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Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty

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Abstract

We address two fundamental critiques of established asset pricing models: that they (1) require a controversial degree of preference for early resolution of uncertainty; and (2) do not match the term structures of risk premia observed in the data. Inspired by experimental evidence, we construct preferences in which risk aversion decreases with the temporal horizon. The resulting model implies term structures of risk premia consistent with the evidence, including time-variations and reversals in the slope, without imposing a particular preference for early or late resolutions of uncertainty or compromising on the ability to match standard moments in the returns distributions.

Key words: risk aversion, early resolution, term structure, volatility risk

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1 Introduction

The finance literature has been successful in explaining many features of observed equilibrium asset prices as well as their dynamics (see Cochrane, 2016). However, recent work has posed new challenges regarding the relation between the timing and the pricing of uncertainty. First, the widely used long-run risk model of Bansal and Yaron (2004) has come under attack on conceptual grounds: Epstein, Farhi, and Strzalecki (2014) show that calibrating the model to match asset pricing moments requires a surprisingly strong preference for early resolutions of uncertainty, difficult to reconcile with the micro evidence and introspection. Second, the empirical evidence shows unexpected patterns in the term structures of risky assets' expected returns, whereby risk premia are sometimes higher for short-term payoffs than for long-term payoffs (e.g. van Binsbergen, Brandt, and Koijen, 2012; Giglio, Maggiori, and Stroebel, 2014; Bansal, Miller, and Yaron, 2017).[1] These findings represent a fundamental critique because they are inconsistent with established asset pricing models: the term structure of risk premia is always upward-sloping in the long-run risk model of Bansal and Yaron (2004) as well as in the habit-formation model of Campbell and Cochrane (1999), whereas it is flat in the rare disaster models of Gabaix (2012) and Wachter (2013).

To address these challenges, we propose a model that relaxes the assumption, standard in the economics literature, that risk aversion is constant across temporal horizons. Inspired by experimental evidence, we let agents be more averse to immediate than to delayed risks. Our first contribution is methodological: we apply this generalization to the standard recursive utility model of Epstein and Zin (1989), which allows us to build on its success at explaining asset pricing moments when it is combined with long-run risk. We show that commonly used recursive techniques can be adapted to a setting of pseudo-recursive preferences with horizon-dependent risk aversion, letting us derive closed-form solutions. Our baseline model can accommodate numerous extensions, be it on the valuation of risk (habit formation, disappointment aversion, loss aversion, etc.), or on the quantity of risk (rare disasters, production-based models, etc.). Further, under our preferences **inter-temporal** decisions for deterministic payoffs are unchanged from the standard, time consistent, model; only **intra-temporal** allocations across risky assets are dynamically time inconsistent. We can therefore study the pricing impact of horizon dependent risk aversion in isolation from quasi-hyperbolic discounting, and in general from models of time inconsistent inter-temporal decisions.

We show that our model resolves all concerns regarding preferences for early or late

[1] For a review of the literature, see van Binsbergen and Koijen (2016).

resolutions of uncertainty, our second contribution. As we mention above, in a standard long-run risk framework with Epstein and Zin (1989) preferences, calibrating the preference parameters as well as the risk in the endowment process so as to match observed asset pricing moments implies that agents have a preference for early resolutions of uncertainty so strong as to be unrealistic—raising doubts as to the validity of its representative agent set-up (Epstein et al., 2014). Our model not only mitigates this result but can even reverse it. Specifically, we formally derive how two consumption streams with identical risk but different timing for the resolution of uncertainty are valued—one where shocks are revealed gradually as they are realized over time, the other where all future shocks are revealed at the same early date. As in the model of Epstein and Zin (1989), our agents value these consumption streams differently, even though the ex-ante distributions of risk are rigorously identical. Whether and how the two valuations differ depends on the wedge in risk aversions for short-horizon payoffs versus for long-horizon payoffs; as well as on their values relative to the elasticity of intertemporal substitution. A consumption stream with early resolution of uncertainty shifts the risk in all future shocks into a short-horizon risk, moving from a risk assessment using the low risk aversion of long-horizon payoffs to a risk assessment using the higher risk aversion of short-horizon payoffs. This lowers the attractiveness of early resolutions of uncertainty, compared to the standard framework with Epstein and Zin (1989) preferences. We find that our model can be calibrated to match the usual asset pricing moments **and** reasonable levels of preferences for either early or late resolution of uncertainty.

As our third contribution, we apply our utility model and methodology to equilibrium asset pricing, and formally derive risk premia consistent with the recent empirical evidence, rationalizing both upward sloping term-structures during normal times as well as steeply downward sloping term-structures during the financial crisis of 2007—2009, as described in van Binsbergen et al. (2013) and Bansal et al. (2017).

We first consider a representative agent who trades and clears the market every period, and, as such, cannot pre-commit to any specific strategy: unable to commit to future behavior but aware of her dynamic inconsistency, in the spirit of Strotz (1955), the agent optimizes in the current period, fully anticipating re-optimization in future periods. Solving our model this way yields a one-period pricing problem in which the Euler equation is satisfied. The stochastic discount factor of our pseudo-recursive model nests the standard Epstein and Zin (1989) case, but with a new multiplicative term that loads on the wedge arising from the preferences' dynamic inconsistency between the continuation value used for optimization at a given time t and the actual valuation at $t+1$.

In a Lucas-tree endowment economy with long-run risk, we derive equilibrium prices, and analyze how this new term in the stochastic discount factor affects them. We find that the pricing of shocks that impact consumption **levels** are unchanged from the standard model —reflecting that the dynamic inconsistency in our model does not concern inter-temporal decisions. One implication is that, if the quantity of risk is constant in the economy, equilibrium asset prices are unaffected by our horizon-dependent risk aversion model. Shocks to consumption **risk** (volatility) on the other hand directly affect intra-temporal decisions, and their pricing changes under horizon-dependent risk aversion: the lower risk aversion for long-horizon payoffs reduces the pricing of volatility shocks, an effect that accumulates over time. In a standard log-normal consumption growth setting with stochastic volatility, our calibrated model can simultaneously match the average level of risk prices and generate a term structure of risk premia with a shape —upward sloping over the short to medium horizon, then flat —that matches the recent empirical work by Bansal et al. (2017) for non-crisis periods, i.e. outside of the the financial crisis of 2007—2009.

We formally show that the one-period classical framework **cannot** match, on the other hand, the sharply downward sloping term-structures documented in Bansal et al. (2017) for the recent financial crisis, and consistent with van Binsbergen et al. (2012) and van Binsbergen and Koijen (2016): under dynamic trading, there is no tautological link between a decreasing term structure of risk aversion and a decreasing term structure of risk premia. However, we hypothesize that the one-period representative agent framework no longer provides a realistic, and useful, approximate structure in which to derive equilibrium asset prices during severe liquidity crises, such as the one experienced in 2007—2009. Accordingly, in the second part of our analysis, we deviate from the representative agent assumption and assume that illiquidity pushes investors to adopt buy-and-hold strategies, such that one-period pricing no longer applies. When sufficiently many investors opt for committed buy-and-hold strategies, in particular for assets with long horizons —a realistic assumption when liquidity breaks down —a downward sloping shape emerges in our calibrated horizon-dependent risk aversion model, consistent in magnitude with the evidence in Bansal et al. (2017) for the December 2007—June 2009 period.

In sum, we develop a new model that can both address the early versus late resolution of uncertainty challenge of Epstein et al. (2014) and generate risk premia consistent with the downward sloping term structure puzzle first emphasized by van Binsbergen et al. (2012), and with the slope reversal dynamics described in van Binsbergen et al. (2013) and Bansal et al. (2017). We show that these hotly debated problems on the timing and pricing of uncertainty can be solved without compromising on the model's ability to match the usual asset pricing moments as in Bansal et al. (2014), and without departing from the methodology of the widely-used preference structure of Epstein and Zin (1989).

After a short overview of the literature, we present our model of preferences in Section 3. We analyze the preference for early or late resolution of uncertainty in Section 4. In Section 5, we derive the asset pricing implications of our model. Section 6 presents and discusses the model's quantitative predictions. Section 7 concludes. All mathematical proofs are in the Appendix.

2 Related literature

This paper is the first to solve for equilibrium asset prices in an economy populated by agents with dynamically inconsistent risk aversions. It complements Luttmer and Mariotti (2003), who show that dynamically inconsistent preferences for inter-temporal trade-offs of the kind examined by Harris and Laibson (2001) have only muted implications for asset pricing, and little power to explain cross-sectional variation in asset returns. Given that cross-sectional asset pricing involves intra-period risk-return tradeoffs, it is indeed quite intuitive that inter-temporal dynamic inconsistency is not suitable to address puzzles related to risk premia.

Our model generalizes Epstein and Zin (1989) preferences by relaxing the dynamic consistency axiom of Kreps and Porteus (1978) to analyze the subtle relationship between the timing and pricing of uncertainty. By contrast, Routledge and Zin (2010), Bonomo et al. (2011) and Schreindorfer (2014) follow Gul (1991) and relax the independence axiom to analyze the asset pricing impact of generalized disappointment aversion within a recursive framework. They find that their models generates endogenous predictability (Routledge and Zin, 2010); matches various asset pricing moments (Bonomo et al., 2011); and prices the cross-section of options better than the standard model (Schreindorfer, 2014). Their models, however, do not address the "excessive preference for early resolutions of uncertainty puzzle" pointed out by Epstein et al. (2014) or quantitatively match the term structure of risk prices — the two questions of interest in our analysis.[2]

The importance of a volatility risk channel, central to our qualitative and quantitative asset pricing results, is supported by Campbell et al. (2016), who show that it is crucial for asset returns in a CAPM framework, and relates to numerous other works on the relation between volatility risk and returns (Ang et al., 2006; Adrian and Rosenberg, 2008; Bollerslev and Todorov, 2011; Menkhoff et al., 2012; Boguth and Kuehn, 2013).

The puzzle of a downward sloping term-structure of excess returns has emerged in the recent empirical literature. Van Binsbergen et al. (2012) show that the expected excess re-

[2] Just like the standard Epstein and Zin (1989) model, our model can accommodate generalized disappointment aversion for the valuation of risk. Such a framework might be of interest for future research.

turns for short-term dividends are higher than for long-term dividends (see also Boguth et al., 2012; van Binsbergen and Koijen, 2011; van Binsbergen et al., 2013). Van Binsbergen and Koijen (2016) document downward sloping Sharpe ratios of risky assets' excess returns, across a variety of assets. Giglio et al. (2014) show a similar pattern exists for discount rates over much longer horizons using real estate data; and Lustig et al. (2016) for currency carry trade risk premia. Weber (2016) sorts stocks by the duration of their cash flows and finds significantly higher returns for short-duration stocks. Dew-Becker et al. (2016) use data on variance swaps to show the volatility risk is priced (crucial to our model), but mostly at very short horizons. Using different methodologies and standard index option data, Andries et al. (2016) also find a negative price of variance risk for maturities up to 4 months, and a strongly nonlinear downward sloping term structure (in absolute value).

These striking empirical findings have triggered a significant literature that aims to explain these patterns. Various models generate the desired implications —downward sloping term-structures of risk premia —by making structural assumptions about the priced shocks affecting the economy. For example, Ai et al. (2015) derive term-structure results in a production-based real business cycle model in which capital vintages face heterogeneous shocks to aggregate productivity. Other production-based models with implications for the term structure of equity risk are, e.g. Kogan and Papanikolaou (2010, 2014), and Gârleanu et al. (2012). Favilukis and Lin (2015), Belo et al. (2015), and Marfe (2015) offer wage rigidities as an explanation why risk levels and thus risk premia could be higher at short horizons. Croce et al. (2015) use informational frictions to generate a downward-sloping equity term structure. Backus et al. (2016) propose the inclusion of jumps to account for the discrepancy between short-horizon and long-horizon returns; while Nzesseu (2018) shows it is sufficient to add negative covariation between the consumption shocks and the volatility shocks to the long-run risk model of Bansal and Yaron (2004). Other models focus, as we do, on the risk prices rather than on the quantity, of risk, such as Andries (2015) and Curatola (2015) who propose preferences with first order-risk aversion to explain the observed term structure patterns; or Khapko (2015) and Guo (2015), who both study other dynamic extensions to Eisenbach and Schmalz (2016).[3]

These papers explicitly focus on matching **downward**-sloping term structures of risk prices. However, the recent work by Bansal et al. (2017) documents that the term-structure

[3]They do so in a time-separable model, which confounds dynamically inconsistent risk preferences with dynamically inconsistent time preferences (hyperbolic discounting). That approach makes the two ingredients' relative contributions opaque. Further, the approach does not accommodate formal solutions, and thus formal interpretations.

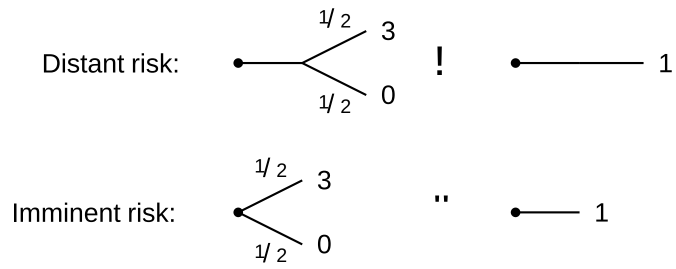

Figure 1: Preferences with horizon-dependent risk aversion.

of expected excess returns may be **upward** sloping on average, though it was sharply downward sloping during the recent financial crisis of 2007 — 2009, sufficiently so to explain the aforementioned empirical term-structure results (most of them derived over short time periods that include the crisis years). None of the theoretical papers cited above matches the slope dynamics described in van Binsbergen et al. (2013) and Bansal et al. (2017); or proposes solutions to the challenge from Epstein et al. (2014) on the excessive preference for early resolutions of uncertainty implied by the standard model. Our paper addresses both sets of puzzles.

3 Preferences with horizon-dependent risk aversion

Field and laboratory experiments document that risk-taking behavior is affected by how far in the future a risk occurs: subjects tend to be more risk averse for risks in the near future than for distant ones. Early work by Jones and Johnson (1973) provides evidence for such horizon-dependent risk aversions from a simulated medical trial. More recent studies use the standard protocol of Holt and Laury (2002) to elicit risk aversion — Noussair and Wu (2006) in a within-subjects design and Coble and Lusk (2010) in an across-subjects design — both finding risk aversion decreases as risk becomes more distant in time. The same pattern is documented by Sagristano, Trope, and Liberman (2002) and Baucells and Heukamp (2010) using binary choice among lotteries, as well as by Onculer (2000) and Abdellaoui, Diecidue, and Onculer (2011) using certainty equivalents.

Figure 1 provides an example of preferences with horizon-dependent risk aversion. Under this illustrative example, all subjects are asked to rank a lottery with payoff $x = 1$ for certain versus a lottery with payoff $x = 3$ with a 50% chance, and $x = 0$ otherwise. All subjects choose their rankings at time $t = 0$, however for some the lottery happens at time $t = 2$ (the "distant risk" case), and for some the lottery happens at time $t = 1$ (the "imminent risk" case).

The experimental evidence shows that subjects may prefer the certain lottery over the risky one when the risk is immediate *and* prefer the same risky lottery over the certain one when the risk is more distant in the future. For a real-life intuitive example, think of someone paying a considerable amount of money for a parachute jumping experience, and then refusing to actually jump once in the plane. This is the notion of *horizon-dependent risk aversion* as introduced by Eisenbach and Schmalz (2016) in a static, time separable, framework.

In the illustrative example above, one subgroup ranks lotteries with horizon $t = 1$ and the other subgroup ranks lotteries with horizon $t = 2$: within each subgroup the ranking is for lotteries that will happen at the same time. That the rankings change with the horizon reveals a dynamic inconsistency in *intra-temporal* choices, not in inter-temporal choices. In particular, the well documented hyperbolic discounting (e.g. Phelps and Pollak, 1968; Laibson, 1997) or other time inconsistencies concerning inter-temporal decisions do not influence, or cause, the evidence discussed above.[4]

3.1 Dynamic preference model

The experimental evidence that subjects are more risk averse for short-horizon than for long-horizon payoffs seems particularly relevant when considering the relation between the timing and the pricing of risk — at the center of the recent challenges to the long-run risk framework. To explore its formal implications in a dynamic framework, we introduce the notion of horizon-dependent risk aversion in the recursive utility preferences of Epstein and Zin (1989), the standard model for long-run risk pricing. The preferences of Epstein and Zin (1989) are dynamically consistent (by definition). We generalize their model by relaxing the dynamic consistency axiom of Kreps and Porteus (1978).

To simplify the exposition, we present the model with only two levels of risk aversion γ and $\boldsymbol{\gamma}$: we assume that the agent treats immediate uncertainty with risk aversion γ, and all delayed uncertainty with risk aversion $\boldsymbol{\gamma}$, where $\gamma > \boldsymbol{\gamma} \geq 1$ in line with the experimental evidence. Our approach with only two levels of risk aversion is analogous to the β-δ framework (Phelps and Pollak, 1968; Laibson, 1997) as a special case of the general non-exponential discounting model of Strotz (1955). Appendix A has the model for general sequences $\{\gamma_h\}_{h \geq 1}$ of risk aversion at horizon h. As long as risk aversions reach a constant level beyond a given horizon, closed-formed solutions similar to those derived in Section 4 and in Section 5 obtain.

[4] Eisenbach and Schmalz (2016) also show horizon-dependent risk aversion is conceptually orthogonal to time-varying risk aversion (Constantinides, 1990; Campbell and Cochrane, 1999).

At any time t, we denote by $E_t[\cdot] = E[\cdot \mid I_t]$ the expectation conditional on I_t, the information set at time t.

Definition 1 (Dynamic horizon-dependent risk aversion). The agent's utility in period t is given by

$$V_t = \left[(1-\beta)\, C_t^{1-\rho} + \beta\, E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}, \tag{1}$$

where the continuation value $\tilde{V}_{t+1}$ satisfies the recursion

$$\tilde{V}_{t+1} = \left[(1-\beta)\, C_{t+1}^{1-\rho} + \beta\, E_{t+1}\left[\tilde{V}_{t+2}^{1-\boldsymbol{\gamma}}\right]^{\frac{1-\rho}{1-\boldsymbol{\gamma}}}\right]^{\frac{1}{1-\rho}}. \tag{2}$$

As in Epstein and Zin (1989), the utility V_t depends on the deterministic current consumption C_t and on the certainty equivalent $E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ of the continuation value $\tilde{V}_{t+1}$, where the aggregation of the two periods occurs with constant elasticity of intertemporal substitution given by $1/\rho$. However, in contrast to Epstein and Zin (1989), the certainty equivalent of consumption starting at $t+1$ is calculated with relative risk aversion γ, wherein the certainty equivalents of consumption starting at $t+2$ and beyond are calculated with relative risk aversion $\boldsymbol{\gamma}$.

This is the concept of horizon-dependent risk aversion applied to the recursive valuation of certainty equivalents, as in Epstein and Zin (1989), but with risk aversion γ for imminent uncertainty and risk aversion $\boldsymbol{\gamma}$ for delayed uncertainty. Our model nests the model of Epstein and Zin (1989) when $\gamma = \boldsymbol{\gamma}$, and, in turn, nests the standard CRRA time-separable model when $\gamma = \boldsymbol{\gamma} = \rho$. Any difference in the results we derive below under the preferences of Definition 1 to those obtained under the standard model of Epstein and Zin (1989) thus hinges on $\boldsymbol{\gamma} \neq \gamma$.

The horizon-dependent valuation of risk implies a dynamic inconsistency, as the uncertain consumption stream starting at $t+1$ is evaluated as $\tilde{V}_{t+1}$ by the agent's self at t and as V_{t+1} by the agent's self at $t+1$:

$$\tilde{V}_{t+1} = \left[(1-\beta)\, C_{t+1}^{1-\rho} + \beta\, E_{t+1}\left[\tilde{V}_{t+2}^{1-\boldsymbol{\gamma}}\right]^{\frac{1-\rho}{1-\boldsymbol{\gamma}}}\right]^{\frac{1}{1-\rho}}$$

$$\neq V_{t+1} = \left[(1-\beta)\, C_{t+1}^{1-\rho} + \beta\, E_{t+1}\left[\tilde{V}_{t+2}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$$

Crucially, this disagreement between the agent's continuation value $\tilde{V}_{t+1}$ at t and the agent's utility V_{t+1} at $t+1$ arises only for uncertain consumption streams. For any deterministic consumption stream the horizon-dependence in Equation (1) becomes irrelevant and we have

$$\tilde{V}_{t+1} = V_{t+1} = \left((1-\beta) \sum_{h=0}^{\infty} \beta^h C_{t+1+h}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Our model implies dynamically inconsistent risk preferences while maintaining dynamically consistent time preferences, focusing strictly on the experimental evidence described above. The results we obtain in the analysis that follows can therefore be attributed to horizon dependent risk aversion, orthogonal to extant models of time inconsistency, such as hyperbolic discounting.

3.2 Timing of risk and dynamic inconsistency

An agent with the time-inconsistent preferences of Definition 1 can be either naive or sophisticated about the disagreement between her temporal selves; she can either commit to multi-period strategies or be compelled to re-optimize every period. The valuation of early versus late resolutions of uncertainty, which we analyze first, is by nature a static problem: its solutions are the same for naive and sophisticated investors, with or without commitment. But these modeling choices matter for dynamic outcomes, in particular the equilibrium asset prices we then derive.

To do so, we follow the tradition of Strotz (1955), and assume the agent is fully rational and sophisticated when making choices in period t to maximize V_t. Self t realizes that its valuation of future consumption, given by $\tilde{V}_{t+1}$, differs from the objective function V_{t+1} which self $t+1$ will maximize. The solution then corresponds to the subgame-perfect equilibrium in the sequential game played among the agent's different selves (see Appendix A.1). We assume no commitment in our general case, as appropriate for a representative agent who trades and clears the market at all times, and as such cannot pre-commit to a given strategy — similar to the framework of Luttmer and Mariotti (2003) for non-geometric discounting. However, we let the sophisticated agents commit to certain strategies when we explore the implications of liquidity crises in which one-period pricing breaks down.

Extending our results to an agent naive about her own dynamic inconsistencies is straightforward, and does not present any conceptual challenge. We briefly discuss and derive formal results for this alternative approach in Appendix A.3.

4 Preference for early or late resolution of uncertainty

To analyze whether agents have a preference for early or late resolutions of uncertainty, two types of consumption streams are evaluated at a given time t. In the first case, consumption shocks are revealed gradually, whenever they are realized: the shock affecting consumption at $t+h$ is revealed at time $t+h$, for all horizons $h \geq 1$. In the second case, all future consumption shocks are revealed in the next period, at time $t+1$, even when they affect consumption at a later period: the shock affecting consumption at $t+h$ is revealed at time $t+1$, for all $h \geq 1$.

Crucially, even when she receives the information about her future risk shocks earlier, at time $t+1$, the agent cannot act on it to change her future consumption stream. From the point of view of time t, when the agent evaluates the two consumption streams with or without an early resolution of uncertainty, the distributions of future risks are therefore exactly the same in both cases. In the expected utility framework, she would assign them the exact same value. However, because risk aversion is disentangled from the elasticity of intertemporal substitution in the preferences of Epstein and Zin (1989), as well as in our pseudo-recursive horizon-dependent risk aversion model of Definition 1, two consumption streams with ex-ante identical risks, but different timing for the resolution of uncertainty, can have different values.

An agent with Epstein and Zin (1989) utility prefers early resolutions of uncertainty if and only if $\gamma > \rho$.[5] How much so depends on the wedge $\gamma - \rho$ and on the magnitude of the uncertainty in the consumption shocks. As Epstein et al. (2014) point out, the parameters used in the long-run risk literature imply a strong preference for early resolutions of uncertainty. For example, in the calibration of Bansal and Yaron (2004), the representative agent would be willing to forgo up to 35% of her consumption stream in exchange for all uncertainty to be resolved the next month instead of gradually over time.[6]

Choosing a consumption stream with an early resolution, i.e. where all shocks are revealed at time $t+1$, rather than the same consumption stream with late resolutions, i.e. where shocks are revealed as they come over time, corresponds to shifting all future risk, short-term and long-term, to a next-period risk. Whether long-term risks are evaluated with the same risk aversion as immediate risks or not will thus matter for the relative val-

[5] To see why, note that in the case where all future shocks are revealed at $t+1$, the shocks to consumption from $t+2$ onward are evaluated with the inverse elasticity of intertemporal substitution ρ since they are no longer uncertain; whereas, when shocks are revealed over time, variations in consumption from $t+2$ onward are still risky at $t+1$ and thus evaluated with risk aversion γ.

[6] In the calibration of Bansal et al. (2009), the timing premium is even greater, at more than 80% — see Figure 2 and Table 2 in Section 6.

ues of the two theoretical consumption streams, and therefore for the preference for early or late resolutions of uncertainty.

To formalize this argument, and derive how an agent with the horizon-dependent risk aversion preferences of Definition 1 assesses the early resolution of uncertainty, we replicate the formal analysis of Epstein et al. (2014). We assume, as they do, a unit elasticity of intertemporal substitution, $\psi = 1$, and log-normal consumption growth with time varying drift, corresponding to long-run risk. Using lower-case letters to denote logs throughout, e.g. $c_t = \log C_t$, we let consumption follow the process

$$
\begin{aligned}
c_{t+1} - c_t &= \mu_c + \varphi_c x_t + \sigma_c \cdot W_{c,t+1}, \\
x_{t+1} &= \rho_x x_t + \sigma_x \cdot W_{x,t+1}.
\end{aligned} \tag{3}
$$

For simplicity x_t, which represents time variations in the average consumption growth, is one-dimensional and the shocks $W_{c,t}$ and $W_{x,t}$ are i.i.d. $N(0,1)$ and orthogonal. The drift is stationary, i.e. ρ_x is contracting.

Denoting by V_t^* the agent's utility at t if all uncertainty — i.e. the entire sequence of shocks $\{W_{c,t+h}, W_{x,t+h}\}_{h \geq 1}$ in the consumption process (3) — is resolved at $t+1$, and V_t the agent's utility when shocks are revealed over time, the timing premium is defined as

$$
TP_t = \frac{V_t^* - V_t}{V_t^*}.
$$

It represents the fraction of utility the agent is willing to forego for an early rather than late resolution of uncertainty.

Proposition 1. An agent with the horizon-dependent risk aversion preferences of Definition 1 with $\psi = 1$, facing the consumption process (3), has a constant timing premium

$$
TP = 1 - \exp\left[\frac{1}{2}\left(1 - \left(\gamma - (1+\beta)(\gamma - \boldsymbol{\gamma})\right)\right)\frac{\beta^2}{1-\beta^2}\sigma_v^2\right], \tag{4}
$$

where $\sigma_v^2 = \sigma_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \sigma_x^2$.

To highlight the role played by horizon-dependent risk aversion, note that an agent with the standard preferences of Epstein et al. (2014) with risk aversion γ has a timing premium given by $TP = 1 - \exp\left(\frac{1}{2}(1-\gamma)\frac{\beta^2}{1-\beta^2}\sigma_v^2\right)$, obtained by setting $\gamma = \boldsymbol{\gamma}$ in Equation (4). When $\gamma > \boldsymbol{\gamma}$, the timing premium is lower since

$$
\gamma - (1+\beta)(\gamma - \boldsymbol{\gamma}) < \gamma.
$$

Corollary 1. For an agent with horizon dependent risk aversion, ! > **!** unambiguously lowers the timing premium.

To understand the result of Corollary 1, observe that a consumption stream with an early resolution of uncertainty concentrates all the risk on the first period, over which the agent is the most risk averse, with immediate risk aversion ! . In contrast, a consumption stream with late resolutions of uncertainty has risk spread over multiple horizons, over some of which the agent is moderately risk averse, with risk aversion **!** < ! .[7]

Consider next cases when the timing premium turns negative, indicating a preference for late resolution. For an Epstein-Zin agent, this happens when ! < $. In our model, with $ = 1 and the consumption process (3), the timing premium is negative if and only if

! < 1 + (1 + ") (! # **!**) . (5)

When ! > **!** , we immediately obtain 1 + (1 + ") (! # **!**) > $ = 1, and the agent with horizon-dependent risk aversion can have a preference for late resolution, even when both risk aversions ! and **!** are greater, even considerably so, than the inverse elasticity of intertemporal substitution Ñ as long as the decline in risk aversion across horizons is sufficiently large. For example, suppose we set immediate risk aversion ! = 10 and " close to 1. Then the agent will prefer uncertainty to be resolved late rather than early according to the condition of Equation (5) as long as **!** < 5.5 which is substantially larger than $ = 1.[8]

Corollary 2. An agent with horizon-dependent risk aversion can prefer a late resolution of uncertainty even when all risk aversions exceed the inverse elasticity of intertemporal substitution, i.e. when ! > **!** > $.

The result of Corollary 2 is of particular interest because extant calibrations of the long-run risk model with Epstein and Zin (1989) preferences require ! greater than $ by an order of magnitude to match equilibrium asset pricing moments Ñ thus resulting in a high

---

[7] The same intuitive argument applies for other dynamic inconsistencies on inter-temporal rather than intra-temporal choices, our focus. In Appendix B.1, we derive the timing premium under hyperbolic discounting, whereby ! = **!** but, at time t, the value V_t is derived with time discount parameter ", and the continuation value V_{t+1} is derived with time discount parameter ! > ". The preference for an early resolution of uncertainty still holds if and only if ! > $, but the magnitude of the timing premium is lower than if the time discount is ! everywhere (and greater than if it is " everywhere). Introducing hyperbolic discounting has, however, a small quantitative e#ect: e.g. under the calibration of Bansal and Yaron (2004) with constant volatility, ! = 10, $ = 1, and " = 0.8, ! = 0.998, the timing premium only goes from 27% (under " = ! = 0.998) to 22.5%.

[8] In the calibrated model of Section 6, we add time varying volatility to the consumption process (3), which a#ects this result: we obtain a preference for late resolution whenever **!** < 4.42.

timing premium. Under the horizon-dependent risk aversion preference model of Definition 1, the same calibration for ! and $ no longer automatically implies such a strong preference for early resolutions of uncertainty. This is true even when the long-run risk aversion !! also remains above the inverse elasticity of intertemporal substitution, in line with the micro evidence. We quantify this result in Section 6, when we consider the joint implications of our calibrated model for asset pricing moments, term structures, and preferences for early or late resolution of uncertainty. We find an equity premium consistent with the data can obtain both under preferences for early resolutions **and** under preferences for late resolutions (see Table 2).

While there is no direct evidence on the values of timing premia — by construction a purely theoretical question — Epstein et al. (2014) argue that the magnitudes implied by calibrations of the long-run risk model with standard Epstein and Zin (1989) preferences are excessive. Since the agent cannot act on early information to modify the consumption stream she will receive, it appears unreasonable that she would be willing to forgo a large fraction of her wealth for earlier resolutions. Besides, in numerous cases in both the empirical and the theoretical literatures, agents prefer not to observe early information, even when they **can** act on it, suggesting a preference for late rather than early resolution of uncertainty (see Golman et al., 2016; Andries and Haddad, 2015). This makes the magnitude of the timing premium under the standard long-run risk model all the more problematic.

A representative agent whose individual optimal decisions appear contrary to commonsense considerations — here on early versus late resolutions of uncertainty — raises doubts as to the legitimacy of the long-run risk model to derive equilibrium asset prices. In this section, we formally showed that introducing the notion of horizon-dependent risk aversion with the preferences of Definition 1 can lower the timing premium to any reasonable range. Our model provides a reasonable answer, grounded in the experimental evidence, to the challenge posed by Epstein et al. (2014), as long as it can also still match asset pricing moments. This is the question we turn to next.

5 Asset prices

The decision to opt for an early, rather than late, resolution of uncertainty is by nature a multi-horizon problem, as the agent chooses how valuable it is to discover all her future risk at the next immediate period, rather than slowly over time. In this multi-horizon problem, introducing a wedge between the immediate risk aversion and the long-horizon risk aversion has a first-order impact on the agent's valuations — as we show in Proposition 1.

In contrast, asset prices are set by agents who can, in general, reduce their risk allocation decisions to a repeated one-period problem. When nothing prevents agents from trading every period, prices at equilibrium must be such that the immediate consumption utility loss from investing a marginal amount of wealth today is strictly offset by the expected **next-period** utility gain when evaluating the investment's payoff. When the conditions for the one-period set-up are satisfied, as in our general case, they naturally limit the impact horizon dependent risk aversion can have on equilibrium asset prices: if all decisions are made one period to the next, how much should investors care about their long-horizon risk aversion at all? This is the question we formally explore next, where we show how and when our model affects risk prices and the term-structure of expected returns.

5.1 One-period pricing

We derive the marginal pricing of risk in our model using a standard consumption-based asset pricing framework. We assume a fully sophisticated representative agent who re-optimizes every period and thus cannot commit. All decisions are made in sequential one-period problems (see Appendix A.1 for details).

For asset pricing purposes, the object of interest is the stochastic discount factor (SDF) under the preferences of Definition 1. The SDF's derivation is based on the intertemporal marginal rate of substitution

$$\text{"}_{t,t+1} = \frac{dV_t / dW_{t+1}}{dV_t / dC_t},$$

which satisfies the Euler equation, whereby the equilibrium price at time t of a future payoff X_{t+1} is given by $P_t = E_t[\text{"}_{t,t+1} X_{t+1}]$.

We decompose the marginal utility of next-period wealth as

$$\frac{dV_t}{dW_{t+1}} = \frac{dV_t}{d\hat{V}_{t+1}} \times \frac{d\hat{V}_{t+1}}{dW_{t+1}}, \tag{6}$$

and appeal to the envelope condition at $t + 1$:

$$\frac{dV_{t+1}}{dW_{t+1}} = \frac{dV_{t+1}}{dC_{t+1}} \tag{7}$$

Note that in our model, the decomposition in Equation (6) involves $\hat{V}_{t+1}$, the value self t attaches to future consumption, while the envelope condition in Equation (7) concerns V_{t+1}, the objective function of self $t + 1$. Nonetheless, due to the homotheticity of our preferences, we can rely on the fact that both $\tilde{V}_{t+1}$ and V_{t+1} are homogeneous of degree one in wealth and therefore

$$\frac{d\tilde{V}_{t+1}/dW_{t+1}}{dV_{t+1}/dW_{t+1}} = \frac{\tilde{V}_{t+1}}{V_{t+1}}.$$

This allows us to formally derive the SDF as:

$$M_{t,t+1} = \frac{dV_{t+1}/dC_{t+1}}{dV_t/dC_t} \times \frac{dV_t}{d\tilde{V}_{t+1}} \times \frac{\tilde{V}_{t+1}}{V_{t+1}}.$$

Proposition 2. **An agent with the horizon-dependent risk aversion preferences of Definition 1 has a one-period stochastic discount factor**

$$M_{t,t+1} = \beta \underbrace{\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}}_{(I)} \times \underbrace{\left(\frac{\tilde{V}_{t+1}}{E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma}}_{(II)} \times \underbrace{\left(\frac{\tilde{V}_{t+1}}{V_{t+1}}\right)^{1-\psi}}_{(III)}. \tag{8}$$

The SDF consists of three multiplicative parts. The first term (I) is standard, capturing the intertemporal substitution between t and $t+1$, and is governed by the time discount factor β and the elasticity of intertemporal substitution $1/\psi$.

The second term (II) captures the unexpected shocks realized in $t+1$ to consumption in the long-run, i.e. beyond $t+1$. It compares the ex-post realized $t+1$ utility $\tilde{V}_{t+1}$ to its ex-ante certainty equivalent $E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$; both the comparison as well as the certainty equivalent are evaluated with immediate risk aversion γ. The same term obtains under standard Epstein-Zin preferences with the difference that, in our model, the $t+1$ utility of self t ($\tilde{V}_{t+1}$) differs from that of self $t+1$ (V_{t+1}).

Finally, the third term (III) captures the dynamic inconsistency in our model by loading on the disagreement between selves t and $t+1$ when evaluating their $t+1$ utilities, given by the ratio $\tilde{V}_{t+1}/V_{t+1}$.

To derive closed-form solutions for the pricing of risk under horizon-dependent risk aversion, we again focus on the case $\psi = 1$, a unit elasticity of intertemporal substitution.[9] We maintain the standard Lucas-tree endowment economy but generalize the consumption process (3) by adding stochastic volatility, in line with the long-run risk literature (e.g.

[9] In Appendix C, we consider $\psi \neq 1$ and the approximation of a rate of time discount close to zero, $\beta \to 1$. We show our main results remain valid as long as the elasticity of intertemporal substitution is greater or equal to one ($1/\psi \geq 1$).

Bansal and Yaron, 2004; Bansal et al., 2009):

$$
\begin{aligned}
c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma_t w_{c,t+1} \\
x_{t+1} &= \rho_x x_t + \alpha_x \sigma_t w_{x,t+1}, \\
\sigma^2_{t+1} &= \sigma^2 + \rho_\sigma \left(\sigma^2_t - \sigma^2 \right) + \alpha_\sigma w_{\sigma,t+1}
\end{aligned}
\tag{9}
$$

For simplicity, we assume that x_t is one-dimensional and the three shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$ are i.i.d. $N(0,1)$ and orthogonal.[10] Both ρ_x and ρ_σ are contracting.

With $\psi = 1$, and taking logs, the SDF in Equation (8) becomes

$$
m_{t,t+1} = \log \beta - (c_{t+1} - c_t) + (1-\gamma) \underbrace{\left(\tilde{v}_{t+1} - E_t \tilde{v}_{t+1} - \frac{1}{2}(1-\gamma)\, \mathrm{var}_t\, \tilde{v}_{t+1} \right)}_{\text{shock to } \tilde{v}_{t+1} \text{ evaluated with risk aversion } \gamma} .
$$

The shocks to the continuation value $\tilde{v}_{t+1}$ are priced with immediate risk aversion γ, as in the standard model of Epstein and Zin (1989). The sole difference is that the SDF involves shocks to $\tilde{v}_{t+1}$ (which evaluates future uncertainty with risk aversion $\tilde{\gamma}$) rather than v_{t+1} (which evaluates future uncertainty with risk aversion γ). To understand the pricing implications of horizon-dependent risk aversion, we consider how the $t+1$ utilities $\tilde{v}_{t+1}$ and v_{t+1} differ.

Lemma 1. ***Under the Lucas-tree endowment process* (9) *and* $\psi = 1$**

$$
\tilde{v}_{t+1} - v_{t+1} = \frac{1}{2} \beta (\gamma - \tilde{\gamma}) \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 + B_v(\tilde{\gamma})^2 \alpha_\sigma^2 \right) \sigma^2_{t+1}, \tag{10}
$$

where ϕ_v ***is independent of both*** γ ***and*** $\tilde{\gamma}$ ***and*** $B_v(\tilde{\gamma}) < 0$ ***is independent of*** γ

$$
\begin{aligned}
\phi_v &= \frac{\beta \phi_c}{1 - \beta \rho_x}, \\
B_v(\tilde{\gamma}) &= \frac{1}{2} \frac{\beta (1 - \tilde{\gamma})}{1 - \beta \rho_\sigma} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right).
\end{aligned}
\tag{11}
$$

Equation (10) reflects that the $t+1$ value of self t ($\tilde{v}_{t+1}$) and that of self $t+1$ (v_{t+1}) only differ in their $t+1$ valuation of uncertain consumption starting in $t+2$ onwards, which is governed by volatility σ^2_{t+1}. Self t evaluates this uncertainty with low risk aversion $\tilde{\gamma}$ while self $t+1$ evaluates it with high risk aversion γ; implying that $\tilde{v}_{t+1} - v_{t+1}$ is positive, and increasing in $\gamma - \tilde{\gamma}$ and in the amount of uncertainty driven by volatility σ^2_{t+1}.

[10] These assumptions can be generalized. We employ them here to make our results comparable to those of Bansal and Yaron (2004) and Bansal et al. (2009).

We obtain the following central result:

Proposition 3. **If volatility is constant i.e. $\sigma_t = \bar{\sigma}\ \forall t$ in the consumption process (9), horizon dependent risk aversion does not affect equilibrium risk prices.**

Under constant volatility, the agent can fully anticipate how her future self will re-optimize, and her time inconsistency does not cause any additional uncertainty in her one-period decision making. Only unanticipated changes in her intra-temporal decisions, when the quantity of risk varies through time, get priced in the risky assets' excess returns. This result crucially hinges on the fact that, in our preference framework, only intra-temporal decisions are time inconsistent: inter-temporal decisions are unchanged from the standard model.

If volatility is constant at all times, $\tilde{V}_{t+1}$ and V_{t+1} only differ by a constant wedge —see Equation (10)— and any shock impacts $\tilde{V}_{t+1}$ and V_{t+1} one-for-one. The difference between the two turns inconsequential for the stochastic discount factor of Equation (8), which variations become unaffected by the dynamic time inconsistency of horizon-dependent risk aversion:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{\tilde{V}_{t+1}}{E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \quad \text{if } \sigma_t = \bar{\sigma}\ \forall t$$

This equality obtains because self t and self $t+1$ disagree only about the risk aversion applied to future uncertainty but not about the deterministic part of the consumption stream starting at $t+1$. The result of Proposition 3 can be extended to any endowment process where uncertainty is constant through time, e.g. jumps or regime switches, such that unexpected shocks affect V and $\tilde{V}$ identically. Proposition 3 is also not specific to the knife-edge case of a unit elasticity of intertemporal substitution, $\rho = 1$, as we show in Appendix C.

When volatility is time varying, on the other hand, Lemma 1 suggests that equilibrium prices **will** depend on both the immediate risk aversion γ and the longer-term one $\bar{\gamma}$. To illustrate the role the parameters κ_v and $\lambda_v(\bar{\gamma})$ —from Lemma 1— play in our model, we decompose the shocks to $\tilde{v}_{t+1}$ into the components due to the three sources of uncertainty in consumption process (9).

Lemma 2. **Under the Lucas-tree endowment process (9) and $\rho = 1$**

$$\tilde{v}_{t+1} - E_t[\tilde{v}_{t+1}] = \left(c_{t+1} - E_t[c_{t+1}]\right) + \kappa_v \left(x_{t+1} - E_t[x_{t+1}]\right) + \lambda_v(\bar{\gamma}) \left(\sigma^2_{t+1} - E_t[\sigma^2_{t+1}]\right).$$

Positive shocks to the immediate consumption, $c_{t+1} \# E_t[c_{t+1}]$, or to the expected consumption growth, $x_{t+1} \# E_t[x_{t+1}]$, naturally increase v_{t+1}, the value of the consumption stream starting at $t + 1$. On the other hand, increases in aggregate uncertainty, $'^2_{t+1} \# E_t['^2_{t+1}]$, reduce the value v_{t+1} Ñ consistent with $*_v(!!) < 0$ in Lemma 1.

Because $\%_v$ does not depend on the risk aversions ! and !!, the pricing of assets that covary only with the immediate consumption shocks or with shocks to the drift is unaffected by horizon-dependent risk aversion, i.e. is unchanged from the standard long-run risk model. Once again, this at-Þrst-glance puzzling result can be understood as follows: these shocks concern inter-temporal consumption smoothing decisions only and, as such, their valuations are governed by the elasticity of intertemporal substitution and not by risk aversion, nor by the dynamic risk inconsistency of our model. Long-run risk aversion !! only matters for the pricing of shocks to the time-varying volatility, as also indicated by Proposition 3.

From Lemma 2 we obtain:

Proposition 4. **nder the Lucas-tree endowment process (9) and $\$ = 1$ the stochastic discount factor satisfies**

$$
\begin{aligned}
)_{t,t+1} \# E_t[)_{t,t+1}] = {} & \# !\&_c '_t W_{c,t+1} + (1 \# !)\%_v \&_x '_t W_{x,t+1} \\
& + (1 \# !) *_v(!!) \& W_{',t+1}.
\end{aligned} \tag{12}
$$

The risk free rate is independent of !!

$$
r_{f,t} = \# \log " + \mu_c + \%_c x_t + \left(\frac{1}{2} \# !\right) \&_c^2 '^2_t \tag{13}
$$

The pricing of the immediate consumption shocks, given by the term $!\&_c '_t W_{c,t+1}$ in Equation (12); the pricing of drift shocks, the term $(1 \# !)\%_v \&_x '_t W_{x,t+1}$ in Equation (12); as well as the risk-free rate in Equation (13); all depend **only** on the immediate risk aversion !.[11] In line with the results of Lemma 2, these shocks hinge on one-period inter-temporal decisions only, and are therefore unaffected by the intra-temporal dynamic inconsistency of horizon-dependent risk aversion. Their pricing is unchanged from the standard long-run risk model.

Our model yields a negative price for volatility shocks, from $(1 \# !) *_v(!!) \& W_{',t+1}$ in Equation (12). Assets with payoffs that covary with aggregate volatility provide valuable insurance, consistent with the existing long-run risk literature and the observed evidence

[11] When $\$ \neq 1$, the risk-free rate can depend on !÷, though not risk prices for immediate consumption shocks and drift shocks — see Appendix C.

(see Dew-Becker et al., 2016; and Andries et al., 2016 for recent examples). The volatility shock prices in Equation (12) depend on both the immediate risk aversion ! , and on the longer-horizon one through $*_{v}(!!)$: shocks to volatility make future intra-temporal decisions uncertain and, for this reason, how risky they are depends on horizon-dependent risk aversion. Due to the lower risk aversion $!! < !$, their implied long-run uncertainty does not ÒfeelÓ as costly, which reduces the value of hedges against volatility shocks, i.e. assets whose payoffs covary positively with shocks to volatility. Consistent with this intuition, we obtain from Equation (11) in Lemma 1, the formal result

$$\frac{*_{v}(!!)}{*_{v}(!)} = \frac{1 \# !!}{1 \# !} < 1. \tag{14}$$

In Section 6, we calibrate our model and show that, despite the reduction in the pricing of volatility shocks highlighted in Equation (14), we can match the usual asset pricing moments Ñsee Tables 1a and 1b.

We now turn to the analysis of term-structure premia, to see if our model can match the observed pricing evidence, when risk varies through time, as successfully as it does the valuations of early versus late resolutions of risk in Section 4.

5.2 Term-structure

As we discussed in the literature review, several recent papers (van Binsbergen et al., 2012; van Binsbergen and Koijen, 2016; Giglio et al., 2014; Dew-Becker et al., 2016; Andries et al., 2016) provide empirical evidence in favor of a downward sloping term-structure of expected excess returns, for various types of risk (equity index, housing, volatility risk). Bansal et al. (2017) on the other hand, find that the term-structure for equity risk premia is upward sloping on average, but sharply downward sloping during the 2007— 2009 financial crisis (see also van Binsbergen et al. (2013)); sufficiently so to drive the downward sloping averages derived over short periods in van Binsbergen et al. (2012) and van Binsbergen and Koijen (2016).

At first glance, introducing the concept of a risk aversion that decreases with the horizon of payoff uncertainty Ñour horizon-dependent risk aversion framework Ñappears perfectly tailored to obtain a downward sloping term-structure of expected returns. However, as Propositions 3 and 4 make clear, the impact of horizon-dependent risk aversion on equilibrium risk prices is far from tautological: it is either null, when volatility is constant in the economy, or it is limited to volatility risk premia, when volatility is time varying. Can our model of preferences, grounded in the experimental evidence that agents are dynamically inconsistent for intra-temporal decisions, nonetheless help explain observed features of the term-structure of expected returns?

To formally derive the implications of our model, we analyze the expected excess returns on dividend strip futures.[12] This allows us to compare our calibrated term structure results to the empirical evidence in van Binsbergen and Koijen (2016) and Bansal et al. (2017).

In line with the long-run risk literature (Bansal and Yaron, 2004; Bansal et al., 2009), and consistent with the consumption growth process (9), we assume that dividends follow a log-normal growth process given by:

$$d_{t+1} - d_t = \mu_d + \phi_d x_t + \rho \varphi_c \sigma_t W_{c,t+1} + \varphi_d \sigma_t W_{d,t+1}, \tag{15}$$

where the shocks $W_{d,t}$ are i.i.d. $N(0,1)$ and orthogonal to the consumption shocks $W_{c,t}$, $W_{x,t}$ and $W_{\sigma,t}$;[13] ϕ_d captures the link between the mean consumption growth and the mean dividend growth; ρ the correlation between immediate consumption and dividend shocks in the business cycle.

We denote the value at time t for a dividend strip with horizon h, i.e. the claim to the aggregate dividend at horizon $t+h$, as $D_{t,h}$; and that of a risk-free zero-coupon bond with horizon $t+h$ as $B_{t,h}$.

The one-period holding returns on dividend strip futures — equivalent to one-period excess returns on dividend strips — are given by:

$$R^F_{t+1,h} = \frac{D_{t+1,h-1}/D_{t,h}}{B_{t+1,h-1}/B_{t,h}}.$$

Lemma 3. **The price of a dividend strip with maturity h at time t is**

$$\frac{D_{t,h}}{D_t} = \exp\left(\mu_{d,h} + \phi_{d,h} x_t + \lambda_{d,h} \sigma_t^2\right),$$

where

- $\mu_{d,h}$ **depends on both h and ψ;** $\lambda_{d,h}$ **depends on h but not on ψ; and** $\phi_{d,h} = (-\phi_c + \phi_d)\frac{1-\kappa_x^h}{1-\kappa_x}$ **depends on neither h nor ψ.**

[12] Under a dividend strip futures contract with horizon h at time t, the dividends paid on an index over the year $t+h$ will be exchanged at time $t+h$ against a fixed payment that is set at time t. The dividend strip futures were first introduced on the Eurostoxx 50 in 2008. A similar analysis on the term-structure of risk-free zero-coupon bond yields can be found in Appendix B.3.

[13] Once again, these assumptions can be generalized, but they are those of Bansal and Yaron (2004) and Bansal et al. (2009).

- $\%_{d,h} > 0$ **is increasing with the horizon** h **when** $\%_d > \%_c$. $\mu_{d,h}$ **and** $*_{d,h}$ **are not monotone in the horizon** h.[14]

The values of claims to the future dividends of the market index are affected by the wedge $! \# !!$ in risk aversions for short-horizon payoffs versus long-horizon payoffs through the constant terms $\{\mu_{d,h}\}_h$. How the term-structure of risk aversions affects the term-structure of dividend strip prices is not one-for-one, and depends on the parameters of the model: $\{\mu_{d,h}\}_h$ is not monotone in h. And their co-variations with the aggregate shocks through the state variables $\{x_t, '_t\}$ depend only on the immediate risk-aversion $!$. This is consistent with the results of Section 5.1: horizon-dependent risk aversion has a muted impact on equilibrium prices under the one-period asset pricing framework.

Even though the dividend strips’ price loadings $\{\%_{d,h}, *_{d,h}\}_h$ on the consumption shocks do not depend on $!!$, the one-period expected returns on these assets may be impacted by horizon-dependent risk aversion, since the pricing of volatility shocks depends on both $!$ and $!!$ — see Proposition 4.

Proposition 5. **hen volatility is time varying the slopes of the term-structures of dividend strips expected returns and expected excess returns**

- **are flatter when** $! > !!$ **but of the same sign than under the standard model** $! = !!$
- **vary over time with volatility without changing sign: they are steeper when** $'_t$ **is high.**

The pricing results of Lemma 3 and Proposition 5 show that horizon-dependent risk aversion affects the pricing of equity assets, in levels and term structures, when volatility is time varying; but a lower risk aversion for long-horizon payoffs than for short-horizon payoffs **does not** result in lower expected returns for long-horizon assets than for short-horizon assets.

Under the standard calibrations of the long-run risk model with Epstein and Zin (1989) preferences (Bansal and Yaron, 2004; Bansal et al., 2009), the term-structure of expected returns of dividend strip futures is upward sloping. Proposition 5 states that relaxing the model, as we do, to let the long-run risk aversion $!!$ be lower than the immediate risk aversion $!$ cannot help reverse the slope to downward sloping, at any point in time — Proposition 5 holds for both conditional and unconditional term-structures. This is in direct contradiction with van Binsbergen et al. (2012) and van Binsbergen and Koijen (2016), who find the term-structure of dividend strip futures’ expected returns is downward sloping across assets and regions.

[14] The closed-form solutions for $\mu_{d,h}$ and $*_{d,h}$ are provided in Appendix B.3.

Interestingly, van Binsbergen and Koijen (2016) find their result to be more robust for the term-structure of Sharpe ratios, and our model **can** generate a downward sloping term-structure of Sharpe ratios for mid to long-term horizons under the standard calibration of long-run risk, as we show in Section 6.[15] Though the slope of the expected returns of dividend futures is more upward sloping in more volatile times — from Proposition 5 — under the standard calibration of the model — that of the Sharpe ratios can become more downward sloping for mid to long-term horizons. Note however our calibrated one-period model does not quantitatively match the Sharpe ratios from van Binsbergen and Koijen (2016): the term-structure is only slightly downward sloping, and quantitatively too flat compared to their data (see Figure 4 in Section 6).

Can our model still help explain equilibrium asset prices, in levels and in the term-structure, over time? The recent evidence in Bansal et al. (2017) suggests an interesting framework to study this question. Bansal et al. (2017) find, using data on dividend strip futures as in van Binsbergen and Koijen (2016), that the term-structure of expected excess returns was increasing most of the time over 2004—2017; but was sharply downward sloping during the financial crisis (December 2007—June 2009) — see also van Binsbergen et al. (2013). The evidence in Gormsen (2016) further indicates that, on average, low price-dividend ratios — e.g. periods of high volatility under the model of Section 5.1 — correspond to steeper **upward** sloping term-structures of expected excess returns — consistent with the time series result of Proposition 5.

Taken together, these empirical results suggest that our model performs quite well: Proposition 4 shows that it qualitatively matches the usual asset pricing moments, and Proposition 5 shows that it matches the term-structure of expected returns on equity, and its variations over time, outside of periods of crisis (how well it performs quantitatively is explored in Section 6); **and** it can do so without implying an un-reasonable preference for early resolutions of uncertainty, in contrast to the standard long-run risk framework — the results of Proposition 1 and Corollary 2.

But can it also propose a rationalization of the slope reversal that happened during the financial crisis of 2007—2009?[16] This is the question we explore next.

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[15] This result can obtain because !! < ! results in a flat term-structure of dividend strip expected excess returns beyond a given horizon, while the quantity of risk keeps rising with the horizon under long-run risk processes.

[16] Bansal et al. (2017) argue the standard long-run risk model can generate a reversal in the slope of expected holding-to-maturity returns if the negative shocks to the aggregate consumption during the crisis, both in drift and volatility, are expected to be followed by a reversal to the mean. However, the calibrated model (e.g. Bansal and Yaron, 2004; Bansal et al., 2009) cannot quantitatively match the slopes of the term-structures in and out of crisis. To explain the slope reversal would, for instance, require introducing a regime shift consumption process with extremely sharp mean-reversions following the crisis shocks.

5.3 Constrained asset pricing – liquidity crunch

Our analysis so far rests on the assumption of a one-period framework: the stochastic discount factor derived in Proposition 2 assumes retrading in every period, appropriate for a representative agent who determines the equilibrium asset pricing moments we consider in Section 6. In dynamically consistent models, one-period pricing is an innocuous assumption: the h-period stochastic discount factor that prices, at time t, an asset with payoff at $t+h$ is the same as the product of all one-period stochastic discount factors between t and $t+h$. For dynamically inconsistent preferences such as the horizon-dependent risk aversion model of Definition 1, the long-horizon stochastic discount factors may differ from the products of the one-period factors and therefore departing from the one-period framework to allow for lower trading frequency **can** affect equilibrium prices. We investigate how much so in this section, focusing on the term-structure of expected returns.

We interpret lower trading frequencies as a form of illiquidity which can be both exogenously imposed, e.g. through infrequent trading opportunities, or endogenously optimal, e.g. when buy-and-hold strategies help avoid rising trading costs. The literature on asset prices with liquidity risk points out the additional risk premium directly attributable to illiquidity (e.g. Acharya and Pedersen, 2005; Lee, 2011).[17] Our approach here is complementary since our focus is on the slope of the term structure of risk premia, not on its level.

We consider the limit case of an investor who prices assets with horizon h under a pure buy-and-hold strategy: she assumes no re-trading at intermediate dates.

Proposition 6. **Under the horizon-dependent risk aversion preferences of Definition 1 and $\rho = 1$ the stochastic discount factor for a buy-and-hold strategy with horizon h is given by**[18]

$$M^{\text{buy-and-hold}}_{t,t+h} = \beta^h \left(\frac{C_{t+h}}{C_t}\right)^{-1} \times \underbrace{\frac{V_{t+1}^{1-\gamma}}{E_t\left(V_{t+1}^{1-\gamma}\right)}}_{\text{high }\gamma} \times \underbrace{\frac{V_{t+2}^{1-\bar{\gamma}}}{E_{t+1}\left(V_{t+2}^{1-\bar{\gamma}}\right)} \times \cdots \times \frac{V_{t+h}^{1-\bar{\gamma}}}{E_{t+h-1}\left(V_{t+h}^{1-\bar{\gamma}}\right)}}_{\text{low }\bar{\gamma}},$$

whereas under one-period trading the horizon h stochastic discount factor is given by

$$M_{t,t+1} \times \cdots \times M_{t+h-1,t+h} = \beta^h \left(\frac{C_{t+h}}{C_t}\right)^{-1} \times \underbrace{\frac{V_{t+1}^{1-\gamma}}{E_t\left(V_{t+1}^{1-\gamma}\right)} \times \cdots \times \frac{V_{t+h}^{1-\gamma}}{E_{t+h-1}\left(V_{t+h}^{1-\gamma}\right)}}_{\text{high }\gamma}.$$

[17] See also Duffie (2010) and Tirole (2011) for surveys of the literature on liquidity.

[18] The more general case with $\rho \neq 1$ is provided in Appendix A.2.

Compared to the one-period investor, with implicit risk aversion ! for future shocks at all horizons, the buy-and-hold agent evaluates the shocks between $t+2$ and $t+h$ with lower risk aversion !! Ñsuggesting a higher willingness to pay for risky assets and therefore lower expected returns than under frequent intermediate trading. [19]

To fully explore the role liquidity crunches can play, letÕs assume equilibrium prices are set by buy-and-hold investors. This implicitly makes several assumptions: that it is internally optimal for investors to choose such buy-and-hold strategies when liquidity falls (e.g. higher transaction costs); and that there are investors to clear the market every period. Sketching a complete equilibrium model to rationalize such assumptions is beyond the scope of this paper and left for future research.

To be able to speak to the empirical evidence, we again consider expected one-period excess returns on dividend strip. At time t, the dividend strip with horizon h is priced by buy-and-hold investors with horizon h, under the stochastic discount factor $"^{\text{buy-and-hold}}_{t,t+h}$. At time $t+1$, the same dividend strip (now with horizon $h\#1$) is priced by buy-and-hold investors with horizon $h\#1$, under the stochastic discount factor $"^{\text{buy-and-hold}}_{t+1,t+1+h\#1} = "^{\text{buy-and-hold}}_{t+1,t+h}$. This implies a one-period return on dividend strip futures between t and $t+1$ given by

$$R^F_{t+1,h} = \frac{E_{t+1}\#"^{\text{buy-and-hold}}_{t+1,t+h} D_{t+h}\$^4 \; E_t\#"^{\text{buy-and-hold}}_{t,t+h} D_{t+h}\$}{E_{t+1}\#"^{\text{buy-and-hold}}_{t+1,t+h}\$^4 \; E_t\#"^{\text{buy-and-hold}}_{t,t+h}\$}.$$

Proposition 7. **nder consumption process (9) and dividend risk (15) buy-and-hold investors with the horizon-dependent risk aversion preferences of Definition 1 have a downward impact on the slope of the term structure of dividend strips expected excess returns.**

The result holds even when volatility is constant as in consumption process (3). If volatility is time varying the downward pressure on the term-structure is greater when the economy is more volatile.

Proposition 7 can explain the evidence of Bansal et al. (2017) that the term-structure of expected excess returns was sharply downward sloping during the Þnancial crisis of 2007— 2009, if we assume that prices over that period were driven, at least partly, by buy-and-hold investors. This seems reasonably realistic, whether driven by actual constraints on trading frequencies in the form of liquidity disruptions, or driven by optimal choices,

[19] We show in Appendix A.3 that naive agents in the one-period standard framework behave as the buy-and-hold investors in Proposition 6: $"^{\text{naive}}_{t,t+1} \% áá á \%"^{\text{naive}}_{t+h\#1,t+h} = "^{\text{buy-and-hold}}_{t,t+h}$, when $\$ = 1$.

so as to avoid higher trading costs (see Brunnermeier, 2009, for a detailed description of the liquidity disruptions during the financial crisis).

The result of Proposition 7 also sheds light on differences in term structures across markets, suggesting a more downward sloping term structure in markets with less liquidity and/or with longer trading horizons. This matches quite naturally the empirical evidence in Giglio et al. (2014) for the housing market — long-horizon assets with high transaction costs.

Horizon-dependent risk aversion preferences (Definition 1) thus formally imply term-structures of expected returns in line with the evidence, not only with the average upward/flat shape (Section 5.2) but also with its time variations in good time — higher slope under higher volatility (Section 5.2) — and with the slope reversal in liquidity crises (Section 5.3). This separates our model from the existing recent literature on term-structures of returns which focuses on deriving and rationalizing downward sloping term structures of expected returns at all times (see our review of the literature) — contrary to the evidence in Bansal et al. (2017) and van Binsbergen et al. (2013).

In the next section, we explore whether our model performs not just qualitatively but also quantitatively.

6 Quantitative results

The consumption and dividend growth processes (9) and (15) are calibrated strictly as in Bansal et al. (2014). This choice, instead of a GMM approach incorporating term structure moments which could improve the fit of Figures 3, 4 and 5, allows us to highlight how the preference model of Definition 1 — rather than changes in the calibration for the endowment process — affects prices.

The calibration of Bansal et al. (2014) fits moments in the macro data, within the constraints of the consumption growth model (Tables 1a and 1b, data source from Shiller's website, annual data 1926—2009). Note that fitting both the strongly positive autocorrelation for consumption growth at the one-year frequency and the strongly negative one at the four-year frequency is difficult when the time varying drift follows an AR(1) process (see Bryzgalova and Julliard, 2015, for a recent analysis of consumption growth in the data). In line with Bansal et al. (2014), we use $\delta = 0.9989$ for the monthly rate of time discount. The elasticity of intertemporal substitution is 1 throughout (see Appendix C for $\psi \neq 1$ results).

Table 1: Calibration.

(a) Parameters.

Process	Parameters
c_t	$\mu_c = 0.15\%$
	$\%_c = 1$
	$\&_c = 1$
x_t	$(_x = 0.975$
	$\&_x = 0.038$
$'_t$	$(_' = 0.999$
	$' = 0.72\%$
	$\&_' = 0.00028\%$
d_t	$\mu_d = 0.15\%$
	$\%_d = 2.5$
	$\&_d = 5.96$
	$+ = 2.6$

(b) Results.

Moment	Data	Calibr.
$E[d_{cons}]$	0.02	0.02
$'[d_{cons}]$	0.03	0.03
$AC_1[d_{cons}]$	0.29	0.21
$AC_2[d_{cons}]$	0.03	0.15
$AC_3[d_{cons}]$	# 0.17	0.12
$AC_4[d_{cons}]$	# 0.22	0.10
$AC_5[d_{cons}]$	0.03	0.07
$E[d_{div}]$	0.01	0.02
$'[d_{div}]$	0.11	0.19
$AC_1[d_{div}]$	0.18	0.05
$\$(d_{div}, d_{cons})$	0.52	0.45

Data is from Shiller's website, annual 1926—2009

6.1 Timing premium

We first study the quantitative implications of horizon dependent risk aversion, under the preferences of Definition 1, on the agent's willingness to pay for an early resolution of all consumption uncertainty.

Figure 2 plots the timing premium for both horizon-dependent risk aversion and for standard Epstein-Zin preferences when ! = 10, using the calibration of Bansal et al. (2014) in Table 1a.[20]

As pointed out by Epstein et al. (2014), calibrating a standard Epstein-Zin representative agent to match asset pricing moments implies an extreme high willingness to pay for early resolution—more than 80% of the value of her expected consumption under Bansal et al. (2014).[21] Under the same calibration, an agent with horizon-dependent risk aversion can have a significantly lower willingness to pay for an early resolution. In fact, for delayed risk aversion !! (4.42, the agent with the utility model of Definition 1 prefers a **late** resolution of risk (negative timing premium).

This result is of particular interest for two reasons. First, as briefly discussed in Sec-

[20] In Section 4, we analyze the timing premium under the constant volatility process (3), as in Epstein et al. (2014), to make the results more readily interpretable. We formally derive the timing premium under the stochastic volatility process (9) in Appendix B.

[21] In the calibration of Bansal and Yaron (2004) with stochastic volatility, but a lesser persistence in the volatility shocks, the timing premium is "just" 30%.

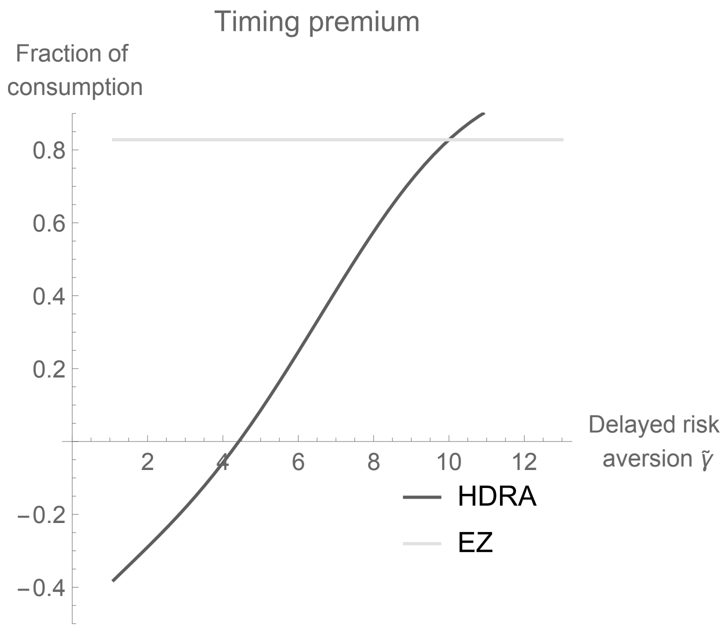

Figure 2: Effect of horizon-dependent risk aversion (HDRA) on willingness to pay for early resolution of uncertainty (timing premium), compared to Epstein-Zin preferences (EZ) with $\gamma = 10$.

tion 4, apart from the fact that a 80% premium seems unrealistically large, there is no clear consensus concerning the "right" value for the timing premium: how large it should be, or whether it should even be positive. With horizon-dependent risk aversion, and the calibration of Table 1a, the possible values for the timing premia range from −38% to +83%: depending on the parametrization of the long-horizon risk aversion $\tilde{\gamma}$, our framework can accommodate any reasonable valuation of early versus late resolutions of uncertainty. Second, and crucially, the average risk free rate and equity premium are mostly determined by the calibration of the immediate risk aversion γ, with $\tilde{\gamma}$ playing a limited role. This is made clear by the results presented in Table 2. Taken together, these two observations show that, under the horizon-dependent risk aversion model of Definition 1, calibrating the usual asset pricing moments no longer precludes a reasonable timing premium.

6.2 Asset prices

We now turn to the pricing of risk in the term structure. We present results for $\tilde{\gamma} \approx 1$, under which horizon-dependent risk aversion is the most impactful. Figures under higher calibrations of $\tilde{\gamma}$ are provided in Appendix D.

Figure 3 depicts the unconditional expected dividend strips one-month returns (annualized) and Figure 4 their unconditional Sharpe ratios, under horizon-dependent risk

Table 2: Equity premium versus timing premium.

		Equity premium	Timing premium
Data		6.64%	—
Calibration	!! & 1	7.01%	—38%
	!! = 2	7.07%	—29%
	!! = 3	7.14%	—18%
	!! = 4.42	7.23%	0
	!! = 5	7.26%	9%
	!! = 7	7.39%	41%
	!! = ! = 10	7.58%	83%

Annualized returns under ! = 10, $ = 1; calibration of Bansal et al. (2009) – Table 1a; data is from Shiller's website, annual 1926—2009.

aversion with ! = 10 and !! & 1, as well as under standard Epstein-Zin preferences with ! = 10. The elasticity of intertemporal substitution is set to 1 in both cases. Both term-structures are increasing all the way through under the standard Epstein-Zin model and the calibration of Bansal et al. (2009) – Table 1a; a well established result in the literature. Under horizon-dependent risk aversion, the term-structure of expected returns is also upward sloping but considerably flatter, as formally established in Proposition 5. Under ! = 10 and !! & 1, and the calibration of Bansal et al. (2009) (Table 1a), the term-structure is almost flat beyond the ten-year horizon. This results in a slightly downward-sloping term-structure of Sharpe ratios for longer-horizon assets, in Figure 4.

As we discuss above, in Section 5.1, the term-structures in Figures 3 and 4 under horizon-dependent risk aversion do not match the evidence in van Binsbergen et al. (2012) and van Binsbergen and Koijen (2016). They find increasing expected returns and Sharpe ratios over the first 7-year horizons, as we do; but with 7-year horizon levels much above the whole index, implying the term-structures must decrease sharply beyond a given horizon – something we cannot replicate. On the other-hand, the term-structures we obtain are consistent with the most recent evidence in Bansal et al. (2017): outside of the crisis years 2007—2009, these authors find increasing term-structures over the first 7-year horizons for dividend strips' expected returns and Sharpe ratios, with just slightly lower levels on the whole index, suggesting a flattening or very slight decrease beyond a given horizon.[22] Such term-structure shapes do not obtain under the standard Epstein-Zin model,

[22]Their term-structure moments are obtained over the short 2005—2017 period, so we do not try to match their levels, just their overall shapes.

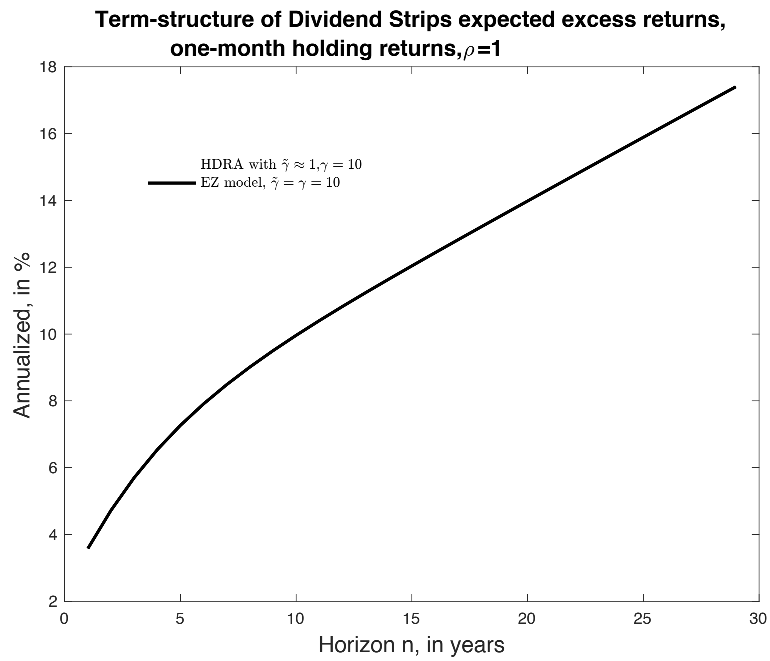

Figure 3: Term structure of dividend strips expected excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) Ñ Table 1a.

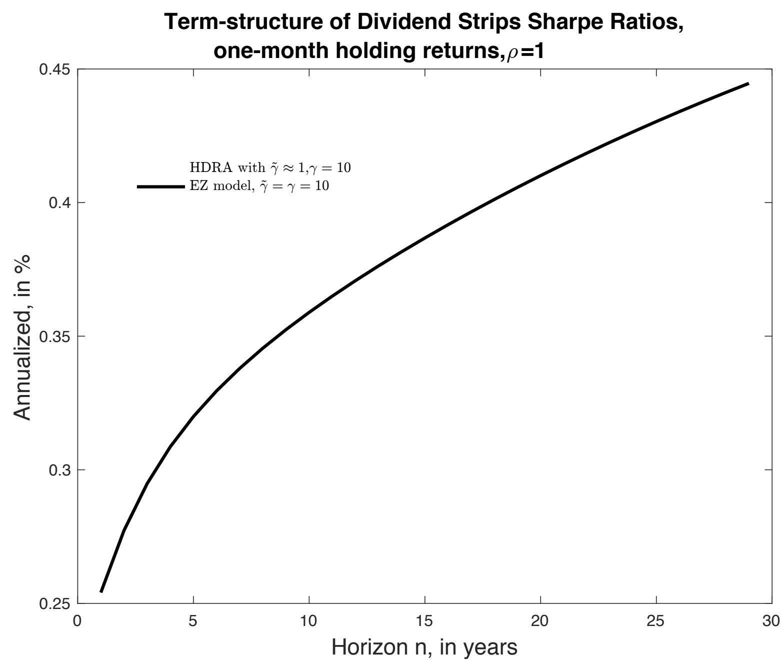

Figure 4: Term structure of Sharpe ratios of dividend strips returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) Ñ Table 1a.

as Figures 3 and 4 make clear. But modifying the long-run risk framework to introduce the notion of horizon-dependent risk aversion, as we do with the preferences of Definition 1, makes the model compatible with the term-structures of risk prices observed in non-crisis, "normal" times.

This can be achieved under horizon-dependent risk aversion without compromising on the model's ability to match the usual asset pricing moments: Table 2 shows the equity premium is barely affected —from 7.58% under the standard Epstein-Zin preferences with ! = 10, to 7.01% under ! = 10 and !! & 1, with the calibration of Bansal et al. (2009); while Proposition 4 formally shows the risk-free rate is left unchanged by the {! , !!} specification.

We now turn to the quantitative implications of horizon-dependent risk aversion for periods that depart from business-as-usual, when one-period pricing no longer prevails for equilibrium prices, e.g. the liquidity crunch of 2007 — 2009. Under the buy-and-hold model sketched out in Section 5.3, assets with payoffs at horizon h " 2 are priced with both immediate risk aversion ! and long-term risk aversion !!, whereas assets with one-period horizon payoffs are priced with high risk aversion ! only. This results in a downward pressure on the term-structure of expected excess returns (Proposition 7). We derive, and illustrate in Figure 5, the conditional expected returns of dividend futures for buy-and-hold investors at short-to-medium horizons when volatility reaches unusually high levels in the consumption and dividend growth processes (9) and (15), under the calibration of Bansal et al. (2009).

Our model implies a sharply downward sloping term-structure of expected excess returns under liquidity crunches. To quantify the slope impact of our model, during liquidity crises, we calculate the conditional expected excess returns for the seven-year horizon dividend risk relative to the next-period horizon dividend risk —corresponding to the empirical analysis in Bansal et al. (2017). As Table 3 makes clear, the standard model of Epstein and Zin (1989) under the calibration of Bansal et al. (2014) fails unambiguously in generating the observed term-structure during the financial crisis of 2007—2009: its seven-year horizon expected one-month excess return is more than three times that of the immediate horizon, whereas it is roughly ten times smaller in the data. In contrast, our model with horizon-dependent risk aversion can generate the correct ratio for the short-horizon relative to the longer-horizon excess returns (for !! & 1).

Note the levels for the expected excess returns in Figure 5 are much lower than those reported in Bansal et al. (2017) (they find more than 10% annualized excess returns at the front end of the curve), suggesting the consumption and dividend growth processes (9)

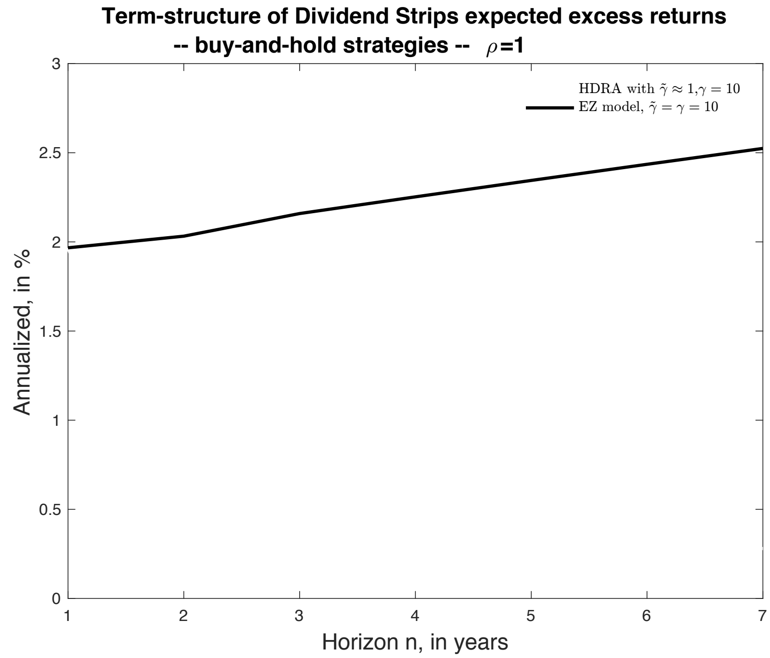

Figure 5: Term structure of dividend strips expected excess returns for buy-and-hold strategies under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a; case with ' t four standard deviations above average.

Table 3: Ratio of immediate versus 7-year dividend strip expected excess returns.

Data (Bansal et al, 2017):		0.09
Calibration:	!! & 1	0.11
	!! = 2	0.39
	!! = 3	0.67
	!! = 5	1.27
	!! = ! = 10	3.46
Annualized returns under ! = 10, $ = 1; calibration of Bansal et al. (2009) (Table 1a); case' $_t^2$ = ' ^{2} + 4& .		

and (15) may require regime shift modifications to rationalize risk premia during the financial crisis; though they may simply be attributable to a liquidity risk level effect. [23] However, the slope impact of horizon-dependent risk aversion buy-and-hold investors matches the evidence under the business-as-usual calibration of Bansal et al. (2009), and does not require reversed-engineered process adjustments for the financial crisis period. [24]

7 Conclusion

Established equilibrium asset pricing models have been criticized because they make counterfactual predictions about the term structure of risk prices (e.g., van Binsbergen et al., 2012, 2013; van Binsbergen and Koijen, 2016; Bansal et al., 2017). Calibrations of the long-run risk model of Bansal and Yaron (2004) are also difficult to reconcile with the microeconomic foundations of the preferences they employ (Epstein et al., 2014). We show that these criticisms do not imply that the whole model needs to be discarded. Instead, relaxing the restriction of Epstein and Zin (1989) that risk preferences be constant across horizons makes it possible to retain the desirable pricing properties of the long-run risk model, obtain reasonable implications for the timing of the resolution of uncertainty, and simultaneously match the slopes of the term structure of risk prices in **and** out of liquidity crises.

Our analysis is accomplished with considerable technical difficulty and is not due to a tautological relationship between risk aversion and risk pricing at different maturities. In particular, we show how to solve for general equilibrium asset prices in an economy populated by agents with dynamically inconsistent risk preferences. In a one-period classical model, the price of risk depends on the horizon, but only if volatility is stochastic. This insight leads to several testable predictions. One prediction we analyze, that the term structure of risk premia be subject to slope reversals in and out of crises, rationalizes the recent empirical literature —as far as we know the only model to do so. Other implications of our framework, in particular how liquidity influences term-structures’ slopes, constitute opportunities for future research. We conclude that relaxing the common assumption that risk preferences are constant across maturities —and specifically, replacing it with the assumption that short-horizon risk aversion is higher than long-horizon risk aversion —is a useful new tool for asset pricing and macro-finance.

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[23] See Muir (2016) for a discussion on the behavior and dramatic increases in risk premia during financial crises.

[24] Our results are illustrated under the high volatility case $'^2_t = '^2 + 4\&$, but similar ratios obtain for $'_t = '$.

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Appendix (For online publication)

A Derivations under general sequence of risk aversions

Let $\{\gamma_h\}_{h \geq 1}$ be a decreasing sequence representing risk aversion at horizon h. In period t, the agent evaluates a consumption stream starting in period $t+h$ by

$$V_{t,t+h} = \left[(1-\beta) C_{t+h}^{1-\rho} + \beta \left(E_{t+h} \left[V_{t,t+h+1}^{1-\gamma_{h+1}} \right] \right)^{\frac{1-\rho}{1-\gamma_{h+1}}} \right]^{\frac{1}{1-\rho}} \quad \text{for all} \quad h \geq 0. \tag{16}$$

The agent's utility in period t is given by setting $h = 0$ in (16) which we denote by $V_t \equiv V_{t,t}$ for all t:

$$V_t = \left[(1-\beta) C_t^{1-\rho} + \beta \left(E_t \left[V_{t,t+1}^{1-\gamma_1} \right] \right)^{\frac{1-\rho}{1-\gamma_1}} \right]^{\frac{1}{1-\rho}}$$

As in the Epstein-Zin model, utility V_t depends on deterministic current consumption C_t and a certainty equivalent $\left(E_t \left[V_{t,t+1}^{1-\gamma_1} \right] \right)^{\frac{1}{1-\gamma_1}}$ of uncertain continuation values $V_{t,t+1}$, where the aggregation of the two periods occurs with constant elasticity of intertemporal substitution given by $1/\rho$, regardless of the horizon h. However, in contrast to the Epstein-Zin model, the certainty equivalent of consumption starting at $t+1$ is calculated with relative risk aversion γ_1, wherein the certainty equivalent of consumption starting at $t+2$ is calculated with relative risk aversion γ_2, and so on. This is the concept of horizon-dependent risk aversion applied to the nested valuation of certainty equivalents, as in the Epstein-Zin model, but with relative risk aversion γ_h for the certainty equivalent formed at horizon h. Our model therefore nests the Epstein-Zin model if we set $\gamma_h = \gamma$ for all h, which, in turn, nests the standard time-separable model for $\gamma = \rho$.

An interesting question is the possibility to axiomatize the horizon-dependent risk aversion preferences we propose. Our dynamic model builds on the functional form of Epstein and Zin (1989) which captures non-time-separable preferences of the form axiomatized by Kreps and Porteus (1978). However, our generalization of Epstein and Zin (1989) explicitly violates Axiom 3.1 (temporal consistency) of Kreps and Porteus (1978) which is necessary for the recursive structure. In contrast to Epstein-Zin, the preference of our model captured by $V_t \equiv V_{t,t}$ is **not** recursive since $V_{t+1} \equiv V_{t+1,t+1}$ does not recur in the definition of V_t.

In order to derive the closed-form solution for $V_t \equiv V_{t,t}$, we assume that risk aversion is decreasing until some horizon H and constant thereafter, $\gamma_h > \gamma_{h+1}$ for $h < H$ and $\gamma_h = \bar{\gamma}$

for $h \geq H$. Starting with $V_{t,t+H}$, our model then corresponds to the standard Epstein-Zin recursion with risk aversion $\tilde{\gamma}$ for which we can use the standard solution. Determining V_t then is just a matter of solving backwards.

A.1 Stochastic discount factor

We present the derivation of the stochastic discount factor with a general sequence of risk aversions $\{\gamma_h\}_{h \geq 1}$. The equations simplify to the ones in the main text by setting $\gamma_1 = \gamma$ and $\gamma_h = \tilde{\gamma}$ for $h \geq 2$.

Proof of Proposition 2. This appendix derives the stochastic discount factor of our dynamic model using an approach similar to the one used by Luttmer and Mariotti (2003) for dynamic inconsistency due to non-geometric discounting. In every period t the agent chooses consumption C_t for the current period and state-contingent levels of wealth $\{W_{t+1,s}\}$ for the next period to maximize current utility V_t subject to a budget constraint and anticipating optimal choice C^*_{t+h} in all following periods ($h \geq 1$):

$$\max_{C_t,\{W_{t+1}\}} \left[(1-\beta) C_t^{1-\rho} + \beta E_t\left[\left(V^*_{t,t+1}\right)^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}\right]^{\frac{1}{1-\rho}}$$

$$\text{s.t.} \quad \pi_t C_t + E_t[\pi_{t+1} W_{t+1}] \leq \pi_t W_t$$

$$V^*_{t,t+h} = \left[(1-\beta)\left(C^*_{t+h}\right)^{1-\rho} + \beta E_{t+h}\left[\left(V^*_{t,t+h+1}\right)^{1-\gamma_{h+1}}\right]^{\frac{1-\rho}{1-\gamma_{h+1}}}\right]^{\frac{1}{1-\rho}} \quad \text{for all } h \geq 1.$$

Denoting by λ_t the Lagrange multiplier on the budget constraint for the period-t problem, the first order conditions are: [25]

- For C_t:

$$\left[(1-\beta) C_t^{1-\rho} + \beta E_t\left[V_{t,t+1}^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}\right]^{\frac{1}{1-\rho}-1} (1-\beta) C_t^{-\rho} = \lambda_t.$$

[25] For notational ease we drop the star from all Cs and Vs in the following optimality conditions but it should be kept in mind that all consumption values are the ones optimally chosen by the corresponding self.

- For each $W_{t+1,s}$:

$$\frac{1}{1-\rho}\left[(1-\beta)C_t^{1-\rho}+\beta\left(E_t V_{t,t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}-1}\beta\frac{d}{dW_{t+1,s}}\beta\left(E_t V_{t,t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}$$
$$=\Pr[t+1,s]\frac{\pi_{t+1,s}}{\pi_t}\lambda_t.$$

Combining the two, we get an initial equation for the SDF:

$$\frac{\pi_{t+1,s}}{\pi_t}=\beta\frac{\frac{1}{1-\rho}\frac{1}{\Pr[t+1,s]}\frac{d}{dW_{t+1,s}}\left(E_t V_{t,t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}}{1}\frac{1}{(1-\beta)C_t^{-\rho}}. \tag{17}$$

The agent in state s at $t+1$ maximizes

$$\left[(1-\beta)C_{t+1,s}^{1-\rho}+\beta\left(E_{t+1,s}\left(V_{t+1,s,t+2}\right)^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}$$

and has the analogous first order condition for $C_{t+1,s}$:

$$\left[(1-\beta)C_{t+1,s}^{1-\rho}+\beta\left(E_{t+1,s}V_{t+1,s,t+2}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}-1}(1-\beta)C_{t+1,s}^{-\rho}=\lambda_{t+1,s}.$$

The Lagrange multiplier $\lambda_{t+1,s}$ is equal to the marginal utility of an extra unit of wealth in state $t+1,s$:

$$\lambda_{t+1,s}=\frac{1}{1-\rho}\left[(1-\beta)C_{t+1,s}^{1-\rho}+\beta\left(E_{t+1,s}V_{t+1,s,t+2}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}-1}$$
$$\times\frac{d}{dW_{t+1,s}}\left[(1-\beta)C_{t+1,s}^{1-\rho}+\beta\left(E_{t+1,s}V_{t+1,s,t+2}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right].$$

Eliminating the Lagrange multiplier $\lambda_{t+1,s}$ and combining with the initial Equation (17) for the SDF, we get:

$$\frac{\pi_{t+1,s}}{\pi_t}=\beta\frac{\frac{1}{\Pr[t+1,s]}\frac{d}{dW_{t+1,s}}\left(E_t V_{t,t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}}{\frac{d}{dW_{t+1,s}}\left[(1-\beta)C_{t+1,s}^{1-\rho}+\beta\left(E_{t+1,s}V_{t+1,s,t+2}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]}\left(\frac{C_{t+1,s}}{C_t}\right)^{-\rho}.$$

Expanding the V expressions, we can proceed with the di□erentiation in the numerator:

$$
\frac{"_{t+1,s}}{"_t} = E_t \left[(1\#")\, C_{t+1}^{1\#\$} + "E_{t+1}[\ldots]^{\frac{1\#\$}{1\#!}} \right]^{\frac{1\#!}{1\#\$}\cdot\frac{1\#\$}{1\#!}\#1}
$$

$$
\% \left((1\#")\, C_{t+1,s}^{1\#\$} + "E_{t+1,s}[\ldots]^{\frac{1\#\$}{1\#!}} \right)^{\frac{1\#!}{1\#\$}\#1}
$$

$$
\%" \frac{\frac{d}{dW_{t+1,s}}\left((1\#")\, C_{t+1,s}^{1\#\$} + "E_{t+1,s}[\ldots]^{\frac{1\#\$}{1\#!}} \right)}{\frac{d}{dW_{t+1,s}}\left((1\#")\, C_{t+1,s}^{1\#\$} + "E_{t+1,s}[\ldots]^{\frac{1\#\$}{1\#!}} \right)} \left(\frac{C_{t+1,s}}{C_t} \right)^{\#\$} . \qquad (18)
$$

For Markov consumption $C = \%W$, we can divide by $C_{t+1,s}$ and solve both di□erentiations:

- For the numerator:

$$
\frac{d}{dW_{t+1,s}} \left\{ (1\#")\, C_{t+1,s}^{1\#\$} + "E_{t+1,s}\left[(1\#")\, C_{t+2}^{1\#\$} + "E_{t+2}[\ldots]^{\frac{1\#\$}{1\#!}} \right]^{\frac{1\#!}{1\#\$}\cdot\frac{1\#\$}{1\#!}} \right\}
$$

$$
= \left\{ (1\#")\, 1 + "E_{t+1,s}\left[(1\#") \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1\#\$} + "E_{t+2}[\ldots]^{\frac{1\#\$}{1\#!}} \right]^{\frac{1\#!}{1\#\$}\cdot\frac{1\#\$}{1\#!}} \right\}
$$

$$
\%\%_{t+1,s}^{1\#\$} W_{t+1,s}^{\#\$}.
$$

- For the denominator:

$$
\frac{d}{dW_{t+1,s}} \left\{ (1\#")\, C_{t+1,s}^{1\#\$} + "E_{t+1,s}\left[(1\#")\, C_{t+2}^{1\#\$} + "E_{t+2}[\ldots]^{\frac{1\#\$}{1\#!}} \right]^{\frac{1\#!}{1\#\$}\cdot\frac{1\#\$}{1\#!}} \right\}
$$

$$
= \left\{ (1\#")\, 1 + "E_{t+1,s}\left[(1\#") \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1\#\$} + "E_{t+2}[\ldots]^{\frac{1\#\$}{1\#!}} \right]^{\frac{1\#!}{1\#\$}\cdot\frac{1\#\$}{1\#!}} \right\}
$$

$$
\%\%_{t+1,s}^{1\#\$} W_{t+1,s}^{\#\$}.
$$

Substituting these into Equation (18) and canceling we get:

$$\frac{\beta_{t+1,s}}{\beta_t} = \frac{\left\{(1-\beta)C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[(1-\beta)C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1-\alpha}{1-\rho}\times\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1-\alpha}{1-\rho}}}{\left\{(1-\beta)C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[(1-\beta)C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1-\alpha}{1-\rho}\times\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1-\alpha}{1-\rho}}}$$

$$\times\beta\left(\frac{C_{t+1,s}}{C_t}\right)^{-\rho}\left(\frac{(1-\beta)C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\alpha}}}{E_t\left\{(1-\beta)C_{t+1,s}^{1-\rho} + \beta E_{t+1}[\dots]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1-\alpha}{1-\rho}\times\frac{1-\rho}{1-\alpha}}}\right)^{\rho-\alpha}.$$

Simplifying and cleaning up notation, we arrive at

$$\beta_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{V_{t,t+1}}{E_t\left[V_{t,t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}\right)^{\rho-\alpha}\beta\left(\frac{V_{t,t+1}}{V_{t+1}}\right)^{1-\rho},$$

as stated in the text. $\square$

A.2 Stochastic discount factor – illiquid markets

To derive the h-period ahead stochastic discount factor, we use the intertemporal marginal rate of substitution

$$\beta_{t,t+h} = \frac{dV_t/dW_{t+h}}{dV_t/dC_t}$$

where

$$\begin{aligned}\frac{dV_t}{dW_{t+h}} &= \frac{dV_t}{dV_{t,t+h}}\times\frac{dV_{t,t+h}}{dW_{t+h}}\\ &= \frac{dV_t}{dV_{t,t+1}}\times\prod_{\tau=1}^{h-1}\frac{dV_{t,t+\tau}}{dV_{t,t+\tau+1}}\times\frac{dV_{t,t+h}}{dW_{t+h}}.\end{aligned}$$

Due to the homotheticity of our preferences, we can rely on the fact that both $V_{t,t+h}$ and V_{t+h} are homogeneous of degree one which implies that

$$\frac{dV_{t,t+h}/\,dW_{t+h}}{dV_{t+h}/\,dW_{t+h}} = \frac{V_{t,t+h}}{V_{t+h}}.$$

This allows us to derive the h-period SDF $"_{t,t+h}$ as

$$"_{t,t+h} = "^{h} \left(\frac{C_{t+h}}{C_t}\right)^{\%\#\$} \left(\frac{V_{t,t+h}}{V_{t+h}}\right)^{\%1\#\$} \prod_{-=1}^{h} \left(\frac{V_{t,t+-}}{E_{t+-\#1}\left(V_{t,t+-}^{1\#!-}\right)^{\frac{1}{1\#!-}}}\right)^{\$\#!-}.$$

A.3 Naive investors

In our analysis so far, we assumed agents are self-aware about their own dynamic inconsistencies. If our agent is naive about it instead, she wrongly assumes she will optimize on $V_{t,t+h}$ instead of V_{t+h} for all $h\ "\ 1$. In particular, the envelope conditions at $t+1$ applies to $V_{t,t+1}$ in her one-period SDF, which becomes:

$$"^{naive}_{t,t+1} = "\left(\frac{C_{t+1}}{C_t}\right)^{\%\#\$} \left(\frac{V_{t,t+1}}{E_t\left(V_{t,t+1}^{1\#!_1}\right)^{\frac{1}{1\#!_1}}}\right)^{\$\#!_1}$$

The following one-period SDFs for $h\ "\ 1$ are then given by:

$$"^{naive}_{t+h,t+h+1} = "\left(\frac{C_{t+h+1}}{C_{t+h}}\right)^{\%\#\$} \left(\frac{V_{t,t+h+1}}{E_{t+h}\left(V_{t,t+h+1}^{1\#!_{h+1}}\right)^{\frac{1}{1\#!_{h+1}}}}\right)^{\$\#!_{h+1}}$$

When $\$ = 1$, naive agents behave as the buy-and-hold investors in Proposition 6:

$$"^{naive}_{t,t+1}\ \%\ \cdots\ \%\ "^{naive}_{t+h\#1,t+h}\big|_{\$=1} = "^{buy\text{-}and\text{-}hold}_{t,t+h}\big|_{\$=1}.$$

B Exact solutions for $\$ = 1$

This appendix presents the exact solutions derived for unit elasticity of intertemporal substitution, $1/\$ = 1$, and log-normal uncertainty. Denoting logs by lowercase letters, our general model (16) becomes

$$v_t = (1-\beta)\, c_t + \beta \left(E_t[v_{t,t+1}] + \frac{1}{2}(1-\alpha_1)\, \mathrm{var}_t(v_{t,t+1}) \right), \tag{19}$$

with the continuation value $v_{t,t+1}$ satisfying the recursion

$$v_{t,t+h} = (1-\beta)\, c_{t+h} + \beta \left(E_{t+1}[v_{t,t+h+1}] + \frac{1}{2}(1-\alpha_{h+1})\, \mathrm{var}_{t+1}(v_{t,t+h+1}) \right).$$

B.1 Valuation of risk and temporal resolution

Proof of Proposition 1. Starting at horizon $t+1$, Equation (19) corresponds to the standard recursion

$$\tilde{v}_{t+1} = (1-\beta)\, c_{t+1} + \frac{\beta}{1-\tilde{\alpha}} \log\left(E_{t+1}\left[\exp\left((1-\tilde{\alpha})\, \tilde{v}_{t+2}\right)\right]\right).$$

If consumption follows process (3), guess and verify that the solution to the recursion satisfies

$$\tilde{v}_t - c_t = \mu_v + \varphi_v x_t.$$

Substituting in and matching coefficients yields

$$\tilde{v}_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\varphi_c}{1-\beta\rho_x} x_t + \frac{1}{2}\frac{\beta(1-\tilde{\alpha})}{1-\beta}\left[\sigma_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \sigma_x^2\right]'^2.$$

From the perspective of period t,

$$v_t = (1-\beta)\, c_t + \frac{\beta}{1-\alpha} \log\left(E_t\left[\exp\left((1-\alpha)\, \tilde{v}_{t+1}\right)\right]\right)$$

and

$$v_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\varphi_c}{1-\beta\rho_x} x_t + \frac{1}{2}\frac{\beta}{1-\beta}\left[\sigma_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \sigma_x^2\right]'^2\left((1-\alpha) + \beta(\alpha-\tilde{\alpha})\right),$$

as stated in the text. $\square$

If all risk is resolved at $t+1$, log continuation utility $v^*_{t,t+1}$ is given by

$$
\begin{aligned}
v^*_{t+1} &= (1-\beta)c_{t+1} + \beta\left[(1-\beta)c_{t+2} + \beta\left((1-\beta)c_{t+3} + \cdots\right)\right] \\
&= c_{t+1} + \sum_{h=1}^{\infty} \beta^h (c_{t+h+1} - c_{t+h}) .
\end{aligned}
$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$
\begin{aligned}
E[v^*_{t+1}] &= c_t + \frac{1}{1-\beta}\mu + \frac{\varphi_c}{1-\beta\rho_x} x_t , \\
\operatorname{var}(v^*_{t+1}) &= \left[\frac{1}{1-\beta^2}\phi_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \phi_x^2\right]\sigma^2 .
\end{aligned}
$$

Using these expressions, we can derive the early resolution utility at t as

$$
v^*_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\varphi_c}{1-\beta\rho_x} x_t + \frac{1}{2}\frac{\beta(1-\gamma)}{1-\beta^2}\left[\phi_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \phi_x^2\right]\sigma^2 .
$$

Subtracting this from the utility v_t under gradual resolution, we arrive at a timing premium given by

$$
TP = 1 - \exp\left[\frac{1}{2}\frac{\beta^2(1-\gamma)}{1-\beta}\left[\phi_c^2 + \left(\frac{\beta\varphi_c}{1-\beta\rho_x}\right)^2 \phi_x^2\right]\sigma^2\left(\frac{\gamma-\alpha}{1-\gamma} + \frac{1}{1+\beta}\right)\right],
$$

as stated in the text. $\square$

Case with stochastic volatility: If consumption follows process (9) with stochastic volatility, guess and verify that the solution to the recursion for $\tilde{v}_t$ satisfies

$$
\tilde{v}_t - c_t = \tilde{\mu}_v + \varphi_v x_t + \omega_v \sigma_t^2
$$

where

$$
\begin{aligned}
\mu_v &= \frac{"}{1\#"}\left[\mu_c + *!_v'^2(1\#(\cdot) + \frac{1}{2}(1\#!!)*!_v^2\&^2\right]\\
\%_v &= \frac{"\%_c}{1\#"(_x}\\
*!_v &= \frac{1}{2}\frac{"(1\#!!)}{1\#"(\cdot}\left(\&_c^2 + \%_v^2\&_x^2\right).
\end{aligned}
$$

We then obtain:

$$
v_t \# v_t^! = \#\frac{1}{2}"(!\#!!)\left[\left(\&_c^2 + \%_v^2\&_x^2\right)'^2_t + *!_v^2\&^2\right]
$$

$$
\begin{aligned}
v_t \# c_t = {} & \frac{"}{1\#"}\left[\mu_c + *!_v'^2(1\#(\cdot) + \frac{1}{2}*!_v^2\big(1\#!) + "(!\#!!)\big)\&^2\right]\\
& + \%_v x_t + \frac{*!_v}{1\#!!}\big(1\#!) + "(\cdot(!\#!!)\big)'^2_t
\end{aligned}
$$

If all risk is resolved at $t+1$, log continuation utility $v^!_{t,t+1}$ is given by

$$
\begin{aligned}
v^!_{t+1} &= (1\#")c_{t+1} + "\left((1\#")c_{t+2} + "\left((1\#")c_{t+3} + \cdots\right)\right)\\
&= c_{t+1} + \sum_{h=1}^{\infty} "^h(c_{t+h+1}\# c_{t+h}).
\end{aligned}
$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$
\begin{aligned}
E_t[v^!_{t+1}] &= c_t + \frac{1}{1\#"}\mu + \frac{\%_c}{1\#"(_x}x_t,\\
\mathrm{var}_t(v^!_{t+1}) &= \frac{1}{1\#"^2(\cdot}\left['^2_t + \frac{"^2}{1\#"^2}'^2(1\#(\cdot)\right]\&_c^2 + \left(\frac{"\%_c}{1\#"(_x}\right)^2\&_x^2.
\end{aligned}
$$

Using these expressions, we can derive the early resolution utility at t as

$$
v^!_t \# c_t = \frac{"}{1\#"}\mu_c + \frac{"\%_c}{1\#"(_x}x_t + \frac{1}{2}\frac{"(1\#!)}{1\#"^2(\cdot}\left[\left(\&_c^2 + \left(\frac{"\%_c}{1\#"(_x}\right)^2\&_x^2\right)'^2_t + \frac{"^2}{1\#"^2}'^2(1\#(\cdot)\right]
$$

and

$$
\begin{aligned}
v_t \# v_t^{!} &= \frac{"}{1\#"} {*}!_v{}'^{\,2}(1\#(') \; 1\# \frac{1\#!}{1\#!!}\frac{1\#"(')}{1\#"^2(')}\frac{"}{1+"} \\
&+ {*}!_v(')'^{\,2}_t \frac{"}{1\#!!} \; (1\#!)\frac{1\#"}{1\#"^2(')} + (!\#!!) \\
&+ \frac{1}{2}" \; \frac{(1\#!)+"(!\#!!)}{1\#"} \; {*}!_v^2 \&^2
\end{aligned}
$$

Time premium under hyperbolic discounting Ò "-#Ó model Assume ! = !!, but " < "!.

$$
\mathrm{v}\!\!\mathrm{l}_t \# c_t = \frac{"!}{1\#"!}\mu_c + \frac{"!\%_c}{1\#"!(_x} x_t + \frac{1}{2}\frac{"!(1\#!)}{1\#"!} \; / \; \&_c^2 + \frac{"!\%_c}{1\#"!(_x}^{\,2} \&_x^2 1\,'^{\,2}
$$

$$
v_t \# c_t = \frac{"}{1\#!} E_t\left[\exp(1\#!)(\mathrm{v}\!\!\mathrm{l}_{t+1} \# c_{t+1} + c_{t+1} \# c_t)\right]
$$

$$
\mathrm{v}\!\!\mathrm{l}_t \# c_t = \frac{"!}{1\#!} E_t\left[\exp(1\#!)(\mathrm{v}\!\!\mathrm{l}_{t+1} \# c_{t+1} + c_{t+1} \# c_t)\right]
$$

$$
\begin{aligned}
v_t \# c_t &= \frac{"}{"!}(\mathrm{v}\!\!\mathrm{l}_t \# c_t) \\
&= \frac{"}{1\#"!}\mu_c + \frac{"\%_c}{1\#"!(_x} x_t + \frac{1}{2}\frac{"(1\#!)}{1\#"!} \; / \; \&_c^2 + \frac{"!\%_c}{1\#"!(_x}^{\,2} \&_x^2 1\,'^{\,2}
\end{aligned}
$$

If all risk is resolved at $t+1$, log continuation utility $v^{!}_{t,t+1}$ is given by

$$
\begin{aligned}
v^{!}_{t+1} &= \& 1\#"!\,' \; c_{t+1} + "! \; \& (1\#")c_{t+2} + "! \left((1\#")c_{t+3} + \text{ááá}\right)' \\
&= c_{t+1} + \sum_{h=1}^{\%} "!^h (c_{t+h+1} \# c_{t+h}) \\
&= c_t + \sum_{h=0}^{\%} "!^h (c_{t+h+1} \# c_{t+h}).
\end{aligned}
$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$E_t[v^!_{t+1}] = c_t + \frac{1}{1\#\ "!}\mu_c + \frac{\%_c}{1\#\ "!(\ _x}x_t,$$

$$\mathrm{var}_t(v^!_{t+1}) = \cdot\ \frac{1}{1\#\ "!}\ '^{2}\ /\ \&^2_c + 2\ \frac{"!\%_c}{1\#\ "!(\ _x}\ 3^2\ \&^2_x 1\ 0 .$$

Using these expressions, we can derive the early resolution utility at t as

$$v^!_t \# c_t = \frac{"}{1\#\ !}E_t \ \#\ \exp(1\#\ !)\ (v^!_{t+1} \# c_t \$$$

$$v^!_t \# c_t = \frac{"}{1\#\ "!}\mu_c + \frac{"\%_c}{1\#\ "!(\ _x}x_t + \frac{1}{2}\frac{"(1\#\ !)}{1\#\ "!^2}\cdot\ /\ \&^2_c + 2\ \frac{"!\%_c}{1\#\ "(\ _x}\ 3^2\ \&^2_x 1\ '^{2}\ 0$$

and

$$v_t \# v^!_t = \frac{1}{2}\frac{""!(1\#\ !)}{1\#\ "!^2}\cdot\ /\ \&^2_c + 2\ \frac{"!\%_c}{1\#\ "!(\ _x}\ 3^2\ \&^2_x 1\ '^{2}\ 0$$

with $" < "!$, $\frac{"!^2}{1\#"!^2} > \frac{""!}{1\#"!^2} > \frac{"^2}{1\#"^2}$.

When $! > \$$, the timing premium under $\{", "!\}$ is greater than under the $"$-only model and lower than under the $"!$-only model.

B.2 Stochastic discount factor

We now specialize to the case of two levels of risk aversion, setting $!_1 = !$ and $!_h = !!$ for $h " 2$.

Proof of Lemma 1. Under the stochastic process (9), we can guess and verify that the solution to the recursion for $v\!_t$ satisfies

$$v\!_t \# c_t = \mu\!_v + \%_v x_t + *!_v '^2_t$$

where we write $\bar{A}_v = A_v(\bar{\theta})$ throughout for simplification, and

$$\bar{\mu}_v = \frac{\kappa}{1-\kappa}\left[\mu_c + \bar{A}_v\bar{\varsigma}^2(1-\rho_\varsigma) + \frac{1}{2}(1-\bar{\theta})\bar{A}_v^2\sigma_\varsigma^2\right]$$

$$\varphi_v = \frac{\kappa\varphi_c}{1-\kappa\rho_x}$$

$$\bar{A}_v = \frac{1}{2}\frac{\kappa(1-\bar{\theta})}{1-\kappa\rho_\varsigma}\left(\sigma_c^2 + \varphi_v^2\sigma_x^2\right).$$

Substituting these into (19), we arrive at the solution for v_t:

$$v_t - \bar{v}_t = -\frac{1}{2}\kappa(\theta-\bar{\theta})\left[\left(\sigma_c^2 + \varphi_v^2\sigma_x^2\right)\varsigma_t^2 + \bar{A}_v^2\sigma_\varsigma^2\right]$$

and

$$v_t - c_t = \frac{\kappa}{1-\kappa}\left[\mu_c + \bar{A}_v\bar{\varsigma}^2(1-\rho_\varsigma) + \frac{1}{2}\bar{A}_v^2\left((1-\theta) + \kappa(\theta-\bar{\theta})\right)\sigma_\varsigma^2\right] + \varphi_v x_t + \frac{\bar{A}_v}{1-\bar{\theta}}\left((1-\theta) + \kappa\rho_\varsigma(\theta-\bar{\theta})\right)\varsigma_t^2$$

$\blacksquare$

Proof of Lemma 2. The result follows directly from the expression for $\bar{v}_{t+1}$ in the proof of Lemma 1. $\blacksquare$

Proof of Proposition 4. Using the results of Lemmas 1 and (19), the expression for the SDF follows from Equation (8):

$$\begin{aligned} m_{t,t+1} = \log\kappa - \mu_c - \varphi_c x_t - \overbrace{\left[(1-\theta)^2\frac{1-\kappa\rho_\varsigma}{\kappa(1-\bar{\theta})}A_v(\bar{\theta})\right]}^{\xi_t}\varsigma_t^2 \\ - \theta\sigma_c\varsigma_t W_{c,t+1} + (1-\theta)\varphi_v\sigma_x\varsigma_t W_{x,t+1} \\ + (1-\theta)A_v(\bar{\theta})\sigma_\varsigma\varsigma_t W_{\varsigma,t+1}, \end{aligned}$$

The risk-free rate is defined as $r_{f,t} = -\log E_t(M_{t,t+1})$ and simplifies to

$$r_{f,t} = -\log\kappa + \mu_c + \varphi_c x_t + \left[\frac{1}{2} - \theta\right]\sigma_c^2\varsigma_t^2$$

as stated in the text. $\blacksquare$

B.3 Term structure of returns

B.3.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t\left[M_{t,t+1} G_{y,t+1} Y_{t+1,h-1}\right],$$

that is

$$y_{t,h} = E_t\left[m_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right] + \frac{1}{2}\mathrm{var}_t\left(m_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right),$$

where

$$\begin{aligned} g_{y,t+1} = {} & \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ & + \lambda_{y,c}\sigma_c\sigma_t W_{c,t+1} + \lambda_{y,x}\sigma_x\sigma_t W_{x,t+1} + \lambda_{y,\sigma}\sigma_\sigma W_{\sigma,t+1} + \lambda_{y,d}\sigma_d\sigma_t W_{d,t+1}, \end{aligned}$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp\left\{\mu_{y,h} + \phi_{y,h} x_t + \psi_{y,h}\sigma_t^2\right\}.$$

Suppose $h \geq 1$, then:

$$\log M_{t,t+1} G_{t,t+1} Y_{t+1,h-1} = \left\{\begin{aligned} & \log\delta - \mu_c - \phi_c x_t - \tfrac{1}{2}(1-\gamma)^2\left[\left(\sigma_c^2 + \phi_v^2\sigma_x^2\right)\sigma_t^2 + \psi_v^2\sigma_\sigma^2\right] \\ & + \mu_y + \phi_y x_t + \psi_y\sigma_t^2 \\ & + \mu_{y,h-1} + \phi_{y,h-1}\rho_x x_t + \psi_{y,h-1}\left(\sigma^2(1-\rho_\sigma) + \rho_\sigma\sigma_t^2\right) \\ & + \left(-\gamma + \lambda_{y,c}\right)\sigma_c\sigma_t W_{t+1} + \left((1-\gamma)\phi_v + \lambda_{y,x} + \phi_{y,h-1}\right)\sigma_x\sigma_t W_{t+1} \\ & + \left((1-\gamma)\psi_v + \lambda_{y,\sigma} + \psi_{y,h-1}\right)\sigma_\sigma W_{t+1} \\ & + \lambda_{y,d}\sigma_d\sigma_t W_{t+1} \end{aligned}\right.$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t:

$$\phi_{y,h} = -\phi_c + \phi_y + \phi_{y,h-1}\rho_x$$

$$\Rightarrow \quad \phi_{y,h} = \left(-\phi_c + \phi_y\right)\frac{1-\rho_x^h}{1-\rho_x}$$

- Terms in σ_t^2:

$$B_{y,h} = -\frac{1}{2}(1-\phi)^2\left(\sigma_c^2+\beta_v^2\sigma_x^2\right) + B_{y,h-1}(\nu + B_y) + \frac{1}{2}\left(\left(-\phi+\sigma_{y,c}\right)^2\sigma_c^2 + \left((1-\phi)\beta_v+\sigma_{y,x}+\beta_{y,h-1}\right)^2\sigma_x^2 + \sigma_{y,d}^2\sigma_d^2\right)$$

and thus the solution, for $h \geq 1$:

$$B_{y,h} = \left[-\frac{1}{2}(1-\phi)^2\left(\sigma_c^2+\beta_v^2\sigma_x^2\right) + B_y + \frac{1}{2}\left(\left(-\phi+\sigma_{y,c}\right)^2\sigma_c^2 + \sigma_{y,d}^2\sigma_d^2\right)\right]\frac{1-\nu^h}{1-\nu} + \frac{1}{2}\sum_{n=0}^{h-1}\nu^n\left((1-\phi)\beta_v+\sigma_{y,x}+\beta_{y,n-1-h}\right)^2\sigma_x^2$$

- Constant:

$$\mu_{y,h}-\mu_{y,h-1} = \log\delta - \mu_c + \mu_y + \sigma^2(1-\nu)B_{y,h-1} + \frac{1}{2}\left(\left((1-\phi)\tilde{\phi}_v+\sigma_{y,\sigma}+B_{y,h-1}\right)^2 - (1-\phi)^2\tilde{\phi}_v^2\right)\sigma_\sigma^2$$

and thus the solution, for $h \geq 1$:

$$\mu_{y,h} = h\left(\log\delta - \mu_c + \mu_y - \frac{1}{2}(1-\phi)^2\tilde{\phi}_v^2\sigma_\sigma^2\right) + \sum_{n=0}^{h-1}\left[\sigma^2(1-\nu)B_{y,n} + \frac{1}{2}\left((1-\phi)\tilde{\phi}_v+\sigma_{y,\sigma}+B_{y,n}\right)^2\sigma_\sigma^2\right]$$

Note only the constant terms $\left\{\mu_{y,h}\right\}$ are affected by the wedge between ϕ and $\tilde{\phi}$.

In line with the specification of van Binsbergen and Koijen (2016), we consider one-period holding returns for these claims of the form

$$1+R^Y_{t+1,h} = \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\delta_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]} = R_{f,t}\frac{E_t\left[\delta_{t,t+1}\right]G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\delta_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]},$$

with the risk-free rate

$$R_{f,t} = \frac{1}{E_t\left[\delta_{t,t+1}\right]}.$$

The conditional Sharpe Ratio is

$$
\begin{aligned}
SR^{Y}_{t,h} &= \frac{E_t\left[1+R^{Y}_{t+1,h}\right]-1}{\sqrt{\mathrm{var}_t\left(1+R^{Y}_{t+1,h}\right)}} \\
&= \frac{E_t\left(1+R^{Y}_{t+1,h}\right)-1}{\sqrt{E_t\left[\left(1+R^{Y}_{t+1,h}\right)^2\right]-\left(E_t\left(1+R^{Y}_{t+1,h}\right)\right)^2}} \\
&\approx \frac{r_{f,t}+\left(\lambda\sigma_{y,c}\sigma_c^2-(1-\lambda)\beta_v\left(\sigma_{y,x}+\beta_{y,h-1}\right)\sigma_x^2\right)\sigma_t^2-(1-\lambda)\gamma_v\left(\sigma_{y,\omega}+\gamma_{y,h-1}\right)\sigma_\omega^2}{\sqrt{\sigma_t^2\left(\sigma_{y,c}^2\sigma_c^2+\left(\sigma_{y,x}+\beta_{y,h-1}\right)^2\sigma_x^2+\sigma_{y,d}^2\sigma_d^2\right)+\left(\sigma_{y,\omega}+\gamma_{y,h-1}\right)^2\sigma_\omega^2}}.
\end{aligned}
$$

In line with the specification of van Binsbergen and Koijen (2016), we also consider one-period holding returns for futures on these claims of the form

$$
\begin{aligned}
R^{F,Y}_{t+1,h}+1 &= \frac{1+R^{Y}_{t+1,h}}{1+R^{B}_{t+1,h}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}}\frac{B_{t,h}}{B_{t+1,h-1}} \\
&= \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left(M_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right)}\frac{E_t\left(M_{t,t+1}B_{t+1,h-1}\right)}{B_{t+1,h-1}},
\end{aligned}
$$

where $B_{t,h}$ is the price of \$1 at horizon h, i.e. the price of a Bond with horizon h.

Their conditional Sharpe Ratio is

$$
\begin{aligned}
SR^{F,Y}_{t,h} &= \frac{E_t\left(1+R^{F,Y}_{t+1,h}\right)-1}{\sqrt{\mathrm{var}_t\left(1+R^{F,Y}_{t+1,h}\right)}} \\
&= \frac{E_t\left(1+R^{F,Y}_{t+1,h}\right)-1}{\sqrt{E_t\left[\left(1+R^{F,Y}_{t+1,h}\right)^2\right]-\left(E_t\left(1+R^{F,Y}_{t+1,h}\right)\right)^2}} \\
&\approx \frac{\sigma_t^2\left(\lambda\sigma_{y,c}\sigma_c^2-\left(\sigma_{y,x}+\beta_{y,h-1}-\beta_{b,h-1}\right)\left((1-\lambda)\beta_v+\beta_{b,h-1}\right)\sigma_x^2\right)-\left(\sigma_{y,\omega}+\gamma_{y,h-1}-\gamma_{b,h-1}\right)\left((1-\lambda)\gamma_v+\gamma_{b,h-1}\right)\sigma_\omega^2}{\sqrt{\sigma_t^2\left(\sigma_{y,c}^2\sigma_c^2+\left(\sigma_{y,x}+\beta_{y,h-1}-\beta_{b,h-1}\right)^2\sigma_x^2+\sigma_{y,d}^2\sigma_d^2\right)+\left(\sigma_{y,\omega}+\gamma_{y,h-1}-\gamma_{b,h-1}\right)^2\sigma_\omega^2}}.
\end{aligned}
$$

For the **unconditional** Sharpe ratio observe that the volatility process

$$'^{2}_{t+1} \# '^{2} = (_{'} \Big\& '^{2}_{t} \# '^{2} \Big' + \&_{'} W_{t+1}$$

is stationary under the constraint $(_{'} < 1$ with normal distribution with mean $'^{2}$ and variance $\&_{'} = \frac{\&^{2}_{'}}{1\# (^{2}_{'}}$.

and therefore $E\left(\exp\left(a'^{2}_{t}\right)\right) = \exp \Big\& a'^{2} + \frac{1}{2} a^{2} \frac{\&^{2}_{'}}{1\# (^{2}_{'}} \Big'$.

B.3.2 Bonds

Bond prices Let the price at time t for \$1 in h periods be $B_{t,h}$ with $B_{t,0} = 1$. For $h \text{"} 1$, we have

$$B_{t,h} = E_{t}[\text{"}_{t,t+1} B_{t+1,h\# 1}]$$

This is the general problem from above with $g_{y,t+1} = 0$ for all t and therefore

$$b_{t,h} = \mu_{b,h} + \%_{b,h} x_{t} + *_{b,h} '^{2}_{t},$$

with

$$\%_{b,h} = \# \%_{c} \frac{1\# (^{h}_{x}}{1\# (_{x}}$$

$$*_{b,h} = \# \frac{1}{2}(1\# !)^{2} \Big\& \&^{2}_{c} + \%^{2}_{v} \&^{2}_{x} \Big' + *_{b,h\# 1} (_{'} + \frac{1}{2} \Big\& !^{2} \&^{2}_{c} + \left(1\# ! \right) \%_{v} + \%_{b,h\# 1})^{2} \&^{2}_{x} \Big'$$

and

$$*_{b,1} = \text{"} ! \# \frac{1}{2} \% \&^{2}_{c} > 0$$

and $*_{b,h} > 0$ for all h, and $*_{b,h}$ increasing in h.

Further,

$$\mu_{b,h} \# \mu_{b,h\# 1} = \log \text{"} \# \mu_{c} + '^{2} (1\# (_{'}) *_{b,h\# 1} + \text{"} (1\# !) *!_{v} *_{b,h\# 1} + \frac{1}{2} *^{2}_{b,h\# 1} \% \&^{2}_{'}$$

increasing in h. But $\mu_{b,h}$ can be decreasing if $\log \text{"} \# \mu_{c} < 0$.

Bond returns The one-period returns are given by:

$$R^B_{t+1,h} = \frac{B_{t+1,h-1}}{B_{t,h}} - 1$$

and therefore

$$\log\left(R^B_{t+1,h} + 1\right) = -\log " + \mu_c - \left[(1-!)*!_v *_{b,h-1} + \frac{1}{2}*^2_{b,h-1}\right]\&^2 + \%_c x_t + (*_{b,h-1}(- *_{b,h})'^2_t$$
$$+ *_{b,h-1}\& W_{t+1} + \%_{b,h-1}\&_x '_t W_{t+1}$$

the term structure of expected returns is given by:

$$E_t\left(R^B_{t+1,h} + 1\right) \& -\log " + \mu_c - (1-!)*!_v *_{b,h-1}\&^2 + \%_c x_t - \left[! - \frac{1}{2}\right]\&^2_c + (1-!)\%_v\%_{b,h-1}\&^2_x '^2_t$$

$$E_t\left(R^B_{t+1,h+1}\right) - E_t\left(R^B_{t+1,h}\right) \& (!-1)*!_v(*_{b,h} - *_{b,h-1})\&^2 + (!-1)\%_v\%_c\frac{(^h_x - (^{h-1}_x}{1-(_x}\&^2_x '^2_t (0.$$

The only impact of !! is through $*!_v$, and makes the slope less decreasing (but not increasing).

Risk-free rate The risk-free rate is given by

$$r_{f,t} = -\log B_{t,1}$$

i.e.

$$r_{f,t} = -\log " + \mu_c + \%_c x_t - \left[! - \frac{1}{2}\right]\&^2_c '^2_t$$

B.3.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h " 1$:

$$\frac{P_{t,h}}{D_t} = E_t\left["_{t,t+1}\frac{D_{t+1}}{D_t}\frac{P_{t+1,h-1}}{D_{t+1}}\right],$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} \# d_t = \mu_d + \%_d x_t + +\&_c{}'_t W_{t+1} + \&_d{}'_t W_{t+1},$$

for all t and therefore

$$p_{t,h} \# d_t = \mu_{p,h} + \%_{d,h} x_t + *_{d,h}{}'^{2}_{t},$$

with

$$\%_{d,h} = (\# \%_c + \%_d) \frac{1 \# (_x^h}{1 \# (_x}$$

$$*_{d,h} = \# \frac{1}{2}(1 \# !)^2 \& \&_c^2 + \%_v^2 \&_x^2 ' + *_{d,h\# 1}(\cdot$$
$$+ \frac{1}{2} \& (\# ! + +)^2 \&_c^2 + (1 \# !)\%_v + \%_{d,h\# 1})^2 \&_x^2 + \&_d^2 '$$

$$*_{d,1} = \frac{1}{2}\&_d^2 + (+ + 1 \# 2!)(+ \# 1) \frac{1}{2}\&_c^2$$

the sign depends on the parameters of the model.

$$\mu_{d,h} \# \mu_{d,h\# 1} = \log " \# \mu_c + \mu_d + '^2 (1 \# (\cdot) *_{d,h\# 1} + " (1 \# !) *!_v *_{d,h\# 1} + \frac{1}{2} *^2_{d,h\# 1} \% \&^2$$

where the sign depends again on the parameters of the model.

For the dividend strips, the spot one-period returns are given by

$$R^P_{t+1,h} + 1 = \frac{P_{t+1,h\# 1}/D_{t+1}}{P_{t,h}/D_t} \frac{D_{t+1}}{D_t},$$

$$\log \& R^P_{t+1,h} + 1 ' = \# \log " + \mu_c \# " (1 \# !) *!_v *_{d,h\# 1} + \frac{1}{2} *^2_{d,h\# 1} \% \&^2$$
$$+ \%_c x_t + (*_{d,h\# 1}(\cdot \# *_{d,h})'^2_t$$
$$+ *_{d,h\# 1}\& W_{t+1} + \%_{d,h\# 1}\&_x{}'_t W_{t+1} + +\&_c{}'_t W_{t+1} + \&_d{}'_t W_{t+1}$$

the conditional expected one-period returns are

$$E_t \left[R^{P}_{t+1,h} + 1 \right] \& \# \log " + \mu_c \# (1 \# !) *!_v *_{d,h\#1} \&^2 + \%_c x_t$$
$$\# \quad ! \, (1 \# +) \# \frac{1}{2} \quad \&^2_c + (1 \# !) \%_v \%_{d,h\#1} \&^2_x \quad '^2_t$$

$$E_t \left[R^{P}_{t+1,h+1} \right] \# E_t \left[R^{P}_{t+1,h} \right] \& \underbrace{(! \# 1)}_{(\;0} *!_v (*_{d,h} \# *_{d,h\#1}) \&^2 + \underbrace{(! \# 1) \%_v (\%_c \# \%_d) \frac{(^h_x \# (^{h\#1}_x}{1 \# (_x}}_{"\;0} \&^2_x \, '^2_t$$

We need $(*_{d,h} \# *_{d,h\#1}) \;"\; 0$ to generate a downward sloping term-structure, but that does not depend on the choice of !!. If $(*_{d,h} \# *_{d,h\#1}) \;(\; 0$, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when $'_t$ is high...

!

The future one-period returns are given by:

$$R^{F,P}_{t+1,h} + 1 = \frac{1 + R^{P}_{t+1,h}}{1 + R^{B}_{t+1,h}}$$

$$\log \left[R^{F,P}_{t+1,h} + 1 \right] = \# \; " \; (1 \# !) *!_v (*_{d,h\#1} \# *_{b,h\#1}) + \frac{1}{2} \left[*^2_{d,h\#1} \# *^2_{b,h\#1} \right] \% \&^2$$
$$+ \left(*_{d,h\#1} \# *_{b,h\#1} \right) (' \# (*_{d,h} \# *_{b,h}) \; '^2_t$$
$$+ (*_{d,h\#1} \# *_{b,h\#1}) \& W_{t+1} + (\%_{d,h\#1} \# \%_{b,h\#1}) \&_x '_t W_{t+1} + +\&_c '_t W_{t+1} + \&_d '_t W_{t+1}$$

$$E_t \left[R^{F,P}_{t+1,h} + 1 \right] = \# \left. \vphantom{\Big|} \right. \left(\underbrace{(1 \# !) *!_v + *_{b,h\#1}}_{"\;0 \text{ and increasing}} \right) (*_{d,h\#1} \# *_{b,h\#1}) \; \&^2$$
$$+ \; !+\&^2_c + \underbrace{(! \# 1) \%_v \# \%_{b,h\#1} (\%_{d,h\#1} \# \%_{b,h\#1})}_{"\;0 \text{ and increasing}} \&^2_x \; '^2_t$$

Note:

$$* _{d,h} \# * _{b,h} = (* _{d,h\#1} \# * _{b,h\#1})(' \; + \left(+ \frac{1}{2} + \# ! \right) \&_c^2 + \left((1 \# !) \%_v + \frac{1}{2} (\%_{d,h\#1} + \%_{b,h\#1}) \right) (\%_{d,h\#1} \# \%_{b,h\#1}) \&_x^2 + \frac{1}{2} \&_d^2 \; 0 \; 1$$

$$\underbrace{\quad}_{(\;0} \qquad \underbrace{\quad}_{(\;0 \text{ for } ! \text{ high enough}} \qquad \underbrace{\quad}_{"\;0} \qquad \underbrace{\quad}_{"\;0}$$

the sign depends on the parameters. But if it is positive increasing, !! reduces the downward impact of it on the term-structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher $'_t$ means a MORE upward sloping term-structure again

!

the Sharpe ratio term structure is given by:

$$SR_{t,n}^{F,P} \& \frac{'^2_t \left(! + \&_c^2 \# (\%_{d,h\#1} \# \%_{b,h\#1}) \left((1 \# !) \%_v + \%_{b,h\#1} \right) \&_x^2 \right) \# (*_{d,h\#1} \# *_{b,h\#1}) \left((1 \# !) *_v + *^1_{b,h\#1} \right) \&^2}{\sqrt{'^2_t \; +^2 \&_c^2 + (\%_{d,h\#1} \# \%_{b,h\#1})^2 \&_x^2 + \&_d^2 \; + (*_{d,h\#1} \# *_{b,h\#1})^2 \&^2}}$$

If the expected returns term-structure is upward sloping with $*_{d,h} \# *_{b,h} (\; 0$ and decreasing, then !! can help make the sharpe ratio term-structure downward sloping.

The unconditional Sharpe ratio term structure is:

$$SR_h^{F,P} \& \frac{'^2 \left(! + \&_c^2 \# (\%_{d,h\#1} \# \%_{b,h\#1}) \left((1 \# !) \%_v + \%_{b,h\#1} \right) \&_x^2 \right) + \frac{1}{2} \frac{\&^2}{1 \# (^2} \left(! + \&_c^2 \# (\%_{d,h\#1} \# \%_{b,h\#1}) \left((1 \# !) \%_v + \%_{b,h\#1} \right) \&_x^2 \right)^2 \# (*_{d,h\#1} \# *_{b,h\#1}) \left((1 \# !) *_v + *^1_{b,h\#1} \right) \&^2}{\sqrt{\begin{array}{l} '^2 \; +^2 \&_c^2 + (\%_{d,h\#1} \# \%_{b,h\#1})^2 \&_x^2 + \&_d^2 \\ + 2 \; (*_{d,h\#1} \# *_{b,h\#1}) (' \# (*_{d,h} \# *_{b,h}) + (\%_{d,h\#1} \# \%_{b,h\#1})^2 \&_x^2 + +^2 \&_c^2 + \&_d^2 \; '^2 \&' \\ \# \; (*_{d,h\#1} \# *_{b,h\#1}) (' \# (*_{d,h} \# *_{b,h}) + \frac{1}{2} \; (\%_{d,h\#1} \# \%_{b,h\#1})^2 \&_x^2 + +^2 \&_c^2 + \&_d^2 \; {}^2 \&' \\ + (*_{d,h\#1} \# *_{b,h\#1})^2 \&^2 \end{array}}}.$$

B.4 Term structure of returns - Illiquid markets

We analyze horizon-dependent dividend claims when markets are illiquid and prices are set by buy-and-hold investors. From above, the SDF for a horizon h investor is (when $\$ = 1$):

$$"_{t,t+h} = "^h \left(\frac{C_{t+h}}{C_t}\right)^{\% - 1} \left(\frac{V_{t+1}}{\left[E_t\, V_{t+1}^{1-!}\right]^{\frac{1}{1-!}}}\right)^{0\, 1-!} \left(\frac{V_{t+2}}{\left[E_{t+1}\, V_{t+2}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \cdots \left(\frac{V_{t+h}}{\left[E_{t+h-1}\, V_{t+h}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!}$$

Consider a dividend with horizon h priced at time t under $"_{t,t+h}$,

$$P_{t,h} = E_t["_{t,t+h} D_{t+h}],$$

$$P_{t,h} = E_t \left\{ "^h \left(\frac{C_{t+h}}{C_t}\right)^{\% - 1} \left(\frac{V_{t+1}}{\left[E_t\, V_{t+1}^{1-!}\right]^{\frac{1}{1-!}}}\right)^{0\, 1-!} \left(\frac{V_{t+2}}{\left[E_{t+1}\, V_{t+2}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \cdots \left(\frac{V_{t+h}}{\left[E_{t+h-1}\, V_{t+h}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} D_{t+h} \right\},$$

The price at time $t+1$ is under $"_{t+1,t+1+(h-1)}$,

$$\frac{P_{t+1,h-1}}{D_{t+1}} = E_{t+1}\left["_{t+1,t+1+h-1} \frac{D_{t+h}}{D_t} \right],$$

$$P_{t+1,h-1} = E_{t+1} \left\{ "^{h-1} \left(\frac{C_{t+h}}{C_{t+1}}\right)^{\% - 1} \left(\frac{V_{t+2}}{\left[E_{t+1}\, V_{t+2}^{1-!}\right]^{\frac{1}{1-!}}}\right)^{0\, 1-!} \left(\frac{V_{t+3}}{\left[E_{t+2}\, V_{t+3}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \cdots \left(\frac{V_{t+h}}{\left[E_{t+h-1}\, V_{t+h}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} D_{t+h} \right\},$$

The one-period return is given by:

$$R^{F,P}_{t+1,h} + 1 = \frac{\frac{P_{t+1,h-1}}{P_{t,h}}}{\frac{B_{t+1,h-1}}{B_{t,h}}}$$

so

$$E_t\left[R^P_{t+1,h}\right] = \frac{E_t \left\{ "^{h-1} \left(\frac{C_{t+h}}{C_{t+1}}\right)^{\% - 1} \left(\frac{V_{t+2}}{\left[E_{t+1}\, V_{t+2}^{1-!}\right]^{\frac{1}{1-!}}}\right)^{0\, 1-!} \left(\frac{V_{t+3}}{\left[E_{t+2}\, V_{t+3}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \cdots \left(\frac{V_{t+h}}{\left[E_{t+h-1}\, V_{t+h}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \frac{D_{t+h}}{D_t} \right\}}{E_t \left\{ "^{h} \left(\frac{C_{t+h}}{C_{t}}\right)^{\% - 1} \left(\frac{V_{t+1}}{\left[E_{t}\, V_{t+1}^{1-!}\right]^{\frac{1}{1-!}}}\right)^{0\, 1-!} \left(\frac{V_{t+2}}{\left[E_{t+1}\, V_{t+2}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \cdots \left(\frac{V_{t+h}}{\left[E_{t+h-1}\, V_{t+h}^{1-!!}\right]^{\frac{1}{1-!!}}}\right)^{0\, 1-!!} \frac{D_{t+h}}{D_t} \right\}}$$

To simplify notations, write:

$$\frac{\left(\frac{D_{t+h}}{D_t}\frac{C_{t+h}}{C_t}\right)^{\# 1}}{E_t\left[\left(\frac{D_{t+h}}{D_t}\frac{C_{t+h}}{C_t}\right)^{\# 1}\right]} = \exp\left[\sum_{j=1}^{h} \Lambda_j W_{t+j}\right]$$

where

$$\Lambda_j W_{t+j} = \Lambda_{t+j\#1}\left[(\%_d \# \%_c)\frac{1\#(_x^{h\#j}}{1\#(_x}\&_x W_{x,t+j} + \&_d W_{d,t+j} + (+\#1)\&_c W_{c,t+j}\right]$$

and

$$\left(\frac{V_{t+j}}{E_{t+j\#1}\left[V_{t+j}^{1\#!!}\right]^{\frac{1}{1\#!!}}}\right)^{1\#!!} = \exp\left[(1\#!!)\&_j W_{t+j} \# \frac{1}{2}\left|(1\#!!)\&_j\right|^2\right]$$

(substitute !! with ! when necessary) where

$$\&_j = \Lambda_{t+j\#1}\left(\%_v \&_x W_{x,t+j} + \&_c W_{c,t+j}\right) + *!_v \& W_{\Lambda,t+j}$$

where W_{t+j} is the $4\%1$ vector of the independent iid shocks at time $t+j$, and $\Lambda_{j,t+j\#1}, \&_{j,t+j\#1}$ is written $\Lambda_j, \&_j$ to simplify the formulas.

We obtain:

$$E_t(R_{t+1,h}) = \frac{E_t\left[\frac{C_{t+1}}{C_t}\exp\left[\sum_{j=3}^{h}\left(\Lambda_j + (1\#!!)\&_j\right)W_{t+j} \# \frac{1}{2}\left|(1\#!!)\&_j\right|^2\right] + (\Lambda_2 + (1\#!)\&_2)W_{t+2} \# \frac{1}{2}|(1\#!)\&_2|^2 + \Lambda_1 W_{t+1}\right]}{"E_t\left[\exp\left[\sum_{j=2}^{h}\left(\Lambda_j + (1\#!!)\&_j\right)W_{t+j} \# \frac{1}{2}\left|(1\#!!)\&_j\right|^2\right] + (\Lambda_1 + (1\#!)\&_1)W_{t+1} \# \frac{1}{2}|(1\#!)\&_1|^2\right]}$$

Because the shocks are iid, we obtain, when volatility is constant:

$$E_t\left(R^P_{t+1,h}\right) = \frac{"^{\#1}E_t\left(\frac{C_{t+1}}{C_t}\right)\exp\left[(\Lambda_2 + (1\#!)\&_2)W_{t+2} \# \frac{1}{2}|(1\#!)\&_2|^2 + \Lambda_1 W_{t+1}\right]}{E_t\exp\left[(\Lambda_2 + (1\#!!)\&_2)W_{t+2} \# \frac{1}{2}|(1\#!!)\&_2|^2 + (\Lambda_1 + (1\#!)\&_1)W_{t+1} \# \frac{1}{2}|(1\#!)\&_1|^2\right]}$$

$$\log E_t\left(R^P_{t+1,h}\right) = \#\log" + \mu_c + \%_c x_t + \frac{1}{2}\&_c^2\Lambda^2 + \mathrm{cov}(\Lambda_1,\&_c) + (!!\#!)\mathrm{cov}(\Lambda_2,\&_2) \# (1\#!)\mathrm{cov}(\Lambda_1,\&_1)$$

$$\log E_t\left(R^P_{t+1,h}\right) = \#\log" + \mu_c + \%_c x_t + \left(+\#\frac{1}{2}\right)\&_c^2\Lambda^2 \# (1\#!)\Lambda^2\left[\%_v(\%_d\#\%_c)\frac{1\#(_x^{h\#1}}{1\#(_x}\&_x^2 + (+\#1)\&_c^2\right] + \underbrace{(!!\#!)\Lambda^2\left[\%_v(\%_d\#\%_c)\frac{1\#(_x^{h\#2}}{1\#(_x}\&_x^2 + (+\#1)\&_c^2\right]}_{<0}$$

Even when volatility is constant, HDRA impacts the term-structure of expected returns when investors choose buy-and-hold strategies. The negative impact of HDRA increases with the horizon.

To obtain the returns on bonds, and the expected excess returns, replace $\%_d$, $\&_d$ and + by 0 in the formula above:

$$\log E_t \, \&\, R^{B}_{t+1,h} \, ' = \# \log " + \mu_c + \%_c x_t + \# \frac{1}{2} \&_c^{2} '^{2} + (1 \# !) '^{2} \; M \, \%_v \%_c \frac{1 \# (_x^{h \# 1}}{1 \# (_x} \&_x^2 + \&_c^2 \, N$$

$$\# (!! \# !) '^{2} \; M \, \%_v \%_c \frac{1 \# (_x^{h \# 2}}{1 \# (_x} \&_x^2 + \&_c^2 \, N$$

and

$$\log E_t \, \&\, R^{P,F}_{t+1,h} \, ' = ! + \&_c^{2} '^{2} \# (1 \# !) '^{2} \; M \, \%_v \%_d \frac{1 \# (_x^{h \# 1}}{1 \# (_x} \&_x^2 \, N$$

$$\underbrace{+ (!! \# !) '^{2} \; M \, \%_v \%_d \frac{1 \# (_x^{h \# 2}}{1 \# (_x} \&_x^2 + + \&_c^2 \, N}_{<0}$$

When volatility is time varying, we can rewrite,

$$\frac{E_t \, \& \, \frac{C_{t+1}}{C_t} \, ' \exp \; ! \, 5 \sum_{j=3}^{h} 5 \left(' _j + (1 \# !!) \&_j \right) W_{t+j} \# \frac{1}{2} \left| (1 \# !!) \&_j \right|^2 6 + (' _2 + (1 \# !) \&_2) W_{t+2} \# \frac{1}{2} |(1 \# !) \&_2|^2 + ' _1 W_{t+1} \, 6}{" E_t \, \exp \; ! \, 5 \sum_{j=2}^{h} 5 \left(' _j + (1 \# !!) \&_j \right) W_{t+j} \# \frac{1}{2} \left| (1 \# !!) \&_j \right|^2 6 + (' _1 + (1 \# !) \&_1) W_{t+1} \# \frac{1}{2} |(1 \# !) \&_1|^2 \, 6} =$$

$$\frac{\exp (\# \log " + \mu_c + \%_c x_t) \, E_t \exp \; ! \, 5 \sum_{j=3}^{h} (!_j '^{2}_{t+j \# 1} + (' _2 + (1 \# !) \&_2) W_{t+2} \# \frac{1}{2} |(1 \# !) \&_2|^2 + (' _1 W_{t+1} + \&_c ' _t W_{c,t+1}) \, 6}{E_t \exp \; ! \, 5 \sum_{j=3}^{h} (!_j '^{2}_{t+j \# 1} + (' _2 + (1 \# !!) \&_2) W_{t+2} \# \frac{1}{2} |(1 \# !!) \&_2|^2 + (' _1 + (1 \# !) \&_1) W_{t+1} \# \frac{1}{2} |(1 \# !) \&_1|^2 \, 6}$$

where

$$(!_j = \frac{1}{2} \, / \, \cdot^{2} \; (\%_d \# \%_c) \frac{1 \# (_x^{h \# j}}{1 \# (_x} \&_x \; 3^{2} + \&_d^2 + (+ \# 1)^2 \&_c^2 \, 0 \, 1 + (1 \# !!) \; M \, \%_v (\%_d \# \%_c) \frac{1 \# (_x^{h \# j}}{1 \# (_x} \&_x^2 + (+ \# 1) \&_c^2 \, N$$

$$(!_\% = \frac{1}{2} \, 2 \, " \; (\%_d \# \%_c) \frac{\&_x}{1 \# (_x} \; \%^{2} + \&_d^2 + (+ \# 1)^2 \&_c^2 \; 3 + (1 \# !!) \; ? \, \%_v (\%_d \# \%_c) \frac{\&_x^2}{1 \# (_x} + (+ \# 1) \&_c^2 \, @$$

replace !! with ! to get ($_j$

$$\frac{E_t \exp\ !\ \sum_{j=3}^{h} (!_j{'}_{t+j\#1}^2 + (' _2 + (1\#\,!)\,\&_2)\,W_{t+2}\#\tfrac{1}{2}|(1\#\,!)\,\&_2|^2}{E_t\ \exp\ !\ \sum_{j=2}^{h} (!_j{'}_{t+j\#1}^2 + (' _2 + (1\#\,!!)\,\&_2)\,W_{t+2}\#\tfrac{1}{2}|(1\#\,!!)\,\&_2|^2 + (1\#\,!)\,*!_v\&\,W_{\cdot,t+1}\#\tfrac{1}{2}(1\#\,!)\,*!_v\&|^2} =$$

$$\frac{E_t \exp\ \&\ ((\cdot W_{\cdot,t+1} + W_{\cdot,t+2})\,!\ \sum_{j=3}^{h}(!_j(^{j\#3} + (' _2 + (1\#\,!)\,\&_2)\,W_{t+2}\#\tfrac{1}{2}|(1\#\,!)\,\&_2|^2}{E_t \exp\ \&\ ((\cdot W_{\cdot,t+1} + W_{\cdot,t+2})\,!\ \sum_{j=3}^{h}(!_j(^{j\#3} + (' _2 + (1\#\,!!)\,\&_2)\,W_{t+2}\#\tfrac{1}{2}|(1\#\,!!)\,\&_2|^2 + (1\#\,!)\,*!_v\&\,W_{\cdot,t+1}\#\tfrac{1}{2}(1\#\,!)\,*!_v\&|^2} =$$

$$\frac{E_t \exp\ \&\ (\cdot W_{\cdot,t+1}\ !\ \sum_{j=3}^{h}(!_j(^{j\#3} + (_2 + (_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2) + \tfrac{1}{2}\,!\ \sum_{j=3}^{h}(!_j(^{j\#3}\ !\ \sum_{j=3}^{h}(!_j(^{j\#3} + 2(1\#\,!)\,*!_v\ \&^2}{E_t \exp\ \tfrac{1}{2}\,!\ \sum_{j=3}^{h}(!_j(^{j\#3}\ !\ \sum_{j=3}^{h}(!_j(^{j\#3} + 2(1\#\,!!)\,*!_v\ \&^2 + (!_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2) + (\cdot\,!\ \sum_{j=3}^{h}(!_j(^{j\#3} + (1\#\,!)\,*!_v + (!_2\ \&\,W_{\cdot,t+1}\#\tfrac{1}{2}(1\#\,!)\,*!_v\&|^2} =$$

$$\frac{\exp\ \&^2\ \tfrac{1}{2}((_2)^2 + \tfrac{1+(^2}{2}\ !\ \sum_{j=3}^{h}(!_j(^{j\#3}{}^2 + (_2(\cdot + (1\#\,!)\,*!_v)\,!\ \sum_{j=3}^{h}(!_j(^{j\#3} + (_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2)}{\exp\ \&^2\ \tfrac{1}{2}\ (!_2{}^2 + \tfrac{1+(^2}{2}\ !\ \sum_{j=3}^{h}(!_j(^{j\#3}{}^2 + (!_2(\cdot + [1\#\,!!) + (\cdot(1\#\,!]\,*!_v\ !\ \sum_{j=3}^{h}(!_j(^{j\#3} + (1\#\,!)\,*!_v(!_2 + (!_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2)} =$$

$$\frac{\exp\ \&^2\ \tfrac{1}{2}((_2)^2 + (_2(\cdot + (1\#\,!)\,*!_v)\,!\ \sum_{j=3}^{h}(!_j(^{j\#3} + (_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2)}{\exp\ \&^2\ \tfrac{1}{2}\ (!_2{}^2 + (!_2(\cdot + [1\#\,!!) + (\cdot(1\#\,!]\,*!_v\ !\ \sum_{j=3}^{h}(!_j(^{j\#3} + (1\#\,!)\,*!_v(!_2 + (!_2\,{}^{,2}(1\#(\cdot) + (\cdot{'}_t^2)} =$$

$$\exp\ \frac{1}{2}\&^2\ \left((_2^2\#(!_2^2 + (_2\#(!_2\right)^2(1\#(\cdot) + (\cdot{'}_t^2 + \&^2\sum_{j=3}^{h}(!_j(^{j\#2} + (!!\#\,!)\,*!_v\&^2\sum_{j=3}^{h}(!_j(^{j\#3}\#(1\#\,!)\,*!_v\&^2\sum_{j=2}^{h}(!_j(^{j\#2}$$

$$\log E_t\ R^P_{t+1,h} = \#\log" + \mu_c + \%_c x_t + \ \ + \# \frac{1}{2}\ \&_c^2{'}_t^2\#(1\#\,!)\,{'}_t^2\ \ \%_v(\%_d\#\%_e)\frac{1\#(_x^{h\#1}}{1\#(_x}\&_x^2 + (+\#1)\,\&_c^2$$
$$+ (!!\#\,!)\ \ \%_v(\%_d\#\%_e)\frac{1\#(_x^{h\#2}}{1\#(_x}\&_x^2 + (+\#1)\,\&_c^2\ \ {}^{,2}(1\#(\cdot) + (\cdot{'}_t^2$$
$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{<0}$$
$$+ \&^2\ \frac{1}{2}\left((_2^2\#(!_2^2 + (_2\#(!_2\right)\sum_{j=3}^{h}(!_j(^{j\#2} + (!!\#\,!)\,*!_v\sum_{j=3}^{h}(!_j(^{j\#3}\#(1\#\,!)\,*!_v\sum_{j=2}^{h}(!_j(^{j\#2}$$

$$\log E_t\ R^P_{t+1,h} = \#\log" + \mu_c + \%_c x_t + \ \ + \# \frac{1}{2}\ \&_c^2{'}_t^2\#(1\#\,!)\,{'}_t^2\ \ \%_v(\%_d\#\%_e)\frac{1\#(_x^{h\#1}}{1\#(_x}\&_x^2 + (+\#1)\,\&_c^2$$
$$+ (!!\#\,!)\ \ \%_v(\%_d\#\%_e)\frac{1\#(_x^{h\#2}}{1\#(_x}\&_x^2 + (+\#1)\,\&_c^2\ \ {}^{,2}(1\#(\cdot) + (\cdot{'}_t^2$$
$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{<0}$$
$$+ \&^2\underbrace{(!\ \#\ 1)\,*!_v}_{<0}\ \underbrace{\sum_{j=2}^{h}(!_j(^{j\#2}}_{}$$

< 0 under ! but >0 for sufficiently low !!

$$+ \&^2\left\{\frac{1}{2}\left((_2^2\#(!_2^2 + (_2\#(!_2\right)\underbrace{\sum_{j=3}^{h}(!_j(^{j\#2} + (!!\#\,!)\,*!_v\sum_{j=3}^{h}(!_j(^{j\#3}}_{<0\text{ for sufficiently low }!!}\right\}$$

Note: we write $)_k = \frac{(!_{h+1\#k}}{(^k} =* \left((^{h\#2} !^{h\#2}_{k=1}\right)_k = !^{h}_{j=3} (!_j (^{j\#3}$ in the matlab document

To obtain the returns on bonds, and their expected excess returns, replace $\%_d$, $\&_d$ and + by 0 in the formula above:

$$\log E_t \left(R^B_{t+1,h} \right) = \# \log " + \mu_c + \%_c x_t \# \frac{1}{2}\&^2_c '^2_t + (1\#!)'^2_t \left[\%_v \%_c \frac{1\#(^{h\#1}_x}{1\#(_x} \&^2_x + \&^2_c \right]$$

$$\# \underbrace{(!!\#!)\left[\%_v\%_c \frac{1\#(^{h\#2}_x}{1\#(_x}\&^2_x + \&^2_c\right] \left('^2(1\#(\cdot)+(\cdot'^2_t \right)}_{<0}$$

$$+ \&^2_\cdot \underbrace{(!\#1)}_{<0} *!_v \underbrace{\sum^{h}_{j=2} (!_{B,j}(^{j\#2}}_{<0 \text{ under } ! \text{ but } >0 \text{ for sufficiently low } !!}$$

$$+ \&^2_\cdot \left\{ \frac{1}{2}\left((^2_{B,2} \# (!^2_{B,2} \right) + \underbrace{\left((_{B,2} \# (!_{B,2} \right) \sum^{h}_{j=3} (!_{B,,j}(^{j\#2} + (!!\#!)*!_v \sum^{h}_{j=3} (!_{B,j}(^{j\#3}}_{<0 \text{ for sufficiently low } !!} \right\}$$

where

$$(!_{B,j} = \frac{1}{2} / \left[\left(\%_c \frac{1\#(^{h\#j}_x}{1\#(_x}\&_x \right)^2 + \&^2_c 1 \# (1\#!!) \left[\%_v\%_c \frac{1\#(^{h\#j}_x}{1\#(_x}\&^2_x + \&^2_c \right] \right]$$

and

$$(!_j \# (!_{B,j} = \frac{1}{2} / \left[\%_d(\%_d \# 2\%_c) \left(\frac{1\#(^{h\#j}_x}{1\#(_x}\&_x \right)^2 + \&^2_d + +(+\#2)\&^2_c 1 + (1\#!!) \left[\%_v\%_c \frac{1\#(^{h\#2}_x}{1\#(_x}\&^2_x + \&^2_c \right] \right]$$

$$\log E_t \left(R^{P,F}_{t+1,h} \right) = !+\&^2_c '^2_t \# (1\#!)'^2_t \left[\%_v\%_d \frac{1\#(^{h\#1}_x}{1\#(_x}\&^2_x \right]$$

$$+ \underbrace{(!!\#!)\left[\%_v\%_d \frac{1\#(^{h\#1}_x}{1\#(_x}\&^2_x + +\&^2_c\right] \left('^2(1\#(\cdot)+(\cdot'^2_t \right)}_{<0}$$

$$+ \&^2_\cdot \underbrace{(!\#1)}_{<0} *!_v \underbrace{\sum^{h}_{j=2} \left((!_j \# (!_{B,j} \right) (^{j\#2}}_{<0 \text{ under } ! \text{ but } >0 \text{ for sufficiently low } !!}$$

$$+ \&^2_\cdot \left\{ \begin{array}{l} \frac{1}{2}\left((^2_{\&^2} \# (!^2_2 \right) + \left((_2 \# (!_2 \right) \sum^{h}_{j=3} (!_j(^{j\#2} \\ \# \frac{1}{2}\left((^2_{B,2} \# (!^2_{B,2} \right) + \left((_{B,2} \# (!_{B,2} \right) \sum^{h}_{j=3} (!_{B,j}(^{j\#2} \end{array} + (!!\#!)*!_v \sum^{h}_{j=3} \left((!_j \# (!_{B,j} \right) (^{j\#3} \right\}$$

C Approximation for $\beta \approx 1$

As in Appendix B, consider the simplified model with only two levels of risk aversion:

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(R_{t,\gamma}\left(\hat{V}_{t+1}\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

$$\hat{V}_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(R_{t,\tilde{\gamma}}\left(\hat{V}_{t+1}\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where

$$R_{t,\alpha}(X) = \left(E_t\left(X^{1-\alpha}\right)\right)^{\frac{1}{1-\alpha}}.$$

Also, as in Appendix B, take the evolutions:

$$c_{t+1} - c_t = \mu + \phi_c x_t + \lambda_c \sigma_t W_{t+1},$$

$$x_{t+1} = \nu_x x_t + \lambda_x \sigma_t W_{t+1},$$

$$\sigma_{t+1}^2 - \sigma^2 = \nu_\sigma\left(\sigma_t^2 - \sigma^2\right) + \lambda_\sigma W_{t+1},$$

and suppose the three shocks are independent. (We can relax this assumption.)

For β close to 1, we have:

$$\left[\frac{\hat{V}_t}{C_t}\right]^{1-\tilde{\gamma}} \approx \beta^{\frac{1-\tilde{\gamma}}{1-\rho}} E_t\left\{\left[\frac{\hat{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right]^{1-\tilde{\gamma}}\right\}.$$

This is an eigenfunction problem with eigenvalue $\beta^{-\frac{1-\tilde{\gamma}}{1-\rho}}$ and eigenfunction $\left(\hat{V}/C\right)^{1-\tilde{\gamma}}$ known up to a multiplier. Let's assume:

$$\hat{v}_t - c_t = \hat{\mu}_v + \phi_v x_t + \kappa_v \sigma_t^2.$$

Then we have:

- Terms in x_t (standard formula with $\beta = 1$):

$$\phi_v = \phi_c\left(I - \nu_x\right)^{-1}$$

- Terms in σ_t^2:

$$\kappa_v = \frac{1}{2}\frac{1-\tilde{\gamma}}{1-\nu_\sigma}\left(\lambda_c^2 + \phi_v^2\lambda_x^2\right) < 0$$

¥ Constant terms:

$$\log " = \# (1\# \$) \left[\mu + *\div_v ' ^2 (1\# (\cdot) + \frac{1}{2}(1\# !\div) *\div_v^2 \&^2 \right]$$

we verify the solution for " is such that " < 1 and " & 1. We find that, as long as !÷ (5, " < 1 + $ < 1; and " & 1 is easily satisfied even for very low levels of $. e.g. in the calibration of Section (6), 1 > " " 0.9988 for $ = 0.2 and !÷ (5.

For " close to 1, we have:

$$\frac{V_t}{V\!\!/_t} \& \frac{R_{t,!}}{R_{t,!!}} \frac{\& V\!\!/_{t+1} '}{\& V\!\!/_{t+1} '} = \frac{E_t \left[?\& \frac{V\!\!/_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} '^{1\#!} @ \right]^{\frac{1}{1\#!}}}{E_t \left[?\& \frac{V\!\!/_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} '^{1\#!!} @ \right]^{\frac{1}{1\#!!}}},$$

and therefore:

$$v_t \# v\!\!/_t = \# \frac{1}{2}(! \# !!) \left[\&_c^2 + \%_v^2 \&_x^2 ' ^2_t + *!^2_v \&^2 \right],$$

The stochastic discount factor becomes:

$$) _{t,t+1} =)\!\!/_t \# !\&_c ' _t W_{t+1} + (\$\# !)\%_v \&_x ' _t W_{t+1}$$
$$+ ?(\$\# !) + (1\# \$)(! \# !!) \frac{1\# (\cdot}{1\# !!} @ *\div_v \& W_{t+1},$$

where

$$)\!\!/_t = \log " \# \$\mu_c \# \$\%_e x_t \# (\$\# !) \frac{1}{2}(1\# !) \left[\&_c^2 + \%_v^2 \&_x^2 ' ^2_t + *!^2_v \&^2 \right]$$
$$+ (1\# \$) \frac{1}{2}(! \# !!) \left[\&_c^2 + \%_v^2 \&_x^2 (\cdot ' ^2_t + ' ^2 (1\# (\cdot) + *!^2_v \&^2 \right]$$

$$)\!\!/_t = \# \mu_c \# \$\%_e x_t \# (1\# \$) \frac{1}{2} \& \&_c^2 + \%_v^2 \&_x^2 ' \left[\frac{1\# !!}{1\# (\cdot} \# (! \# !!) ' ^2 (1\# (\cdot) \right]$$
$$\# \frac{1}{2}(1\# !)^2 *\div_v^2 \&^2$$
$$\# \frac{1}{2}\left(\$\# !)(1\# !) \# (1\# \$)(! \# !!)(\cdot) \right] \&_c^2 + \%_v^2 \&_x^2 ' ^2_t$$

The risk-free rate is defined as $r_{f,t} = \# \log E_t(" _{t,t+1})$:

$$
\begin{aligned}
r_{f,t} = {} & \mu_c + \$\%_e x_t + (1\#\$)\frac{1}{2}\Big\&\, \&_c^2 + \%_v^2\&_x^2 \; '\,"\, \frac{1\#!!}{1\#(\cdot} \#(!\#!!)\Big\%\, '^{\,2}(1\#(\cdot) \\
& + \frac{1}{2}\, M\, (1\#!)^2 \# \;?\; (\$\#!) + (1\#\$)(!\#!!)\frac{1\#(\cdot}{1\#!!} \, @_2 N\; *\div^2_v \&^2 \\
& + \frac{1}{2}\big(\$\#!\big)\big(1\#!\big)\#(1\#\$)\big(!\#!!\big)(\cdot)\; \& \; \&_c^2 + \%_v^2\&_x^2 \;'\; 6\; '^{\,2}_t \\
& \# \frac{1}{2}\,\&\; !^2\&_c^2\,'^{\,2}_t + (\$\#!)^2\%_v^2\&_x^2\,'^{\,2}_t \;'
\end{aligned}
$$

Note the risk-free rate now depends on $!\div$. !

C.1 Term structure of returns

C.1.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t \# " _{t,t+1} G_{y,t+1} Y_{t+1,h\#1} \$,$$

that is

$$y_{t,h} = E_t \#) _{t,t+1} + g_{y,t+1} + y_{t+1,h\#1} \$ + \frac{1}{2}\mathrm{var}_t \big( ) _{t,t+1} + g_{y,t+1} + y_{t+1,h\#1} \big),$$

where

$$
\begin{aligned}
g_{y,t+1} = {} & \mu_y + \%_y x_t + *_y\,'^{\,2}_t \\
& + \&_{y,c}\&_c\,'_t W_{c,t+1} + \&_{y,x}\&_x\,'_t W_{x,t+1} + \&_{y,'}\&\,'_t W_{',t+1} + \&_{y,d}\&_d\,'_t W_{d,t+1},
\end{aligned}
$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp \& \mu_{y,h} + \%_{y,h} x_t + *\div_{y,h}\,'^{\,2}_t \,' .$$

Suppose h " 1, then:

$$
\begin{aligned}
) _{t,t+1} = \; &)\!\!\!\!l_t \# !\& _c ' _t W_{t+1} + (\$\# !)\,\%_v \&_x ' _t W_{t+1} \\
& ? \qquad\qquad\qquad\qquad @ \\
& + \;\; (\$\# !) + (1\# \$)(!\# !!)\,\frac{1\# (\cdot}{1\# !!} \;\; *\div_v \& W_{t+1},
\end{aligned}
$$

$$
\log "! _{t,t+1} G_{t,t+1} Y_{t+1,h\# 1} =
\begin{array}{l}
A \\
)\!\!\!\!l_t \\
+ \mu_y + \%_y x_t + *_y ' ^2_t \\
+ \mu_{y,h\# 1} + \%_{y,h\# 1} (_x x_t + *_{y,h\# 1} \left(' ^2 (1\# (\cdot) + (\cdot ' ^2_t \right) \\
+ \left(\# ! + \&_{y,c} \right) \&_c ' _t W_{t+1} + \left((\$\# !)\,\%_v + \&_{y,x} + \%_{y,n\# 1} \, (_x \right) ' _t W_{t+1} \\
\& \qquad\qquad 6 \\
+ \;\; (\$\# !) + (1\# \$)(!\# !!)\,\frac{1\# (\cdot}{1\# !!} \;\; *!_v + \&_{y,\cdot} + *_{y,h\# 1} \;\; \& W_{t+1} \\
+ \&_{y,d} \&_d ' _t W_{t+1}
\end{array}
$$

where

$$
\begin{aligned}
)\!\!\!\!l_t = \; & \# \mu_c \# \$\%_c x_t \# (1\# \$)\,\frac{1}{2} \;\& \;\&^2_c + \%^2_v \&^2_x \;\; ' \; " \; \frac{1\# !!}{1\# (\cdot} \# (!\# !!) \;\; \% \; ' ^2 (1\# (\cdot) \\
& \# \frac{1}{2} (1\# !)^2 *\div^2_v \&^2 \\
& \# \frac{1}{2} \left(\$\# ! \right)\left(1\# ! \right) \# (1\# \$)(!\# !!)\,(\cdot) \;\; \& \;\; \&^2_c + \%^2_v \&^2_x \;\; ' \; 6 \; ' ^2_t
\end{aligned}
$$

Matching coe□cients, we find the recursions, for h " 1:

- Terms in x_t:

$$
\begin{aligned}
& \%_{y,h} = \# \$\%_c + \%_y + \%_{y,h\# 1} (_x \\
* \quad & \%_{y,h} = \left(\# \$\%_c + \%_y \right) \frac{1\# (^h_x}{1\# (_x}
\end{aligned}
$$

- Terms in $' ^2_t$:

$$
\begin{aligned}
*\div_{y,h} = \; & \# \frac{1}{2} \left(\$\# ! \right)\left(1\# ! \right) \# (1\# \$)(!\# !!)\,(\cdot) \;\; \& \;\; \&^2_c + \%^2_v \&^2_x \;\; ' \; + *\div_{y,h\# 1} (\cdot + *_y \\
& + \frac{1}{2} \;\& \left(\# ! + \&_{y,c} \right)^2 \&^2_c + \left((\$\# !)\,\%_v + \&_{y,x} + \%_{y,h\# 1} \right)^2 \&^2_x + \&^2_{y,d} \&^2_d \;\; '
\end{aligned}
$$

¥ Constant:

$$\mu_{y,h} \# \mu_{y,h\#1} = \# \mu_c \# (1\#\$) \frac{1}{2} \& \&_c^2 + \%_v^2\&_x^2 \; ' \; " \; \frac{1\#!!}{1\#('} \# (!\#!!) \; \% \; '^2(1\#(')$$

$$\text{M} \; ? \; + \frac{1}{2} \; (\$\#!) + (1\#\$)(!\#!!) \frac{1\#('}{1\#!!} \; @ \; *!_v + \&_{y,'} + *\div_{y,h\#1} \; \%_2 \; \# (1\#!)^2 *!_v^2 \; \&^2 \; \text{N}$$

$$+ \mu_y + '^2(1\#(') *\div_{y,h\#1}$$

Note only both the constant terms $\text{E}\,\mu_{y,h}\,\text{F}$ and the loadings on the volatility shocks $\text{E}\,*!_{y,h}\,\text{F}$ are aected by the wedge between $!$ and $!!$.

!

In line with the specication of van Binsbergen and Koijen (2016), we consider one-period holding returns for these claims of the form

$$1 + R^Y_{t+1,h} = \frac{G_{y,t+1}Y_{t+1,h\#1}}{Y_{t,h}} = \frac{G_{y,t+1}Y_{t+1,h\#1}}{E_t \, \# \, "_{t,t+1}G_{y,t+1}Y_{t+1,h\#1} \, \$}$$

$$= R_{f,t} \frac{E_t["_{t,t+1}]\,G_{y,t+1}Y_{t+1,h\#1}}{E_t \, \# \, "_{t,t+1}G_{y,t+1}Y_{t+1,h\#1} \, \$},$$

with the risk-free rate

$$R_{f,t} = \frac{1}{E_t["_{t,t+1}]}.$$

In line with the specication of van Binsbergen and Koijen (2016), we also consider one-period holding returns for futures on these claims of the form

$$R^{F,Y}_{t+1,h} + 1 = \frac{1 + R^Y_{t+1,h}}{1 + R^B_{t+1,h}} = \frac{G_{y,t+1}Y_{t+1,h\#1}}{Y_{t,h}} \frac{B_{t,h}}{B_{t+1,h\#1}}$$

$$= \frac{G_{y,t+1}Y_{t+1,h\#1}}{E_t \left("_{t,t+1}G_{y,t+1}Y_{t+1,h\#1} \right)} \frac{E_t \left("_{t,t+1}B_{t+1,h\#1} \right)}{B_{t+1,h\#1}},$$

where $B_{t,h}$ is the price of \$1 at horizon h, i.e. the price of a Bond with horizon h.

Their conditional Sharpe Ratio is

$$SR^{F,Y}_{t,h} = G\frac{E_t \& 1 + R^{F,Y}_{t+1,h} ' \# 1}{var_t \& 1 + R^{F,Y}_{t+1,h} '}$$

$$= H\frac{E_t \& 1 + R^{F,Y}_{t+1,h} ' \# 1}{" E_t \& 1 + R^{F,Y}_{t+1,h} '^2 \# \& E_t \& 1 + R^{F,Y}_{t+1,h} ' '^2 \%}$$

$$\& G\frac{\begin{array}{l} A \\ C \; '^2_t \left(!\&_{y,c}\&^2_c \# \left(\&_{y,x} + \%_{y,h\#1} \# \%_{b,h\#1} \right) (\$ \# !) \%_v + \%_{b,h\#1}) \&^2_x \right) 6 \\ D \; \# \left(\&_{y,'} + *÷_{y,h\#1} \# *÷_{b,h\#1} \right) \& \; (\$ \# !) + (1 \# \$)(! \# !!) \frac{1\#(\cdot}{1\#!!} *!_v + *÷_{b,h\#1} \; \&^2 \end{array} '}{\& \; '^2_t \; \&^2_{y,c}\&^2_c + \left(\&_{y,x} + \%_{y,h\#1} \# \%_{b,h\#1} \right)^2 \&^2_x + \&^2_{y,d}\&^2_d \; ' + \left(\&_{y,'} + *÷_{y,h\#1} \# *÷_{b,h\#1} \right)^2 \&^2}.$$

C.1.2 Bonds

Let the price at time t for $1 in h periods be $B_{t,h}$ with $B_{t,0} = 1$. For $h "\ 1$, we have

$$B_{t,h} = E_t[" _{t,t+1} B_{t+1,h\#1}]$$

This is the general problem from above with $g_{y,t+1} = 0$ for all t and therefore

$$b_{t,h} = \mu_{b,h} + \%_{b,h} x_t + *÷_{b,h} '^2_t,$$

with

$$\%_{b,h} = \# \$ \%_e \frac{1 \# (^h_x}{1 \# (_x}$$

$$*÷_{b,h} = \# \frac{1}{2} (\$ \# !)(1 \# !) \# (1 \# \$)(! \# !!) (\cdot) \& \&^2_c + \%^2_v \&^2_x ' + *÷_{b,h\#1} (\cdot$$
$$+ \frac{1}{2} \& !^2 \&^2_c + (\$ \# !) \%_v + \%_{b,h\#1})^2 \&^2_x '$$

C.1.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t \left[\text{"}_{t,t+1} \frac{D_{t+1}}{D_t} \frac{P_{t+1,h-1}}{D_{t+1}} \right],$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} - d_t = \mu_d + \%_d x_t + +\&_c{'}_t W_{t+1} + \&_d{'}_t W_{t+1},$$

for all t and therefore

$$p_{t,h} - d_t = \mu_{p,h} + \%_{d,h} x_t + *\div_{d,h}{'}^2_t,$$

with

$$\%_{d,h} = (-\$\%_c + \%_d) \frac{1 - (^h_x}{1 - (_x}$$

$$\begin{aligned} *\div_{d,h} = & -\frac{1}{2}\Big(\$ - !\Big)\Big(1 - !\Big) - (1 - \$)\Big(! - !!\Big)(\text{'}) \left(\&_c^2 + \%_v^2 \&_x^2\right) + *\div_{d,h-1}(\text{'} \\ & + \frac{1}{2} \left((-! + +)^2 \&_c^2 + \Big(\$ - !\Big)\%_v + \%_{d,h-1}\right)^2 \&_x^2 + \&_d^2 \end{aligned}$$

$$\begin{aligned} \mu_{y,h} - \mu_{y,h-1} = & -\mu_c - (1-\$) \frac{1}{2} \left(\&_c^2 + \%_v^2 \&_x^2\right) \left[\frac{1 - !!}{1 - (\text{'}} - (! - !!) \right] {'}^2 (1 - (\text{'})) \\ & + \frac{1}{2} \left((\$ - !) + (1 - \$)\Big(! - !!\Big) \frac{1 - (\text{'}}{1 - !!} *!_v + *\div_{d,h-1} - (1 - !)^2 *!^2_v \right)^2 \&_{\text{'}}^2 \\ & + \mu_d + {'}^2 (1 - (\text{'})) *\div_{d,h-1} \end{aligned}$$

For the dividend strips, the spot one-period returns are given by

$$R^P_{t+1,h} + 1 = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t} \frac{D_{t+1}}{D_t},$$

$$\log \left(R^{P}_{t+1,h} + 1 \right) = \mu_c + (1 \# \$) \frac{1}{2} \left(\&_c^2 + \%_v^2 \&_x^2 \left(\frac{1 \# !!}{1 \# ('} \# (! \# !!) \right)^2 (1 \# (') \right)$$

$$\# \frac{1}{2} \left((\$ \# !) + (1 \# \$)(! \# !!) \frac{1 \# ('}{1 \# !!} *!_v + *\div_{d,h\#1} \# (1 \# !)^2 *!_v^2 \right) \&^2$$

$$+ \$\%_e x_t + (*\div_{d,h\#1}(' \# *\div_{d,h})'^2_t$$

$$+ *\div_{d,h\#1} \& W_{t+1} + \%_{d,h\#1} \&_x '_t W_{t+1} + +\&_c '_t W_{t+1} + \&_d '_t W_{t+1}$$

the conditional expected one-period returns are

$$E_t \left(R^{P}_{t+1,h} + 1 \right) \& \text{constant (in h)} \# \left((\$ \# !) + (1 \# \$)(! \# !!) \frac{1 \# ('}{1 \# !!} *!_v *\div_{d,h\#1} \&^2 \right)$$

$$+ (*\div_{d,h\#1}(' \# *\div_{d,h})'^2_t + \frac{1}{2} \left(\%^2_{d,h\#1} \&_x^2 '^2_t \right)$$

$$E_t \left(R^{P}_{t+1,h} + 1 \right) \& \text{constant (in h)} \# \left((\$ \# !) + (1 \# \$)(! \# !!) \frac{1 \# ('}{1 \# !!} *!_v *\div_{d,h\#1} \&^2 \right)$$

$$\# \left(\$ \# ! \right) \%_v \%_{d,h\#1}) \&_x^2 '^2_t$$

$$E_t \left(R^{P}_{t+1,h} \right) \# E_t \left(R^{P}_{t+1,h\#1} \right) \& \underbrace{(! \# \$) + (1 \# \$)(! \# !!) \frac{1 \# ('}{1 \# !!} *!_v (*\div_{d,h} \# *\div_{d,h\#1}) \&^2}_{(\, 0}$$

$$+ \underbrace{(! \# \$) \%_v (\$\%_e \# \%_d) \frac{(_x^h \# (_x^{h\#1}}{1 \# (_x} \&_x^2 '^2_t}_{"\, 0}$$

We need $(*\div_{d,h} \# *\div_{d,h\#1})$ " 0 to generate a downward sloping term-structure. If $(*\div_{d,h} \# *\div_{d,h\#1})$ (0, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when $'_t$ is high...

The future one-period returns are given by:

$$R^{F,P}_{t+1,h} + 1 = \frac{1 + R^{P}_{t+1,h}}{1 + R^{B}_{t+1,h}}$$

$$\log \left(R^{P}_{t+1,h} + 1 \right) = \mu_c + (1\#\$)\,\frac{1}{2}\left(\&^2_c + \%^2_v \&^2_x \left(\frac{1\#!!}{1\#(\cdot} \#(!\#!!) \right)^2 (1\#(\cdot) \right.$$
$$\# \frac{1}{2} \left((\$\#!) + (1\#\$)(!\#!!)\,\frac{1\#(\cdot}{1\#!!}\, *!_v + *\div_{d,h\#1} \right)^2 \#(1\#!)^2 *!^2_v\ \&^2$$
$$+ \$\%_e x_t + (*\div_{d,h\#1}(\cdot \# *\div_{d,h})\ '^2_t$$
$$+ *\div_{d,h\#1} \&\, W_{t+1} + \%_{d,h\#1} \&_x\, '_t W_{t+1} + +\&_c\, '_t W_{t+1} + \&_d\, '_t W_{t+1}$$

$$\log \left(R^{F,P}_{t+1,h} + 1 \right) = \# \left((\$\#!) + (1\#\$)(!\#!!)\,\frac{1\#(\cdot}{1\#!!}\, *!_v (*\div_{d,h\#1} \# *\div_{b,h\#1}) + \frac{1}{2}\left(*\div^2_{d,h\#1} \# *\div^2_{b,h\#1} \right) \right.$$
$$+ \left(*\div_{d,h\#1} \# *\div_{b,h\#1} \right)(\cdot \# (*\div_{d,h} \# *\div_{b,h})\ '^2_t$$
$$+ (*\div_{d,h\#1} \# *\div_{b,h\#1})\,\&\, W_{t+1} + (\%_{d,h\#1} \# \%_{b,h\#1})\,\&_x\, '_t W_{t+1} + +\&_c\, '_t W_{t+1} + \&_d\, '_t W_{t+1}$$

$$E_t \left(R^{F,P}_{t+1,h} + 1 \right) = \# \left(\underbrace{ (\$\#!) + (1\#\$)(!\#!!)\,\frac{1\#(\cdot}{1\#!!}\, *!_v + *\div_{b,h\#1} }_{\text{" 0 and increasing}} (*\div_{d,h\#1} \# *\div_{b,h\#1}) \right) \&^2$$
$$+ \left(!+\&^2_c + \underbrace{(!\#\$)\%_v \# \%_{b,h\#1}}_{\text{" 0 and increasing}} (\%_{d,h\#1} \# \%_{b,h\#1})\,\&^2_x \right) '^2_t$$

Note:

$$*\div_{d,h} \# *\div_{b,h} = (*\div_{d,h\#1} \# *\div_{b,h\#1})(\cdot$$
$$+ \left(+ \underbrace{\left(\frac{1}{2} + \#! \right)}_{(\ 0} \&^2_c + \underbrace{\left((\$\#!)\%_v + \frac{1}{2}(\%_{d,h\#1} + \%_{b,h\#1}) \right)}_{(\ 0 \text{ for } !\ \text{high enough}} \underbrace{(\%_{d,h\#1} \# \%_{b,h\#1})}_{\text{" 0}} \&^2_x + \underbrace{\frac{1}{2}}_{\text{" 0}} \&^2_d \right)$$

the sign depends on the parameters. But if it is positive increasing, !! reduces the downward impact of it on the term-structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher $'_t$ means a MORE upward sloping term-structure again. !

D Additional Þgures

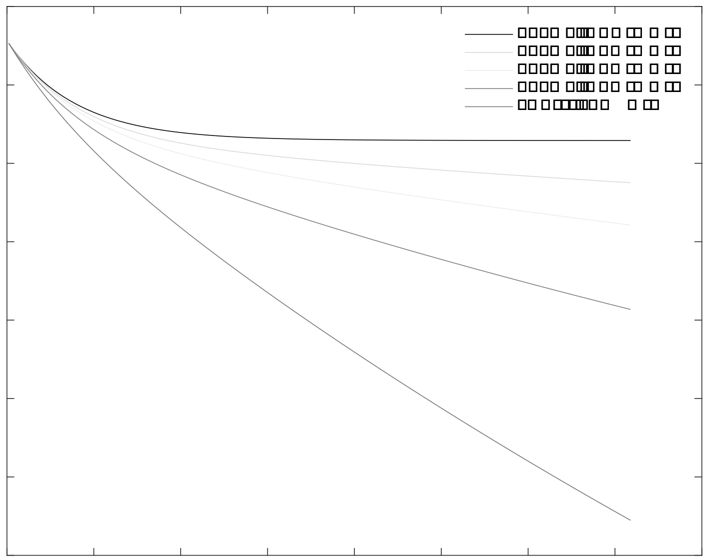

Figure 6: Term structure of bond returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a. Returns are conditional, with state variables set at their means: $x_t = 0$ and $'_t = '$.

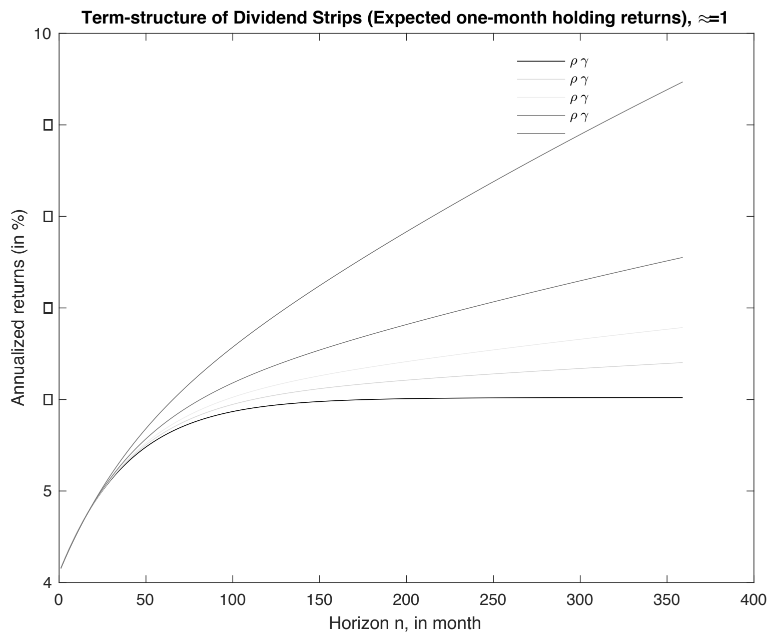

Figure 7: Term structure of dividend strip expected returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a. Returns are conditional, with state variables set at their means: $x_t = 0$ and $'_t = '$.

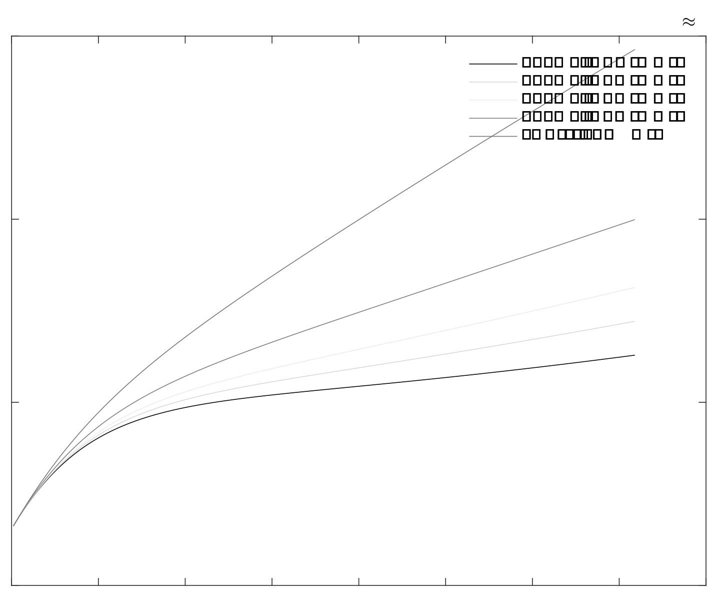

Figure 8: Term structure of dividend strip expected excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a. Returns are conditional, with state variables set at their means: $x_t = 0$ and $'_t = '$.

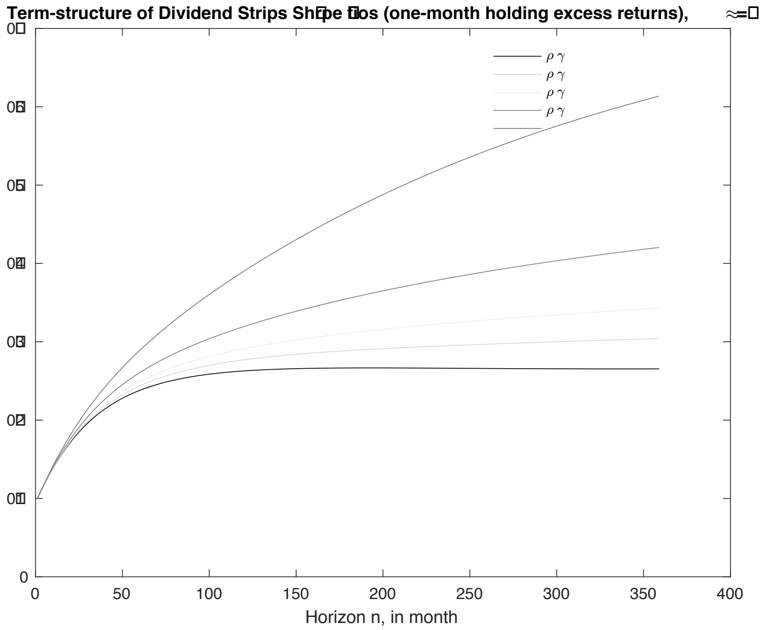

Figure 9: Term structure of dividend strip unconditional Sharpe ratios of excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a.

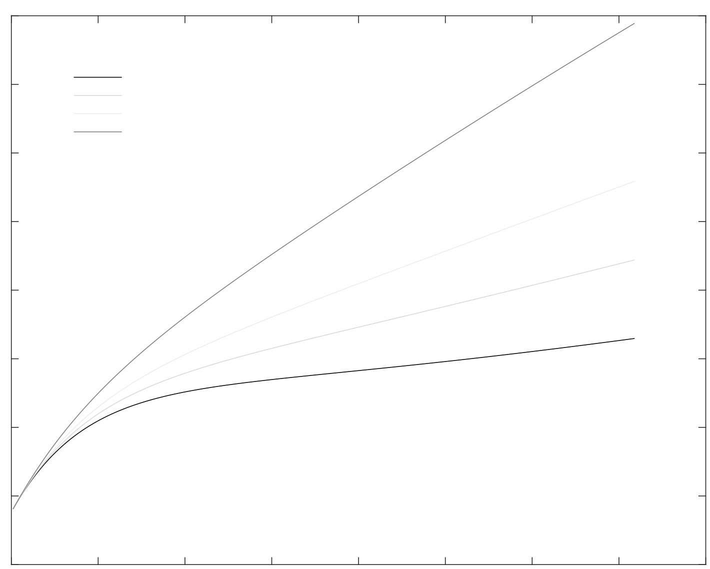

Figure 10: Term structure of dividend strip expected excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a and $ = 0.8.

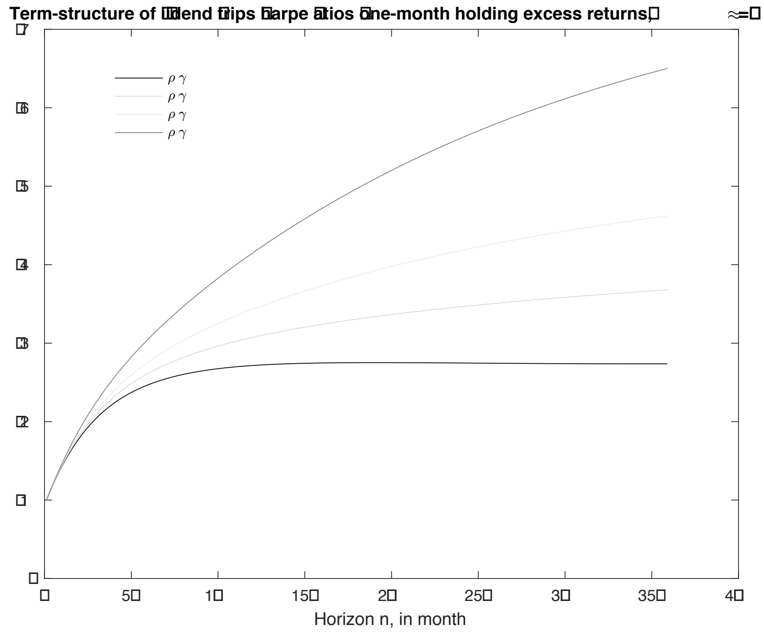

Figure 11: Term structure of dividend strip unconditional Sharpe ratios of excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a and $ = 0.8.

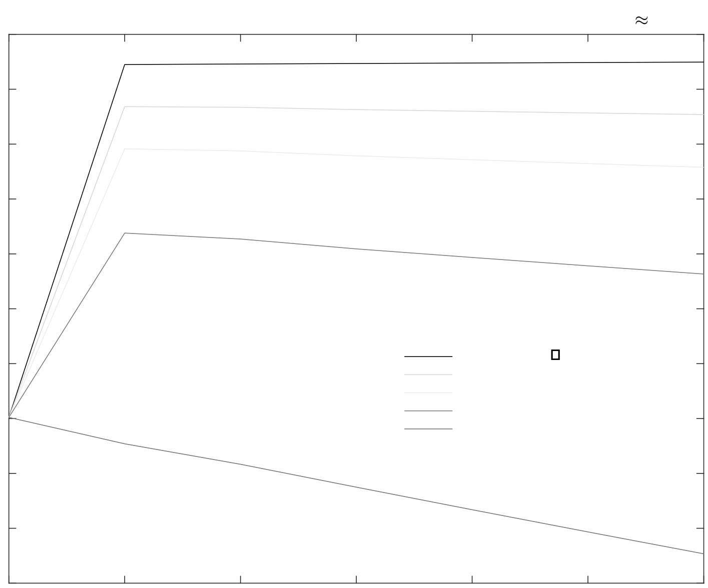

Figure 12: Term structure of of bond returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a.

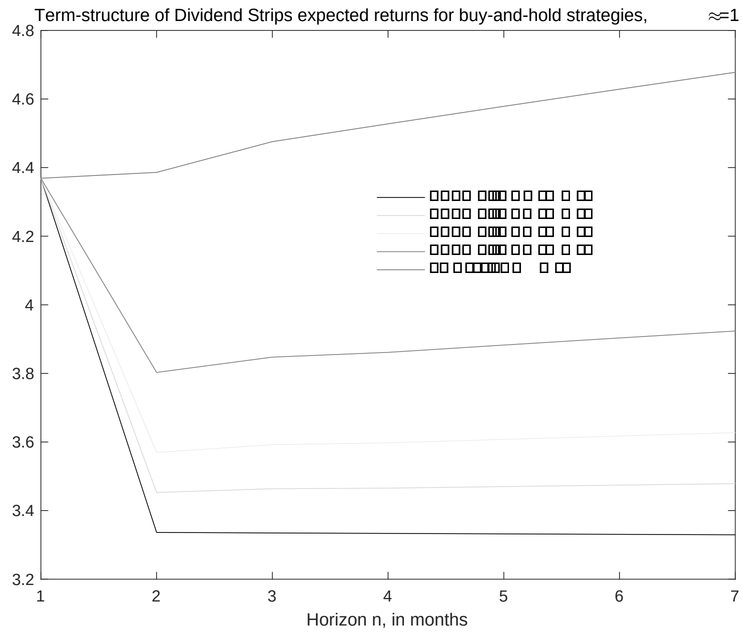

Figure 13: Term structure of dividend strip expected excess returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Bansal et al. (2014) ÑTable 1a.