A., Eickmeier, S., Lemke, W., and Marcellino, M. (2016). The changing international transmission of financial shocks: evidence from a classical time-varying FAVAR. *Journal of Money, Credit and Banking*, 48(4), 573-601.

[3] Adrian T., N. Boyarchenko and D. Giannone (in press). Vulnerable growth. *American Economic Review*.

[4] Ahn, S. and Horenstein A. (2013). Eigenvalue ratio test for the number of factors. *Econometrica*, 81(3), 1203-1227.

[5] Alessi, L., Barigozzi, M. and Capasso, M. (2010). Improved penalization for determining the number of factors in approximate factor models. *Statistics and Probability Letters*, 80(1), 1806-1813.

[6] Alonso, A.M., Pea, D. and Rodrguez, J. (2008). A methodology for population projections: an application to Spain. *Statistics and Econometrics Series 12.* WP 08-45. Universidad Carlos III de Madrid.

[7] Alonso, A.M., Rodriguez, J., Sanchez, M. J. and Garca-Martos, C. (2011).

Dynamic factor analysis and bootstrap inference: application to electricity market forecasting. *Technometrics*, 53(2), 137-151.

[8] Alvarez, R., Camacho, M. and Perez-Quiros, G. (2016). Aggregate versus disaggregate information in dynamic factor models. *International Journal of Forecasting*, 32, 680-694.

[9] Ando, T. and Tsay, R.S. (2011). Quantile regression models with factor-augmented predictors and information criterion. *Econometrics Journal*, 14, 1-24.

[10] Ando, T. and Tsay, R.S. (2014). A predictive approach for selection of diffusion index models. *Econometric Reviews*, 33(1-4), 68-99.

[11] Arouba, S.B., Diebold, F.X. and Scotti, C. (2009). Real-time measurement of business conditions. *Journal of Business and Economic Statistics*, 27(4), 417-427.

[12] Artis, M.J., A. Banerjee and M. Massimiliano (2005). Factor forecasts of the UK. *Journal of Forecasting*, 24, 279-298.

[13] Azam, M. and Kingdon, G. (2013). Are girls the fairer sex in India? revisiting intra-household allocation of education expenditure. *World Development*, 42(C), 143-164.

[14] Azzalini, A. and A. Capitanio (2003). Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution. *Journal of the Royal Statistical Association, Series B (Statistical Methodology)*, 65, 367-389.

[15] Babamoradi, H., Van den Berg, F. and Rinnan, A. (2013). Bootstrap based confidence limits in principal component analysis - A case study. *Chemometrics and Intelligent Laboratory Systems*, 120, 97-105.

[16] Bai, J. (2003). Inferential theory for factor models of large dimensions. *Econometrica*, 71(1), 135-171.

[17] Bai, J. (2004). Estimating cross-section common stochastic trends in nonstationary panel data. *Journal of Econometrics*, 122(1), 137-183.

[18] Bai, J. and Li, K. (2012). Statistical analysis of factor models of high dimension. *The Annals of Statistics*, 40, 436-465.

[19] Bai, J. and Liao, Y. (2016). Efficient estimation of approximate factor model via penalized maximum likelihood. *Journal of Econometrics*, 191, 1-18.

[20] Bai, J. and Ng, S. (2004). A PANIC attack on unit roots and cointegration. *Econometrica*, 72, 1127-1177.

[21] Bai, J. and Ng, S. (2006). Confidence intervals for diffusion index forecasts and inference for factor-augmented regressions. *Econometrica*, 74(4), 1133-1150.

[22] Bai, J. and Ng, S. (2008). Large dimensional factor analysis. *Foundations and Trend in Econometrics*, 3, 89-163.

[23] Bai, J. and Ng, S. (2010). Instrumental variable estimation in a data rich enviroment. *Econometric Theory*, 26, 1577-1606.

[24] Bai, J. and Ng, S. (2013). Principal components estimation and identification of static factors. *Journal of Econometrics*, 176, 18-29.

[25] Bai, J. and Ng, S. (2008a). Large dimensional factor analysis. *Foundations and Trends in Econometrics*, 3, 89-163.

[26] Bai, J. and Ng, S. (2008b). Extreme estimation when the predictors are estimated from large panels. *Annals of Economics and Finance*, 9, 202-222.

[27] Bai, J. and Wang, P. (2016). Econometric analysis of large factor models. *Annual Review of Economics*, 8, 53-80.

[28] Bai, J., Kunpeng, L. and Lina, L. (2016). Estimation and inference of FAVAR models. *Journal of Business & Economic Statistics*, 34(4), 620-641.

[29] Banerjee, A., Marcellino, M. and Masten, I. (2008). Forecasting macroeconomic variables using diffusion indexes in short samples with structural chang, in David E. Rapach and Mark E. Wohar, eds.. *Forecasting in the Presence of Structural Breaks and Model Uncertainty*, Vol. 3 of Frontiers of Economics and Globalization (Emerald Group Publishing Limited)

[30] Banerjee, A., Marcellino, M. and Masten, I. (2014). Forecasting with factor-augmented error correction models. *International Journal of Forecasting*, 30(3), 589-614.

[31] Barcellos, H. S., Carvalho, L. S. and Lleras-Muney, A. (2014). Child gender and parental investments in India: Are boys and girls treated differently?. *American Economic Journal: Applied Economics*, 6(1), 157-189.

[32] Bargain, O. and Donni, O. (2012). Expenditure on children: A Rothbarth-type method consistent with scale economies and parents' bargaining. *European Economic Review*, 56, 792-813.

[33] Barigozzi, M. and Luciani, M. (2017). Common factors, trends, and cycles in large datasets. *Manuscript*.

[34] Barigozzi, M. and Moneta, A. (2016). Identifying the independent sources of consumption variation *Journal of Applied Econometrics*, 31, 420-449.

[35] Barigozzi, M., Lippi, M. and Luciani, M. (2016). Non-stationary dynamic factor models for large datasets, Finance and Economics Discussion Series; Division of Research and Statistics and Monetary Affairs, Federal Reserve Bank: Washington, DC.

[36] Bell, W.R. (1997). Comparing and assesing time series methods for forecasting age-specific fertility and mortality rates. *Journal of Official Statistics*, 13(3), 279-303.

[37] Beran, R., and Srivastra, M.S. (1985). Bootstrap tests and confidence regions for functions of a covariance matrix. *Annals of Statistics*, 13, 95-115.

[38] Bernanke, B.S., Boivin, J. and Eliasz, P. (2005). Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. *The Quarterly Journal of Economics*, 120, 387-422.

[39] Bertsimas, D., Brown, D.B. and Caramanis, C. (2013). Theory and applications of robust optimization. *SIAM Review*, 53(3), 464-501.

[40] Bhalotra, S. and Attfield, C. (1998). Intrahousehold resource allocation in rural Pakistan: A semiparametric analysis. *Journal of Applied Econometrics*, 13(5), 463-480.

[41] Bjornland, H.C., Ravazzolo, F. and Thorsrud, L.A. (2017). Forecasting GDP with global components: This time is different. *International Journal of Forecasting*, 33(1), 153-173.

[42] Blundell, R., Duncan, A. and Pendakur, K. (1998). Semiparametric estimation and consumer demand. *Journal of Applied Econometrics*, 13, 435-461.

[43] Boivin, J. and Ng, S. (2006). Are more data always better for factor analysis?. *Journal of Econometrics*, 132, 169-194.

[44] Booth, H. (2006). Demographic forecasting: 1980 to 2005 in review. *International Journal of Forecasting*, 22(3), 547-581.

[45] Booth, H. and Tickle, L. (2008). Mortality modelling and forecasting: a review of methods. *Annals of Actuarial Sciences*, 3(I/II), 3-43.

[46] Booth, H., Hyndman, R. and Tickle, L. (2006). Lee-Carter mortality forecasting: A

multi-country comparison of variants and extensions. *Demographic Research*, 15(9), 289-310.

[47] Booth, H., Maindonald, J. and Smith, L. (2002). Applying Lee-Carter under conditions of variable mortality decline. *Population Studies*, 56(3), 325-336.

[48] Borger, M. and Schupp, J. (2018). Modeling trend processes in parametric mortality models. *Insurance: Mathematics and Economics*, 78, 369-380.

[49] Bräuning, F. and Koopman, S.J. (2014). Forecasting macroeconomic variables using collapsed dynamic factor analysis. *International Journal of Forecasting*, 30, 573-584.

[50] Breitung, J. and Choi, I. (2013). Factor Models. In: *Handbook of Research Methods and Applications in Empirical Macroeconomics*. Cheltenham:Edwar Elgar, pp. 249-265.

[51] Breitung, J. and Eickmeier, S. (2006). Dynamic Factor Models. In: *Modern Econometric Analysis*. Heidelberg: Springer, pp. 25-40.

[52] Breitung, J. and Eickmeier, S. (2011). Testing for structural breaks in dynamic factor models. *Journal of Econometrics*, 163, 71-84.

[53] Breitung, J. and Eickmeier, S. (2016). Analyzing international business and financial cycles using multi-level factor models: A comparison of alternative approaches. In: *Advances in Econometrics*. Bingley: Emerald Group Publishing Limited, pp. 177-214.

[54] Breitung, J. and Tenhofen, J. (2011). GLS estimation of dynamic factor models. *Journal of the American Statistical Association*, 106, 1150-1166.

[55] Brillinger, D.R. (1981). Time Series Data Analysis and Theory. San Francisco: Holden-Day.

[56] Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epsein, D. and Khalaf-Allah,

M. (2011). Mortality density forecasts: An analysis of six stochastic mortality models. *Insurance: Mathematics and Economics*, 48(3), 355-367.

[57] Cakmakli, C. and van Dijk, D. (2016). Getting the most out of macroeconomic information for predicting excess stock returns. *International Journal of Forecasting*, 32, 650-668.

[58] Camacho, M., Perez-Quiros G. and Poncela, P. (2015). Extracting nonlinear signals from several economic indicators. *Journal of Applied Econometrics*, 30, 1073-1089.

[59] Cardoso, J. and Souloumiac, A. (1993). Blind beamforming for non-Gaussian signals. *IEE Proceedings Part F: Radar and Singal Processing*, 140, 362-370.

[60] Carter, L.R. (1996). Forecasting US mortality: A comparison of Box-Jenkins ARIMA and structural time series models. *Sociological Quarterly*, 37, 127-144.

[61] Case, A. and Deaton, A. (2003). Consumption, Health, Gender and Poverty. Policy Research Working Paper; No. 3020. World Bank, Washington, DC.

[62] Cavalu, H. (1999). A review of the Asian crisis causes, consequences and policy responses. *The Australian Economic Review*, 32(3), 304-313.

[63] Chai, A. and Moneta, A. (2010). Retrospectives: Engel curves. *The Journal of Economics Perspectives*, 24(1), 225-240.

[64] Chassein, A. and Goerigk, M. (2017). Minmax regret combinatorial optimization problems with ellipsoidal uncertainty sets. *European Journal of Operational Research*, 258(1), 58-69.

[65] Cheng, X. and Hansen, B.E. (2015). Forecasting with factor-augmented regression: A frequentist model averaging approach. *Journal of Econometrics*, 186, 280-293.

[66] Chiou, J.M. and Muller, H.G. (2009). Modeling hazard rates as functional data for

the analysis of cohort lifetables and mortality forecasting. *Journal of the American Statistical Association*, 104(486), 572-585.

[67] Choi, J. and Lee, S-H. (2006). Does prenatal care increase access to child immunization? Gender bias among children in India. *Social Science and Medicine*, 63(1), 107-117.

[68] Ciccarelli, M. and Mojon, B. (2010). Global inflation. *Review of Economic Statistics*, 92(3), 524-535.

[69] Comon, P. (1994). Independent component analysis: a new concept?. *Signal Processing*, 36, 287-314.

[70] Connor, G. and Korajczyk, R.A. (1986). Performance measurement with arbitrage pricing theory: A new framework for analysis. *Journal of Financial Economics*, 15, 373-394.

[71] Corona, F., Poncela, P.and Ruiz, E. (2017). Determining the number of common factors after stationary univariate transformations. *Empirical Economics*, 53, 351-373.

[72] Corona, F., Poncela, P.and Ruiz, E. (2017b). Estimating non-stationary common factors: implications for risk-sharing, WP17-05 (Statistics and Econometrics), Universidad Carlos III de Madrid.

[73] Das Gupta, M. (1987). Selective discrimination against female children in rural Punjab, India. *Population and Development Review*, 13(1), 77-100.

[74] Das Gupta, M., Zhenghua, J., Bohua, L., Zhenming, X., Chung, W. and Hwa-ok, B. (2003). Why is Son Preference so Persistent in East and South Asia? A Cross-Country Study of China, India and the Republic of Korea *The Journal of Development Studies*,40(2), 153-187.

[75] Deaton, A. (1989). Looking for boy-girl discrimination in household expenditure data. *World Bank Economic Review*, 3, 1-15.

[76] Deaton, A. (1997). The analysis of Household Surveys: A Microeconometric Approach to Development Policy. Baltimore: John Hopkins.

[77] Deaton, A., Ruiz-Castillo, J. and Duncan, T. (1989). The influence of Household Composition on Household Expenditure Patterns: Theory and Spanish Evidence. *The Journal of Political Economy*, 97(1), 179-200.

[78] Djogbenou, A., Gonçalves, S. and Perron, B. (2015). Bootstrap inference in regressions with estimated factors and serial correlation. *Journal of Time Series Analysis*, 36, 481-502.

[79] Duflo, E. (2012). Women Empowerment and Economic Development. *Journal of Economic Literature*, 50(4), 1051-1079.

[80] Dunbar, G., Lewbel, A. and Pendakur, K. (2013). Children's resources in collective households: Identification, estimation and an application to child poverty in Malawi . *American Economic Review*, 103(1), 438-471.

[81] El Karoui, N. and Purdom, E. (2015). Can we trust the bootstrap in high-dimension?. *Manuscript*.

[82] Engel, E. (1987). Die Production- und Consumtionsverhlnitsse des Knigreichs Sachsen. *Bulletin de l'Institut International de la Statistique*, 9.

[83] Favero, C.A., Marcellino, M. and Neglia, F. (2005). Principal components at work: the empirical analysis of monetary policy with large datasets. *Journal of Applied Econometrics*, 20(5), 603-620.

[84] Feldstein, M. (1971). The error of forecast in econometric models when the forecast-period exogenous variable is stochastic. *Econometrica*, 39, 55-60.

[85] Fisher, A., Caffo, B., Schwartz, B. and Zipunnikov, V. (2015). Fast, exact bootstrap principal component analysis for p > 1 million. *Journal of the American Statistical Association*, 111(514), 846-860.

[86] Foester, A., Sarte, P.D.G. and Watson M.H. (2011). Sectoral vesus aggregate shocks: a structural factor analysis of industrial production. *Journal of Political Economy*, 119, 1-38.

[87] Forni, M., Gambetti, L. and Sala, L. (2014). No news in business cycles. *Economic Journal*, 124(581), 1168-1191.

[88] Forni, M., Hallin, M., Lippi, M. and Reichlin, L. (2000). The generalized dynamic factor model: identification and estimation. *Review of Economics and Statistics*, 82, 540-554.

[89] Forni, M., Hallin, M., Lippi, M. and Reichlin, L. (2005). The generalized dynamic factor model. *Journal of the American Statistical Association*, 100(471), 830-840.

[90] Fossati, S. (2016). Dating US business cycle with macroeconomic factors. *Studies in Nonlinear Dynamics of Econometrics*, 20(5), 529-549.

[91] French, D. and O'Hare, C. (2013). A dynamic factor approach to mortality modeling. *Journal of Forecasting*, 32(7), 587-599.

[92] Fuwa, N., Ito, S., Kubo, K., Kurosaki, T. and Sawada, Y. (2006). Gender discrimination, intrahousehold resource allocation, and importance of spouses' fathers: evidence on household expenditure from rural india. *The Developing Economies*, 44(4), 398-439.

[93] Geweke, J. (1977). The dynamic factor analysis of economic time series. In: *Latent Variables in Socio-Economic Models.* North-Holland:Amsterdam, pp. 365-382.

[94] Gibson, J. and Rozelle, S. (2004). Is it better to be a boy? A disaggregated Outlay Equivalent Analysis of gender bias in Papua New Guinea. *The Journal of Development Studies*, 40(4), 115-136.

[95] Giglio, S., Kelly, B. and Pruitt, S. (2016). Systemic risk and the macroeconomy: An empirical evaluation. *Journal of Financial Economics*, 119(3), 457-471.

[96] Gneiting, T. and Raftery, A.E. (2007). Strictly proper scoring rules, prediction and estimation. *Journal of the American Statistical Association*, 102(477), 359-378.

[97] Gonçalves, S. and Perron, B. (2014). Bootstrapping factor-augmented regression models. *Journal of Econometrics*, 182, 156-173.

[98] Gonçalves, S. and White, H. (2005). Bootstrap standard error estimates for linear regression. *Journal of the American Statistical Association*, 100(471), 970-979.

[99] Gonçalves, S., McCraken, M.W. and Perron, B. (2017). Tests for equal accuracy for nested models with estimated factors. *Journal of Econometrics*, 198(2), 231-252.

[100] Gonçalves, S., Perron, B. and Djogbenou, A. (2017). Bootstrap prediction intervals for factor models. *Journal of Business & Economic Statistics*, 35(1), 53-69.

[101] Gong, X., Van Soest, A. and Zhang, P. (2005). The effects of the gender of children on expenditure patterns in rural China: a semiparametric analysis. *The Journal of Development Studies*, 37(1), 73-92.

[102] González-Rivera, G. (2003). Value in Stress: A Coherent Approach to Stress Testing. *The Journal of Fixed Income*, 13(2), 7-18.

[103] Gospodinov, N. and Ng, S. (2013). Commodity prices, convenience yields and inflation. *The Review of Economics and Statistics*, 95(1), 206-219.

[104] Haberman, S. and Renshaw, A. (2011). A comparative study of parametric mortality projection models. *Insurance: Mathematics and Economics*, 48, 35-55.

[105] Haddad, L. and Reardon, T. (1993). Gender bias in the allocation of resources within household in Burkina Faso: A disaggregated outlay equivalent analysis. *The Journal of Development Studies*, 29 (2), 260-276.

[106] Haldrup, N. (1998). An econometric analysis of I(2) variables. *Journal of Economic Surveys*, 12(5), 595-650.

[107] Hamilton, J.D. (1986). A standard error for the estimated state vector of a state-space model. *Journal of Econometrics*, 33(3), 387-397.

[108] Hansen, L.P. (1982). Large sample properties of Generalized Method of Moments Estimators. *Econometrica*, 50(4), 1029-1054.

[109] Harvey, A.C. (1989). Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, Cambridge.

[110] Hazarika, G. (2000). Gender differences in children's nutrition and access to health care in Pakistan. *Journal of Applied Econometrics*, 20, 509-527.

[111] Henzel, S.R. and Rengel, M. (2017). Dimensions of macroeconomic uncertainty: a common factor analysis. *Economic Inquiry*, 55(2), 843-877.

[112] Himaz, R. (2010). Intrahousehold allocation of education expenditure: The case of Sri Lanka. *Economic Development and Cultural Change*, 58 (2), 231-258.

[113] Hofman, B. (2009). Do monetary indicators lead euro area inflation?. *Journal of International Money and Finance*, 28, 1165-1181.

[114] Holly, A. and Sargan, J. (1982). Testing for exogeneity in a limited information framework. *American Economic Review*, 70, 268-272.

[115] Horrell, S. and Oxley, D. (1999). Crust or crumb?: Intrahousehold resource allocation and male breadwinning in late Victorian Britain *Economic History Review*, 52 (3), 494-522.

[116] Hunt, A. and Blake, D. (2014). A general procedure for constructing mortality models. *North American Actuarial Journal*, 18(1), 116-138.

[117] Hyndman, R.J. and Booth, H. (2008). Stochastic population forecasts using functional data models for mortality, fertility and migration. *International Journal of Forecasting*, 24(3), 323-342.

[118] Hyndman, R.J. and Ullah, M.S. (2007). Robust forecasting of mortality and fertility rates: A functional data approach. *Computational Statistics & Data Analysis*, 51(10), 4942-4956.

[119] Hyndman, R.J., Booth, H. and Yasmeen, F. (2013). Coherent mortality forecasting: the product-ratio method with functional time series models, *Demography*, 50(1), 261-283.

[120] Imbs, J. (2010). The first global recession in decades. *IMF Economic Review*, 58(2), 327-354.

[121] Jackson, L.E., Kose, M.A., Otrok, C. and Owyan, M.T. (2016). Specification and estimation of bayesian dynamic factor models: A Monte Carlo analysis with an application to global house price comovement. In: *Advances in Econometrics*. Bingley: Emerald Group Publishing Limited, 361-400.

[122] Jiang, Y., Guo, Y. and Zhang, Y. (2017). Forecasting China's GDP growth using dynamic factors and mixed-frequency data. *Economic Modelling*, 66, 132-138.

[123] Jungbacker, B. and Koopman, S. J. (2015). Likelihood-based dynamic factor analysis for measurement and forecasting. *Econometric Journal*, 18, 1-21.

[124] Jurado, K., Ludvigson, S.C. and Ng, S. (2015). Measuring uncertainty. *American Economic Review*, 105(3), 1177-1216.

[125] Kapetanios, G. and Marcellino, M. (2010). Factor-GMM estimation with large sets of possibly weak instruments. *Computational Statistics and Data Analysis*, 54(11), 2655-2675.

[126] Kebede, B. (2008). Intra-household allocations in rural Ethiopia: A demand system approach. *Review of Income and Wealth*, 54 (1), 1-26.

[127] Keilman, N. (2001). Demography: Uncertainty population forecasts. *Nature*, 412, 490-492.

[128] Kingdon, G. (2005). Where has all the bias gone? Detecting gender bias in the intrahousehold allocation of educational expenditure. *Economic Development and Cultural Change*, 53(2), 409-451.

[129] Klasen, S. (1996). Nutrition, Health and Mortality in Sub-Saharan Africa: Is there a Gender Bias?. *The Journal of Development Studies*, 32(6), 913-932.

[130] Kneip, A. (1994). Nonparametric estimation of common regressors for similar curve data. *Annals of Statistics*, 22, 1386-1427.

[131] Koenker, R. and Bassett, G. (1978). Regression quantiles. *Econometrica*, 46, 33-50.

[132] Koenker, R. and d'Orey, V. (1987). Algorithm AS229: Computing regression quantiles. *Journal of the Royal Statistical Association. Series C (Applied Statistics)*, 36(3), 383-393.

[133] Koenker, R. and Machado, J.A.F. (1999). Goodness of fit and related inference processes for regression quantile. *Journal of the American Statistical Association*, 94(448), 1296-1310.

[134] Koissi, M., Shapiro, A.F. and Hognas, G. (2006). Evaluating and extending the Lee-Carter model for mortality forecasting: Bootstrap confidence interval. *Insurance: Mathematics and Economics*, 38(1), 1-20.

[135] Kose, M.A., Otrok, C. and Prasad, E. (2012). Global business cycles: convergence or decoupling?. *International Economic Review*, 53(2), 511-538.

[136] Kose, M.A., Otrok, C. and Whiteman, C.H. (2003). International business cycles: world, region and country-specific factors. *American Economic Review*, 93(4), 1216-1239.

[137] Kristensen, J.T. (2014). Factor-based forecasting in the presence of outliers: Are factors better selected and estimated by the median than by the mean?. *Studies in Nonlinear Dynamics and Econometrics*, 18(3), 309-338.

[138] Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin Y. (1992). Testing the null hypothesis of a unit root. *Journal of Econometrics*, 54, 159-178.

[139] Lee, R.D. and Carter, L.R. (1992). Modeling and forecasting U.S. mortality. *Journal of the American Statistical Association*, 87(419), 659-671.

[140] Lee, R.D. and Miller, T. (2001). Evaluating the performance of the Lee-Carter method for forecasting. *Demography*, 38(4), 537-549.

[141] Lee, Y.D. (2008). Do families spend more on boys than girls? Empirical evidence from rural China. *China Economic Review*, 19, 80-100.

[142] Leser, C. (1963). Form of Engel curves. *Econometrica*, 31, 694-703.

[143] Lewbel, A. (1991). The rank of demand systems: Theory and non-parametric estimation. *Econometrica*, 59, 711-730.

[144] Li, J.S.H. and Chan, W.S. (2011). Time-simultaneous prediction bands: A new look at the uncertainty involved in forecasting mortality. *Insurance Mathematics and Economics*, 49, 81-88.

[145] Li, J.S.H., Chan, W.S. and Zhou, R. (2015). Semicoherent multipopulation

mortality modeling: The impact of longevity risk securitization. *Journal of Risk and Insurance*, 84(3), 1025-1065.

[146] Li, N. and Lee, R. (2005). Coherent mortality forecasts for a group of populations: an extension of the Lee-Carter method. *Demography*, 42(3), 575-594.

[147] Lopez, H. and Wodon, Q. (2005). The economic impact of armed conflict in Rwanda. *Journal of African Economies*, 14(4), 586-602.

[148] Ludvigson, S.C. and Ng, S. (2007). The empirical risk-return tradeoff: A factor analysis approach. *The Journal of Financial Economics*, 83, 171-222.

[149] Ludvigson, S.C. and Ng, S. (2009). Macro Factors in Bond Risk Premia. *The Review of Financial Studies*, 22(12), 5027-5067.

[150] Ludvigson, S.C. and Ng, S. (2010). A Factor Analysis of Bond Risk Premia. In: *Handbook of Empirical Economics and Finance.* Boca Raton: Chapman and Hall, 313 - 372.

[151] Marcellino, M., Stock, J.H. and Watson, M.W. (2003). Macroeconomic forecasting in the euro area: country specific versus euro wide information. *European Economic Review*, 47, 1-18.

[152] McKenzie, D. J. (2006). The consumer response to the Mexican Peso crisis in 1994. *Economic Development and Cultural Change*, 55(1), 139-172.

[153] Mitchell, D., Brockett, P., Mendoza-Arriaga, R. and Muthuraman, K. (2013). Modeling and forecasting mortality rates. *Insurance: Mathematics and Economics*, 52(2), 275-285.

[154] Moneta, F. and Ruffler, R. (2009). Business cycle synchronization in east Asia. *Journal of Asian Economics*, 20, 1-12.

[155] Neely, C.J., Rapach, D.E., Tu, J. and Zhan, G. (2014). Forecasting the equity risk premium: the role of technical indicators. *Management Science*, 60(7), 1772-1791.

[156] Ohno, S. and Ando, T. (2018). Stock return predictability: A factor-augmented predictive regression system with shrinkage method. *Econometric Reviews*, 37(1), 29-60.

[157] Onatski, A. (2010). Determining the number of factors from empirical distribution of eigenvalues. *Review of Economics and Statistics*, 92(4), 1004-1016.

[158] Ouysse, R. (2006). Approximate factor models: finite sample distribution.*Journal of Statistical Computation and Simulation*, 76(4), 279-303.

[159] Ozturk, E.O. and Sheng, X.S. (in press). Measuring global and country-specific uncertainty. *Journal of International Money and Finance*.

[160] Pascual, L., Romo, J. and Ruiz, E. (2005). Bootstrap prediction intervals for power-transformed time series. *International Journal of Forecasting*, 21, 219-235.

[161] Peña, D. and Poncela, P. (2016). Nonstationary dynamic factor analysis. *Journal of Statistical Planning and Inference*, 136(4), 1237-1257.

[162] Pfeffermann, D. and Tiller, R. (2005). Bootstrap approximation to prediction MSE for state-space models with estimated parameters. *Journal of Time Series Analysis*, 26, 893-916.

[163] Pitacco, E., Denuit, M., Haberman, S. and Olivieri, A. (2009). Modelling Longevity Dynamics for Pensions and Annuity Business, Oxford University Press, Oxford.

[164] Politis, D. N. (2003). The impact of bootstrap methods on time series analysis. *Statistical Science*, 18(2), 219-230.

[165] Politis, D. N. and Romano, J. P. (1994). Large sample confidence regions based on subsamples under minimal assumptions. *Annals of Statistics*, 22, 2031-2050.

[166] Politis, D. N., Romano, J. P. and Wolf, M. (2001). On the asymptotic theory of subsampling. *Statistica Sinica*, 11, 1105-1124.

[167] Poncela, P. and Ruiz, E. (2016). Small versus big data factor extraction. In: *Advances in Econometrics*. Bingley:Emerald Group Publishing Limited, pp. 401-434.

[168] Proietti, T., Marczak, M. and Mazzi, G. (2016). Euromind-D: A density estimate of monthly gross domestic product for the euro area. *Journal of Applied Econometrics*, 32, 683-703.

[169] Radelet, S., Sachs, J.D., Cooper, R.N. and Bosworth, B.P. (1998). The East Asian financial crisis: diagnosis, remedies, prospects. *Brooking Papers on Economic Activity*, 1998(1), 1-90.

[170] Rodriguez, A. and Ruiz, E. (2012). Bootstrap prediction mean squared errors of unobserved states based on the Kalman filter with estimated parameters. *Computational Statistics and Data Analysis*, 56, 6274.

[171] Rodriguez, L. (2016). Intrahousehold inequalities in child rights and well-being. A barrier to progress?. *World Development*, 83, 111-134.

[172] Rose, E. (2000). Gender bias, credit constraints and time allocation in rural India. *The Economic Journal*, 110(465), 738-758.

[173] Rossi, B. and Sekhposyan, T. (2015). Macroeconomic uncertainty indices based on nowcast forecast errors. *American Economic Review*, 105(5), 650-655.

[174] Rothbarth, E. (1943). Note on a Method of Determining Equivalent Income for Families of Different Composition. *Wartime Pattern of Saving and Spending* in C. Madge (ed.), Cambridge University Press, Cambridge.

[175] Sargent, T.J. and Sims, C.A. (1977). Business cycle modeling without pretending to have too much a-priori economic theory, New Methods in Business Cycle Research. Federal Reserve Bank of Minneapolis: Minneapolis.

[176] Sen, A. (1990). More than 100 Million Women are missing *New York Review of Books*, 37(20).

[177] Sen, A. and Sengupta, S. (1983). Malnutrition of rural children and the sex bias. *Economic and Political Weekly*, 18, 855-864.

[178] Shang, H.L. and Hyndman, R. (2017). Grouped functional time series forecasting: an application to age-specific mortality rates. *Journal of Computational and Graphical Statistics*, 26(2), 330-343.

[179] Shang, H.L., Booth, H. and Hyndman, R.J. (2011). Point and interval forecasts of mortality rates and life expectancy: a comparison of ten principal components methods. *Demographic Research*, 25, 173-214.

[180] Shintani, M. and Guo, Z. (2015). Improving the finite sample performance of autoregression estimators in dynamic factor models: A bootstrap approach. *Econometric Reviews*, forthcoming.

[181] Song, L. (2000). Gender effects on household resource allocation in rural China. *Chinese Economy*, 33, 69-95.

[182] Stauffer, D. O., Garton, E.O. and Kirk Steinhorst R. (1985). A comparison of Principal Components from real and random Data. *Ecology*, 66(6), 1693-1698.

[183] Stock, J.H. and Watson, M.W. (1999). Forecasting inflation. *Journal of Monetary Economics*, 44, 293-335.

[184] Stock, J.H. and Watson, M.W. (2002). Forecasting using Principal Components

from a large number of predictors. *Journal of the American Statistical Association*, 97(460), 1167-79.

[185] Stock, J.H. and Watson, M.W. (2006). Forecasting with many predictors. In: *Handbook of Economic Forecasting*. Elsevier, pp. 515-554.

[186] Stock, J.H. and Watson, M.W. (2011). Dynamic factor models. In:*The Oxford Handbook of Economic Forecasting*. Oxford: Oxford University Press.

[187] Stock, J.H. and Watson, M.W. (2002a). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, 97(460), 1167-1179.

[188] Stock, J.H. and Watson, M.W. (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*, 20, 147-162.

[189] Subramanian, S. (1994). Gender Discrimination in Intra-household Allocation in India. Cornell University: Department of Economics.

[190] Sweeting, P. (2011). A trend-change extension of the Carins-Blake-Dowd model. *Annals of Acturial Science*, 5(2), 143-162.

[191] Timmerman, M.E., Kiers, H.A.L. and Smilde, A. K. (2007). Estimating confidence intervals for principal component loadings: a comparison between the bootstrap and asymptotic results. *British Journal of Mathematical and Statistical Psychology*, 60, 295-314.

[192] Vaillant, J., Grimm, M., Lay, J. and Rouband, F. (2014). Informal sector dynamics in times of fragile growth: the case of Madagascar. *European Journal of Development Research*, 26(4).

[193] Van Aelst, S. and Willems, G. (2013). Fast and robust bootstrap for multivariate inference: the R package FRB. *Journal of Statistical Software*, 53(3), 1-32.

[194] Wang, M-C. (2009). Comparing the DSGE model with the factor model: An out of sample forecasting experiment. *Journal of Forecasting*, 28, 167-182.

[195] Working, H. (1943). Statistical laws of family expenditure. *Journal of the American Statistical Association*, 38, 43-56.

[196] World Mortality Report, Highlights (2015). United Nations Department of Economic and Social Affairs/Population Division.

[197] Yamamoto, Y. (2016). Bootstrap inference for impulse response functions in factor-augmented vector autoregressions. HIAS Discussion Paper E-26.

[198] Yang, S.S., Yue, J.C. and Huang, H. (2010). Modeling longevity risks using a principal component approach: a comparison with existing stochastic mortality models. *Insurance: Mathematics and Economics*, 46(1), 254-270.

[199] Zimmermann, L. (2012). Reconsidering gender bias in intrahousehold allocation in India. *Journal of Development Studies*, 48(1), 151-163.

Appendix A

Appendix to Chapter 2

A.1. Derivation of the MSE of the estimated factors

In this section, we derive the procedure for obtaining the MSE due to parameter and disturbance uncertainty ant the covariance between them.

A.1.1. Derivation of the MSE attributed to parameter uncertainty

$$
\begin{aligned}
\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(\widehat{f}_t - f_t\right)^{'}\right] = &\\
&= \underset{t}{E}[(\frac{1}{N}\tilde{P}^{'}Y_{.t} - \frac{1}{N}P^{'}Y_{.t})(\frac{1}{N}\tilde{P}^{'}Y_{.t} - \frac{1}{N}P^{'}Y_{.t})^{'}]\\
&= \frac{1}{N^2}\underset{t}{E}[(\tilde{P}^{'}Y_{.t} - P^{'}Y_{.t})(Y_{.t}^{'}\tilde{P} - Y_{.t}^{'}P)^{'}]\\
&= \frac{1}{N^2}\underset{t}{E}[(\tilde{P}^{'} - P^{'})Y_{.t}Y_{.t}^{'}(\tilde{P} - P)]
\end{aligned}
$$

A.1.2. Derivation of the MSE attributed to disturbance uncertainty

$$
\begin{aligned}
&\underset{t}{E}\left[\left(f_t - HF_t\right)\left(f_t - HF_t\right)'\right] = \\
&\qquad = \underset{t}{E}[(\frac{1}{N}P^{'}Y_{.t} - (\frac{1}{N}P^{'}Y_{.t} - \frac{1}{N}P^{'}\varepsilon_{.t}t))(\frac{1}{N}P^{'}Y_{.t} - (\frac{1}{N}P^{'}Y_{.t} - \frac{1}{N}P^{'}\varepsilon_{.t}))^{'}] \\
&\qquad = \frac{1}{N^2}\underset{t}{E}[(P^{'}\varepsilon_{.t})(P^{'}\varepsilon_{.t}))^{'}] \\
&\qquad = \frac{1}{N^2}\underset{t}{E}[P^{'}\varepsilon_{.t}\varepsilon_{.t}^{'}P]
\end{aligned}
$$

A.1.3. Derivation of the covariance between the parameter uncertainty and the covariance uncertainty

$$
\begin{aligned}
&\underset{t}{E}\left[\left(\widehat{f}_t - f_t\right)\left(f_t - HF_t\right)'\right] = \\
&\qquad = \underset{t}{E}[(\frac{1}{N}\tilde{P}^{'}Y_{.t} - \frac{1}{N}P^{'}Y_{.t})(\frac{1}{N}P^{'}Y_{.t} - (\frac{1}{N}P^{'}Y_{.t} - \frac{1}{N}P^{'}\varepsilon_{.t}))^{'}] \\
&\qquad = \frac{1}{N^2}\underset{t}{E}[(\tilde{P}^{'}Y_{.t} - P^{'}Y_{.t})(P^{'}\varepsilon_{.t})^{'}] \\
&\qquad = \frac{1}{N^2}\underset{t}{E}[(\tilde{P}^{'} - P^{'})\varepsilon_{.t}\varepsilon_{.t}^{'}P]
\end{aligned}
$$

Appendix B

Appendix to Chapter 3

Table B.1: LS estimates of the parameters of factor augmented predictive regressions (p-values in parenthesis), coefficient of determination, R^2, and Box-Ljung statistic for the joint significance of the first four autocorrelations of the corresponding residuals.

Africa

	DZA	BEN	BWA	BFA	CMR	COG	EGY	GAB	GMB	GHA	KEN	LSO	MDG	MLI	MRT	MUS	MAR	MOZ	NGA	RWA	SEN	SYC	ZAF	TZA	TGO	TUN	UGA	ZWE
μ	2.97	4.56	4.99	7.05	1.94	2.91	2.87	2.22	4.21	3.50	1.82	5.29	3.92	6.57	3.95	5.52	6.65	8.77	4.01	5.58	5.25	2.68	1.43	3.44	2.54	4.33	5.56	0.68
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.05)	(0.00)	(0.04)	(0.00)	(0.00)	(0.65)
ϕ	-0.12	-0.16	0.08	-0.41	0.06	0.02	0.32	-0.06	-0.25	0.34	0.51	-0.30	-0.36	-0.56	-0.10	-0.16	-0.67	-0.20	0.21	-0.01	-0.54	0.33	0.43	0.31	0.07	-0.12	0.13	0.35
	(0.64)	(0.43)	(0.78)	(0.02)	(0.78)	(0.93)	(0.11)	(0.78)	(0.22)	(0.12)	(0.01)	(0.16)	(0.12)	(0.01)	(0.66)	(0.47)	(0.00)	(0.31)	(0.28)	(0.96)	(0.01)	(0.12)	(0.1)	(0.13)	(0.75)	(0.54)	(0.47)	(0.07)
β_1	-0.25	0.08	0.50	0.14	-0.95	-0.77	0.17	-0.12	-0.42	-0.18	-0.69	0.36	1.18	-0.02	-0.07	0.59	0.25	0.63	-0.78	-0.40	-0.07	0.24	-0.19	-0.24	-0.47	0.26	0.42	-1.15
	(0.47)	(0.86)	(0.69)	(0.74)	(0.07)	(0.23)	(0.55)	(0.91)	(0.44)	(0.63)	(0.09)	(0.31)	(0.03)	(0.98)	(0.93)	(0.15)	(0.65)	(0.48)	(0.49)	(0.59)	(0.84)	(0.79)	(0.63)	(0.44)	(0.67)	(0.55)	(0.29)	(0.43)
β_2	1.38	0.84	-1.06	1.73	2.95	0.78	0.31	-1.11	0.14	0.20	0.13	-1.22	2.06	1.14	1.07	-0.46	0.52	1.71	1.43	3.33	1.59	-0.70	0.40	1.02	1.10	0.27	0.16	-1.51
	(0.02)	(0.07)	(0.27)	(0)	(0)	(0.23)	(0.28)	(0.33)	(0.8)	(0.59)	(0.74)	(0.00)	(0.00)	(0.09)	(0.23)	(0.23)	(0.35)	(0.09)	(0.25)	(0.00)	(0.00)	(0.39)	(0.36)	(0.02)	(0.32)	(0.55)	(0.69)	(0.33)
β_3	-0.67	0.25	-0.59	0.65	0.15	-0.19	-0.18	1.16	-0.85	0.50	0.16	0.74	-0.78	-0.82	1.01	-0.23	-0.10	0.21	-1.03	0.04	-0.33	0.57	-0.14	-0.32	0.20	-0.28	0.22	1.50
	(0.07)	(0.58)	(0.52)	(0.12)	(0.77)	(0.76)	(0.51)	(0.31)	(0.14)	(0.23)	(0.68)	(0.06)	(0.12)	(0.25)	(0.23)	(0.51)	(0.85)	(0.81)	(0.37)	(0.95)	(0.33)	(0.5)	(0.67)	(0.3)	(0.85)	(0.53)	(0.58)	(0.31)
R^2	0.39	0.14	0.11	0.43	0.64	0.13	0.23	0.08	0.13	0.29	0.32	0.34	0.40	0.29	0.11	0.14	0.49	0.14	0.17	0.46	0.39	0.19	0.31	0.55	0.06	0.06	0.10	0.26
$Q(4)$	4.95	2.09	2.27	1.37	5.27	3.05	3.92	3.05	4.92	2.02	3.34	0.73	1.71	3.29	2.10	0.93	5.52	5.25	0.96	6.74	1.94	7.13	7.97	0.95	8.05	0.32	0.39	2.92
	(0.29)	(0.72)	(0.69)	(0.85)	(0.26)	(0.55)	(0.42)	(0.55)	(0.30)	(0.73)	(0.5)	(0.95)	(0.79)	(0.51)	(0.72)	(0.92)	(0.24)	(0.26)	(0.92)	(0.15)	(0.75)	(0.13)	(0.09)	(0.92)	(0.09)	(0.99)	(0.98)	(0.57)

America

	ARG	BOL	BRA	CAN	CHL	COL	CRI	DOM	ECU	SLV	GTM	HND	MEX	NIC	PAN	PRY	PER	TTO	USA	URY	VEN
μ	2.25	3.46	2.11	0.83	5.61	3.51	4.47	4.13	3.79	1.46	3.60	4.39	2.15	2.19	3.19	3.75	2.54	1.54	-0.15	2.31	2.50
	(0.09)	(0.00)	(0.02)	(0.29)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)	(0.01)	(0.03)	(0.04)	(0.00)	(0.02)	(0.11)	(0.84)	(0.01)	(0.05)
ϕ	0.11	0.13	0.11	0.64	-0.11	0.04	-0.02	0.16	-0.19	0.50	0.02	-0.23	0.31	0.16	0.34	-0.04	0.24	0.57	1.05	0.23	-0.13
	(0.65)	(0.39)	(0.63)	(0.04)	(0.65)	(0.88)	(0.95)	(0.47)	(0.40)	(0.02)	(0.94)	(0.37)	(0.21)	(0.58)	(0.22)	(0.84)	(0.22)	(0.01)	(0.00)	(0.23)	(0.56)
β_1	-1.12	0.01	-0.56	-0.72	0.68	0.00	0.28	-0.62	-0.01	-0.08	0.28	0.58	-0.27	-0.63	-0.18	-1.01	-1.72	-0.54	-1.16	-0.61	1.82
	(0.33)	(0.95)	(0.28)	(0.14)	(0.21)	(1.00)	(0.53)	(0.40)	(0.98)	(0.83)	(0.23)	(0.27)	(0.63)	(0.34)	(0.84)	(0.18)	(0.04)	(0.42)	(0.01)	(0.32)	(0.12)
β_2	-0.20	0.06	0.04	-0.10	-1.64	-0.37	-0.40	0.17	0.14	-0.64	-0.05	0.01	-0.41	1.63	0.77	-1.22	1.49	0.72	-0.31	-0.34	0.53
	(0.86)	(0.81)	(0.94)	(0.79)	(0.00)	(0.39)	(0.3)	(0.81)	(0.77)	(0.07)	(0.79)	(0.99)	(0.37)	(0.13)	(0.43)	(0.09)	(0.10)	(0.45)	(0.21)	(0.58)	(0.64)
β_3	2.02	0.65	0.30	-0.21	0.33	0.52	0.30	0.64	1.13	-0.03	0.18	0.44	-0.36	0.40	0.68	0.69	1.47	-1.03	-0.15	1.76	2.37
	(0.17)	(0.02)	(0.57)	(0.54)	(0.49)	(0.36)	(0.46)	(0.37)	(0.05)	(0.94)	(0.37)	(0.40)	(0.47)	(0.6)	(0.55)	(0.37)	(0.13)	(0.12)	(0.57)	(0.02)	(0.08)
R^2	0.20	0.35	0.07	0.24	0.41	0.11	0.08	0.09	0.16	0.38	0.12	0.07	0.09	0.34	0.27	0.21	0.45	0.53	0.44	0.41	0.20
Q(4)	2.44	1.57	0.89	3.42	2.79	1.04	4.37	2.18	1.33	2.82	4.78	1.32	2.61	0.27	5.01	3.83	0.67	0.36	3.56	1.33	1.89
	(0.65)	(0.81)	(0.93)	(0.49)	(0.59)	(0.90)	(0.36)	(0.70)	(0.86)	(0.59)	(0.31)	(0.86)	(0.63)	(0.99)	(0.29)	(0.43)	(0.95)	(0.99)	(0.47)	(0.86)	(0.76)

Table 1 cont.: LS estimates of the parameters of factor augmented predictive regressions (p-values in parenthesis), coefficient of determination, R^2, and Box-Ljung statistic for the joint significance of the first four autocorrelations of the corresponding residuals.

	Europe and Oceania																			
	AUT	BEL	DNK	FIN	FRA	DEU	GRC	ISL	IRL	ITA	LUX	NLD	NOR	PRT	ESP	SWE	CHE	GBR	AUS	NZL
μ	2.52	3.42	0.90	1.21	2.71	1.93	0.39	2.53	1.14	1.35	4.36	2.07	0.86	1.25	1.66	1.11	1.69	-0.04	3.03	0.95
	(0.00)	(0.00)	(0.12)	(0.31)	(0.00)	(0.01)	(0.50)	(0.01)	(0.51)	(0.02)	(0.00)	(0.01)	(0.11)	(0.08)	(0.03)	(0.24)	(0.01)	(0.96)	(0.00)	(0.09)
ϕ	-0.24	-0.75	0.36	0.34	-0.50	-0.11	0.58	0.02	0.93	-0.34	-0.09	0.05	0.60	0.33	0.29	0.51	0.04	1.01	0.04	0.62
	(0.45)	(0.06)	(0.21)	(0.51)	(0.24)	(0.71)	(0.00)	(0.95)	(0.01)	(0.47)	(0.74)	(0.88)	(0.01)	(0.26)	(0.30)	(0.22)	(0.89)	(0.00)	(0.86)	(0.00)
β_1	0.99	1.30	-0.36	0.02	1.18	0.62	0.53	1.38	-1.50	1.26	0.88	0.85	-0.14	0.47	0.65	-1.09	0.34	-1.11	0.32	-0.52
	(0.05)	(0.02)	(0.48)	(0.99)	(0.06)	(0.30)	(0.39)	(0.07)	(0.19)	(0.16)	(0.29)	(0.16)	(0.63)	(0.48)	(0.26)	(0.20)	(0.44)	(0.03)	(0.21)	(0.10)
β_2	-0.18	-0.04	0.00	0.22	-0.16	-0.55	0.14	1.56	-0.19	-0.61	-0.52	-0.10	-0.16	-0.47	0.09	0.12	0.30	-0.28	0.30	-0.03
	(0.51)	(0.85)	(1.00)	(0.81)	(0.49)	(0.18)	(0.80)	(0.03)	(0.83)	(0.08)	(0.36)	(0.75)	(0.54)	(0.28)	(0.77)	(0.84)	(0.36)	(0.37)	(0.21)	(0.94)
β_3	-0.51	-0.83	-0.30	-0.90	-0.97	-0.40	-0.62	0.00	-0.76	-1.11	-1.28	-0.60	-0.05	-1.02	-1.08	-0.75	-0.36	0.03	-0.22	0.00
	(0.06)	(0.00)	(0.40)	(0.23)	(0.00)	(0.32)	(0.33)	(1.00)	(0.38)	(0.01)	(0.04)	(0.08)	(0.85)	(0.06)	(0.02)	(0.09)	(0.23)	(0.92)	(0.38)	(1.00)
R^2	0.33	0.37	0.09	0.25	0.42	0.16	0.55	0.37	0.38	0.44	0.24	0.38	0.31	0.54	0.62	0.27	0.16	0.41	0.19	0.39
Q(4)	4.55	4.22	1.17	3.80	7.76	4.40	2.64	2.81	1.75	5.35	5.89	1.21	0.28	3.77	2.09	2.89	1.75	1.66	7.52	0.11
	(0.34)	(0.38)	(0.88)	(0.43)	(0.10)	(0.35)	(0.62)	(0.59)	(0.78)	(0.25)	(0.21)	(0.88)	(0.99)	(0.44)	(0.72)	(0.58)	(0.78)	(0.80)	(0.11)	(1.00)

	Asia																	
	BGD	CHN	HKG	IND	IDN	IRN	ISR	JPN	KOR	MYS	NPL	PAK	PHL	SGP	LKA	SYR	THA	TUR
μ	5.40	3.18	1.59	5.72	3.87	2.11	3.51	1.03	6.56	5.68	5.86	2.38	2.29	4.72	5.18	6.36	3.12	4.79
	(0.00)	(0.08)	(0.23)	(0.00)	(0.01)	(0.02)	(0.01)	(0.09)	(0.00)	(0.00)	(0.00)	(0.02)	(0.02)	(0.02)	(0.00)	(0.00)	(0.02)	(0.00)
ϕ	-0.08	0.64	0.58	0.10	0.20	0.35	0.15	0.34	-0.16	0.03	-0.35	0.43	0.47	0.26	-0.04	-0.46	0.37	-0.07
	(0.78)	(0.00)	(0.03)	(0.61)	(0.4)	(0.03)	(0.59)	(0.24)	(0.56)	(0.89)	(0.1)	(0.04)	(0.03)	(0.35)	(0.87)	(0.01)	(0.09)	(0.81)
β_1	-0.17	-0.11	-1.39	-0.60	-0.42	0.06	-0.08	-0.27	0.64	0.04	0.11	0.05	-0.87	-1.30	-0.56	-0.44	-0.49	-0.56
	(0.31)	(0.78)	(0.10)	(0.13)	(0.56)	(0.94)	(0.88)	(0.67)	(0.41)	(0.96)	(0.69)	(0.89)	(0.04)	(0.17)	(0.15)	(0.5)	(0.49)	(0.58)
β_2	0.66	0.02	-0.04	0.53	-0.85	-0.24	-0.52	-0.71	-1.83	-1.46	-0.44	-0.08	0.20	-1.34	0.47	-1.05	-1.55	0.23
	(0.01)	(0.96)	(0.95)	(0.21)	(0.30)	(0.75)	(0.29)	(0.13)	(0.03)	(0.07)	(0.16)	(0.82)	(0.61)	(0.08)	(0.25)	(0.13)	(0.07)	(0.81)
β_3	0.52	-0.12	-0.87	0.25	0.08	-2.15	0.03	-0.82	-1.35	-0.34	0.20	-0.13	-0.17	-0.91	0.70	2.21	-1.04	0.22
	((0.04)	(0.78)	(0.22)	(0.51)	(0.92)	(0.01)	(0.95)	(0.04)	(0.03)	(0.65)	(0.48)	(0.69)	(0.64)	(0.24)	(0.20)	(0.00)	(0.11)	(0.85)
R^2	0.53	0.38	0.22	0.19	0.14	0.35	0.10	0.37	0.33	0.17	0.16	0.21	0.28	0.27	0.22	0.36	0.47	0.03
$Q(4)$	4.01	5.43	5.41	2.82	1.41	3.52	3.27	3.55	4.05	4.11	3.59	2.99	5.38	3.18	1.88	2.50	0.59	4.69
	(0.41)	(0.25)	(0.25)	(0.59)	(0.84)	(0.48)	(0.51)	(0.47)	(0.40)	(0.39)	(0.46)	(0.56)	(0.25)	(0.53)	(0.76)	(0.64)	(0.96)	(0.32)

Table B.2: Estimated parameters of factor augmented quantile predictive regressions with p-values in parenthesis and fit measure, R^1

	Africa	DZA	BEN	BWA	BFA	CMR	COG	EGY	GAB	GMB	GHA	KEN	LSO	MDG	MLI	MRT	MUS	MAR	MOZ	NGA	RWA	SEN	SYC	ZAF	TZA	TGO	TUN	UGA	ZWE
$\tau = 0.95$	μ	5.05	6.72	12.61	7.87	4.70	6.41	5.89	6.83	6.06	8.48	5.84	6.42	6.71	7.84	11.58	8.04	6.76	18.36	17.23	11.42	5.81	9.10	4.25	7.30	12.56	6.70	9.89	10.27
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	0.26	-0.39	1.14	-0.49	0.13	0.09	0.12	-0.15	0.01	1.29	0.26	-0.27	0.15	-0.95	0.33	0.17	-0.78	-0.70	-0.04	0.00	0.03	0.12	0.75	-0.01	0.17	-0.04	-0.41	0.01
		(0.10)	(0.00)	(0.00)	(0.07)	(0.00)	(0.30)	(0.41)	(0.00)	(0.91)	(0.00)	(0.00)	(0.05)	(0.51)	(0.00)	(0.19)	(0.29)	(0.00)	(0.02)	(0.76)	(0.97)	(0.88)	(0.14)	(0.00)	(0.94)	(0.47)	(0.69)	(0.00)	(0.94)
	β_1	0.36	-0.91	-2.31	0.28	-0.62	-0.66	0.12	0.27	-0.32	-0.36	-0.71	0.28	-0.81	0.90	0.50	0.78	0.79	1.73	-7.71	0.66	-0.14	0.75	-0.34	0.16	0.91	0.38	1.03	-0.41
		(0.1)	(0.00)	(0.01)	(0.65)	(0.00)	(0.03)	(0.57)	(0.1)	(0.16)	(0.14)	(0.00)	(0.21)	(0.13)	(0.00)	(0.58)	(0.01)	(0.00)	(0.17)	(0.00)	(0.00)	(0.62)	(0.04)	(0.01)	(0.33)	(0.45)	(0.14)	(0.00)	(0.69)
	β_2	0.70	0.23	-0.41	1.81	1.04	1.56	0.55	-1.87	0.30	0.47	-0.39	-0.71	1.73	0.94	0.71	0.57	-0.32	6.11	4.78	2.44	0.38	0.51	-0.33	0.46	0.67	-0.48	-0.37	-3.81
		(0.04)	(0.12)	(0.51)	(0.01)	(0.00)	(0.00)	(0.01)	(0.00)	(0.19)	(0.05)	(0.00)	(0.01)	(0.01)	(0.00)	(0.47)	(0.05)	(0.16)	(0.00)	(0.00)	(0.00)	(0.29)	(0.12)	(0.01)	(0.05)	(0.58)	(0.07)	(0.01)	(0.00)
	β_3	-0.98	-0.20	-2.33	1.54	-0.43	-0.17	0.05	0.87	-0.47	1.57	1.05	0.68	-1.22	-3.00	3.80	-0.09	0.04	1.07	-6.32	-0.24	-1.00	-0.93	-0.13	0.17	6.28	-0.05	0.57	4.24
		(0.00)	(0.16)	(0.00)	(0.02)	(0.00)	(0.54)	(0.79)	(0.00)	(0.05)	(0.00)	(0.00)	(0.01)	(0.02)	(0.00)	(0.00)	(0.71)	(0.85)	(0.4)	(0.00)	(0.25)	(0.00)	(0.01)	(0.17)	(0.29)	(0.00)	(0.84)	(0.00)	(0.00)
	R^1	0.42	0.36	0.34	0.15	0.41	0.40	0.40	0.47	0.23	0.52	0.47	0.37	0.36	0.62	0.29	0.24	0.63	0.21	0.39	0.22	0.24	0.18	0.42	0.28	0.13	0.30	0.24	0.31
$\tau = 0.50$	μ	2.40	4.08	5.59	5.15	2.57	3.58	4.03	3.40	4.18	5.11	4.44	4.29	3.02	5.29	3.36	4.94	4.04	7.01	4.40	6.00	3.63	4.33	2.64	4.76	1.98	3.84	5.95	1.30
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.09)
	ϕ	-0.22	0.10	-0.09	-0.45	0.41	-0.01	0.37	0.22	-0.17	0.40	0.56	-0.45	-0.12	-0.63	0.04	0.21	-0.80	0.17	-0.11	0.23	-0.55	0.40	0.27	0.03	0.54	-0.04	0.34	0.32
		(0.60)	(0.56)	(0.83)	(0.04)	(0.02)	(0.97)	(0.18)	(0.02)	(0.22)	(0.02)	(0.01)	(0.13)	(0.51)	(0.00)	(0.86)	(0.47)	(0.00)	(0.02)	(0.39)	(0.34)	(0.07)	(0.14)	(0.44)	(0.89)	(0.00)	(0.86)	(0.06)	(0.00)
	β_1	-0.53	0.44	0.69	0.14	-0.59	-0.34	0.44	-0.16	-0.62	-0.33	-0.42	0.95	0.80	0.08	-0.72	0.50	-0.02	0.02	-0.84	-0.68	-0.20	-0.21	-0.38	-0.59	-0.56	0.02	0.13	-1.46
		(0.38)	(0.24)	(0.7)	(0.77)	(0.16)	(0.67)	(0.28)	(0.73)	(0.1)	(0.26)	(0.35)	(0.05)	(0.06)	(0.84)	(0.41)	(0.35)	(0.98)	(0.95)	(0.29)	(0.45)	(0.67)	(0.85)	(0.49)	(0.12)	(0.14)	(0.96)	(0.74)	(0.05)
	β_2	2.21	0.37	-0.72	1.66	1.78	0.74	0.36	-1.35	0.45	0.07	0.22	-1.58	1.12	1.23	0.67	-0.33	0.73	0.57	2.05	2.60	1.63	-0.88	0.12	1.72	0.35	0.69	0.18	-1.85
		(0.03)	(0.34)	(0.6)	(0.01)	(0.01)	(0.36)	(0.36)	(0.01)	(0.22)	(0.8)	(0.62)	(0.01)	(0.03)	(0.01)	(0.47)	(0.51)	(0.28)	(0.11)	(0.02)	(0.03)	(0.01)	(0.4)	(0.84)	(0.00)	(0.36)	(0.22)	(0.64)	(0.02)
	β_3	-0.79	-0.30	-0.39	0.51	0.34	-0.18	-0.38	0.74	-0.69	0.25	0.13	0.88	-0.26	-2.12	1.14	-0.59	-0.35	-0.18	0.43	-0.21	-0.59	0.54	-0.01	-0.50	0.70	-0.28	-0.07	-0.05
		(0.20)	(0.43)	(0.76)	(0.3)	(0.4)	(0.81)	(0.32)	(0.13)	(0.08)	(0.43)	(0.78)	(0.09)	(0.5)	(0.00)	(0.20)	(0.21)	(0.60)	(0.59)	(0.6)	(0.82)	(0.21)	(0.62)	(0.98)	(0.18)	(0.07)	(0.6)	(0.85)	(0.94)
	R^1	0.20	0.15	0.06	0.31	0.37	0.20	0.17	0.17	0.16	0.20	0.29	0.26	0.29	0.23	0.16	0.19	0.35	0.14	0.18	0.29	0.31	0.22	0.18	0.38	0.12	0.18	0.21	0.25
$\tau = 0.05$	μ	0.59	0.58	-2.63	2.05	-1.52	-2.56	2.08	-9.24	-1.65	3.63	0.24	1.30	-1.03	-1.04	-2.42	2.51	-0.29	2.20	-2.88	-0.58	1.15	-3.07	-0.71	3.14	-5.28	-0.53	3.58	-12.23
		(0.01)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.51)	(0.00)	(0.00)	(0.17)	(0.00)	(0.00)	(0.34)	(0.00)	(0.01)	(0.48)	(0.00)	(0.00)	(0.08)	(0.00)	(0.00)	(0.12)	(0.00)	(0.00)
	ϕ	-0.01	0.46	0.27	-0.06	0.34	1.12	-0.34	0.63	-0.85	0.08	-0.28	-0.44	-0.92	-0.12	-0.63	-0.04	-0.37	0.30	0.12	-0.50	-0.58	0.08	-0.40	0.23	0.23	-0.01	-0.07	1.58
		(0.95)	(0.00)	(0.34)	(0.31)	(0.01)	(0.00)	(0.00)	(0.01)	(0.00)	(0.19)	(0.1)	(0.00)	(0.00)	(0.59)	(0.00)	(0.59)	(0.00)	(0.00)	(0.43)	(0.03)	(0.00)	(0.54)	(0.2)	(0.03)	(0.02)	(0.96)	(0.48)	(0.00)
	β_1	-0.76	-0.48	4.27	0.47	-1.84	0.26	1.06	-5.79	-1.15	0.11	-0.70	0.02	0.63	-1.66	-0.45	-0.22	-1.54	-1.40	-3.78	2.53	0.47	1.50	0.85	-0.88	-2.70	-0.65	0.55	0.95
		(0.00)	(0.05)	(0.00)	(0.00)	(0.00)	(0.5)	(0.00)	(0.00)	(0.00)	(0.30)	(0.07)	(0.88)	(0.02)	(0.03)	(0.14)	(0.13)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.01)	(0.08)	(0.00)	(0.00)	(0.06)	(0.01)	(0.27)
	β_2	1.30	0.56	-3.18	1.55	3.24	-1.85	0.63	3.24	-0.63	0.33	1.08	-0.27	3.86	0.83	-0.80	-1.18	2.59	3.97	4.25	5.54	1.96	-1.87	1.20	1.01	5.56	-0.85	0.11	1.14
		(0.00)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.03)	(0.00)	(0.01)	(0.15)	(0.00)	(0.28)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)	(0.02)	(0.62)	(0.21)
	β_3	-0.49	1.00	0.36	1.28	-0.04	-1.16	-0.36	4.61	0.19	0.10	0.43	1.23	-0.63	-0.53	-0.64	0.10	1.13	1.46	2.18	1.46	0.17	0.81	-0.39	-0.25	-0.41	-0.97	0.31	-1.56
		(0.04)	(0.00)	(0.69)	(0.00)	(0.89)	(0.00)	(0.03)	(0.00)	(0.50)	(0.39)	(0.25)	(0.00)	(0.02)	(0.52)	(0.04)	(0.43)	(0.00)	(0.00)	(0.03)	(0.08)	(0.37)	(0.12)	(0.31)	(0.11)	(0.40)	(0.01)	(0.15)	(0.08)
	R^1	0.54	0.49	0.33	0.49	0.66	0.43	0.28	0.37	0.35	0.21	0.21	0.25	0.55	0.29	0.22	0.37	0.51	0.56	0.49	0.50	0.45	0.24	0.29	0.58	0.47	0.22	0.45	0.36

Table 2 cont.: Estimated parameters of factor augmented quantile predictive regressions with p-values in parenthesis and fit measure, R^1

	America	ARG	BOL	BRA	CAN	CHL	COL	CRI	DOM	ECU	SLV	GTM	HND	MEX	NIC	PAN	PRY	PER	TTO	USA	URY	VEN
$\tau = 0.95$	μ	9.91	5.34	5.36	4.39	7.83	5.95	6.72	9.47	6.47	4.87	4.85	6.03	5.84	5.61	9.19	7.9	7.83	8.72	4.00	7.31	12.07
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	0.04	0.43	-0.17	0.26	0.06	-0.11	-0.12	0.18	-0.65	0.27	0.32	0.14	0.14	0.46	-0.23	-0.49	0.51	0.26	0.71	0.47	-0.79
		(0.81)	(0.00)	(0.26)	(0.05)	(0.56)	(0.36)	(0.37)	(0.2)	(0.00)	(0.00)	(0.05)	(0.29)	(0.41)	(0.00)	(0.06)	(0.00)	(0.00)	(0.07)	(0.00)	(0.00)	(0.00)
	β_1	0.32	-0.10	-0.57	-0.31	-0.40	0.48	0.51	0.16	0.51	0.72	0.16	0.41	-0.16	-0.29	0.72	-0.84	-1.76	-1.39	-0.44	-0.35	2.55
		(0.70)	(0.34)	(0.09)	(0.14)	(0.08)	(0.02)	(0.04)	(0.72)	(0.00)	(0.00)	(0.34)	(0.13)	(0.68)	(0.29)	(0.07)	(0.01)	(0.00)	(0.01)	(0.05)	(0.34)	(0.04)
	β_2	-1.57	-0.16	0.36	0.19	-1.22	0.25	-0.58	-1.06	1.79	-1.20	0.14	0.01	0.91	0.12	0.19	-2.75	-0.88	3.46	-0.59	0.44	2.18
		(0.06)	(0.15)	(0.26)	(0.22)	(0.00)	(0.17)	(0.01)	(0.02)	(0.00)	(0.00)	(0.35)	(0.96)	(0.01)	(0.78)	(0.65)	(0.00)	(0.00)	(0.00)	(0.00)	(0.23)	(0.06)
	β_3	-0.23	0.56	0.78	0.01	-0.25	0.24	1.27	1.36	0.69	-0.05	0.34	-0.15	-0.36	-0.31	1.75	2.60	1.17	-0.81	-0.14	1.61	7.55
		(0.83)	(0.00)	(0.03)	(0.93)	(0.21)	(0.32)	(0.00)	(0.00)	(0.00)	(0.61)	(0.03)	(0.58)	(0.30)	(0.33)	(0.00)	(0.00)	(0.00)	(0.08)	(0.28)	(0.00)	(0.00)
	R^1	0.21	0.49	0.43	0.27	0.48	0.23	0.44	0.15	0.31	0.55	0.49	0.15	0.25	0.3	0.34	0.54	0.40	0.49	0.27	0.20	0.20
$\tau = 0.50$	μ	3.05	4.22	2.84	2.48	5.17	4.11	4.28	5.26	3.43	3.02	3.71	3.97	3.15	3.24	5.24	3.46	4.15	3.25	2.58	3.01	2.11
		(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.27)
	ϕ	0.44	0.29	0.18	0.33	-0.43	-0.20	-0.02	0.39	-0.11	0.46	0.07	0.03	0.30	0.01	-0.02	0.12	0.46	0.35	1.26	0.00	0.09
		(0.12)	(0.02)	(0.68)	(0.42)	(0.21)	(0.50)	(0.90)	(0.22)	(0.71)	(0.08)	(0.73)	(0.91)	(0.22)	(0.95)	(0.90)	(0.73)	(0.07)	(0.20)	(0.00)	(0.99)	(0.81)
	β_1	-2.90	-0.02	-0.31	-0.58	0.80	0.06	-0.15	-0.8	-0.17	-0.14	0.15	0.04	0.12	-0.67	0.43	-0.58	-1.28	-0.74	-1.63	-0.18	2.03
		(0.03)	(0.91)	(0.74)	(0.38)	(0.29)	(0.90)	(0.59)	(0.44)	(0.78)	(0.78)	(0.47)	(0.94)	(0.84)	(0.05)	(0.36)	(0.63)	(0.23)	(0.4)	(0.00)	(0.85)	(0.31)
	β_2	1.07	-0.11	0.09	0.13	-1.63	-0.05	-0.05	-0.18	0.30	-0.70	0.00	-0.15	-0.51	1.81	1.14	-1.06	-0.07	1.14	-0.25	-0.51	-1.33
		(0.39)	(0.56)	(0.92)	(0.8)	(0.02)	(0.91)	(0.81)	(0.86)	(0.62)	(0.12)	(0.99)	(0.77)	(0.26)	(0.00)	(0.04)	(0.36)	(0.95)	(0.35)	(0.3)	(0.58)	(0.48)
	β_3	0.31	0.3	-0.08	-0.15	0.71	0.61	0.23	-0.55	0.93	0	0.11	0.09	-0.18	0.19	1.62	0.09	0.25	-0.9	0.00	1.96	1.81
		(0.85)	(0.15)	(0.93)	(0.74)	(0.29)	(0.32)	(0.35)	(0.58)	(0.19)	(1)	(0.56)	(0.88)	(0.71)	(0.61)	(0.02)	(0.94)	(0.84)	(0.28)	(0.99)	(0.08)	(0.43)
	R^1	0.21	0.44	0.1	0.12	0.23	0.09	0.08	0.11	0.11	0.24	0.21	0.09	0.23	0.36	0.23	0.09	0.26	0.41	0.28	0.27	0.16
$\tau = 0.05$	μ	-5.12	2.14	-1.57	-1.03	1.27	0.1	0.94	-0.72	-1.06	-0.69	1.62	-0.06	-2.14	-5.21	-2.98	-1.4	-1.98	-1.52	0.61	-1.74	-4.58
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.71)	(0.03)	(0.27)	(0.01)	(0.08)	(0.00)	(0.83)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)	(0.00)	(0.00)
	ϕ	0.56	-0.30	0.6	0.77	-0.26	1.15	0.30	0.45	0.63	0.48	0.38	-0.39	0.40	0.31	2.25	0.32	-0.30	0.89	1.79	0.33	-0.21
		(0.00)	(0.04)	(0.00)	(0.00)	(0.18)	(0.00)	(0.25)	(0.04)	(0.00)	(0.04)	(0.11)	(0.02)	(0.26)	(0.28)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.11)
	β_1	2.51	-0.10	0.85	0.60	2.89	-1.57	1.26	-0.71	1.17	1.63	0.94	3.36	0.75	1.28	0.95	2.68	-0.37	1.42	-2.26	0.47	4.85
		(0.00)	(0.64)	(0.02)	(0.04)	(0.00)	(0.00)	(0.01)	(0.29)	(0.00)	(0.00)	(0.00)	(0.00)	(0.36)	(0.06)	(0.28)	(0.00)	(0.29)	(0.07)	(0.00)	(0.17)	(0.00)
	β_2	1.21	0.61	-0.45	-0.31	-2.96	-0.25	-0.88	0.91	-0.86	0.51	-0.77	0.13	-2.68	7.26	-1.67	-2.00	4.67	-1.09	-0.52	1.70	2.17
		(0.13)	(0.01)	(0.21)	(0.17)	(0.00)	(0.37)	(0.04)	(0.18)	(0.04)	(0.18)	(0.00)	(0.64)	(0.00)	(0.00)	(0.1)	(0.00)	(0.00)	(0.3)	(0.01)	(0.00)	(0.00)
	β_3	0.51	1.76	1.99	-0.28	1.44	0.26	-0.52	2.13	1.91	-0.63	0.46	1.76	-0.34	-0.63	-3.05	0.13	3.64	-1.97	-0.53	2.41	4.06
		(0.62)	(0.00)	(0.00)	(0.18)	(0.00)	(0.5)	(0.22)	(0.00)	(0.00)	(0.12)	(0.05)	(0.00)	(0.64)	(0.41)	(0.01)	(0.65)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)
	R^1	0.43	0.67	0.39	0.36	0.43	0.46	0.29	0.37	0.38	0.29	0.41	0.4	0.33	0.38	0.50	0.33	0.67	0.39	0.58	0.53	0.44

Table 2 cont.: Estimated parameters of factor augmented quantile predictive regressions with p-values in parenthesis and fit measure, R^1

	Asia	BGD	CHN	HKG	IND	IDN	IRN	ISR	JPN	KOR	MYS	NPL	PAK	PHL	SGP	LKA	SYR	THA	TUR
$\tau = 0.95$	μ	6.09	11.79	7.54	9.16	6.65	7.92	6.80	3.99	8.68	8.91	6.31	6.18	6.51	10.17	7.35	8.45	8.74	9.62
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	0.33	0.91	0.75	0.27	0.29	0.52	-0.33	0.18	-0.31	0.12	-0.91	0.82	0.49	0.07	0.6	-0.32	0.05	0.28
		(0.14)	(0.00)	(0.00)	(0.14)	(0.00)	(0.00)	(0.05)	(0.24)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.48)	(0.00)	(0.00)	(0.73)	(0.07)
	β_1	0.15	0.98	-1.04	0.09	0.27	0.62	1.01	0.10	1.91	0.32	-0.10	-0.99	-0.56	-1.01	-0.2	0.67	0.81	-0.15
		(0.25)	(0.00)	(0.00)	(0.79)	(0.01)	(0.27)	(0.00)	(0.77)	(0.00)	(0.04)	(0.69)	(0.00)	(0.02)	(0.01)	(0.29)	(0.02)	(0.08)	(0.78)
	β_2	0.10	-1.01	0.23	-0.50	-0.49	0.52	-1.21	-2.04	-2.75	-0.34	-1.03	-0.22	-0.46	-0.85	-0.33	-2.45	-2.5	-0.76
		(0.59)	(0.00)	(0.24)	(0.18)	(0.00)	(0.36)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.13)	(0.06)	(0.00)	(0.1)	(0.00)	(0.00)	(0.13)
	β_3	-0.02	-0.12	-1.92	-0.10	-0.23	-1.99	-0.61	-0.31	-1.98	0.13	-0.22	-0.39	0.38	0.22	-0.24	0.48	-0.06	0.06
		(0.92)	(0.66)	(0.00)	(0.77)	(0.04)	(0.00)	(0.06)	(0.13)	(0.00)	(0.33)	(0.39)	(0.01)	(0.09)	(0.43)	(0.38)	(0.12)	(0.88)	(0.92)
	R^1	0.39	0.36	0.57	0.18	0.39	0.53	0.27	0.50	0.48	0.23	0.43	0.40	0.26	0.47	0.35	0.54	0.48	0.1
$\tau = 0.50$	μ	4.87	8.94	4.7	6.56	5.48	2.82	5.08	1.64	5.82	6.69	4.41	4.57	4.45	6.35	4.98	4.94	5.22	5.30
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	-0.21	0.98	0.48	0.04	0.20	0.09	-0.01	0.45	0.15	-0.12	-0.4	0.57	0.45	0.22	0.07	-0.23	0.41	-0.29
		(0.55)	(0.00)	(0.20)	(0.87)	(0.00)	(0.66)	(0.96)	(0.32)	(0.52)	(0.52)	(0.21)	(0.00)	(0.03)	(0.64)	(0.86)	(0.13)	(0.03)	(0.31)
	β_1	-0.09	0.03	-1.12	-0.88	0.32	-0.3	0.32	-0.74	0.25	0.59	0.02	-0.07	-1.06	-0.91	-0.46	0.27	-0.67	-0.38
		(0.67)	(0.93)	(0.34)	(0.09)	(0.13)	(0.77)	(0.45)	(0.45)	(0.71)	(0.33)	(0.96)	(0.75)	(0.01)	(0.56)	(0.39)	(0.63)	(0.28)	(0.71)
	β_2	0.64	0.23	-0.32	0.61	-0.42	-0.37	-1.31	-0.38	-1.57	-1.69	-0.48	-0.25	0.23	-0.78	0.46	-1.3	-1.43	1.04
		(0.03)	(0.43)	(0.75)	(0.26)	(0.07)	(0.72)	(0.00)	(0.60)	(0.04)	(0.00)	(0.31)	(0.22)	(0.56)	(0.53)	(0.42)	(0.04)	(0.06)	(0.28)
	β_3	0.83	-0.23	-0.59	-0.13	0.22	-2.01	-0.34	-1.00	-1.40	-0.17	-0.04	-0.09	-0.29	-1.45	0.7	1.36	-0.89	0.83
		(0.01)	(0.49)	(0.55)	(0.79)	(0.35)	(0.06)	(0.44)	(0.1)	(0.02)	(0.75)	(0.92)	(0.64)	(0.44)	(0.25)	(0.36)	(0.04)	(0.12)	(0.49)
	R^1	0.35	0.37	0.22	0.21	0.27	0.28	0.1	0.18	0.34	0.27	0.06	0.27	0.24	0.19	0.17	0.24	0.33	0.08
$\tau = 0.05$	μ	3.9	6.21	-2.19	3.25	4.18	-3.52	0.90	-1.67	-1.71	-0.87	2.14	1.57	0.67	1.15	1.54	-0.2	1.62	-4.83
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.08)	(0.16)	(0.00)	(0.00)	(0.04)	(0.00)	(0.01)	(0.56)	(0.00)	(0.00)
	ϕ	0.58	0.53	1.38	0.27	0.17	0.73	-0.16	1.31	2.19	-0.19	0.00	-0.36	0.66	0.87	-0.64	-0.43	0.86	-0.13
		(0.01)	(0.00)	(0.00)	(0.22)	(0.00)	(0.00)	(0.08)	(0.00)	(0.00)	(0.40)	(0.95)	(0.01)	(0.00)	(0.00)	(0.10)	(0.00)	(0.00)	(0.58)
	β_1	-0.05	-0.97	-4.09	-1.8	-0.38	-2.01	0.07	-0.23	-6.04	-0.42	0.04	0.36	-1.67	-4.1	-1.58	-1.38	-1.97	-2.05
		(0.67)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.67)	(0.53)	(0.00)	(0.58)	(0.59)	(0.08)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.02)
	β_2	0.63	1.8	-2.1	0.97	-0.71	-0.42	-0.2	1	2.73	-4.84	-0.13	-0.99	-0.49	-2.19	-0.29	0.22	-1.79	-1.73
		(0.00)	(0.00)	(0.01)	(0.04)	(0.00)	(0.33)	(0.19)	(0.00)	(0.04)	(0.00)	(0.09)	(0.00)	(0.16)	(0.00)	(0.61)	(0.53)	(0.00)	(0.03)
	β_3	0.27	0.03	-0.41	0.55	0.19	-2.78	0.72	-1.59	3.38	-0.78	1.1	-0.24	-1.51	-1.41	1.73	2.28	-1.83	-2.85
		(0.11)	(0.92)	(0.58)	(0.19)	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.00)	(0.20)	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)	(0.01)
	R^1	0.56	0.46	0.34	0.3	0.12	0.47	0.23	0.44	0.23	0.24	0.43	0.17	0.27	0.37	0.39	0.63	0.51	0.12

Table 2 cont.: Estimated parameters of factor augmented quantile predictive regressions with p-values in parenthesis and fit measure, R^1

		Europe and Oceania																			
		AUT	BEL	DNK	FIN	FRA	DEU	GRC	ISL	IRL	ITA	LUX	NLD	NOR	PRT	ESP	SWE	CHE	GBR	AUS	NZL
$\tau = 0.95$	μ	3.42	3.40	4.32	5.10	3.28	4.2	4.62	6.88	13.47	2.90	7.79	3.62	4.23	3.94	4.18	4.40	3.54	3.94	4.64	5.26
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	0.16	-1.02	0.1	1.27	-0.66	-0.36	-0.03	0.77	1.47	0.52	0.24	0.4	0.77	0.51	0.54	0.06	0.53	0.51	-0.05	-0.26
		(-0.26)	(0.00)	(-0.52)	(0.00)	(-0.03)	(-0.2)	(-0.8)	(0.00)	(0.00)	(0.00)	(-0.15)	(-0.02)	(0.00)	(0.00)	(0.00)	(-0.55)	(-0.01)	(0)	(-0.72)	(-0.07)
	β_1	0.17	1.41	-2.46	-2.85	1.44	0.92	0.86	-0.88	-5.63	-0.42	0.30	-0.01	-1.1	0.13	0.15	-0.51	-0.34	-0.04	0.80	-0.81
		(-0.42)	(0.00)	(0.00)	(0.00)	(0.00)	(-0.1)	(-0.03)	(-0.09)	(0.00)	(-0.16)	(-0.54)	(-0.98)	(0.00)	(-0.67)	(-0.49)	(-0.01)	(-0.26)	(-0.82)	(0.00)	(0.00)
	β_2	0.25	-0.23	0.15	-0.38	-0.33	-1.12	1.23	0.85	2.09	-0.34	-0.13	0.07	0.03	-0.86	-0.42	0.28	-0.02	-0.38	-0.17	-0.72
		(-0.04)	(0.00)	(-0.45)	(-0.43)	(-0.05)	(-0.01)	(0.00)	(-0.08)	(0.00)	(-0.01)	(-0.71)	(-0.6)	(-0.86)	(0.00)	(0.00)	(-0.05)	(-0.92)	(0.00)	(-0.25)	(-0.01)
	β_3	-0.4	-0.58	-0.07	0.51	-1.32	0.12	-0.95	0.32	-4.64	-0.42	-0.12	-0.27	-0.22	-0.46	-0.45	-0.04	-0.24	-0.78	-0.45	-0.54
		(0.00)	(0.00)	(-0.71)	(-0.19)	(0.00)	(-0.75)	(-0.02)	(-0.43)	(0.00)	(0.00)	(-0.74)	(-0.09)	(-0.27)	(-0.06)	(-0.01)	(-0.68)	(-0.25)	(0.00)	(-0.01)	(-0.03)
	R^1	0.35	0.4	0.22	0.22	0.47	0.27	0.24	0.31	0.43	0.37	0.30	0.46	0.32	0.59	0.39	0.33	0.29	0.48	0.34	0.21
$\tau = 0.50$	μ	1.94	2.27	1.58	2.10	2.07	1.85	0.69	2.53	5.37	1.24	4.17	2.25	2.40	2.50	2.60	2.69	2.05	2.37	3.40	2.31
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(-0.23)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	ϕ	0.06	-0.79	0.55	0.81	-0.48	-0.05	0.7	-0.25	0.55	-0.13	-0.42	0.09	0.57	0.57	0.20	0.18	0.13	0.74	-0.22	0.68
		(-0.84)	(-0.17)	(-0.01)	(-0.03)	(-0.27)	(-0.82)	(0)	(-0.19)	(-0.03)	(-0.76)	(-0.28)	(-0.8)	(-0.04)	(-0.13)	(-0.44)	(-0.77)	(-0.71)	(-0.04)	(-0.4)	(0.00)
	β_1	1.03	1.31	-0.85	-0.57	0.77	0.29	0.65	1.63	-0.28	0.73	1.18	0.68	-0.13	-0.42	0.74	-0.41	0.04	-0.91	0.47	-0.25
		(-0.03)	(-0.12)	(-0.02)	(-0.59)	(-0.21)	(-0.56)	(-0.33)	(-0.01)	(-0.72)	(-0.35)	(-0.31)	(-0.29)	(-0.76)	(-0.62)	(-0.18)	(-0.74)	(-0.94)	(-0.14)	(-0.10)	(-0.36)
	β_2	-0.24	-0.08	0.00	-0.44	0.05	-0.43	0.4	2.12	0.14	-0.22	-0.65	-0.07	-0.25	-0.43	0.35	0.40	0.63	-0.10	0.02	0.00
		(-0.32)	(-0.82)	(-0.99)	(-0.51)	(-0.82)	(-0.21)	(-0.49)	(0.00)	(-0.82)	(-0.48)	(-0.42)	(-0.83)	(-0.49)	(-0.45)	(-0.23)	(-0.65)	(-0.1)	(-0.80)	(-0.93)	(-0.99)
	β_3	-0.46	-0.89	-0.75	-0.14	-0.92	-0.57	-0.48	0.62	-0.73	-0.73	-1.56	-0.63	0.15	-0.30	-0.7	-0.60	-0.57	-0.23	-0.24	-0.04
		(-0.07)	(-0.03)	(0.00)	(-0.79)	(-0.01)	(-0.09)	(-0.47)	(-0.19)	(-0.24)	(-0.04)	(-0.07)	(-0.08)	(-0.67)	(-0.65)	(-0.09)	(-0.35)	(-0.11)	(-0.57)	(-0.39)	(-0.87)
	R^1	0.21	0.22	0.21	0.22	0.26	0.12	0.34	0.36	0.32	0.25	0.19	0.28	0.23	0.33	0.39	0.13	0.12	0.15	0.14	0.38
$\tau = 0.05$	μ	-0.68	-0.26	-2.08	-3.80	-0.33	-1.7	-2.97	-3.11	0.06	-2.11	-0.53	-0.26	0.21	-0.76	-0.42	-1.42	-0.36	-0.53	1.51	0.18
		(-0.02)	(-0.38)	(0.00)	(0.00)	(-0.02)	(0.00)	(0.00)	(0.00)	(-0.92)	(0.00)	(-0.38)	(-0.32)	(-0.36)	(0.00)	(-0.09)	(0.00)	(-0.24)	(0.00)	(0.00)	(-0.56)
	ϕ	-1.75	-2.12	0.41	-1.33	-2.37	-1.4	1.02	0.31	0.37	0.89	0.18	-1.03	0.78	0.13	0.05	1.22	-0.2	2.43	0.05	0.64
		(0.00)	(0.00)	(-0.15)	(0.00)	(0.00)	(0.00)	(0.00)	(-0.35)	(-0.15)	(-0.01)	(-0.54)	(0.00)	(0)	(-0.26)	(-0.81)	(0.00)	(-0.51)	(0.00)	(-0.71)	(0.00)
	β_1	3.08	3.69	2.07	6.92	4.81	4.17	-0.04	3.2	-0.52	1.32	1.43	3.80	0.12	1.79	2.54	-0.05	1.55	-1.88	-0.04	-0.57
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(-0.94)	(0.00)	(-0.54)	(-0.04)	(-0.12)	(0.00)	(-0.65)	(0.00)	(0.00)	(-0.86)	(0.00)	(0.00)	(-0.73)	(-0.07)
	β_2	0.21	-0.25	-0.43	4.50	-0.45	-0.31	-0.63	1.53	-1.1	-0.16	-1.49	0.16	0.24	-0.43	-0.04	-1.41	0.46	0.23	1.25	-0.43
		(-0.45)	(-0.39)	(-0.24)	(0.00)	(0.00)	(-0.05)	(-0.22)	(-0.13)	(-0.09)	(-0.49)	(-0.02)	(-0.53)	(-0.3)	(-0.02)	(-0.86)	(0.00)	(-0.17)	(-0.20)	(0.00)	(-0.21)
	β_3	-1.94	-1.29	0.21	-2.59	-2.63	-1.3	0.24	-2.05	-2.69	-0.44	-1.42	-0.69	-0.79	-1.33	-1.02	-0.23	-0.8	1.46	-0.54	-0.62
		(0.00)	(0.00)	(-0.55)	(0.00)	(0)	(0.00)	(-0.69)	(-0.02)	(0)	(-0.09)	(-0.04)	(-0.02)	(0.00)	(0.00)	(-0.01)	(-0.13)	(-0.01)	(0.00)	(0.00)	(-0.05)
	R^1	0.43	0.44	0.34	0.43	0.49	0.44	0.62	0.36	0.46	0.48	0.43	0.54	0.44	0.55	0.54	0.47	0.42	0.51	0.50	0.39

Figure B.1: Resampling ellipsoids for the three factors in 1998 (blue) and 2004 (red). Predicted iso-growth surfaces in 1999 and 2005 based on predictive regressions for USA, Germany and Greece (industrialized: first row), Brazil, China and India (emerging: second row), and Bolivia, Uganda and Nepal (developing: third row). For each year and country, the GiS is the tangency point between the ellipsoid and the corresponding surface.

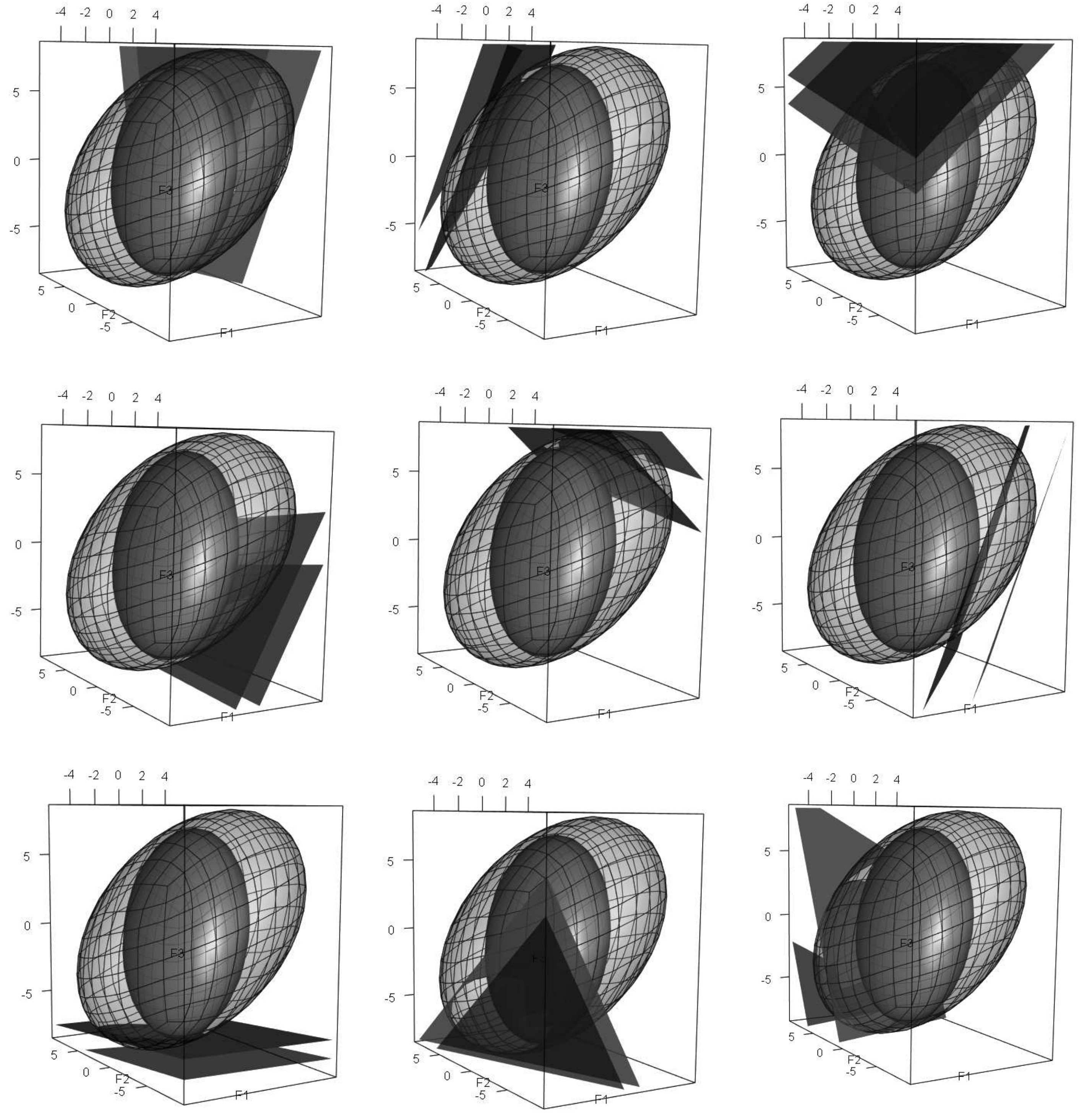


Figure B.2: GiS for each industrialized (red), emerging (blue) and other developing (gray) country in Africa (top left panel), America (top right panel), Asia (bottom left panel) and Europe and Oceania (bottom right panel).

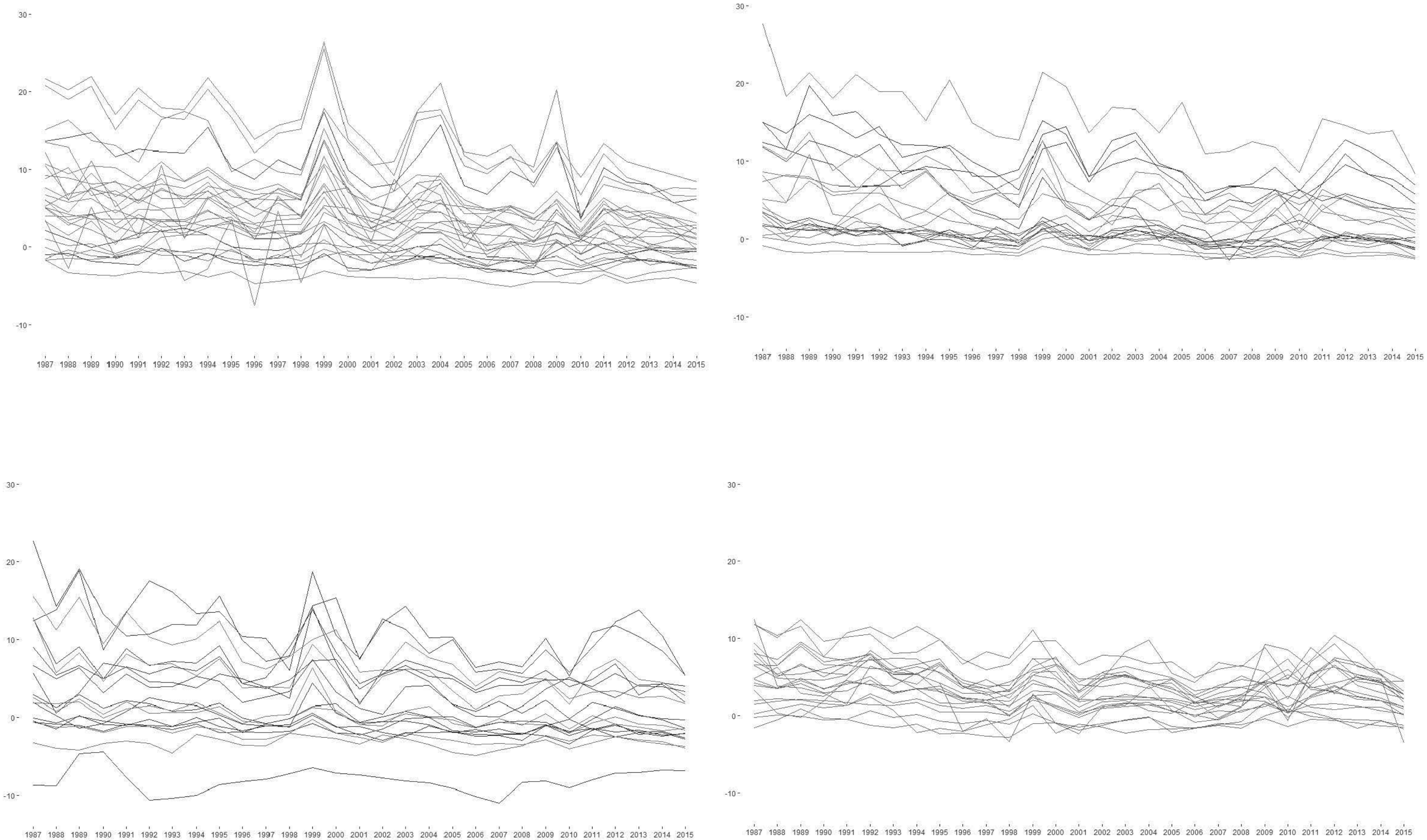


Figure B.3: Histograms of estimated parameters of the factor augmented quantile predictive regressions for $\tau = 0.5$ corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).

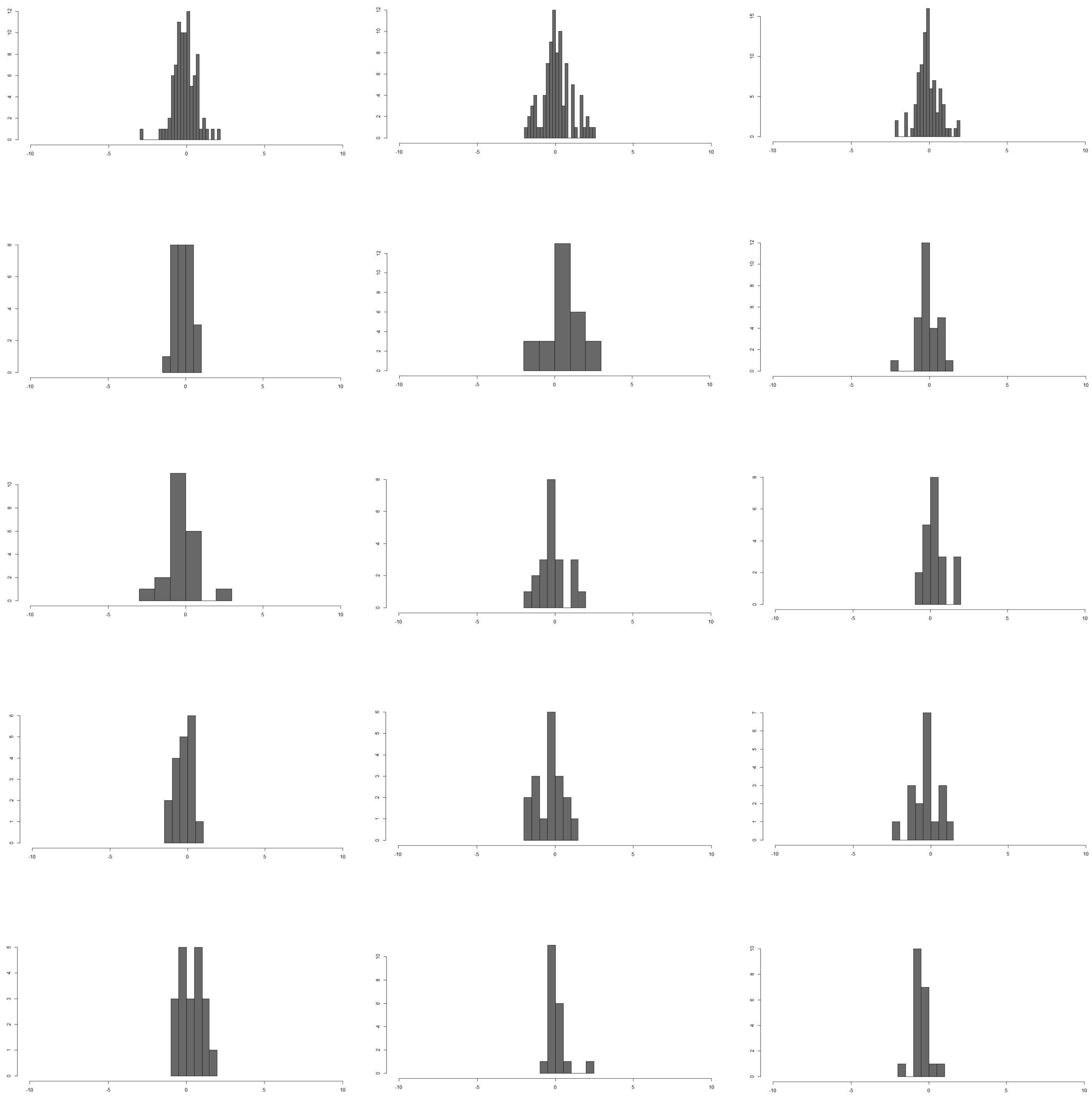


Figure B.4: Histograms of estimated parameters of the factor augmented quantile predictive regressions for $\tau = 0.95$ corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).