Computationally-efficient structural models for analysis of woven composites

Ibrahim Kaleel[1], Alberto G. de Miguel[2], Marco Petrolo[3], Alfonso Pagani[4] and Erasmo Carrera[5]
Politecnico di Torino, Torino, Italy, 10129

Trenton M. Ricks[6], Evan J. Pineda[7], Brett A. Bednarcyk[8] and Steven M. Arnold[9]
NASA Glenn Research Center, Cleveland, OH, 44135, United States of America

The paper presents a novel approach to model woven composite using the computationally-efficient one-dimensional models. The framework is built within the scheme of the Carrera Unified Formulation (CUF), a generalized hierarchical formulation that generates variable kinematic structural theories. Various components of the woven composite unit cell are modeled using a combination of straight and curved one-dimensional CUF models. By employing a component-wise approach, a modeling technique within CUF, the complex geometry of the woven composite components is modeled precisely. The ability of CUF models to accurately resolve stress and strain fields are exploited to capture complex deformation within a woven composite unit cell. Numerical results include analyses of a non-crimped textile composite, a curved tow under tension, and a dry woven textile unit cell.

I. Introduction

Three-dimensional (3D) woven composites exhibit distinct advantages over traditional laminated composites such as improved inter-laminar properties due to the introduction of through-the-thickness yarns and increased ease of manufacturing. However, the lack of a robust numerical tool for modeling such composites structures have restricted their wide-spread adoption.

The challenges associated with numerical modeling of woven composites can be threefold, namely, (a) difficulties in defining the geometry of the textile unit cell due to inherent complexity in the architecture, (b) development of robust and computationally-efficient numerical models and (c) issues associated with scale separation. Specialized pre-processing tools such as TexGen provides an idealized representation of the textiles based on the specified user inputs [1]. Such approaches often neglect realistic features such as yarn waviness, which could significantly alter the strength of the woven textile [2, 3]. Even though analytical models provide solution instantaneously, they are often restricted to specific weave geometry configuration and often lack the proper resolution of stress and strain fields [4]. Bednarcyk and Arnold developed a two-step homogenization procedure based on Generalized Method of Cells (GMC) to predict elastic properties of woven composites [5]. Most of the approaches for numerical modeling of woven composite found in the literature are based on standard finite element (FE) approach [3, 6]. Voxel-based FE methods are quite popular as it drastically reduces the pre-processing time for unit cell generation, but they often produce inaccurate stress fields, especially along the interfaces [7]. Ricks et al. compared different modeling strategies to predict the effective properties of various 3D orthogonal woven composites and compared it against traditional FE approach [8]. Even though most of the available tools provide reasonable estimates for the overall moduli of the textile composites, lack of high-fidelity stress resolution within the individual constituents can lead to unreliable strength predictions [9].

The paper presents a novel modeling strategy based on one-dimensional beam models for modeling woven composites. Various components of the woven composite are modeled via the Component-wise approach (CW) adopted within the

[1]Postdoctoral Researcher, Department of Mechanical and Aerospace Engineering, Corso duca delgi Abruzzi 24, 10129, Torino, Italy
[2]Postdoctoral Researcher, Department of Mechanical and Aerospace Engineering, Corso duca degli Abruzzi 24, 10129, Torino, Italy
[3]Assistant Professor, Department of Mechanical and Aerospace Engineering, Corso duca degli Abruzzi 24, 10129, Torino, Italy
[4]Assistant Professor, Department of Mechanical and Aerospace Engineering, Corso duca degli Abruzzi 24, 10129, Torino, Italy
[5]Professor, Department of Mechanical and Aerospace Engineering, Corso duca degli Abruzzi 24, 10129, Torino, Italy
[6]Research Aerospace Engineer, Multiscale and Multiphysics Modeling Branch, AIAA Member
[7]Research Aerospace Engineer, Multiscale and Multiphysics Modeling Branch, AIAA Senior Member
[8]Research Aerospace Engineer, Multiscale and Multiphysics Modeling Branch, AIAA Associate Fellow
[9]Technical Lead, Multiscale and Multiphysics Modeling Branch, AIAA Member

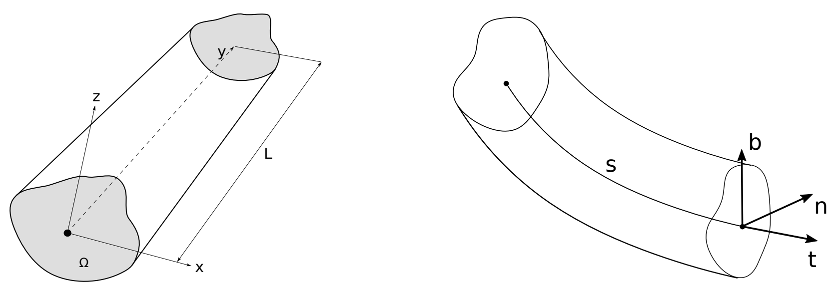
scheme of Carrera Unified Formulation (CUF) [10]. The ability of CUF-CW models to accurately resolve stress and strain fields is exploited to model the woven composite unit cell.

II. One-dimensional CUF theories and finite element formulation

CUF decomposes the 3D displacement field into [10]:

$$\mathbf{u}(x, y, z) = F_\tau(x, z)\, \mathbf{u}_\tau(y) \qquad \forall \qquad \tau = 1, \ldots, M \tag{1}$$

where $\mathbf{u}$ is the generic displacement vector, F_τ is the cross-section expansion function of the beam defined over the plane $x - z$ with M terms and Einstein notation acts on τ. The class of function adopted as F_τ determines the theory of structure and its order, for instance, F_τ can use Taylor polynomials, trigonometric or exponential expansions, and combination thereof. This work adopts Lagrange-based polynomials as expansion functions, henceforth referred to as LE models [11]. The CW approach, an extension of LE models, is adopted to model various beam configurations of the textile weave. A nine-node (L9) Lagrange polynomial function is utilized to describe the kinematic field over the cross-section for different components of a given beam element. Based on the curvature of the textile weave components, two kinds of beam models are formulated: (a) Straight beams for components without curvature and (b) Curved beam for woven components.



(a) Straight beams **(b) Curved beams**

Fig. 1 Coordinate systems for one-dimensional CUF models

A. Straight beams

The cartesian coordinate system adopted for a generic straight beam is depicted in Fig. 1a with cross-section Ω overlayed on $x - z$ plane with beam along the y axis. By adopting standard FE shape functions, the displacement field in Eqn. II can be reformulated as:

$$\mathbf{u}(x, y, z) = F_\tau(x, z)\, N_i(y)\mathbf{u}_{\tau i} \quad \forall \quad \tau = 1, \ldots, M, i = 1, \ldots, p + 1 \tag{2}$$

where N_i is the i^{th} shape function of a beam element of order p and $\mathbf{u}_{\tau i}$ is the generic nodal displacement vector. The virtual variation of strain energy is reformulated using the Principle of Virtual Work:

$$\delta L_{int} \qquad = \delta \mathbf{u}_{sj} \int_V \left\{ \mathbf{D}^T\ \mathbf{C}\, \mathbf{D}\, dV \right\} \mathbf{u}_{\tau i} \tag{3}$$

$$= \delta \mathbf{u}_{sj}\ \mathbf{k}_{\tau sij}\ \mathbf{u}_{\tau i} \tag{4}$$

where $\mathbf{k}_{ij\tau s}$ is termed as the Fundamental Nucleus (FN) of the structural stiffness for 1D CUF FE model, $\mathbf{C}$ is the material matrix and $\mathbf{D}$ is the differential operator defining strain-displacement relationship. The expansion of indices i, j, τ, s leads to construction of the structural stiffness matrix of a single 1D CUF finite element.

B. Curved beams

As illustrated in Fig. 1b, a Frenet-Serret coordinate system is adopted. The displacement vector for the curved beam can be defined as

$$\mathbf{u} = u_s \mathbf{t} + u_\xi \mathbf{n} + u_\eta \mathbf{b}, \tag{5}$$

with $\mathbf{u} = \{u_s \; u_\xi \; u_\eta\}^T$ in the Frenet-Serret frame. With small strain assumptions, the geomtrical relations read:

$$\boldsymbol{\varepsilon}_C = (\boldsymbol{D}_M + \boldsymbol{D}_S)\,\mathbf{u}, \qquad \boldsymbol{\varepsilon}_\Omega = \boldsymbol{D}_\Omega\,\mathbf{u} \tag{6}$$

where $\boldsymbol{D}_M$ is the differential operator accounting for the axial terms, $\boldsymbol{D}_S$ is that of the shear terms and $\boldsymbol{D}_\Omega$ is the one for the cross-sectional deformations. Within the scheme of CUF, the displacement vector can be expressed as [12]:

$$\mathbf{u}(s,\xi,\eta) = N_i(s)F_\tau(\xi,\eta)\mathbf{u}_{\tau i}(s), \qquad \tau = 1,\dots,M, \quad i = 1,\dots,p+1 \tag{7}$$

where $\mathbf{u}_{\tau i} = \{u_{s_{\tau i}} \; u_{\xi_{\tau i}} \; u_{\eta_{\tau i}}\}^T$ is the generalized displacement vector containing the beam unknowns and N_i is the i^{th} shape function of beam element of order p. Using the PVW, the variation of virtual internal work with contributions from longitudinal strains (δL_{int_C}) and sectional strains (δL_{int_Ω}) can be expressed as:

$$\delta L_{\text{int}} = \delta L_{int_C} + \delta L_{int_\Omega} = \delta \mathbf{u}_{\tau i}^T(\,\mathbf{k}_{CC}^{\tau\varsigma ij} + \mathbf{k}_{C\Omega}^{\tau\varsigma ij} + \mathbf{k}_{\Omega C}^{\tau\varsigma ij} + \mathbf{k}_{\Omega\Omega}^{\tau\varsigma ij}\,)\mathbf{u}_{\varsigma j}\,, \tag{8}$$

where the $\mathbf{k}^{\tau\varsigma ij}$ matrices are the fundamental nuclei of the stiffness matrix containing contributions from the integrals of the expansion functions, indicated by the indexes τ and ς, and those of the 1D interpolating functions, indicated by i and j. Detailed information on the locking-free curved beam formulation within CUF can be found in [12].

C. Modeling approach

In this work, two classes of FE are adopted to model various components of the woven textiles, namely (a) higher-order straight and curved beams and (b) standard 3D FE. As illustrated in Fig. 2, a generic textile is decomposed into two components: tows and filling matrix. The tows are modeled as an assembly of 1D higher-order straight and curved beams. Based on the tow geometry, the cross-section of the beam is modeled using the CW approach with multiple LE elements. Standard 3D elements are implemented within the framework to model the filling matrix. In this manner, computational cost should be reduced by replacing 3D elements for the tow, a standard approach, with significantly more efficient higher-order beam elements. Displacement continuity across the interface of matrix and wefts are enforced through the introduction of tie-constraints. In this work, the Tex-Gen tool is used to the generate textile weave architecture [1].

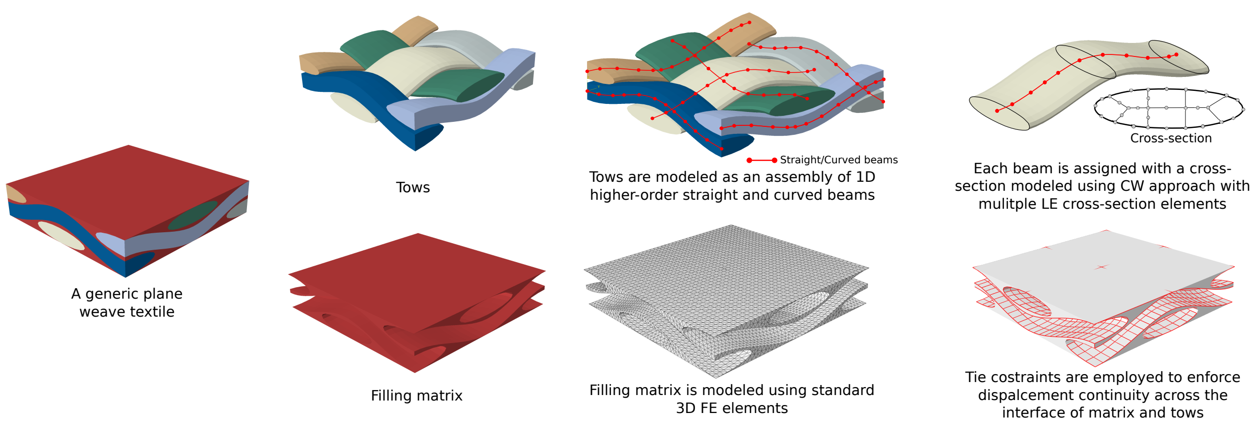


Fig. 2 Illustration of the modeling approach adopted for a generic textile weave composite using CUF-CW models

III. Numerical results

This section presents three sets of results concerning the analysis of a non-crimp textile composite, a curved tow subjected to tensile loading, and a dry woven textile unit cell.

A. Non-crimp textile composite

The CW technique adopted to model the non-crimp textile composite is illustrated in Fig. 3. Two sets of beam assembly using 5 four-node cubic beam elements (B4) were used to model the textile with a degrees of freedom (DOFs)

Table 1 Material properties for the analysis of non-crimp textile composite

	E_{11}	E_{22}/E_{33}	ν_{12}/ν_{13}	ν_{23}	G_{12}/G_{13}
	(GPa)	(GPa)	(-)	(-)	(GPa)
Tow	184.5	10.13	0.26	0.26	6.95
Matrix	3.5	3.5	0.35	0.35	1.30

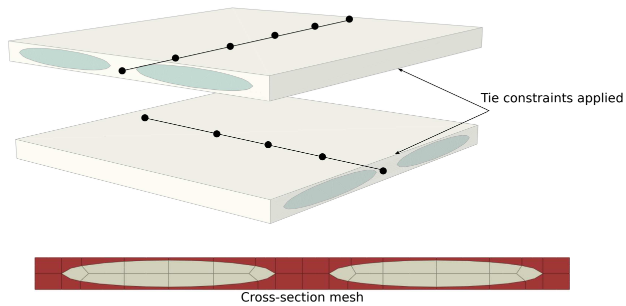
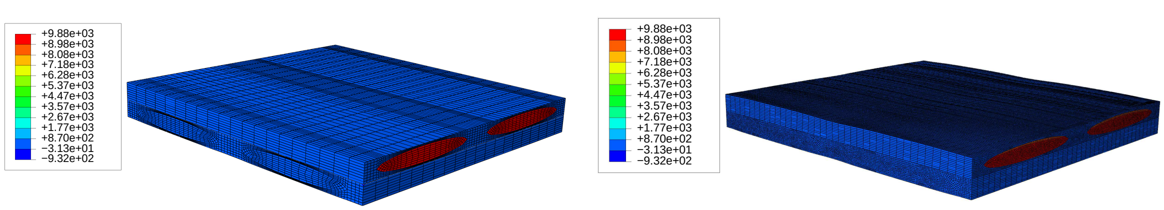


Fig. 3 CW modeling of a non-crimp textile under uniaxial tension

of 23,520. The material properties for the transversely isotropic fiber tows and isotropic matrix are tabulated in Table 1. A similar model was developed in ABAQUS using 229,040 linear brick elements amounting to 756,696 DOFs. The mesh density of the ABAQUS model was chosen after a mesh convergence study to produce similar fields in comparison to CUF-CW model. A displacement of 0.1 mm is applied on the faces of the RUC. Figure 4 compares the stress σ_{11} for CUF-CW and ABAQUS model. The analysis time of CUF-CW was less than a minute, whereas the ABAQUS model required 7 minutes to complete the analysis. It is evident from Fig. 4 that CUF-CW and ABAQUS 3D model produces similar stress fields, but the DOFs of the former is an order of magnitude fewer than the latter.

B. Curved tows under tension

The second set of numerical results deal with the modeling of two curved tows under tension using curved beams. The study intends to highlight the efficiency of curved CUF-CW models to produce high-fidelity solution in comparison to standard 3D FE approach. As depicted in Fig. 5a, each tow is modeled using 4 four-node cubic curved beam element amounting to 4134 DOFs. The tow is assumed to be transversely isotropic with material properties tabulated in Table 1. The tows are clamped at one end and displacement of 0.1 mm is applied at the other. To emphasis upon the efficiency of the CW models, three sets of ABAQUS 3D model with varying mesh density are modeled. Table 2 lists the model information for different models along with analysis time and maximum stress. Figure 6 compares the normal stress σ_{11} of CUF-CW against various ABAQUS 3D models. It can be observed that the absence of aspect ratio constraints in CUF-CW models along with the ability to capture curvature accurately yields high-fidelity stress solutions. CUF-CW

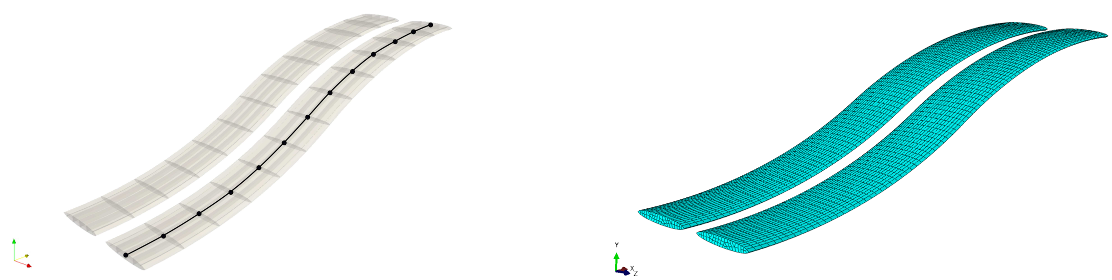


(a) CUF-CW **(b) ABAQUS 3D**

Fig. 4 Comparison of σ_{11} (kPa) for different models at applied displacement of 0.1 mm

Table 2 Model information along with maximum stress σ_{11} (Pa) for the analysis of curved tows under tension loading

Model	Information	DOF	σ_{11}^{max} (MPa)	Analysis time [s]
CUF-CW	4 four-node cubic curved beam element per tow	4,134	338.3	5
ABAQUS 3D - Coarse	Linear brick element (C3D8) with an average element size of 0.10	32,640	246.8	10
ABAQUS 3D - Medium	Linear brick element (C3D8) with an average element size of 0.05	120,204	309.7	17
ABAQUS 3D - Refined	Linear brick element (C3D8) with an average element size of 0.01	1,260,699	328.9	265



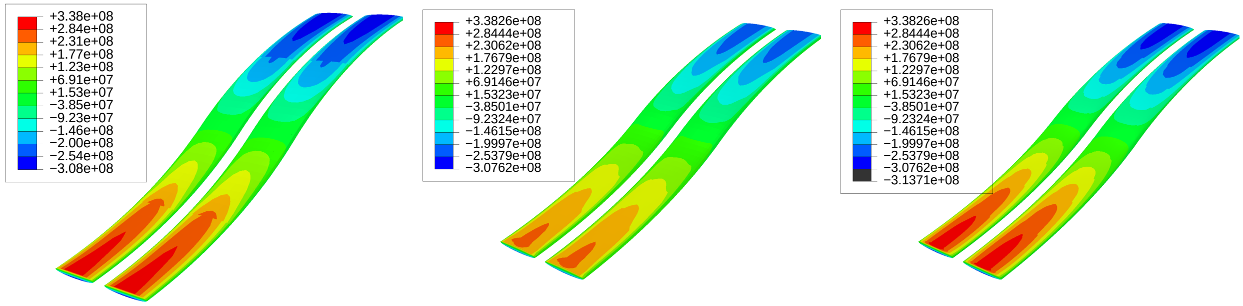
(a) CUF-CW **(b) ABAQUS 3D - Coarse**

Fig. 5 Model configuration of the curved tow under tension

models tends to outperform standard 3D FEM with multi-fold efficiency in terms of problem size and analysis times. It clearly illustrates that for woven systems, current formulation extracts most, if not all, of the speedup by replacing 3D elements for tows with CUF beam elements.

C. Dry woven textile unit cell

The last numerical example deals with the analysis of a dry woven textile composite under biaxial loading. The architecture and loading conditions are depicted in Fig. 7 with u^* of 0.001 mm. The planar dimensions are $0.44 \times 0.44 mm$ with an elliptical tow of 0.18 and 0.038 mm. The orthotropic material property assumed for the tow is listed in Table 3. To compare the accuracy of the CUF-CW models, similar three-dimensional models are developed using ABAQUS. Table 3 enlists the information for the different numerical models along with peal stress observed in the tow. Figure 3 compares the longitudinal tow stress for CUF-CW and ABAQUS-refined model. Once again, it is evident from the results that CUF-CW shows multi-fold efficiency in capturing the complex deformation state and stress fields for dry textile.



(a) CUF-CW **(b) ABAQUS 3D - Coarse** **(c) ABAQUS 3D - Refined**

Fig. 6 Comparison of σ_{11} (Pa) for curved tows under tension

Table 3 Material properties for the analysis of dry textile composite

E_{11} (GPa)	E_{22}/E_{33} (GPa)	ν_{12}/ν_{13} (-)	ν_{23} (-)	G_{12}/G_{13} (GPa)
165.0	9.0	0.34	0.34	5.6

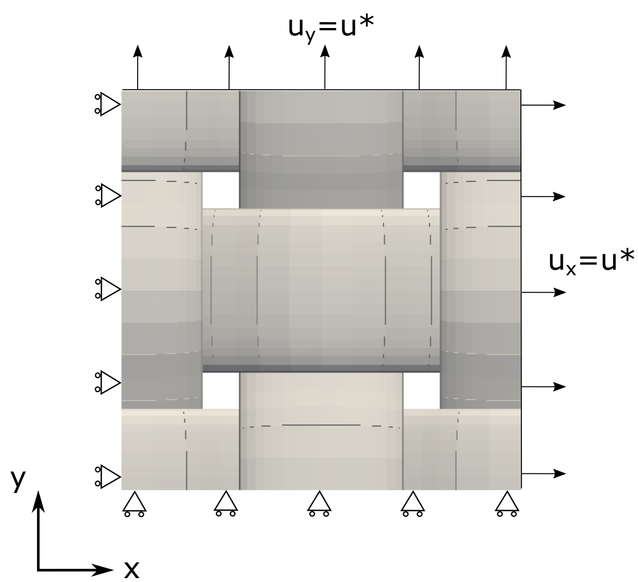
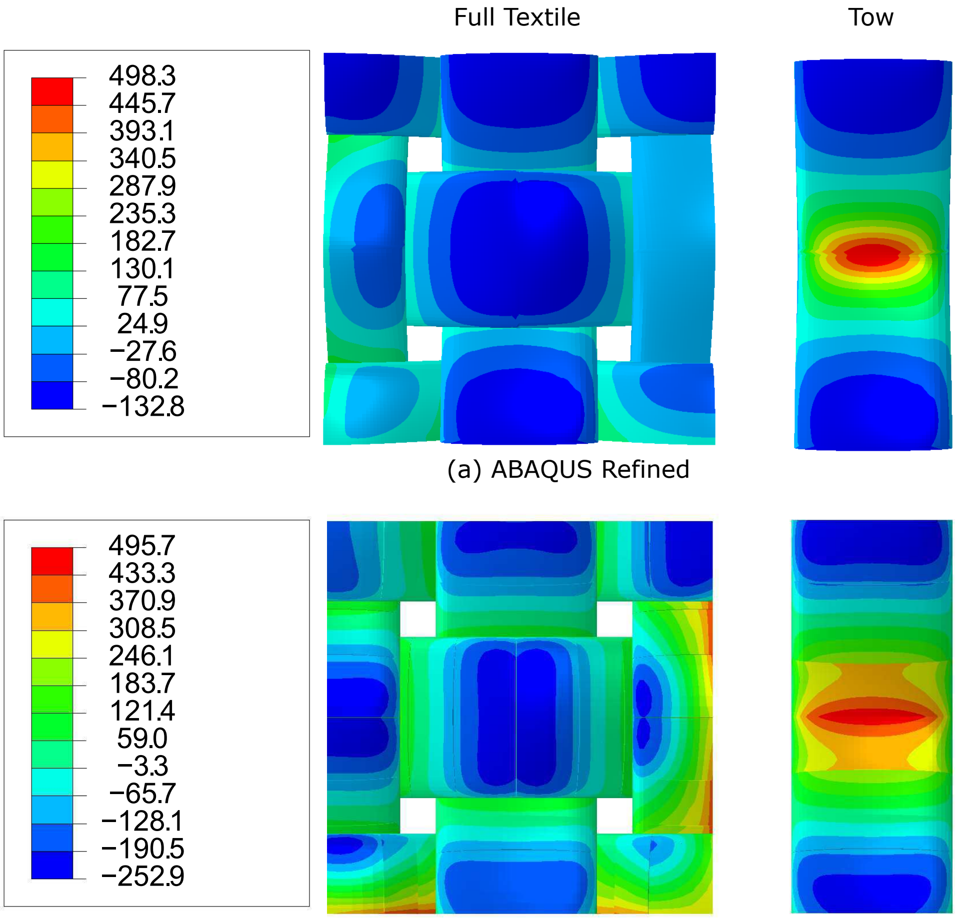


Fig. 7 Textile architecture and the boundary condition for dry textile under biaxial tension



(b) CUF

Fig. 8 Comparison of longitudinal stress σ_{11} (MPa) for dry textile under biaxial tension

Table 4 Model information along with maximum stress σ_{11} for the analysis of dry textile under biaxial loading

Model	Information	Peak longitudinal stress [MPa]	DOF	Analysis time [s]
ABAQUS Coarse	Linear brick element (C3D8) with average element size of 0.0075 mm	392.6	105,042	33
ABAQUS Refined	Linear brick element (C3D8) with average element size of 0.004 mm	498.3	638,940	129
CUF-CW	8 three-noded quadratic curved beam per tow with a cross-section with 18 L9 elements	495.7	19,482	25

IV. Conclusion

The paper presented a novel approach to modeling woven composites within a computationally efficient framework. Various classes of one-dimensional finite element models were built within the scheme of Carrera Unified Formulation (CUF) to model different components of the woven composite unit cell. The ability of CUF models to accurately resolve stress and strain fields efficiently was exploited to capture complex deformations. The computational cost of CUF models is some 10-20 times less than 3D elements in terms of computational times. Future developments should deal with the modeling of the complete woven cell, i.e., by considering the matrix component. Furthermore, the present framework could be of interest for multiscale nonlinear analyses in which the reduction of computational times is of significant importance.

V. Acknowledgments

The Politecnico di Torino authors acknowledge funding from the European Union Horizon 2020 Research and Innovation program under the Marie Sklodowska-Curie grant agreement No. 642121, project FULLCOMP (FULLy analysis, design, manufacturing, and health monitoring of COMPosite structures). The NASA authors wish to acknowledge the support of NASA's Transformational Tools and Technologies (TTT) program.

References

[1] Lin, H., Zeng, X., Sherburn, M., Long, A. C., and Clifford, M. J., "Automated geometric modelling of textile structures," *Textile Research Journal*, Vol. 82, No. 16, 2012, pp. 1689–1702. doi:10.1177/0040517511418562.

[2] Cox, B. N., Dadkhah, M. S., and Morris, W., "On the tensile failure of 3D woven composites," *Composites Part A: Applied Science and Manufacturing*, Vol. 27, No. 6, 1996, pp. 447 – 458. doi:https://doi.org/10.1016/1359-835X(95)00053-5, URL `http://www.sciencedirect.com/science/article/pii/1359835X95000535`.

[3] Green, S., Matveev, M., Long, A., Ivanov, D., and Hallett, S., "Mechanical modelling of 3D woven composites considering realistic unit cell geometry," *Composite Structures*, Vol. 118, 2014, pp. 284 – 293. doi:https://doi.org/10.1016/j.compstruct.2014.07.005, URL `http://www.sciencedirect.com/science/article/pii/S0263822314003146`.

[4] Crookston, J. J., Long, A. C., and Jones, I. A., "A summary review of mechanical properties prediction methods for textile reinforced polymer composites," *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, Vol. 219, No. 2, 2005, pp. 91–109. doi:10.1243/146442005X10319.

[5] Bednarcyk, B. A., and Arnold, S. M., "Micromechanics-Based Modeling of Woven Polymer Matrix Composites," *AIAA Journal*, Vol. 41, No. 9, 2003, pp. 1788–1796. doi:10.2514/2.7297.

[6] Tsukrov, I., Bayraktar, H., Giovinazzo, M., Goering, J., Gross, T., Fruscello, M., and Martinsson, L., "Finite Element Modeling to Predict Cure-Induced Microcracking in Three-Dimensional Woven Composites," *International Journal of Fracture*, Vol. 172, No. 2, 2011, pp. 209–216. doi:10.1007/s10704-011-9659-x, URL `https://doi.org/10.1007/s10704-011-9659-x`.

[7] Kim, H. J., and Swan, C. C., "Voxel-based meshing and unit-cell analysis of textile composites," *International Journal for Numerical Methods in Engineering*, Vol. 56, No. 7, 2003, pp. 977–1006. doi:10.1002/nme.594.

[8] Ricks, T. M., Farrokh, B., Bednarcyk, B. A., and Pineda, E. J., *A Comparison of Different Modeling Strategies for Predicting Effective Properties of 3D Woven Composites*, 2019. doi:10.2514/6.2019-0692, URL `https://arc.aiaa.org/doi/abs/10.2514/6.2019-0692`.

[9] Bahei-El-Din, Y., Rajendran, A., and Zikry, M., "A micromechanical model for damage progression in woven composite systems," *International Journal of Solids and Structures*, Vol. 41, No. 9, 2004, pp. 2307 – 2330. doi:https://doi.org/10.1016/j.ijsolstr.2003.12.006, URL `http://www.sciencedirect.com/science/article/pii/S0020768303007054`.

[10] Carrera, E., Cinefra, M., Zappino, E., and Petrolo, M., *Finite Element Analysis of Structures Through Unified Formulation*, 2014. doi:10.1002/9781118536643.

[11] Carrera, E., and Petrolo, M., "Refined Beam Elements with only Displacement Variables and Plate/Shell Capabilities," *Meccanica*, Vol. 47, 2011, pp. 537–556.

[12] de Miguel, A., de Pietro, G., Carrera, E., Giunta, G., and Pagani, A., "Locking-free curved elements with refined kinematics for the analysis of composite structures," *Computer Methods in Applied Mechanics and Engineering*, Vol. 337, 2018, pp. 481 – 500. doi:https://doi.org/10.1016/j.cma.2018.03.042, URL `http://www.sciencedirect.com/science/article/pii/S004578251830166X`.