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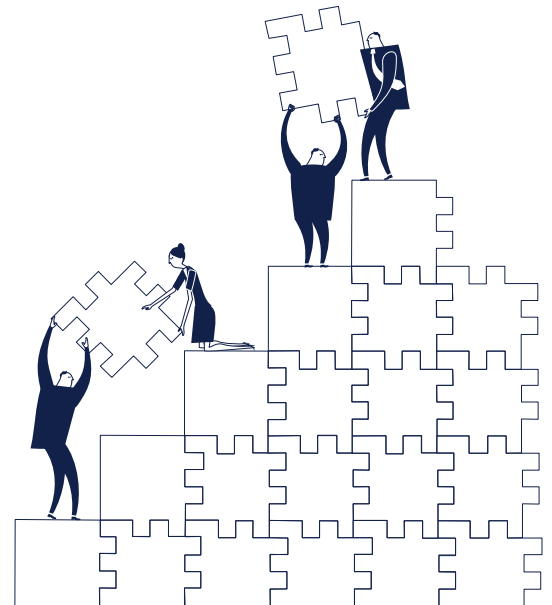
Uncovering Vote Trading through Networks and Computation

Omar A. Guerrero
*CABDyN Complexity Centre and Centre for Business Taxation,
University of Oxford*

Ulrich Matter
School of Economics and Political Science, University of St.Gallen

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Omar A. Guerrero[†] and Ulrich Matter[‡]

[†]CABDyN Complexity Centre and Centre for Business Taxation, University of Oxford

[‡]School of Economics and Political Science , University of St.Gallen

Abstract

We develop a new methodological framework for the empirical study of legislative vote trading. Building on the concept of reciprocity in directed weighted networks, our method facilitates the measurement of vote trading on a large scale, while estimating the micro-structure of trades between individual legislators. In principle, it can be applied to a broad variety of voting data and refined for various specific contexts. It allows, for example, to study how vote trading in a specific legislative assembly varies over time. We validate our method with a computational model in which we control the level of vote trading. Finally, we demonstrate our framework in an analysis of four decades of roll call voting in the U.S. Congress.

1 Introduction

Vote trading, also commonly known as logrolling, is a cornerstone of the positive analysis of politicians' behavior in collective decision-making. It may influence economic policy-making and can, therefore, have broad consequences for the rules that guide citizens' lives in a democratic society. A typical example carrying a negative connotation is when legislators favor special interest groups in exchange for campaign finance. Despite a negative stigma, the possibility of vote trading also enables compromise and the consideration of minorities with strong preferences. The former is a central aspect of rent seeking by special interest groups. The latter might play an important role in times of political polarization and policy of obstruction.

Anecdotal evidence of vote trading has been reported in several legislative assemblies around the world. In the United States such reports can be found across the entire congress history. In fact, historical records mentioning explicit vote trading go back to the first congress in 1789. According to Bordewich (2016, p. 148), vote trading was substantial when deciding the permanent location of the congress: "No other issue that had come before congress had produced the same frenzy of backroom bartering and vote trading."[1] In the early 19th century, James K. Polk (U.S. President and former Speaker of the House), reportedly "stood against the practice of logrolling, or vote trading, considering it a form of corruption" (Schraufnagel, 2011, p. 170). More reports of recent occurrences of vote trading in the U.S. Congress often mention these activities in the context of special interest politics. A well-known example is the (failed) congressional attempt to revise tariffs in order to boost the economy during the Great Depression. It led President Herbert Hoover to conclude that "[c]ongressional [tariff] revisions are not only disturbing to business, but with all their necessary collateral surroundings in lobbies, logrolling and the activities of group interests, are disturbing to public confidence" (Pastor, 1982, p. 69). Finally, qualitative

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[1]For example, Congressman Fisher Ames is described as having "descried 'this vile and unreasonable business' of feverish vote swapping but nevertheless put his weight and his eloquence into the battle." (Bordewich, 2016, p. 148).

studies based on interviews of Members of Congress suggest that they are willing to talk about vote trading, however, only anonymously. Kingdon (1989, p. 100), for example, quotes a Member of Congress' statement regarding his traded vote on a bill deregulating cigarette advertisement: "This will be sort of a buddy vote. I know cigarettes are harmful and I wouldn't touch them myself. But a lot of my friends are concerned about this, because tobacco means a lot to the economy of their areas. They do things for me when I need it, and I'll do this for them. Frankly, it's just a matter of helping out your friends."

In terms of quantitative evidence, we know little about logrolling because vote-trading agreements are not directly observable. While the topic is broadly covered in the theoretical literature of political science and economics (Buchanan and Tullock, 1962; Wilson, 1969; Tullock, 1970; Haefele, 1970; Riker and Brams, 1973; Bernholz, 1974, 1978; Mueller, 2003; Casella et al., 2014), statistical evidence is almost nonexistent, besides a handful of important exceptions (see Stratmann 1992, 1995 and Cohen and Malloy 2014).[2] Insights into the relative importance of this hidden cooperation in legislative assemblies, however, are crucial in order to understand the success and failure of alternative institutional arrangements used to address economic and social problems. Without a widely applicable framework that allows the empirical estimation of vote trading, our understanding of its determinants, mechanisms, and consequences is limited.

The challenge of empirically assessing the prevalence of logrolling lies in its 'hidden' nature. Legislators on both sides of a vote-trading agreement are keen to keep such a deal secret. Strategically offering one's vote to a fellow legislator with opposing political views may contrast with the expectation of voters and peers. Hence, empirically capturing vote trading requires us to measure something that is not directly observable. Theoretical considerations are thus essential to guide the empirical methodology.

When thought of as a systemic phenomenon, vote trading is driven by fundamental incentives set by a common democratic procedure: voting under the simple majority rule.

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[2]We discuss these contributions in more detail and generalize the previous approaches to detecting vote trading in the next section.

From a positive perspective, we can look at voting decisions under simple majority rule as being shaped by incentives that are determined by the legislators' policy preferences and the constraint of having only one (unweighted) vote per roll call. The rule, therefore, cannot take into account legislators' preference intensities for different bills. Vote trading, however, can serve as a mechanism for expressing (weighting) these preferences. Legislators can give up their inherent position on a bill and 'sell' their vote in order to get support in the passage of their own favorite bills. It is thus mutually beneficial for legislators who are strongly interested in the passage of specific bills. It also implies that when trading their votes, legislators do not vote sincerely, i.e., their vote is misaligned with their inherently preferred policy position. Importantly, offering to vote in favor of bills one dislikes in order to get support for one's most favored bill(s) primarily makes sense in the context of narrow voting outcomes (Stratmann, 1992; Cohen and Malloy, 2014).[3] In narrow votes, the marginal support is particularly valuable for those legislators with strong preferences for the bill.[4]

With this framework in mind, we have developed a methodology to estimate the prevalence of vote trading in roll call data. The theoretical pillars of the method can be summarized in four principles that are consistent with previous findings from specific voting

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[3]In roll calls, legislators engaged in vote trading can observe each others' actions once the votes are cast and, thus, know with certainty whether their partner kept his or her part of the bargain. However, as they have strong incentives to keep such deals secret, they cannot directly punish defection (e.g., publicly blame or sue a legislator who does not keep his or her part of the bargain). Under these theoretical assumptions and the protocol of sequential vote casting, logrolling in the form of a one shot game would present a sequential prisoner's dilemma. In the case of a potential trade between two legislators, the legislator whose preferred bill is first voted would always be better off by defecting after the other legislator holds his or her part of the bargain. Knowing this, the other legislator is better off defecting in the first place anyway. Hence, cooperation in the form of logrolling is likely to evolve as reciprocal behavior over repeated interactions (Axelrod, 1984). This potential adoption of a "tit-for-tat" strategy to overcome the prisoner's dilemma in the case of repeated interactions in congressional vote trading is also pointed out in the context of historical anecdotal records about vote trading (Jillson and Wilson, 1994).

[4]If a legislator is fairly certain that his or her most favored bills will pass or fail by a large margin, there are almost no incentives to exchange favours in order to increase the margin of the vote outcome. In other words the marginal benefit from increasing the chances of passing the favored bill is lower than the marginal cost of arranging the trades that would be needed. Thus, the trade of votes is present if the voting decision is expected to be very close. Note that the narrowness of vote outcomes might be considered at different yes-share thresholds in the context of logrolling. Here, we focus on the most obvious one: the simple majority rule at the 50% threshold. However, in general, other thresholds could also be of interest for the study of logrolling. For example, the procedural rules in the U.S. Senate might set incentives for logrolling to pass bills with a yes-share greater than just 60% or a 2/3 majority in order to override a filibuster or a presidential veto, respectively.

contexts (Stratmann, 1992, 1995):

1. Incentives to trade votes are stronger the *narrower* the vote outcomes are.

2. If a legislator trades a vote, then he or she votes in the opposite direction to what would be predicted, i.e. the legislator *deviates* from his or her preferred position.

3. A deviation is considered a potential trade if it is *directed* to benefit a legislator with a clear interest in passing the bill that is being voted on.

4. Directed deviations are considered traded votes only if they are *reciprocal*.

The method consists of three components: (i) predicting legislators' voting decisions based on a set of observable factors $\mathbf{X}$ that represent the legislators' usual policy positions (e.g., DW-Nominate scores, party, etc.) and coding deviations from the predicted decisions; (ii) identifying the beneficiaries of those deviations; and (iii) measuring reciprocity between deviators and beneficiaries. From components (i) and (ii), we build a network of legislators who deviate from their ideal positions in roll call votes, benefiting other legislators. We call it the *directed-deviation network* (DDN), and a link from one legislator to another means that the former voted 'Yes' in a bill in which he or she was expected to vote 'No', while the latter benefits from the passage of such bill. In section 5 of this paper, we use (co-)sponsorships data in order to identify beneficiaries; however, the methodology allows the usage of any other type of data indicating strong specific policy preferences. Finally, in (iii) we extract the reciprocal deviations from the DDN and obtain the *vote-trading network* (VTN). The prevalence of vote trading is captured in an aggregate index ℓ and its anatomy is revealed through the VTN. The modular design of our methodology provides great flexibility to generalize it to various voting contexts where the simple majority rule determines voting outcomes.

The paper is structured in the following way. In section 2 we elaborate on the problem of estimating the prevalence of vote trading and discuss the existing approaches, their

advantages and limitations. Section 3 introduces the methodology. Section 4 presents a simulation study in order to understand the power and limitations of our method. In section 5, we demonstrate the application of the method to a 40-years dataset from the U.S. Congress. Finally, we discuss our results and conclude in section 6.

2   On the Estimation of Vote Trading

In order to establish a better foundation for our methodological framework, we first summarize and generalize the core problem of detecting and measuring the prevalence of vote trading as well as the previously suggested empirical strategies that address this problem. The literature on vote trading distinguishes two particular forms: implicit and explicit trades. The former refers to exchanging favors at the drafting stage of so-called 'omnibus bills', resembling a package of policies favoring different groups and, thereby, ensuring passage of all policies in one vote. The latter refers to legislators voting in favor of each others' favorite bills with the aim of ensuring minimal winning coalitions in all roll calls involved in the trade. It is this type of *explicit* vote trading that most previous empirical contributions, as well as the method presented here, focus on (we use the terms 'vote trading' and 'logrolling' interchangeably for explicit vote trading in what follows).

First, allow us to introduce some notation convention that we will use throughout the paper. A 'tuple' (i, k) denotes a legislator i and a roll call k. We use tuples to indicate relationships between legislators and roll calls, for example, a Yes vote. A tuple of tuples such as $((i, k), (j, l))$ represents a pair of decisions of the same type, for example, legislator i voting in roll call k and legislator j voting in roll call l. We denote sets with bold capitalized characters such as $\mathbf{A}$. Usually, we use sets to indicate collections of tuples. Finally, we employ hollow capitalized characters to denote matrices. For example, $\mathbb{A}$ indicates a matrix such that $\mathbb{A}_{ik}$ is the entry in row i and column k.

The empirical problem of estimating the prevalence of vote trading in roll call data can be

generalized as follows. For a specific legislature we want to assess whether vote trading can explain some of the voting decisions cast in the K roll call votes taken during a specific period of time. The starting point is thus a set of Yes roll call decisions $\mathbf{V}$ from N legislators voting in K roll calls. Each element in $\mathbf{V}$ has the form (i, k), meaning that legislator i voted Yes in roll call k. In addition, we have a data set $\mathbf{X}$ with information on legislator characteristics, bill characteristics, constituency characteristics and any other available information that we consider relevant to explaining usual voting behavior (independent of trades). Then, the task at hand is to assess whether $\mathbf{V}$ contains cases where (i, k) and (j, l) because of a vote trading agreement in which legislator i votes in favor of policy k that he or she dislikes but j strongly favors, and vice versa in the case of bill l.[5]

Following this line of thought, the perfect method to detect vote trading would thus take $\mathbf{V}$ (and $\mathbf{X}$) as an input and return a subset $\mathbf{V}_{trades} \subseteq \mathbf{V}$ containing all voting decisions $(i, k) \in \mathbf{V}$ involved in all trades. Based on this subset, the phenomenon of vote trading could be studied in depth, as it would reveal who traded his or her vote in which roll call on which bill.[6] In essence, we want to categorize the $(i, k) \in \mathbf{V}$ cases into traded votes and not traded votes, given the information in $\mathbf{V}$ and $\mathbf{X}$.

A crucial constraint to developing a methodology that comes close to the outlined ideal approach is the non-observable nature of vote-trading agreements. In other words, there is no training data set available and it is practically illusive to try building one.[7] Hence, any approach has to build substantially on fundamental theories of vote trading to inform how to employ $\mathbf{X}$ when 'filtering' $\mathbf{V}$ in order to build a statistical test. Given such a test, the

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[5]Note that a favor of i to j could also be repaid by several favors of j to i. However, as long as $\mathbf{V}$ contains the entire universe of Yes votes ever cast by the N legislators in it, a traded vote would have, by definition, some counterpart of one or several traded votes. In practice, it can be that some $(i, k) \in \mathbf{V}$ does not actually have such a counterpart because the observation period caps it or legislators leave office before returning the favor.

[6]One could, for example, assess who the frequent traders were, what type of bills were involved in trading, and during what periods trading was particularly relevant.

[7]Surveying legislators about their vote trading activities is likely illusive as they have strong incentives to keep such deals secret. Qualitative research on the U.S. Congress is in line with this view (see section 1). While legislators might not be reluctant to reveal that they and others engage in vote trading from time to time, they avoid giving explicit information on which vote they traded with whom.

assessment of whether any $(i, k) \in \mathbf{V}$ is actually a traded vote relies on ruling out alternative explanations for the observed voting pattern.[8]

So far, a few approaches have been suggested to achieve this. All prominent suggestions (Stratmann, 1992, 1995; Cohen and Malloy, 2014) are essentially a combination of directly filtering $\mathbf{V}$ based on $\mathbf{X}$ and then regressing a subset of $\mathbf{V}$ on a subset of $\mathbf{X}$, whereby one coefficient (or a linear combination of several coefficients) indicates whether vote trading is likely the reason for some $(i, k) \in \mathbf{V}$. Specifically, the null hypothesis of absent vote trading would be rejected if these coefficients are positive and statistically significantly different from 0. The logic of such an approach is that a systematic partial correlation between certain factors and some $(i, k) \in \mathbf{V}$, holding all other factors constant, can only be attributed to vote trading. This, of course, does not directly provide $\mathbf{V}_{trades}$. However, given the relative size of regression coefficients, it is straightforward to assess how substantial the 'vote trading factor' is in explaining the observed voting decisions (in comparison to, for example, party affiliation). Also, post-estimation techniques can be applied to approximately assess how many legislators likely traded how many votes in which policy contexts.

In the case of Stratmann (1992, 1995), the author selects a subset of $\mathbf{V}$ and $\mathbf{X}$ based on anecdotal reports on vote trading coalitions being active in the context of a handful of bills.[9]

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[8]This problem becomes more apparent when considering that alternative explanations can be very close to what we consider vote trading. For example, i schedules a meeting with j with the goal of getting j's support for policy l. We do not observe what i and j discuss in their meeting, but we see that j later votes in favor of l. It could be that they actually struck a deal and i promised to return the favor ('in the same currency') and thus to support policy k that j is particularly in favor of at some point in the future. It could also be the case that j managed to convince i to vote that way, that i promised to repay j in a currency other than votes (e.g. campaign support), or even that j simply does i a favor with no strings attached. Note that each of these stories becomes equally more likely the better j and i know and trust each other (be it from sitting in the same committee(s), being in office for a long time, or from knowing each other from outside politics) despite their political differences. Of course, this is equally relevant for previous empirical studies on vote trading as well as for our approach.

[9]In the case of Stratmann (1992), the emphasis is on specific votes on five amendments to the Food Security Act of 1985 (i.e., the so-called "Farm Bill") in the first session of the 99th U.S. House. Whether or not vote trading took place in the remaining 900 bills during the same congress (or in all the roll calls in any other congress) remains open. Stratmann (1995) focuses on three roll call votes taking place during the 86th U.S. Congress (1959-1960) and three votes in the 87th U.S. Congress. The votes were on bills covering urban, labor, and farm interests. Importantly, the selection of these few specific roll call votes in both studies was guided by rumors documented in secondary sources Mayhew (1966); Quarterly (1986), indicating the presence of vote trading in these roll calls. Without this prior information about logrolling presence, the method developed in Stratmann (1992) would not be applicable.

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The specific setting in combination with a straightforward theory of vote trading reveals what combination of explanatory variables in $\mathbf{X}$ can be used to generate the ceteris paribus conditions in order to interpret the positive effect of voting Yes in one vote and voting Yes in the other vote as an indicator of some $(i, k) \in \mathbf{V}$ and $(j, l) \in \mathbf{V}$ being driven by vote trading. Thus, the author selects, based on anecdotal evidence, a very specific subset out of $\mathbf{V}$ and then tests whether a part of the voting decisions in this subset actually reflect vote trading. The advantage of this approach is that, given the strong prior information of vote trading being present in the specific subset of $\mathbf{V}$, one can convincingly assess whether controlling for factors distinct to each of the votes (such as campaign finance contributions from specific agricultural sectors) is sufficient to rule out alternative explanations. However, it is hardly possible to scale this approach to a multitude of roll call votes and policy areas.[10] The applicability of this approach is thus conditional on the availability of concise prior information on vote trading taking place in a specific setting.

Cohen and Malloy (2014) select $\mathbf{V}$ and $\mathbf{X}$ in order to exploit a specific setting that is theoretically favorable for vote trading: Senators sharing personal ties (for example, being in the same University alumni organization) and their incentives to support economic policies that favor important industries in their states. The selection of a subset of $\mathbf{V}$ is then driven by whether a policy k is relevant to either i's or j's state.[11] While the selection of both bills and Senators to identify vote trading is very intuitive and theoretically straightforward, the test does not explicitly take into account that $(i, k) \in \mathbf{V}$ *and* $(j, l) \in \mathbf{V}$ are due to a trade, as the model does not explicitly consider which specific votes were traded for which other votes.[12]

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[10]This is clearly pointed out by Stratmann (1992, p. 1164): "To test for the presence of logrolling using [this method], one must be able to identify the particular issues on which trading takes place. [...] Thousands of votes are taken during a session of Congress, many of which involve no logrolling. Moreover, the potential patterns of trades are limitless."

[11]Whether or not such trades take place in the context of other bills and/or between senators that do not have ties through an alumni organization remains open.

[12]Thus, while the empirical results clearly indicate that Senators seem to exchange favors in the context of bills that are very relevant to specific states' major industries, the test cannot reject the null hypothesis that favors are always rewarded in another currency. That is, the test cannot distinguish the case of observing only $(i, k) \in \mathbf{V}$ due to other favors than vote trading (e.g., campaign finance support) and observing $(i, k) \in \mathbf{V}$

Clearly, the scant existing approaches to estimating vote trading have limitations with respect to scalability or with regards to discarding relevant alternative hypotheses. We address these issues explicitly by bringing insight from Network Science and Computational Social Science into the study of vote trading. In the next section, we present our method in detail.

3   Methodology

There are three main components or modules in the methodology: (i) predicting legislators' voting behavior and detecting deviations from these predictions; (ii) relating legislators to the beneficiaries (also legislators) of these deviations; and (iii) measuring reciprocity. This modular design gives enough flexibility to incorporate different methods that may be better suited to study particular legislatures (in different institutional settings).[13] The general logic of our procedure consists of predicting the outcome of each individual vote, given the observable factors that might explain voting decisions in general ($\mathbf{X}$). When the observed outcome is considerably different from the prediction, we say that the legislator is deviating from his or her preferred policy position.[14] We then relate each deviation to those legislators who have a strong attitude toward the passage of the bill. We build such relations through a directed network of legislators where an edge indicates that the sending node deviated his or her vote in a bill where the receiving node has a strong attitude for its passage. In other words, we say that the sender is the 'deviator' and the receiver is the 'beneficiary'. The network of deviators and beneficiaries is called the *directed-deviation network* (DDN), and

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and $(j, l) \in \mathbf{V}$ due to vote trading.

[13]For example, the approximation of preferred policy positions to predict voting behavior in step (i) can, in theory, be done through the computation of ideal points (Poole and Rosenthal, 1985), through text mining of discourses or manifestos (Budge et al., 1987), or through legislator characteristics and constituency characteristics (Peltzman, 1984).

[14]Of course, as long as we cannot be sure that we include all potential explanatory factors (unrelated to trading) to predict voting decisions, there are other potential reasons for such deviations than vote trading. Therefore deviations cannot be considered trades yet, however, they are a first indicator for the potential presence of trades. Importantly, by controlling for more factors in the step of predicting voting decisions, our method can be arbitrarily refined in order to discriminate between different types of deviations (and potentially different types of motivations to trade votes).

we use it to infer the prevalence of vote trading by measuring its level of reciprocity. This means that, when deviators and beneficiaries draw edges in opposite directions, an aggregate pattern of reciprocity emerges. We construct a logrolling index from such patterns. If the index is higher than what we would expect under a null hypothesis, we interpret it as a situation in which vote trading is prevalent. As with any aggregate measure, our approach does not allow us to identify exactly which individual votes are being traded. However, the method allows extracting a network of potential vote traders which we term the *vote-trading network* (VTN). The VTN facilitates the micro-level study of vote trading because it results from discovering patterns that fulfill our theoretical considerations about vote trading. Without relying on ex-ante knowledge about potential situations of vote trading, the VTN represents a scalable way to find the prevalence of voting patterns that are consistent with vote-trading activity. In general, our logrolling index provides a normalized measure to compare the prevalence and evolution of vote trading across different settings (different legislative assemblies, different time frames, etc.).

Figure 1 presents a sketch of the methodology. There are three bills labeled according to their political position: L for left and R for right. There are two left-wing (L1 and L2) and two right-wing (R1 and R2) legislators who vote on these bills. Examining figure 1 from left to right, the first panel from left to right illustrates the structure of the roll call data. For example, L1 voted Yes on all bills (thumbs up symbol). Since L1 was expected to vote No in the R bill (thumbs down symbol), this vote is considered a deviation (dashed line). In the second panel, legislators signal strong preferences toward specific bills (badge symbol). For example, L1 only signals the top L bill, but not the middle L. By joining the roll call data with the signaling data, we construct the DDN, shown in the third panel. In this step, we compute the logrolling index. Finally, we extract the reciprocal part of the DDN in order to obtain the VTN in the fourth panel. For example, L1 deviates on the bill signaled by R2 (L1→R2) but R2 never deviates, so the latter does not reciprocate L1's deviation. Hence, the resulting VTN only contains the reciprocation between L2 and R1, and L1 with R1.

Figure 1: Sketch of the methodology.

1 - Voting Data   2 - Signaling Data   3 - Directed-Deviation Network (DDN)   4 - Vote-Trading Network (VTN)
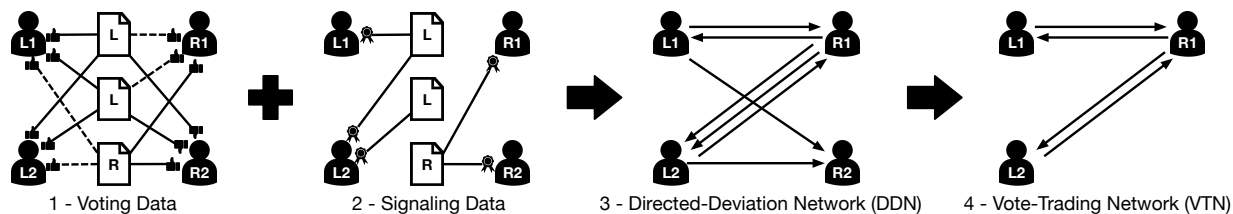
In the remainder of this section, we elaborate on the details of each module. For this, we introduce some notation to describe the structure of the data prepared for an application of our method. First, we encode the roll call data into a matrix $\mathbb{V}$. Let entry $\mathbb{V}_{ik} = 1$ if legislator i voted Yes in roll call k and $\mathbb{V}_{ik} = 0$ if he or she voted No. $\mathbb{V}$ has dimension $N \times K$, where N is the number of legislators and K the number of roll calls. The data capturing legislators' preference signals (e.g., speeches, sponsorships, press releases, etc.) is encoded in a matrix $\mathbb{S}$ with dimensions $N \times K$, where entry $\mathbb{S}_{ik} = 1$ if legislator i signals strong preferences for the bill voted on in roll call k and $\mathbb{S}_{ik} = 0$ otherwise.

3.1   Detecting Deviations

Suppose we are given the probability that legislator i votes Yes in roll call k. For all legislators and roll calls, we encode this information in a matrix $\mathbb{Q}$. Naturally these probabilities reflect the information in data set $\mathbf{X}$, which helps to explain legislators' voting behavior in the absence of vote trading. Given $\mathbf{X}$, $\mathbb{Q}$ could be estimated in various ways, for example, a probit model. Whichever estimation strategy the researcher chooses, it must fulfill one requirement: it should allow predicting the outcome probability of each individual vote. Each individual prediction is an entry in matrix $\mathbb{Q}$, which, consequently, has dimension $N \times K$.[15]

The usefulness of $\mathbb{Q}$ becomes evident when we concentrate on $\mathbb{V}^s$, a sub-matrix of $\mathbb{V}$ containing only roll calls with narrow outcomes. First, let m^s denote the maximum number

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[15]A way to account for uncertainty is constructing a matrix $\mathbb{E}$ where each entry is the standard error of $\mathbb{Q}_{ik}$. Then, instead of applying the method to $\mathbb{Q}$, we can use random matrices with entries drawn from $[\mathbb{Q}_{ik} - \mathbb{E}_{ik}, \mathbb{Q}_{ik} + \mathbb{E}_{ik}]$.

of votes necessary to consider a vote outcome narrow. Second, let $\tau \in [0,1]$ denote a probability threshold to consider that a Yes vote is a deviation. For example, if $\tau = 0.1$ and all roll calls k were passed or turned down by m^s or fewer votes, then all entries $\mathbb{Q}_{ik} \leq 0.1$ given that $\mathbb{V}_{ik}^s = 1$ are considered deviations.[16] We denote the Yes votes that are deviations as $\mathbb{V}_{ik}^d$. This means that legislator i is deviating because he or she was expected with 90% probability to vote No in roll call k, and yet the observed vote was a Yes. Intuitively, the deviation threshold τ should be such that No votes are expected with more probability. Otherwise, it would not make sense to consider deviations where we expect a Yes more often than a No. Once deviations are detected, we want to know who are the potential beneficiaries from the passage of their corresponding bills.

3.2 Networks and Reciprocity

We have reached the most distinctive part of the methodology. Here, we introduce insights from network science in order to address the issue of discriminating between explicit vote trading and deviations that may be paid in "other currencies" (and those that might be simply due to alternative explanations). Furthermore, by thinking about vote trading in a legislature as a network of deviations, we can construct the logrolling index and devise a statistical test.

Let us begin with an empty graph represented by the $N \times N$ adjacency matrix $\mathbb{W}$. Consider the legislators' signals encoded in matrix $\mathbb{S}$. Whenever we detect a deviation, we say that the legislators signaling strong preferences for the corresponding bill are its beneficiaries. More specifically, if legislator i deviates in roll call k while legislator j signals strong preferences for the bill voted in roll call k, we say that i is deviating in benefit of j. Therefore $\mathbb{W}_{ij} = 1$. More generally, $\mathbb{W}_{ij} = n$ indicates that legislator i deviated in n different

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[16]Of course, No votes can also be considered deviations, for example, if a legislator attempts to block the passage of a bill he or she is expected to be in favor of. However, we do not explicitly discuss them in this paper because identifying the potential beneficiaries of a No in empirical data is not as straightforward as with a Yes vote. Note that the procedure could be equally applied to a situation where $\mathbb{S}$ encodes who has strong attitudes *against* specific bills.

roll calls associated to bills that were strongly favored by j. Doing this for all deviations produces a directed network with weights encoded in $\mathbb{W}$. We call this the directed-deviation network (DDN).

In order to consider a vote traded, it has to have a corresponding deviation in some part of $\mathbb{V}$, i.e., it has to be reciprocated. In the DDN, some deviations may be reciprocated and some others not. Therefore, by evaluating the systemic level of reciprocity in the DDN we can detect if there is a systematic pattern of reciprocal deviations that we would not expect under a null hypothesis of 'no vote trading'. We achieve this by adapting a method to measure reciprocity in directed, weighted networks to the context of vote trading. In particular, we build on Garlaschelli and Loffredo (2004); Squartini et al. (2013). It is important to briefly elaborate on this literature in order to understand its limitations in our context and motivate the construction of our logrolling index.

Consider the adjacency matrix $\mathbb{W}$ of the DDN. The level of reciprocity between legislators i and j is given by $w_{ij}^{\leftrightarrow} = \min\left[\mathbb{W}_{ij}, \mathbb{W}_{ji}\right] = w_{ji}^{\leftrightarrow}$. Counting over all legislators, the reciprocity estimator is

$$r = \frac{\sum_i \sum_j w_{ij}^{\leftrightarrow}}{\sum_i \sum_j \mathbb{W}_{ij}}, \tag{1}$$

which is the ratio of reciprocal edges to total number of edges.

Equation 1 is quite intuitive, but in the context of vote trading, it may be problematic when the signaling patterns generate false positives. To better understand these situations, consider four legislators: i, j, x and y. Suppose that two roll calls are voted in the floor: k and l. Let us assume that i and j trade their votes in k and l, while independently at the other side of the floor, x and y trade their votes in the same roll calls. In this situation, there are four votes being traded. Suppose that the records show $\mathbb{S}_{il} = 1$, $\mathbb{S}_{jk} = 1$, $\mathbb{S}_{xk} = 1$ and $\mathbb{S}_{yl} = 1$; and we detect deviations (i, k), (j, l), (x, l), (y, k). Without knowing which ones are the true trades, we obtain a DDN with reciprocal links (i, j), (x, y), (i, x) and (j, y), totalling eight, double the true number of votes traded. Figure 2 provides a graphical illustration of

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this bias.

Figure 2: Bias due to (co-)sponsorship structure.

independent trades
among two coalitions (four links)

inflation bias due to
(co)sponsorship (eight links)
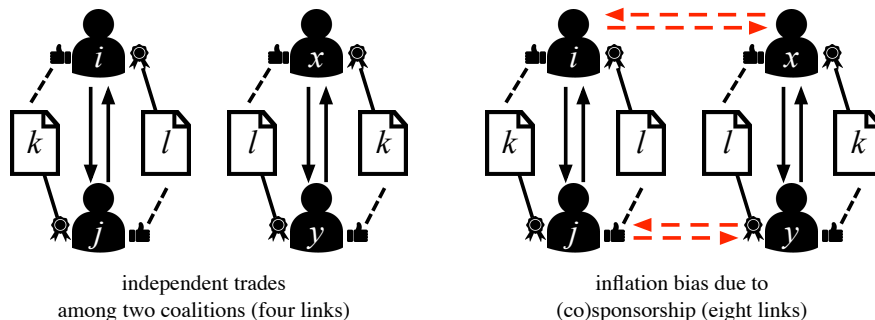
Left side: true trades of votes. Right side: inflated observation of reciprocal deviations.

Another bias originates in the unlikely recyclability of votes. This means that, when a legislator trades a vote, it is highly unlikely that he or she would reuse that vote on a different deal. Although this situation might be plausible in the case of coalition trading, the cognitive and coordination costs (and the uncertainty of the agenda) for a single individual would be prohibitive.

Therefore, instead of r, we propose the usage of a custom estimator ϕ. This estimator could be a function of r or other source of information. Its selection will depend on the structure of the data at hand. In the next section, we will show how we can estimate ϕ via Monte Carlo simulation, exploiting the structure of $\mathbb{V}$, $\mathbb{Q}$, and $\mathbb{S}$.

Once ϕ has been determined, we need to assess whether its level is statistically significant or not. For this, we test the alternative hypothesis that deviations are the result of random errors[17] rather than intentional behavior to benefit other legislators. More specifically, each deviation in $\mathbb{V}$ can be considered an independent decision that took place for reasons that have nothing to do with reciprocating other deviations. In this case, we can model them as independent Bernoulli random variables with success probabilities given by $\mathbb{Q}$.[18] This

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[17]This is a close analogy to the null hypothesis assumed in the previous econometric approaches discussed above.

[18]Note that this automatically takes into account the individual voting behavior characteristics of legislators (i.e., who deviates more or less often, also depending on the factors that are considered to compute $\mathbb{Q}$ in the first place). In the same vein, the null is taking into account who signals what preferences for bills (captured in $\mathbb{S}$).

hypothesis implies that the observed level of ϕ may be just a pattern caused by these unrelated events. We simulate these random variables and generate a null DDN, from which we compute its corresponding estimator ϕ_0. Performing this procedure T times allows us to compute the null expected reciprocity estimator

$$\bar{\phi}_0 = \frac{1}{T} \sum_i^T \phi_{0,i}. \tag{2}$$

Finally, we construct the logrolling index[19]

$$\ell = \frac{\phi - \bar{\phi}_0}{1 - \bar{\phi}_0}. \tag{3}$$

This index has an interpretation similar to a correlation coefficient. If ℓ is positive, it means that the DDN has more systemic reciprocity than what we would expect from the null. The higher the index, the more reciprocity. In order to establish if a positive index is statistically significant, we can generate a sample of null indices. This is done by bootstrapping the ensemble of null reciprocity estimators, in order to obtain a new expectation $\bar{\phi}_0'$ under the null. Then, we compute a new logrolling index ℓ'. By repeating this, we construct a sample of logrolling indices. The resulting distribution is consistent with the idea of reciprocity being generated by chance in a world where there is no vote trading. If zero lies inside this interval, we say that the level of reciprocity in the network is not significant.

Finally, if we find a significant ℓ for given parameters m^s and τ, we can extract the reciprocal edges in the DDN. These edges represent votes that were potentially trades, and can be extremely valuable to filter data in the search of specific cases of vote trading. Altogether, these filtered edges and nodes constitute the vote-trading network (VTN). The VTN provides us with information about the micro-structure of vote trading, and other characteristics (temporal composition, frequent traders, important dyads, etc.). We will present some

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[19]This index construction is due to Garlaschelli and Loffredo (2004) and it offers several advantages over alternative specifications (Katz and Powell, 1955; Achuthan et al., 1982; Wang et al., 2013; Akoglu et al., 2012).

results regarding an empirical VTN in section 5. However, it is important to first understand the performance and limitations of the methodology. For this, we conduct a simulation in the next section.

4 Simulation Study

In the context of vote trading, most mathematical theories are extremely abstract or highly specific game-theoretic models. This limits our ability to generate a vote trading 'training data set'. Therefore, our strategy is to build a computational model that takes into account our theoretical pillars and allows us to control the extent of vote trading. Note that other empirical studies of vote trading are subject to the same limitation, and, to the extent of our knowledge, nobody has previously attempted to validate a method aimed at detecting vote trading based on simulated data with a controlled level of vote trading. Therefore, this section also provides a way to subject other empirical frameworks to the same evaluation.

Our computational model generates synthetic micro-data in the form of $\mathbb{V}$, $\mathbb{Q}$, and $\mathbb{S}$. It assumes as little as possible about the mechanisms that generate vote trading. In fact, it first generates matrices from random voting behavior, and then allows us to manipulate these data through an algorithm that induces vote trading. Through this approach, we do not need to assume anything about the motivations of why agents engage in vote trading. Instead, we only need to introduce assumptions about types of agents and their consistency when signaling (their policy preferences), which is common in agent-computing models (Epstein and Axtell, 1996; Laver and Sergenti, 2011). In fact, our strongest assumption is having two types of agents. Therefore, we will study the method under the scope of bipartisan institutions that could generate $\mathbb{V}$, $\mathbb{Q}$, and $\mathbb{S}$. Once we have introduced the method, we perform Monte Carlo simulations and performance evaluation.

4.1 A Computational Model of the Legislative Process

Assume a population of N agents voting in K roll calls. There are two types of agents: *left* and *right*. Every agent has probability δ of being either type. Hence $\delta \approx 1/2$, assumes that, on average, no type dominates. Each bill has an affinity $\alpha_k \sim U(0,1)$ to each type of agent, indicating type left when $\alpha_k = 0$ and type right when $\alpha_k = 1$. Agents are more likely to vote Yes in bills that are closer to their types. This likelihood depends on the distance between the bill's affinity, the agent's type and a behavioral factor $1 - \beta_i^b$, where $\beta_i \sim U(0,1)$ and $b \geq 1$. This factor captures the "openness" of agents to bills that are less affine to their types. Parameter β_i provides behavioral heterogeneity, while b allows to control the average extent of such openness. In other words, we can control the number of narrow vote outcomes through b. Formally, agent i's vote in roll call k is determined by function

$$
\mathbb{V}_{ik} = \begin{cases} \text{no} & \text{with probability } \alpha_k(1 - \beta_i^b) \text{ for } \textit{left} \text{ and } (1 - \alpha_k)(1 - \beta_i^b) \text{ for } \textit{right} \\ \text{yes} & \text{with probability } 1 - \alpha_k(1 - \beta_i^b) \text{ for } \textit{left} \text{ and } 1 - (1 - \alpha_k)(1 - \beta_i^b) \text{ for } \textit{left} \end{cases} \tag{4}
$$

Note that, in this form, the model tends to produce passage outcomes, which are consistent with real-world legislatures. By modifying the behavioral factor, we can easily produce legislatures with a tendency for blockage.

In order to produce signals that are consistent with the agents' behavior, we assume that legislators reveal their preference in specific bills according to the function

$$
\mathbb{S}_{ik} = \begin{cases} \text{no} & \text{with probability } 1 - \gamma[1 - \alpha_k(1 - \beta_i^b)] \text{ for } \textit{left} \text{ and } 1 - \gamma[1 - (1 - \alpha_k)(1 - \beta_i^b)] \text{ for } \textit{right} \\ \text{yes} & \text{with probability } \gamma[1 - \alpha_k(1 - \beta_i^b)] \text{ for } \textit{left} \text{ and } \gamma[1 - (1 - \alpha_k)(1 - \beta_i^b)] \text{ for } \textit{left} \end{cases} ,
$$
$$\tag{5}$$

where γ is a parameter that controls the overall amount of signaling.

So far, this model can generate matrices $\mathbb{V}$, $\mathbb{Q}$, and $\mathbb{S}$ from purely random behavior.

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By tuning two parameters (b and γ), it allows control of the amount of narrow outcomes and signaling. These properties of $\mathbb{V}$ and $\mathbb{S}$ are very important because they have a direct incidence in ℓ. For example, if we generate a highly dense matrix $\mathbb{S}$, the method would construct highly reciprocal DDNs just from the fact that there is a lot of signaling. In this case, the method would likely estimate a significant ℓ, even in the absence of vote trading. Therefore, by controlling the density of $\mathbb{V}$ and $\mathbb{S}$, we can assess whether our method would be useful to estimate vote trading in specific data sets.

Figure 3 shows the behavior of the model for different levels of parameters b and γ, while randomizing δ. The left panel shows the fraction of roll calls that had a narrow outcome for multiple realizations of the model, and conditional on b. As shown, this parameter allows control of the number of narrow outcomes, which is one of the necessary conditions to elicit vote trading. The panel on the right shows the number of votes in which each legislator signaled strong preferences for specific bills. Clearly, this quantity is highly sensitive to γ, which allows control of the overall level of signaling.

Figure 3: Parameter exploration of computational model.
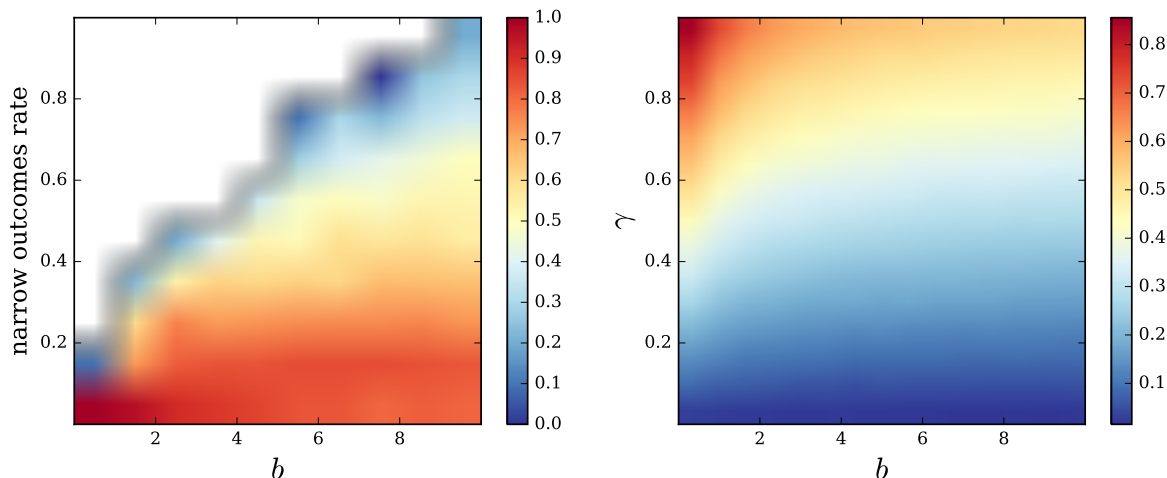
The left heat map is a histogram with frequencies normalized as ratios of logarithms $(\log x_i / \sum_j \log x_j)$, This is because most of the Monte Carlo simulations produce outcomes that fall in the lower left corner of the plot. The right panel shows the quantity $(\sum_i \sum_k \mathbb{S}_{ik})/(NK)$ as a function of b and γ.

The data generated from this model does not contain vote trading. Therefore, it serves

as a benchmark for measuring false positives. In order to induce vote trading, we could introduce different theoretical considerations about the various motivations of why pairs of legislators engage in repeated reciprocal deviations. This, however, would require multiple additional parameters; reduce our control over the density of $\mathbb{V}$ and $\mathbb{S}$ and constrain the data generating process to a particular theory; and hence, restrict the matrices with vote trading that we can generate. In order to overcome these limitations, we first generate the synthetic matrices without logrolling, and then, induce trades by manipulating the entries of $\mathbb{V}$ and $\mathbb{S}$ such that their densities are preserved. The next section presents the algorithm that we develop for this purpose.

4.2   Reciprocity Algorithm

Let $\mathbb{V}$, $\mathbb{S}$ and $\mathbb{Q}$ be synthetic matrices generated by the computational model. We introduce vote trading by randomly selecting pairs of votes that could qualify as trades. An example of such pairs is the tuple $((i, k), (j, l))$, where i signals in l and is expected to vote Yes in k with probability $\mathbb{Q}_{ik} \leq \tau$; in return, j signals in k and is expected to vote Yes in j with probability $\mathbb{Q}_{jl} \leq \tau$. In addition, both roll calls have to have a narrow outcome. Let us call these tuples 'potential trades'. A potential trade implies that its votes are not necessarily Yes votes in $\mathbb{V}$. However, if we turn them into Yes votes, we can induce logrolling by selecting them and turning other Yes votes into No votes in order to preserve the density of $\mathbb{V}$ and its narrow outcomes. The same logic applies to the signaling matrix $\mathbb{S}$.

Selecting potential votes and their counterparts (to preserve matrix density) is done at random. Therefore, our approach is to build samples of potential trades, and induce as much trade as possible.[20] The reason why we say as much as possible is because not every vote in a sample of potential trades can become a trade. For example, if we induce a trade through the tuple $((i, k), (j, l))$, then we cannot induce anymore through the tuple $((i, k), (j, x))$ because using (i, k) again would violate the non-recyclability assumption. Therefore, the sample

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[20]Note that constructing the list of all potential trades can require too much computation for large data sets.

size provides some degree of control over the amount of vote trading that we can induce. Algorithm 1 provides the pseudocode for inducing vote trading. By providing a sample size s, we can induce different levels of vote trading. Before evaluating the performance of the index, let us get back to the reciprocity estimator ϕ which has not been specified so far.

Algorithm 1: Inducing vote trading.

Input: $\mathbb{V}$, $\mathbb{Q}$, $\mathbb{S}$, s

1 build set **L** of s potential trades;
2 build set **V** of Yes votes that happened in narrow roll calls;
3 build set **S** of signals that happened in narrow roll calls;
4 **while** $|\mathbf{L}| > 0$ *and* $|\mathbf{N}| > 0$ **do**
5   randomly pick a potential trade $((i,k),(j,l))$ and remove it from **L**;
6   **if N** *has Yes votes different from* $((i,k),(j,l))$ **then**
7        $\mathbb{V}_{ik} = 1$;
8        $\mathbb{V}_{jl} = 1$;
9   randomly take two votes (a,k) and (d,l) from **N**;
10       $\mathbb{V}_{ak} = 0$;
11       $\mathbb{V}_{dl} = 0$;
12   remove (a,k) and (d,l) from **N**;
13   remove all potential trades from **N** that involve (i,k) or (j,l);
14   **if** $(i,l) \in \mathbf{S}$ **then**
15   remove (i,l) from **S**;
16   **else**
17         $\mathbb{S}_{ik} = 1$;
18   randomly pick a signal (x,y) from **S**;
19         $\mathbb{S}_{xy} = 0$;
20   remove (x,y) from **S**;
21   **if** $(j,k) \in \mathbf{S}$ **then**
22   remove (j,k) from **S**;
23   **else**
24         $\mathbb{S}_{jl} = 1$;
25   randomly pick a signal (w,z) from **S**;
26         $\mathbb{S}_{wz} = 0$;
27   remove (w,z) from **S**;

4.3 Reciprocity Estimator

Until now we have indicated that the reciprocity estimator conventionally used in the literature of complex networks can be problematic in the study of vote trading. How problematic it is depends on the particular structure of $\mathbb{V}$ and $\mathbb{S}$. Therefore it is convenient to use a custom estimator ϕ for each application. Here, we show an example of such an estimator, using the data generated from our computational model. The following procedure is applicable to any roll call and signaling data sets.

Suppose we are given a voting matrix $\mathbb{V}$ and the corresponding signaling matrix $\mathbb{S}$. Like in the estimation of the logrolling index, we will focus on those entries of $\mathbb{V}$ that are deviations in narrow outcomes. With this information, we can construct a set of all potential trades, and filter those that comply with the non-recyclability assumption. In order to create more variation from $\mathbb{V}$, we can apply this procedure for arbitrarily smaller subsets of potential trades. For each set of potential trades, we call the resulting number of filtered elements the 'true trades'. This procedure allows us to construct a sample of alternative voting matrices on which we can compute $w^{\leftrightarrow}$. Since we know the true number of trades T, this procedure allows us to compute the difference between $w^{\leftrightarrow}$ and T, which is informative of the bias produced by a standard reciprocity estimator.

Algorithm 2 provides the pseudocode to generate a sample of alternative matrices $\mathbb{V}'$ that can be used to find a suitable estimator of the level of reciprocity in the DDN. For large enough data sets, it is possible to create a rich sample of DDNs. We perform Monte Carlo simulations where we randomize the following parameters from the computational model and the methodology: $b \in [1, 10]$, $\gamma \in (0, 1)$, $\delta \in (0, 1)$, $m^s \in [1, 10]$ and $\tau \in (0, 1/2)$. For each of these networks, we compute $w^{\leftrightarrow}$ and the number of true votes traded T.

Figure 4 shows the results. The left panel contrasts the conventional reciprocity estimator $w^{\leftrightarrow}$ against the true number of votes traded T (the 45 degree line). Clearly, $w^{\leftrightarrow}$ overestimates the amount of reciprocity in most of the simulations (note the logarithmic scale). This is due to the fact that some legislators signal in many roll calls, even for sparse $\mathbb{S}$. The challenge,

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Algorithm 2: Generating variation for $\mathbb{V}$.

    Input: $\mathbb{V}$, $\mathbb{Q}$, $\mathbb{S}$, h

1 build set $\mathbf{M}$ of h potential trades;

2 build a matrix $\mathbb{V}'$, which is a copy of $\mathbb{V}$;

3 initialize $T = 0$;

4 **for** *legislator i* **do**

5   **for** *roll call k* **do**

6   **if** (i, k) *is in an element of* $\mathbf{M}$ **then**

7 $\mathbb{V}'_{ik} = 0$;

8 **while** $|\mathbf{M}| > 0$ **do**

9   pick an element $((i, k), (j, l))$ from $\mathbf{M}$ at random;

10   **if** $((i, k), (j, l))$ *does not violate non-recyclability* **then**

11 $\mathbb{V}'_{ik} = 1$;

12 $\mathbb{V}'_{jl} = 1$;

13   increase T by 2;

14   remove $((i, k), (j, l))$ from $\mathbf{M}$;

15 compute estimators;

---

therefore, is to use the information available from the methodology to construct an estimator Φ that fits the 45 degree line as close as possible. More specifically, we will aim to minimize the root-mean-square error (RMSE)

$$\text{RMSE} = \frac{1}{n} \left[\sum_{i}^{n} (T_i - \Phi_i)^2 \right]^{\frac{1}{2}}, \tag{6}$$

where n is the number of data points.

The right panel in Figure 4 shows the performance of different estimators with a free parameter c (normalized in the plot for illustration purposes). For example, a simple estimator consists of re-scaling $w^{\leftrightarrow}$ by a factor c, so that $\Phi = cw^{\leftrightarrow}$. Alternatively, we can also specify a non-linear relationship $\Phi = (w^{\leftrightarrow})^c$. The performance of each estimator depends on the structure of $\mathbb{V}$ and $\mathbb{S}$, so the results that we obtain in this example might not be optimal in a specific data set. Hence, algorithm 2 should be always used in order to find its most suitable estimator. For our synthetic data, the non-linear estimator $\Phi = (w^{\leftrightarrow})^c$ provides the best performance, so the logrolling index is built using $\phi = (w^{\leftrightarrow})^c / \sum_i \sum_j \mathbb{W}_{ij}$.

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Figure 4: Finding custom estimator ϕ.
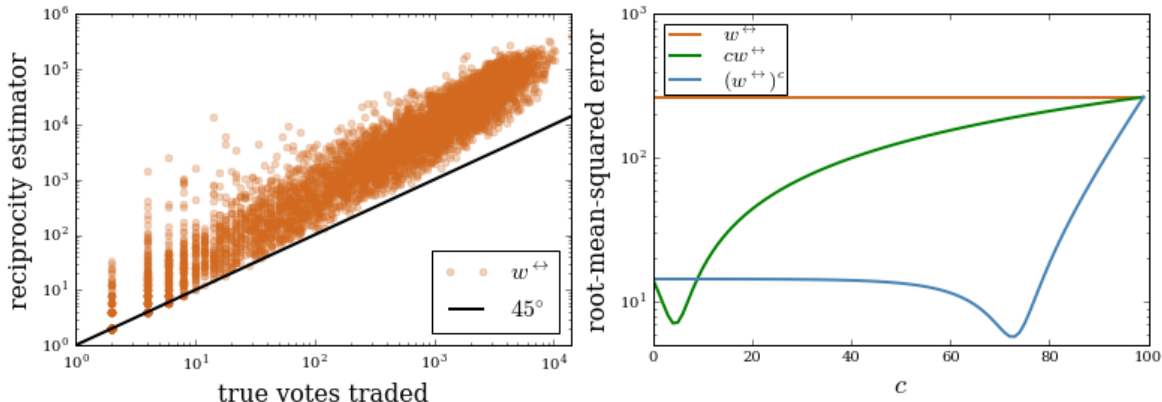
Left panel: each dot is a simulation's estimated level of reciprocity ($w^{\leftrightarrow}$). Their distance from the 45° indicates the size of the bias (note the logarithmic scale). Right panel: Bias size as a function of parameter c.

4.4 Performance and Validation

The next step is to compute the logrolling index for our synthetic data sets. We sample different synthetic matrices via Monte Carlo simulations of our computational model. This allows us to identify the conditions under which the logrolling index yields significant levels when there is vote trading, and non-significant when there is no trading. In fact, we show that the method works well under matrices $\mathbb{V}$ and $\mathbb{S}$ that are sparse and thus consistent with roll call data from the U.S. Congress (see section 5 for details).

We evaluate the method's performance in terms of the VTN. That is, we compare the similarity between the edges obtained from the estimated VTN and the ones induced via algorithm 1, such that $Q_{ik} \leq \tau$ for all (j, k) in the set of true trades. Let us denote the number of true positive edges as TP, true negatives as TN, false positives as FP and false negatives as FN. We employ Matthew's correlation coefficient

$$MCC = \frac{(TP \times TN) - (FP \times FN)}{[(TP + FP) \times (TP + FN) \times (TN + FP) \times (TN + FN)]^{1/2}}. \tag{7}$$

The MCC behaves like other correlation coefficients. $MCC = 1$ means a perfect

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prediction, $MCC = 0$ means that the method performs no better than randomness and $MCC = -1$ means that the prediction is entirely the opposite to the data. Let us consider a high performing outcome if its $MCC \geq 0.5$ (at least 50% performance).

Figure 5 presents our main result: the method exhibits high performance if matrix $\mathbb{V}$ has low density of narrow outcomes and $\mathbb{S}$ is sparse. The left panel shows a negative relationship between density in narrow outcomes and the MCC. Let n denote the number of narrow outcomes in roll calls. We say that n/K is the density of narrow outcomes. A similar relationship between signaling density and performance is shown in the middle panel. In this case, $\sum_k \sum_i \mathbb{S}_{ik}/(NK)$ measures the density of signals. The right pane shows performance as a function of both types of density. Here, 'joint density' is the product between both types: $\frac{n \sum_k \sum_i \mathbb{S}_{ik}}{NK^2}$.

Clearly, high-performing outcomes are those with low density.[21] This result has different implications about the strengths and limitation of the method. First, if the data at hand is sparse, the method is well-suited to analyze it. Second, if the data has low density in narrow outcomes but high density in signals, it is possible to improve performance by pruning $\mathbb{S}$. In other words, a highly dense matrix $\mathbb{S}$ implies excess of information that exacerbates the inflation bias and, hence, generates more false positives. Therefore, carefully choosing the signals that matter for vote trading is critical. Third, if the data have sparse matrices $\mathbb{V}$ and $\mathbb{S}$, a positive index is a strong indication of reciprocal deviations. Arguably, vote trading is the most plausible explanation of these patterns, given that the method identifies them by taking into account a set of theoretical considerations that are consistent with vote-trading behavior. Any alternative hypothesis would need to explain why these patterns are systematic, why they happen in narrow outcomes, and why the votes involved are deviations from the expected behavior of each legislator.[22]

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[21]These results are significant in the sense that the high-performing outcomes are statistically significant.

[22]Note that these conditions for an alternative explanation are very similar to the conditions regarding the previously suggested approaches to empirically tackle vote trading discussed above. However, one important additional condition idiosyncratic to the method presented here is the necessity that an alternative explanation must account for systematic reciprocity in deviations.

Figure 5: Density and Performance.
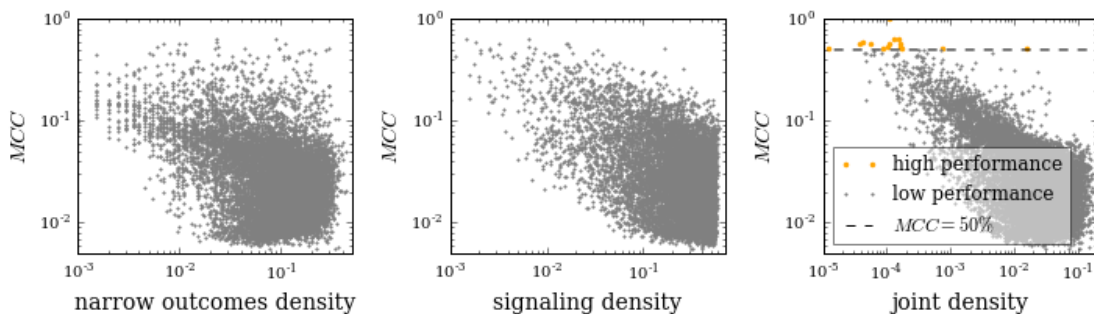
Each dot corresponds to an individual Monte Carlo simulation.

Figure 6 shows the distribution of the logrolling index and the free parameters. The left panel shows the kernel densities of ℓ for high-performing and low-performing outcomes respectively. Clearly, high-performing outcomes exhibit higher logrolling indices. Furthermore, most of the distribution covers positive indices, validating our method. The middle panel shows the distribution of the narrow margin m^s. Recall that this parameter was uniformly sampled between 1 and 10. This panel suggests that high-performing outcomes have a different distribution of m^s. In fact, one can expect more high-performing outcomes for narrow margins in the neighborhood of four votes, which is consistent with the narrowness of voting outcomes in other studies. The right panel shows that the distribution of τ under high-performing outcomes is very different from the uniform one used to sample this parameter.

The simulation study has shown us that our method is better suited for data sets with low-density voting and signaling matrices. In such cases, it is able to detect reciprocal deviations with a high performance. Arguably, these deviations are the result of vote trading. Therefore, the next step is to apply the methodology to an empirical data set with densities of the same order of magnitude as the ones where the method proved to perform well. Our data set on the last 40 years of roll call voting in the U.S. House is very much in line with this requirement. The next section presents our empirical application based on this data.

Figure 6: Outcomes and High Performance.
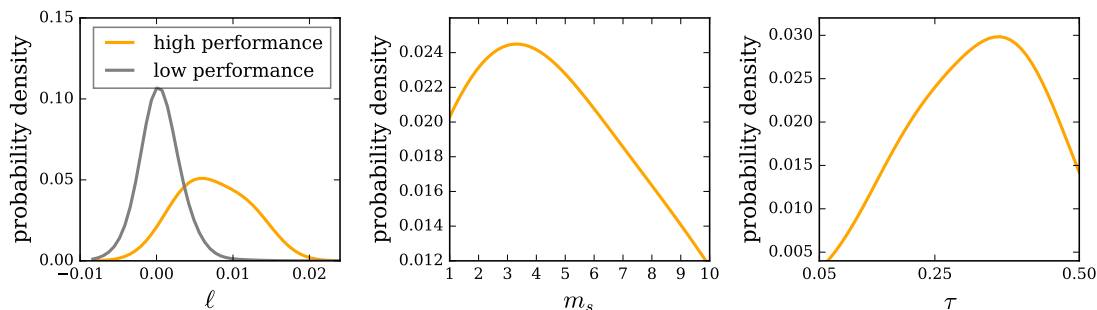
Each panel shows the probability density function of a different statistic, conditional on performance. The left panel shows the unconditional distribution of the logrolling index against the high-performance conditional one. The other two panels are conditional on high-performance (unconditional events follow a uniform distribution).

5 Application

We present an application of the methodology to data on the U.S. House over the last four decades. The results give an overview of the prevalence and variability of vote trading among U.S. Representatives. Although a detailed investigation of the drivers of vote trading in the U.S. House is beyond the scope of this paper, this application can well serve as a basis for future research.

5.1 Data

Our data set includes all roll calls and bill (co-)sponsorship information in the U.S. House from 1973 to 2016. For recent congresses the data are based on the official roll call records published on the U.S. House's website. For less recent congresses, the data is based on Voteview.com. Apart from roll call data, a key aspect of investigating vote trading empirically is constructing the signaling matrix $\mathbb{S}$. Here, we employ information on bill (co-)sponsorship as a proxy for Members of Congress' signaling of strong preferences for specific bills. This data is based on the official bill data published by the Library of Congress (on its website www.thomas.gov). All raw data on roll calls and bill (co-)sponsorship were provided by LLC.

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(2015).[23] We restrict our study to 40 years because (co-)sponsorship dates back to 1973 only. Overall, the data contains every roll call from 1973 to 2016 (from the 93rd to the 114th U.S. Congress) that was not decided unanimously. Table 1 summarizes the characteristics of the data.

Table 1: Data

Variable	Data
Number of legislators (Dem./Ind.) (Rep.)	1,930 (986) (944)
Number of roll calls	20,911
Number of bills	7,757
Number of votes	10,052,647
Number of (co-)sponsors	1,861
Number of (co-)sponsorships	120,142
Average duration in office (years)	10.38
Average number of votes	5,313.24
Average number of (co-)sponsorships	64.56

Encoding the data follows the previous conventions. Let $\mathbb{V}$ denote the voting matrix, such that entry $\mathbb{V}_{ik} = 1$ if legislator i voted Yes in roll call k and $\mathbb{V}_{ik} = 0$ if he or she voted No ($\mathbb{V}_{ik} = -1$ if observation is missing, e.g., if the legislator is not in office). $\mathbb{V}$ has dimension $N \times K$, where N is the number of legislators and K the number of roll calls. Similarly, $\mathbb{S}$ is an $N \times K$ matrix where entry $\mathbb{S}_{ik} = 1$ if legislator i sponsored or co-sponsored the bill voted on in roll call k and $\mathbb{S}_{ik} = 0$ otherwise. We observe a substantial number of roll calls per year (between 100 and over 800) and (co-)sponsorships per year (between 977 and 5,401). Note that the densities of $\mathbb{V}$ and $\mathbb{S}$ observed in the data for the U.S. House for these years are thus very much in line with the densities of the matrices in the simulation study that yield high performance.[24]

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[23]The Civic Impulse LLC. supplies information extracted from official U.S. government websites as Java Script Object Notation (JSON) data files via their webservice (`www.govtrack.us`). All data are freely accessible through GovTrack's application programming interface.

[24]Narrow outcomes density is 10^{-3} and (co-)sponsorship density is 10^{-2}. See Figures A1 and A2 in the Appendix for details on the variation in the number of roll calls and (co-)sponsorships over time.

5.2 Empirical Strategy

In a first step, we aim to determine the legislators' actual or *preferred policy position*, given their voting decisions in the absence of incentives to trade. That is, we predict their usual voting behavior based on ideal point estimates, taking into account that incentives to trade votes are higher the narrower the vote outcome is anticipated to be. With higher incentives to trade votes, roll calls in narrow outcomes do not necessarily reveal legislators' actual political positions on an issue but might also be the result of exchanging favors. Conversely, legislators' observed decisions in roll calls decided by a wide margin are more likely to reveal their preferred policy positions (with a few exceptions such as protest voting Sherif and Sherif 1953; Asch 1956).[25] We thus compute the Members of Congress' ideal points based on those votes that were decided by a wide margin (where incentives to trade are arguably rather not present) and then employ the ideal points to predict their voting decisions in roll calls decided by a narrow margin (where incentives to trade were present).

Formally, each roll call in $\mathbb{V}$ is thus associated with a specific bill incorporating a policy that can be located in a one-dimensional policy space.[26] Each legislator has an ideal point in this policy space, taking into account his or her individual policy preference. Each legislator's Yes or No vote on a particular bill can also be considered as a point in this policy space. In the implied spatial voting model, legislators suffer disutility the further their vote-decision

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[25]For our method to work, inherently preferred policy positions do not need to be perfectly measured. The key point is that they need to be representative of legislators' voting behavior in the absence of incentives to trade votes. Voting out of protest might not be in line with a legislator's true policy preferences. But it is arguably neither in line with vote trading. Recall that the crucial aspect of detecting potential trades based on our method is the systematic reciprocal pattern in deviations, not deviations per se. Thus while factors other than vote trading might cause a deviation, reciprocity is hard to explain by alternative factors (as discussed in the previous sections).

[26]Theoretically, our method can consider an n-dimensional Euclidean policy space. However, as previous empirical work has shown for U.S. national politics (Poole and Rosenthal, 1985, 2007), the first dimension usually captures over 70% of the variation in voting decisions in most congresses (whereas the second dimension only sporadically has much explanatory power). The first dimension, often referred to as 'ideology', is a widely applied measure used to empirically account for a Member of Congress' political position. Increasing the number of dimensions is a trade-off between potentially marking too many votes as deviations, which might introduce noise, and missing the traded votes all together because one of the higher dimensions is likely to already capture vote trading (however, we do not know which one). Finally, it is not necessarily the case that some logrolling activities are inconsistent with a one-dimensional spatial model, though (see Poole and Rosenthal (1997) for a discussion of this).

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is from their overall ideal point (their preferred policy position). If legislators vote in each roll call exclusively according to their individual policy preferences, the resulting $\mathbb{V}$ would be the basis for inferring their ideal points (relative to each other). However, if we accept that their voting decisions might also be motivated by logrolling, $\mathbb{V}$ is the result of both the legislators' honest policy preferences as well as their strategic cooperation. Inferring their ideal points based on $\mathbb{V}$ would, then, not necessarily be representative of their actual policy preferences or, in other words, their preferred policy positions.

In line with Stratmann (1992); Cohen and Malloy (2014) we assume that legislators' perceptions about the narrowness of vote outcomes can be approximated by the actually observed outcomes. Thus the narrower an observed vote outcome $\mathbb{V}_k$, the more likely is the presence of incentives to trade votes. We use this logic to define two matrices: $\mathbb{V}^s$, containing roll calls decided by a *small* margin (narrow vote outcomes), and $\mathbb{V}^b$, containing roll calls decided by a *broad* margin. However, narrow vote outcomes are rare and legislators arranging a trade of votes might prefer to arrange coalitions greater than the minimal winning coalition due to the uncertainty about other legislators' support (Weingast, 1979; Groseclose and Snyder, 1996). Choosing the margin is thus a trade-off between the size of the set of potentially traded votes and the inclusion of only those votes posing strong incentives to trade. Therefore, we need to choose two margins m^s and m^b in order to define $\mathbb{V}^s$ and $\mathbb{V}^b$ respectively.

In our baseline estimations, we choose m^s to be rather small (five votes) because we are interested in those cases where incentives to trade were most apparent, and legislators relied on a few other representatives to actually pass the bill.[27] In contrast, m^b needs to be rather

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[27]What is considered a 'narrow' vote outcome is arguably rather arbitrary. We therefore guide our selection of a threshold value for our baseline specification according to the regression discontinuity design (RDD) literature in the context of legislative voting (Lee, 2001; Lee et al., 2004; Lee and Lemieux, 2010). This literature attempts to exploit narrow election or vote outcomes as random treatments in a quasi-experimental setting. In this, vote or election outcomes that were decided by a margin of up to 5% are considered 'narrow'. Note that in this literature, the selection of a winning margin that is arguably very close to 0 is central to the identification assumptions of the empirical strategy. RDD in the context of voting/election outcomes relies on the idea that vote outcomes just above the passage margin of 50% could just as likely have failed with a very small margin. In contrast, the necessity of narrow outcomes in our context is simply driven by the theoretical arguments outlined above. By choosing a larger m^s one might also capture trades in which

large because, arguably, in these situations there should be fewer incentives to trade votes but more incentives to vote in accordance with one's own political preferences. We thus set m^b such that $\mathbb{V}^b$ only contains votes that were decided by an absolute winning margin of 20% or more.

Based on $\mathbb{V}^b$, we then estimate the legislators' ideal points. In this step, we follow the ideal point estimation procedure suggested by Clinton et al. (2004).[28] We choose this method due to its flexibility with respect to potential extensions of our methodology and the freely available and straightforward implementation of their method provided in Jackman (2008). Importantly, Clinton et al. (2004) show that, when applied to congressional voting data, their baseline implementation of ideal point estimation provides results that are quantitatively very similar to Poole and Rosenthal (1985) but qualitatively superior to both Poole and Rosenthal (1985) and Heckman and Snyder (1997) (as their method can better discriminate among extreme observations). A detailed account of the ideal point estimation can be found in the Appendix A.II.

We use the ideal points $\hat{x}_i$ in order to predict how legislators would vote in each roll call (except for those in which the legislators were not in office). More specifically, we estimate for each roll call the probit model

$$\Pr(\mathbb{V}_{ik}^s = 1 | \hat{x}_i) = \Phi(\hat{x}_i \beta_k). \tag{8}$$

and then employ the estimated coefficient $\hat{\beta}_k$ to compute predictions of legislator i's probability to vote Yes in each individual roll call k, which we then collect in $\mathbb{Q}$.

legislators arranging a deal were not very certain as to whether their trade partners would keep their part of the bargain. The total extent of explicit vote trading (in accordance with our theoretical considerations) may thus well be higher. For the sake of demonstrating our method, we choose a rather conservative value for the margin.

[28]Alternative approaches to estimating the legislators' positions based on roll call data would be the DW-Nominate method suggested by Poole and Rosenthal (1985), the principal component factor analysis suggested by Heckman and Snyder (1997) or the singular value decomposition approach taken by Porter et al. (2005).

5.3 Results

Figure 7 presents the logrolling index ℓ for different values of the deviation threshold τ. The index shows a concave pattern with respect to τ, suggesting that there is a τ that maximizes ℓ. The rise and decay of the index is quite intuitive. On the one hand, a positive relationship between these parameters suggests that, by increasing τ, we gain reciprocal deviations that cannot be explained by the null. On the other hand, a negative relationship means that additional information is detrimental to detect patterns of reciprocity that are not expected under the null. If we think about the method as a filter, then τ controls its bandwidth; too narrow misses important information and too wide allows too much noise.

Figure 7: Logrolling index of the U.S. House: parameter sensitivity.
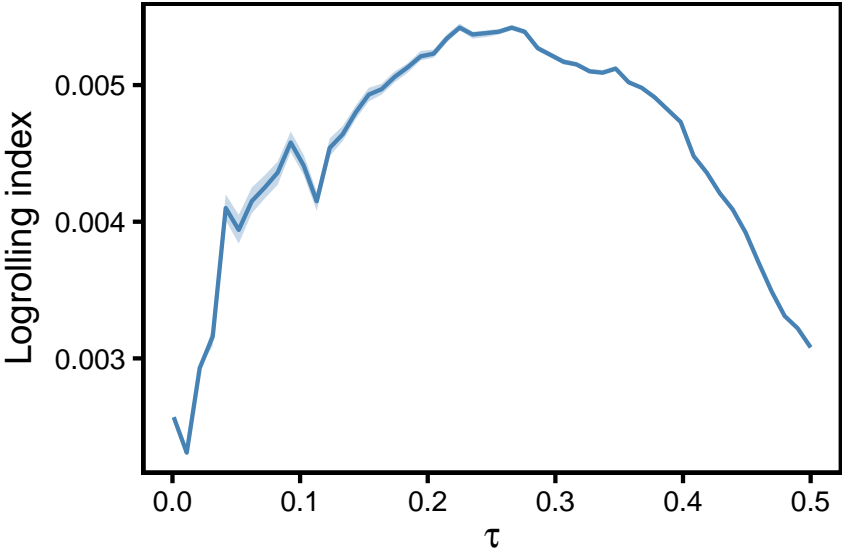
Logrolling index values for the U.S. House (1973-2016) computed for different specifications of the parameter τ. The computed ℓ on the y-axis are then plotted against the corresponding level of τ (on the x-axis) used in the corresponding computation with a 95% confidence band (light blue area around the plotted line). Specifically, the index was computed by generating samples of 10^5 null DDNs for each level of τ. The 95% percentile intervals were built from samples of 1,000 bootstraps for each value of τ.

We present the results corresponding to the τ that yields the maximum logrolling index. Figure 8 shows the VTN of the U.S. House based on our entire observation period (four decades of roll call voting). Each node represents a Member of Congress and each edge

represents a vote-trading relationship between two Members of Congress. On the left-hand side we show the partisan component of the VTN with blue (red) edges indicating trades between Democrats (Republicans). The graph on the right-hand side indicates the bipartisan trades with grey edges. The size of nodes captures the number of potential trades in which a Member of Congress was involved. The graphs reveal that while trades among members of the same party are common, a large part of vote trading is bipartisan.[29]

Figure 8: Vote-trading network of the U.S. House (1973-2016).
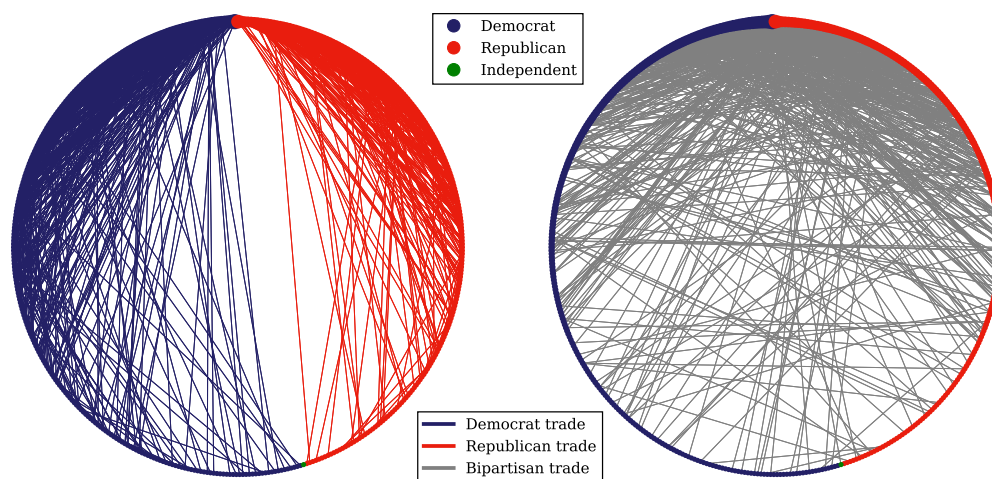
Plots of the VTN for the U.S. House based on roll call data and (co-)sponsorship data covering the years 1973 to 2016. Nodes (Representatives) are colored according to the party of longest affiliation (red: Republican; blue: Democrat; green: Independent). Their sizes are proportional to their degree in the VTN.The left panel depicts all partisan edges (trades) captured in the VTN, colored according to the party affiliation. The panel on the right depicts the bipartisan component of the same VTN (edges colored gray). See section 5.1 for details on the data.

Characterizing the structure of vote trading through a VTN is highly informative for the study of vote trading. For example, the number of connections (or the degree) of a node is indicative of the amount of trades in which he or she was involved. We compute summary statistics that capture stylized facts about logrolling in the U.S. House, derived from the

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[29]Of course, one could further filter the DDN and the VTN by imposing additional constraints such as the time-limits between trades. This, however, would require additional theoretical considerations, specific to the particular context and legislature under study, something beyond the scope of this paper.

extracted VTN. Table 2 shows the summary statistics of the VTN with respect to party affiliation and network characteristics.

Table 2: VTN summary statistics.

Logrolling Index	0.00542 ± 0.00003
Members of Congress	433 (22.77%)
Bills	197
Votes	245
Average degree	21.06
Average trading partners	8.82
Highest degree (partners)	231
Average trades per partnership	2.39
Most central party (votes traded)	Democrat (1756)
Democrat trades	1756
Republican trades	1012
Bipartisan trades	1838

Description of the House VTN with the highest ℓ, covering all potential trades from 1973 to 2016.

Using the VTN, we can extract how vote trading varies over the observation period. Figure 9 presents the total number of edges in each year of our observation period, normalized by the corresponding number of narrow roll calls. In any application of our method focusing on prevalence of vote trading over time, it is imperative to consider that trades might happen over long time spans. That is, a Representative might return a favor months or even years after receiving the favor from his or her trading partner. Thus at the beginning and the end of an observation period it is likely that some trades are missed (because one part of the trade is missing in the data). In order to investigate whether such truncation substantially affects the results, we split our data set in the middle of the observation period (1994/1995) and recompute VTNs for both samples. We then look at how the number of edges per year around the cut point (±7 years) is affected when comparing counts based on the two separate VTNs with those of the whole sample. These additional results are shown in the Appendix (Figure A3). They indicate that, while there is (not surprisingly) a numerical change around the cutting point, the overall picture of the variation of vote trading over time does not change

substantially around the cutting point.[30] The graphic suggests that vote trading is common in most congresses but varies substantially over time. Interestingly, bipartisan cooperation in the form of vote trading seems to be a common feature of the political process during most congresses. In recent congresses, both partisan and bipartisan trading has strongly declined. Importantly, the robustness check discussed above suggests that this finding is not simply an artifact of the truncation of observation periods. Note that the decrease in Republican trades stands in stark contrast to previous congresses with a Republican majority in the House, indicating that inner-party struggles related to the advent of the Tea Party Caucus within the Republican Party might have substantially hindered within-party cooperation (Kabaservice, 2013). A more sophisticated study building on the suggested methodological approach could investigate what factors might explain variation in partisan and bipartisan vote trading over time.

Figure 9: Prevalence of vote trading in the U.S. House between 1973 and 2016.
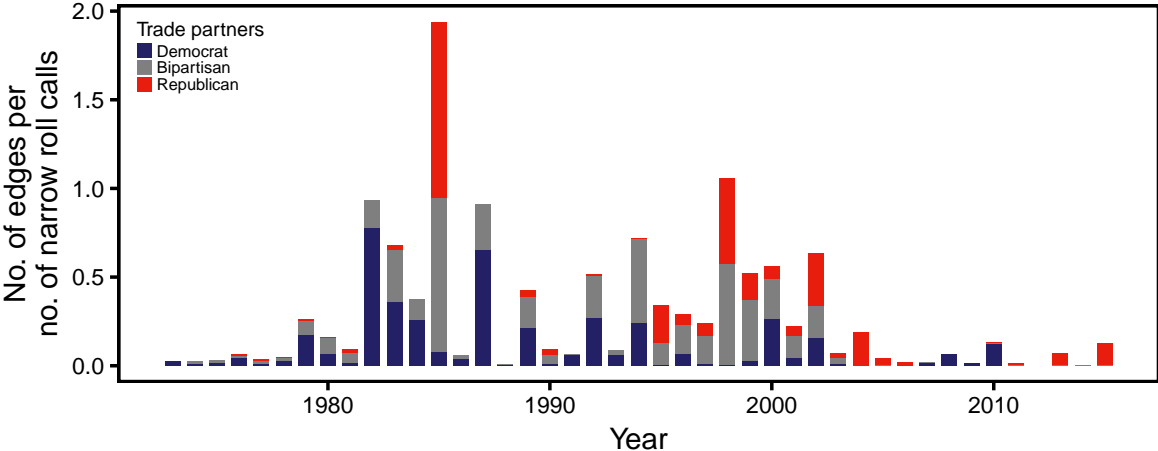
The y-axis indicates the number of potential trades (VTN-edges) per the number of narrow roll calls in a given year (x-axis). The bars are colored according to the proportion of trades captured in the VTN between Democrats (blue area), between Republicans (red area), or bipartisan trades (gray area). See section 5.1 for details on the data.

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[30]Note that this depends, of course, on the length of the observation period on both side of the cutting point. If the method were applied to estimate the prevalence of vote trading based on data for only one congress, it can, by construction, only detect trades happening within this one congress. Thus computing one VTN for two congresses together in comparison to one VTN for each congress, is likely to result in larger differences in the number of edges.

6 Conclusions

While broadly covered in the theoretical literature from political science and economics, only little is known about the prevalence, variability, and underlying mechanisms of legislative vote trading in the real world. Our suggested methodological approach aims to contribute to a broader empirical assessment and understanding of this form of hidden cooperation in politics. Following the core assumptions about the incentives to trade votes underlying previous empirical contributions (Stratmann, 1992, 1995; Cohen and Malloy, 2014), our approach builds on four theoretical pillars, characterizing the minimal requirements to consider an observed vote as traded. These pillars are integrated at the heart of the suggested empirical approach. By building on the concept of reciprocity in directed, weighted networks, we can incorporate the very micro-structure of potential trades between individual members of a legislature.

The suggested framework also has limitations that must be considered when applying it and could be addressed in future research. Although we can test the statistical significance of logrolling at the aggregate level, we cannot tell whether all the reciprocal deviations revealed in the VTN are true trades in a specific empirical setting. As in all previous approaches to assess the prevalence of vote trading in roll call data, there might, of course, be false positives (and like all previous approaches, we cannot compare the results with trades in the real world). Any conclusions with respect to low-degree legislators in the VTN thus need to be considered with care. In spite of this limitation, we developed a computational approach to generate synthetic roll call and signaling data, allowing us to assess the performance and validity of our method. For this, we identify the density levels of the data matrices under which our method performs well. These levels are consistent with those from our empirical application where we study the prevalence of vote trading in the U.S. House of Representatives. Finally, our simulation study can be further extended to evaluate previous and future empirical approaches to the study of explicit vote trading.

On a more general level, it is important to keep in mind that there might well be other

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forms of vote trading. Two aspects are of particular relevance: first regarding the presented application with data of the U.S. Congress, (co-)sponsorships are not the only signal of strong preferences toward bills. Alternative observable signals for policy preferences might additionally reveal tendencies to trade votes. Second, it might well be that some deals are arranged between groups. In those cases, our method would likely capture the roll calls/bills affected by such trades, but not each individual participating. Considering both of these points, our method is likely to provide a lower-bound estimate of the number of legislators engaged in vote trading. Therefore, and in line with the previous approaches, our framework should rather not be applied in order to assess the *absolute* prevalence of vote trading. Instead, the focus should lie on assessing whether there is any prevalence of vote trading and/or investigating relative differences and changes over time. That considered, as the suggested logrolling index is built from bottom up, it is straightforward to investigate how the statistical evidence for vote trading might have emerged based on the underlying individual voting behavior (by extracting and analyzing the VTN as demonstrated in the previous section).

Altogether, we think the suggested framework provides a valuable tool for future empirical work on vote trading. Particularly, it can be the basis for two important directions of research. (*i*) The flexibility and scalability of the framework can help to study the prevalence of vote trading at various levels of government, across time, and across different jurisdictions. Such a line of research focusing on the macro-level could help to better understand what factors drive the overall tendency to trade votes. (*ii*) The explicit consideration of the underlying structure of the vote-trading network opens opportunities to test hypotheses of the micro-dynamics of vote trading. More sophisticated computational models of the trading process can be developed in order to test specific null hypotheses. Such models can straightforwardly be integrated in our framework. This might enhance our understanding of the mechanisms of vote trading at the level of individual legislators.

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Appendix

A.I   Data appendix

Figure A1: Number of roll call votes per year.
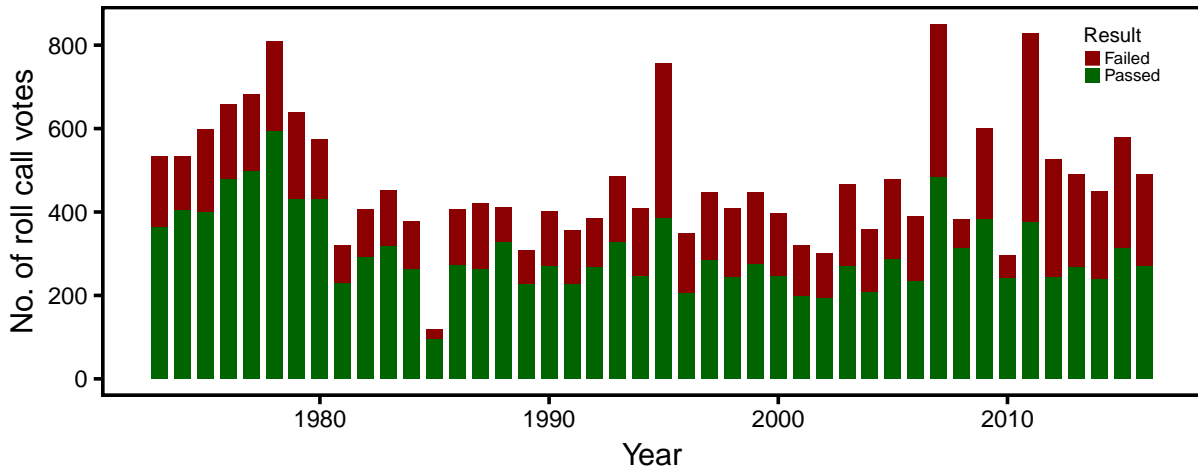
Bar plot of the number of roll call votes (y-axis) per year (x-axis) recorded in the U.S. House between 1973 and 2016. The share of roll call votes that failed to pass are indicated with red colored areas, the share that passed with green colored areas. See section 5.1 for details on the data.

Figure A2: Number of sponsorships and cosponsorships per year.
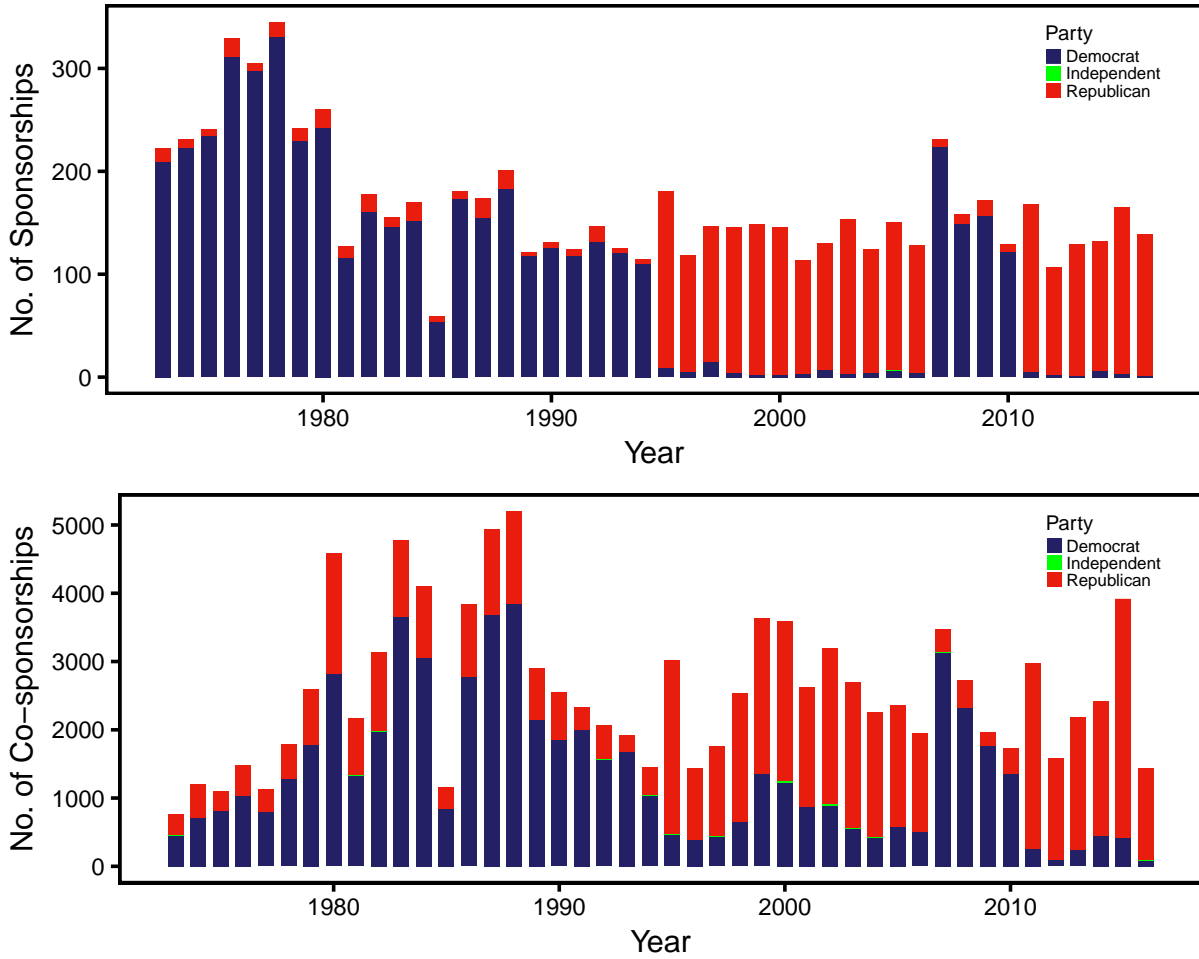
The upper panel shows a bar plot of the number of bill sponsorships by Representatives in the U.S. House (y-axis) per year (x-axis) for the years 1973 to 2016. The lower panel shows the same bar plot for (co-)sponsorships by Representatives in the U.S. House for the same period. The proportion of (co-)sponsorships issued by either party (or Independents) is indicated with by the colored areas on each bar: blue for Democrats, Red for Republicans, green for Independents. See section 5.1 for details on the data.

A.II   Estimation of preferred policy positions

A Member of Congress i is confronted with the choice of voting in favor or against a bill in each roll call k .[31] Both possible stances on the bill are represented as positions in a one-dimensional policy space. Let the ν_k represent the No position and the ψ_k the Yes position. The Member of Congress' utility declines the further this position is from his or her *ideal position*. Let x_i be a Member of Congress' ideal point and U_i his or her utility function. The relative position of ψ_k and ν_k to x_i is then formally defined as

$$d(x_i, \psi_k) = |x_i - \psi_k|. \tag{9}$$

$$d(x_i, \nu_k) = |x_i - \nu_k|. \tag{10}$$

Following Clinton et al. (2004), we assume the Member of Congress' utility to be affected by a given decision in a roll call such that

$$U_i(\psi_k) = -|x_i - \psi_k|^2 \tag{11}$$

$$U_i(\nu_k) = -|x_i - \nu_k|^2. \tag{12}$$

Thus, ceteris paribus, a Member of Congress votes in favor of a bill if $U_i(\psi_k) > U_i(\nu_k)$ and otherwise against it in order to maximize the utility derived from his or her voting decision. Following Clinton et al. (2004) and Jackman (2008), we estimate the empirical spatial voting model

$$P(\mathbb{V}_{ik}^b = 1) = P(U_i(\psi_k) > U(\nu_k)). \tag{13}$$

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[31]For simplicity, we refrain from discussing absenteeism and abstention from voting. These real-world alternatives in many voting decisions might, of course, also bear economically relevant incentives and reveal insights into strategic behavior in the legislative process. However, these aspects would need a detailed consideration that exceeds the scope of this paper.

Let $\epsilon_{i\psi_k}$ and $\epsilon_{i\nu_k}$ denote the error terms from estimating 11 and 12. Then, assuming that $\epsilon_{i\psi_k}$ and $\epsilon_{i\nu_k}$ are jointly normally distributed, we can derive the empirical representation in the form of a probit model:

$$\Pr(\mathbb{V}_{ik}^b = 1) = \Phi(\beta_k x_i - \alpha_k), \tag{14}$$

where $\beta_k = 2(\psi_k - \nu_k)/\sigma_k$, $\alpha_k = (\psi_k^2 - \nu_k^2)/\sigma_k$, $\sigma_k^2 = \mathrm{var}(\epsilon_{i\nu_k} - \epsilon_{i\psi_k})$, and Φ is the standard normal CDF. As the roll call records describing the dependent variable are the only data available, both the ideal point coordinates as well as the coefficients β_k and α_k must be estimated. As starting values for the Bayesian estimation procedure, we use the ideal points derived from a principal-component factor analysis of the roll call data (i.e., the first d eigenvectors resulting from a principal component analysis of $\mathbb{V}^b$). For the roll call-specific parameters β_k and α_k, we use values from estimates based on K probit models of the observed votes run on the starting values for the ideal points. This results in all N Members of Congress's estimated ideal points (preferred policy positions), each one represented by $\hat{x}_i$.

A.III   Robustness to truncation

Figure A3: Robustness check of the prevalence of vote trading in the U.S. House.
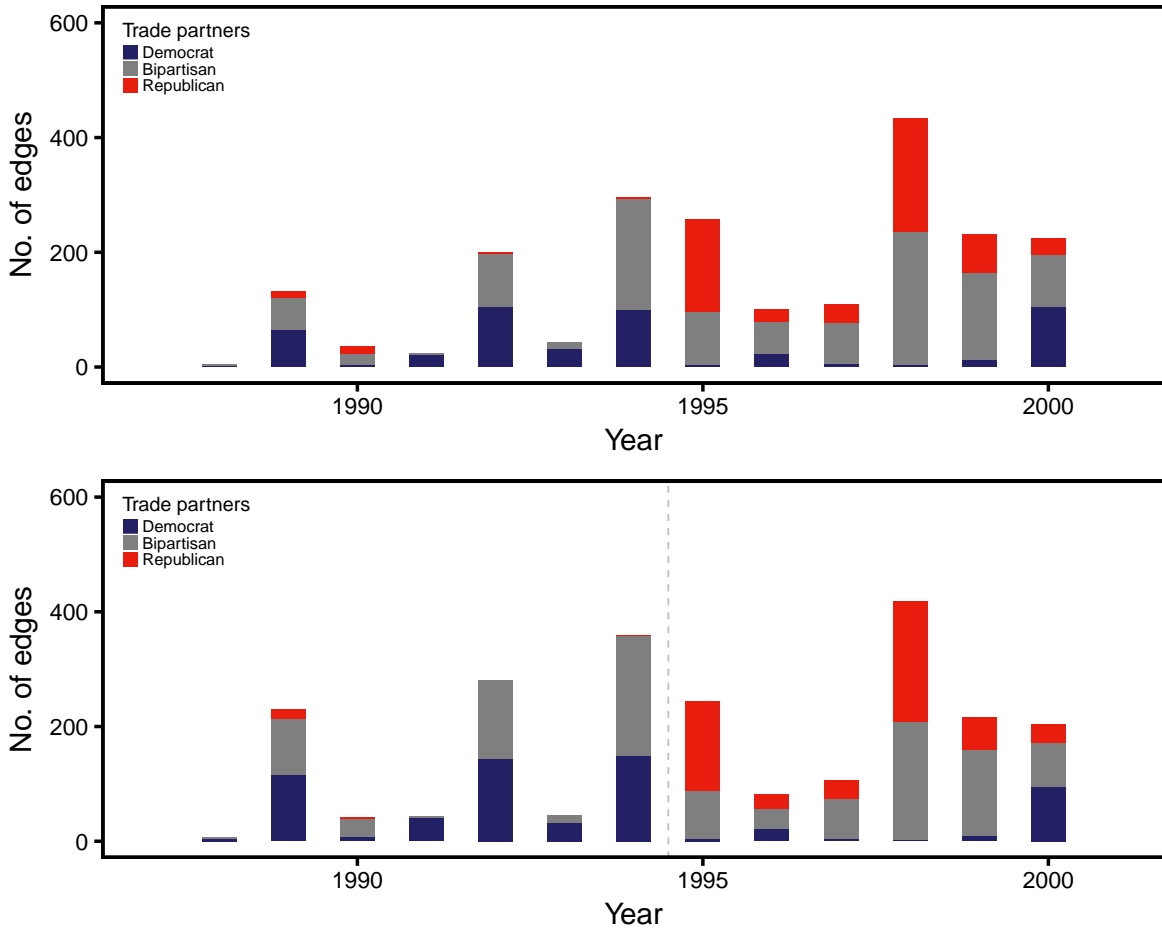
Comparison of the number of VTN edges per year around the cutting point (± 7 years) when splitting the sample in the middle of the observation period (1994/1995). The upper panel shows the number of edges per year when computing the VTN based on the entire observation period (as in the main results presented in section 5.3). The lower panel shows the results when computing two VTNs, one for the years up to and including 1994, and the other for the years 1995 and after. The gray dashed vertical line indicates the cutting point. See section 5.1 for details on the data.