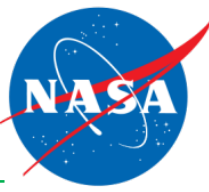# Acoustic Analysis of Sensor Ports

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Outline

- Background

- Design Guidelines

- Analytical Theory
  - Thermoviscous Effects
  - Radiation Acoustics

- Numerical Modeling
  - Comparison to Analytical Model
  - Nonlinear Effects

- Specialized Filter Design for Data Correction

Background

- **Acquiring data that is representative of an environment can often be difficult due to a multitude of influences:**
  - acquisition system electronics, sensing components, cables
  - contamination from mains electrical noise
  - sensitivity to other environments

- Another source of contamination that is often not considered – **it may not always be possible to install a dynamic pressure sensor so that it is mounted flush with the environment of interest**
  - Mounting accessibility
  - Thermal isolation
  - Shock reduction
  - Prevention of debris impingement

- The recess produces an acoustic cavity referred to as a sensor port
  - The sensor port may be small; its contribution to collected data may be very significant
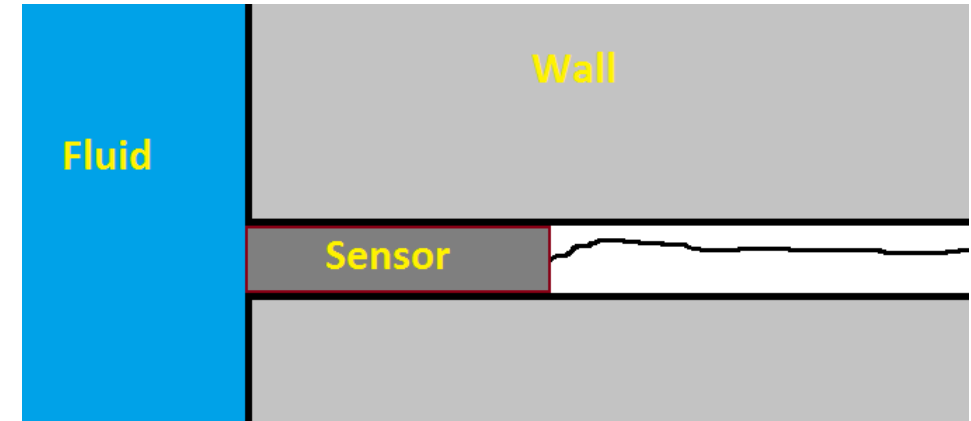
The sensor port is part of the system being measured, but it is not part of the system that is intended to be measured

Background

- The cavity acoustics produce a frequency-dependent amplification and phase deviation that directly affects the data collected

- **Goal**
  - A sensor port acoustic response is desired
    - for port design, data interpretation, or data correction

- **Approach**
  - Develop a theoretical acoustic model that gives the acoustic response of a sensor port
  - Develop a numerical methodology to verify the analytical theory, and that can be extended for more complex port designs
    - Apply numerical modeling to understand more complex effects
  - Compare analytical and numerical models
  - Design a specialized filter that can be used for data correction

Frequency Response Function
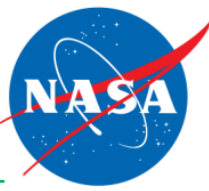
Gain

frequency

Phase
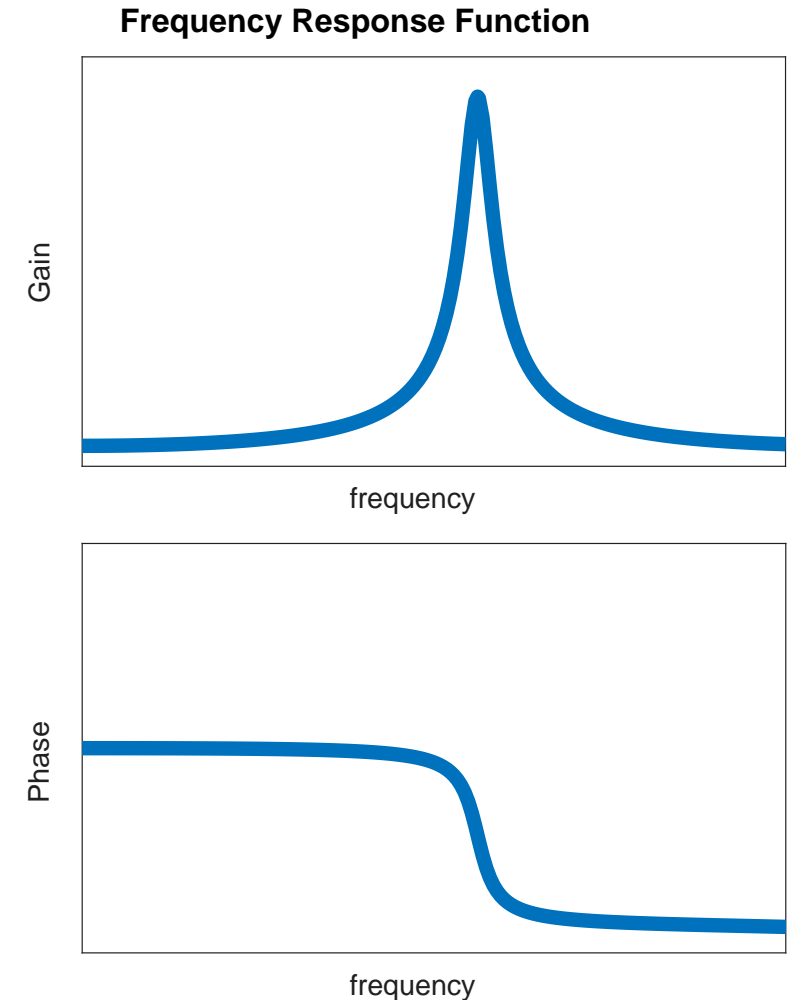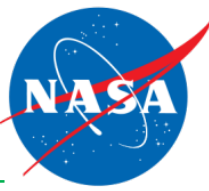
frequency

Design Guidelines

- A desired port configuration is usually flush mounted
  - Acceptable configurations can be close coupled such that frequencies-of-interest are negligibly affected

- Traditional form derived from wave equation at resonance conditions
  - Open boundary is not truly a pressure release boundary
  - A general impedance exists due to the local pressure radiating from the end into the fluid outside the port

$$f_n = \frac{\overline{c}}{4(L + \Delta L)} \qquad \Delta L \approx \frac{8R}{3\pi}$$

- Traditional criteria range between 3x to 5x lower than the natural frequency.
  - Criteria based on a forced undamped single-degree-of-freedom (SDOF) model

Improvements to Design Guidelines

- Using an undamped SDOF model
  - The **gain and gain error** can be written in terms of the frequency multiplication factor, $M = f_n / f_i$

$$D = \frac{M^2}{M^2 - 1} \qquad \varepsilon = \frac{1}{M^2 - 1}$$

  - For $M = 3x$ the amplification is 1.125 (error of 12.5%), and for $M = 5x$ the amplification factor is 1.042x (error of 4.2%).
  - This general approach has been used as a guide to represent the upper bound of the pressure amplification factor.
  - The improvement is describing this in a convenient form.

- A **design length** can also be written in a convenient form

$$L_i \leq \frac{\overline{c}}{4 f_i M} - \frac{8R}{3\pi}$$

- Or more useful is a **design length** written in terms of a *specified flat useable bandwidth* (gain relative error and max frequency-of-interest)

$$L_i \leq \sqrt{\frac{\varepsilon}{\varepsilon + 1} \cdot \frac{\overline{c}}{4 f_i}} - \frac{8R}{3\pi}$$

- Simple Design Estimate (pros/cons)
  - Good for a preliminary estimate
  - Not always guaranteed (limiting assumptions and complex multi-port designs)
  - Can be very conservative, where a longer port may be adequate
  - Simple design estimates do not provide the frequency response for data correction and/or data interpretation

# Sensor Port Frequency Response Model
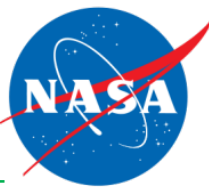
- As an advancement to the undamped SDOF model, a theory is developed for obtaining the acoustic frequency response of a sensor port

- There are three critical advancements
  - Application of the distributed acoustic models
    - Classic lumped acoustic element approach relies on the long wavelength limit
  - Development an exact solution to the thermoviscous wave equation applicable to the framework
  - Formulation of acoustic radiation impedance into an acoustic propagation constant

- Pressure transfer function (complex pressure ratio) must be obtained to extract the amplification factor and relative phase

$$H = \frac{\hat{p}_{out}}{\hat{p}_{in}} \qquad X(f) = \left| \frac{\hat{p}_U}{\hat{p}_D} \right| \qquad \phi(f) = \angle \left(\frac{\hat{p}_U}{\hat{p}_D} \right)$$

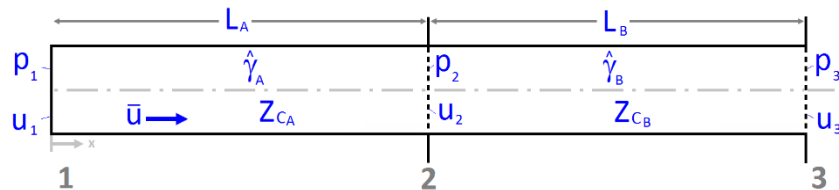# Sensor Port Framework

Wave Equation Model
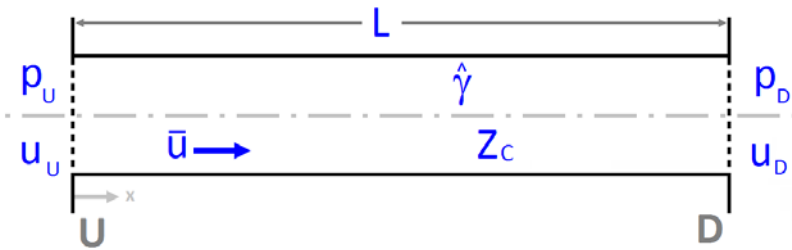
- Impedance model or transfer matrix system. Solution form given as

$$p' = \bar{P}e^{i\omega t}e^{-\hat{\gamma}x}$$

$$\hat{\gamma}_{+} = \alpha + i\beta$$

$$Z = \frac{p'}{u'} \qquad Z_C = -\frac{\bar{\rho}\,\bar{c}^2\hat{\gamma}}{\omega g_c}i$$

- 2-line model is *required* to include both thermoviscous damping and radiation acoustics

$$\frac{p_1}{p_3} = \left(\cosh(\hat{\gamma}_A L_A)\cosh(\hat{\gamma}_B L_B) + \frac{Z_{C,B}}{Z_{C,A}}\sinh(\hat{\gamma}_A L_A)\sinh(\hat{\gamma}_B L_B) \right)^{-1}$$

Thermoviscous Dissipation

- As sound propagates through tubes, losses occur in the acoustic viscous and thermal boundary layer.
    - Other losses to random thermal energy are negligible, e.g., intrinsic thermal and viscous absorption and molecular thermal relaxation

$$\delta_{\mu} = \sqrt{\frac{2\mu}{\omega\bar{\rho}}} \qquad \delta_{\kappa} = \sqrt{\frac{2\mu}{\omega\bar{\rho}\,\text{Pr}}}$$
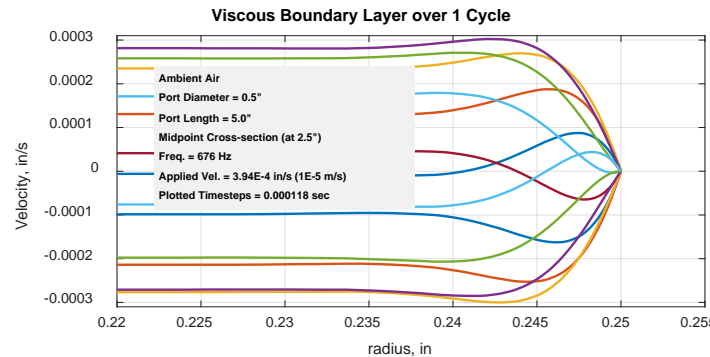
- For a sensor port, the thermoviscous effects are valid over the physical length of the port

$$\gamma_{\mu\kappa} = \frac{1}{\bar{c}R}\cdot\sqrt{\frac{\mu\omega}{2\bar{\rho}}}\left(1+\frac{\gamma-1}{\sqrt{\text{Pr}}}\right)+i\cdot\frac{\omega}{\bar{c}}\cdot\left(1-\frac{1}{R}\sqrt{\frac{\mu}{2\omega\bar{\rho}}}\left(1+\frac{\gamma-1}{\sqrt{\text{Pr}}}\right)\right)^{-1}$$

Note this propagation constant is based on the traditional form of absorption. The exact form advancement is described in backup.

Statement A: Approved for public release; distribution is unlimited.
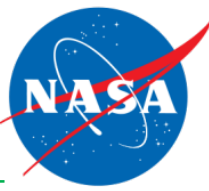
Radiation Acoustics

- An effective damping mechanism is due to the local pressure in the sensor port radiating into the fluid outside of the port
    - Surrounding medium imposes an impedance on the propagating waves which results in attenuation and an added effective length to the port
    - Attenuation due to the geometry is not truly a damping mechanism as no absorption takes place and no energy is lost

- Piston Vibration Theory used to approximate the solution of a pressure wave exiting a flanged port, however results are traditionally linearized
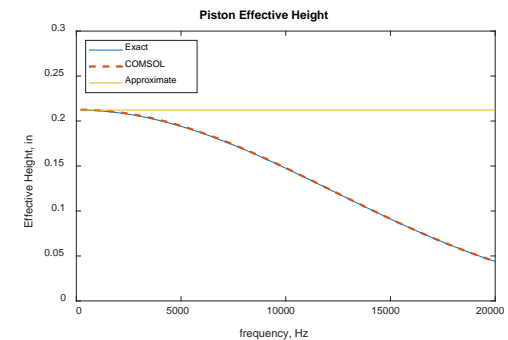
- True solution reveals the end correction is freq. dependent. Resistive component can be significant

- Furthermore formulated as a propagation constant for use in theory

$$\gamma_{rad} = \frac{R\omega^2/\bar{c}^2}{2H_1(2\omega R/\bar{c})}\cdot\left(1-\frac{\bar{c}\cdot J_1(2\omega R/\bar{c})}{\omega R}\right)+i\frac{\omega}{\bar{c}}$$

End Correction

Piston Vibration Theory and the End Correction

- Acoustic radiation that arises from a moving surface is well understood

  - It is described by the radiation impedance, which represents the **resistance** and **mass loading** that is confronted by a pressure wave exiting the port

  - For a circular rigid diaphragm

$$\breve{Z}_{rad} = \breve{R}_{rad} + i\,\breve{X}_{rad} = \overline{\rho c}A\cdot\left[\left(1 - \frac{J_1(2kR)}{kR}\right) + i\left(\frac{H_1(2kR)}{kR}\right)\right]$$

  - Note that the piston reactance term is a function of the 1st order Struve function

Exact End Correction

- Piston vibration theory shows the reactance term contributes to an added mass (see paper)

  - Can be represented by a short continuation of the port

$$m_{rad} = \overline{\rho}A\Delta L = \frac{\breve{X}_{rad}}{\omega}$$

$$\Delta L = \frac{\breve{X}_{rad}}{\overline{\rho}A\omega}$$

  - In terms of mathematical functions

$$\boxed{\Delta L = \frac{H_1(2kR)}{k^2 R}}$$

Note that the end correction is frequency dependent!

Approximate End Correction

- 1st order Struve function can be described as a power series

$$H_1(x) = \sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma\left(m+\frac{3}{2}\right)\Gamma\left(m+\frac{5}{2}\right)}\left(\frac{x}{2}\right)^{(2m+2)}$$
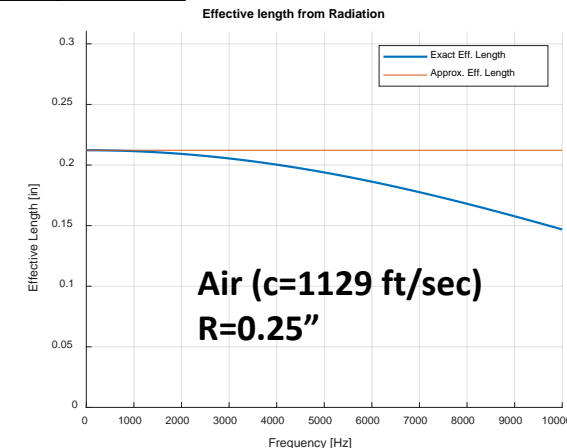
- Note the low frequency approximation (m = 0)

$$H_1(x) \approx \frac{2x^2}{3\pi} \quad \rightarrow \quad H_1(2kR) \approx \frac{8k^2R^2}{3\pi}$$

- Well known end correction – *note there is no frequency dependence*

$$\Delta L \approx \frac{8R}{3\pi}$$

End Correction Comparison

Effective length from Radiation

Air (c=1129 ft/sec)
R=0.25"

Note that the radiation impedance also has a resistance term, that has not yet been considered
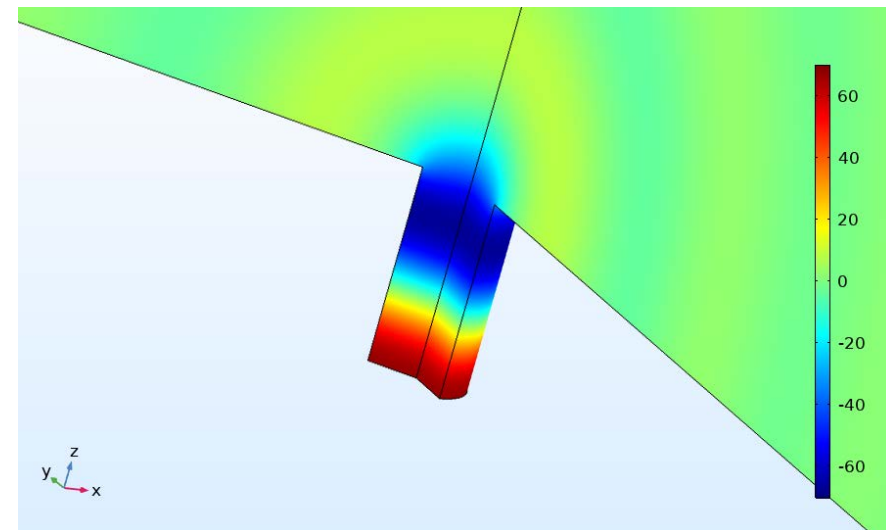
Numerical Modeling

- Break model down to fundamental physics
  - Model piston functions and compare to analytical model
  - Model thermoviscous effects and compare to numerical model
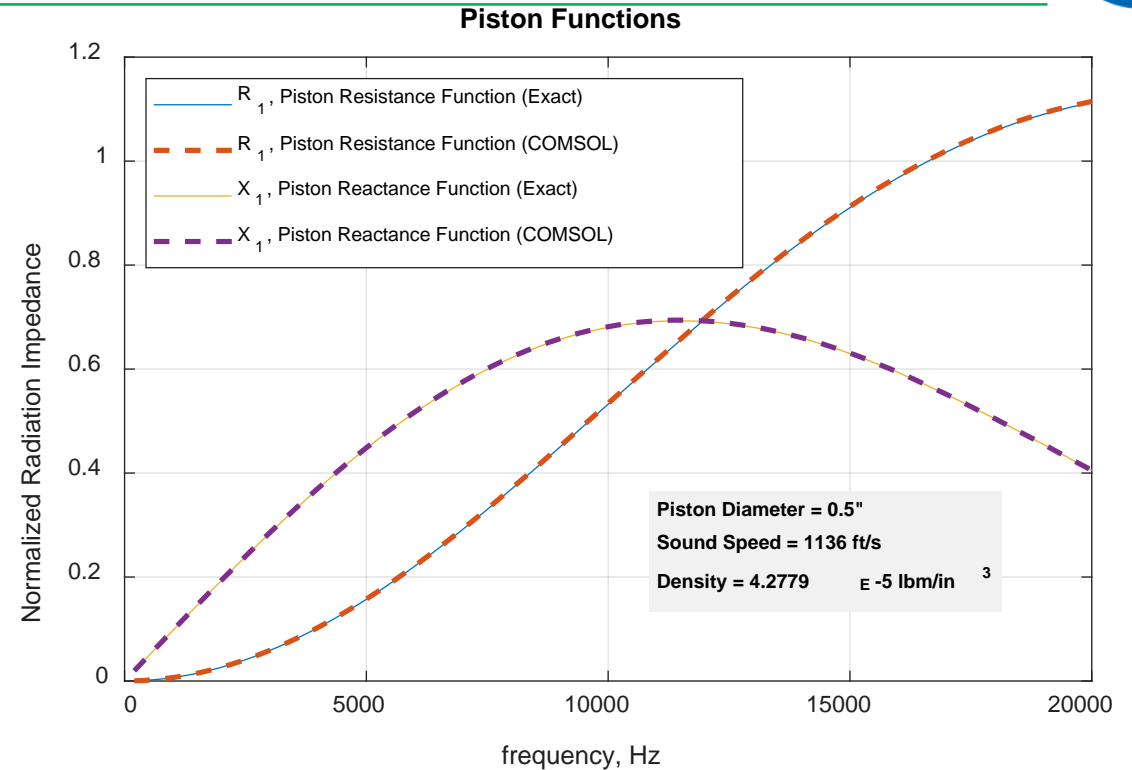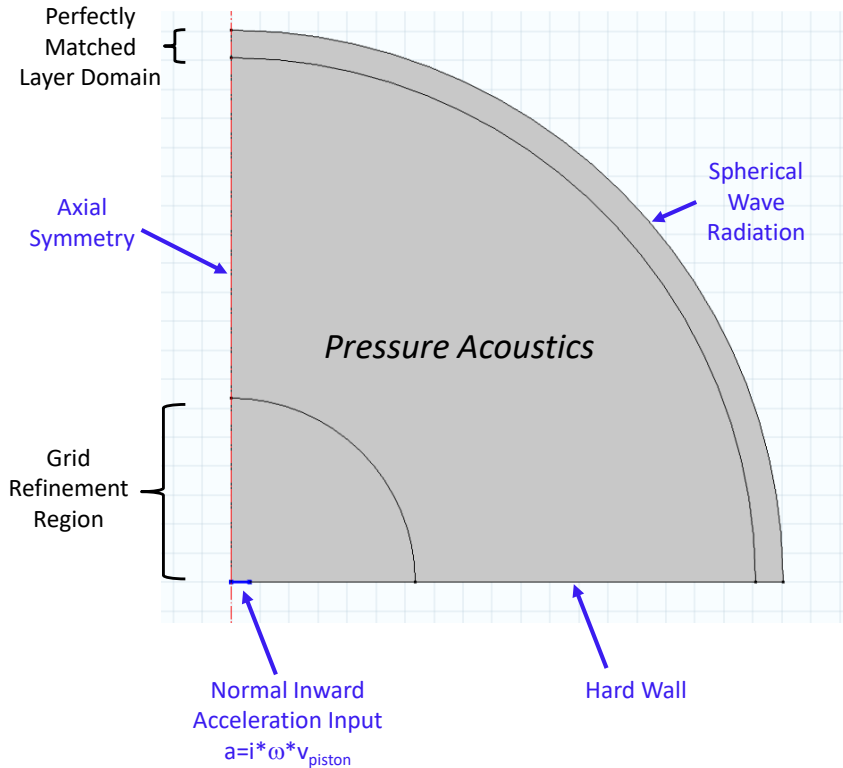 - Model the combined system and compare to theory
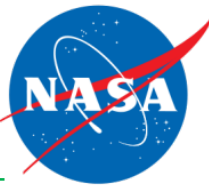
Example Showing Combined System
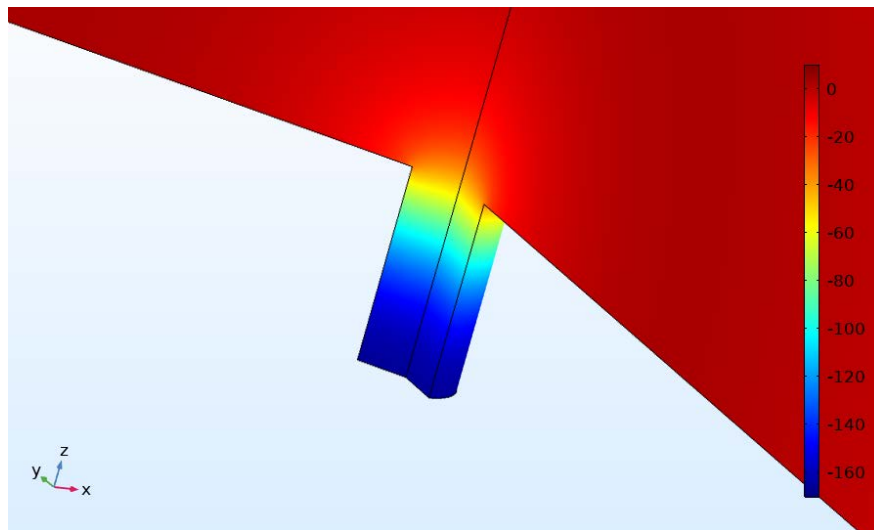

Total Pressure Contour Plot: Quarter-Wave Mode

Total Pressure Contour Plot: Three-Quarter-Wave Mode

Verification for Radiation Physics

Perfectly Matched Layer Domain

Axial Symmetry

Spherical Wave Radiation

Pressure Acoustics

Grid Refinement Region

Normal Inward Acceleration Input
$a = i * \omega * v_{piston}$

Hard Wall

Piston Functions

- R$_1$, Piston Resistance Function (Exact)
- R$_1$, Piston Resistance Function (COMSOL)
- X$_1$, Piston Reactance Function (Exact)
- X$_1$, Piston Reactance Function (COMSOL)

Normalized Radiation Impedance

frequency, Hz

Piston Diameter = 0.5"
Sound Speed = 1136 ft/s
**Density = 4.2779** $_E$ **-5 lbm/in** ^{3}

- Piston Vibration Theory is well established

- Model using COMSOL and compare to theoretical model

- This establishes that the mesh and domain design is adequate
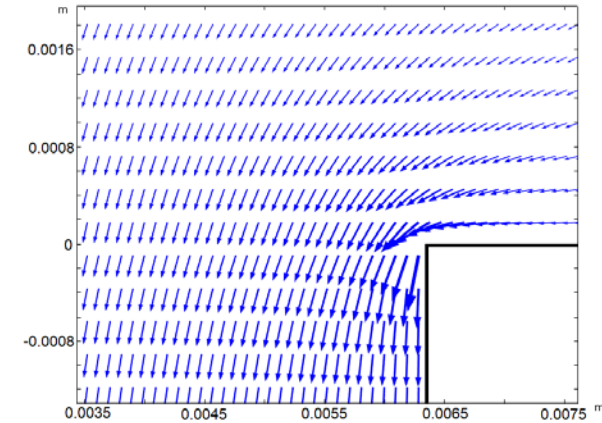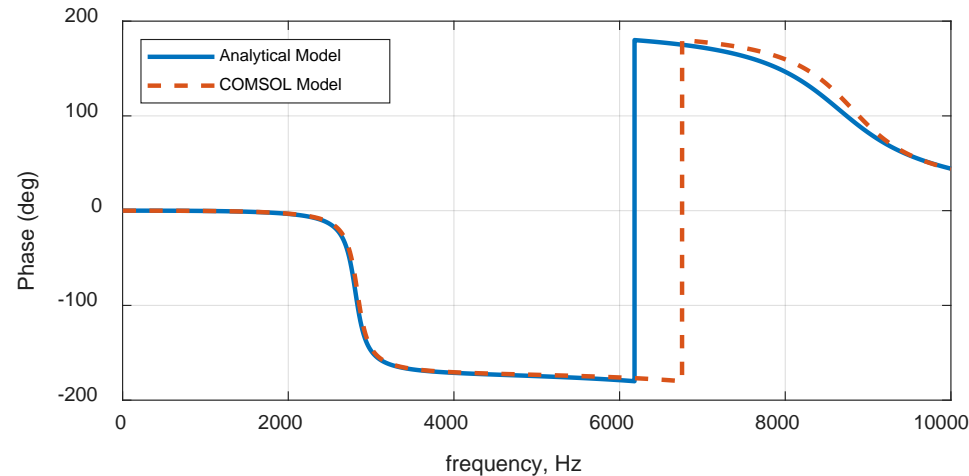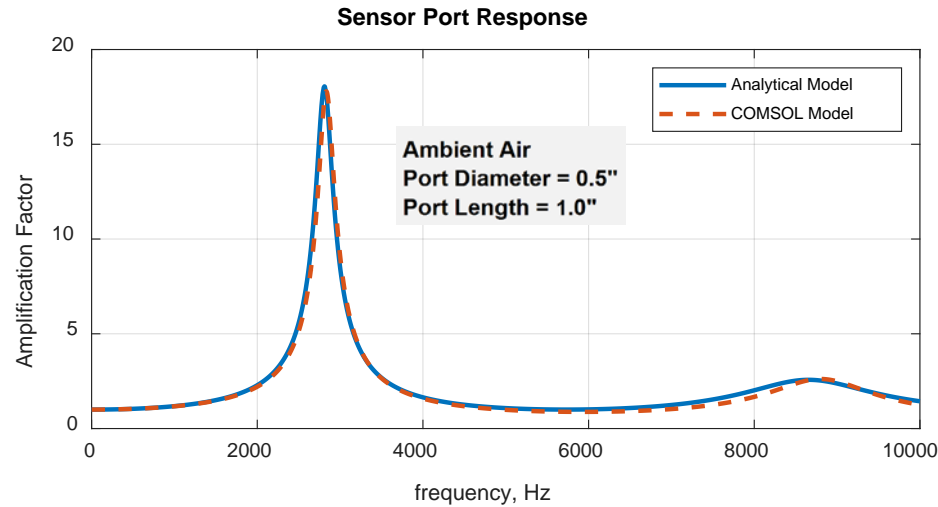
Verification for Thermoviscous Physics
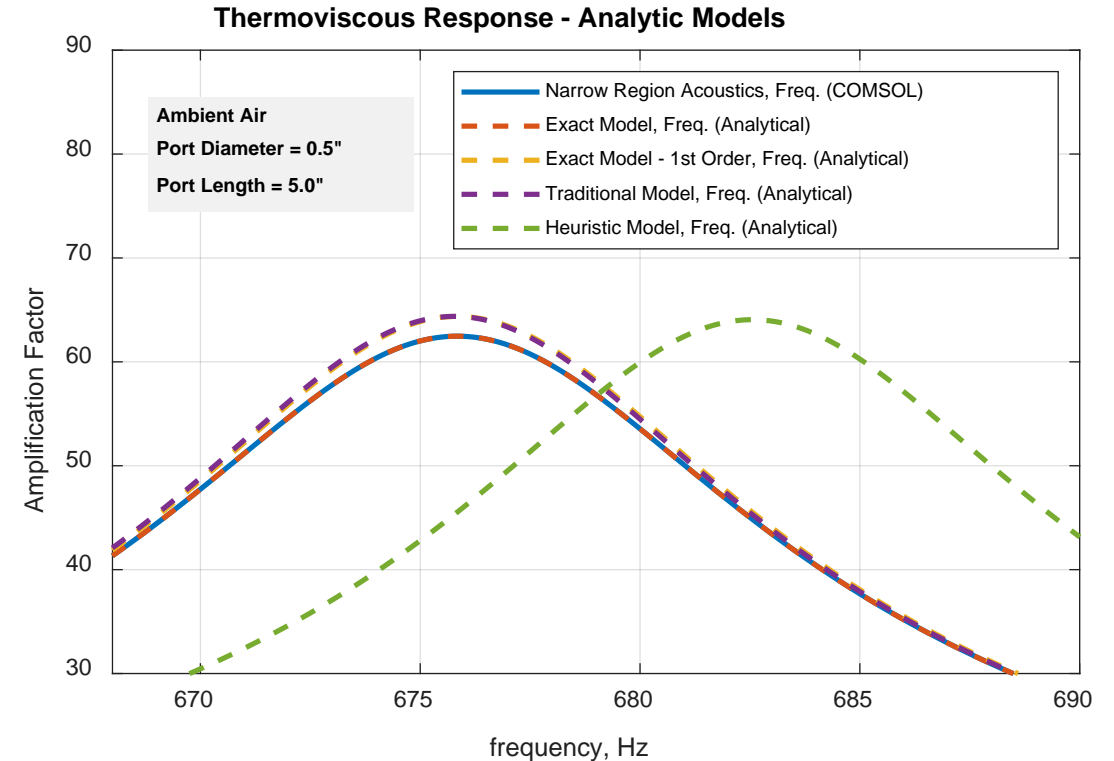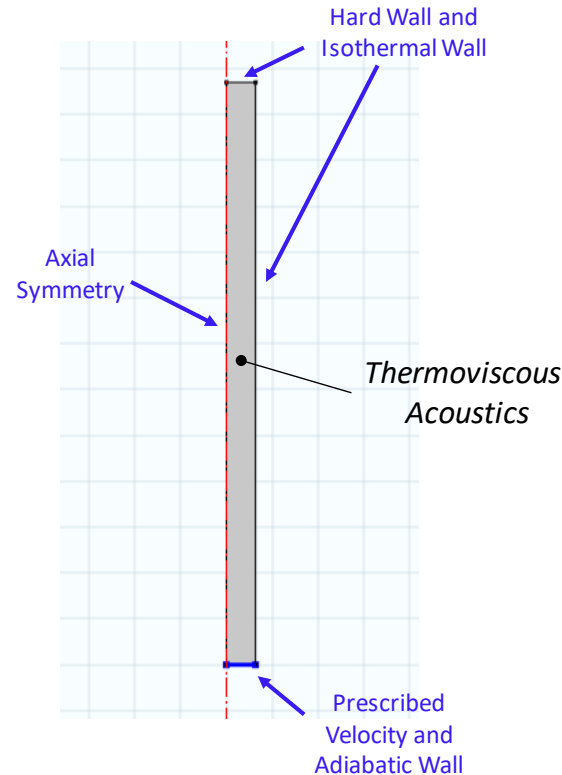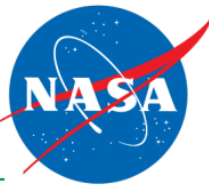
- Thermoviscous influence is well established

- Model using COMSOL and compare to theoretical model

- This establishes that the mesh and domain design is adequate

Sensor Port Modeling
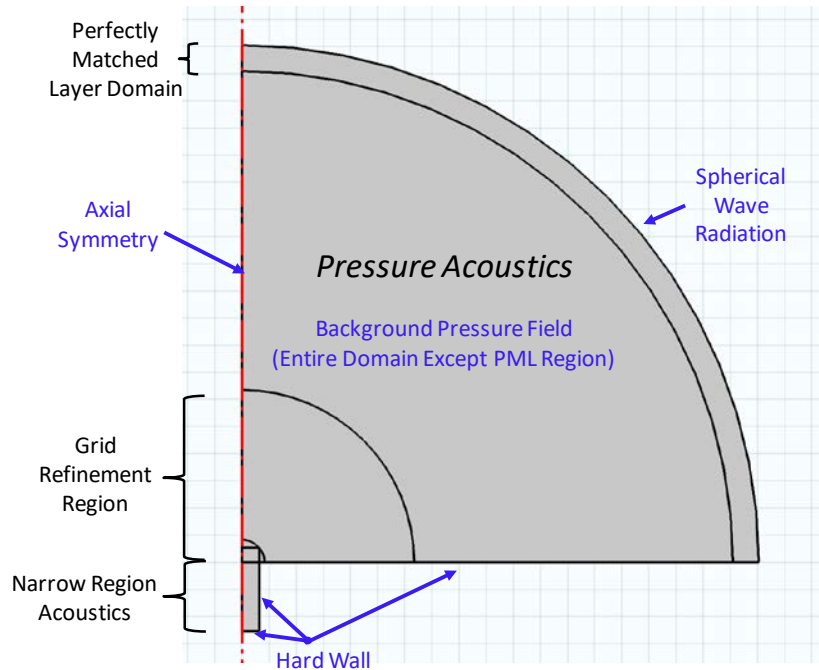
- Theoretical model compares very well to a computational acoustic numeric simulation

- Deterministic numerics is a big advantage

Sensor Port Response

Ambient Air
Port Diameter = 0.5"
Port Length = 1.0"

For this example, note that without radiation acoustics, the analytic model predicts at 142x rather than 18x

- Instantaneous velocity contours zoomed in near port corner

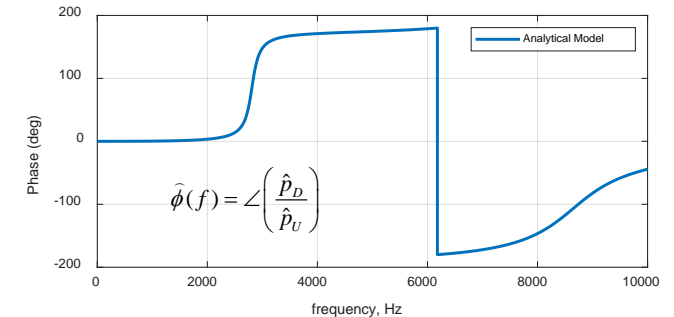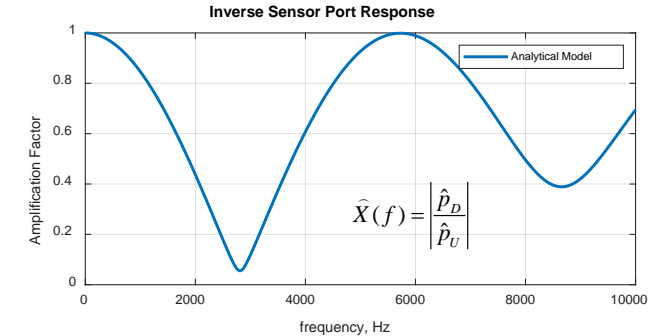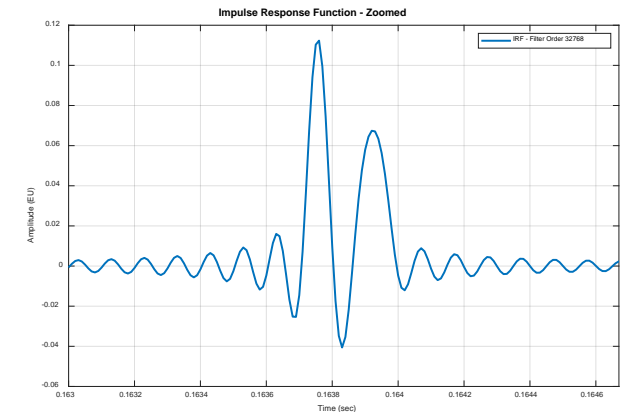- Partly responsible for deviation observed

Multiphysics Modeling
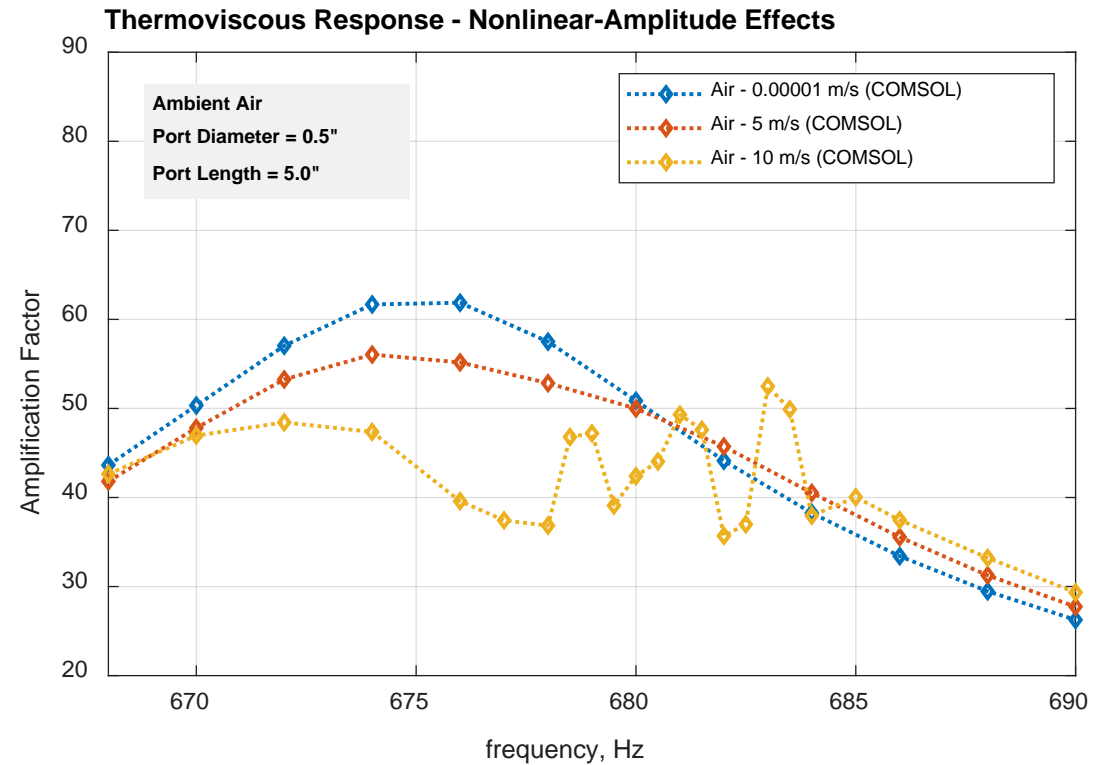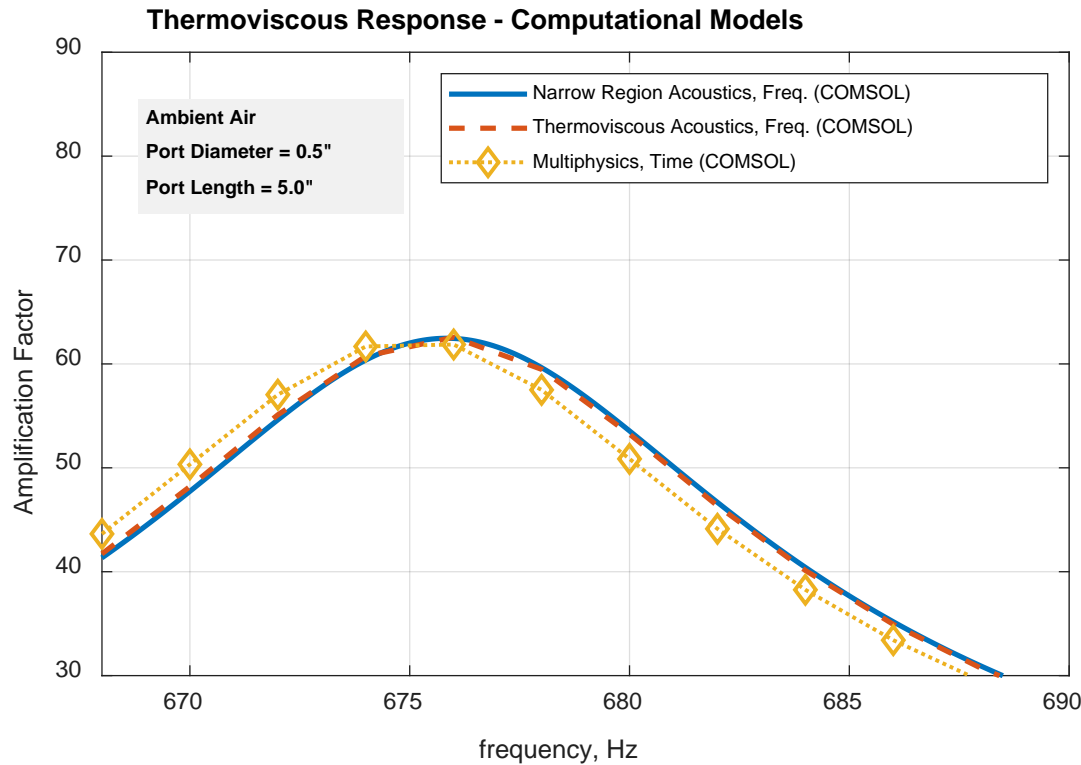
- Numerical models can capture thermoviscous effects more accurately
- Nonlinear thermoviscous effects – secondary resonances
- Can be extended to model liquids, thru-flow, cross-flow, etc.
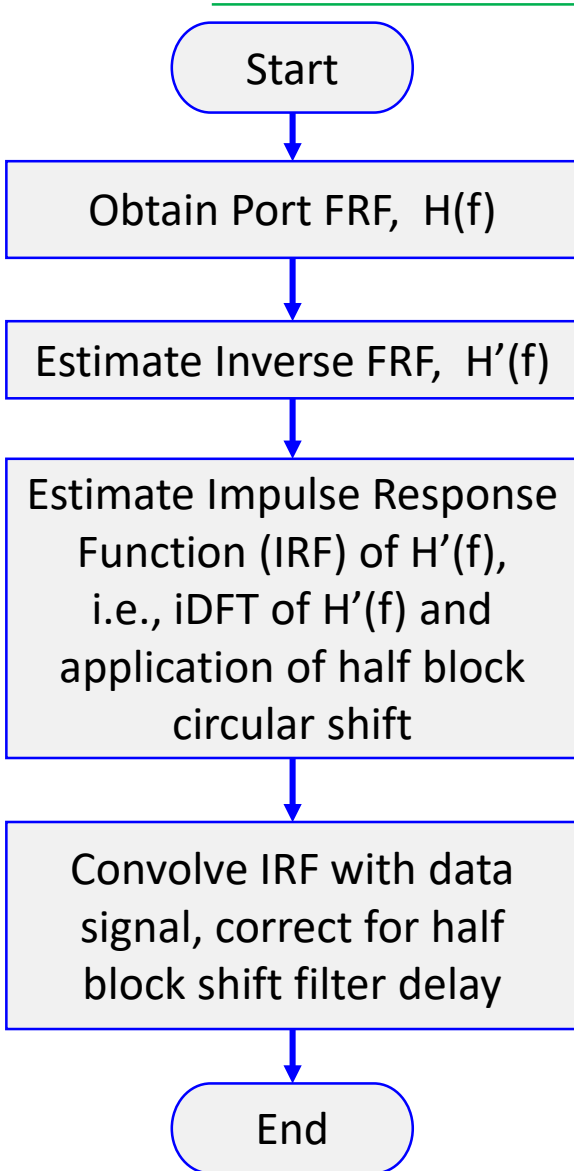
Specialized Filter Design

```
┌─────────────┐
│    Start     │
└─────────────┘
       │
       ▼
┌─────────────────────┐
│ Obtain Port FRF, H(f)│
└─────────────────────┘
       │
       ▼
┌─────────────────────┐
│Estimate Inverse FRF, H'(f)│
└─────────────────────┘
       │
       ▼
┌─────────────────────────┐
│ Estimate Impulse Response│
│ Function (IRF) of H'(f), │
│ i.e., iDFT of H'(f) and  │
│ application of half block│
│    circular shift         │
└─────────────────────────┘
       │
       ▼
┌─────────────────────────┐
│ Convolve IRF with data   │
│ signal, correct for half │
│ block shift filter delay │
└─────────────────────────┘
       │
       ▼
┌─────────────┐
│    End       │
└─────────────┘
```

- In most cases, a filter is used to pass certain frequencies and reject others, but in general a filter is a device that modifies certain frequencies relative to others

- Flowchart describes the use of a frequency response function (transfer function) as a filter, so that the data is modified accordingly

- Process can be applied to correct data known to be contaminated with a sensor port response

Inverse Frequency Response Function

$$\hat{X}(f) = \left| \frac{\hat{p}_D}{\hat{p}_U} \right|$$

$$\hat{\phi}(f) = \angle \left(\frac{\hat{p}_D}{\hat{p}_U} \right)$$

Impulse Response Function

Specialized Filter Design Application
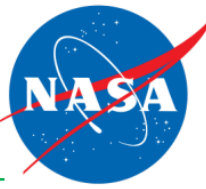
- Simulated data is first produced to show procedure
  - Sinusoid at peak resonance frequency
  - Uniform random
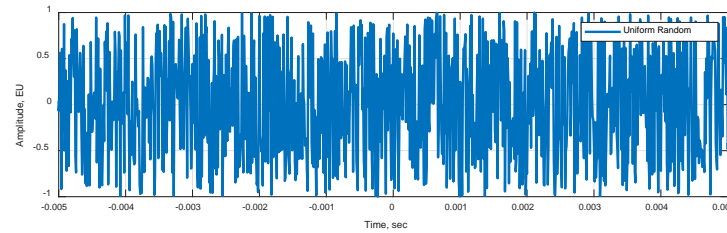- The filtering operation is performed and the processed data is analyzed

PSD of Uniform Random Data and Filtered Random Data

Original and Filtered Sinusoid

Original Uniform Random Data
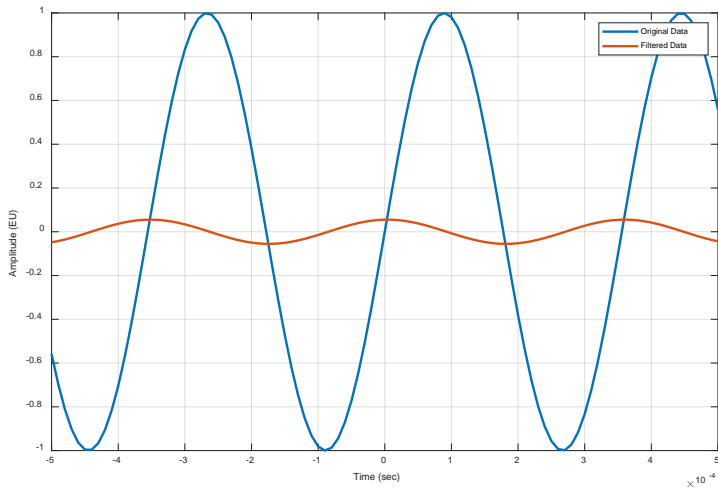
Sinusoid
- Analytic TF gives an amplitude reduction of 18.1x and phase shift of +86˚ - verified from data signal comparison
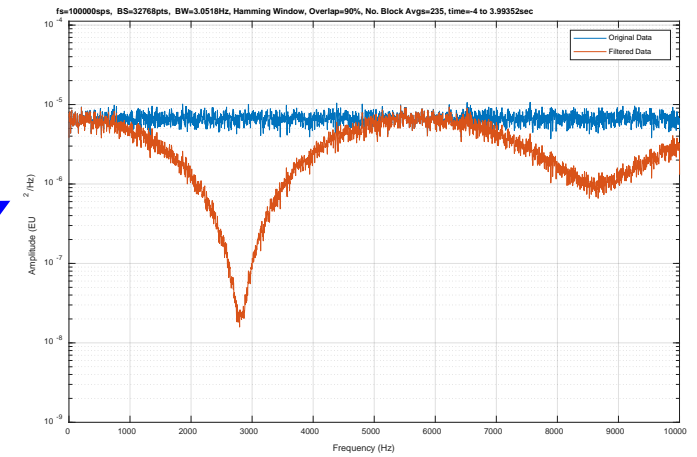
Uniform Random
- Overlay original inverse TF analytical model with the data transfer function
- Data TF estimate made by computing the ratio of the cross power spectral density and reference power spectral density
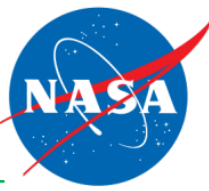- Such good agreement I showed through a higher frequency where I truncated the analytical model

**Overlay of TFs: Original Analytical and Computed from Random Data**
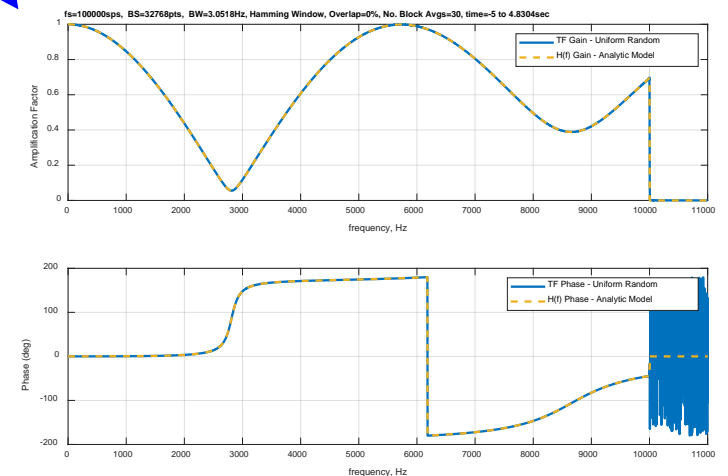
Summary

- A sensor port is part of the system being measured, but not part of the system that is intended to be measured.
  - The acoustic resonance within the sensor port produces a frequency-dependent amplification and phase deviation that directly affects the data collected

- Several theoretical and numerical advancements
  - Traditional design criteria is improved to directly incorporate an end user's acceptable error
  - Theoretical framework developed as a concise tractable solution to model a frequency response
    - 3 critical analytic improvements needed to combine as a response model
  - Numerical methodology developed to determine sensor port response
    - Methodology takes advantage of numerical deterministic modeling

- Application/Verification of a specialized filter based on a sensor response to simulated data
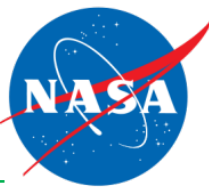
Backup
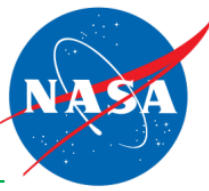
**Thermoviscous Boundary Layer
Quasi-Plane-Wave Equation**

Traditional Absorption Coefficient

$$\alpha_{\mu\kappa} = \frac{1}{cR} \cdot \sqrt{\frac{\mu\omega}{2\overline{\rho}}}\left(1 + \frac{\gamma-1}{\sqrt{\text{Pr}}}\right)$$

Wave Equation

$$p_{xx} - \frac{1}{\overline{c}^2}\,p_{tt} = 2\overline{c}\,\alpha_{\mu\kappa}\sqrt{\frac{8}{\omega\pi}} \cdot \int\limits_{0}^{\infty}\frac{p_{xx}(x,t-\tau)}{\sqrt{\tau}}\,d\tau$$

Complex Wave Number

$$\hat{k}_{BL} = \frac{\omega/\overline{c}}{\sqrt{\left(1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right) + i\cdot\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}}}$$

Linearized

$$\gamma_{\mu\kappa} = \frac{1}{cR}\cdot\sqrt{\frac{\mu\omega}{2\overline{\rho}}}\left(1 + \frac{\gamma-1}{\sqrt{\text{Pr}}}\right) + i\cdot\frac{\omega}{\overline{c}}\cdot\left(1 - \frac{1}{R}\sqrt{\frac{\mu}{2\omega\overline{\rho}}}\left(1 + \frac{\gamma-1}{\sqrt{\text{Pr}}}\right)\right)^{-1}$$

Exact

$$R_{den} = \sqrt[4]{\left(\left(1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 + \left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2\right)} \cdot \cos\left(\frac{1}{2}\tan2^{-1}\left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}, 1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)\right)$$
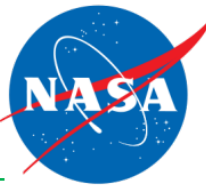
$$I_{den} = \sqrt[4]{\left(\left(1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 + \left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2\right)} \cdot \sin\left(\frac{1}{2}\tan2^{-1}\left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}, 1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)\right)$$
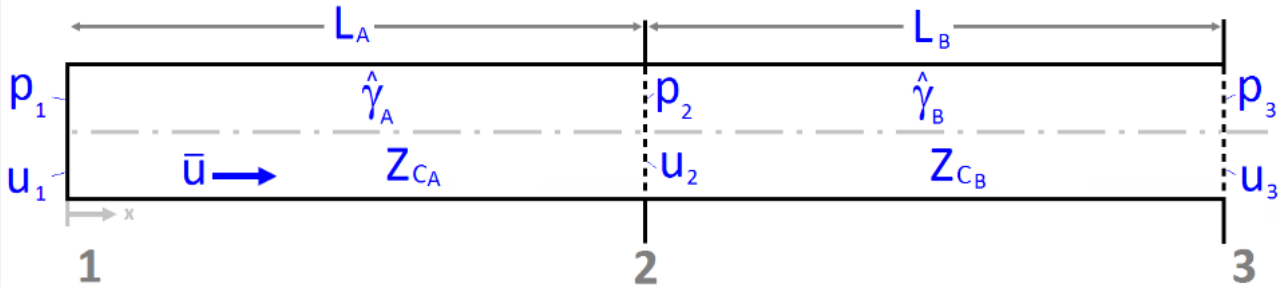
$$\hat{\gamma}_{BL} = \left(\frac{\omega}{\overline{c}}\cdot\frac{I_{den}}{\left(R_{den}\right)^2 + \left(I_{den}\right)^2}\right) + i\cdot\left(\frac{\omega}{\overline{c}}\cdot\frac{R_{den}}{\left(R_{den}\right)^2 + \left(I_{den}\right)^2}\right)$$

Everything Needed for a Sensor Port Model

Single-diameter Port*

$$X(f) = \left| \frac{\hat{p}_U}{\hat{p}_D} \right| \quad \phi(f) = \angle \left(\frac{\hat{p}_U}{\hat{p}_D} \right) \quad Z_C = -\frac{\overline{\rho}\,\overline{c}^2 \hat{\gamma}}{\omega g_c} i$$

$$\frac{p_1}{p_3} = \left(\cosh(\hat{\gamma}_A L_A)\cosh(\hat{\gamma}_B L_B) + \frac{Z_{C,B}}{Z_{C,A}}\sinh(\hat{\gamma}_A L_A)\sinh(\hat{\gamma}_B L_B) \right)^{-1}$$

*See paper for more complex systems

Thermoviscous Acoustics

$$\alpha_{\mu\kappa} = \frac{1}{\overline{c}R} \cdot \sqrt{\frac{\mu\omega}{2\overline{\rho}}} \left(1 + \frac{\gamma - 1}{\sqrt{\mathrm{Pr}}} \right)$$

$$R_{den} = \sqrt[4]{\left(\left(1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 + \left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 \right)} \cdot \cos\left(\frac{1}{2}\tan2^{-1}\left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}, 1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa} \right) \right)$$

$$I_{den} = \sqrt[4]{\left(\left(1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 + \left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}\right)^2 \right)} \cdot \sin\left(\frac{1}{2}\tan2^{-1}\left(\frac{2\overline{c}}{\omega}\alpha_{\mu\kappa}, 1 - \frac{2\overline{c}}{\omega}\alpha_{\mu\kappa} \right) \right)$$

$$\hat{\gamma}_{BL} = \left(\frac{\omega}{\overline{c}} \cdot \frac{I_{den}}{\left(R_{den}\right)^2 + \left(I_{den}\right)^2} \right) + i \cdot \left(\frac{\omega}{\overline{c}} \cdot \frac{R_{den}}{\left(R_{den}\right)^2 + \left(I_{den}\right)^2} \right)$$

Radiation Acoustics

$$\gamma_{rad} = \frac{R\,\omega^2/\overline{c}^2}{2H_1(2\omega R/\overline{c})} \cdot \left(1 - \frac{\overline{c} \cdot J_1(2\omega R/\overline{c})}{\omega R} \right) + i\frac{\omega}{\overline{c}}$$

Frequency Response Function