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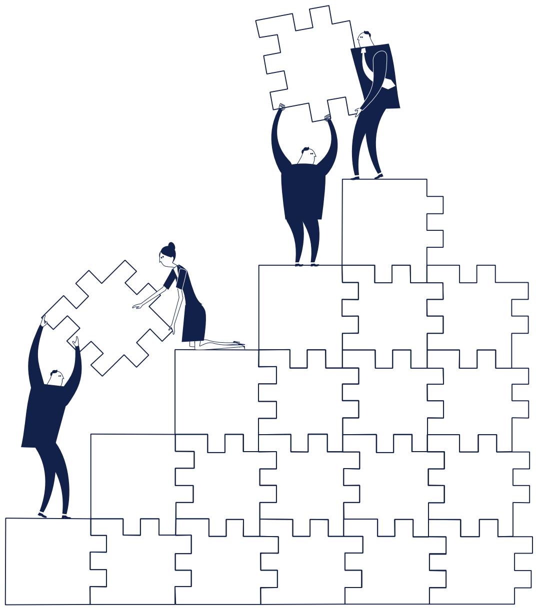

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Diffusing Workers in a Multiplex World

Catherine Tong, Omar Guerrero, Eduardo López and Felix Reed-Tsochas

***Abstract*—The study of labor mobility across firms is crucial to understand economic performance, unemployment, skills reallocation and other aspects that shape the economic life of nations. Modeling labor flows between firms has been a challenge due to the complexity arising from the distributed and heterogeneous nature of labor flows. In this paper, we introduce a discrete-time model of labor flowing on a multi-layered network (i.e. a multiplex graph). By introducing multiple layers, the model accounts for different mobility patters (e.g. industries, geographies, occupations, etc.), which is important to understand the reallocation of human capital, skills and knowledge. We apply the model to UK empirical micro-data and find that our measure of regional preferences for low versus high skilled workers vary significantly from a single to a multi-layer representation of the world.**

***Index Terms*—labor flows, networks, firms, multiplex, Markov chains, random walks**

I. Introduction

Every day, people change jobs, moving from one firm to another. This process is crucial for economic performance because it allows human capital, skills and knowledge to be reallocated among the firms that need them. The movement of millions of heterogeneous individuals across hundreds of thousands of firms (or even millions) is a distributed process. Naturally, decentralization and heterogeneity lead to considerable complexity. By using aggregate matching functions, traditional models neglect such complexities [1]. Aggregation destroys information about the origins and destinations of labor flows.

Recent modeling frameworks have tackled the challenge of modeling labor mobility as a distributed process through the idea of labor flows on networks [2]. Here, nodes represent firms and edges mean that workers can flow between companies. In the absence of an edge, one would not expect labor flows between disconnected firms due to high mobility barriers in the labor market (e.g. unrelated industries, geographically distance, competing firms, etc.). In consequence, the network topology has direct incidence in the concentration of employment, skills and knowledge among specific groups of firms.

In this paper, we generalize of the 'labor flow network' (LFN) framework. This generalization allows for heterogeneous patterns of labor flows.In other words, it takes into account the fact that different groups of people exhibit different mobility patterns. For example, people in different occupations (e.g. lawyers and chefs) perform specific tasks that shape their career paths. These trajectories reflect the specificities of their professions, which translate into different network topologies between occupations. One would expect chefs to be more mobile than lawyers and that the LFN of lawyers should be more clustered than that of chefs.

Clearly, a single LFN is not representative of this fact because everyone is equally subjected to the same topology. Alternatively, one could think of having different networks (one for each group), but that would ignore the interaction between them. We solve this problem by modeling labor flows on multiplex networks, i.e. in networks with multiple layers. Our generalization consists of using Markov chains to model random walks on multiplex graphs. This can be thought as a mixture of intra-layer and inter-layer processes of labor reallocation. We obtain the steady-state solutions that predict the concentration of employment in each firm, decomposed into each layer. Our solutions are explicit and compact, facilitating empirical applications. Using survey micro-data from the UK, we fit our model to study the differences between high-skilled and low-skilled labor flows. Finally, we show that our understanding, and hence policy implications, of labor mobility between these two groups differs substantially when we account for their respective mobility patterns (layers).

The paper is organized in the following way. Section II introduces the model, develops the methodology of Markov chains on multiplex networks, and presents the steady-state solution. Section III demonstrates an empirical application and the main results. Finally, we discuss our results in section IV.

II. Proposed Method

In this section, we first propose a dynamic model for labour mobility on a multiplex network and present its solution. We subsequently estimate the model parameters based on any given dataset. Although the multi-layer nature of the model allows considering any kind of groups of workers, we will concentrate on skills due to their high relevance to policy applications. Table I shows the basic notations used in the rest of this paper.

TABLE I
Basic Notations

$G(V, E, L)$	A multiplex network with L layers, V as node set, E as edge set
$\Gamma_{i,\alpha}$	A set of neighbouring nodes of node i within layer α
$k_{i,\alpha}$	Number of neighbours connected to node i within layer α
$r_{i,\alpha}$	Probability of worker becoming employed in node i in layer α
$s_{i,\alpha}$	Probability of worker becoming unemployed in node i in layer α
$\lambda_{i,\alpha}$	Probability of worker becoming separated from node i in layer α
$h_{i,\alpha}$	Probability of worker becoming hired into node i in layer α
$D_i^{\alpha\beta}$	Probability of switching from layer α to layer β while at node i

we assume edges capture the affinity between firms such that frictions are low in both directions. We assume that

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an unweighted graph because such affinity has a qualitative character.

A. Problem Definition

Let N denote the number of firms (nodes) in an undirected unweighted multiplex graph[1] G with L layers. Each node is present in all layers, and there are no inter-layer edges in G. Each layer represents a skill-specific mobility pattern, so G^α is the sub-network corresponding to the layer of skill level α. Assume that each firm (node) i has workers belonging to L different skill levels (layers). We write (i, α) for a given level of a given firm. Every worker is associated to a firm. While associated, the worker is in one of two states: employed or unemployed. Association to a firm during unemployment means that the firm was the worker's last employer. We denote the probability of being employed at firm i in layer α as $r_{i,\alpha}$. Similarly, $s_{i,\alpha}$ is the probability of being unemployed.

The labor mobility problem for a multiplex graph is to define a set of rules which govern the movement of workers between employment and unemployment, between firms at the same level, as well as between firms at different levels. To do this, we propose a discrete-time stochastic process. Let $M = [(M^{\alpha\beta}_{xy})_{ij}], \alpha, \beta \in \{1, ...L\}, i \in \{1, ..., N\}, x \in \{r, s\}$, be a $NL \times NL$ *supra-transition matrix* describing a multi-layer Markov Chain on the graph G. The elements of the matrix give the transition probabilities from one specified state to another. This allows us to define the evolution equation of the probability of employment of the ith node in layer α as

$$\begin{aligned} r_{i,\alpha}(t+1) = (M^{\alpha\alpha}_{rr})_{ii} r_{i,\alpha}(t) + \sum_{\beta=1,\beta\neq\alpha}^{L} (M^{\alpha\beta}_{rr})_{ii} r_{i,\beta}(t) \\ + (M^{\alpha\alpha}_{sr})_{ij} s_{j,\alpha}(t), \end{aligned} \tag{1}$$

where

(i) $(M^{\alpha\alpha}_{rr})_{ii}$ is the term that defines the contribution of the employment state of node i in layer α to $r_{i,\alpha}(t+1)$, i.e. $r_{i,\alpha}(t) \to r_{i,\alpha}(t+1)$. With reference to a single worker, this is the probability of he or she remaining employed in node i in layer α;

(ii) $(M^{\alpha\beta}_{rr})_{ii}$ defines the contribution $r_{i,\beta}(t) \to r_{i,\alpha}(t+1)$. With reference to a single worker, this is the probability that he or she switches its state of employment from i in layer β to node i in layer $\alpha \neq \beta$;

(iii) $(M^{\alpha\alpha}_{sr})_{ij}$ defines the contribution $s_{i,\alpha}(t) \to r_{i,\alpha}(t+1)$. With reference to a single worker, this is the probability that he or she changes from a state of unemployment at node i in layer α to employment at node j in layer α.

In a similar way, we construct the evolution equation for the probability of unemployment as

$$s_{i,\alpha}(t+1) = (M^{\alpha\alpha}_{rs})_{ii} r_{i,\alpha}(t) + (M^{\alpha\alpha}_{ss})_{ii} s_{i,\alpha}(t), \tag{2}$$

where

(iv) $(M^{\alpha\alpha}_{rs})_{ii}$ defines the contribution $r_{i,\alpha}(t) \to s_{i,\alpha}(t+1)$. With reference to a single worker, this is the probability that he or she changes from a state of employment to unemployment at node i in layer α;

(v) $(M^{\alpha\alpha}_{ss})_{ii}$ defines the contribution $s_{i,\alpha}(t) \to s_{i,\alpha}(t+1)$. With reference to a single worker, this is the probability of he or she remaining unemployed in node i in layer α.

In defining the term (ii), we restrict that workers only move between layers *if and only if* they are employed at the same node. This is motivated by the fact that workers usually hone their skills as they receive training during employment, e.g. salesmen might be promoted to management in the course of their employment at a firm. Naturally, this assumption can be relaxed, in which case we obtain solutions in matrix-forms that can be solved numerically. Since we present an empirical application, we prefer to hold this assumption in order to obtain explicit solutions.

The system (1), (2) can be rewritten in matrix form as

$$\mathbf{X}(t+1) = M\mathbf{X}(t) = M^{t+1}\mathbf{X}(0) \tag{3}$$

where $\mathbf{X} = [\mathbf{x}_\alpha]$ is a vector composed of the states (employed or unemployed) of all nodes in the different layers of the multiplex network with $\mathbf{x}_\alpha = (r_{1,\alpha}, \ldots, r_{N,\alpha}, s_{1,\alpha}, \ldots, s_{N,\alpha})^T$.

B. Parameters

We next define the matrix M as a function of some parameters with real-world connotations and explore the steady state dependence of employment and unemployment probabilities on these parameters.

We introduce node-specific parameters $h_{i,\alpha}, \lambda_{i,\alpha}$ and $D^{\alpha\beta}_i$ representing the probabilities for hiring, separation and layer-switching respectively. We propose a set of rules for workers to move in the multiplex, shown in Algorithm 1.

```
for period t do
    for each worker do
        if employed then
            become unemployed from firm (i,α) with
              probability λ_{i,α};
            if still employed then
                move to layer β with probability D_i^{αβ};
            end
        else
            select neighbouring firm (j,α) where j ≠ i at
              random;
            become hired with probability h_{j,α};
        end
    end
end
```

Algorithm 1: Labour mobility model.

Based on the above, the elements of the supra-transition matrix M are defined as:

[1] We assume edges capture the affinity between firms such that frictions are low in both directions. We assume that an unweighted graph because such affinity has a qualitative character.

$$(\mathbf{M}_{rr}^{\alpha\alpha})_{ij} = D_i^{\alpha\alpha}(1-\lambda_{i,\alpha})\delta_{ij} \quad (4)$$
$$(\mathbf{M}_{sr}^{\alpha\alpha})_{ij} = h_{i,\alpha}\sum_{j\in\Gamma_{i,\alpha}}\frac{1}{k_{j,\alpha}} \quad (5)$$
$$(\mathbf{M}_{rs}^{\alpha\alpha})_{ij} = \lambda_{i,\alpha}\delta_{ij} \quad (6)$$
$$(\mathbf{M}_{ss}^{\alpha\alpha})_{ij} = [1-\sum_{j\in\Gamma_{i,\alpha}}\langle h\rangle_{\Gamma_{i,\alpha}}]\delta_{ij} \quad (7)$$
$$(\mathbf{M}_{rr}^{\alpha\beta})_{ij} = D_i^{\alpha\beta}(1-\lambda_{i,\alpha})\delta_{ij} \quad (8)$$

where δ is the Kronecker delta.

C. Steady state

In the steady state, $r_{i,\alpha}(t+1) = r_{i,\alpha}(t)$ and $s_{i,\alpha}(t+1) = s_{i,\alpha}(t)$. The steady state equations no longer have time dependence, i.e. $r_{i,\alpha}(t) \to r_\infty(i,\alpha)$ and $s_{i,\alpha}(t) \to s_\infty(i,\alpha)$. The system of equations is solved to produce analytical solutions for a multiplex of any number of layers, L, such that $\alpha = \{1,..,L\}, i = \{1,...,N\}$. The steady state solutions are

$$r_\infty(i,\alpha) = \frac{\chi h_{i,\alpha}\langle h\rangle_{\Gamma_{i,\alpha}} k_{i,\alpha}}{\lambda_{i,\alpha}}, \quad (9)$$

$$s_\infty(i,\alpha) = \chi h_{i,\alpha}k_{i,\alpha}, \quad (10)$$

$$\chi = \frac{1}{\sum_{\alpha=1}^{L}\sum_{i=1}^{N}\langle h\rangle_{\Gamma_{i,\alpha}}h_{i,\alpha}k_{i,\alpha}\left(\frac{1}{\lambda_{i,\alpha}}+\frac{1}{\langle h\rangle_{\Gamma_{i,\alpha}}}\right)}, \quad (11)$$

where χ is the normalization such that the condition $\sum_\alpha\sum_i[r_{i,\alpha}+s_{i,\alpha}] = 1$ is fulfilled.

From Equation 9, we see that steady-state employment at a node is not only dependent on its own hiring rate, but also the hiring rates of its neighbouring firms *on the same layer*; this highlights the importance of the network topology in each layer of the labour flow multiplex. In addition, the layer-switching parameter $D_i^{\alpha\beta}$ drops out entirely in the steady state solutions; this eliminates the need for estimating the empirical value of $D_i^{\alpha\beta}$.

III. Application

In this section, we present an application of the method proposed onto a novel dataset. We first describe the data and document the procedure taken to construct a two-layer network, with high-skilled and low-skilled workers in different layers. Finally, we present a comparison of the hiring patterns of employers towards different workers using the estimated model parameters.

Through this empirical application, our main findings are as follows:

- High-skilled workers and low-skilled workers demonstrate very different mobility patterns and thus network topologies (with Jaccard index of edge sets at 0.17)
- Different regional hiring patterns revealed for UK employers towards high-skilled and low-skilled workers

A. Data

We use anonymized data from the Quarterly Labour Force Survey (QLFS) [6], conducted by the UK Office for National Statistics (ONS). The QLFS is based on the resident population in the United Kingdom, with each quarterly sample containing approximately 100,000 individuals. Members of randomly selected households are asked to complete a questionnaire relating to the worker's employer, its employment situation and other personal characteristics.

We perform the analysis using a five-quarter longitudinal QLFS dataset [7]. The workers included in this dataset have completed 5 successive quarters of surveys, meaning that they responded to each question five times. Analysis of changes in a worker's five-quarter responses forms the baseline for tracking its employment over time. The data covers the period from January 2009 to June 2016. In total, the data contain 138,400 individuals.

Responses to the surveys are recorded in the dataset in form of a highly specific code system specified by the ONS [8]. Workers do not identify their firms but give the firms' details, e.g. industry, location. Since tracking of individual firm in this dataset is not allowed, we group firms by industry and location in our investigation. This means that in the following model application, nodes in the network would refer to a group of firms belonging to a certain industry and region in the UK.[2] We identify job transitions between nodes by comparing workers' five-quarter responses regarding their firms systematically (as documented in Appendix B).

Workers also report their personal details in the survey. We base our following investigation on workers' reported occupational groups (a total of nine groups), which give indications of their skill level.

B. Network Construction

Traditionally, labor studies and policy design focus on two types of skills: low and high. For this reason, we concentrate our application on a two-layer model that captures the mobility patterns of high and low-skilled workers. In order to identify both types, we propose a data-driven method to group workers based on their mobility patterns.

The method considers job transitions recorded in the dataset, specifically the occupational groups of workers before and after each transition, o_I and o_F. We expect that an optimal division will produce the two *skill groups* of workers, which will have the smallest number of workers moving from one skill group to another. Therefore, we define a cost function R associated with inter-skill-group edges. This method seeks to iteratively minimize the cost function to produce a division of occupational groups{1,..,9} into two skill groups S and $\overline{S}$.

$$R(S,\overline{S}) = \frac{t_{inter}(S,\overline{S})}{min(t_{intra}(S), t_{intra}(\overline{S}))} \quad (12)$$

where $t_{inter}(S,\overline{S})$ is the number of inter-skill-group transitions, i.e. transitions where $o_I \in S$ and $o_F \in \overline{S}$; and $t_{intra}(S)$ is the number of intra-skill-group transitions, i.e. transitions

[2]The ONS code system lists 21 regions and 615 industries.

where $o_I, o_F \in S$. This ratio R is defined in the same spirit of a graph conductance measure introduced in [9].

The result of this iteration is an optimal classification of $S^* = \{1, 2, 3, 4\}$, $\overline{S^*} = \{5, 6, 7, 8, 9\}$, with $R(S^*, \overline{S^*}) = 0.25$. This topology-driven partition is remarkably intuitive with accepted notions of high- and low-skilled workers (e.g., managers versus trades people). As a cross-check, the k-means clustering algorithm from a Machine Learning context [10] produces the same classification.

A resulting two-layer network G consists of transitions of workers in skill group S^* (high-skilled layer) and $\overline{S^*}$ (low-skilled layer). In total, G has 2,380 nodes on both layers, 5,246 edges on the high-skilled layer and 4,754 edges on the low-skilled layer. For comparison purposes, we also construct a single-layer network (multiplex with $L = 1$) using the same procedure.

The sets of edges of the two layers of this multiplex have Jaccard index 0.17, which is a low value meaning that the topology of the two layers are distinctive. Thus, workers on the two layers flow in different ways. It is interesting to note that although the degree distributions for both layers look quite similar (ref. Figure 1), the layers share such dissimilar set of edges.

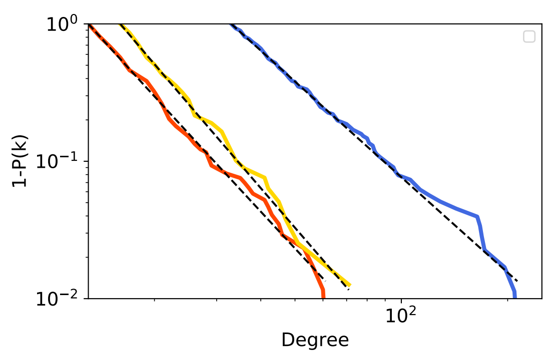

Fig. 1. Complementary Cumulative Probability $1-P(k)$ plotted as a function of degree k (solid lines) of the labour flow network on log-log scales, with fitted power law (dashed lines). The blue line shows the degree distribution of a single-layer network, where power-law fitting [11] finds $b = 3.30$. The red and yellow lines show the degree distributions of high-skilled and low-skilled layer of the two-layer multiplex network, power-law fitting finds $b = 3.74$ and $b = 3.94$ respectively.

C. Parameter Estimation

Direct retrieval of the parameters (hiring and separation probabilities) from data is often not possible as they are neither observable nor recorded. Therefore, we use a method previously developed to estimate parameters in Markov chains [12].

The estimation method works in a similar fashion to the gradient descent algorithm [13], seeking to minimize an error function, which is defined as the difference between flows observed from data and predicted from our model. Labor flows are computed by counting the job transitions captured in the data. From our model, the number of flows is a function of the hiring rate $h_{i,\alpha}$, separation rate $\lambda_{i,\alpha}$. The method explores the parameter space $(\lambda_{i,\alpha}, h_{i,\alpha})$ until it reaches a fixed point that minimizes the error function. Details of the method is documented in [12].

D. Results

One of the most interesting features of the model is its ability to obtain the hiring probability $h_{i,\alpha}$, which is not observable (in empirical data one can observe hires, but not the number of applicants who failed to be hired). This parameter serves as a proxy for the preference of employers to hire job applicants. In our context, different $h_{i,\alpha}$ for the same firm across different layers reveal the relative preference towards high vs low-skilled workers.

We summarize our results by computing the average hiring rate for all nodes inside a geographical region. This allows us to study how different regions exhibit different 'hiring preferences' for certain skills, and how this differs from the single-layer view. For each layer, we rank the regions according to their average $h_{i,\alpha}$, as shown in Figure 2.

We see considerable difference in the rankings produced in the different networks, which confirms our motivation that a multi-layer view. The difference in rankings implies the differentiating employer's attitude towards high-skilled and low-skilled workers, allowing us to identify regions which have higher demand for high-skilled or low-skilled workers. We highlight some interesting results.

The rankings align with the common perception that big cities (e.g. London) have a higher demand for workers (both high-skilled and low-skilled). This analysis also presents a more complete picture as we look at cases where hiring behaviour differ significantly for the high-skilled and low-skilled layers. One example is Wales, from a single-layer point of view we would have missed that high-skilled workers have a much better chance of getting hired in Wales than low-skilled workers.

The analysis presented here is a preliminary approach to utilize the model developed in this paper, but nevertheless an important one. For instance policymakers may use this ranking to judge the investment or training opportunities with high-skilled or low-skilled labour in each regions.

IV. Conclusion

In this paper, we introduced a method to study heterogeneous patterns of labour flows using Markov Chains in multiplex network. This framework allows the study of the behaviour of different groups of workers in an economy as well as the behaviour of firms towards different workers. Explicit analytic solutions are obtained for at the steady state of labour dynamics model.

Our method is applied to the Labour Force Survey data of United Kingdom. The dissimilar topologies of networks describing workers of different skills provide empirical support for the study of heterogeneous patterns of labor flows. We examine the hiring behaviour of employers in different regions of UK towards high-skilled and low-skilled workers; revealing that some regions have significant difference between the probabilities of hiring of high-skilled versus low-skilled workers.

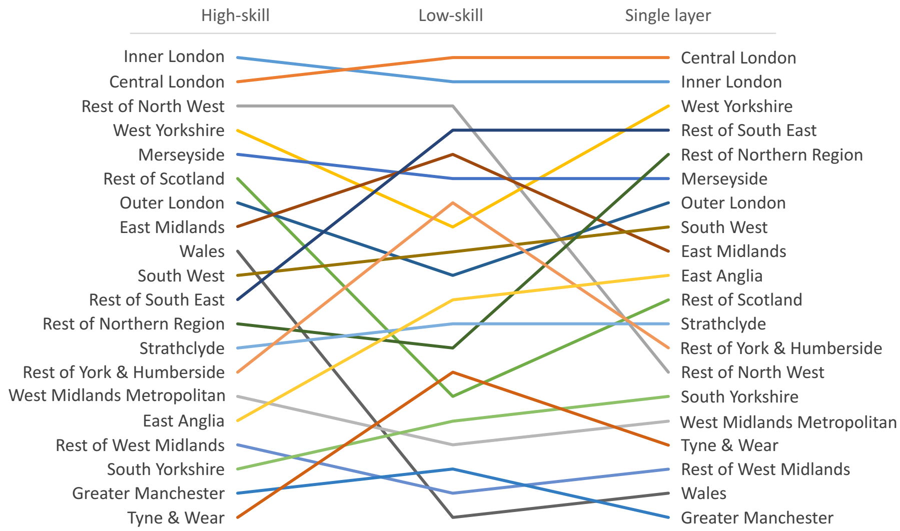

Fig. 2. The three lists ranking a region's average hiring rates; from left to right are the ranking for the high-skilled layer, low-skilled layer, and single layer. The top of each list has the highest ranking and therefore has the highest average hiring rate. (a) High-skilled layer: ranges from 54% in Inner London to 45% in Tyne & Wear. (b) Low-skilled layer: ranges from 49% in Central London to 39% in Wales. (c) Single layer: ranges from 74% in Central London to 65%.

Overall, we are confident that this paper can serve as a foundation for valuable future work involving labour studies of heterogeneous mobility patters.

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APPENDIX A
PROOF OF STEADY STATE SOLUTIONS

We substitute matrix elements [4-8] into Eq. 1 and 2 to obtain the following at steady state,

$$\begin{aligned} r_\infty(i,\alpha) &= D_i^{\alpha\alpha}(1-\lambda_{i,\alpha})r_\infty(i,\alpha) \\ &+ \sum_{\beta=1,\beta\neq\alpha}^{L} D_i^{\beta\alpha}(1-\lambda_{i,\beta})r_\infty(i,\beta) \\ &+ h_{i,\alpha}\sum_{j\in\Gamma_{i,\alpha}}\frac{s_\infty(j,\alpha)}{k_{j,\alpha}} \end{aligned} \tag{13}$$

$$s_\infty(i,\alpha) = \lambda_{i,\alpha}r_\infty(i,\alpha) + s_\infty(i,\alpha)\left[1-\langle h\rangle_{\Gamma_{i,\alpha}}\right] \tag{14}$$

We re-arrange Eq. 14 for $s_\infty(i,\alpha)$ to get,

$$s_\infty(i,\alpha) = \frac{\lambda_{i,\alpha}r_\infty(i,\alpha)}{\langle h\rangle_{\Gamma_{i,\alpha}}} \tag{15}$$

Substituting Eq. 15 into Eq. 13 and noting that $D_i^{\alpha\alpha} = 1-(\sum_{\beta\neq\alpha}D_i^{\alpha\beta})$,

$$\begin{aligned} r_\infty(i,\alpha) &= \left(1-(\sum_{\beta\neq\alpha}D_i^{\alpha\beta})\right)(1-\lambda_{i,\alpha})r_\infty(i,\alpha) \\ &+ \sum_{\beta\neq\alpha}\left(D_i^{\beta\alpha}(1-\lambda_{i,\beta})r_\infty(i,\beta)\right) \\ &+ h_{i,\alpha}\sum_{j\in\Gamma_{i,\alpha}}\frac{\lambda_{i,\alpha}r_\infty(j,\alpha)}{\langle h\rangle_{\Gamma_{j,\alpha}}k_{j,\alpha}} \end{aligned} \tag{16}$$

Rearranging, this gives

$$\begin{aligned} &\sum_{\beta\neq\alpha}D_i^{\alpha\beta}\left(r_\infty(i,\alpha)(1-\lambda_{i,\alpha})\right) - \sum_{\beta\neq\alpha}\left(D_i^{\beta\alpha}(1 \right. \\ &\left. -\lambda_{i,\beta})r_\infty(i,\beta)\right) = \sum_{j\in\Gamma_{i,\alpha}}\left[\left(\frac{h_{i,\alpha}}{\langle h\rangle_{\Gamma_{j,\alpha}}k_{j,\alpha}}\right.\right. \\ &\left.\left. -\delta_{ij}\right)\lambda_{j,\alpha}r_\infty(j,\alpha)\right] \end{aligned} \tag{17}$$

Eq. 17 concerns layer α in particular, summing over all layers, we obtain

$$\begin{aligned} &\sum_\alpha\sum_{\beta\neq\alpha}D_i^{\alpha\beta}\left(r_\infty(i,\alpha)(1-\lambda_{i,\alpha})\right) - \sum_\alpha\sum_{\beta\neq\alpha}\left(D_i^{\beta\alpha}(1 \right. \\ &\left. -\lambda_{i,\beta})r_\infty(i,\beta)\right) = \sum_\alpha\sum_{j\in\Gamma_{i,\alpha}}\left[\left(\frac{h_{i,\alpha}}{\langle h\rangle_{\Gamma_{j,\alpha}}k_{j,\alpha}}\right.\right. \\ &\left.\left. -\delta_{ij}\right)\lambda_{j,\alpha}r_\infty(j,\alpha)\right] \end{aligned} \tag{18}$$

As the first and second terms on the left hand side of Eq. 18 cancel, we obtain

$$\sum_\alpha\left[\sum_{j\in\Gamma_{i,\alpha}}\left(\frac{h_{i,\alpha}}{\langle h\rangle_{\Gamma_{j,\alpha}}k_{j,\alpha}}\lambda_{j,\alpha}r_\infty(j,\alpha)\right) - \lambda_{i,\alpha}r_\infty(i,\alpha)\right] = 0 \tag{19}$$

Using the ansatz,

$$r_\infty(i,\alpha) = \frac{h_{i,\alpha}\langle h\rangle_{\Gamma_{i,\alpha}}k_{i,\alpha}}{\lambda_{i,\alpha}} \tag{20}$$

the left hand side of Eq. 19 simplifies to

$$\sum_\alpha\left[h_{i,\alpha}\sum_{j\in\Gamma_{i,\alpha}}h_{j,\alpha} - h_{i,\alpha}\langle h\rangle_{\Gamma_{i,\alpha}}k_{i,\alpha}\right] \tag{21}$$

Using the definition of average hiring rate,

$$\langle h\rangle_{\Gamma_{i,\alpha}} = \frac{\sum_{j\in\Gamma_{i,\alpha}}h_{j,\alpha}}{k_{i,\alpha}} \tag{22}$$

Terms in Eq. 21 cancel, and this ansatz solves the steady state equations. Substituting the ansatz Eq. 19 into the steady state unemployment probability given in Eq. 15, we have

$$s_\infty(i,\alpha) = h_{i,\alpha}\langle h\rangle_{\Gamma_{i,\alpha}}k_{i,\alpha} \tag{23}$$

A normalization factor χ should be included such that

$$\sum_\alpha\sum_i r_\infty(i,\alpha) + s_\infty(i,\alpha) = 1 \tag{24}$$

This gives

$$\chi = \frac{1}{\sum_{\alpha=1}^{L}\sum_{i=1}^{N}\langle h\rangle_{\Gamma_{i,\alpha}}h_{i,\alpha}k_{i,\alpha}\left(\frac{1}{\lambda_{i,\alpha}}+\frac{1}{\langle h\rangle_{\Gamma_{i,\alpha}}}\right)} \tag{25}$$

Steady state solutions are

$$r_\infty(i,\alpha) = \frac{\chi h_{i,\alpha}\langle h\rangle_{\Gamma_{i,\alpha}}k_{i,\alpha}}{\lambda_{i,\alpha}} \tag{26}$$

$$s_\infty(i,\alpha) = \chi h_{i,\alpha}k_{i,\alpha} \tag{27}$$

APPENDIX B
NETWORK CONSTRUCTION FROM DATASET

An edge (i,j) is generated if any one worker performs a job transition from i to j. Job transitions are identified by considering workers' five-quarter responses in two ways. First, if a worker is employed at a node i in a quarter but employed at another node j in a succeeding quarter, this is a job transition from i to j so an edge (i,j) is generated. Second, if a worker is employed at the same node i in successive quarters but reports that he or she has changed job recently, this is a job transition within i so an edge (i,i), i.e. a self-loop, is constructed. Examples of edge construction are found in Table II.

TABLE II

EXAMPLE FIVE-QUARTER RESPONSE OF A WORKER IN THE DATASET. NULL RESPONSES ARE DENOTED BY '-' IN THIS TABLE. HIGHLIGHTED IN RED ARE RESPONSES INDICATING TWO JOB TRANSITIONS WHICH RESULT IN THE CONSTRUCTION OF EDGES ('1000_1' ,'2000_1') AND ('2000_1', '1000_2'), NOTE THE LATTER IS COUNTED DESPITE AN INTERMEDIATE PERIOD OF UNEMPLOYMENT IN QUARTER 3. RESPONSES IN QUARTER 5 ARE HIGHLIGHTED IN BLUE TO INDICATE A JOB TRANSITION WITHIN A NODE, LEADING TO THE CONSTRUCTION OF A SELF-LOOP ('1000_2','1000_2'). THE PAIRS OF OCCUPATION ASSOCIATED TO EACH EDGE ARE (9,9), (9,9), (9,3) RESPECTIVELY.

Quarter	1	2	3	4	5
Industry	1000	2000	-	1000	1000
Region	1	1	-	2	2
Employed?	Yes	Yes	No	Yes	Yes
Left job?	No	Yes	Yes	No	Yes
Occupation	9	9	-	9	3